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SELECTION ON THE BASIS OF PRIOR TESTING

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Abstract

We establish that under mild conditions, testing for the individual significance of an impulse indicator in the conditional model, selected on the basis of prior testing of its significance in the impulse saturated marginal model does not require bootstrapping critical values. Extensive Monte Carlo evidence shows that the real size of a joint F test in the conditional on the block of dummies retained from the marginal is independent of nominal size used for impulse saturation used in the marginal model. The findings are shown to hold for a plethora of dynamic models and sample sizes. Such results are fundamental not only in model selection theory, but also for the emerging class of automatically computable super exogeneity tests.

JEL Codes: C52; C22; C15

Keywords: model selection; impulse saturation, super exogeneity; bootstrapping

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1 Introduction

Impulse saturation (Hendry, Johansen and Santos, 2007) is a major recent development in model selection. It entails the possibility of testing an individual impulse indicator for each observation in a sample. Groups of indicators are entered in the econometric model in feasible subsets of either halves ($T/2$), thirds ($T/3$) or any other even or uneven sample partition. Subset selection is then used to retain the relevant indicators from each terminal model into a union model. The authors derive the asymptotic distribution of the sample mean and the sample variance in a simple location-scale model with IID observations, after saturation. Monte Carlo evidence shows that under the null that no indicator is in the DGP, the average retention rate matches the binomial result αT, showing no signs of spurious retention. Furthermore, the number of sample splits is also shown to be irrelevant, under the null, for the number of indicators retained.

Following the seminal work of Hendry et. al. (2008), major extensions have been developed: Santos and Hendry (2006) and Nielsen and Johansen (2007) show that the procedure can be extended to certain classes of dynamic models. Santos (2008) evaluates impulse saturation as a test for multiple breaks with unknown locations, and concludes that the procedure has good power properties both against mean and variance shifts. Hendry and Santos (2007) extend this idea to develop a new class of automatically computable super exogeneity tests (see, inter alia, Engle, Hendry and Richard (1983) and Engle and Hendry (1993)).

In the new super exogeneity tests, marginal models are subject to the impulse saturation break test, and the conditional is augmented with the resulting block of dummies. The significance of this set of dummies is then tested for in the conditional model, either by means of a joint F test or by means of index based tests (see Hendry and Santos (2005) for the theory of indices of indicators).

The procedure allows the researcher to test for super exogeneity, whilst, at the same time, it avoids the criticisms of classical Engle-Hendry type of super exogeneity tests, advocated by Lindé (2001). In particular, Engle-Hendry type of testing is often reduced to testing in the conditional hand picked dummies from the marginal. These dummies are selected on the basis of dates of events of economic relevance. In fact, the new test can be made fully automatic, precluding any intervention from the researcher, as the included indicators in the conditional are only those assessed as significant in the marginal, when an indicator has been tested for every sample observation.

Notwitstanding, the validity of the new super exogeneity test rests upon the use of correct critical values when testing in the conditional dummies retained on the basis of prior testing in the marginal. Some authores (see, inter alia, Christiano (1992) and Hansen (2005)) argue in favour of the need to bootstrap critical values when testing with variables selected on the basis of prior testing.

In this paper, we establish that for a wide range of dynamic conditional and marginal models there is in fact no need to use bootstrapped critical values. Thus, automatically computable super exogeneity tests can easily be built. In section 2 we show that, under mild requirements for a bivariate VAR, the usual critical values can be used in this framework. Section 3 provides extensive Monte Carlo evidence showing a plethora of dynamic models derived from the bivariate VAR DGP where the key result of the previous section - irrelevance of conditioning on selection - holds. Section 4 concludes.

2 A theoretical approach to factorizing and conditioning in dynamic bivariate models

Theorem 1 *Consider the sequential, bivariate normal:*

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} | X_{t-1} \sim \mathsf{N}_2 \left[\begin{pmatrix} \pi_{10} \\ \pi_{20} \end{pmatrix} + \begin{pmatrix} \pi_{11} & \pi_{12} \\ \pi_{21} & \pi_{22} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix}, \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} \right] \tag{1}$$

where $\boldsymbol{x}_t = (y_t : z_t)'$, $t = 1, ..., T$, *and hence* X_{t-1} *is the information set containing the history of* y_t *and* z_t, *such that:*

$$X_{t-1} = (Y_{t-1} : Z_{t-1})' \tag{2}$$

Consider the conditional econometric model:

$$y_t | z_t, X_{t-1} = \beta_0 + \beta_1 y_{t-1} + \beta_2 z_t + \beta_3 z_{t-1} + \zeta_t \tag{3}$$

where $\zeta_t \sim \mathsf{IN}[0, \sigma_{11.2}]$, *with* $\sigma_{11.2} = \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{12}$, *and the marginal model*

$$z_t | X_{t-1} = \pi_{20} + \pi_{21} y_{t-1} + \pi_{22} z_{t-1} + \omega_t \tag{4}$$

with $\omega_t \sim \mathsf{IN}[0, \sigma_{22}]$. *Let the parameter vector of the conditional be* $\phi_1 = (\beta_0 : \beta_1 : \beta_2 : \beta_3 : \sigma_{11.2})'$ *and the parameter vector of the marginal be* $\phi_2 = (\pi_{20} : \pi_{21} : \pi_{22} : \sigma_{22})'$. *In general, neither strong nor super exogeneity hold. Assume that* $|\beta_1| \leq 1$, *and* $|\pi_{22}| \leq 1$. *There are no indicators in the DGP. Consider augmenting the marginal model with an impulse indicator* $I_{t\{1:t=t^*\}}$, *which turns out to be statistically significant and suppose adding such an indicator to the conditional. Suppose further there are no past shocks either in the history of* ζ_t *or in the history of* ω_t.

Under the null of the indicator having a zero coefficient in the conditional, we claim that size in the conditional is independent from the significance level used in the marginal, that is:

$$\mathsf{P}\left(\left|\mathsf{t}_{I_{t^*},conditional}\right| > c_{\alpha_2} \middle| \left|\mathsf{t}_{I_{t^*},marginal}\right| > c_{\alpha_1}\right) = \mathsf{P}\left(\left|\mathsf{t}_{I_{t^*},conditional}\right| > c_{\alpha_2}\right) \tag{5}$$

where α_1 is the significance level used in the marginal, α_2 is the significance level used in the conditional and t *is the relevant* t*-ratio. Hence, there is no need to condition on selection.*

Proof. Start by noticing that keeping an impulse indicator in the marginal means

$$\mathsf{P}\left(\left|\mathsf{t}_{I_{t^*},\text{marginal}}\right| > c_{\alpha_1}\right) \tag{6}$$

We shall assume this arises due to a rare draw from the error distribution: $|\omega_t| > m^*$, where it follows from (4) that ω_t is the error in the marginal model.

Define the random variable S_t,

$$S_t|X_{t-1} = y_t|X_{t-1} - \sigma_{12}\sigma_{22}^{-1} z_t|X_{t-1} \tag{7}$$

Then, the random vector $(S_t|X_{t-1} : z_t|X_{t-1})'$ can be written as

$$\begin{pmatrix} S_t|X_{t-1} \\ z_t|X_{t-1} \end{pmatrix} = \begin{pmatrix} 1 & -\sigma_{12}\sigma_{22}^{-1} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_t|X_{t-1} \\ z_t|X_{t-1} \end{pmatrix} \tag{8}$$

where $\mathbf{M} = \begin{pmatrix} 1 & -\sigma_{12}\sigma_{22}^{-1} \\ 0 & 1 \end{pmatrix}$. Given gaussianity of $\begin{pmatrix} y_t|X_{t-1} \\ z_t|X_{t-1} \end{pmatrix}$ due to (1):

$$\begin{pmatrix} S_t|X_{t-1} \\ z_t|X_{t-1} \end{pmatrix} \sim \mathsf{N}\left[\mathbf{M}\boldsymbol{\mu}, \mathbf{M}\boldsymbol{\Sigma}\mathbf{M}'\right]$$

where $\boldsymbol{\mu}$ is the vector of means in (1), so that

$$\mathbf{M}\boldsymbol{\mu} = \begin{pmatrix} \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}z_{t-1} + \sigma_{12}\sigma_{22}^{-1}\left(\pi_{20} - \pi_{21}y_{t-1} - \pi_{22}z_{t-1}\right) \\ \pi_{20} + \pi_{21}y_{t-1} + \pi_{22}z_{t-1} \end{pmatrix} \tag{9}$$

and

$$\mathbf{M}\boldsymbol{\Sigma}\mathbf{M}' = \begin{pmatrix} \sigma_{11} - \sigma_{12}\sigma_{22}^{-1}\sigma_{12} & 0 \\ 0 & \sigma_{22} \end{pmatrix} \tag{10}$$

It then follows that the marginal for $S_t|X_{t-1}$ is gaussian:

$$S_t|X_{t-1} \sim \mathsf{N}\left[\pi_{10} + \pi_{11}y_{t-1} + \pi_{12}z_{t-1} + \sigma_{12}\sigma_{22}^{-1}\left(\pi_{20} - \pi_{21}y_{t-1} - \pi_{22}z_{t-1}\right), \sigma_{11.2}\right] \tag{11}$$

which is to say

$$y_t|X_{t-1} - \sigma_{12}\sigma_{22}^{-1}z_t|X_{t-1} = \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}z_{t-1} + \sigma_{12}\sigma_{22}^{-1}\left(\pi_{20} - \pi_{21}y_{t-1} - \pi_{22}z_{t-1}\right) + \zeta_t \tag{12}$$

and given independence of $S_t|X_{t-1}$ and z_t, the density of $S_t|z_t, X_{t-1}$ is also that of (12), so

$$y_t|z_t, X_{t-1} - \sigma_{12}\sigma_{22}^{-1}z_t|X_{t-1} = \pi_{10} + \pi_{11}y_{t-1} + \pi_{12}z_{t-1} + \sigma_{12}\sigma_{22}^{-1}\left(\pi_{20} - \pi_{21}y_{t-1} - \pi_{22}z_{t-1}\right) + \zeta_t \tag{13}$$

where $\sigma_{12}\sigma_{22}^{-1}z_t|X_{t-1}$ is a constant due to conditioning on z_t. So, if (13) is independent from z_t, $y_t|z_t, X_{t-1}$ which only differs from $S_t|X_{t-1}$ by a constant, is also independent of z_t. Rewriting in the parameters of interest,

$$y_t|z_t, X_{t-1} = \beta_0 + \beta_1 y_{t-1} + \beta_2 z_t + \beta_3 z_{t-1} + \zeta_t \tag{14}$$

where we now know that ζ_t is independent of ω_t in marginal. So, assuming an impulse will be retained in the conditional if $|\zeta_t| > b^*$,

$$\mathsf{P}\left(|\zeta_t| > b^* \right| |\omega_t| > m^*) = \mathsf{P}\left(|\zeta_t| > b^*\right) \tag{15}$$

$\mathsf{P}\left(|\zeta_{t^*}| > b^*\right)$ is therefore necessary for an indicator to be retained in the conditional at time t^*. ■

3 Monte Carlo evidence

In order to provide simulation evidence in support of the irrelevance of conditioning on selection in some dynamic models, we have conducted a series of Monte Carlo experiments. Several DGP designs were tried: the next subsection refers to stationary marginal models. Neither strong not super exogeneity hold (there is an ADL (1,1) structure both in the marginal and in the conditional, precluding strong exogeneity; stationarity in both models is not incompatible with weak exogeneity as was recently shown by Santos (2007) in a clear contradiction with the results in Psaradakis and Sola (1996)); subsection 2 assumes the same conditional model, whilst the marginal was designed to be strongly exogenous for y_t and to have a unit root; subsection 3 differs from the previous one in that the marginal is stationary, and from the first in that the marginal is strongly exogenous with respect to y_t. In subsection 4, the marginal is still strongly exogenous for y_t but the conditional has a unit root. Finally, in subsection 5, a unit root is imposed both in the marginal and in the conditional, albeit the marginal still being strongly exogenous for y_t. In nearly all scenarios, experiments were conducted for samples of sizes $T = 300$, $T = 200$, $T = 100$ and $T = 50$. $M = 10000$ replications were conducted for every experiment. α_1, the significance level used for impulse saturation in the marginal model, took values from the set $\{0.1; 0.05; 0.025; 0.01\}$. α_2 is the empirical rejection frequency (real size) in the conditional. For each table, irrelevance of conditioning on selection exists if, for any given column, there is no systematic change in α_2 with the value chosen for α_1. Two words of caution should be placed here:

- the theorem of the previous section refers to t-testing of an individual indicator retained from the marginal. Here, we allow for the possibility that several indicators are retained from the marginal, as the marginal has effectively been impulse saturated in every replication. Hence, a joint F-test on the retained indicators is used in the conditional (as in Hendry and Santos, 2007). The automatic super exogeneity test should not conduct an individual significance test in the conditional for every dummy retained from the marginal: size would not be kept under control, as the probability of spurious retention would be $(1 - \alpha_2)$;

- it is well known in the model selection literature (see, e.g., Hendry and Krolzig, 2001) that nominal and real sizes diverge when stringent nominal significance levels are used for small samples. This should be taken into account when analysing the tables. The rule suggested by Hendry and Krolzig (2001) is that the significance level should be such that $\alpha_2 T > 3$. Therefore, even if we report a wider variety of results these should be the ones deserving special attention.

3.1 Weak exogeneity and stationarity

Consider the DGP:

$$\begin{pmatrix} y_t \\ z_t \end{pmatrix} |X_{t-1} \sim \mathsf{N}_2 \left[\begin{pmatrix} 2.75 \\ 1.5 \end{pmatrix} + \begin{pmatrix} 0.9 & 0.125 \\ 0.4 & 0.55 \end{pmatrix} \begin{pmatrix} y_{t-1} \\ z_{t-1} \end{pmatrix}, \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix} \right] \tag{16}$$

yielding the marginal and conditional models:

$$z_t|X_{t-1} = 1.5 + 0.4y_{t-1} + 0.55z_{t-1} + \varepsilon_{z,t} \tag{17}$$

$$y_t|z_t, X_{t-1} = 2 + 0.7y_{t-1} - 0.15z_{t-1} + 0.5z_t + \varepsilon_t \tag{18}$$

WE holds, since we are interested in the parameters of the vector $\phi_1 = (\beta_0 : \beta_1 : \beta_2 : \beta_3 : \sigma_{11.2})'$. Neither strong nor super exogeneity hold. Tables (1) to (4) report the nominal significance levels used in the marginal for impulse saturation and the real significance levels when F-testing is used in the conditional. The sample split was defined at $T/2$ and $M = 10000$ replications were conducted.

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.01	0.023	0.046	0.093
$\alpha_1 = 0.025$	0.01	0.024	0.049	0.094
$\alpha_1 = 0.05$	0.01	0.025	0.048	0.098
$\alpha_1 = 0.1$	0.01	0.023	0.048	0.099

Table 1: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 300$, marginal and conditional in (17) and (18)

Clearly, for no sample size does it seem that empirical rejection frequencies of the null joint hypothesis of none of the selected indicators being significant in the conditional are being influenced in any systematic way by the choice of α_1. .

$\alpha_1\backslash\alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.01	0.022	0.042	0.081
$\alpha_1 = 0.025$	0.01	0.024	0.048	0.097
$\alpha_1 = 0.05$	0.01	0.023	0.048	0.093
$\alpha_1 = 0.1$	0.01	0.023	0.046	0.093

Table 2: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 200$, marginal and conditional in (17) and (18)

$\alpha_1\backslash\alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.01	0.016	0.039	0.061
$\alpha_1 = 0.025$	0.01	0.023	0.042	0.087
$\alpha_1 = 0.05$	0.01	0.024	0.048	0.098
$\alpha_1 = 0.1$	0.01	0.023	0.049	0.096

Table 3: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 100$, marginal and conditional in (60) and (61)

3.2 Strong exogeneity and unit roots

For the simulations in this section, we consider the same conditional process as above. However, the marginal process is now:

$$z_t|X_{t-1} = 1.5 + z_{t-1} + \varepsilon_{z,t} \tag{19}$$

Hence, we simultaneously consider the imposition of strong exogeneity and a unit root in the marginal. Tables (5)-(8) report the results for the same sample sizes as previously.

The tables reveal that under strong exogeneity, the presence of a unit root in the marginal does not have a significant impact on real size in the conditional. Again, this seems to be independent from size in the marginal.

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.01	0.012	0.023	0.045
$\alpha_1 = 0.025$	0.01	0.018	0.035	0.068
$\alpha_1 = 0.05$	0.01	0.023	0.043	0.086
$\alpha_1 = 0.1$	0.01	0.023	0.045	0.091

Table 4: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 50$, marginal and conditional in (17) and (18)

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.0096	0.023	0.046	0.0931
$\alpha_1 = 0.025$	0.0112	0.0236	0.0489	0.0943
$\alpha_1 = 0.05$	0.01	0.0248	0.0485	0.098
$\alpha_1 = 0.1$	0.01	0.023	0.045	0.1

Table 5: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 300$, marginal in (19), conditional in (18) (strong exogeneity and unit root in marginal)

3.3 Strong exogeneity with stationarity in marginal and conditional

Consider strong exogeneity without imposing a unit root in the marginal. For the simulations in this section, the DGP will entail the following marginal and conditional models:

$$z_t | X_{t-1} = 1.5 + 0.55 z_{t-1} + \varepsilon_{z,t} \tag{20}$$

and

$$y_t | z_t, X_{t-1} = 1.25 + 0.7 y_{t-1} - 0.15 z_{t-1} + 0.5 z_t + \varepsilon_t \tag{21}$$

assuming the same variance-covariance matrix in the bivariate normal. Tables (9)-(12) report the results on real size in the conditional for the usual sample sizes.

It is once more clear that real α_2 behaves independently of nominal α_1.

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.0096	0.0225	0.0426	0.0873
$\alpha_1 = 0.025$	0.0097	0.0248	0.0483	0.097
$\alpha_1 = 0.05$	0.0096	0.0245	0.0488	0.096
$\alpha_1 = 0.1$	0.0104	0.0242	0.048	0.091

Table 6: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 200$, marginal in (19), conditional in (18) (strong exogeneity and unit root in marginal)

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.0064	0.0161	0.032	0.064
$\alpha_1 = 0.025$	0.0099	0.023	0.045	0.091
$\alpha_1 = 0.05$	0.0111	0.024	0.051	0.098
$\alpha_1 = 0.1$	0.0097	0.0263	0.049	0.096

Table 7: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 100$, marginal in (19), conditional in (18) (strong exogeneity and unit root in marginal)

3.4 Strong exogeneity with stationarity in the marginal and a unit root in the conditional

For the pilot simulations in this section, the DGP will entail the following marginal and conditional models:

$$z_t | X_{t-1} = 1.5 + 0.55 z_{t-1} + \varepsilon_{z,t} \tag{22}$$

and

$$y_t | z_t, X_{t-1} = 1.25 + y_{t-1} - 0.15 z_{t-1} + 0.5 z_t + \varepsilon_t \tag{23}$$

and the same variance-covariance matrix as in the bivariate normal. Table (13) reports the results on real size in the conditional, in a pilot Monte Carlo experiment for $T = 200$. From table (13), we conclude that conditioning on selection is irrelevant for the models considered here as well.

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.0064	0.0147	0.027	0.048
$\alpha_1 = 0.025$	0.0083	0.02	0.037	0.072
$\alpha_1 = 0.05$	0.0082	0.022	0.043	0.088
$\alpha_1 = 0.1$	0.0098	0.0214	0.044	0.086

Table 8: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 50$, marginal in (19), conditional in (18) (strong exogeneity and unit root in marginal)

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.0103	0.0238	0.0468	0.0952
$\alpha_1 = 0.025$	0.0096	0.0254	0.0462	0.0892
$\alpha_1 = 0.05$	0.0099	0.023	0.0475	0.0981
$\alpha_1 = 0.1$	0.0106	0.0248	0.048	0.0984

Table 9: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 300$, marginal and conditional in (20) and (21) (strong exogeneity with stationarity)

3.5 Strong exogeneity with a unit root in marginal and another in conditional

For the simulations in this section, the DGP will entail the following conditional and marginal models:

$$z_t | X_{t-1} = 1.5 + z_{t-1} + \varepsilon_{z,t} \tag{24}$$

and

$$y_t | z_t, X_{t-1} = 1.25 + y_{t-1} - 0.15 z_{t-1} + 0.5 z_t + \varepsilon_t \tag{25}$$

and the same variance-covariance matrix as in the bivariate normal. Table (14) reports the results on real size in the conditional, in a pilot Monte Carlo experiment for $T = 200$. The same conclusion as to irrelevance of conditioning applies.

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.008	0.02	0.042	0.09
$\alpha_1 = 0.025$	0.011	0.026	0.048	0.096
$\alpha_1 = 0.05$	0.0093	0.024	0.047	0.099
$\alpha_1 = 0.1$	0.0098	0.025	0.047	0.096

Table 10: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 200$, marginal and conditional in (20) and (21) (strong exogeneity with stationarity)

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.007	0.0165	0.034	0.071
$\alpha_1 = 0.025$	0.0097	0.0235	0.046	0.095
$\alpha_1 = 0.05$	0.011	0.025	0.05	0.104
$\alpha_1 = 0.1$	0.011	0.028	0.052	0.103

Table 11: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 100$, marginal and conditional in (20) and (21) (strong exogeneity with stationarity)

4 Conclusion

In this paper we have established that two-stage super exogeneity tests where indicators in the conditional model are tested for after prior selection in the marginal can be conducted without the need for bootstrapped critical values. Monte Carlo results extend the baseline result to wider classes of models: including non stationary ones.

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$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.006	0.0142	0.029	0.055
$\alpha_1 = 0.025$	0.009	0.0213	0.04	0.08
$\alpha_1 = 0.05$	0.01	0.023	0.048	0.089
$\alpha_1 = 0.1$	0.009	0.022	0.047	0.088

Table 12: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 50$, marginal and conditional in (20) and (21) (strong exogeneity with stationarity)

$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.0082	0.021	0.041	0.084
$\alpha_1 = 0.025$	0.009	0.026	0.05	0.095
$\alpha_1 = 0.05$	0.0101	0.023	0.047	0.097
$\alpha_1 = 0.1$	0.0099	0.026	0.0472	0.096

Table 13: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 200$, marginal and conditional in (22) and (23) (strong exogeneity with unit root in conditional)

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$\alpha_1 \backslash \alpha_2$	$\alpha_2 = 0.01$	$\alpha_2 = 0.025$	$\alpha_2 = 0.05$	$\alpha_2 = 0.1$
$\alpha_1 = 0.01$	0.007	0.0184	0.0361	0.08
$\alpha_1 = 0.025$	0.009	0.023	0.0433	0.092
$\alpha_1 = 0.05$	0.0091	0.0231	0.0462	0.094
$\alpha_1 = 0.1$	0.008	0.021	0.042	0.09

Table 14: Real significance levels in the conditional with retained dummies from impulse saturated marginal model: $T = 200$, marginal and conditional in (24) and (25) (strong exogeneity with unit roots in marginal and conditional)

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