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LEAST ORTHOGONAL DISTANCE ESTIMATION OF SIMULTANEOUS EQUATIONS: A SIMULATION EXPERIMENT

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DIPARTIMENTO DI ECONOMIA

LEAST ORTHOGONAL DISTANCE ESTIMATION OF SIMULTANEOUS EQUATIONS: A SIMULATION EXPERIMENT

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Least Orthogonal Distance Estimation of simultaneous equations: a simulation experiment

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Abstract: the aim of this work is to estimate the structural parameters of a simultaneous equation system using both the Limited and Full Information Least Orthogonal Distance Estimator (Pieraccini, 1988; Naccarato, 2007). We compare the results - via simulation experiments – of LODE estimates with those obtained by other methods (Maximum Likelihood, Least Squares). LODE estimators appear to be unbiased and (nearly always) more efficient.

Keywords: Simultaneous equations models, Orthogonal distance, Principal Components.

Introduction

This paper aims at evaluating the features of the Least Orthogonal Distance Estimator (LODE) for the structural parameters of simultaneous equations systems (Pieraccini, 1988, Naccarato, 2007). Such evaluation has been conducted by comparing the results of LODE with Least Squares and Maximum Likelihood.

In literature there are two main approaches to this kind of comparison: analytical (that focuses on searching the theoretical distribution of parameter estimators), or computational (based on Monte Carlo simulations).

As is well known, the difficulty in simultaneous equations estimation is the nonlinear relationship between Reduced Form (RF) and Structural Form (SF) coefficients. Least Squares, as well as Maximum Likelihood derives estimators under the hypothesis of identification restrictions. Thus the analytical approach refers to models that satisfy some sort of identification restrictions. This makes the analytical results unsuitable for more general applications.

The computational approach it is suited to handle more general models. It consists in choosing a model and assuming one or more structures by assigning specific numerical values to the parameters and to the variance-covariance matrix of the SF errors. Subsequently, samples of different sizes are extracted from the assumed error distribution and from each of the predetermined structures. Exogenous variable are generated randomly and vary with each sample.

In this paper we show, by means of a computational approach, that LODE estimators perform better than Least Squares and Maximum Likelihood estimators. In particular we compare Limited Information LODE with 2SLS and LIML, and Full Information LODE with 3SLS and FIML.

The outline of the paper is the following. After a brief introduction on the estimation of systems of simultaneous equations (§ 1) and the LODE estimator (§ 2), we describe the plan of experiments (§ 3). We discuss the results in § 4, mainly that LODE estimator is unbiased and although not efficient it performs as well as (or not worse than) the other estimators we have considered. Finally in § 5 we draw some conclusions and suggest future developments.

1. The simultaneous equations model

Making use of standard notations, the structural form of a simultaneous equations model can be defined as follows:

$$\underset{n,m}{Y}\,\underset{m,m}{\Gamma} + \underset{n,k}{X}\,\underset{k,m}{\mathrm{B}} + \underset{n,m}{U} = \underset{n,m}{0} \tag{1}$$

where Y is the $n \times m$ matrix of endogenous variables and Γ is the corresponding $m \times m$ matrix of structural parameters, X is the $n \times k$ matrix of exogenous variables and B is the $k \times m$ matrix of their structural parameters. Finally U is the $n \times m$ matrix of disturbances for which standard hypotheses are supposed to hold:

$$\begin{aligned} &E(vecU) = 0 \\ &E\left(vecU(vecU)^T\right) = \Omega \otimes I \end{aligned} \tag{2}$$

where

$$\underset{m,m}{\Omega} = \begin{bmatrix} \sigma_1^2 & \cdots & \sigma_{1m} \\ \vdots & \ddots & \vdots \\ \sigma_{m1} & \cdots & \sigma_m^2 \end{bmatrix}$$

is the variance-covariance matrix of the disturbances U, constant for all the observations.

Furthermore it is generally assumed that:

$$\begin{aligned} & p\lim_{n\to\bullet}\frac{1}{n}U^T U = \Omega \\ & p\lim_{n\to\bullet}\frac{1}{n}X^T U = \underset{k,m}{0} \\ & p\lim_{n\to\bullet}\frac{1}{n}X^T X = \underset{k,k}{\Sigma_x}. \end{aligned} \tag{3}$$

Under non singularity condition for Γ the *reduced form* of the equations is derived as:

$$\underset{n,m}{Y} = \underset{n,k}{X}\,\underset{k,m}{\Pi} + \underset{n,m}{V} \tag{4}$$

where:

$$\begin{aligned} \underset{k,m}{\Pi} &= -\underset{k,m}{\mathrm{B}}\,\underset{m,m}{\Gamma^{-1}} \\ \underset{n,m}{V} &= -\underset{n,m}{U}\,\underset{m,m}{\Gamma^{-1}}. \end{aligned} \tag{5}$$

The last equation in (5) represents the matrix of reduced form disturbances, for which it is possible to write:

$$\begin{aligned} & E(V) = 0 \\ & E(V^T V) = n\left(\Gamma^{-1}\right)^T \Omega\,\Gamma^{-1}. \end{aligned} \tag{6}$$

Post-multiplying by Γ the first equation in (5) we obtain:

$$\underset{k,m}{\Pi}\ \underset{m,n}{\Gamma} = -\underset{k,m}{\mathrm{B}} \tag{7}$$

which represents the relation between reduced and structural form parameters.

Since (7) is a system of k equations with $m+k$ unknowns, usual exclusion constraints are introduced in order to find the solution with respect to Γ and B in terms of Π.

If – as it usually happens – each equation does not include all the endogenous and exogenous variables, it is possible to consider the following partition of the overall matrix of endogenous variables with respect to *i-th* structural form equation:

$$\underset{n,m}{Y} = \left[\underset{n,m_{1i}}{Y_{1i}} \quad \vdots \quad \underset{n,m_{2i}}{Y_{2i}} \right]$$

where the first m_{1i} columns refer to the endogenous variables included in *i-th* equation and the last m_{2i} columns refer to those excluded. In the same way the vectors of Γ's in *i-th* equation can be reordered as:

$$\underset{m,1}{\Gamma_i} = \begin{bmatrix} \underset{m_{1i,1}}{\Gamma_{1i}} \\ \cdots \\ \underset{m_{2i,1}}{0} \end{bmatrix}$$

where the first m_{1i} elements of Γ_i refer to endogenous variables included in the *i-th* equation. Notice that defining the vector Γ_i no normalization rule has yet been introduced.

Similarly, let us consider the partition:

$$\underset{n,k}{X} = \left[\underset{n,k_{1i}}{X_{1i}} \quad \vdots \quad \underset{n,k_{2i}}{X_{2i}} \right]$$

where X_{1i} and X_{2i} are the sub-matrices corresponding to the exogenous variables included in and excluded from the *i-th* equation respectively. Accordingly let us define

$$\underset{k,1}{\mathrm{B}_i} = \begin{bmatrix} \underset{k_{1i},1}{\mathrm{B}_{1i}} \\ \cdots \\ \underset{k_{2i},1}{0} \end{bmatrix}$$

where the first k_{1i} parameters are related to the exogenous variables included in the *i-th* equation.

Therefore the *i-th* structural equation can be expressed as:

$$Y_{1i}\Gamma_{1i} + X_{1i}\mathrm{B}_{1i} = U_i .$$

Notice that different orderings of variables correspond to each equation of the system.

2. Limited information and full information LODE

LODE estimator is – in its original formulation – a limited information method, i.e. an estimator equation by equation of structural parameters (Pieraccini, 1988). Since it is well known that Full Information estimators are asymptotically more efficient than Limited Information ones, (Goldberger, 1964, pp. 346-356, Judge et al., 1985) it is worthwhile to generalize LODE method to a full information context.

Defining:

$$\underset{k,m_{1i}+k_{1i}}{\hat{\Pi}^i_*} = \begin{bmatrix} \underset{k_{1i},m_{1i}}{\hat{\Pi}_{1i}} & I_{k_{1i}} \\ \underset{k_{2i},m_{1i}}{\hat{\Pi}_{2i}} & \underset{k_{2i},k_{1i}}{0} \end{bmatrix}, \quad \underset{m_{1i}+k_{1i},1}{\delta_i} = \begin{bmatrix} \underset{m_{1i},1}{\Gamma_{1i}} \\ \underset{k_{1i},1}{\mathrm{B}_{1i}} \end{bmatrix}, \quad \underset{k,1}{\varepsilon_i} = \begin{bmatrix} \underset{k_{1i},1}{\varepsilon_{1i}} \\ \underset{k_{2i},1}{\varepsilon_{2i}} \end{bmatrix},$$

we have:

$$\hat{\Pi}^i_* \delta_i = \varepsilon_i \tag{8}$$

where:

$$\underset{k,1}{\varepsilon_i} = \left(X^T X\right)^{-1} X^T U_i \,. \tag{9}$$

Limited information LODE is given by the vector δ_i which minimizes the following quadratic form:

$$\sigma_i^2 \delta_i^T \hat{\Pi}_*^{i\,T} \left(X^T X\right) \hat{\Pi}_*^i \delta_i \tag{10}$$

and it is then given by the eigenvector associated to the smallest eigenvalue of the matrix $\sigma_i^2 \hat{\Pi}_*^{i\,T} \left(X^T X\right) \hat{\Pi}_*^i$, divided by the element corresponding to the endogenous variable at r. h. s. in the SF equation after introducing the normalization rule.

It can be easily shown that (10) reduces to:

$$\underset{m_{1i}+k_{1i},k}{\hat{\Pi}_*^{i\,T}} \; \sigma^{ii} \underset{k,k}{\left(X^T X\right)} \; \underset{k,m_{1i}+k_{1i}}{\hat{\Pi}_*^i} = \sigma^{ii} \begin{bmatrix} \underset{m_{1i},m_{1i}}{Y_{1i}^T X \left(X^T X\right)^{-1} X^T Y_{1i}} & \underset{m_{1i},k_{1i}}{Y_{1i}^T X_{1i}} \\ \underset{k_{1i},m_{1i}}{X_{1i}^T Y_{1i}} & \underset{k_{1i},k_{1i}}{X_{1i}^T X_{1i}} \end{bmatrix} = \underset{m_{1i}+k_{1i},m_{1i}+k_{1i}}{A_{ii}} \tag{11}$$

(where the meaning of the symbol A_{ii} will become clear in few lines), and to

$$\sigma_i^2 \delta_i^T A_{ii} \delta_i \tag{12}$$

so that LODE estimator $\hat{\delta}_i$ is defined in terms of the eigenvalues and eigenvectors of matrix A_{ii}. Notice that σ_i^2, being a constant, does not influence the minimization of the quadratic form (12).

Relations between reduced and structural form parameters for the whole system of equations are given by:

$$\begin{bmatrix} \underset{k,m_{11}+k_{11}}{\hat{\Pi}_*^1} & \underset{k,m_{12}+k_{12}}{0} & \cdots & \underset{k,m_{1m}+k_{1m}}{0} \\ \underset{k,m_{11}+k_{11}}{0} & \underset{k,m_{12}+k_{12}}{\hat{\Pi}_*^2} & \cdots & \underset{k,m_{1m}+k_{1m}}{0} \\ \vdots & \vdots & \ddots & \vdots \\ \underset{k,m_{11}+k_{11}}{0} & \underset{k,m_{12}+k_{12}}{0} & \cdots & \underset{k,m_{1m}+k_{1m}}{\hat{\Pi}_*^m} \end{bmatrix} \begin{bmatrix} \underset{m_{11}+k_{11,1}}{\delta_1} \\ \underset{m_{12}+k_{12,1}}{\delta_2} \\ \vdots \\ \underset{m_{1m}+k_{1m,1}}{\delta_1} \end{bmatrix} = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_m \end{bmatrix} \tag{13}$$

or in a more compact form, using a self evident notation:

$$\underset{mk,s}{\hat{\Pi}_*} \underset{s,1}{\delta} = \underset{mk,1}{\varepsilon} \tag{14}$$

where $s = \sum_{i=1}^{m} (m_{1i} + k_{1i})$.

From equation (9) applied to the vector ε, the variance-covariance matrix of the error component is:

$$E(\varepsilon\varepsilon^T) = \underset{mk,mk}{\Sigma} = \underset{m,m}{\Omega} \otimes \left(\underset{k,k}{X^T X} \right)^{-1}. \tag{15}$$

Full Information LODE is obtained by minimizing the quadratic form:

$$\underset{1,S}{\delta^T} \underset{S,mk}{\hat{\Pi}_*^T} \underset{mk,mk}{\left(\Omega \otimes \left(X^T X\right)^{-1}\right)^{-1}} \underset{mk,S}{\hat{\Pi}_*} \underset{S,1}{\delta} = \delta^T \hat{\Pi}_*^T \left(\Omega^{-1} \otimes \left(X^T X\right)\right) \hat{\Pi}_* \delta \tag{16}$$

i.e. by considering the eigenvector associated with the smallest eigenvalue of the matrix:

$$\underset{S,S}{A} = \hat{\Pi}_*^T \left(\Omega^{-1} \otimes \left(X^T X\right)\right) \hat{\Pi}_* \tag{17}$$

The block-diagonal elements of $\underset{S,S}{A}$ are of the form (11) – now it is clear the reason for using the proposed notation – whereas the extra-diagonal block elements are:

$$A_{ij} = \sigma^{ij} \begin{bmatrix} \underset{m_{1i}, m_{1j}}{Y_{1i}^T X \left(X^T X\right)^{-1} X^T Y_{1j}} & \underset{m_{1i}, k_{1j}}{Y_{1i}^T X_{1j}} \\ \underset{k_{1i}, m_{1j}}{X_{1i}^T Y_{1j}} & \underset{k_{1i}, k_{1j}}{X_{1i}^T X_{1j}} \end{bmatrix}. \tag{18}$$

The eigenvector associated with the smallest eigenvalue of matrix $\underset{S,S}{A}$ will then minimize the quadratic form (16).

Full Information LODE is given by this eigenvector multiplied through m constants defined as the reciprocal of the elements corresponding to the endogenous variables at right hand sides in each SF equation.

It has to be noticed that Full Information LODE could have computational advantages with respect to FIML which, in non standard problems, converges slowly to solutions or may achieve a local maximum instead of the absolute one.

Equation (16), which defines explicitly the quadratic form to be minimized, is a function of disturbances variance-covariance matrix Ω which is unknown. Then it is necessary to estimate it.

As usual it is possible to go through a two stage procedure: in the first stage estimates of the parameters are obtained through Limited Information LODE and used to calculate $\hat{U}$ i. e. the matrix of disturbances of SF:

$$\hat{U} = -\hat{V}\,\hat{\Gamma}$$

where $\hat{V}$ is the matrix of residuals of OLS estimators of RF equations.

In the second stage structural parameters estimates are obtained introducing $\hat{\Omega}$ in equation (16). Then Full Information LODE is proportional to the eigenvector associated to the smallest eigenvalue of :

$$\hat{A} = \hat{\Pi}_*^T \left(\hat{\Omega}^{-1} \otimes \left(X^T X\right)\right) \hat{\Pi}_*$$

It is possible to prove that Full Information LODE consistently estimates the parameters of the structural form (Naccarato, 2007).

3. The design of the experiment

The simulation experiment has been conducted using the three equation model proposed by Cragg in 1967:

$$\begin{cases} y_1 = -0.89y_2 - 0.16y_3 + 44 + 0.74x_2 + 0.13x_5 \\ y_2 = -0.74y_1 + 62 + 0.70x_3 + 0.96x_5 + 0.06x_7 \\ y_3 = -0.29y_2 + 40 + 0.53x_3 + 0.11x_4 + 0.56x_6 \end{cases}$$

The three equations show a different degree of parameter over-identification. Such a feature, as well as the number of equations of the system, surely has an influence on the results and therefore must be considered as a factor whose variability affects simulation outcomes.

Once values of γ and β parameters are fixed, the problem of generating exogenous variables and disturbances must be addressed. In our simulation, exogenous variables are generated from random uniform distributions and vary with sample dimension and other simulation conditions.

Exogenous values are randomly generated and kept constant for each sample size. It will be observed that values taken by the endogenous variables are randomly generated from random uniform distributions in the intervals: $X_2 = [10-20]$, $X_3 = [15-27]$, $X_4 = [3-12]$, $X_5 = [3-7]$ $X_6 = [11-24]$, $X_7 = [7-13]$.

This problem does not relate to the endogenous variables, since they can be obtained from the relation

$$Y^* = X\Pi \tag{19}$$

once that exogenous variables and parameters are known. Equation (19) gives values of the endogenous variables unaffected by error.

In order to obtain the observed endogenous variable values it is necessary to add to (19) the error component generated from a multivariate Normal distribution with given variance-covariance matrix

According to the relation

$$Y\Gamma = -X\mathrm{B} - U$$

we know that the variance of U is part of the variance of $Y\Gamma = Z$.

Thus, starting from $Y^*\Gamma$ values, error variances can be obtained by imposing the relation

$$\omega_{ii} = \sigma_Z^2 \, S_i$$

namely by assuming that disturbance variances are given by the variability of each exogenous variable multiplied by a proportionality coefficient S_i. In our simulation proportionality coefficients S_i have been chosen randomly from three different intervals: $[0{,}2 - 0{,}25]$, $[0{,}4 - 0{,}5]$, $[0{,}75 - 0{,}8]$.

However, a comparison among different estimation methods should be carried out as the disturbance variances change, i.e. repeating the experiment as the proportionality coefficients change.

On the other hand, error covariances can be obtained from variances and correlation coefficients that can be obtained by generating $m(m-1)/2$ (m is the number of equations) random numbers in the intervals $[0{,}1 - 0{,}2]$, $[0{,}4 - 0{,}5]$, $[0{,}8 - 0{,}9]$ to each of them is assigned a random sign.

Now it is possible to construct the extra-diagonal elements of the SF disturbances variance-covariance matrix from the relation

$$\omega_{ij} = \rho_{ij} \left(\omega_{ii} \, \omega_{jj}\right)^{1/2}$$

And subsequently the RF variance-covariance matrix from the relation

$$\Sigma = \left(\Gamma^{-1}\right)^T \Omega \Gamma^{-1}$$

Once that Σ is known, the matrix V of the RF disturbances has to be generated from a Normal multivariate distribution

$$V \approx N(0, \Sigma).$$

According to the Spectral Decomposition Theorem the symmetric matrix Σ can be expressed as:

$$\Sigma = P \Lambda P^T$$

Where P e Λ are respectively the matrix of eigenvectors and the diagonal matrix of eigenvalues matrices.

Let :

$$Q = P\Lambda^{1/2}P^T$$

$$\Sigma = Q^T Q$$

and let the matrix C be generated with normally independently distributed columns $N(0,1)$. Then $V = CQ$ is a (multivariate) normally distributed matrix with a variance-covariance matrix Σ.

Adding the columns of C to the r.h.s. of (19), the matrix of observed endogenous variables is obtained.

To each S_i and ρ_{ij} samples of different size are taken from the assumed error distribution; in particular simulations have been conducted using samples of 20, 30, 50 and 100 observations.

After reiterating the procedure for 500 samples, the features of the different estimation methods are analyzed and compared for each scenario.

The experiment differ by the following factors:

1. the percentage of unexplained variance, represented by the proportionally coefficient S_i
2. correlation coefficients ρ_{ij} ;
3. sample sizes

The simulation design can be reassumed as follows:

ρ_{ij} \ S_i	***0.20-0.25***	***0.4-0.5***	***0.75-0.80***
0.1-0.2	N=20	N=20	N=20
	N=30	N=30	N=30
	N=50	N=50	N=50
	N=100	N=100	N=100
0.4-0.5	N=20	N=20	N=20
	N=30	N=30	N=30
	N=50	N=50	N=50
	N=100	N=100	N=100
0.8-0.9	N=20	N=20	N=20
	N=30	N=30	N=30
	N=50	N=50	N=50
	N=100	N=100	N=100

As a matter of fact, two further factors should be enlisted among those whose variability could affect results obtained by different methods. Such factors are the different degree of over-identification among equations and the type (endogenous or exogenous????) of estimated parameter.

In this study, LODE estimators have been compared with other simultaneous equations estimators both in Limited Information and in Full Information context. Such estimators differ both in terms of estimation technique and in computational difficulty.

All methods have been compared by varying the three factors that surely have an influence on their features, i.e. the disturbances' variance, degree of correlation among them and the sample size.

4. Results of the experiment

The simulation analysis has been driven by two objectives: to compare different methods and to evaluate the effects of different experimental factors.

As for the former, we have taken into consideration:

- Bias (divided by the fixed initial parameter value)

$$\varphi = \left(\hat{\theta} - \theta\right)/\theta$$

where $\hat{\theta}$ is the average of estimated parameter over the 500 samples where θ is one of the γ or β parameters;

- Root Mean Square Error (RMSE) (divided by the initial parameter value)

$$\psi = RMSE/\theta$$

.

In order to study the behaviour of LODE when simulation conditions vary we have set $S_i \in [0{,}2 - 0{,}25]$ and $\rho_i \in [0{,}1 - 0{,}2]$ as a reference *scenario* for comparisons. In the following when referring to this situation, we will call it the *basic experiment*.

Considering this *scenario* it has to be stressed that, apart from one exception, both Full Information and Limited Information LODE feature a lower bias than other estimators.

The exception occurs in the third equation, only for a sample of size 20, when both Limited and Full Information LODE estimators have higher bias than the others. The bias converges to zero as the sample size increases.

Table 1a – Relative frequency distribution of FI LODE estimators presenting a lower φ than 3SLS estimators, grouped by S_i , ρ_i and sample sizes.

S_i	**0.2-0.25**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.73	0.73	0.93	0.80	0.67	0.87	0.93	1.00	0.73	0.87	0.67	0.73

S_i	**0.4-0.5**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.73	0.73	0.60	0.87	0.73	0.73	0.93	0.93	0.60	0.93	0.80	1.00

S_i	**0.75-0.8**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.93	0.73	0.93	1.00	0.80	0.67	1.00	0.80	0.87	0.93	0.60	0.53

Table 1b – Relative frequency distribution of LI LODE estimators presenting a lower φ than 3SLS estimates, grouped by S_i , ρ_i and sample size.

S_i	**0.2-0.25**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.67	0.73	0.93	0.93	0.73	0.93	0.67	1.00	0.47	0.93	0.60	0.93

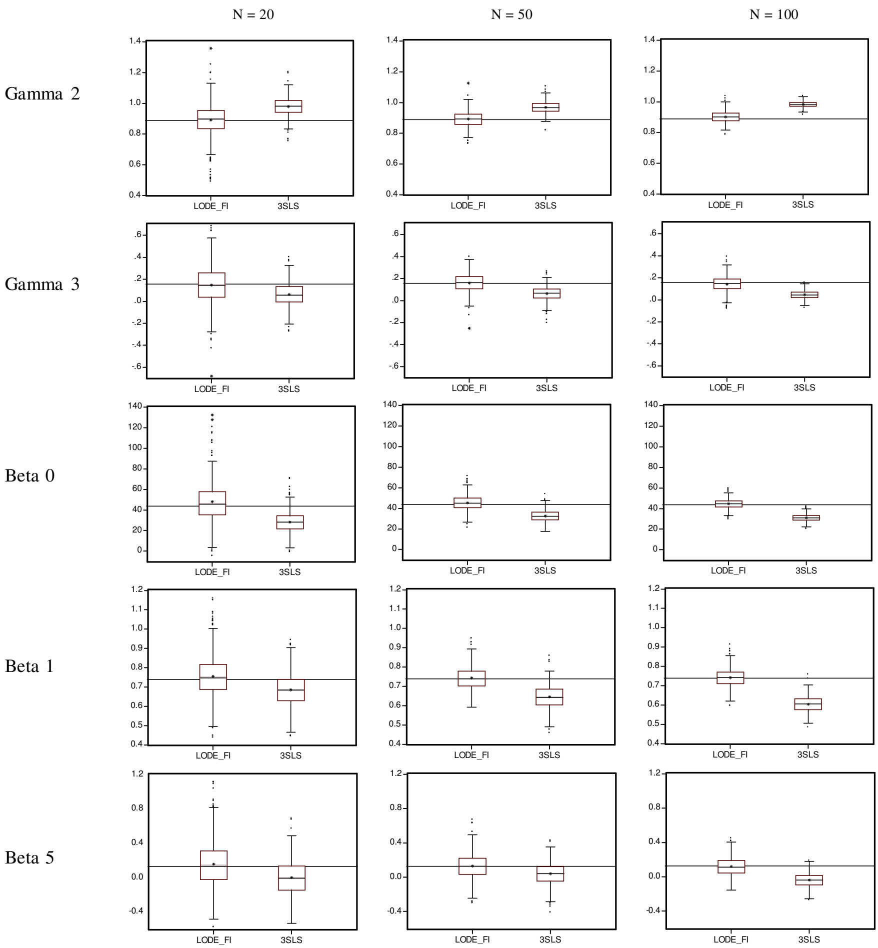
S_i	**0.4-0.5**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.67	0.67	0.67	0.93	0.47	0.60	1.00	1.00	0.93	0.80	0.80	0.93

S_i	**0.75-0.8**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.53	0.67	0.73	0.87	0.93	0.73	0.73	0.67	0.33	0.80	0.80	0.87

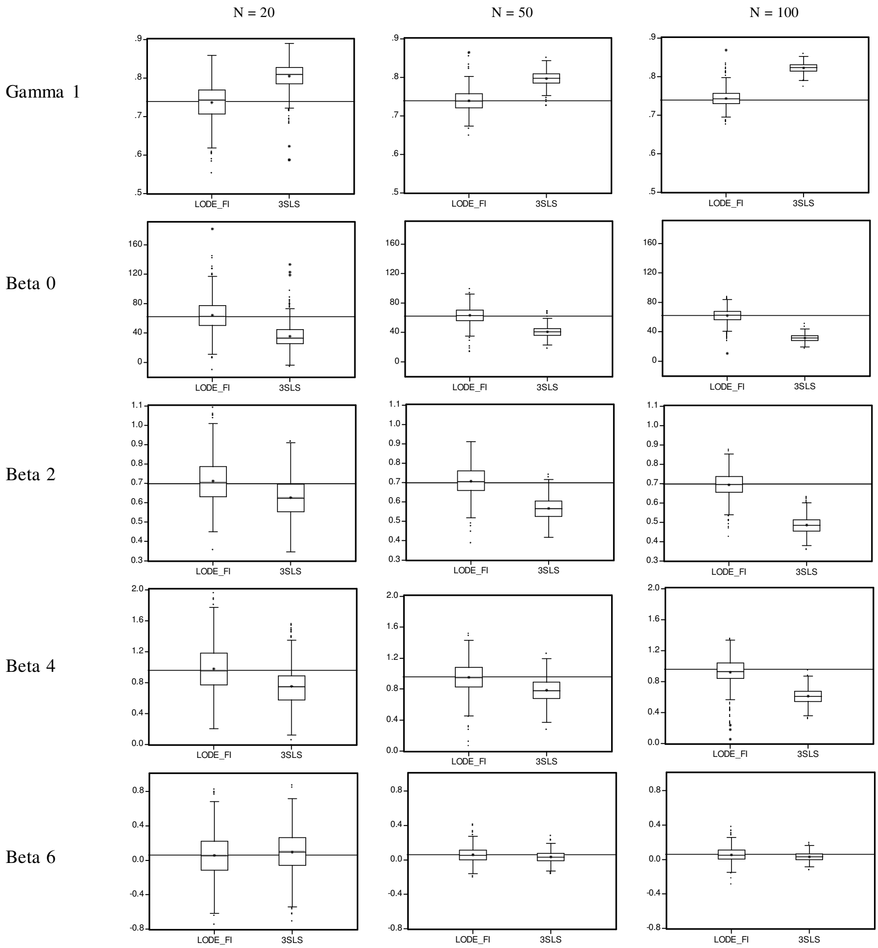
Graphs 1-2 show box-plots of equations’ parameters for the basic experiment.

They clearly show that LODE performs better than the other methods.

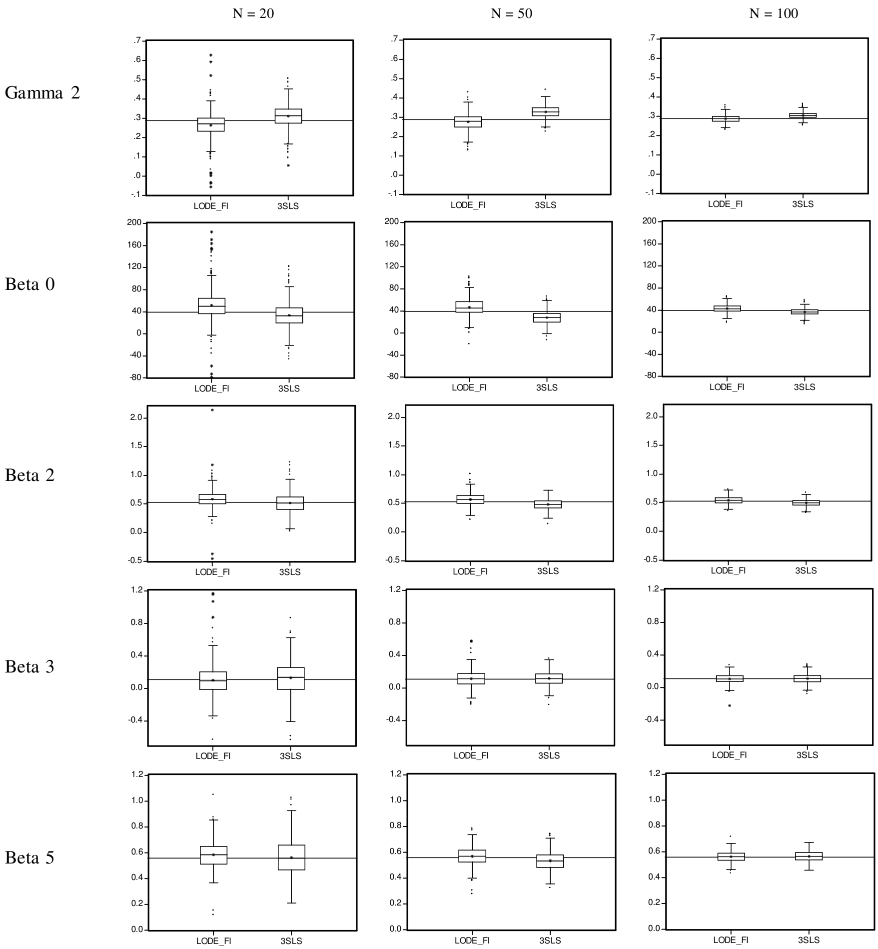
Graph 1a – Full Information, Equation 1



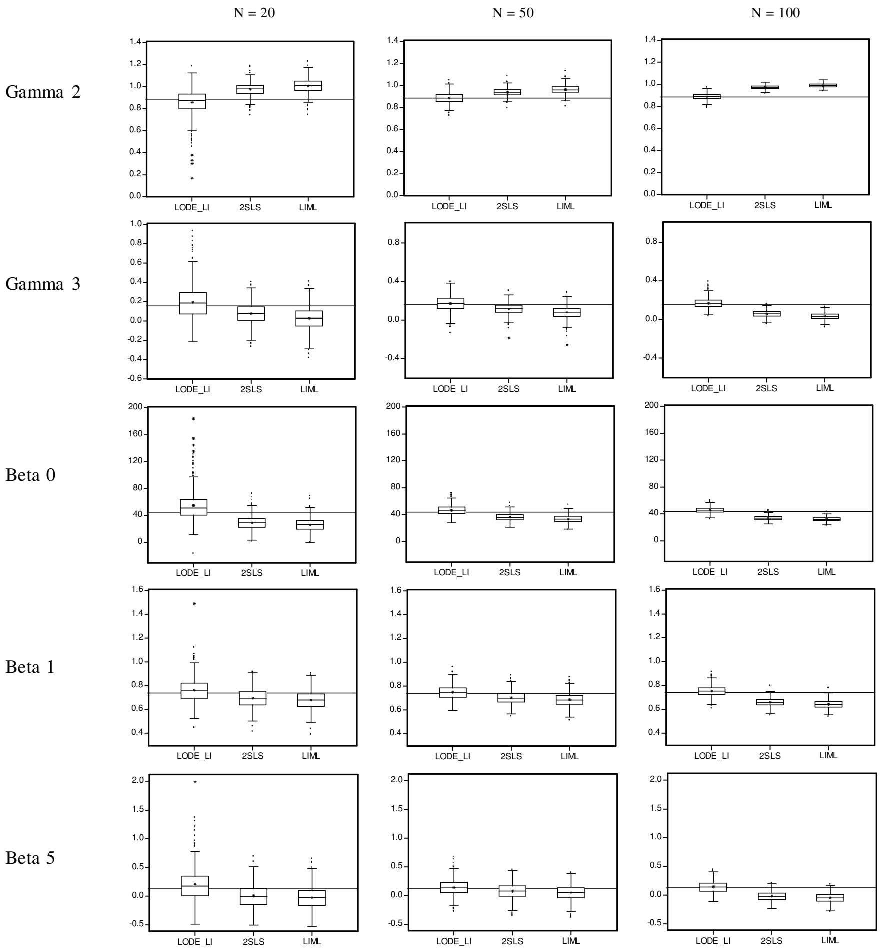
Graph 1b – Full Information, Equation 2



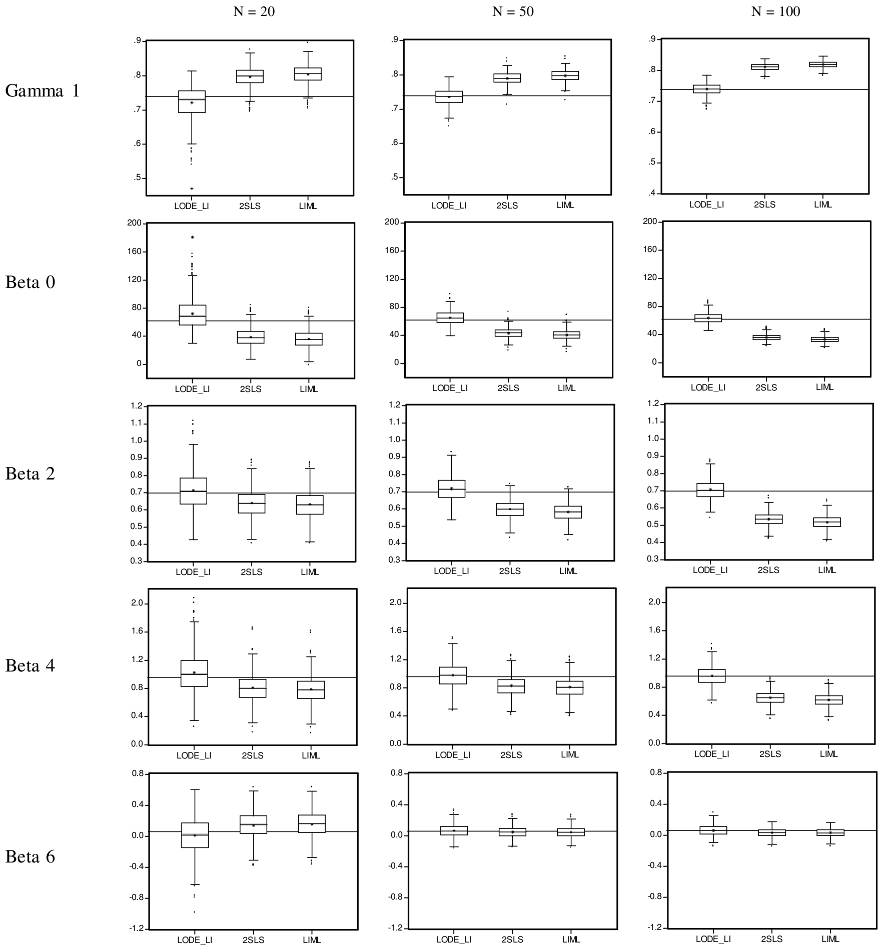
Graph 1c – Full Information, Equation 3



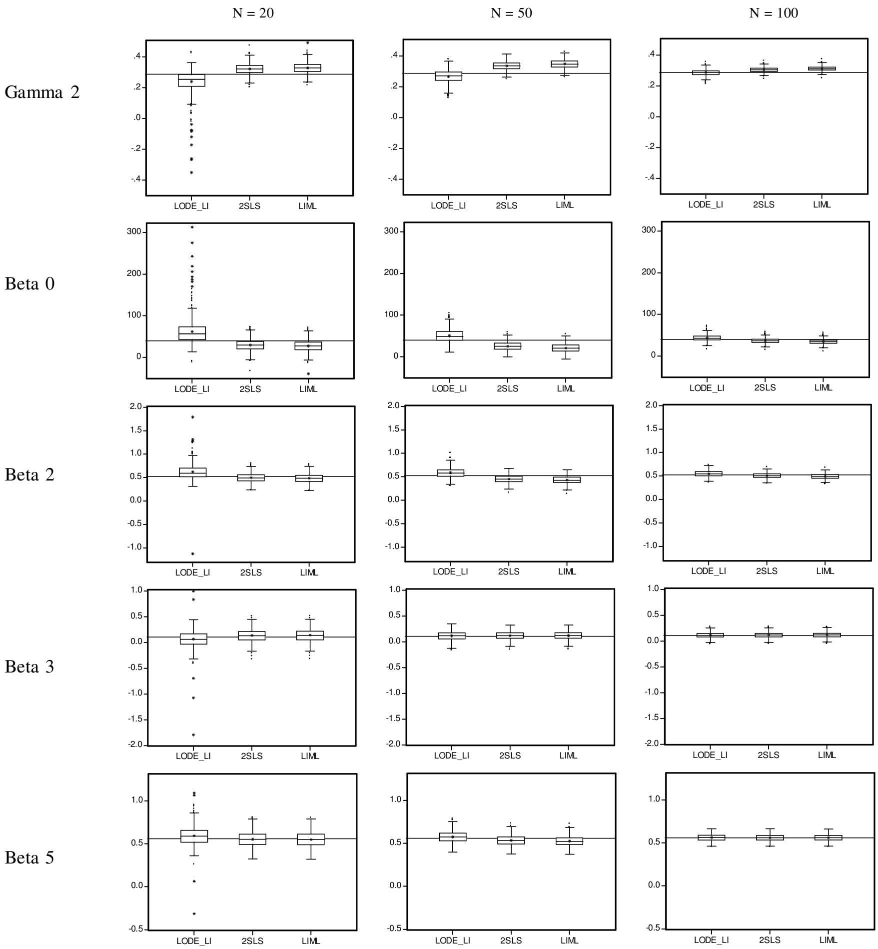
Graph 2a – Limited Information, Equation 1



Graph 2b – Limited Information, Equation 2



Graph 2c – Limited Information, Equation 3



Similar results have been obtained for the other *scenarios* (8 combinations of S_i and ρ_i and other sample sizes).

In all these *scenarios* LODE estimators are unbiased or less biased than the others. When it comes to evaluating the MSE it is necessary to make a distinction between Full Information and Limited Information LODE.

In the *basic experiment*, the number of FI LODE estimators showing a lowest MSE is always greater than (or at least equal to) 3SLS estimates. In other word comparing relative efficiency FI LODE performs better than 3SLS.

In particular, when the sample size is 20 3SLS and FI LODE have the same behaviour. As the sample size increases, the number of FI LODE estimators with lower MSE increases too.

Vice-versa, when comparing MSE of LI LODE and of 2SLS, 2SLS presents lower MSE for samples of size 20 and 30, where as LI LODE is more efficient as soon as the sample size is greater or equal to 50.

Extending the analysis to the other *scenarios*, FI LODE estimators show features similar to those observed in the *basic experiment*. As a matter of fact it must be noticed that their MSE decreases not only for increasing sample sizes, but also as S_i increases (see Tab. 2a).

Also LI estimates show features similar to those observed in the *basic experiment*

Moreover, results show that LI LODE estimators with a lower MSE then 2SLS or LIML decreases as S_i increase (see Tab. 2b).

Table 2a – Relative frequency distribution of FI LODE estimates presenting a lower ψ than 3SLS estimates, grouped by S_i , ρ_i and sample size.

S_i	**0.2-0.25**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.47	0.73	0.87	0.93	0.60	0.73	0.73	0.80	0.40	0.53	0.27	0.67

S_i	**0.4-0.5**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.60	0.33	0.60	0.40	0.40	0.33	0.67	0.80	0.60	0.53	0.67	0.73

S_i	**0.75-0.8**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.27	0.40	0.53	0.73	0.27	0.27	0.53	0.60	0.53	0.40	0.60	0.33

Table 2b – Relative frequency distribution of LI LODE estimates presenting a lower ψ than 2SLS estimates, grouped by S_i , ρ_i and sample size.

S_i	**0.2-0.25**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.13	0.40	0.67	0.67	0.13	0.47	0.60	0.87	0.07	0.53	0.60	0.73

S_i	**0.4-0.5**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.00	0.27	0.53	0.60	0.00	0.33	0.60	0.80	0.00	0.33	0.67	0.80

S_i	**0.75-0.8**											
ρ_i	**0.1-0.2**				**0.4-0.5**				**0.8-0.9**			
Sample size	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*	*20*	*30*	*50*	*100*
Relative frequency	0.00	0.20	0.33	0.53	0.00	0.00	0.53	0.47	0.00	0.07	0.20	0.40

When it come to comparing the three limited information methods (LI LODE, 2SLS and LIML) it is important to notice that it is 2SLS that shows a lower MSE. However LI LODE is more efficient than LIML.

It should also be noticed that in many cases where 2SLS is more efficient than LI LODE, FI LODE is more efficient than 3SLS.

It is as though when moving from limited to full information estimation (and from 2 to 3 stages) there is an "efficiency gain" in the LODE estimators compared with Least Squares estimators.

In order to study in more depth the $RMSE/\theta$ we have constructed three regression models, one for each equation.
The dependant variable is represented by the values taken by the $RMSE/\theta$ of each parameter in each of the 9 *scenarios*.
On the other hand explanatory variables are represented by

- the disturbance variance of considered equation
- the covariance with the other two equations
- the sample size
- a dummy variable that enables us to assess the extent to which the method is affected by the unknown parameter being of endogenous or exogenous variable.

The regression outputs are shown in Tables 3a – 3c. The effect of the variance, the dummy variable and the sample size are all significantly different from zero. In particular, the sign (>0) of the variance coefficient confirms a result already mentioned in the descriptive analysis i.e. a direct relation between the disturbance variance and the MSE . As for the dummy coefficient (<0), it indicates that endogenous variable estimators are more efficient than their exogenous counterparts. Moreover , all three regressions show an inverse relation between MSE and sample size (negative, significant, coefficient for the sample size).

Table 3a

Dependent Variable: MSE EQ1; Method: Least Squares; Included observations: 180				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.950897	0.190809	4.983507	0.0000
VARIANCE1	0.080289	0.033263	2.413761	0.0168
DUMMY1	-0.324763	0.134593	-2.412926	0.0169
SAMPLE SIZE	-0.008586	0.002216	-3.874478	0.0002
R-squared	0.115335	Mean dependent var		0.782795
Adjusted R-squared	0.100256	S.D. dependent var		0.932620
S.E. of regression	0.884636	Akaike info criterion		2.614690
Sum squared resid	137.7341	Schwarz criterion		2.685644
Log likelihood	-231.3221	F-statistic		7.648459
Durbin-Watson stat	2.864256	Prob(F-statistic)		0.000078

Table 3b

Dependent Variable: MSE EQ2; Method: Least Squares; Included observations: 180				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.878926	0.401426	2.189508	0.0299
VARIANCE2	0.185314	0.083208	2.227120	0.0272
DUMMY2	-1.189987	0.309598	-3.843648	0.0002
SAMPLE SIZE	-0.008604	0.004157	-2.069855	0.0399
R-squared	0.111733	Mean dependent var		1.039321
Adjusted R-squared	0.096592	S.D. dependent var		1.748046
S.E. of regression	1.661479	Akaike info criterion		3.875265
Sum squared resid	485.8501	Schwarz criterion		3.946219
Log likelihood	-344.7738	F-statistic		7.379567
Durbin-Watson stat	2.072380	Prob(F-statistic)		0.000110

Table 3c

Dependent Variable: MSE EQ3; Method: Least Squares; Included observations: 180				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.463829	0.172651	2.686517	0.0079
VARIANCE3	0.339445	0.078677	4.314393	0.0000
DUMMY3	-0.479939	0.115272	-4.163519	0.0000
SAMPLE SIZE	-0.007810	0.001510	-5.170919	0.0000
R-squared	0.245591	Mean dependent var		0.655691
Adjusted R-squared	0.232731	S.D. dependent var		0.706233
S.E. of regression	0.618617	Akaike info criterion		1.899310
Sum squared resid	67.35285	Schwarz criterion		1.970265
Log likelihood	-166.9379	F-statistic		19.09837
Durbin-Watson stat	2.870870	Prob(F-statistic)		0.000000

As for the second objective of our simulation experiment – namely evaluate the effects of different experimental factors - it has been pursued by means of a Multivariate Analysis of Variance.

This analysis enabled us to assess whether the estimation of each single structural parameter is influenced by the simulation conditions. In the rest of the paragraph we will discuss results for FI LODE . Note that results shown can be extended to LI LODE also.

We considered the 15 structural coefficient estimators obtained using FI LODE as the dependant variables. The six different intervals from which S_i and ρ_i values have been generated and the different sample sizes are considered as "effects".

We assume that the number of equations, their specification as well as their degree of over-identification are all factors that could have an impact on parameter estimation. Thus we have included a further factor that takes into account “the nature” of the system, i.e. which equation the parameter belongs to.

Results obtained from 54000 observations – 500 samples generated for each of the 4 sample sizes and for each of the 9 simulation conditions – confirm the hypothesis that parameter estimation is influenced by all the above mentioned factors.

The analysis was carried out in two steps. First of all an ANOVA was carried out considering all 15 parameters estimated with FI LODE separately; secondly a Multivariate ANOVA was performed also, considering all 15 parameters simultaneously. ANOVA results indicate that all factors have an impact on the estimation of each single parameter, expect for parameter b7 in second equation – that seems to be uninfluenced by both S_i and ρ_i – and parameter b4 in third equation – that seems to be uninfluenced by S_i .

Results of the Multivariate ANOVA are shown in tables 4a – 4d and confirm that all four considered factors have an influence on the simultaneous estimation of all 15 parameters. Note that the Multivariate ANOVA carried out in this study only takes into account the main effects of factors, i.e. it ignores all interactions of order higher than the 1^{st} .

It goes without saying that further analysis in this direction could lead to interesting conclusions on the joint effects of all factors.

Table 4a - MANOVA Test Criteria and F Approximation for Hypothesis of Overall Sample Size Effect

N=26987					
Statistic	**Value**	**F Value**	**Num DF**	**Den DF**	**Pr > F**
Wilks ' Lambda	0.977	27.12	45	160350	<.0001
Pillai's Trace	0.022	27.05	45	161934	<.0001
Hotellin-Lawley Trace	0.022	27.19	45	133510	<.0001
Roy's Greatest Root	0.016	58.09	15	53978	<.0001

Table 4b - MANOVA Test Criteria and F Approximation for Hypothesis of Overall S_i Effect

N=26987					
Statistic	**Value**	**F Value**	**Num DF**	**Den DF**	**Pr > F**
Wilks ' Lambda	0.983	29.27	30	107952	<.0001
Pillai's Trace	0.161	29.22	30	107954	<.0001
Hotellin-Lawley Trace	0.016	29.32	30	95953	<.0001
Roy's Greatest Root	0.013	48.57	15	53977	<.0001

Table 4c - MANOVA Test Criteria and F Approximation for Hypothesis of Overall ρ_i Effect

N=26987					
Statistic	**Value**	**F Value**	**Num DF**	**Den DF**	**Pr > F**
Wilks ' Lambda	0.961	43.72	30	107952	<.0001
Pillai's Trace	0.023	43.54	30	107954	<.0001
Hotellin-Lawley Trace	0.024	43.9	30	95953	<.0001
Roy's Greatest Root	0.222	80.08	15	53977	<.0001

Tabella 4d - MANOVA Test Criteria and F Approximation for Hypothesis of Equation Effect

N=26987					
Statistic	**Value**	**F Value**	**Num DF**	**Den DF**	**Pr > F**
Wilks ' Lambda	0	5.08E+09	30	107952	<.0001
Pillai's Trace	2	1.47E+09	30	107954	<.0001
Hotellin-Lawley Trace	3196072.531	5.75E+09	30	95953	<.0001
Roy's Greatest Root	2351362.867	8.49E+09	15	53977	<.0001

5. Conclusions

The simulation experiment on the estimation of a simultaneous equation system using LODE has highlighted some good features of the proposed method both in terms of MSE and in terms of unbiasedness. As a matter of fact, we show that in all our simulations LODE has always a lower bias.

As for the MSE, FI LODE is more efficient than 3SLS in almost all cases, whereas LI LODE are more efficient only for samples greater or equal to 50 (unless the explained variance is low, in which case LI LODE is more efficient than 2SLS even for a sample size of 30).

Having found some evidence that the various simulation conditions have a significant impact on the LODE it is our intention to expand this study e.g. by including more equations into the system with a higher degree of over-identification, generating observation from other types of distributions and considering the numerical value taken by the parameter estimate as a possible influencing factor.

Lastly it is to be noted that in this study a comparison between FI LODE and FIML could not be carried out. This lack is due to the fact that in our simulations Maximum Likelihood Full Information estimation algorithm did not converge to a maximum (it did not provide estimates). Besides the well known difficulties related to maximization of the likelihood function, this problem was likely due to the algorithm applied by the chosen Soft Ware application.

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Appendice

Table 1a $S_i \in [0.2 - 0.25]$ $\rho_i \in [0.1 - 0.2]$

n=20						
Equation 1		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g2	φ	-0.002	0.097	-0.042	0.091	0.125
	ψ	*0.133*	*0.119*	*0.141*	*0.113*	*0.146*
g3	φ	-0.104	-0.634	0.187	-0.575	-0.885
	ψ	*1.238*	*0.956*	*1.146*	*0.905*	*1.187*
b0	φ	0.080	-0.368	0.222	-0.363	-0.434
	ψ	*0.462*	*0.443*	*0.543*	*0.436*	*0.498*
b2	φ	0.018	-0.077	0.025	-0.066	-0.088
	ψ	*0.152*	*0.138*	*0.149*	*0.126*	*0.139*
b5	φ	0.201	-1.029	0.537	-1.018	-1.241
	ψ	*2.038*	*1.799*	*2.281*	*1.786*	*1.907*
Equation 2		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g1	φ	-0.006	1.173	-0.027	0.074	0.084
	ψ	*0.070*	*0.102*	*0.071*	*0.084*	*0.092*
b0	φ	0.027	0.123	0.150	-0.389	-0.438
	ψ	*0.384*	*0.541*	*0.388*	*0.442*	*0.484*
b3	φ	0.014	0.784	0.014	-0.091	-0.102
	ψ	*0.159*	*0.190*	*0.159*	*0.151*	*0.156*
b5	φ	0.013	0.554	0.063	-0.163	-0.184
	ψ	*0.297*	*0.351*	*0.297*	*0.270*	*0.280*
b7	φ	-0.085	1.993	-0.898	1.279	1.450
	ψ	*4.315*	*4.046*	*4.043*	*3.245*	*3.253*
Equation 3		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g2	φ	-0.100	0.063	-0.183	0.099	0.119
	ψ	*0.297*	*0.229*	*0.611*	*0.158*	*0.173*
b0	φ	0.273	-0.186	0.520	-0.282	-0.339
	ψ	*0.841*	*0.643*	*1.748*	*0.456*	*0.496*
b3	φ	0.091	-0.042	0.150	-0.081	-0.099
	ψ	*0.324*	*0.329*	*0.509*	*0.200*	*0.208*
b4	φ	-0.083	0.159	-0.480	0.164	0.199
	ψ	*1.697*	*1.865*	*2.002*	*1.161*	*1.163*
b6	φ	0.040	0.001	0.056	-0.017	-0.024
	ψ	*0.204*	*0.243*	*0.249*	*0.154*	*0.155*

Table 1b $S_i \in [0.2 - 0.25]$ $\rho_i \in [0.1 - 0.2]$

		n=30				
Equation 1		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
	φ	-0.003	0.062	-0.010	0.043	0.058
g2	ψ	*0.056*	*0.075*	*0.057*	*0.059*	*0.073*
	φ	-0.032	-0.454	0.030	-0.322	-0.444
g3	ψ	*0.489*	*0.595*	*0.494*	*0.500*	*0.604*
	φ	0.046	-0.175	0.061	-0.125	-0.166
b0	ψ	*0.207*	*0.238*	*0.207*	*0.203*	*0.231*
	φ	0.012	-0.154	0.018	-0.065	-0.088
b2	ψ	*0.126*	*0.203*	*0.127*	*0.125*	*0.139*
	φ	0.150	-1.193	0.266	-0.724	-0.991
b5	ψ	*1.606*	*1.833*	*1.615*	*1.557*	*1.700*
Equation 2		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
	φ	-0.003	1.203	-0.019	0.081	-0.605
g1	ψ	*0.063*	*0.110*	*0.057*	*0.088*	*0.607*
	φ	0.015	0.054	0.093	-0.381	-0.369
b0	ψ	*0.304*	*0.527*	*0.273*	*0.415*	*0.430*
	φ	0.005	0.391	0.045	-0.225	-0.245
b3	ψ	*0.201*	*0.340*	*0.186*	*0.259*	*0.307*
	φ	-0.012	0.632	0.033	-0.181	-0.883
b5	ψ	*0.271*	*0.273*	*0.257*	*0.271*	*0.889*
	φ	0.081	0.146	-0.063	0.166	8.406
b7	ψ	*2.774*	*2.477*	*2.606*	*2.148*	*8.464*
Equation 3		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
	φ	-0.115	-0.016	-0.139	-0.026	0.007
g2	ψ	*0.218*	*0.163*	*0.351*	*0.126*	*0.126*
	φ	0.304	0.039	0.371	0.066	-0.022
b0	ψ	*0.586*	*0.443*	*0.946*	*0.339*	*0.342*
	φ	0.177	0.030	0.214	0.049	-0.002
b3	ψ	*0.369*	*0.325*	*0.536*	*0.246*	*0.245*
	φ	-0.109	0.033	-0.136	-0.011	0.023
b4	ψ	*1.030*	*1.351*	*0.941*	*0.890*	*0.890*
	φ	0.009	0.010	0.006	0.009	0.008
b6	ψ	*0.114*	*0.139*	*0.113*	*0.107*	*0.107*

Table 1c $S_i \in [0.2 - 0.25]$ $\rho_i \in [0.1 - 0.2]$

n=50						
Equation 1		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g2	φ	0.000	0.086	-0.009	0.049	0.076
	ψ	*0.057*	*0.096*	*0.056*	*0.063*	*0.088*
g3	φ	-0.027	-0.623	0.051	-0.300	-0.533
	ψ	*0.533*	*0.743*	*0.521*	*0.489*	*0.684*
b0	φ	0.021	-0.272	0.044	-0.194	-0.260
	ψ	*0.166*	*0.304*	*0.170*	*0.235*	*0.293*
b2	φ	0.003	-0.130	0.006	-0.057	-0.080
	ψ	*0.082*	*0.155*	*0.082*	*0.093*	*0.110*
b5	φ	-0.008	-0.717	0.044	-0.454	-0.668
	ψ	*1.118*	*1.222*	*1.100*	*1.089*	*1.197*
Equation 2		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g1	φ	-0.002	1.151	-0.008	0.065	0.075
	ψ	*0.038*	*0.079*	*0.034*	*0.069*	*0.079*
b0	φ	0.009	0.294	0.037	-0.314	-0.361
	ψ	*0.182*	*0.385*	*0.165*	*0.334*	*0.378*
b3	φ	0.007	0.610	0.021	-0.151	-0.174
	ψ	*0.107*	*0.212*	*0.100*	*0.168*	*0.189*
b5	φ	-0.009	0.623	0.018	-0.140	-0.162
	ψ	*0.210*	*0.251*	*0.188*	*0.210*	*0.223*
b7	φ	-0.061	0.028	0.035	-0.280	-0.327
	ψ	*1.477*	*1.198*	*1.361*	*1.199*	*1.190*
Equation 3		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g2	φ	-0.054	0.120	-0.086	0.147	0.188
	ψ	*0.152*	*0.164*	*0.164*	*0.175*	*0.212*
b0	φ	0.146	-0.330	0.234	-0.394	-0.504
	ψ	*0.411*	*0.451*	*0.444*	*0.472*	*0.569*
b3	φ	0.060	-0.104	0.093	-0.163	-0.209
	ψ	*0.206*	*0.199*	*0.212*	*0.223*	*0.260*
b4	φ	0.018	0.028	-0.026	-0.004	-0.006
	ψ	*0.901*	*0.820*	*0.775*	*0.725*	*0.720*
b6	φ	0.012	-0.052	0.022	-0.053	-0.067
	ψ	*0.128*	*0.139*	*0.125*	*0.121*	*0.127*

Table 1d $S_i \in [0.2 - 0.25]$ $\rho_i \in [0.1 - 0.2]$

n=100						
Equation 1		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g2	φ	0.010	0.101	-0.005	0.087	0.106
	ψ	*0.043*	*0.103*	*0.034*	*0.089*	*0.108*
g3	φ	-0.134	-0.740	0.027	-0.662	-0.832
	ψ	*0.452*	*0.772*	*0.333*	*0.694*	*0.861*
b0	φ	0.005	-0.307	0.022	-0.254	-0.293
	ψ	*0.108*	*0.316*	*0.109*	*0.264*	*0.302*
b2	φ	0.001	-0.186	0.013	-0.116	-0.140
	ψ	*0.066*	*0.194*	*0.065*	*0.125*	*0.147*
b5	φ	-0.093	-1.312	0.069	-1.186	-1.417
	ψ	*0.864*	*1.456*	*0.809*	*1.347*	*1.551*
Equation 2		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g1	φ	0.004	1.220	-0.002	0.094	0.104
	ψ	*0.032*	*0.111*	*0.026*	*0.095*	*0.105*
b0	φ	-0.013	-0.007	0.013	-0.434	-0.481
	ψ	*0.148*	*0.526*	*0.120*	*0.440*	*0.486*
b3	φ	-0.010	0.380	0.004	-0.242	-0.268
	ψ	*0.096*	*0.317*	*0.084*	*0.249*	*0.273*
b5	φ	-0.045	0.263	-0.002	-0.328	-0.362
	ψ	*0.197*	*0.382*	*0.143*	*0.343*	*0.375*
b7	φ	-0.134	-0.102	-0.053	-0.526	-0.577
	ψ	*1.414*	*0.990*	*1.144*	*1.019*	*1.027*
Equation 3		**FI LODE**	**3SLS**	**LI LODE**	**2SLS**	**LI ML**
g2	φ	-0.016	0.042	-0.020	0.039	0.063
	ψ	*0.068*	*0.073*	*0.070*	*0.066*	*0.083*
b0	φ	0.047	-0.109	0.058	-0.102	-0.167
	ψ	*0.184*	*0.194*	*0.190*	*0.176*	*0.221*
b3	φ	0.012	-0.074	0.018	-0.059	-0.091
	ψ	*0.121*	*0.139*	*0.120*	*0.121*	*0.140*
b4	φ	-0.056	-0.036	-0.036	-0.030	-0.033
	ψ	*0.532*	*0.537*	*0.511*	*0.509*	*0.509*
b6	φ	-0.001	0.002	-0.003	-0.005	-0.006
	ψ	*0.070*	*0.072*	*0.068*	*0.067*	*0.068*