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Negative and Positive Effects of Competition in a Preemption Game*

Toru Suzuki[†]

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Abstract

Agents compete to acquire a limited economic opportunity of uncertain profitability. Each agent decides how much he acquires public signals before making investment under fear of preemption. I show that equilibria have various levels of efficiency under mild competition. The effect of competition on the equilibrium strategy is different depending on which class of equilibrium we focus on. However, when competitive pressure is sufficiently high, there exists a unique equilibrium. Finally, I show that the effect of competition on efficiency is different between the common value and the private value setting. Strong competition leads to the least efficient equilibrium for the common value setting but efficiency can be improved by competition in the private value setting.

Keywords: Competition, Preemption game, Strategic real option

JEL Code: C73, D83

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1 Introduction

There are many economic situations where agents have to decide how much to wait to receive more information under a time pressure. For example, consider a firm's entry problem. The firm can receive more information about the profitability of the market if he delays the timing of the entry. On the other hand, when the firm delays the entry too much, the market might be less profitable after having too many entries by other firms. Another example is a clearance sale. If consumers wait until the final day of the sale, they may receive more information about new alternative products. On the other hand, the product might be sold out if the consumers wait until the final day of the sale.

This paper investigates a preemption game in which each agent decides how much to acquire public information before investment under fear of preemption. Especially, I compare the effect of competition in two cases: the investment opportunity has (i) common value, (ii) private value. I show that even though the effect of competition on the equilibrium strategy is similar for both cases, the effect of competition on efficiency can be quite different. Concretely, in both cases, strong competition discourages information acquisition and induces a quicker decision making. In the common value setting, the quicker decision leads to the least efficient equilibrium. However, in the private value setting, the quicker decision can improve efficiency through allocation of opportunities. That is, opportunities tend to be obtained by agents who value opportunities higher, i.e. there is a sorting effect.

In Section 2, I introduce the game. I employ a discrete time framework to analyze the preemption problem in which opportunities have common value. Section 3 provides the equilibrium analysis. The key parameter of the model is the competition level, which is measured by the ratio between the number of agents and the number of opportunities. Since the equilibrium investment strategy determines the level of information utilization, our interest is in the relationship between the level of competition and the information efficiency of the set of equilibria. First, I introduce a class of strategies with an important property which is called myopia. Myopic strategies "invest" whenever the expected profit from immediate investment is strictly positive and "wait" whenever the expected payoff from immediate investment is negative. I show a myopic equilibrium exists for any competition level. Importantly, the efficiency of these equilibria is the worst possible level that any equilibrium can have. Then, I analyze the other possible equilibria under mild and strong competition levels. I show that equilibria with "forward looking behavior," that is, where agents wait even though the expected payoff is positive, exist under mild competition. Moreover, there is a wide range of equilibrium outcomes from the least efficient to the first best under mild competition. However, under strong competition, the set of equilibria loses its variety. That is, all equilibria are myopic. Moreover, I show that the effect of competition on the equilibrium strategy depends on the class of equilibrium we focus on. For example, if we focus on symmetric mixed strategy equilibrium, competition can increase the probability of waiting in equilibrium. On the other hand, competition discourages waiting if we focus on the pure strategy symmetric equilibrium.

In Section 4, I analyze the case where the value of opportunities for each agent depends on private information, e.g., technology or willingness to pay. First, I show that all equilibria are monotonic in the sense that whenever one type invests with positive probability, "more profitable" type also invests. However, unlike the common value setting, myopic equilibrium does not always exist. Second, the effect of competition in equilibrium strategy is similar to that in the common value case. That is, equilibria with full use of information can exist under mild competition but not for sufficiently strong competition. In fact, the myopic equilibrium is the unique equilibrium when competition is sufficiently strong. The effect of competition can improve efficiency through better allocation of opportunities. That is, the myopic equilibrium implies that the higher profitability type has a greater chance to get the investment opportunity. I provide a sufficient condition in which the net effect of competition is positive.

Related literature

Unlike the usual preemption games, e.g., Fudenberg and Tirole (1985), Hoppe and Lehmann-Grube (2005), information acquisition is the key element of this model. Thus, this paper is close to strategic real option models which study the timing of investment under uncertain profitability and fear of preemption. Grenadier (2002) analyzes an oligopoly game in which each firm chooses when to increase the production level when facing uncertainty. Especially, he shows that the option premium is monotonically decreasing in competition and vanishes in the limit. Lambrecht and Perraudin (2003) and Anderson et al (2009) study a winner-take-all preemption game with incomplete information, which is closer to my model. Especially, Anderson et al (2009) analyzes a N-player game and shown how the threshold value of the equilibrium trigger strategy changes with respect to the level of competition.

There are several key differences between my model and others. In my model, the degree of competition is defined as the ratio between the number of opportunities and the number of competitors. In Lambrecht and Perraudin (2003) and Anderson et al (2010), the game has one opportunity and the game immediately finishes after the opportunity is taken. On the other hand, in my model, the game continues after one opportunity is taken and thus the history of actions can play a key role. To analyze the effect of competition more generally, I employ a discrete time framework and allow all behavioral strategies given a history of public signals and actions. The richer preemption structure and the strategy space make it possible to provide more insights into the effect of competition. I show that the effect of competition on the equilibrium strategy depends on which class of strategy we focus on. For example, when I focus on symmetric mixed strategy, competition can make agents more "forward looking" in some range of competition. On the other hand, if we focus on pure strategy symmetric equilibria, competition discourages "forward looking" behaviors. Thus, my model refines understanding about the effect of competition on equilibrium strategies.

Finally, unlike Grenadier (2002) and Anderson et al (2010) which focus on the effect of competition on the option premium and the threshold value of the trigger strategy, my paper also analyzes the effect of competition on efficiency. I show that even though the effect of competition on the equilibrium strategy is similar between the common value and the private value setting, the effect of competition on efficiency can be quite different. That is, strong competition decreases efficiency in the common value setting but efficiency can be improved by strong competition in the private value setting.

2 The model

Basics. There are $K \geq 1$ identical investment opportunities. Let $\mathcal{I} = \{1, 2, .., I\}$ be the set of agents. In each period, each agent chooses either $a_i = B$, "invest/buy" or $a_i = W$, "wait". Thus, the set of actions is $A_i = \{B, W\}$. Let $S = \times_{n=1}^{N} S_n$ be the finite set of states of the world. The profitability of the investment is state dependent, that is, the profit function is $v : S \to \mathbb{R}$. Finally, $f(s)$ is the probability distribution over S which is common knowledge. For simplicity, I assume the following.

Assumption 1. *There exists* $s, s' \in S$ *such that* $v(s) > 0$, $v(s') < 0$.
Assumption 2. $\text{supp}(f) = S$.

Time is discrete, $n = 1, 2, ...N$. Given state $s = (s_1, s_2, .., s_N)$, agents observe signal s_n at period n. That is, agents face a sequence of public signals $(s_1, s_2, .., s_N)$. Then, processed signals at n given state s is denoted by s^n, which is the projection of s on $\times_{n'=1}^{n} S_{n'}$. Thus, if the agent waits until period N, he can make a decision based on s. For simplicity, I assume no agent discounts the future payoff.[1]

Assumption 3. *For any* s^n, $\sum_s v(s)f(s|s^n) \neq 0$.

This assumption is for simplicity. Note that, when the model has s^n such that $\sum_s v(s)f(s|s^n) = 0$, Assumption 3 can be always satisfied by perturbing the model slightly.

Preemption game. In the beginning of each period, agents observe a public signal and a history of investments. Then, the agents who have not previously invested simultaneously choose actions. When the number of agents who choose B is less than the number of investment opportunities in the period, each agent who chose B gets an opportunity. When the number of agents who choose B is larger than the number of available opportunities in the period, the investment opportunities are allocated by a fair lottery. More preciously, let x_n be the total number of investors who decide to invest in period n, i.e., $\#\{j \in \mathcal{I} | a_{jn} = B\}$, and x^{n-1} denote the total number of opportunities taken by agents by period $n-1$. Then, the payoff of player i from $a_{in} = B$ given state s and h^n is $\min\left\{1, \frac{K - x^{n-1}}{x_n}\right\} v(s)$. The game ends when all K opportunities are taken or at the end of period N, whichever comes first. When the agent play W until the game ends, his payoff is 0.

Strategy and Equilibrium. Let $H_i^{n-1} \subset A_i^{n-1}$ be the set of possible histories of i's actions, $h_i^{n-1} = (a_{i1}, a_{i2}, .., a_{in-1})$. Each history of actions has the properties that (i) $h_i^{n-1} = \varnothing$ if

[1] It is easy to modify the results with impatient agents. Moreover, all results are preserved when all agents are sufficiently patient.

$n = 1$ and (ii) h_i^{n-1} includes at most one $a_i = B$. Let $H^{n-1} \subset \times_i H_i^{n-1}$ denote the set of possible h_i^{n-1} of all agents at n. H^{n-1} has the property that the number of B in h^{n-1} is at most K. Moreover, let $\Delta(A_i)$ be the set of probability distributions over A_i. Then, a *behavioral strategy* of agent i is a mapping from set of histories to the set of distributions over actions, that is,

$$\sigma_i : \bigcup_{n \in \{1,2,..,N\}} (S^n \times H^{n-1}) \to \Delta(A_i)$$

such that $\sigma_i(s^n, h^{n-1})(B) = 0$ if h_i^{n-1} includes $a_{in'} = B$ for some $n' < n$. The behavioral strategy is *nondegenerate* if $\sigma_i(s^n, h^{n-1})(B) \in (0, 1)$ for some histories.

The game is analyzed using the concept of *Subgame Perfect Equilibrium (SPE).* Formally, let $u_i(\sigma_i, \sigma_{-i}|s^n, h^{n-1})$ be the expected utility when agent i chooses σ_i and the other agents $-i$ choose σ_{-i} at a history (s^n, h^{n-1}). Then, $\sigma^* = ({\sigma_i}^*)_{i \in \mathcal{I}}$ is subgame perfect equilibrium if, for all histories $(s^n, h^{n-1}) \in \bigcup_{n \in \{1,2,..,N\}} (S^n \times H^{n-1})$,

$$u(\sigma_i^*, \sigma_{-i}^*|s^n, h^{n-1}) \geq u(\sigma_i, \sigma_{-i}^*|s^n, h^{n-1})$$

for every strategy σ_i.

Efficiency. I suppose that efficiency of the economy I with strategy profile σ is measured by

$$V(\sigma) = \sum_s \sum_{i \in \mathcal{I}} v(s) \chi_\sigma^i(s) f(s)$$

where $\chi_\sigma^i(s)$ is the allocation rule that is determined by the outcome of the game given strategy profile σ and state s, i.e., $\chi_\sigma^i(s) = 1$ if agent i gets an investment opportunity at s and $\chi_\sigma^i(s) = 0$ otherwise.

3 The equilibrium analysis

Before starting the analysis, I provide an important observation about the role of competition.

Observation 1. *Equilibrium always has the highest possible efficiency level if there is no competition, i.e., $I \leq K$.*

The reasoning is simple. If there is no competition, then whenever an agent chooses to invest, he gets the opportunity with probability 1. Thus, each agent waits until period N. As a result, the decision is based on the sign of $v(s)$ so the allocation is efficient. Thus, the interesting case of the model is with competition. Henceforth, I focus on the case $I > K$.

Now, turning to the main analysis, I introduce a class of strategies which plays a crucial role in this analysis.

Definition 1. *A strategy $\sigma_i(s^n, h^{n-1})$ is myopic if*

$$\sigma_i(s^n, h^{n-1})(B) = \begin{cases} 1 \text{ whenever } E[v(s)|s^n] > 0 \\ 0 \text{ whenever } E[v(s)|s^n] < 0. \end{cases}$$

Moreover, an equilibrium is myopic equilibrium if all agents play myopic strategy in the equilibrium.

In short, under this strategy, the agent invests whenever the expected payoff from the immediate investment is positive.

Let σ_{myo} be the myopic strategy profile and $\Sigma^*(K, I)$ be the set of all equilibria given (K, I). The next proposition claims that myopic equilibrium always exists and this is always least efficient equilibrium.

Proposition 1. *Myopic equilibrium always exists for any $I > K$. Moreover, $V(\sigma_{myo}) \leq V(\sigma)$* for all $\sigma \in \Sigma^*(K, I)$.

Proof. Consider any history (s^n, h^{n-1}) in which $E[v(s)|s^n] > 0$. Then, the expected payoff from the myopic strategy, i.e., $a_{in} = B$, is $\frac{K-x^{n-1}}{I-x^{n-1}}E[v(s)|s^n] > 0$. Suppose an agent deviates to $a_{in} = W$. Since $I > K$, the game is terminated at n. This means the payoff from this deviation is zero, so there is no incentive to deviate. On the other hand, consider any history (s^n, h^{n-1}) in which $E[v(s)|s^n] < 0$. Obviously, the payoff from the myopic strategy, i.e., $a_{in} = W$, is at least 0. On the other hand, the deviation to $a_{in} = B$ yields weakly negative profit. Thus, again, the deviation is not profitable.

For the last part of the proposition, suppose $\sigma', \sigma'' \in \Sigma^*(K, I)$ in which, for any i, $\sigma''_i(s^n, h^{n-1})(B) \geq \sigma'_i(s^n, h^{n-1})(B)$ for all (s^n, h^{n-1}) and the inequality is strict for some

(s^n, h^{n-1}). Then, the period at which the game is terminated with σ' is equal or later than that with σ'' given s. Then, since $\sigma', \sigma'' \in \Sigma^*(K, I)$ and no one invest when the expected payoff is negative, $V(\sigma') \geq V(\sigma'')$.

Now, suppose $V(\sigma_{myo}) > V(\sigma)$ for $\sigma \in \Sigma^*(K, I)$. Then, there must be some agents and some states in which investment under σ is earlier than under myopic strategies. Hence, some agents choose $a_i = B$ for some histories in which s^n satisfies $E[v(s)|s^n] < 0$. But such a strategy cannot be optimal given s^n since the expected payoff from W is always non negative. A contradiction. Q.E.D.

An intuition of Proposition is simple. For the existence, note that when all agents follow myopic strategy and they invest at the current period, no opportunity is left in the next period. Thus, the deviation cannot be profitable. For the second part, observe that, in any equilibrium, no one invests at any history where the expected payoff is negative. Thus, in any other equilibria, the game ends at later period than myopic equilibrium and more information is utilized.

3.1 Mild Competition

Our next question is whether the game has more efficient equilibria than myopic. Thus, I introduce a class of strategy with "forward looking behavior," i.e., choosing W at a history in which the expected payoff from the immediate investment is strictly positive.

Definition 2. *A strategy $\sigma_i(s^n, h^{n-1})$ is forward looking (FL) if*
(i) $\sigma_i(s^n, h^{n-1})(B) = 0$ if s^n is such that $E[v(s)|s^n] < 0$,
(ii) $\sigma_i(s^n, h^{n-1})(B) < 1$ for some (s^n, h^{n-1}) in which s^n is such that $E[v(s)|s^n] > 0$.
Moreover, an equilibrium is FL equilibrium if some agents play FL strategies in the equilibrium.[2]

Thus, when an agent plays a forward looking strategy, the agent may not invest even though immediate investment yields a strictly positive expected payoff. That is, the agent shows "forward looking behavior" for some s^n such that $E[v(s)|s^n] > 0$. Obviously, FL

[2] FL equilibrium can be asymmetric equilibrium.

strategies utilize more information than myopic strategies and FL equilibria have higher information efficiency.

Now, the question is under which conditions FL equilibria exist.

Proposition 2. *Suppose there exists $\hat{s} \in S$ such that $v(\hat{s}) < 0$ and there exists n^* such that $E[v(s)|s^{n^*}(\hat{s})] > 0$. Then, for sufficiently small $I/K > 1$, there always exists a FL equilibrium which exhibits the forward looking behavior in period $n^* \in \{1, 2, .., N\}$ in state $\hat{s}$.*

In short, under weak competition, the existence of forward looking equilibria is guaranteed whenever there are signals which imply positive expected payoffs at period $n < N$ but it does not guarantee positive payoffs, i.e., strictly positive option value. Thus, there are many equilibrium outcomes under mild competition from the least efficient to the first best outcome.

Proof. The proof is by construction. Consider the following strategy such that agents exhibit a forward looking behavior at $(s^{n^*}(\hat{s}), h^{n-1})$; that is,

$$\sigma_i(s^n, h^{n-1})(B) = \left\{ \begin{array}{lc} 0 & \text{for } (s^n, h^{n-1}) \text{ s.t. } E[v(s)|s^n] < 0 \\ 0 & \text{for } (s^n(\hat{s}), h^{n-1}) \text{ where } n \leq n^* \text{ and } h^{n-1} \text{ is such that } x^{n-1} = 0 \\ 1 & \text{Otherwise} \end{array} \right\}.$$

I claim there is an equilibrium where all agents follow this strategy.

Notice that except for history $(s^n(\hat{s}), h^{n-1})$ where $x^{n-1} = 0$ for $n \leq n^*$, this strategy is myopic. Thus, to see whether this strategy constitutes a symmetric equilibrium, it is enough to check the incentive to deviate at $(s^n(\hat{s}), h^{n-1})$ where $x^{n-1} = 0$ for $n \leq n^*$.

Let $1(s^n)$ be indicator function which equals 1 for any s^n such that $n > n^*$ and $E[v(s)|s^n] > 0$, 0 otherwise. Then, observe that we have an equilibrium only if there is no incentive to deviate to B at (s^{n^*}, h^{n^*-1}) where $s^{n^*} = s^{n^*}(\hat{s})$ and h^{n^*-1} has $x^{n^*-1} = 0$. That is,

$$E[v(s)|s^{n^*}] \leq \frac{K}{I} \sum_{k=1}^{N-n^*} \sum_{s \in S} v(s) 1(s^{n^*+k}(s)) f(s|s^{n^*}).$$

If $K/I = 1$, the property of $\hat{s}$ implies that this inequality holds strictly. Thus, there always exists small $K/I > 1$ for which these strategies form an equilibrium. The same argument works for with $n < n^*$. Q.E.D.

The intuition of this proof is simple. Under mild competition, if there is a chance that profitability is negative, the benefit from information acquisition can be larger than the risk of losing a potentially profitable opportunity. As a result, a forward looking behavior is possible in equilibrium. The next result is immediate from Proposition 2.

Corollary 1. *Suppose, for any s^{N-1}, there exists $s_N \in S_N$ which makes the value of the opportunity strictly negative. Then, for sufficiently small $I/K > 1$, there exists FL equilibrium where all agents wait until the last period.*

Remark 1. It is important to note that the mild competition is characterized by I/K rather than I. To see the reason, suppose $K = 1$. Then, the mildest competition level is $I/K = 2$ since competition means $K < I$. On the other hand, if $K = 10$, the mildest competition level is $11/10$. Thus, when K is small, there is a limitation to how mild competition can be. In other words, the existence of an FL equilibrium depends on K as well as I.

3.2 Strong Competition

Now we know that, under mild competition, there are many equilibrium outcomes from the least efficient to the first best outcome. In this subsection, I focus on equilibria when competition is strong in the sense that I/K is large. The next result says that, when competition is sufficiently strong, the multiplicity of equilibria vanishes and only the least efficient equilibrium exists.

Proposition 3. *If I/K is sufficiently large, all equilibria are myopic.*

Proof. See appendix.

The proof is by induction. At the final period, it is easy to see that the equilibrium strategy is myopic. Given this fact, we can show that, conditional on reaching stage $N - 1$, the expected payoff from the immediate investment is higher than that from waiting whenever the expected payoff from the immediate investment is strictly positive and competition is sufficiently strong. The intuition is the following. Strong competition lowers the probability of getting a profitable opportunity by waiting. As a result, the benefit from waiting is discounted even though waiting improves the accuracy of decision making. It is not hard to confirm the same argument at earlier periods by similar reasoning. Thus, by induction, this is true for all periods, and agents play myopic strategy.

Next, I analyze how equilibrium forward looking behaviors change as the level of competition increases. Let $\Sigma^*(I,K)$ be the set of all equilibria given (I,K) and $s^n(s)$ denote the projection of s on $\times_{n'=1}^{n} S_{n'}$. Let $S(I,K)$ be the set of states in which some agents exhibit a forward looking behavior in some equilibria given (I,K). That is,

$$S(I,K) = \left\{ s' \in S : \begin{array}{c} \text{(i) } \exists \sigma^* \in \Sigma^*(I,K) \text{ such that } \sigma_i^*(s^n(s'), h^{n-1})(B) < 1 \text{ for some } i \\ \text{(ii) } E[v(s)|s^n(s')] > 0 \end{array} \right\}.$$

The next proposition says that the set of states in which some agents exhibit a forward looking behavior in some equilibria monotonically shrinks as competition becomes stronger.

Proposition 4. *$S(I,K) \subset S(I',K)$ for $I > I'$. Moreover, $\lim_{I\to\infty} S(I,K) = \emptyset$.*

Proof. See appendix.

The intuition behind the proof is similar to that for Proposition 3. First of all, we can show by induction that any forward looking behavior in equilibrium vanishes for sufficiently large I. The more complex part of the proof is to show that if $\hat{s} \notin S(I,K)$, then, $\hat{s} \notin S(I',K)$ for $I' > I$. I show if (i) immediate investment has a positive expected payoff (ii) there is no nondegenerate behavioral strategy equilibrium at any history which includes $s^n(\hat{s})$, then B always dominates W. Next, I show that if B dominates W, then the dominance is preserved for larger I. Intuitively, this is because, given the number of other agents choosing B in the current period, an increase in I does not affect the payoff from B but lowers the payoff from W. Thus, if the dominance of B is established at I, it is preserved for larger I.

Remark 2. Note that Proposition 4 does not imply that efficiency of the most efficient equilibrium is monotonically decreasing in I given K. However, if we focus on symmetric pure strategy equilibria, the effect of competition on efficiency is monotonic. To see the claim, given a symmetric pure strategy FL equilibrium, consider a history in which no one has not invested. Then, observe that, larger K/I makes the value of waiting higher but it does not affect the payoff from immediate investment. Thus, for larger K/I, the FL behavior can be supported on the equilibrium path. On the other hand, by Proposition 4, any FL equilibrium vanishes for sufficiently small K/I. Hence, *efficiency of the most efficient equilibrium is monotonically decreasing in I given K if we focus on pure strategy symmetric FL equilibria.*

When we focus on the symmetric mixed strategy equilibrium, efficiency can be improved by competition in some range. The next example demonstrates the claim.

3.3 Example

Consider a two-period model with one investment opportunity, i.e. $N = 2$ and $K = 1$. Let $C(s_1) = \sum_{s\in S} \max\{v(s), 0\} f(s|s_1)$ be the payoff from W without rationing and $D(s_1) = \sum_{s\in S} v(s)f(s|s_1)$ is the payoff from B without rationing. Henceforth, I suppress s_1 from $C(s_1)$ and $D(s_1)$ for notational simplicity. Notice that C/D measures the relative value of the information s_2. Suppose s_1 is such that $D > 0$; otherwise the analysis is trivial. Let q_I be the behavioral strategy $\sigma_i(s^1, h^0)(B)$ when the number of agents is I. The behavioral strategy q_I solves the following condition in a symmetric FL equilibrium,

$$\sum_{j=0}^{I-1} \begin{pmatrix} I-1 \\ j \end{pmatrix} q_I^j (1-q_I)^{I-1-j} \frac{1}{j+1} D = \frac{1}{I}(1-q_I)^{I-1} C.$$

The condition says the expected payoff from B and the expected payoff from W are same.

We can rewrite the above condition as

$$\sum_{j=0}^{I-1} \begin{pmatrix} I-1 \\ x_1 \end{pmatrix} \left(\frac{q_I}{1-q_I}\right)^j \frac{I}{j+1} = \frac{C}{D}.$$

Then, define $G(I,q) := \sum_{j=0}^{I-1} \begin{pmatrix} I-1 \\ j \end{pmatrix} \left(\frac{q}{1-q}\right)^j \frac{I}{j+1}$. It is easy to see that $G(I,q)$ is strictly increasing in q for all $I \geq 2$. Moreover, since $\lim_{q\to 1} G(I,q) > \frac{C}{D}$, there exists a symmetric

FL equilibrium, which is unique, if and only if $\lim_{q\to 0} G(I,q) < \frac{C}{D}$ or

$$I < \frac{C}{D}.$$

Now, we are interested in the behavior of the equilibrium q with respect to I. That is, what happens to $G(I,q)$ as I increases.

$$\begin{aligned} G(I+1,q) - G(I,q) &= \left(\frac{q}{1-q}\right)^I + \sum_{j=0}^{I-1}\left(\frac{q_I}{1-q_I}\right)^j \left[\binom{I}{j}\frac{I+1}{j+1} - \binom{I-1}{j}\frac{I}{j+1}\right] \\ &= \left(\frac{q}{1-q}\right)^I + \sum_{j=0}^{I-1}\left(\frac{q}{1-q}\right)^j \binom{I}{j+1}\left[\frac{I+1}{j+1} - 1\right]. \end{aligned}$$

Since $\frac{I+1}{j+1} > 1$, $G(I+1,q) - G(I,q) > 0$ for all $q \in (0,1)$. Thus, it is obvious that, for any $I' < I''$ such that $I', I'' < \frac{C}{D}$, we have $q_{I'} < q_{I''}$. Hence, for any I_1, I_2, and I_3 such that $I_1 < I_2 < \frac{C}{D} < I_3$, we have

$$q_{I_2} < q_{I_1} < q_{I_3} = 1.$$

In other words, equilibrium behavior becomes more forward looking as competition increases as long as $I < C/D$ but the strategy jumps to the myopic strategy when $I \geq C/D$. In short, *as long as $I < C/D$, the behavior in the symmetric equilibrium becomes more forward looking as competition increases.*

The next question is whether, in the range $I < C/D$, larger competition increases the probability that the game reaches second period and thus utilizes all information. That is, we want to know the behavior of $(1-q_I)^I$ given s_1, i.e., probability that the game reaches the final period when the expected payoff given s_1 is positive. Notice the equilibrium condition implies

$$(1-q_I)^I = \frac{D}{C}\sum_{j=0}^{I-1}\binom{I-1}{j} q_I^j (1-q_I)^{I-j}\frac{I}{j+1}.$$

Since $\binom{I-1}{j}\frac{I}{j+1} = \binom{I}{j+1}$, we have

$$(1-q_I)^I = \frac{D}{C}\sum_{j=0}^{I-1}\binom{I-1}{j+1} q_I^j (1-q_I)^{I-j}.$$

Then, let $j' = j + 1$ and we have

$$\begin{aligned}(1-q_I)^I &= \frac{D}{C}\sum_{j'=1}^{I}\begin{pmatrix} I-1 \\ j' \end{pmatrix} q_I^{j'-1}(1-q_I)^{I-j'-1} \\ &= \frac{D}{C}\begin{pmatrix} 1-q_I \\ q_I \end{pmatrix}\sum_{j'=1}^{I}\begin{pmatrix} I \\ j' \end{pmatrix} q_I^{j'}(1-q_I)^{I-j'}.\end{aligned}$$

Note that $\sum_{j=1}^{I}\begin{pmatrix} I \\ j' \end{pmatrix} q_I^{j'}(1-q_I)^{I-j'} = 1-(1-q_I)^I$, substituting and rearranging yields

$$(1-q_I)^I = \frac{(1-q_I)D}{q_I C + (1-q_I)D}.$$

Since $C > D$, the denominator of the right hand side is strictly increasing in q_I. Thus, the right hand side is strictly decreasing in q_I. Hence, for any I_1, I_2, and I_3 such that $I_1 < I_2 < \frac{C}{D} < I_3$, we have

$$0 = (1-q_{I_3})^{I_3} < (1-q_{I_1})^{I_1} < (1-q_{I_2})^{I_2}.$$

Thus, *stronger competition improves efficiency as long as the level of competition is in the range* $I < C/D$.

The intuition behind this result is the following. Larger I decreases the payoff from waiting since the probability to get the object decreases. On the other hand, mixing is an equilibrium only if the expected payoff from waiting and investing are the same. Hence, the probability of immediate investment has to be decreased to balance them. Obviously, an additional investor with a positive probability of immediate investment increases the probability of termination of the game at the first period given this strategy. However, this effect is dominated by the new equilibrium randomization which puts a larger probability on W.

4 Private values

In many cases, profitability depends on the characteristics of the agent. This section analyzes our model in the private value setting. Let $T \subset \mathbb{R}$ be a finite set of types with typical element

t. Then, the payoff is determined by a mapping $v : S \times T \rightarrow \mathbb{R}$. Let $g(t)$ denote the probability of t and t_i is drawn randomly from $g(t)$ for each i. In addition to Assumption 1, 2 and 3, I assume that (i) t is independent of s (ii) $v(s,t)$ is strictly increasing in t. Then, a behavioral strategy of agent i is defined as

$$\sigma_i : \bigcup_{n=1,2,..,N} (S^n \times H^{n-1}) \times T \rightarrow \Delta(A_i).$$

I analyze the game with the concept of *Perfect Bayesian Equilibrium (PBE).* Since we are interested in the effect of competition, I focus on symmetric PBE, i.e., $\sigma_i(s^n, h^{n-1}, t_i) = \sigma_j(s^n, h^{n-1}, t_j)$ if $t_i = t_j$.

In the private value setting, I say a strategy is *myopic* if

$$\sigma_i(s_n, h^{n-1}, t) = \begin{cases} B \text{ for } E[v(s,t)|s_n, h^{n-1}] > 0 \\ W \text{ for} E[v(s,t)|s_n, h^{n-1}] < 0 \end{cases}.$$

The next observation clarifies the key difference between the common value and the private value setting.

Observation 2. *Myopic equilibrium does not always exist.*

To see Observation 2, consider s_1' such that $E[v(s_1', t'))|s_1'] > 0$. Suppose all agents play myopic strategy. Now, consider the case where s_2 is so informative that

$$\sum_s \max\{0, E[v(s,t')|s_1', s_2(s)]\} f(s|s_1') > E[v(s_1', t'))|s_1'].$$

Then, whenever I/K is close to 1 and $\sum_{\{t|E[v(s_1',t'))|s_1']>0\}} g(t)$ is small, type t' prefers to wait given signal s_1'. Note that, in the common value setting, all agents take the same action in myopic strategy. Thus, whenever $I > K$, no opportunity is left after a signal which makes the expected value positive. On the other hand, in the private value setting, when $\sum_{\{t|E[v(s_1',t'))|s_1']>0\}} g(t)$ is small and I/K is close to 1, the probability that objects are left in the second period is high. Then, if the vale of information from later periods is sufficiently high, it is profitable to deviate from myopic strategy.

An equilibrium is *monotonic* if, for any t, t' such that $t > t'$, whenever $\sigma_i(s_n, h^{n-1}, t')(B) > 0$, we have $\sigma_i(s_n, h^{n-1}, t)(B) > 0$. The next proposition says that all equilibria share one property.

Proposition 5. *All equilibria are monotonic.*

To provide an intuition, observe that the value of waiting is higher for lower types. This is because the set of signals in which the payoff becomes negative is larger for lower types and the level of loss given a state is always higher for lower types. On the other hand, it is easy to see that the gain from immediate investment is always higher for higher types. Thus, whenever lower types invest in equilibrium, a higher type always prefers to invest.

Proof. See appendix.

The following proposition says that the effect of competition on equilibrium is similar to that in the common value setting.

Proposition 6. *If K/I is sufficiently small, all equilibria are myopic.*

Proof. See appendix.

The proof of this result is similar to that in the common value case.

Now, I analyze efficiency of equilibria. Let $\mathbf{t} = (t_1, t_2, .., t_I)$ and $\hat{g}(\mathbf{t}) = \prod_{i \in \mathcal{I}} g(t_i)$. Moreover, suppose efficiency of the economy I with strategies σ is measured by

$$V(\sigma) = \sum_{\mathbf{t}} \sum_{s} \sum_{i \in \mathcal{I}} v(s, t_i) \chi^i_\sigma(s, \mathbf{t}) \hat{g}(\mathbf{t}) f(s)$$

where $\chi^i_\sigma(s, \mathbf{t})$ is the allocation rule determined by the game given strategy profile σ, i.e., $\chi^i_\sigma(s) = 1$ if agent i gets an investment opportunity in state s and $\chi^i_\sigma(s) = 0$, if the agent does not gets an opportunity.

Let σ_{myo} denote myopic strategy profile.

Observation 3. *If $\max_{t,t'} |v(s, t) - v(s, t')|$ is sufficiently small for any s, $V(\sigma_{myo}) \leq V(\sigma)$ for all $\sigma \in \Sigma^*$.*

By continuity, this result is immediate from Proposition 1 in the common value setting. This implies that if technological difference among firms or willingness to pay among consumers are small enough, the effect of competition is always negative.

The rest of this section investigates the positive effect of competition. In order to introduce a benchmark efficiency level, let σ_{full} be the strategy profile such that, for all i, $\sigma_i(s_n, h^{n-1}, t) = W$ for all $n < N$ and

$$\sigma_i(s_N, h^{N-1}, t) = \begin{cases} B \text{ for } E[v(s,t)|s_N, h^{N-1}] > 0 \\ W \text{ for} E[v(s,t)|s_N, h^{N-1}] < 0 \end{cases}.$$

Moreover, let $S_{1,t} = \{s_1 | E[v(s,t)|s_1] \geq 0\}$, $S_t = \{s | v(s,t) \geq 0\}$, and $\hat{S}_t = \{s \in S_t | s^1(s) \in S_{1,t}\}$ where $s^1(s)$ is the projection of s on S_1. Thus, if $\hat{S}_t = S_t$, $s^1(s)$ perfectly reveals the profitability in state s. Finally, let (T_L, T_H) be a partition of set T where $t \in T_L$ implies $t < t'$ for all $t' \in T_H$.

The next proposition provides a sufficient condition in which competition improves efficiency.

Proposition 7. *Suppose $S_t \neq \emptyset$ for all t. Given a partition (T_L, T_H), if $\sum_{t \in T_L} \sum_{s \in S_{1,t}} f(s)$ and $\sum_{t \in T_H} \sum_{s \in S_t \setminus \hat{S}_t} f(s)$ are sufficiently small and I/K is sufficiently large, then $V(\sigma_{full}) < V(\sigma_{myo})$.*

Proof. See appendix.

To provide an intuition, suppose there are two types, L and H. When I/K is sufficiently large, all agents play myopic strategy. Then, when type H finds the opportunity profitable at period 1 but type L does not, all opportunities are taken at the first period by type H as I/K goes to infinity. On the other hand, when all agents wait until the final period, the probability that type L gets opportunities become positive . Thus, opportunities are allocated to more profitable agents under strong competition. This sorting effects can dominates the negative effect of competition when s_1 is very "informative" signal for type H. This is because when the optimal decision based on s_1 is consistent with the optimal decision based on s with higher probability, inefficiency from the quicker decision making becomes smaller.

5 Discussion

This model can also be interpreted as a model of bounded rationality where agents have a limited information processing capacity, e.g., Lipman (1995), Rubinstein (1998), Dow (1991). Suppose each agent needs "time to think" to obtain the logical implication, e.g., the computation result, given available information. Then, we can interpret s_n as the result of computation based on s_{n-1}. Thus, all agents face a sequence of public signals, $s_1, s_2, .., s_N$. In this formulation, the agent can make a decision based on all the implications of the initial data as long as he spends a sufficient time. On the other hand, the decision must be based only on coarse information in order to make a quicker decision. Thus, this model can be less ad-hoc than models whose coarseness of perception is fixed under any situations.

The prediction of this model is consistent with experimental observations in cognitive science. Experiments show that decision making changes qualitatively under "time pressure." In other words, the way to process information is determined by "economic situation" rather than being fixed. For example, agents are more likely to ignore some information and make decisions based only on cues. Then, when an agent faces a decision problem under time pressure, he tends to make a decision based on coarser information, e.g., Svenson and Maule (1993).

6 Conclusion

This paper has investigated the effect of competition in a preemption game under uncertainty. The following summarizes the basic results.

In the common value setting,

1. The game always has a myopic equilibrium which is the least efficient equilibrium.
2. The game has multiple equilibria with a wide range of efficiency levels under mild competition.
3. All equilibria are myopic under strong competition. Thus, competition causes a deterioration of efficiency.

In the private value setting,

1. All equilibria are monotonic but myopic equilibrium does not always exist.

2. Under strong competition, all equilibria are myopic.

3. Because of the sorting effect, competition can improve efficiency in some cases.

7 Appendix

7.1 Proof of Proposition 3 and 4

First, I prove Proposition 3 by establishing the following lemma.

Lemma 1. *Fix any $\hat{s} \in S(I, K)$. Then, $\hat{s} \notin S(I', K)$ for sufficiently large I'.*

Proof. First, I show that if I is sufficiently large, the equilibrium strategy at $N-1$ must be myopic. Then, I prove that whenever the equilibrium strategies for periods after $N-m$ are myopic, the equilibrium strategy at $N-m$ is myopic as well if I is sufficiently large.

It is obvious that, for all I, agents must play a myopic strategy at $n = N$. So, consider period $N-1$. If the game continues to this period, we must have $K - x^{N-2} > 0$. Let x_{N-1} be $\#\{j \in \mathcal{I} | a_{jN-1} = B\}$ and $\hat{s}^{N-1}$ be the projection of $\hat{s}$ on $\times_{n'=1}^{N-1} S_{n'}$. Then, define $U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ as the expected payoff from B given $x_{N-1}, \hat{s}^{N-1}, x^{N-2}$. So

$$U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) = \min\left\{\frac{K - x^{N-2}}{x_{N-1}+1}, 1\right\} E[v(s)|\hat{s}^{N-1}].$$

Thus, if $E[v(s)|\hat{s}^{N-1}] > 0$, the choice B brings a strictly positive expected payoff for any $x_{N-1} \in \{0, 1, .., I - x^{N-2} - 1\}$.

Since we know that each agent plays a myopic strategy in the next period, the expected payoff from W is simply

$$U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) = \sum_{s \in S} \min\left\{\frac{K - x^{N-2} - x_{N-1}}{I - x^{N-2} - x_{N-1}}, 1\right\} \max\{v(s), 0\} f(s|\hat{s}^{N-1})$$

Notice that W yields 0 if $x_{N-1} \geq K - x^{N-2}$ and a strictly positive payoff for $x_{N-1} < K - x^{N-2}$. Hence

$$U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) > U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) = 0$$

for $x_{N-1} \geq K - x^{N-2}$.

Obviously, $U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ does not depend on I. On the other hand, $U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ is strictly decreasing in I when $x_{N-1} < K - x^{N-2}$. Thus, for sufficiently large I,

$$U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) > U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$$

for $x_{N-1} < K - x^{N-2}$. Hence, $U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) > U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ for all $x_{N-1} \in \{0, 1, .., X - x^{N-2} - 1\}$. Thus, B strictly dominates W at this period if $\hat{s}^{N-1}$ is such that $E[v(s)|\hat{s}^{N-1}] > 0$ and I is sufficiently large. Therefore, the equilibrium strategy at any history in this period is myopic for sufficiently large I.

I claim whenever the equilibrium strategies at all periods after $N - m$ are myopic, the equilibrium strategies at $N - m$ are also myopic for sufficiently large I. As above, the game reaches this period, we must have $K - x^{N-m-1} > 0$. Hence,

$$U(B|x_{N-m}, \hat{s}^{N-m}, x^{N-m-1}) = \min\left\{\frac{K - x^{N-m-1}}{x_{N-m} + 1}, 1\right\} E[v(s)|\hat{s}^{N-m}],$$

and B yields a strictly positive payoff for all x_{N-m} whenever $E[v(s)|\hat{s}^{N-m}] > 0$. On the other hand, when agents follow myopic strategies in future periods, the expected payoff from waiting is

$$U(W|x_{N-m}, \hat{s}^{N-m}, x^{N-m-1}) = \sum_{k=1}^{m}\sum_{s\in S} \min\left\{\frac{K - x^{N-m-1+k} - x_{N-m}}{I - x^{N-m-1+k} - x_{N-m}}, 1\right\} v(s) 1_{\sigma}(s^{N-m+k}(s)) f(s|\hat{s}^{N-m})$$

where $s^{N-m+k}(s)$ is the projection of s on $\times_{n'=1}^{N-m+k} S_{n'}$ and $1_{\sigma}(s^{N-m+k}(s))$ is an index function which assigns 1 when $\sum_{s\in S} v(s) f(s|s^{N-m+k}(s)) > 0$ and 0 otherwise.

Thus, W yields 0 if $x_{N-m} \geq K - x^{N-m-1}$ and a strictly positive payoff for $x_{N-m} < K - x^{N-m-1}$. So,

$$U(B|x_{N-m}, \hat{s}^{N-m}, x^{N-m-1}) > U(W|x_{N-m}, \hat{s}^{N-m}, x^{N-m-1})$$

for $x_{N-m} \geq K - y^{N-m-1}$.

Since $U(B|x_{N-m}, s^{N-m}, x^{N-m-1})$ is independent of I and $U(W|x_{N-m}, s^{N-m}, x^{N-m-1})$ is strictly decreasing in I when $x_{N-m} < K - x^{N-m-1}$, it is easy to see that, for sufficiently large I,

$$U(B|x_{N-m}, \hat{s}^{N-m}, x^{N-m-1}) > U(W|x_{N-m}, \hat{s}^{N-m}, x^{N-m-1})$$

for all $x_{N-m} \in \{0, 1, .., X - x^{N-m-1} - 1\}$. Thus, B strictly dominates W for sufficiently large I when $E[v(s)|\hat{s}^{N-m}] > 0$. Therefore, the equilibrium strategy in any history in this period is myopic when I is sufficiently large.

Therefore, by induction, for sufficiently large I, all equilibria are myopic. Q.E.D.

Now, Proposition 4 is proved by establishing the following lemma.

Lemma 2. *Fix any $\hat{s} \notin S(I, K)$. Then, $\hat{s} \notin S(I', K)$ for $I' > I$.*

I show that a necessary condition for $\hat{s} \notin S(I, K)$ is that, for histories $(\hat{s}^{N-1}, h^{N-2})$ such that $E[v(s)|\hat{s}^{N-1}] > 0$,

$$\sum_{s \in S} \min\left\{\frac{K - x^{N-2}}{I - x^{N-2}}, 1\right\} \max\{v(s), 0\} f(s|\hat{s}^{N-1}) < E[v(s)|\hat{s}^{N-1}]. \qquad \text{(Inequality 1)}$$

To see this claim, suppose it is not true. Then, I can construct a symmetric mixed equilibrium which is forward looking. Let q_{-i} be mixed strategies of all agents except for i. Then, when $q_{-i} = 1$, i.e. all other agents invest immediately, the expected payoffs are

$$\begin{aligned} \lim_{q_{-i} \to 1} U(B|\hat{s}^{N-1}, x^{N-2}) &= \sum_{s \in S} \min\left\{\frac{K - x^{N-2}}{I - x^{N-2}}, 1\right\} v(s) f(s|\hat{s}^{N-1}) \\ \lim_{q_{-i} \to 1} U(W|\hat{s}^{N-1}, x^{N-2}) &= 0. \end{aligned}$$

Thus, $\lim_{q_{-i} \to 1} [U(B|\hat{s}^{N-1}, x^{N-2}) - U(W|\hat{s}^{N-1}, x^{N-2})] > 0$. On the other hand, when $q_{-i} = 0$, i.e. all other agents wait, the expected payoffs are

$$\begin{aligned} \lim_{q \to 0} U(B|\hat{s}^{N-1}, x^{N-2}) &= E[v(s)|\hat{s}^{N-1}] \\ \lim_{q \to 0} U(W|\hat{s}^{N-1}, x^{N-2}) &= \sum_{s \in S} \min\left\{\frac{K - x^{N-2}}{I - x^{N-2}}, 1\right\} \max\{v(s), 0\} f(s|\hat{s}^{N-1}). \end{aligned}$$

Hence, if the inequality in the claim does not hold, then $\lim_{q_{-i} \to 0} [U(B|\hat{s}^{N-1}, x^{N-2}) - U(W|\hat{s}^{N-1}, x^{N-2})] \leq 0$. Thus we can find nondegenerate behavioral strategy which constitutes symmetric equilibrium by the continuity of $U(.|\hat{s}^{N-1}, x^{N-2})$ with respect to q. Hence, we have a contradiction.

Next, I show, for histories with s^n such that $E[v(s)|s^n] > 0$, if Inequality 1 holds, each player must play B at this history.

To see the claim, first observe that, for $x_{N-1} \in \{0, 1, .., K - x^{N-2}\}$, $U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) = E[v(s)|\hat{s}^{N-1}]$ which is constant and $U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ is strictly decreasing in x_{N-1}. Moreover, for $x_{N-1} \geq K - x^{N-2}$,

$$\begin{aligned} U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) &> 0 \\ U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) &= 0. \end{aligned}$$

Now, notice that $U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})|_{x_{N-1}=0} = \lim_{q_{-i}\to 0} U(W|\hat{s}^{N-1}, x^{N-2})$. Thus, Inequality 1 guarantees $U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2}) > U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ for all x_{N-1}. Hence, for any history with $\hat{s}^{N-1}$ such that $E[v(s)|\hat{s}^{N-1}] > 0$, each player must play B.

Thus, we can see that, for any history at which $\hat{s}^{N-1}$ is such that $E[v(s)|\hat{s}^{N-1}] > 0$,each player must play B at this history and thus the agent plays a myopic strategy. Since larger I lowers only $U(W|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$ but not $U(B|x_{N-1}, \hat{s}^{N-1}, x^{N-2})$, it is easy to see that, if $\hat{s} \notin S(I, K)$, then $U(B|\hat{s}^{N-1}, x^{N-2}) > U(W|\hat{s}^{N-1}, x^{N-2})$ for larger I, and thus each player must play B at this history.

We can show that this is also true for earlier periods by essentially the same argument. Hence, by induction, $\hat{s} \notin S(I, K)$ for larger I. Q.E.D.

7.2 Proof of Proposition 5

Let $U(a_n|\sigma_{-i}, s^n, h^{n-1}, t)$ be the expected payoff of type t from action a_n given s^n, h^{n-1} and σ_{-i}. First, since $v(s,t)$ is increasing in t, in any equilibrium, the strategy at period N is monotonic. Then, let $s_N(t)$ be s_N such that $v(s,t) \geq 0$ for all $s \geq s_N(t)$. Since the strategy is monotonic at period N, $s_N(t') \geq s_N(t)$.

Turning to period $N-1$, note that

$$\begin{aligned} U(B|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) &= \sum_{x_{N-1}} \sum_{s} \min\left\{\frac{K - x^{N-2}}{x_{N-1}}, 1\right\} q(x_{N-1}|s^{N-1}, h^{N-2}, \sigma^*_{-i}) v(s,t) f(s|s^{N-1}) \\ U(W|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) &= \sum_{x_N} \sum_{x_{N-1}} \sum_{s} \min\left\{\frac{K - x^{N-1}}{x_N}, 1\right\} q(x_N|s^{N-1}, h^{N-2}, \sigma^*_{-i}) \\ &\quad \cdot q(x_{N-1}|s^{N-1}, h^{N-2}) v(s,t) 1_{\{s_N(s) > s_N(t)\}} f(s|s^{N-1}) \end{aligned}$$

where $q(x_n|s^n, h^{n-1}, \sigma^*_{-i})$ is the probability that the demand is x_n given $s^n, h^{n-1}, \sigma^*_{-i}$.

Then, let $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t) = U(B|\sigma_{-i}^*, s^{N-1}, h^{N-2}, t) - U(W|\sigma_{-i}^*, s^{N-1}, h^{N-2}, t)$. Note that $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t)$ is

$$\sum_{x_{N-1}} \sum_{s} \left[\min\left\{\frac{K - x^{N-2}}{x_{N-1}}, 1\right\} - \min\left\{\frac{K - x^{N-1}}{x_N}, 1\right\} q(x_N|s^{N-1}, h^{N-2}, \sigma_{-i}^*) 1_{\{s_N(s) > s_N(t)\}} \right]$$
$$\cdot q(x_{N-1}|s^{N-1}, h^{N-2}, \sigma_{-i}^*) v(s,t) f(s|s^{N-1}).$$

Now, suppose the strategy is not monotonic in this period, that is, $\sigma_i(s^{N-1}, h^{N-2}, t')(B) > 0$ and $\sigma_i(s^{N-1}, h^{N-2}, t)(B) = 0$. Then, $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t') \geq 0$ and $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t) \leq 0$.

Let $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t|s_N)$ be $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t)$ given s_N. First, observe that, if $s_N \geq s_N(t')$, since $v(s,t') > 0$ and $\min\left\{\frac{K-x^{N-2}}{x_{N-1}}, 1\right\} - \min\left\{\frac{K-x^{N-1}}{x_N}, 1\right\} > 0$, we have $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t'|s_N) \geq 0$. Then, since $v(s,t)$ is strictly increasing in t, $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t|s_N) > \Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t'|s_N)$. If $s_N \in \{s_N(t), s_N(t')\}$, then $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t|s_N) > 0$ and $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t'|s_N) < 0$. Thus, $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t|s_N) > \Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t'|s_N)$. If $s_N \leq s_N(t)$, then the loss that type t' can avoid by waiting is always larger for t' since $v(s,t)$ is strictly increasing in t. Thus, $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t|s_N) > \Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t'|s_N)$. Hence, $\Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t') < \Delta U(\sigma_{-i}^*, s^{N-1}, h^{N-2}, t)$. Therefore, any equilibrium strategy at $N-1$ is monotonic.

Since any equilibrium strategy is monotonic at period $N-1$, we can apply the same argument to establish the claim for period $N-2$. Then, by induction, all equilibrium strategies are monotonic in any period. Q.E.D.

7.3 Proposition 6

Proof. The proof consists of two parts.

Claim 1. For sufficiently large I, there exists myopic equilibrium.

Obviously, all agents follow myopic strategy at period N. Then, consider period $N-1$.

The expected payoff from each action at period $N-1$ is

$$\begin{aligned} U(B|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) &= \sum_{x_{N-1}} \sum_{s} \min\left\{\frac{K - x^{N-2}}{x_{N-1}}, 1\right\} q_m(x_{N-1}|s^{N-1}, h^{N-2}) v(s,t) f(s|s^{N-1}), \\ U(W|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) &= \sum_{x_N} \sum_{x_{N-1}} \sum_{s} \min\left\{\frac{K - x^{N-1}}{x_N}, 1\right\} q_m(x_N|s^{N-1}, h^{N-2}) \\ &\quad \cdot q_m(x_{N-1}|s^{N-1}, h^{N-2}) v(s,t) 1_{\{s_N(s) > s_N(t)\}} f(s|s^{N-1}) \end{aligned}$$

where

$$\begin{aligned} q_m(x_n|s^n, h^{n-1}) &= \binom{I - x^{n-1}}{x_n} \lambda(s^n, h^{n-1})^{x_n} (1 - \lambda(s^n, h^{n-1}))^{I - y_{n-1} - x_n}, \\ \lambda(s^n, h^{n-1}) &= \sum_{\{t : E[v(s,t)|s^n, h^{n-1}] > 0\}} g(t|h^{n-1}). \end{aligned}$$

Observe that, whenever $E[v(s,t)|s^{N-1}, h^{N-2}] > 0$, the expected payoff from W given $x_{N-1} \geq K - x^{N-2}$ is 0. Now, note that, by choosing large I, we can make

$$\sum_{x_{N-1} > K - x_{N-2}} q_m(x_{N-1}|s^{N-1}, h^{N-2})$$

arbitrarily close to 1. Thus, if $U(B|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) > 0$ and I is sufficiently large, $U(W|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) < U(B|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t)$. Then, there is no incentive to deviate from myopic strategy. The same argument holds for any period if I is sufficiently large.

Claim 2. For sufficiently large I, all equilibria are myopic.

Since all agents follow myopic strategy at period N, consider period $N-1$. Suppose type t does not follow myopic strategy in equilibrium. Then, for some (s^{N-1}, h^{N-2}) such that $E[v(s,t)|s^{N-1}, h^{N-2}] > 0$, type t plays W.

First, when some type plays B with positive probability given (s^{N-1}, h^{N-2}), the probability that all objects are taken at period $N-1$ becomes arbitrarily large by choosing large I. Thus, $U(B|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t) > U(W|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t)$.

Second, suppose no type plays B at (s^{N-1}, h^{N-2}). Observe that, by choosing large I, $U(W|\sigma^*_{-i}, s^{N-1}, h^{N-2}, t)$ becomes arbitrarily small. Since the expected payoff from deviation, B, is independent of I and positive, it is profitable to deviate when I is sufficiently large.

Hence, when I is sufficiently large, any strategy which assigns positive probability on W given such (s^{N-1}, h^{N-2}) cannot be equilibrium.

Thus, any equilibrium strategy at $N-1$ is myopic for sufficiently large I/K. Then, we can prove that any equilibrium strategy at $N-2$ is myopic for sufficiently large I/K. By induction, the same argument holds for any period. Q.E.D.

7.4 Proof of Proposition 7

Let $V(\sigma|s)$ be $V(\sigma)$ given s. Note that when $\sum_{t\in T_L}\sum_{s\in S_{1,t}} f(s) = 0$, this is the same as $\sum_{t\in T_L} S_{1,t} = \emptyset$. Thus, first, I establish the above statement when $\sum_{t\in T_L} S_{1,t} = \emptyset$.

Case 1-A. $s \in \bigcup\limits_{t\in T_H} S_t$, $s \in \bigcup\limits_{t\in T_L} S_t$ and $s_1 \in \bigcup\limits_{t\in T_H} S_{1,t}$. Suppose all agents follow myopic strategy. Then, as I/K goes to infinity, all opportunities are taken by some types in T_H. On the other hand, if all agents wait until the last period, all agents invest and opportunities can be acquired by some types in T_L. Thus, $V(\sigma_{full}|s) < V(\sigma_{myo}|s)$.

Case 1-B. $s \in \bigcup\limits_{t\in T_H} S_t$, $s \in \bigcup\limits_{t\in T_L} S_t$ and $s_1 \notin \bigcup\limits_{t\in T_H} S_{1,t}$. Suppose all agents follow myopic strategy. Then, opportunities can be acquired by some types in T_L. If all agents wait until the last period, all agents invest and opportunities can be acquired by some types in T_L.

Case 2-A. $s \in \bigcup\limits_{t\in T_H} S_t$, $s \notin \bigcup\limits_{t\in T_L} S_t$ and $s_1 \in \bigcup\limits_{t\in T_H} S_{1,t}$. Suppose all agents follow myopic strategy. Then, as I/K goes to infinity, all opportunities are taken by some types in T_H. If all agents wait until the last period, all opportunities are obtained by some type in T_H. Thus, as I/K goes to infinity, $V(\sigma_{full}|s) = V(\sigma_{myo}|s)$.

Case 2-B. $s \in \bigcup\limits_{t\in T_H} S_t$, $s \notin \bigcup\limits_{t\in T_L} S_t$ and $s_1 \notin \bigcup\limits_{t\in T_H} S_{1,t}$. Suppose all agents follow myopic strategy. Then, opportunities can be obtained by some types in T_L. On the other hand, if all agents wait until the last period, all opportunities are obtained by some types in T_H. Thus, $V(\sigma_{full}|s) \geq V(\sigma_{myo}|s)$.

Case 3-A. $s \notin \bigcup\limits_{t\in T_H} S_t$, $s \notin \bigcup\limits_{t\in T_L} S_t$ and $s_1 \in \bigcup\limits_{t\in T_H} S_{1,t}$. Suppose all agents follow myopic strategy. Then, as I/K goes to infinity, all opportunities are taken by some types in T_H. On the other hand, if all agents wait until the last period, no one takes any opportunities. Thus, $V(\sigma_{full}|s) > V(\sigma_{myo}|s)$.

Case 3-B. $s \notin \bigcup\limits_{t\in T_H} S_t$, $s \notin \bigcup\limits_{t\in T_L} S_t$ and $s_1 \notin \bigcup\limits_{t\in T_H} S_{1,t}$. Suppose all agents follow myopic strategy. Then, no one invests at the initial period but, at some periods, opportunities can be obtained by some types. On the other hand, if all agents wait until the last period, no one takes any opportunity. Thus, $V(\sigma_{full}|s) > V(\sigma_{myo}|s)$.

Observe that the probability that we have case 1-B, 2-B, 3-A and 3-B goes to 0 as $\sum_{t\in T_H}\sum_{s\in S_t\setminus\hat{S}_t} f(s)$ goes to 0. Thus, if I/K is sufficiently large and $\sum_{t\in T_H}\sum_{s\in S_t\setminus\hat{S}_t} f(s)$ is sufficiently small, $V(\sigma_{full}) < V(\sigma_{myo})$. Finally, by continuity, the argument holds when $\sum_{t\in T_L}\sum_{s\in S_{1,t}} f(s)$ is sufficiently small. Q.E.D.

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