Open loop and feedback solutions to an institutional game under non-quadratic preferences

F. Ngendakuriyo

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Institut de Recherches Économiques et Sociales
de l'Université catholique de Louvain

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Fabien Ngendakuriyo[1]
Economics School of Louvain, Catholic University of Louvain
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Abstract

Until now most research in dynamic games focus on models with quadratic objective functions because of practical considerations. But in reality, all problems are not quadratic. In this paper, we solve a differential game where players have non-quadratic preferences. In particular we consider an institutional game governing a permanent interaction between civil society organizations and Government in the economy in the presence of corruption. At the first stage, we compute analytically and solve numerically the open loop and cooperative outcome of the differential game. At the second stage, we approximated analytically and solved numerically the feedback strategies at equilibrium. As results, we found that both open loop and cooperative solution are unique and stable while multiple feedback Nash equilibria should arise. As economic implications, we found that under cooperative play the magnitude of the civil monitoring effort is lower than the one in open loop game. This in turn is smaller than the magnitude of effort associated to the best feedback equilibrium. Total factor productivity effects always dominate the detrimental effect of individual effort devoted to production in almost all situations. Furthermore, institutions improve much faster under cooperative scenario than in open loop game. These results have a similar format with the ones obtained under linear quadratic differential game at least for open loop and cooperative games.

Keywords: Institutions, corruption, civil society, dynamic games, dynamic programming, non-quadratic preferences, Markovian strategies
JEL Classification: C61, C62, C71, C72, C73, O43

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1 Introduction

There has been considerable use of differential games in analyzing economic problems. Many areas such environmental economics namely pollution problems and resources management, lobbying activities, industrial organization, etc are concerned. In dynamic game, information available to players at the time of their decisions matters. Two major cases are distinguished in non cooperative games: Open loop information versus Feedback information. We based ourselves on Basar and Olsder (1982). Following these authors, open loop and feedback nash solutions differ by their information structure. Under open loop information structure, all players know just the initial state of the process and the game structure. It is assumed that players simultaneously determine their actions for the whole planning horizon of the process before it starts. Binding commitments are required to avoid any deviation during the evolution of the process. Then, open loop solution depends on time and initial state of the system. It assumes precommittement of players and is weakly time-consistent since players can deviate during the game. In contrast, feedback information structure allows all players to observe at every point in time the current state of the process and determine their actions based on this observation. Then, feedback controls depend on time and current state. They are time-consistent and the markov subagme perfectness property holds. Additionally, Basar and Olsder (1982), and in the same line, Kossioris et al (2008) state that if the time variable does not appear explicitly in the model (except in discounting factor), the model is autonomous (time invariant) and optimal solution is time-stationary. It is convenient to add the cooperative mood in dynamic games despite the difficulty to justify it in some conflict games. Under the cooperation scenario, players communicate and agree to cooperate in order to achieve their objectives.

Several studies in differential games extensively focus on problems with quadratic objectives (as pay-offs of players) and linear state system. The so-called " Linear Quadratic Differential Games". This formulation helps analytical tractability in particular for computing the Feedback strategies. Non quadratic structure brings more difficulties in solving a differential game. Under quadratic differential games, findings have almost similar format. On one side, some studies pointed out the existence and uniqueness of both Open loop and feedback equilibrium (Wirl, 94; Piga, 2000). On the other side, without boundary condition, multiple feedback Nash equilibria may exist [Dockner et al (1993), Kossioris et al (2008)]. This multiplicity arises the question of selecting the best feedback Nash equilibrium. Kossioris et al (2008) introduce non-linearities in dynamic state equation and consider the steady state as a boundary condition. In sum, due to practical considerations, numerous studies in dynamic games focus on models with quadratic preferences as payoffs of players.

In this work, we model permanent interaction between Government and Civil society as an institutional differential game. The novelty of the work is that we model this dynamic interaction under linear non quadratic structure i.e non quadratic objectives functions and linear state equation. Clearly, we will consider logarithmic preferences and show that by using linear approximation around the steady state, we can handle analytically the markovian strategies for this differential game. Before solving numerically the feedbacks, we will compute analytically the properties of existence of the feedback solutions. Open loop and cooperative outcomes are taken in this framework as benchmark solutions. In our previous work (Ngendakuriyo , 2009), we solved a dynamic institutional game between a corrupt government and an active civil society under linear quadratic structure. Civil society had two strategies: voice or loyalty. On its side, Government could repress or tolerate any attempt at revolt by handling its punishment mechanisms. When an active civil society faces an unrestrictive or passive government, we model this interaction as a one player differential game or a simple optimal control. However, if the corrupt government is an oppressive one, the two-player game takes place. Theoretical results show that open loop Nash equilibrium and cooperative outcome are both unique and stable saddle point. This technical note aims to investigate if the properties of the equilibrium (existence, uniqueness and stability of the open loop Nash and cooperative outcome) are preserved under nonquadratic preferences.

The rest of the paper is organized as follows: section 2 presents the structure of the game and section 3 the model of competitive play (open loop game) while section 4 is devoted to the model of cooperative interaction. In the section 5, we attempt to compute the feedback solutions and section 6 concludes.

2 Structure of the game

We construct a two-agent differential game in which an active civil society faces an authoritarian government which make pressure on civil society organizations in order to reinforce its corruption technology. Time is continuous and the game is played on infinite horizon between a representative consumer and Government in one developing economy. We assume that the representative consumer is a member of civil society and then spend a part of her time on monitoring activities and the other part of time is devoted to production sector. Total amount of time is normalized to one such that $L = 1 - w$ where w is the consumer's monitoring effort and L the labor supply. Government is a quasi-unique employer and is a corrupt entity who exerts pressures on civil society to halt monitoring. We assume that government obtains, per unit of output Y_t, the rent $\phi = \phi(x, w)$ depending negatively on the civil effort w and positively on the pressure x it exerts to civil society. Formally, $\phi_x > 0$ and $\phi_w < 0$. However, the Government supports the costs by implementing and enforcing the sanctioning mechanisms. Let us name $g(x)$ the cost function which is increasing and convex: $g_x > 0$ and $g_{xx} \geq 0$. The representative consumer receives the amount $C_t = (1 - \phi(x, w))Y_t$ and Government the amount of rent $G_t = \phi(x_t, w_t)Y_t$. The production function where only labor L is a production factor is the following: $Y_t = A_t F(L)$, where A_t is the total factor productivity. Total factor productivity is modeled as a function of the institutional quality I_t and other variables z_t interacting in the economy. Briefly, we maintain the same framework as in Ngendakuriyo (2009) except the fact that we consider now non-quadratic objective functions in the controls x_t and w_t.

In summary, each player is choosing his strategy in order to maximize the present value of net benefits over an infinite time horizon, strategies of others players taken as given. Government is choosing a strategy x_t and

$$Max_{x_t} \int_0^\infty exp(-\rho t)[U(\phi(x_t, w_t)Y_t) - g(x_t)]dt, \tag{1}$$

while a representative consumer chooses a strategy w_t or

$$Max_{w_t} \int_0^\infty exp(-\rho t)U((1 - \phi(x_t, w_t)Y_t))dt; 0 \leq w_t \leq 1 \tag{2}$$

subject to

$$\dot{I}_t = bw_t - \delta I_t, \tag{3}$$

$I(0) = I_0$ is given.

3 Open loop equilibrium

It is well known that open loop Nash equilibrium in two-players differential game is obtained by solving two optimal control problems. The current-value Hamiltonian H for government's problem is given by

$$H = U(\phi(x_t, w_t)Y_t) - g(x) + \eta(bw_t - \delta I_t) \tag{4}$$

where η is the costate variable. The necessary optimality conditions are given by

$$H_x \equiv U_G \phi_x Y - g_x = 0, \tag{5}$$

$$H_I \equiv \rho\eta - \dot{\eta} = U_G \phi A_I F(L) - \delta\eta, \tag{6}$$

$$H_\eta \equiv \dot{I} = bw_t - \delta I. \tag{7}$$

The augmented lagrangian function associated to consumer's problem can be written as

$$L = U((1 - \phi(x_t, w_t)Y_t)) + \lambda_t(bw_t - \delta I_t) + \mu_{1t}w_t + \mu_{2t}(1 - w_t) \tag{8}$$

and the corresponding necessary conditions for interior solution are

$$L_w \equiv -U_c\phi'(w)Y - U_c(1 - \phi(w))AF'_L + \lambda b = 0, \tag{9}$$

$$L_I \equiv \rho\lambda - \dot{\lambda} = (1 - \phi(w))U_c F(1 - w)A_I - \delta\lambda, \tag{10}$$

$$L_\lambda \equiv \dot{I} = bw - \delta I. \tag{11}$$

Open loop strategies require that $H_x = 0$ and $L_w = 0$ hold simultaneously. As we seek for explicit solutions, it is convenient to specify the functional forms to be used in modeling this interaction. We take logarithmic utility functions for consumers and Government coupled with linear corruption technology and a linear cost function for implementing the governmental hostility. We use an AK type production function and a linear function of total factor productivity with respect to institutions. Concretely, we consider the following functions:

$$U(C_t) = lnC_t = ln[(1 - \phi(x_t, w_t))Y_t], \tag{12}$$

$$U(G_t) = lnG_t = ln[\phi(x_t, w_t)Y_t], \tag{13}$$

$$\phi(x_t, w_t) = \kappa(1 - w_t + x_t), \tag{14}$$

$$g(x_t) = \alpha x_t, \tag{15}$$

$$Y_t = A_t L_t = A_t(1 - w_t), \tag{16}$$

$$A_t = z_t(A_0 + I_t); A_0 > 0. \tag{17}$$

Reasonably, we can defines the exogenous parameter κ as the capacity of the government to extract rents. It captures the scale of corruption in the economy. The coefficient α is the marginal cost the government supports in implementing one additional unit of pressures or intimidations on civil society organizations. α and κ represent two opposite forces which influence the choices of players. We allow poor institutions ($I_t \to 0$) to have a low level of total factor productivity equating A_0. Given the explicit functions above, problems (1) and (2) respectively can be summarized as follows. Government chooses her strategy x_t and

$$Max_{x_t} \int_0^\infty exp(-\rho t)\{ln[\kappa z_t(A_0+I_t)(w_t^2-2w_t-x_tw_t+x_t-1)]-\alpha x_t\}dt, \quad (18)$$

and a representative consumer chooses her strategy w_t, or

$$Max_{w_t} \int_0^\infty exp(-\rho t)ln[z_t(A_0+I_t)(-\kappa w_t^2+(2\kappa-1)w_t+\kappa x_tw_t-\kappa x_t+1-\kappa)]dt \quad (19)$$

subject to (3) and where $0 < w < 1$.

Proposition 1: *Under logarithmic preferences and linear cost function for implementing governmental hostility, there exists at least one open loop Nash equilibrium* (w^*, x^*) *such that* $0 < w^* < 1$ *for* $\kappa < \dfrac{\alpha(A_0(\delta+\rho)-b)}{(1+\alpha)A_0(\delta+\rho)-b}$. *For* A_0 *small enough, the open loop nash equilibrium exists if and only if* $\kappa < \alpha$. *Furthermore, this equilibrium is unique and displays a stable saddle point.*

Proof. Under the above functional forms, Pontryagin's maximum principle (5) - (7) can be rewritten as

$$H_x \equiv \frac{1}{(1-w+x)} - \alpha = 0, \quad (20)$$

$$H_\eta \equiv \dot{I} = bw - \delta I, \quad (21)$$

$$H_I \equiv \dot{\eta} = (\delta+\rho)\eta - \frac{1}{(A_0+I)}. \quad (22)$$

Following the Mangasarian sufficiency theorem, these necessary conditions will also be sufficient for maximization of the Hamiltonian if it is jointly concave in both the state and control variables. This requires that the 2 X 2 Hessian matrix, comprising the second order partial derivatives of the Hamiltonian with respect to the state and control variables, is negative semi-definite. The negative semi-definiteness is well established since the principal minors have discriminants that alternate in sign and the first one is negative or equal to zero. The Hessian matrix can be derived from (4) as follows

$$\begin{pmatrix} -\dfrac{1}{(1-w+x)^2} & 0 \\ 0 & -\dfrac{1}{(A_0+I)^2} \end{pmatrix}$$

It follows that the first principal minor is $\mid M_1 \mid= -\dfrac{1}{(1-w+x)^2} < 0$ and the second one is $\mid M_2 \mid= \dfrac{1}{(1-w+x)^2(A_0+I)^2} > 0$. Hence, the necessary conditions (20) - (22) are also sufficient for a maximum.

For the representative consumer, the necessary optimality conditions (9) - (11) for interior solution can be rewritten as

$$L_w \equiv \frac{2\kappa(1-w)-1+\kappa x}{(1-\kappa(1-w+x))(1-w)} + \lambda b = 0, \tag{23}$$

$$L_I \equiv \dot{\lambda} = (\delta+\rho)\lambda - \frac{1}{(A_0+I)}, \tag{24}$$

$$L_\lambda \equiv \dot{I} = bw - \delta I. \tag{25}$$

The second order sufficiency conditions are derived from the following matrix

$$\begin{pmatrix} \frac{-2\kappa(1-w)(1-\kappa(1-w-x))-(2\kappa(1-w)+\kappa x-1)^2}{(1-w)^2(1-\kappa(1-w+x))^2} & 0 \\ 0 & -\frac{1}{(A_0+I)^2} \end{pmatrix}$$

We observe that the two principal minors have the following format:

$$\mid M_1 \mid= \tfrac{-2\kappa(1-w)(1-\kappa(1-w-x))-(2\kappa(1-w)+\kappa x-1)^2}{(1-w)^2(1-\kappa(1-w+x))^2} < 0;$$
$$\mid M_2 \mid= \tfrac{2\kappa(1-w)(1-\kappa(1-w-x))+(2\kappa(1-w)+\kappa x-1)^2}{(1-w)^2(1-\kappa(1-w+x))^2(A_0+I)^2} > 0$$

implying that the above necessary conditions are also sufficient for a maximum since the requirement for negative semi-definiteness is verified.

Let us determine that equilibrium. The equations (20) and (23) consitute a system of equations whose solutions give the open loop Nash equilibrium. Then, solving equation (20) for x and substituting the result in the equation (23) yield

$$x = w - 1 + \frac{1}{\alpha}, \tag{26}$$

$$\frac{(1+\alpha)\kappa - \alpha - \alpha\kappa w}{(\alpha-\kappa)(1-w)} + b\lambda = 0. \tag{27}$$

At steady state $\dot{\lambda} = \dot{\eta} = \dot{I} = 0$ and it follows that

$$\lambda^* = \eta^* = \frac{1}{(\delta+\rho)(A_0+I^*)}, \tag{28}$$

$$I^* = \frac{b}{\delta} w^*. \tag{29}$$

Since we know that at steady state, the costate λ is a function of w, the expression (27) can be rewritten as

$$\frac{(1+\alpha)\kappa - \alpha - \alpha\kappa w}{(\alpha-\kappa)(1-w)} + \frac{b}{(\delta+\rho)(A_0 + \frac{b}{\delta}w)} = 0. \tag{30}$$

Denote the left hand side of expression (30) as $P(w)$ which is a second-order polynomial function in w. To prove existence and uniqueness of the Open loop Nash Equilibrium, we apply jointly the intermediate values and bijection theorems on the equation (30) on the interval $]0,1[$. In particular and simple words, these theorems state that if a function f is continuous and strictly monotonic on the interval $J =]a,b[$ and if $f(a)f(b) < 0$, thus the equation $f(x) = 0$ admits a unique solution on J. This result implicitly requires that the null element belongs to the interval $]f(a), f(b)[$ for increasing function or $]f(b), f(a)[$ in case of decreasing function; while $f(a)$ and $f(b)$ have opposite sign[2]. Accordingly and from equation (30), it follows that

$$P(0) = \frac{(1+\alpha)\kappa - \alpha(\kappa-1)}{\alpha - k} + \frac{b}{A_0(\delta+\rho)}, \tag{31}$$

$$P(1) = -\infty. \tag{32}$$

Furthermore, the function $P(w)$ is strictly decreasing since

$$P'(w) = -\frac{1}{(w-1)^2} - \frac{\delta b^2}{(bw + A_0\delta)^2(\delta+\rho)} < 0, \forall w \in]0,1[\tag{33}$$

We can state that there exists one and only one solution w^* such that $0 < w^* < 1$ if and only if $P(0)P(1) < 0$. This condition requires that $P(0) > 0$, which is the case if and only if $\kappa < \dfrac{\alpha(A_0(\delta+\rho) - b)}{A_0(\delta+\rho)(1+\alpha) - b}$. For A_0 small enough, $P(0) > 0$ for all $\kappa < \alpha$. Additionally, the function $P(w)$ is strictly monotonic on the interval $]0,1[$. By this, we had proven the existence and uniqueness of the equilibrium (w^*, x^*). At steady state, the open loop Nash equilibrium is obtained by solving the system of equations (26) - (27) as follows

[2] Intermediate Values Theorem states that if a function f is continuous on the interval $J =]a,b[$, then for all real $\alpha \in]f(a), f(b)[$, there exists at least a real $\beta \in J$ such that $f(\beta) = \varepsilon$. Accordingly, the equation $f(x) = \varepsilon$ admits at least one solution on J. Furthermore, from the bijection theorem, we learn that if a function f is differentiable (thus continuous) and strictly monotonic on the interval $J =]a,b[$, then for any real $\alpha \in]f(a), f(b)[$, the equation $f(x) = \alpha$ admits one and only one solution on J.

$$w = 1 + \frac{\kappa - \alpha}{\alpha\kappa + b(\alpha - \kappa)\lambda} \tag{34}$$

$$x = w - 1 + \frac{1}{\alpha} \tag{35}$$

Replacing λ by its steady state value in the equation (34); we obtain the following stationnary solutions determining the Open-loop Nash equilibrium of this game:

$$w^* = \frac{-b^2\delta(\alpha-\kappa) - A_0\alpha\delta\kappa(\delta+\rho) + b\delta(\alpha(\kappa-1)+\kappa)(\delta+\rho) + \Theta}{2b\kappa(\delta+\rho)}, \tag{36}$$

$$x^* = w^* - 1 + \frac{1}{\alpha}. \tag{37}$$

Where

$$\Theta = \sqrt{4b\alpha\delta\kappa(\delta+\rho)(b^2(\alpha-\kappa) + A_0(\alpha(\kappa-1)+\kappa)(\delta+\rho)) + \Xi}$$

$$\text{with } \Xi = (b^2\delta(\alpha-\kappa) + A_0\alpha\delta\kappa(\delta+\rho) - b(\alpha(\kappa-1)+\kappa)(\delta+\rho))^2$$

Corollary: An increase in marginal cost α will reduce both the effort devoted by individuals to improving institutions and the amount of governmental hostility: $\frac{\partial w^*}{\partial \alpha} < 0$ and $\frac{\partial x^*}{\partial \alpha} < 0$. Furthermore, an increase in κ constrains individuals to invest more effort in civil society activities: $\frac{\partial w^*}{\partial \kappa} > 0$. It can also be noted that when the marginal cost for implementing punishment mechanisms equals one; the amount of civil monitoring effort equals exactly the amount of governmental pressure at steady state. This result is compatible to the findings of Acemoglu and Robinson (2008).

We now study the stability of this open loop nash equilibrium in the state-costate space, (I, η, λ). The equations (21), (22) and (24) constitute the dynamic system of the game and expressing them in terms of (I, λ, η) leads to:

$$\dot{I} = b\left(\frac{\alpha - \kappa}{-\alpha\kappa + \lambda b(\kappa - \alpha)} + 1\right) - \delta I, \tag{38}$$

$$\dot{\eta} = (\delta + \rho)\eta - \frac{1}{(A_0 + I)}, \tag{39}$$

$$\dot{\lambda} = (\delta + \rho)\lambda - \frac{1}{(A_0 + I)}. \tag{40}$$

It follows that the jacobian matrix $J(I^*, \eta^*, \lambda^*)$ is

$$\begin{pmatrix} -\delta & 0 & \dfrac{b(\alpha-\kappa)^2}{(\alpha\kappa+b\lambda^*(\alpha-\kappa))^2} \\ \dfrac{1}{(A_0+I^*)^2} & (\delta+\rho) & 0 \\ \dfrac{1}{(A_0+I^*)^2} & 0 & (\delta+\rho) \end{pmatrix}$$

This jacobian matrix has three real eigenvalues:

$$\xi_{1,2} = \frac{1}{2}(\rho \pm \frac{\sqrt{(A_0+I^*)^2(2\delta+\rho)^2(\alpha^2\kappa^2+b^2(\alpha-\kappa)^2\lambda^2)+\Delta}}{(A_0+I^*)(\alpha\kappa+b(\alpha-\kappa)\lambda)})$$

$$\xi_3 = \delta+\rho$$

where $\Delta = 2b(\alpha-\kappa)(-2\kappa+\alpha(2+(A_0+I^*)^2\kappa\lambda(2\delta+\rho)^2))$

It follows that the unique open loop Nash equilibrium is a stable saddle point since one eigenvalue is negative whereas the others are positive. ■

Let us now numerically determine the open loop Nash equilibrium and do sensitivity analysis for κ. We take the following values: $z = 1, A_0 = \delta = 0.1; b = 0.5$. Those values are chosen in light of conditions to be met for existence and uniqueness of the equilibrium. For the discount rate, we take $\rho = 0.08$ according to existing literature in differential games[3]. The most values used in the literature vary between 0.03 to 0.10. We will maintain this parameterization value along our work.

We consider first the simple case when $\alpha = 1$. Given the parameters of the model, figure 1 shows that the equilibrium exists and is unique since the function $P(w)$ intersects once the X-axis. An increase in κ shifts the curve $P(w)$ to the right meaning that both the civil monitoring effort and the governmental hostility increases also.

As indicated in the Table 1 in appendix, the steady state values of the institutional quality I^* always increase after an increase of the government's capacity to create rent κ. For the optimal output, there exists a threshold value $\bar{\kappa}$ under which optimal output increases. Above this value, output at steady state decreases. Therefore, for $\kappa < \bar{\kappa} = 0.5$, total factor productivity effects dominate the direct detrimental effects of the civil society activities. If $\kappa > \bar{\kappa}$, total factor productivity effets are dominated as illustrated in figure 2. In absolute value, the speed of convergence to long run equilibrium decreases meaning that institutions deteriorate as κ increases.

[3] In the literature, we can cite some authors who calibrate the discount factor. Kossioris et al (2000): 0.03; List and Mason (2001): 0.04; Kunkel and Vondem Hagen (2000): 0.08 and Piga (2000) : 0.10.

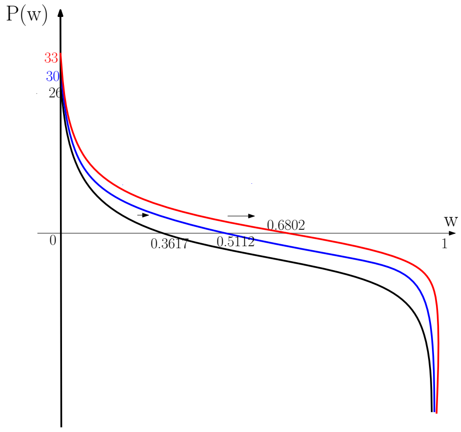

Figure 1: Sensitivity analysis of κ: from the left to the right, the three curves are such that κ equals respectively 0.1; 0.5 and 0.7;civil monitoring effort at steady state increases if κ increases.

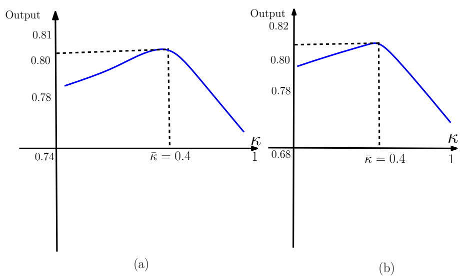

Figure 2: $\bar{\kappa}$ is the value of κ which maximizes the output. Under $\bar{\kappa}$, TFP effects dominate the direct detrimental effects of w on output.

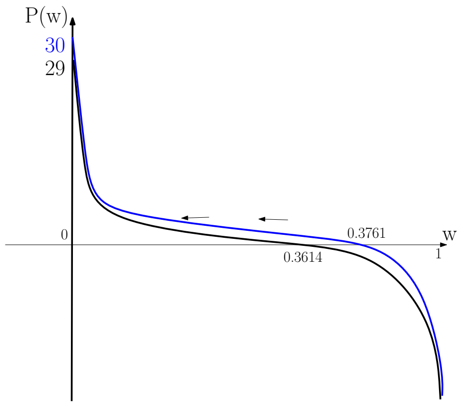

Figure 3: From the right to the left, the two curves illustrate the cases when $\alpha = 0.2$ and 1.2. Individual effort decreases as α increases.

Taking any value of α and allowing it to increase for fixed κ, namely $\kappa = 0.1$, yield following impacts (see Table 4 in appendix): Optimal civil monitoring effort as well as optimal institutional quality and output will decrease, so that total factor productivity effects dominate. Speed of convergence in absolute value will increase meaning that institutions will improve themselves. Governmental pressures decrease as it is costly to implement punishment mechanisms. These implications are plausible: consumers know that government's dignitaries support high cost in implementing pressures and accordingly they decrease their time devoted to monitoring activities. Note that when we take low or high value of κ, the results are not altered.

4 Cooperative arrangement

Intuitively, the competitive play between the government and the civil society in corruption field can lead the government to search of a negotiated solution. Indeed, by mobilizing the opinion against the corrupt government, the civil society can push this one to calm the anger of the population by cooperating with her. In a nutshell, government's fear to lose the power can increase its willingness to cooperate. Furthermore, Governance refers to the role of citizens in the policy process and how groups within a society organize to make and implement decisions on matters of great concern as pointed out by Brinkerhoff (1999). He stipulates that, in the developing countries, democratic governance will take place through the State-Civil society networks which can be defined as cross-sectoral collaborations whose purpose is to achieve convergent objectives through the combined efforts of both sets of actors, but where the respective roles and responsibilities of the actors involved remain distinct. The cooperation way is also possible in our framework since the interactions between the state and the civil society may generate positive synergistic effects in fighting rent seeking activities. The cooperative strategies are pareto optimal. Thus, the objective function to be maximized is the sum of the two individuals payoffs, or

$$max_{w,x} \int_0^\infty exp(-\rho t)\{(1-\pi)U(C_t) + \pi[U(G_t) - g(x_t)]\}dt, 0 < w < 1 \quad (41)$$

subject to (3) and where π is the cooperation weight of the Government.

The augmented lagragian function to be maximized is written as follows[4]

$$L = (1-\pi)U(C_t)+\pi[U(G_t)-g(x_t)]+\lambda_t(bw_t-\delta I_t)+\mu_{1t}w_t+\mu_{2t}(1-w_t) \quad (42)$$

Proposition 2: *Under logarithmic preferences and linear cost for implementing governemental hostility, there exists a unique Pareto strategy* (w^*, x^*) *such that* $0 < w^* < 1$ *and this stationary solution is a stable saddle point.*

Proof. Pontryagin's maximum principle yields the necessary conditions for interior solution

$$\frac{\pi(w-1) + (w-1-x)[1-2\kappa+2\kappa w - x\kappa + b(w-1)(1+(-1+w-x)\kappa)\lambda]}{(w-1)(w-1-x)(1+(w-1-x)\kappa} = 0 \quad (43)$$

$$\pi(-\alpha + \frac{1}{1-w+x}) + \frac{\kappa(\pi-1)}{1+(w-1-x)\kappa} = 0 \quad (44)$$

$$\dot{I} = bw - \delta I \quad (45)$$

$$\dot{\lambda} = (\delta+\rho)\lambda - \frac{1}{(A_0+I)} \quad (46)$$

At this stage, we look for solutions to euler equations (43) and (44). Solving the equation (44) and substituting the result in equation (43) lead to

$$x = \frac{\kappa + \alpha\pi(1+2(w-1)\kappa) + \sqrt{\pi^2\alpha(\alpha-4\kappa)+2\pi\alpha\kappa+\kappa^2}}{2\pi\alpha\kappa}, \quad (47)$$

$$\frac{1+(b\lambda-\pi\alpha)(w-1)}{w-1} = 0, \quad (48)$$

[4]Recall that $U(C_t) = ln[z_t(A_0+I_t)(-\kappa w_t^2 + (2\kappa-1)w_t + \kappa x_t w_t - \kappa x_t + 1 - \kappa)]$ and $U(G_t) = ln[\kappa z_t(A_0+I_t)(w_t^2 - 2w_t - x_t w_t + x_t - 1)]$

Solving expression (48) for w implies that

$$w = 1 + \frac{1}{b\lambda - \pi\alpha}. \tag{49}$$

Let us denote the left hand side of the equation (48) by $F(w)$. Knowing that at steady state $\lambda^* = \dfrac{1}{(\delta+\rho)(A_0+I^*)}$ and $I^* = \frac{b}{\delta}w^*$, it can be easily shown that

$$F(w) = \frac{1}{w-1} + \frac{b}{(A_0 + \frac{b}{\delta}w)(\delta+\rho)} - \pi\alpha \tag{50}$$

One can observe that $F(1) = -\infty$ and $F(0) = -1 - \pi\alpha + \frac{b}{A_0(\delta+\rho)}$. Then, applying the intermediate values and bijection theorems, we can assert that $F(0)$ and $F(1)$ have opposite sign if and only if $F(0) > 0$ which is the case for $\pi < \dfrac{b - A_0(\delta+\rho)}{A_0(\delta+\rho)\alpha}$. Furthermore, the function $F(w)$ is strictly monotonic on the interval $]0,1[$ since

$$F'(w) = -\frac{1}{(w-1)^2} - \frac{\delta b^2}{(bw + A_0\delta)^2(\delta+\rho)} < 0, \forall w \in]0,1[\tag{51}$$

Accordingly, the cooperative outcome (w^*, x^*) exists and is unique. It can be explicitly computed by solving the expression (50) for w, and using (47). After some algebra, one find the following optimal strategies under cooperation

$$w^* = \frac{-b\rho + \pi\alpha(b - A_0\delta)(\delta+\rho) + \sqrt{-4b\pi\alpha\delta(\delta+\rho)(b - A_0(\pi\alpha-1)(\delta+\rho)) + \Gamma}}{2b\pi\alpha(\delta+\rho)}, \tag{52}$$

$$x^* = \frac{\kappa + \alpha\pi(1 + 2(w^*-1)\kappa) + \sqrt{\pi^2\alpha(\alpha - 4\kappa) + 2\pi\alpha\kappa + \kappa^2}}{2\pi\alpha\kappa}, \tag{53}$$

where $\Gamma = (A_0\pi\alpha\delta(\delta+\rho) + b(\rho - \pi\alpha(\delta+\rho)))^2$.

Corollary: An increase in the cooperation weight of the government will reduce both the amount of the governmental pressure and the civil monitoring effort: $\dfrac{\partial x^*}{\partial \pi} < 0$ and $\dfrac{\partial w^*}{\partial \pi} < 0$.

Knowing (52) and (53) enables us to compute the equilibrium (w^*, I^*, λ^*). Let us now study the stability of this equilibrium in the space (I, λ). Plugging (49) in equation (45), the dynamic system (45) - (46) is expressed in terms of (I, λ) and the associated jacobian matrix $J(I^*, \lambda^*)$ is

$$\begin{pmatrix} -\delta & \dfrac{b^2}{(\pi\alpha - b\lambda^*)^2} \\ \dfrac{1}{(A_0 + I^*)^2} & (\delta + \rho) \end{pmatrix}$$

This jacobian matrix evaluated at steady state displays two eigenvalues with opposite sign. Indead, we obtain the following characteristic polynomial

$$\nu^2 - \rho\nu - \delta(\delta + \rho) - \frac{b^2}{(\pi\alpha - b\lambda^*)^2 (A_0 + I^*)^2} = 0 \tag{54}$$

which admits two eigenvalues, given by

$$\nu_{1,2} = \frac{1}{2}(\rho \pm \frac{\sqrt{(\pi\alpha - b\lambda^*)^2 (A_0 + I^*)^2 (2\delta + \rho)^2) + 4b^2}}{(\pi\alpha - b\lambda^*)(A_0 + I^*)}). \tag{55}$$

Thus, the cooperative outcome is a stable saddle point as ν_1 is negative and ν_2 positive. ■

For given parameters, numerical cooperative solution is derived. This solution is unique since the function $F(w)$ has one intersection with the X-axis on the interval $]0, 1[$. The crucial parameter is inescapably the cooperation weight π. Firstly, we set $\alpha = 1$. Figure 4 shows that an increase in π will shifts $F(w)$ to the left, logically implying that both civil monitoring effort and governmental pressure decrease as pointed out by table 3 in appendix.

The speed of convergence in absolute value increases while, however, both the optimal institutional quality and output decreases. Total factor productivity effects dominate. Let us now consider the case where α can take any positive value under fixed level of willingness of government to cooperate[5]. Following the results reported in table 4 in appendix, we see that as α increases, both the civil monitoring effort and the governmental hostility at steady state decrease. The environment becomes hostile to governmental corruption.

We observe however a decrease in optimal institutional quality and output at steady state. Total factor productivity effects dominate. In contrast the speed of convergence to long run equilibrium increase.

[5]Fixing π and ρ at low or high level do not change the trend of the results, except the magnitude of the effects.

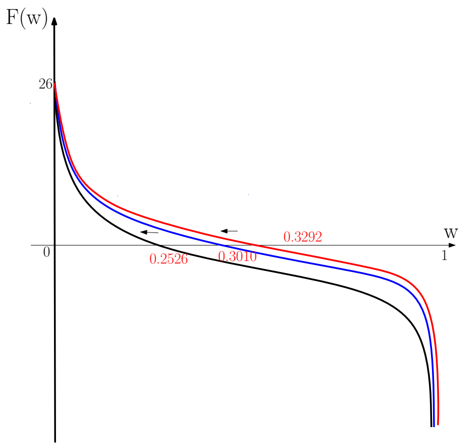

Figure 4: As π increases ($\pi = 0.1; 0.3$ and 0.7) civil monitoring effort decreases as indicated in this figure. The curve of the function $F(w)$ shifts leftward.

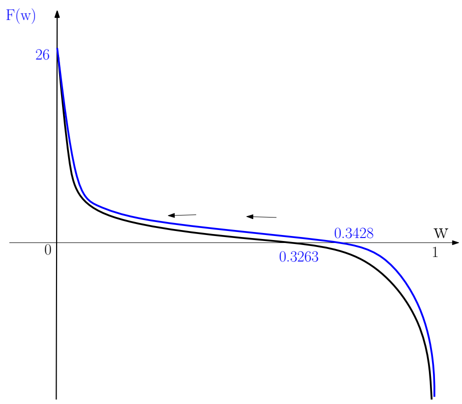

Figure 5: An increase in α (α takes the following values: 0.2 and 1.2) will decrease the amount individuals devote to civil society activities. Again, the curve of the function $F(w)$ shifts leftward.

5 Feedback Nash Equilibrium

In this section, we analyze the game by deriving the feedback strategies for the two control variables. Recall that the important feature of a feedback Nash equilibrium is that it verifies the time consistency property which corresponds to subgame perfectness (Basar and Olsder, 1982). Thus, we have to solve the Hamilton-jacobi-Bellman equations, as in dynamic programming, rather than the hamiltonian maximizing conditions.Then, the pair of strategies (w^*, x^*) in the feedback equilibrium must satisfy the following set of Bellman's equations:

$$\rho V_1(I) = max_w\{U(C_t) + V_1'(I)[bw - \delta I]\} \tag{56}$$
$$\rho V_2(I) = max_x\{U(G_t) - g(x_t) + V_2'(I)[bw - \delta I]\} \tag{57}$$

where $V_i(I)$ is the value function of the two players (Representative consummer and Government); $i = 1, 2$

Taking the first order conditions by maximizing the right hand side of the equations (56) and (57), and rearranging respectively yield[6]

$$w^* = 1 + \frac{\kappa - \alpha}{\alpha\kappa + b(\alpha - \kappa)V_1'} \tag{58}$$
$$x^* = \frac{1}{\alpha} + \frac{\kappa - \alpha}{\alpha\kappa + b(\alpha - \kappa)V_1'} \tag{59}$$

Substituting (58) and (59) into (56) and (57), we obtain:

$$\rho V_1(I) = ln[\frac{A(\alpha - \kappa)}{\alpha(\alpha\kappa + b(\alpha - \kappa)V_1'}] + V_1'[b(1 + \frac{\alpha - \kappa}{\alpha\kappa + b(\alpha - \kappa)V_1'}) - \delta I] \tag{60}$$
$$\rho V_2(I) = ln[\frac{A\kappa(\alpha - \kappa)}{\alpha^2\kappa + b\alpha(\alpha - \kappa)V_1'}] - (1 + \frac{\alpha(\kappa - \alpha)}{\alpha\kappa + b(\alpha - \kappa)V_1'}) + V_2'[b(1 + \frac{\kappa - \alpha}{\alpha\kappa + b(\alpha - \kappa)V_1'}) - \delta I] \tag{61}$$

The expressions (60) and (61) constitute a system of equations whose unknowns are the value functions $V_i(I)$. Once these are found, it becomes possible to derive the optimal equilibrium strategies by substituting the value functions into (58) and (59). Let us propose the following linear value function

$$V_i(I) = \gamma_i I, \forall i = 1, 2 \tag{62}$$

[6]Again, recall $U(C_t) = ln[z_t(A_0 + I_t)(-\kappa w_t^2 + (2\kappa - 1)w_t + \kappa x_t w_t - \kappa x_t + 1 - \kappa)]$ and $U(G_t) = ln[\kappa z_t(A_0 + I_t)(w_t^2 - 2w_t - x_t w_t + x_t - 1)]$.

implying that

$$V_i'(I) = \gamma_i \tag{63}$$

Given the formulation of expressions (58) and (59), only the Bellman equation (60) is relevante for determining the feedback Nash strategies at equilibrium. We can ignore the Bellman equation (61) since feedback startegies don't depend on γ_2. Indead, plugging expression (63) in equation (58) and (59) , feedback stategies candidates become[7]

$$w^* = 1 + \frac{\kappa - \alpha}{\alpha\kappa + b(\alpha - \kappa)\gamma_1}, \tag{64}$$

$$x^* = \frac{1}{\alpha} + \frac{\kappa - \alpha}{\alpha\kappa + b(\alpha - \kappa)\gamma_1}, \tag{65}$$

We can easily show that our dynamic programming equations have solution iff $\alpha > \kappa$ and $\gamma > \frac{\alpha\kappa}{b(\kappa-\alpha)}$ since $0 < w^* < 1$

To determine γ, we must substitute the expressions (62) and (63) in equation (60). We obtain

$$\rho\gamma_1 I = ln(A_0+I)+ln[\frac{z(\alpha-\kappa)^2}{\alpha(\alpha\kappa - b(\alpha-\kappa)\gamma_1)}]+\gamma_1[b(1+\frac{\kappa-\alpha}{\alpha\kappa+b(\alpha-\kappa)\gamma_1})-\delta I]. \tag{66}$$

As we restricted ourselves on local analysis, we compute the value function by using a linear aprroximation method[8] around the steady state value I^s. After some algebra, the equation (66) can be rewritten as

$$(\delta+\rho)\gamma I^s - b\gamma = \frac{I^s}{A_0} + ln(z(\alpha-\kappa)^2) - ln(\alpha^2\kappa - b\alpha(\kappa-1)\gamma) + \frac{b\gamma(\kappa-\alpha)}{\alpha\kappa+b(\alpha-\kappa)\gamma}. \tag{67}$$

The steady state value I^s can be deduced from the law of motion (3) and the expression (58). Then, we can easily show that I^s depends on γ and is equal to

$$I^s = \frac{b}{\delta}(1 + \frac{\kappa-\alpha}{\alpha\kappa + b(\alpha-\kappa)\gamma}) \tag{68}$$

[7]In the following lines, we remove the subscript one to the weight γ even if for the rest of the work we keep in mind that γ means γ_1.

[8]See appendix 1 for more details

Combining the two previous relations and rearranging terms with respect to γ yields:

$$\frac{b\rho}{\delta}\gamma+\frac{b\rho A_0(\kappa-\alpha)\gamma-b(\kappa-\alpha)}{\delta A_0[b(\alpha-\kappa)\gamma+\alpha\kappa]}-\frac{b}{\delta A_0}-ln(z(\alpha-\kappa)^2)+ln(\alpha^2\kappa+b\alpha(\alpha-\kappa)\gamma)=0 \tag{69}$$

The Left Hand Side of this expression can be formulated as a function of γ as follows:

$$P(\gamma)=A\gamma+\frac{B\gamma+H}{C\gamma+D}-E+Log(F+G\gamma) \tag{70}$$

Where

$A=\frac{b\rho}{\delta}, B=b\rho A_0(\kappa-\alpha), C=b\delta\alpha A_0(\alpha-\kappa), D=\delta\alpha A_0\kappa$
$E=\frac{b}{\delta A_0}+Log[z(\alpha-\kappa)^2], F=\alpha^2\kappa, G=b\alpha(\alpha-\kappa)$ and $H=-b(\kappa-\alpha)$

The function $P(\gamma)$ is well defined if and only if $\gamma\neq\frac{-D}{C}=\frac{\alpha\kappa}{b(\kappa-\alpha)}$ and $\gamma>\frac{-F}{G}=\frac{\alpha\kappa}{b(\kappa-\alpha)}$. Recall that γ is the first derivative of the value function with respect to state variable corresponding to the costate variable. In our case, we impose a nonnegativity restriction on γ. Thus, the above implies that $\gamma\in[0;+\infty[$.

Proposition 3: *For any $\gamma\in]0,+\infty[$, the game displays multiple feedback Nash equilibria corresponding to linear Markov Subgame perfect equilibria.*

Proof. To prove existence of multiple feedback equilibria, it is convenient to use again the intermediate values and bijection theorems to refute uniqueness.

We can easily show that the function $P(\gamma)$ is continuous on the interval $]0,+\infty[$ and

$$lim_{\gamma\to 0}P(\gamma)=\frac{b(\alpha-\kappa)}{\delta A_0\alpha\kappa}-\frac{b}{\delta A_0}-ln[(\alpha-\kappa)^2+ln(\alpha^2\kappa), \tag{71}$$

$$lim_{\gamma\to\infty}P(\gamma)=+\infty. \tag{72}$$

The function $P(\gamma)$ admits at least one solution if and only if $P(0)<0$ and $P(\gamma)$ is monotonic. Let us check whether the monotonicity property is verified. The first derivative of $P(\gamma)$ with respect to γ is given by

$$P'(\gamma)=\frac{b(A_0b^2\gamma^2(\alpha-\kappa)^2\rho+A_0\alpha\kappa(\kappa(\rho-\delta)+\alpha(\delta+(\kappa-1)\rho))+\Omega}{A_0\delta(b\gamma(\alpha-\kappa)+\alpha\kappa)^2} \tag{73}$$

where $\Omega = b(\alpha - \kappa)(\kappa - A_0\gamma\delta\kappa + \alpha(-1 + A_0\gamma(\delta + 2\kappa\rho)$. The partial derivative P' vanishes at following critical point[9]

$$\gamma^* = \frac{-\delta(\alpha-\kappa) - 2\alpha\kappa\rho + \frac{\sqrt{(\alpha-\kappa)(A_0\delta^2(\alpha-\kappa)+4b\rho(\alpha-\kappa)+4A_0\alpha\kappa\rho^2)}}{\sqrt{A_0}}}{2b\rho(\alpha-\kappa)} \tag{74}$$

It follows that $P(\gamma)$ is non-monotonic. Furthemore, it can be easily shown that the second derivative

$$P''(\gamma) = -\frac{b^2(\alpha-\kappa)^2(b(A_0\gamma\delta-2)(\alpha-\kappa)+A_0\alpha\kappa(\delta-2\rho))}{A_0\delta(b\gamma(\alpha-\kappa)+\alpha\kappa)^3} \tag{75}$$

is always positive on the interval $[0, +\infty[$. Then, the critical point is a minimum. Since the function $P(\gamma)$ is non-monotonic, there is a possibility of multiple equilibria as it shall intersect at least twice the X-axis. We will compute accurately these equilibria under numerical assessment. Once we find the expression of $\bar{\gamma}$, we derive the feedback Nash strategies at equilibrium from expressions (58) and (59). These policy functions can be rewritten as[10]

$$w^{FNE} = 1 + \frac{\kappa-\alpha}{\alpha\kappa + b(\alpha-\kappa)\bar{\gamma}} \tag{76}$$

$$x^{FNE} = \frac{1}{\alpha} + \frac{\kappa-\alpha}{\alpha\kappa + b(\alpha-\kappa)\bar{\gamma}} \tag{77}$$

and the steady state values of the institutional quality index and output are given by

$$I^{FNE} = \frac{b}{\delta}(1 + \frac{\kappa-\alpha}{b(\alpha-\kappa)\bar{\gamma}+\alpha\kappa}), \tag{78}$$

$$Y^{FNE} = z(A_0 + I^{FNE})(1 - w^{FNE}). \tag{79}$$

Since we can not explicitly compute the expression of $\bar{\gamma}$, we determine it numerically. We distinguish two major cases. First, we impose $\alpha = 1$ and we conduct a sensitivity analusis κ, and secondly, we consider any $\alpha > 0$ and we do sensitivity analysis of α.

The function $P(\gamma)$ is none monotonic and may intersect twice the X-axis. In figure 5, we observe the possibility of having two feedback nash equilibria:

[9] We rejected the negative root because it doesn't belong into the domain of the function $P(\gamma)$.

[10] Particular case: if $\alpha = 1$, $w^{FNE} = x^{FNE}$

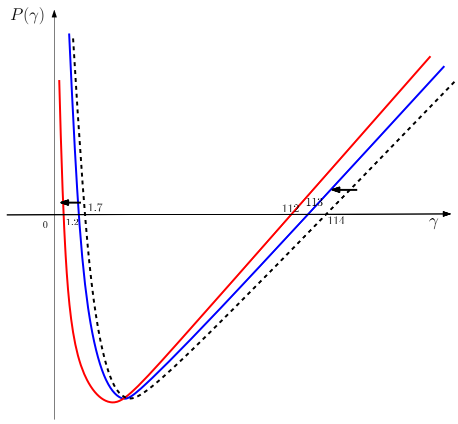

Figure 6: An increase in κ ($\kappa = 0.1; 0.3$ and 0.5) will reduce both the weight $\bar{\gamma}^L$ and $\bar{\gamma}^H$ but with small impact in decrease of $\bar{\gamma}^H$. For $\kappa = 0.1; 0.3$ and 0.5, the pair $(\bar{\gamma}^L; \bar{\gamma}^H)$ respectively equals (1.78; 114.43), (1.16; 113.79) and (0.029; 112.93). At low equilibrium, the amount of effort individuals devote to civil society actions increases while it remains constant at high equilibrium.

a low equilibrium associated with small $\bar{\gamma}$ and high one linked to high value of $\bar{\gamma}$ (Henceforth, $\bar{\gamma}^L$ and $\bar{\gamma}^H$)

When consumers choose the low (high) equilibrium as the best value of the objective function, they devote less (high) effort to improving institutions. We observe that a positive shock on κ induces the civil society to increase its monitoring effort to fight against corruption at both two steady state levels. Furthermore, we observe an increase in institutional quality index while output increases at low equilibrium and decreases at high equilibrium. Total factor productivity effects dominate in the first situation and are dominated in the second one (see table 5 in appendix).

When we take any positive value of α, it is also possible to have multiple feedback Nash equilibria. Namely, two feedback nash equilibria shall arise since the function $P(\gamma)$ will intersect twice the X-axis. This is depicted in figure 6.

As α increases, optimal values of civil monitoring effort and institutional index decrease for both two scenarios. In contrast, optimal output at low feedback equilibrium increases and decreases at high steady state. Total factor productivity effects dominate the direct detrimental effect at low equilibrium and are dominated at high equilibrium. Governmental hostility decreases (See table 6 in the appendix).

Let us now select the best equilibrium. To proceed, we assume that the

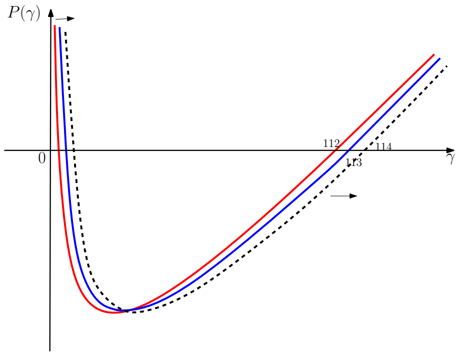

Figure 7: An increase in α ($\alpha = 0.2; 0.4$ and 1) increases both the weight $\bar{\gamma}^L$ and $\bar{\gamma}^H$. $\bar{\gamma}^L$ is near the origin. For $\alpha = 0.2; 0.4; 1$, the pair $(\bar{\gamma}^L, \bar{\gamma}^H)$ is equivalent to (1.63; 112.96), (1.74; 113.97) and (1.78; 114.43) respectively. The civil monitoring effort associated to $\bar{\gamma}^L$ decreases while the one associated to $\bar{\gamma}^H$ remains unchanged.

best equilibrium corresponds to the one which have high value function at steady state. After fixing α to one or allowing it to vary and fixing κ, the best feedback Nash equilibrium is the one associated to high weight $\bar{\gamma}^H$ since $V_1^H(I^*) > V_1^L(I^*)$. Hence, the best choice for consumer is $\bar{\gamma}^H$ despite the fact she devotes almost all her time to monitoring activities. However, at $\bar{\gamma}^H$ the steady state value of the institutional quality index is higher than it is at $\bar{\gamma}^L$.

6 Concluding remarks

This paper has presented a simple institutional dynamic game under nonquadratic preferences. In reality, preferences are not always quadratic. The novelty of this work is that we can handle an important class of differential games under nonquadratic preferences. We demonstrated analytically existence and uniqueness of both open loop and cooperative outcome as benchmarks. By taking logarithmic preferences and using a linear approximation around the steady state, we have solved analytically and numerically our dynamic game for linear feedback Nash equilibria. We found that multiple (at least two) feedback Nash equilibria shall arise. In the general case, it is hard to compare those solutions since the parameters taken into account are not the same in all three situations. Nevertheless, we compared the economic implications inherent to these three cases. under cooperative play the magnitude of the civil monitoring effort is lower than the one in open loop game. This in turn is smaller than the magnitude of effort associated to the best feedback equilibrium. Total factor productivity effects always dominate the detrimental effect of individual's effort to production in all situations, except the case $\alpha = 1$ where there exists a threshold value $\bar{\kappa}$ beyond which TFP effects are dominated. Institutions improve much faster under cooperative scenario than in open loop game since the speed of convergence to the long run equilibrium in absolute value is always greater under cooperation.

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Appendix 1

Without loss of generality, equation (66) can be rewritten as

$$(\rho+\rho)\gamma_1(I-I^s+I^s) = ln(A_0+I-I^s+I^s)+\chi. \tag{80}$$

where $\chi = ln[\frac{z(\alpha-\kappa)^2}{\alpha(\alpha\kappa-b(\alpha-\kappa)\gamma_1)}]+b\gamma_1+\frac{b\gamma_1(\kappa-\alpha)}{\alpha\kappa+b(\alpha-\kappa)\gamma_1}]$. Assuming that $(I-I^s)$ is small enough and the ratio $\frac{I^s}{A_0}$ is small, we can easily show that the equation (80) becomes

$$(\rho+\rho)\gamma_1 I^s = \frac{I^s}{A_0}+ln[\frac{z(\alpha-\kappa)^2}{\alpha(\alpha\kappa-b(\alpha-\kappa)\gamma_1)}]+b\gamma_1+\frac{b\gamma_1(\kappa-\alpha)}{\alpha\kappa+b(\alpha-\kappa)\gamma_1}] \tag{81}$$

Substituting (68) in (81) and rearranging leads to the expression (69).

Appendix 2

Table 1: Open loop Nash equilibrium - Sensitivity analysis of κ under the case $\alpha = 1$

κ	$w^* = x^*$	I^*	y^*	λ^*	v_1
0.1	0.3617	1.8084	1.2182	2.9110	-0.2123
0.3	0.4144	2.0719	1.2719	2.5580	-0.1380
0.5	0.5112	2.5560	1.2983	2.0917	-0.0943
0.7	0.6802	3.4009	1.1197	1.5869	-0.0944

Table 2: Open loop Nash equilibrium - Sensitivity analysis of α under the case $\kappa = 0.1$

α	w^*	x^*	I^*	y^*	λ^*	v_1
0.2	0.3761	4.3761	1.8802	1.2356	2.8054	-0.2231
0.4	0.3652	1.8652	1.8262	1.2227	2.8842	-0.2318
0.6	0.3631	1.0298	1.8155	1.2199	2.9002	-0.2336
0.7	0.3626	0.7911	1.8129	1.2193	2.9043	-0.2341
1	0.3617	0.3617	1.8084	1.2182	2.9110	-0.2348
1.2	0.3614	0.1947	1.8068	1.2178	2.9135	-0.2351

Table 3: Cooperative outcome - Sensitivity analysis of π under the case $\kappa = 0.1$ and $\alpha = 1$

π	w^*	x^*	I^*	y^*	λ^*	υ_1
0.1	0.3292	18.8161	1.6461	1.1713	3.1816	-0.1977
0.2	0.3148	13.6155	1.5740	1.1470	3.3188	-0.2080
0.3	0.3010	11.8366	1.5051	1.1219	3.4613	-0.2189
0.5	0.2755	10.3745	1.3774	1.0704	3.7604	-0.2424
0.7	0.2526	9.7264	1.2630	1.0187	4.0759	-0.2678

Table 4: Cooperative outcome - Sensitivity analysis of α under $\pi = 0.1$ and $\kappa = 0.1$

α	w^*	x^*	I^*	y^*	λ^*	υ_1
0.1	0.3428	108.4260	1.7138	1.1921	3.0630	-0.1890
0.2	0.3412	58.4960	1.7061	1.1898	3.0759	-0.1899
0.4	0.3382	33.6087	1.6909	1.1853	3.1020	-0.1918
0.6	0.3352	25.3615	1.6759	1.1807	3.1283	-0.1937
1	0.3292	18.8161	1.6461	1.1713	3.1816	-0.1977
1.2	0.3263	17.1932	1.6314	1.1665	3.2086	-0.1997

Table 5: Feedback Nash Equilibria - Sensitivity analysis of κ under the case $\alpha = 1$

κ	γ_1	γ_2	$w_1^* = x_{1^*}$	$w_2^* = x_2^*$	I_1	I_2	y_1	y_2
0.1	1.7822	114.43	0.0022	0.9826	0.0109	4.9127	0.1107	0.0874
0.2	1.5093	114.131	0.0046	0.9826	0.0230	4.9128	0.1225	0.0874
0.3	1.1577	113.791	0.0073	0.9826	0.0367	4.9128	0.1357	0.0874
0.4	0.6879	113.397	0.0105	0.9826	0.0524	4.9128	0.1508	0.0873
0.5	0.0287	112.929	0.0142	0.9826	0.0708	4.9128	0.1683	0.0872

Table 6: Feedback Nash Equilibria - Sensitivity analysis of α under the case $\kappa = 0.1$

α	γ_1	γ_2	w_1^*	x_{1^*}	w_2^*	x_2^*	I_1	I_2	y_1	y_2
0.2	1.6291	112.961	0.0143	4.0143	0.9824	4.9824	0.0717	4.9118	0.1692	0.0884
0.4	1.7453	113.975	0.0059	1.5059	0.9825	2.4825	0.0298	4.9125	0.1290	0.0877
0.6	1.7676	114.238	0.0038	0.6704	0.9825	1.6492	0.0189	4.9126	0.1184	0.0876
0.7	1.7731	114.308	0.0032	0.4318	0.9825	1.4111	0.0159	4.9127	0.1156	0.0875
1	1.7822	114.43	0.0022	0.0022	0.9826	0.9826	0.0109	4.9128	0.1107	0.0874

Institut de Recherches Économiques et Sociales
Université catholique de Louvain

Place Montesquieu, 3
1348 Louvain-la-Neuve, Belgique

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