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NATIONAL PRODUCTIVE STRUCTURE AND INNOVATIVE DYNAMICS: FINDING THE (ENDOGENOUS) PATH TO CONVERGENCE

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National Productive Structure and Innovative Dynamics: Finding the (Endogenous) Path to Convergence*

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1st Version

Abstract

We extend the model presented in Barro and Sala-i-Martin (1997) by allowing for two types of economies - more developed and in transition to European Union integration - to both imitate and innovate varieties of intermediate goods. Besides depending on research and development expenditures, we also allow for the stochastic nature of innovation by making it also dependent on a random component. We do this by Monte Carlo simulation, using a Box-Muller process, and solve a three differential equation model by using numerical methods. Two situations are presented: a leading economy with greater institutions and more labour than the transition economy versus a situation where an institutional advance is given to the transition economy.

Keywords: stochastic innovation; transition economies; growth; technology; diffusion; convergence

JEL classification: O40, O30, O11

1 Introduction

Neoclassical growth theory presents long-run economic growth as simply being dependent on technological progress. Many have argued that since

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it is exogenous all efforts to understand and influence growth are reduced to being futile. However, recent endogenous growth theory, initiated by Romer (1987, 1990) and extended by Grossman and Helpman (1995) and Aghion and Howitt (1992), explains long-term growth by modeling technological progress, determined pretty much by private research and development (R&D) that underlies commercial discovery and is motivated along Schumpeterian lines by the flow of profit acumulated by the innovator.

The endogenization of the rate of technical change, a variable that is unexplained in the neoclassical growth model, has produced great incite on the dynamics of the transition path to convergence. Barro and Sala-i-Martin (1997) were amongst those who combined elements of endogenous growth with the convergence implications of the neoclassical growth model. In their model, long run world growth rate is driven by discoveries in the technologically leading economies, and knowledge is diffused and absorved by followers through immitation.

We build on the model for technological diffusion, convergence and growth of Barro & Sala-i-Martin (1997). Barro and Sala-i-Martin discuss how certain countries are selected to be technological leaders, and others destined to be mere followers. We argue that in reality this may not be so, take the case of transition economies towards European integration, for instance. This is the context we have in mind for the model we propose. We add to Barro and Sala-i-Martin's model the possibility for both transition, less developed as well as developed economies to make use of Ricardian specialization and innovate themselves. We argue that this ability is a crucial dimension in the creation of an opportunity for developing or transition economies to 'change the twist of fate' and not only converge more rapidly in growth, but also possibly invert roles, or adopt a possible combination of innovative and imitation strategies. Hence, we allow for both leading and transition economies to assume compensating production structures and particularly diverse institutional settings. This is particularly relevant given that the transition from socialism to capitalism in Central and Eastern Europe is not only an economic, but also a political process (see e.g. Blanchard et al (1991); Portes (1993), Hare et al (1999)). An important aspect of the latter is the possibility of reintegration into Europe symbolised for many countries by prospective membership of the European Union (EU) (see Grabbe and Hughes (1998), Mayhew (1998)).

We also take account for long-term growth rate being an endogenous variable that depends on the underlying parameters and also disturbances in the model. We in fact model innovation as dependent on R&D investment, whilst explicitly allowing for the partially random nature in a discovery process. We simulate these disturbances in the innovative process by Monte Carlo. Besides this, the rates of invention and growth reflect the forces described by Romer (1990). Followers convergence toward the leaders is then guaranteed because copying is cheaper than innovation over some range. A tendency for copying costs to increase reduces followers' growth rates and thereby generates a pattern of conditional convergence. The path to convergence will now be determined by the assortment strategy of innovative and copied intermediate goods to be incorporated in the national productive process. The assortment will in turn depend on the economy's productive structure.

2 Setup of the Model

We consider two type of economies, denoted by i=1,2. In each economy, the production function is of Spence (1976)/Dixit and Stiglitz (1977) type, that is:

$$Y_i = A_i (L_i)^{1-\alpha} \sum_{j=1}^{N_i} (X_{ij})^{\alpha}, \tag{1}$$

where $0 < \alpha < 1$, Y_i represents the output, L_i is labor input, X_{ij} is the quantity employed of the jth type of intermediate good, and N_i is the total number of types of intermediates available in economy i.

Moreover, this technology can be accessed by all agents in economy i and production occurs under competitive conditions. As in Barro and Sala-i-Martin (1997), the quantities of labor in each economy, denoted here by L_1 and L_2, are constants and correspond to the populations of each economy, able to work.

The productivity parameter A_i represents, in general, differences across economies in the level of technology, that is, differences in output that arise for given values of N_i, L_i and the X_{ij}'s. In our model, we associate these differences mainly to differences in the operationality and quality of institutions in each type of economy. In a context of European reintegration of transition economies, many of which characterized by very distictive political regimes, this may reflect variations in government policies, as reflected in infrastructure services, tax rates, the degree of maintenance of property rights, and the rule of law. The effects of these policies on outcomes are analogous to those from pure differences in the levels of technology.

As in Barro and Sala-i-Martin (1997), trade is assumed to be balanced between the two economies, which means that total domestic output Y_i equals total domestic expenditures. These expenditures are for consumption, C_i, production of intermediate goods, X_{ij}, and for R&D activities in order to learn about new varieties of intermediate goods. We consider that, for both economies, any agent may acquire knowledge by inventing a new variety of intermediate good or by imitating a product which is known in the other economy.

Since both economies can imitate or innovate by developing new varieties of intermediate goods, we define

$$N_i^I + N_i^C = N_i, \quad i = 1, 2 \tag{2}$$

where N_i^I is the number of innovated (I) intermediate goods in economy i, N_i^C stands for the number of imitated (C) intermediate goods in economy i and N_i is the total number of varieties in economy i. We also consider that

$$N_i^I = \lambda_i N_i, \quad i = 1, 2 \tag{3}$$

so that the proportion of varieties which are innovated is given by:

$$\lambda_1 = \frac{N_1^I}{N_1} = \left(\frac{N_1}{N_2}\right)^{-\beta/2}, \tag{4}$$

and, for economy 2,

$$\lambda_2 = \frac{N_2^I}{N_2} = \left(\frac{N_2}{N_1}\right)^{\beta}, \quad \beta \geq 0 \tag{5}$$

This specification ensures $\lambda_2 \leq \lambda_1$, which is reasonable, since economy 1 is the leader, more developed and, therefore, with a higher proportion of innovated goods.

Units of C_i or X_{ij} each require one unit of Y_i. The invention of a new variety of intermediate good in economy i has a cost of $\eta_i$units of Y_i and the imitation of a variety of intermediate good known in the other economy is associated with a cost of v_i. We will specify these costs later.

We will assume that, in a first stage, economy 1 is the technological leader and economy 2 is the follower, that is, $N_2(0) < N_1(0)$.

3 Innovation and Imitation in both economies

In each economy, an inventor of an intermediate good of type j in economy i retains a perpetual monopoly over the use of this good for production in economy i. Let P_{ij} be the price of intermediate good j in economy i. The flow of monopoly profit to the inventor of variety j in economy i is given by

$$\pi_{ij} = (P_{ij} - 1) X_{ij}, \tag{6}$$

where the 1 inside the parentheses represents the marginal cost of production for the intermediate j.

Given the production function in equation (1), the marginal product of the intermediate j in the production of output is

$$\frac{\partial Y_i}{\partial X_{ij}} = A_i \alpha L_i^{1-\alpha}(X_{ij})^{\alpha-1} \tag{7}$$

Equating this marginal product to P_{ij}gives the demand function for intermediate good j from all producers of goods in economy i:

$$X_{ij} = L_i(A_i\alpha/P_{ij})^{1/(1-\alpha)} \tag{8}$$

By substituting this result in equation (6) and maximizing π_{ij} with respect to P_{ij}, we get the monopoly price

$$P_{ij} = P_i = 1/\alpha > 1 \tag{9}$$

The monopoly price is the same at all points in time and for all types of intermediates.

This result implies that the total quantity produced of intermediate j in economy i is

$$X_{ij} = X_i = L_i A_i^{1/(1-\alpha)} \alpha^{2/(1-\alpha)}. \tag{10}$$

This quantity is the same for all intermediate goods and at all points in time. If we substitute this result in the production function, we get:

$$Y_i = A_i^{1/(1-\alpha)} \alpha^{2\alpha/(1-\alpha)} L_i N_i \tag{11}$$

If we substitute (9) and (10) in equation (6), we conclude that profits are given by the following expression:

$$\pi_{ij} = \pi_i = (1-\alpha) L_i A_i^{1/(1-\alpha)} \alpha^{(1+\alpha)/(1-\alpha)} \tag{12}$$

For each economy, the profit flow is constant and therefore the present value of profits from date t onward is:

$$V_i^I = V_i^C = V_i = \pi_i \int_t^{\infty} e^{-\int_t^s r_i(v)dv} ds, \tag{13}$$

where $r_i(v)$ is the real interest rate at time v in economy i.

Note that we are assuming that this present value is equal for both imitated (C) and innovated (I) varieties. This means that we are considering that the profits of an intermediate good are independent from how it arises (by imitation or innovation), which is reasonable since a producer of an intermediate good only sells the good developed by him in his economy. This fact explains why, within an economy, there is no difference between a variety imitated or innovated from a profitable point of view; all that matters in the market (i.e., demand for an intermediate good) is that it is a new intermediate good, which grants a monopoly power and respective profits to the firm.

Furthermore, the weighted average cost of economy i's assortment of innovated an immitated varieties will be given by:

$$e_i = (1 - \lambda_i)v_i + \lambda_i \eta_i \tag{14}$$

Since both economies can innovate or imitate, e_i has two components: the first one reflects the cost of imitation v_i, weighted by the proportion of varieties which are imitated; the second component reveals the cost of innovation η_i with a weight given by the proportion of varieties innovated.

As in Barro and Sala-i-Martin (1997) we assume that the goods that are easier to imitate will be copied first, and therefore the cost of imitation will increase with the number already imitated. This property will be guaranteed by assuming that v_i is an monotonically increasing function of $\frac{N_i}{N_k}$:

$$v_i = \eta_i \left(\frac{N_i}{N_k}\right)^{\sigma}, \quad i = 1, 2,\ k \neq i \tag{15}$$

Given equations (5), (14) and (15), the weighted average cost of economy 2's assortment is given by:

$$e_2 = \eta_2 \left\{ \left(\frac{N_2}{N_1}\right)^{\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta} \right\}, \quad \sigma \geq 0 \tag{16}$$

which implies

$$\frac{\dot{e_2}}{e_2} = \frac{\dot{\eta_2}}{\eta_2} + \frac{\dot{\left(\frac{N_2}{N_1}\right)}}{\frac{N_2}{N_1}} \frac{\left\{ \sigma \left(\frac{N_2}{N_1}\right)^{\sigma} - (\beta+\sigma)\left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \beta \left(\frac{N_2}{N_1}\right)^{\beta} \right\}}{\left(\frac{N_2}{N_1}\right)^{\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta}} \tag{17}$$

Applying the same procedure for economy 1, we get

$$e_1 = \eta_1 \left\{ \left(\frac{N_2}{N_1}\right)^{-\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta/2} \right\}, \quad \sigma \geq 0 \tag{18}$$

and

$$\frac{\dot{e_1}}{e_1} = \frac{\dot{\eta_1}}{\eta_1} + \frac{\dot{\left(\frac{N_2}{N_1}\right)}}{\frac{N_2}{N_1}} \frac{\left\{ -\sigma \left(\frac{N_2}{N_1}\right)^{-\sigma} - (\beta/2-\sigma)\left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \beta/2 \left(\frac{N_2}{N_1}\right)^{\beta/2} \right\}}{\left(\frac{N_2}{N_1}\right)^{-\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta/2}} \tag{19}$$

Assuming that there is free entry into the R&D and imitation business, the present value of profits must equal the weighted average cost of economy's assortment at each point of time, that is:

$$V_i(t) = e_i \left(\frac{N_2}{N_1}\right) \tag{20}$$

By substituting equation (13) in (20), and differentiating both sides of equation (20), we get

$$r_i = \frac{\pi_i}{e_i} + \frac{\dot{e_i}}{e_i}, \quad i = 1, 2 \tag{21}$$

Consumers in each economy are assumed to be of the usual Ramsey type with infinite horizons. At time 0, these consumers seek to maximize

$$U_i = \int_0^\infty e^{-\rho t}. \left[(C_i^{1-\theta} - 1)/(1-\theta)\right] dt, \tag{22}$$

where $\rho > 0$ is the rate of time preference and $\theta > 0$ is the magnitude of the elasticity of the marginal utility of consumption. (The inter-temporal elasticity of substitution is $1/\theta$). The number of consumers, i.e. population, is assumed to be constant over time. Maximization of utility, subject to a standard budget constraint, leads to:

$$\frac{\dot{C}_i}{C_i} = \frac{1}{\theta}(r_i - \rho), \tag{23}$$

and, given (21), we can write

$$\frac{\dot{C}i}{C_i} = \frac{1}{\theta}(\frac{\pi_i}{e_i} + \frac{\dot{e}_i}{e_i} - \rho) \tag{24}$$

4 The Dynamic Path to Convergence

Recall that the product of each economy can be disaggregated as follows:

$$Y_i = C_i + N_i X_i + e_i \dot{N}_i, \quad i = 1, 2 \tag{25}$$

Therefore, by manipulation the expression, and given equations (10), (11) and (12), we get

$$\frac{\dot{N}_i}{N_i} = \frac{1}{e_i}\left(\pi_i \frac{1+\alpha}{\alpha} - \chi_i\right), \tag{26}$$

where

$$\chi_i = \frac{C_i}{N_i}, \quad i = 1, 2 \tag{27}$$

Let be $\hat{N} = {}^{N_2}/_{N_1}$, meaning that

$$\frac{\dot{\hat{N}}}{\hat{N}} = \frac{\dot{\left(\frac{N_2}{N_1}\right)}}{\frac{N_2}{N_1}} \tag{28}$$

and, given (26), we obtain

$$\frac{\dot{\hat{N}}}{\hat{N}} = \frac{1}{e_2}\left(\pi_2 \frac{1+\alpha}{\alpha} - \chi_2\right) - \frac{1}{e_1}\left(\pi_1 \frac{1+\alpha}{\alpha} - \chi_1\right) \tag{29}$$

This expression, together with (17), (19) and (24), gives the consumption dynamics in each economy:

$$\frac{\dot{C}_1}{C_1} = \frac{1}{\theta}\left(\frac{\pi_1}{e_1} + \frac{\dot{\eta}_1}{\eta_1}\right) + \frac{1}{\theta}\left[\frac{1}{e_2}\left(\pi_2\left(\frac{1+\alpha}{\alpha}\right) - \chi_2\right) - \frac{1}{e_1}\left(\pi_1\left(\frac{1+\alpha}{\alpha}\right) - \chi_1\right)\right] \times$$

$$\times \frac{\left\{-\sigma\left(\frac{N_2}{N_1}\right)^{-\sigma} - (\beta/2 - \sigma)\left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \beta/2\left(\frac{N_2}{N_1}\right)^{\beta/2}\right\}}{\left(\frac{N_2}{N_1}\right)^{-\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta/2}} - \frac{1}{\theta}\rho \tag{30}$$

and

$$\frac{\dot{C}_2}{C_2} = \frac{1}{\theta}\left(\frac{\pi_2}{e_2} + \frac{\dot{\eta}_2}{\eta_2}\right) + \frac{1}{\theta}\left[\frac{1}{e_2}\left(\pi_2\left(\frac{1+\alpha}{\alpha}\right) - \chi_2\right) - \frac{1}{e_1}\left(\pi_1\left(\frac{1+\alpha}{\alpha}\right) - \chi_1\right)\right] \times$$

$$\times \frac{\left\{\sigma\left(\frac{N_2}{N_1}\right)^{\sigma} - (\beta + \sigma)\left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \beta\left(\frac{N_2}{N_1}\right)^{\beta}\right\}}{\left(\frac{N_2}{N_1}\right)^{\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta}} - \frac{1}{\theta}\rho \tag{31}$$

Recall that we have defined $\chi_i = C_i/N_i, \quad i = 1, 2$, so, by considering the previous expression together with (29), we get

$$\frac{\dot{\chi}_1}{\chi_1} = \frac{1}{\theta}\left(\frac{\pi_1}{e_1} + \frac{\dot{\eta}_1}{\eta_1}\right) + \frac{1}{\theta}\left[\frac{1}{e_2}\left(\pi_2\left(\frac{1+\alpha}{\alpha}\right) - \chi_2\right) - \frac{1}{e_1}\left(\pi_1\left(\frac{1+\alpha}{\alpha}\right) - \chi_1\right)\right] \times$$

$$\times \frac{\left\{-\sigma\left(\frac{N_2}{N_1}\right)^{-\sigma} - (\beta/2 - \sigma)\left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \beta/2\left(\frac{N_2}{N_1}\right)^{\beta/2}\right\}}{\left(\frac{N_2}{N_1}\right)^{-\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta/2-\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta/2}} - \frac{1}{\theta}\rho - \frac{1}{e_1}\left(\pi_1\frac{1+\alpha}{\alpha} - \chi_1\right) \tag{32}$$

and, for economy 2,

$$\frac{\dot{\chi}_2}{\chi_2} = \frac{1}{\theta}\left(\frac{\pi_2}{e_2} + \frac{\dot{\eta}_2}{\eta_2}\right) + \frac{1}{\theta}\left[\frac{1}{e_2}\left(\pi_2\left(\frac{1+\alpha}{\alpha}\right) - \chi_2\right) - \frac{1}{e_1}\left(\pi_1\left(\frac{1+\alpha}{\alpha}\right) - \chi_1\right)\right] \times$$

$$\times \frac{\left\{\sigma\left(\frac{N_2}{N_1}\right)^{\sigma} - (\beta + \sigma)\left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \beta\left(\frac{N_2}{N_1}\right)^{\beta}\right\}}{\left(\frac{N_2}{N_1}\right)^{\sigma} - \left(\frac{N_2}{N_1}\right)^{\beta+\sigma} + \left(\frac{N_2}{N_1}\right)^{\beta}} - \frac{1}{\theta}\rho - \frac{1}{e_2}\left(\pi_2\frac{1+\alpha}{\alpha} - \chi_2\right) \tag{33}$$

Equations (29), (32) and (33) form a system of differential equations in the variables $\hat{N}$, χ_1 and χ_2.

Given that it is not possible to solve this system of differential equations analytically, we will use numerical methods to solve the system and reach conclusions.

We will present two different situations: Firstly we will present a scenario where economy 1 is definitively more developed than economy 2, that is, economy 1 has better institutions and more labor force. This is the typical situation discussed in the baseline model, presented by Barro and Sala-i-Martin (1997). We associate economy 1 with the more developed countries in the EU, or any group of advanced economies, such as the United States, Germany, Japan and so on. We identify economy 2 as the (potentially or not) transition economies into the EU or any less-developed country or group of countries.

In addition, we discuss a case in which economy 1 has more labor force than economy 2 and a more aggresive investment policy in R&D, but the latter has better functioning institutions. This may be the case when both types of economies considered are reasonably developed, but with compensating structural characteristics. In a EU context, take the case of Ireland as an example for economy of type 2, allready a EU member, which started up with handicaps in certain domains, but who recently revealed an extraordinary performance in cummunity fund management and political response to its limitations.

However, we have yet not discussed the stochastic nature of the innovative process. Furthermore, we have not specified $\dot{\eta}_i/\eta_i$. In fact, we will consider two different cases: we analyze the case in which $\dot{\eta}_i/\eta_i$ is constant, that is, the cost of innovation is each economy increases over time at a constant rate. This rate is (or can be) different for the two economies and it is equal to the increment in expenses on R&D for each economy.

Alternatively, we consider that $\dot{\eta}_i/\eta_i$ is the same as before, but accrued with a random component, which may be different in each economy. This is a particularly relevant situation, given the natural uncertainty in the innovation process, which is far from being deterministic and solely dependent on R&D investment. In fact a discovery may come about in any particularly inspired moment, and in order to model this stochastic behavior, we have considered that both countries are subject to this randomness, each following an independent Normal distribution. The precise method is described in the relevant sections.

The following sections proceed with analyzing each of these situations.

5 Economy 1 has more Labor and better Institutions

5.1 Cost of Innovation increases at a constant rate

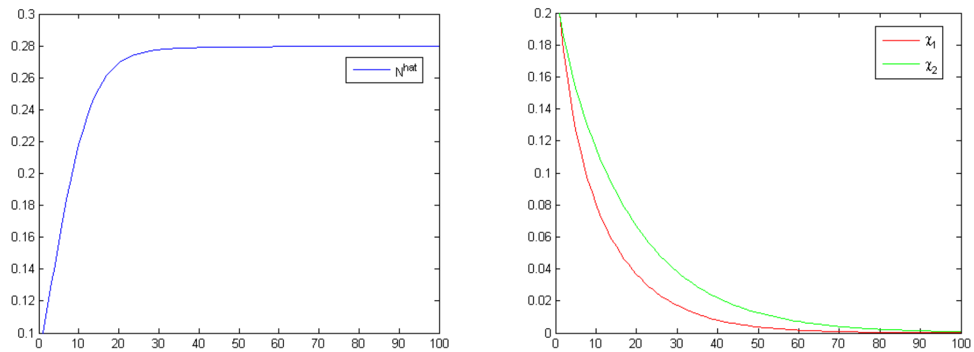

Figure 1: Output results for ratio of varieties and consumption per variety in each economy

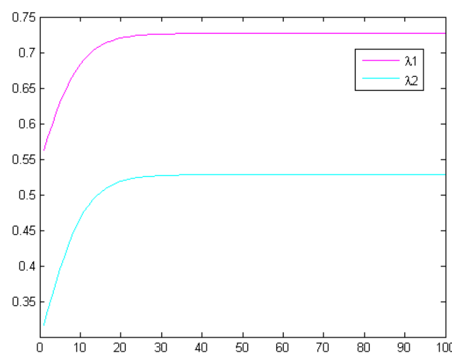

Figure 2: Proportion of innovated intermediate goods in each economy

We observe the trajectory of the ratio of varieties to be quite similar to the baseline model, eventhough convergence is reached later.

However, the most significant difference in relation to Barro and Sala-i-Martin's model is obviously the trajectory of consumption per variety (fig. 1), which now is given by two trajectories (one for each economy), without an initial 'lump' in any of them. This is easily understood in the light of our model, which now allows for both economies to innovate and imitate. Therefore, economy 2 now has no longer an initial boom (followed by a subsequent downfall) in consumption per (low) varieties, as it had before. This was due to varieties only increasing via the imitation of the discoveries in the steady-state leading economy 1, which would take their time to occur.

Now, our model allows for even the less developed economy 2 to innovate ever since the very first instance (fig. 2), therefore increasing varieties along time, even though at a lower rate than the developed economy, with a greater proportion of innovated goods.

Another way the difference in innovative proportions reflect themselves over the dynamics of consumption is that the differential in consumption per variety is fairly significant. Given the strictly convex aspect of the trajectory, with the same starting and finishing points, this difference increases gradually until the convergence point of the innovative proportions (see fig. 2), only then decreasing until steady-state extremely small consumption per varieties level. This ratio clearly decreases at a higher rate in a leading economy, which is only natural given that the number of varieties is (initially) especially incremented in this economy with a higher proportion of innovated goods, always above 50% (fig. 2), until around period 20. From there onwards, however, as its innovative behavior reaches steady-state (as well as the developing economy's), always at a decreasing marginal rate, it gradually runs out of the other country's innovations to imitate, while the other country still has plenty of varieties to imitate. Hence, from period 20 onwards, the differential in consumption per variety is straightened until approximately 0 (and the steady-state in consumption is reached just before period 100).

5.2 Cost of Innovation increases at a constant rate plus a random component

We introduce a random component on the growth rate of the cost of innnovation:

$$\frac{\dot{\eta}_i}{\eta_i} = R\&D_i + \varepsilon_i, \;\; i = 1, 2 \tag{34}$$

where $R\&D_i$ stands for the growth rate of expenditures in R&D and ε_i is the random component.

In order to generate ε_i, we use Monte Carlo simulation. Monte Carlo allows us to generate two random variables, say w_1 and w_2, from a uniform distribution on the unit interval. To generate independant and identically distributed standard Normals we use a Box-Muller transformation, and multiply by a standard deviation term to create:

$$\varepsilon_1 = \sigma\sqrt{-2 \times ln(w_1)} \times cos(w_2) \tag{35}$$

$$\varepsilon_2 = \sigma\sqrt{-2 \times ln(w_2)} \times sin(w_1) \tag{36}$$

which follow a two-dimentional bivariate Normal distribution with zero mean and variance given by σ^2. This allows us to account for the stochastic nature of innovation, given a certain variability in the discovery process.

We use a Normal distribution to describe the additive stochastic term because of its bell-like shape, which allows for more probable, but less radical innovations, to be determined roughly by R&D (because the expected value of ε_i is null), whilst rarer cases will either decrease or potentiate R&D investment with a lower probability of occurence (represented by the left and right tail of the distribution, respectively).

The results obtained are given by the following figures:

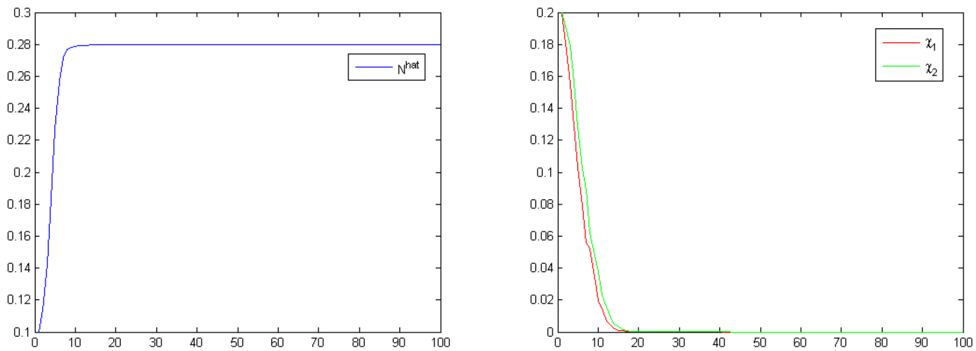

Figure 3: Output results for ratio of varieties and consumption per variety in each economy

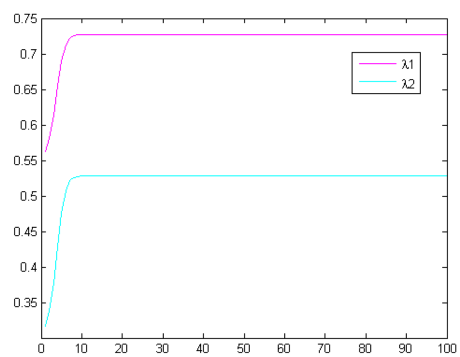

Figure 4: Proportion of innovated intermediate goods in each economy

Interestingly, the main difference observed with innovation partially dependent on a simulated stochastic term is the rate at which convergence is achieved. Uncertainty in the innovative process seems to accelerate the rate of innovation toward steady-state from a certain inflection point onwards (fig.2, right-hand-side(rhs)), and this naturally reflects itself in a more rapid convergence of the variety ratio, and consumptions per variety, which in turn are much closer between countries, along the path to convergence. Similarly to what has been described in the previous section, this is certainly because all innovations (in either economy) are rapidly captured and immitated by both countries.

6 Economy 1 has more Labor but worse Institutions

6.1 Cost of Innovation increases at a constant rate

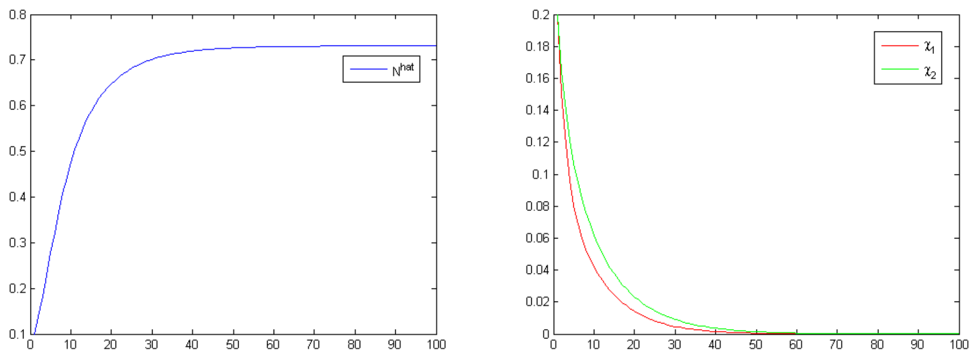

Figure 5: Output results for ratio of varieties and consumption per variety in each economy

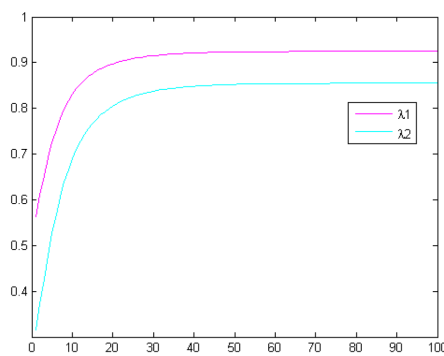

Figure 6: Proportion of innovated intermediate goods in each economy

We now observe the trajectory of the ratio of varieties to converge to much higher ratio levels, i.e., the differential in varieties in both economies is not as significant as before, which is precisely in context with the case of EU economies which we are considering, with a lower development gap.

This situation is quite similar to what has been described in section 5.1, though it is noticeable that the gap between economies straightenes. This is visible in terms of the innovative proportions trajectory, which are naturally closer given their dependance on the productive process, and country 2's *catching up* in terms of innovative behavior (fig. 2, rhs) This is only natural since the disparities in economic development between countries are compensated by different strenghths of each economy at the production level (economy 2 now has better institutional infrastructure).

6.2 Cost of Innovation increases at a constant rate plus a random component

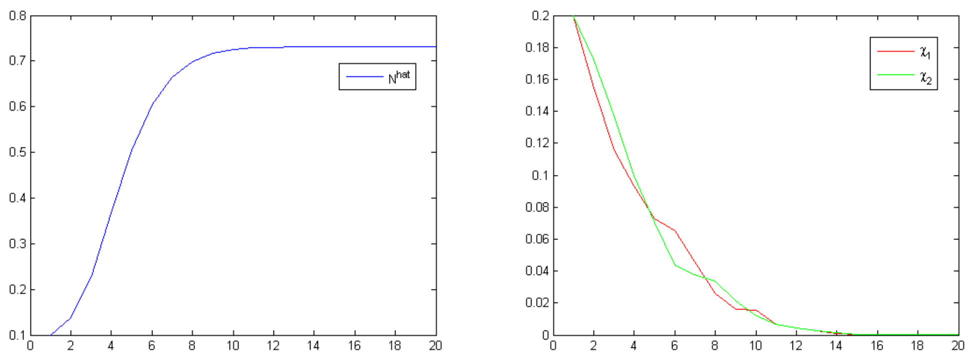

Figure 7: Output results for ratio of varieties and *spiraling* consumption per variety in each economy

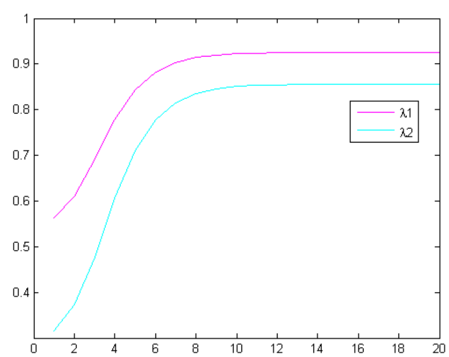

Figure 8: Proportion of innovated intermediate goods in each economy

It is in the light of this context that the introduction of a random component in the discovery process seems to be of greatest interest. The fact that an advance is given to the transition economy, which seems reasonable in any context where development gaps are small, and economies have compensating productive inputs, leads to what we have called a *spiral effect*: consumption per variety reveals itself to be extremely volatile, directly dependent on the innovation variability (σ^2). This reflects an alternation in consumption behavior between countries, as opposed to the stable behavior observed in the case where there is a dominant leading economy (with consumption constantly higher). A sensitivity experiment was conducted by decreasing the volatility of the random process over innovation (e.g.,$\sigma = 1/14$). The result is that the switching of consumption behavior remains, though attenuated (both in number and in intensivity) and eventually vanishes, as randomness disappears.

7 Conclusions

We find evidence that when we account for the stochastic nature of the innovative process, something which is quite consensual in empirical growth literature, but rarely modeled, the paths to convergence seem to assume a different behavior. This is particularly true in more realistic settings where the development gap between economies isn't as strong.

Furthermore, we find our extension of Barro and Sala-i-Martin's model (1997) to be particularly relevant to explain consumption differences throughout time between both types of economies considered, which might raise important welfare considerations. We conclude the consumption gap may be straightened by two possible occurrences: either when an advance (e.g., institutional) is given to the developing or transition economy; or particularly when the random nature of innovation is considered.

Appendix A

Matlab code for section 5.1.

Principal M-File

```
disp('——————————————————————————-');
disp(' National Productive Structure and Innovative Dynamics: ');
disp(' Finding the (Endogenous) Path to Convergence ');
disp(' Implemented by ');
disp(' Luis Pina Rebelo and Jorge Cerdeira ');
disp('——————————————————————————-');

clear all; clf;

%% Country 1 has more labor and better institutions: With innovation
%% growing at a constant rate (equal to the increase in R&D expenditures)

%Initial values and time period
```

```
tend=100;
init_values= [0.1; 0.2; 0.2];
time_period=(1:tend);

%solve system of ODE's
[t,Y] = ode45(@ode,time_period,init_values);

% Plot the solution of system of ODEs
figure (1)
plot(t,Y(:,1),'b-')
legend('N^{hat}')

figure (2)
plot(t,Y(:,2),'r-',t,Y(:,3),'g-')
legend('\chi_1','\chi_2')

%% Computing the trajectory of the proportion of innovated intermediate goods
global beta

% creating vector lambda1 and lambda2
for i=1:tend
 lambda1(i,1)=Y(i,1)^(beta/2);
 lambda2(i,1)=Y(i,1)^beta;
end

% plot the proportion of intermediate goods
figure (3)
plot(1:tend,lambda1,'m',1:tend,lambda2,'c')
legend('\lambda1','\lambda2')
```

ode M-File

```
function dydt = ode(t,Y)
%======================
% ode system of equations
% input: t = must be there by definition
% Y = vector of the rhs of the system
% output: t, Y
%======================

global beta

% parameters
eta1=100;
eta2=100;
tx_cresc_ID1 = 0.01; %this is equal to the growth rate of eta1
tx_cresc_ID2 =0.06; %this is equal to the growth rate of eta2
L1=1.5;
```

```
L2=1.05;
A1=4;
A2=2.8;
alpha=0.75;
theta=2.5;
sigma=1.8;
beta=0.5; %parameter associated with proportion of innovations
rho=0.02;
Pi_1=(1-alpha)*L1*A1^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));
Pi_2=(1-alpha)*L2*A2^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));

% ... system of ordinary diferential equations
%N{hat} is Y(1), \chi1 is Y(2) and \chi2 is Y(3); tx_cresc_ID1 and tx_cresc_ID2
% are equal to the growth rate of eta 1 and eta2

dydt = [...

Y(1)* ( ...
(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))^(-1)*...
[Pi_2*(1+alpha)/alpha-Y(3)]- ...
(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))^(-1)*...
[Pi_1*(1+alpha)/alpha-Y(2)] ...
); ...

Y(2)*( ...
1/theta*(Pi_1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))+tx_cresc_ID1)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3))...
-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*(Pi_1*(1+alpha)/alpha-
Y(2)))*...
(-1*sigma*Y(1)^(-1*sigma)-(beta/2-sigma)*Y(1)^(beta/2-sigma)+beta/2*Y(1)^(beta/2))/...
(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2))-...
1/theta*rho-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)));...

Y(3)*( ...
1/theta*(Pi_2/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))+tx_cresc_ID2)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/...
alpha-Y(3))-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)))*...
(sigma*Y(1)^(sigma)-(beta+sigma)*Y(1)^(beta+sigma)+beta*Y(1)^beta)/...
(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta)-...
1/theta*rho-1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3)))...
];
```

Appendix B

Matlab code for section 5.2.

Principal M-File

```
disp('——————————————————————————-');
disp(' National Productive Structure and Innovative Dynamics: ');
disp(' Finding the (Endogenous) Path to Convergence ');
disp(' Implemented by ');
disp(' Luis Pina Rebelo and Jorge Cerdeira ');
disp('——————————————————————————-');

clear all; clf;

%% Country 1 has more labour and better institutions: with innovation
%% growing at a constant rate (equal to the increase in R&D expeditures)
%% plus a random component

global random1 random2 beta

%Initial values and time period
tend=100;
init_values= [0.1; 0.2; 0.2];
tx_cresc_ID1 = 0.01;
tx_cresc_ID2 = 0.06;
time_period=(1:tend);

txcresc_eta=zeros(tend,2); %initialization matrix for growth rate of eta: we
%will save the values generated by the random process in this matrix

S=zeros(tend,3); %final solution of the ODE system matrix

for i=1:tend

 %Método Box Muller: (random process which will generate the growth rate of eta)
 w1=rand(1,1);
 w2=rand(1,1);
 dp=1/14;
 epsylon1=dp*sqrt(-2*log(w1))*cos(w2);
 epsylon2=dp*sqrt(-2*log(w2))*sin(w1); % these 2 variable follow Normal(0,dp^2)iid

 %Creating random variables:
 random1= tx_cresc_ID1 + epsylon1;
 random2= tx_cresc_ID2 + epsylon2;

 %Identifying the random variables to the growth rate of eta at each run:
 txcresc_eta(i,1)=random1;
 txcresc_eta(i,2)=random2;
```

```
 %solve system of ODE's
 [t,Y] = ode45(@ode,time_period,init_values);

 % Testing the time index
  for j=2:tend
   if (t(j)-t(j-1))~=1
   disp('cuidado'); return
   end
  end
 % Saving the optimal value in each period in matrix S:
 for k=1:3
  if size(Y,1)<i
  S(i,k)=init_values(k,1);
  else
  S(i,k)=Y(i,k); %if Y has already reach convergence, matrix S
%get the initial value; otherwise, matrix S get the value obtained
%in matrix Y
  end
 end

 % Preparing initial values for next run:
 init_values=[S(i,1);S(i,2);S(i,3)];
end

% Plot the solution of system of ODEs
figure (1)
plot(t,S(1:size(t,1),1),'b-')
legend('N^{hat}')

figure (2)
plot(t,S(1:size(t,1),2),'r-',t,S(1:size(t,1),3),'g-')
legend('\chi_1','\chi_2')

%% Computing the trajectory of Lambda, the proportion of innovated
%% intermediate goods

% creating vectors lambda
for i=1:tend
 lambda1(i,1)=S(i,1)^(beta/2);
 lambda2(i,1)=S(i,1)^beta;
end

%plot the proportion of innvated intermediate goods
figure (3)
plot(1:tend,lambda1,'m',1:tend,lambda2,'c')
legend('\lambda1','\lambda2')
```

ode M-File

```
function dydt = ode(t,Y)
%======================
% ode system of equations
% input: t = must be there by definition
% Y = vector of the rhs of the system
% output: t, Y
%======================

global random1 random2 beta

% parameters
eta1=100;
eta2=100;
L1=1.5;
L2=1.05;
A1=4;
A2=2.8;
alpha=0.75;
theta=2.5;
sigma=1.8;
beta=0.5;
rho=0.02;
Pi_1=(1-alpha)*L1*A1^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));
Pi_2=(1-alpha)*L2*A2^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));

% ... system of ordinary diferential equations
%N{hat} is Y(1), \chi1 is Y(2) and \chi2 is Y(3); random1 and random2 are
% equal to the growth rate of eta1 and eta2

dydt = [...

Y(1)* ( ...
(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))^(-1)*...
[Pi_2*(1+alpha)/alpha-Y(3)]- ...
(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))^(-1)*...
[Pi_1*(1+alpha)/alpha-Y(2)] ...
); ...

Y(2)*( ...
1/theta*(Pi_1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))+random1)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3))...
-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*(Pi_1*(1+alpha)/alpha-
Y(2)))*...
(-1*sigma*Y(1)^(-1*sigma)-(beta/2-sigma)*Y(1)^(beta/2-sigma)+beta/2*Y(1)^(beta/2))/...
(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2))-...
1/theta*rho-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
```

```
(Pi_1*(1+alpha)/alpha-Y(2)));...

Y(3)*( ...
1/theta*(Pi_2/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))+random2)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/...
alpha-Y(3))-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)))*...
(sigma*Y(1)^(sigma)-(beta+sigma)*Y(1)^(beta+sigma)+beta*Y(1)^beta)/...
(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta)-...
1/theta*rho-1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3)))...
];
```

Appendix C

Matlab code for section 6.1.

Principal M-File

```
disp('——————————————————————————-');
disp(' National Productive Structure and Innovative Dynamics: ');
disp(' Finding the (Endogenous) Path to Convergence ');
disp(' Implemented by ');
disp(' Luis Pina Rebelo and Jorge Cerdeira ');
disp('——————————————————————————-');

clear all; clf;

%% Country 1 has more labou but worse institutions: With innovation
%% growing at a constant rate (equal to the increase in R&D expenditures)

%Initial values and time period
tend=100;
init_values= [0.1; 0.2; 0.2];
time_period=(1:tend);

%solve system of ODE's
[t,Y] = ode45(@ode,time_period,init_values);

% Plot the solution of system of ODEs
figure (1)
plot(t,Y(:,1),'b-')
legend('N^{hat}')

figure (2)
plot(t,Y(:,2),'r-',t,Y(:,3),'g-')
legend('\chi_1','\chi_2')

%% Computing the trajectory of Lambda, the proportion of innovated
```

```
%% intermediate goods

global beta

% computing vector lambda
for i=1:tend
 lambda1(i,1)=Y(i,1)^(beta/2);
 lambda2(i,1)=Y(i,1)^beta;
end

% plot lambda 1 and lambda 2
figure (3)
plot(1:tend,lambda1,'m',1:tend,lambda2,'c')
legend('\lambda1','\lambda2')
```

ode M-File

```
function dydt = ode(t,Y)
%=====================
% ode system of equations
% input: t = must be there by definition
% Y = vector of the rhs of the system
% output: t, Y
%=====================

global beta

% parameters
eta1=180;
eta2=240;
tx_cresc_ID1 = 0.01; %this is equal to the growth rate of eta1
tx_cresc_ID2 = 0.06; %this is equal to the growth rate of eta2
L1=4;
L2=2.5;
A1=4;
A2=5;
alpha=0.65;
theta=2.5;
sigma=1.8;
beta=0.5;
rho=0.02;
Pi_1=(1-alpha)*L1*A1^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));
Pi_2=(1-alpha)*L2*A2^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));

% system of ordinary diferential equations
%N{hat} is Y(1), \chi1 is Y(2) and \chi2 is Y(3); tx_cresc_ID1 and tx_cresc_ID2
% are equal to the growth rate of eta 1 and eta2
```

```
dydt = [...

Y(1)* ( ...
(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))^(-1)*...
[Pi_2*(1+alpha)/alpha-Y(3)]- ...
(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))^(-1)*...
[Pi_1*(1+alpha)/alpha-Y(2)] ...
); ...

Y(2)*( ...
1/theta*(Pi_1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))+tx_cresc_ID1)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3))...
-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*(Pi_1*(1+alpha)/alpha-
Y(2)))*...
(-1*sigma*Y(1)^(-1*sigma)-(beta/2-sigma)*Y(1)^(beta/2-sigma)+beta/2*Y(1)^(beta/2))/...
(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2))-...
1/theta*rho-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)));...

Y(3)*( ...
1/theta*(Pi_2/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))+tx_cresc_ID2)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/...
alpha-Y(3))-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)))*...
(sigma*Y(1)^(sigma)-(beta+sigma)*Y(1)^(beta+sigma)+beta*Y(1)^beta)/...
(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta)-...
1/theta*rho-1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3)))...
];
```

Appendix D

Matlab code for section 6.2.

Principal M-File

```
disp('——————————————————————————-');
disp(' National Productive Structure and Innovative Dynamics: ');
disp(' Finding the (Endogenous) Path to Convergence ');
disp(' Implemented by ');
disp(' Luis Pina Rebelo and Jorge Cerdeira ');
disp('——————————————————————————-');

clear all; clf;

%% Country 1 has more labour but worse institutions: with innovation
%% growing at a constant rate (equal to the increase in R&D expeditures)
%% plus a random component
```

```
global random1 random2 beta

%Initial values and time period
tend=20;
init_values= [0.1; 0.2; 0.2];
tx_cresc_ID1 = 0.01;
tx_cresc_ID2 = 0.06;
time_period=(1:tend);

txcresc_eta=zeros(tend,2); %initialization matrix for growth rate of eta: we
%will save the values generated by the random process in this matrix

S=zeros(tend,3); %final solution of the ODE system matrix

for i=1:tend
 %Método Box Muller: (random process which will generate the growth rate of eta)
 w1=rand(1,1);
 w2=rand(1,1);
 dp=1/7; % sensitivity was conducted with 1/14; 1/21.
 epsylon1=dp*sqrt(-2*log(w1))*cos(w2);
 epsylon2=dp*sqrt(-2*log(w2))*sin(w1); % these 2 variable follow Normal(0,dp^2)iid

 %Creating random variables:
 random1= tx_cresc_ID1 + epsylon1;
 random2= tx_cresc_ID2 + epsylon2;

 %Identifying the random variables to the growth rate of eta at each run:
 txcresc_eta(i,1)=random1;
 txcresc_eta(i,2)=random2;

 %solve system of ODE's
 [t,Y] = ode45(@ode,time_period,init_values);

 % Testing the time index
 for j=2:tend
  if (t(j)-t(j-1))~=1
  disp('cuidado'); return
  end
 end

 % Saving the optimal value in each period in matrix S:
 for k=1:3
  if size(Y,1)<i
  S(i,k)=init_values(k,1);
  else
  S(i,k)=Y(i,k); %if Y has already reach convergence, matrix S
  %get the initial value; otherwise, matrix S get
  %the value obtained in matrix Y
```

```
  end
 end

 % Preparing initial values for next run:
 init_values=[S(i,1);S(i,2);S(i,3)];
end

% Plot the solution of system of ODEs
figure (1)
plot(t,S(1:size(t,1),1),'b-')
legend('N^{hat}')

figure (2)
plot(t,S(1:size(t,1),2),'r-',t,S(1:size(t,1),3),'g-')
legend('\chi_1','\chi_2')

%% Computing the trajectory of Lambda, the proportion of innovated
%% intermediate goods

% creating vectors lambda
for i=1:tend
 lambda1(i,1)=S(i,1)^(beta/2);
 lambda2(i,1)=S(i,1)^beta;
end

%plot the proportion of innovated intermediate goods
figure (3)
plot(1:tend,lambda1,'m',1:tend,lambda2,'c')
legend('\lambda1','\lambda2')
```

ode M-File

```
function dydt = ode(t,Y)
%======================
% ode system of equations
% input: t = must be there by definition
% Y = vector of the rhs of the system
% output: t, Y
%======================

global random1 random2 beta

% parameters
eta1=180;
eta2=240;
L1=4;
L2=2.5;
A1=4;
```

```
A2=5;
alpha=0.65;
theta=2.5;
sigma=1.8;
beta=0.5;
rho=0.02;
Pi_1=(1-alpha)*L1*A1^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));
Pi_2=(1-alpha)*L2*A2^(1/(1-alpha))*alpha^((1+alpha)/(1-alpha));

% system of ordinary diferential equations
%N{hat} is Y(1), \chi1 is Y(2) and \chi2 is Y(3); random1 and random2
% are equal to the growth rate of eta 1 and eta2

dydt = [...

Y(1)* ( ...
(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))^(-1)*...
[Pi_2*(1+alpha)/alpha-Y(3)]- ...
(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))^(-1)*...
[Pi_1*(1+alpha)/alpha-Y(2)] ...
); ...

Y(2)*( ...
1/theta*(Pi_1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))+random1)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3))...
-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*(Pi_1*(1+alpha)/alpha-
Y(2)))*...
(-1*sigma*Y(1)^(-1*sigma)-(beta/2-sigma)*Y(1)^(beta/2-sigma)+beta/2*Y(1)^(beta/2))/...
(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2))-...
1/theta*rho-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)));...

Y(3)*( ...
1/theta*(Pi_2/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))+random2)+...
1/theta*(1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/...
alpha-Y(3))-1/(eta1*(Y(1)^(-1*sigma)-Y(1)^(beta/2-sigma)+Y(1)^(beta/2)))*...
(Pi_1*(1+alpha)/alpha-Y(2)))*...
(sigma*Y(1)^(sigma)-(beta+sigma)*Y(1)^(beta+sigma)+beta*Y(1)^beta)/...
(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta)-...
1/theta*rho-1/(eta2*(Y(1)^sigma-Y(1)^(beta+sigma)+Y(1)^beta))*(Pi_2*(1+alpha)/alpha-
Y(3)))...
];
```

References

[1] Aghion, P. and P. Howitt (1992), *A Model of Growth through Creative Destruction*, Econometrica 60: 323–351.

[2] Barro, R. and Sala-i-Martin, X. (1995), *Economic Growth*, New York, McGraw Hill.

[3] Barro, R and Sala-i-Martin, X. (1997), *Technological Diffusion, Convergence and Growth*, Journal of Economic Growth, 2: 1-27.

[4] Blanchard, O., Dornbusch, R., Krugman, P., Layard, R. and Summers, L. (1991), *Reform in Eastern Europe*, MIT Press, Cambridge and London

[5] Box, G. E. P. and Muller, M. E. (1958) *A Note on the Generation of Random Normal Deviates*, The Annals of Mathematical Statistics, 29, 610-611

[6] Dixit, A. K., and J. E. Stiglitz (1977), *Monopolistic Competition and Optimum Product Diversity*, American Economic Review 67: 297–308.

[7] Grabbe, H. and Hughes, K. (1998), *Enlarging the EU Eastwards*, Chatham House Papers, The Royal Institute of International Affairs, London

[8] Grossman, G.M. and Helpman, E. (1995) *Innovation and Growth in the Global Economy*, MIT Press, Cambridge

[9] Hare, P.G., Batt, J. and Estrin, S. (1999), *Reconstituting the Market: The Political Economy of Microeconomic Transformation*, Harwood Academic Publishers, Amsterdam, excerpts from the internet

[10] Mayhew, A. (1998) *Recreating Europe : The European Union's Policy Towards Central and Eastern Europe*, Cambridge University Press, Cambridge

[11] Portes, R. (ed.) (1993) *Economic Transformation in Central Europe : a Progress Report*, Centre for Economic Policy Research, London

[12] Romer, P. M. (1987), *Growth Based on Increasing Returns Due to Specialization*, American Economic Review 77: 56–62.

[13] Romer, P. M. (1990), *Endogenous Technological Change*, Journal of Political Economy, 98: 71–102.

[14] Schmidt-Dengler, P. (2007), *Lecture notes on Advanced Economic Analysis*, Faculty of Economics - University of Porto

[15] Spence, M. (1976), *Product Selection, Fixed Costs, and Monopolistic Competition*, Review of Economic Studies, 43: 217–235.