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ALLAIS CHARACTERISATION OF PREFERENCE STRUCTURES AND THE STRUCTURE OF DEMAND

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Abstract

Concepts like complementarity and substitution have intuitive appeal. One would like to use them either to evaluate estimated responses in demand systems as to their plausibility or to incorporate them in the estimation or the specification of demand. The usual formal counterparts of these notions turn out to have some defects. Those proposed by Allais (1943) are free of these. This paper traces their relation to the coefficients of regular and inverse demand systems. It also investigates their use to characterise separability of preferences. A similar type of characterisation can be formulated for preference shifting factors. A numerical example is supplied to illustrate how Allais coefficients can be retrieved from coefficients of a demand system.

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1. Introduction

It is a postulate of empirical economics that model constants should be readily interpretable entities. The justification of this position is not so much their probable stability across different observation units. There is very little known about the true functional form of economic relations. A more important reason is the ability to formulate prior ideas about the sign and perhaps even about the size of coefficients to be estimated from the data. Such prior ideas may be based on introspection, casual observation or comparable empirical studies. Introspection, intuition, is usually in terms of relatively simple concepts.

In view of shortcomings of the data and specification errors in the models pure estimates are frequently not very reliable. The usual standard errors may only partly reflect their lack of precision. Prior information about the value of the coefficients is needed to form an opinion about the plausibility of the outcomes. This prior information more often than not is based on intuition. Apart form being used in an informal or formal test it can also be incorporated in the estimation procedure, for example, in a Bayesian approach.

In demand analysis, the concepts of complementarity, substitution and independence between commodities seem at first sight to be straightforward and close to intuition. If more of good x enhances the desirability of good y one speaks of complementarity. If more of x reduces the attractiveness of y, goods x and y are said to be substitutes. The neutral position is that of independence. Examples are easy to find: wine and beer are substitutes, wine and cheese are complements, while wine and shoes, say, are mutually independent. Adjectives like strong and weak can be applied quite naturally to substitution and complementarity.

In view of the intuitive appeal of these notions one might have thought that they would be playing a major role in empirical demand analysis. This, however, is not the case. Even in the rare case that they are used it is not quite in accordance with the intuitive meaning just given. The least one can say is that there is a considerable degree of confusion.

This leads Samuelson (1947) to consider the concept of complementarity (and substitution and independence) as being essentially unimportant, a statement which contrasts the one by Houthakker (1960), who considers the analysis of substitution and complementarity to be 'one of the most cherished achievements of consumption theory'.

It is clear that there is some need for a definition of complementarity and substitution that agrees well with intuition and at the same time can be useful for demand analysis. It is the contention of this paper that the mathematical expression given by Allais (1943) to characterise these concepts meets these requirements. At the same time it can be easily employed to identify groupwise interaction structures. Furthermore, a comparable formulation can be given to the effects of preference changing variables such as health or age.

The next section takes up two of the most commonly used expressions for complementarity and substitution and discusses their defects. This clears the way for the introduction of the Allais coefficients in Section 3. Section 4 shows how these Allais coefficients can be found back in a regular demand system, i.e. a system which explains the quantities demanded as a function of the budget and the prices. Section 5 does the same for an inverse demand system in which the prices are being explained and the quantities are given, next to the budget. The role of these coefficients to characterise separability structures of preferences is discussed in Section 6. The next section proposes a characterisation of the effects of other determinants than the budget, prices or quantities quite similar to the Allais one. A numerical example is next given to help forming an idea of the possibilities and limitations of the approach. Some concluding remarks end the paper.

2. Complementarity, substitution and independence

To set the stage for the discussion we will equip the consumer with an (at least) twice differentiable strongly quasi-concave utility function representing a well behaved preference order over the n-dimensional compact commodity space. Let this utility function be $u(q)$, with first-order derivatives $u_i(q)$, also known as the marginal utility of good i, and second-order derivatives $u_{ij}(q)$.

Associating the marginal utility u_i with desirability one can define complementarity by the positive sign of u_{ij}: the desirability of i increases if more of j is available. Substitution corresponds with a negative u_{ij} and independence with $u_{ij} = 0$. Unfortunately, the value of u_{ij} and its sign is not invariant under monotone increasing transformations of the utility function. Otherwise said, the sign of u_{ij} is not determined by the preference order but depends on a particular representation of the preference order. This sign can then obviously not be used to characterise the structure of preferences.

Is is of some importance for what follows to derive the lack of invariance of u_{ij}. Let

(2.1) $$v(q) = F(u(q))$$

be a monotone increasing, i.e. orderpreserving, twice differentiable transformation of u(q). Both v(q) and u(q) are equally valid representations of the same preference order. Hence,

(2.2) $$F' = dv/du$$

is strictly positive. Correspondingly, one has

(2.3) $$v_i(q) = \partial v(q)/\partial q = (dv/du)(\partial u(q)/\partial q_i) = F'u_i(q)$$

The marginal utilities change proportionally in the transition from u(q) to v(q) but they preserve their positive sign. Next let:

(2.4) $$F'' = dF'/du = d^2v/du^2$$

be the second-order derivative of the transformation. Its sign depends on the nature of the transformation and does not depend on the preference order. One then has as the counterpart of u_{ij}

(2.5) $$\begin{aligned} v_{ij}(q) &= F'\partial u_i(q)/\partial q_j + F''u_i(q)u_j(q) \\ &= F'u_{ij}(q) + F''u_i(q)u_j(q). \end{aligned}$$

Given the positive nature of F' the first term has the same sign as $u_{ij}(q)$. The presence of F" in the second term makes its sign dependent on the transformation. This then is also true for v_{ij}. The second-order derivatives are not adequate representations of interactions among goods in the preference order.

There is a strong tradition in demand analysis to work with invariant concepts only. Indeed, the utility function is not necessary to derive the main results of demand theory. Only properties of the preference order and of the budget set matter. Consequently, properties of the utility function that are not invariant do not play a role, are irrelevant. They do not leave a trace in observable demand behaviour. Once this was realised the use of (the sign of) the u_{ij} to characterise preference interactions was abandoned. A search was set in to find an invariant way to represent the notion of complementarity, substitution and independence.

The best known characterisation is the one attributed to Allen and Hicks. Let $f_i(m,p)$ be a regular demand function, explaining the quantity demanded of good i as a function of the total budget, m, and of the vector of all prices $p' = (p_i, \ldots, p_n)$. Next let

$$(2.6) \qquad k_{ij} = [\partial f_i(m,p)/\partial p_j]_{u\ constant}$$

be the income compensated price effect, also known as the Slutsky-effect. The negativity of demand implies $k_{jj} < 0$. Now, for $i \neq j$, a positive value of k_{ij} means that an increase in the price of j leads to an increase in the demand for i to substitute for the drop in the demand for j. Hence $k_{ij} > 0$ characterises substitution. If the drop in the demand for j, because of the increase in its price, entails also a drop in the demand for i, goods i and k move in a parallel fashion, are complements. Hence, $k_{ij} < 0$ is associated with complementarity. Clearly, $k_{ij} = 0$ is associated with independence.

There are several advantages to this choice. In principle, the k_{ij} can be easily measured. Complementarity and substitution are symmetric concepts in the sense that if i is a substitute for j, j is also a substitute for i, while the same holds for complementarity. The k_{ij} are also symmetric in i and j. The k_{ij} describe changes in the composition of demand bundles which occupy the same position in the preference order. They reflect the structure of the preference order. They are invariant.

There are certain disadvantages too. The adding up condition of demand states that $\Sigma_i p_i k_{ij} = 0$ while the homogeneity condition implies that $\Sigma_j k_{ij} p_j = 0$. Given the property that $k_{ii} < 0$ there must be a dominance of positive k_{ij} because prices are taken to be strictly positive. In the case of two commodities k_{12} is always positive irrespective of what intuition says about their mutual interaction. Houthakker (1960) considers the relative dominance of substitutuin a 'minor blemish', precisely because it appears to contradict intuition. Another problem is the negativity of k_{ii}. Since a commodity is its own perfect substitute a positive value of k_{ii} would have been more natural.

Furthermore, the adding-up condition and the homogeneity condition reflect the presence of an effective budget constraint. In fact, the k_{ij} only arise as the result of selecting the most preferred bundle on the frontier of the budget set. Intuitive notions about preference interactions are part of the theory of choice which is, to quote from Frisch (1959), 'assumed to be independent of the particular organisational form of the market'. Otherwise said the k_{ij} are not fundamental, not general enough to be used to characterise preference structures. Of course, they reflect such a preference structure but in an imperfect and possibly misleading way.

There have been other proposals for the characterisation of the interactions. With the exception of the Allais coefficients they share some of the disadvantages of the k_{ij}. We will meet one alternative, the sign of the elements of the Antonelli matrix, when discussing inverse demand in Section 5. However, it is appropriate to turn our attention now to the formulation of Allais.

3. The Allais coefficients

One way to derive the Allais coefficients is to start off from (2.5), rewritten here as

$$(3.1)\qquad v_{ij} = F'u_{ij} + F''u_i u_j$$

As noted when this expression was derived the sign of the second component was the source of the possible lack of correspondence in the sign of v_{ij} and u_{ij}. To handle this issue, first divide both sides of (3.1) by $v_i v_j$ to obtain

$$(3.2)\qquad \frac{v_{ij}}{v_i v_j} = \frac{F'u_{ij}}{F'u_i F'u_j} + \frac{F''u_i u_j}{F'u_i F'u_j} = \frac{u_{ij}}{F'u_i u_j} + \frac{F''}{F'}$$

Use has been made of (2.3). The second component has been reduced to a constant independent of i and j. Next, take the difference between the left-hand side of (3.2) and $v_{rs}/v_r v_s$ where r, s is another pair of commodities:

$$(3.3)\qquad \frac{v_{ij}}{v_i v_j} - \frac{v_{rs}}{v_r v_s} = \frac{1}{F'}\left[\frac{u_{ij}}{u_i u_j} - \frac{u_{rs}}{u_r u_s}\right]$$

The sign of this difference is invariant. We would next like to get rid of the F'. For this purpose multiply both sides of (3.3) by $\Sigma_h v_h q_h = F'\Sigma_h u_h q_h >$ 0 to obtain the Allais coefficients:

$$(3.4)\qquad a_{ij} = \Sigma_h v_h q_h \left[\frac{v_{ij}}{v_i v_j} - \frac{v_{rs}}{v_r v_s}\right] = \Sigma_h u_h q_h \left[\frac{u_{ij}}{u_i u_j} - \frac{u_{rs}}{u_r u_s}\right]$$

The a_{ij} are clearly invariant. They are moreover free of units of measurement, as is not too difficult to verify.

The sign of the a_{ij} is determined already in (3.3). Note that the choice of r, s is free. Let it be some standard pair and define its interaction to be neutral. One may write

$$(3.5) \qquad a_{ij} = \mu(q)\, u_{ij}/(u_i u_j) - \alpha(q)$$

with $\mu(q) = \Sigma_h u_h q_h$ and $\alpha(q) = \mu(q) u_{rs}/(u_r u_s)$. One also has

$$(3.6) \qquad u_{ij} = u_i u_j a_{ij} \,/\, \mu(q) + \alpha(q) u_i u_{ij}$$

Consider a change in the marginal utility of good i:

$$du_i = \Sigma_j u_{ij} dq_j = (u_i/\mu(q))\, \Sigma_j a_{ij} u_j dq_j$$

$$+ \alpha(q) u_i \Sigma_j u_j dq_j$$ or in relative terms

$$(3.7) \qquad d\ln u_i = (1/\mu(q))\, \Sigma_j a_{ij} (u_j q_j)\, d\ln q_j + \alpha(q) du$$

$$= \Sigma_j\, a_{ij} \nu_j\, d\ln q_j + \alpha(q)\, du$$

with $\nu_j = u_j q_j \,/\, \mu(q) = u_j q_j \,/\, (\Sigma_h u_h q_h) > 0$. Observe that $\Sigma \nu_k = 1$ and that the ν_j are invariant.

Expression (3.7) shows that the relative change in the marginal utility of good i can be decomposed in a part, $\alpha(q)du$, which is general and not invariant and a part which is invariant and specifically involves the i, j interactions. Here, the a_{ij} capture the impact of a (relative) change in q_j, weighted by ν_j or the relative desirability of j as represented by its marginal utility. It is then natural to associate positive a_{ij} with complementarity, negative a_{ij} with substitution and zero a_{ij} with independence. There is no formal objection against requiring a_{ii} to be negative.

The a_{ij} represent the type of interaction in terms of a difference from that of a standard pair. Changing the standard pair will change the a_{ij}. While the sign of the u_{ij} depends on the rather arbitrary choice of the utility indicator, the sign of the a_{ij} depends on the choice of the standard pair. It appears, however, to be easier to identify a neutral, independent, pair than to identify a particular utility indicator as the appropriate one.

Summing up, one may say that the a_{ij} can describe the interaction among commodities in the preference order in a way that comes close to one's intuition. When one says that cheese makes wine more attractive, it may be taken to mean that more of cheese makes wine more attractive than more of shoes and hence that cheese and wine are complements. An analogous statement can be made about beer and wine being more substitutable than shoes and wine. The choice of the standard pair is admittedly crucial but on first sight not too difficult.

As was mentioned earlier the a_{ij} are free of units of measurement. This still leaves their order of magnitude open. From (3.4) or (3.5) one can say very little about this. The a_{ij} can differ considerably from pair to pair or from the corresponding a_{ii} and a_{jj}. To make prior statements about weak or strong degrees of interaction in terms of values of the a_{ij} is then not too easily feasible. Allais introduces therefore the interaction <u>intensities</u> defined as

$$(3.8) \qquad a^{*}_{ij} = a_{ij}/\surd(a_{ii}\; a_{jj})$$

where it is assumed that the a_{ii} are negative. Thus a^{*}_{ii} = -1. which characterises perfect substitution. Is is then natural to require the a^{*}_{ij} to be on the interval (-1, +1) with +1 representing perfect complementarity.

A word about the ν_j in (3.7). As already said the ν_j are invariant, positive and add up to one. The second law of Gossen defines the consumer equilibrium as the proportionality of the vector of marginal utilities, u_q, with that of prices:

$$(3.9) \qquad u_q = \lambda p$$

where λ is a positive factor of proportionality, interpretable as the marginal utility of the budget. If (3.9) holds $u_j q_j = \lambda p_j q_j$ and $\nu_j = p_j q_j/(\Sigma_h p_h q_h) = w_j$, the share of expenditure on j out of the total budget m $= \Sigma_h p_h q_h$. The ν_j obviously represents the willingness of the consumer to spend on commidity j. One can also say that ν_j expresses the importance of commodity j for the choice problem of the consumer.

One can organise the a_{ij} in a n x n matrix A. It follows from (3.5) that

$$(3.10) \qquad A = \mu(q)\, \grave{u}_q^{-1}\, U\, \grave{u}_q^{-1} - \alpha(q)\iota\iota'.$$

The use of ` over a vector indicates a diagonal matrix with the elements of the vector on the diagonal. This convention is also employed elsewhere in this paper. Since the axiom of desirablity requires that all elements of u_q are strictly positive $\grave{u}_q^{-1}$ is defined. Here, and later on too, ι is a vector of all elements equal to one. The assumption of strong quasi-concavity of the utility function requires - see Barten and Böhm (1982) -

$$(3.11) \qquad x'U\,x < 0 \text{ for all } x \text{ such that } u_q'x = 0$$

By defining $y = \grave{u}_q x$ one obtains

$$(3.12) \qquad y'Ay = \mu(q)\, x'Ux - \alpha(q)(x'u_q)^2$$

which is negative for all x such that $u_q'x = 0$ or, equivalently, for all y such that $\iota'y = 0$. Condition (3.11) implies that the rank of the matrix U is at least n-1. Full rank of the Hessian matrix of the utility function cannot be guaranteed for all possible representations of the preference order. The strong quasi-concavity condition also requires A to be at least of rank n-1, but the property of full rank of A is a property of the preference order. Its validity can be empirically verified, at least in principle. A further strengthening of the properties of A would be to require it to be negative definite.

4. Allais coefficients and the specification of a regular demand system

Basically, there are two types of demand systems. One, the regular system explains the quantities consumed, q, as a function of m, the budget and p, the prices. The other system explains the relative prices one is willing to pay for a given bundle of quantities and a fixed budget. This is the inverse demand system. Its specification is taken up in the next section. Here the focus will be on the regular demand system.

Starting off from (3.9) one has

$$\text{(4.1)} \qquad d\ln u_q = d\ln p + (d\ln\lambda)\,\iota$$

It follows from (3.7) that

$$\text{(4.2)} \qquad d\ln u_q = A\hat{w}\,d\ln q + (\alpha du)\,\iota$$

where use is made of $\nu_j = w_j$ and $\hat{w}$ is the diagonalisation of the vector of budgetshares w. Combining (4.1) and (4.2) one obtains

$$A\hat{w}\,d\ln q = (d\ln\lambda - \alpha du)\iota + d\ln p$$

or, assuming full rank of A.

$$\text{(4.3)} \qquad \hat{w}\,d\ln q = A^{-1}(d\ln\lambda - \alpha du) + A^{-1}d\ln p$$

The quantities demanded have to satisfy the budget:

$$p'q = m$$

which in differential logarithmic form can be written as

$$\text{(4.4)} \qquad d\ln m = w'd\ln q + w'd\ln p$$

Since $w'd\ln q = \iota'\hat{w}\,d\ln q$ (4.3) and (4.4) can be combined to yield

$$d\ln\lambda - \alpha du = (\iota'A^{-1}\iota)^{-1}\,[(d\ln m - w'd\ln p) - \iota A^{-1}d\ln p)]$$

Using this result in (4.3) gives

$$(4.5)\qquad \hat{w}\,\mathrm{dln}q = A^{-1}\iota(\iota'A^{-1}\iota)^{-1}(\mathrm{dln}m - w'\mathrm{dln}p) + [A^{-1} - A^{-1}\iota(\iota'A^{-1}\iota)^{-1}\iota'A^{-1}]\,\mathrm{dln}p$$

$$= b(\mathrm{dln}m - w'\mathrm{dln}p) + S\mathrm{dln}p$$

with

$$(4.6)\qquad b = A^{-1}\iota(\iota A^{-1}\iota)^{-1}$$

$$(4.7)\qquad S = A^{-1} - A^{-1}\iota(\iota A^{-1}\iota)^{-1}\ \iota A^{-1}$$

System (4.5) with b and s constant is precisely the specification of a demand system proposed by Theil (1965). It later became known as the Rotterdam system.

The b_i are the marginal propensities to spend the budget on good i. As is easily seen from (4.6) $\iota'b = 1$. The matrix S is a simple transformation of K, the matrix of Slutsky coefficients k_{ij}, as given by (2.6):

$$(4.8)\qquad S = (1/m)\hat{p}K\hat{p}$$

It is clear that $s_{ij} = p_i k_{ij} p_j/m$ has the same sign as k_{ij}. Expression (4.7) provides the link between the Allais and the Hicks-Allen characterisation. This relation is not very straightforward, in the sense that there is no simple correspondence between the signs of the s_{ij} and the corresponding a_{ij}.

Equation (4.7) expresses S as a function of A. For practical purposes the inverse relation is of some interest. Estimation of (4.5) yields estimates of b and S which can be used to obtain values for A. These could be evaluated for their plausibility. In this indirect way the plausibility of the estimates for S and b can be analysed.

Let

$$(4.9)\qquad \varphi = \iota'A\iota$$

Then

$$(4.10) \qquad b = (\iota/\varphi)\, A^{-1}\iota$$

and

$$(4.11) \qquad S = A^{-1} - \varphi bb'$$

which can also be written as

$$(4.12) \qquad AS + \iota b' = I$$

As is obvious from (4.9) through (4.11) $\iota'b = 1$, $\iota'S = 0$. One can combine these results into the following expression

$$(4.13) \qquad \begin{bmatrix} A - \frac{1}{\varphi}\iota\iota' & \iota \\ \iota' & 0 \end{bmatrix} \begin{bmatrix} S & b \\ b' & 0 \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & 1 \end{bmatrix}$$

Let M be the nxn NW block of the inverse of the second matrix in (4.13). Then

$$(4.14) \qquad A = M + \frac{1}{\varphi}\iota\iota'$$

Here $1/\varphi$ is unknown. One may select its value in such a way that $a_{rs} = 0$. With e_r being the r-th column of the identity matrix, then $(1/\varphi) = -e_r'Me_s$

and

$$(4.15) \qquad A = M - e_r'Me_s\iota\iota'$$

This expression is rather straightforward. One calculates M and subtracts from all its elements the value of $e_r'Me_s$. There is one degree of freedom which is used up by the determination of the standard pair. Otherwise said, given observed values for S and b, A cannot be determined unless one adds as an identifying restriction or normalisation that $a_{rs} = 0$. For that matter one may also choose for a_{rs} another value than zero. As a corrolary to this statement one has that S and b are invariant for the choice of the standard pair and the value of that interaction.

5. Allais coefficients and the specification of an inverse demand system

Inverse demand systems explain the relative prices a consumer is willing to pay given his budget m and the quantities of the commodities. Inverse demand occurs, for example, in the case of quickly perishable goods like fresh vegetables and fresh fish, where the supply is basically fixed and the supplier is a price taker.

The dependent variable in inverse demand relations is usually taken to be the normalised price vector

$$\pi = (1/m)p \tag{5.1}$$

Here π_i is the fraction of the budget paid for one unit of good i. Note that it follows from $p'q = m$ that $\pi'q = 1$. The consumer equilibrium (3.9) can be expressed in terms of π as

$$u_q = \lambda m\pi = \mu(q)\pi \tag{5.2}$$

or as

$$\pi = (1/\mu(q))\ u_q$$

Take differentials

$$\begin{aligned} d\pi &= (1/\mu(q))(-\ \pi u_q'dq + (I-\pi q')du_q) \\ &= -\ \pi\pi'dq + (I-\pi q')(1/\mu(q))Udq \end{aligned}$$

A minor rearrangement yields the inverse demand system in differential form

$$\begin{aligned} d\pi &= -\ (\pi-(I-\pi q')(1/\mu(q))Uq)\pi'dq \\ &\quad + (I-\pi q')(1/\mu(q))U(I-q\pi')dq \\ &= g\pi'dq + Gdq \end{aligned} \tag{5.3}$$

with

$$(5.4) \qquad g = -\pi + (I-\pi q')(1/\mu(q))Uq$$

and

$$(5.5) \qquad G = (I-\pi q')(1/\mu(q))U(I-q\pi')dq$$

The change in prices is explained as the result from two shifts. The first one, $g\pi'dq$, is a scale effect. It represents the move from one indifference surface to the other. The second one, Gdq, represents the move along an indifference surface - see Anderson (1980).

The matrix G is known as the Antonelli matrix. It is the counterpart of the Slutsky matrix of regular demand systems. It also is a symmetric matrix and its diagonal elements are negative. Its rank is likewise n-1.

The signs of the elements of G are sometimes also used to characterise interactions of the complementarity/substitution type. If goods i and j are substitutes more of good i reduces the price one is willing to pay for good j. Substitution means then $g_{ij} < 0$. A good being its own substitute corresponds then nicely with $g_{ii} < 0$. Complementarity corresponds with $g_{ij} > 0$.: more of good i makes good j more attractive and increases the price one is willing to pay for it. However, complementarity will dominate. As is easily checked $Gq = 0$ and $q'G = 0$. With negative g_{ii} and positive q there must be at least one complementarity interaction even when intuition would consider all goods to be substitutes. This dominance of complementarity is of the same nature as the dominance of substitution in the case of the Allen-Hicks definition. The signs of the Antonelli coefficients are equally unsuitable as characterisations of preference interactions.

To establish the relation between the Allais matrix and the Antonelli matrix it is convenient to first transform the latter by multiplying its elements by $q_i q_j$:

$$(5.6) \qquad H = \dot{q}G\dot{q} = (I-w\iota')(1/\mu(q))\dot{q}U\dot{q}(I-\iota w')$$

and correspondingly to work with

$$(5.7) \qquad h = \dot{q}g = -w + (I-w\iota')(1/\mu(q))\dot{q}Uq$$

Note that use is made of $\dot{q}\pi = w$. Note also that $q'\pi = 1$. It follows from (3.10) and from (5.2) that

$$(5.8) \qquad U = (1/\mu(q)[\dot{u}_q A\dot{u}_q - \alpha(q)u_q u_q'] \\ = \mu(q)[\dot{\pi}A\dot{\pi} - \alpha(q)\pi\pi']$$

Inserting this result in (5.6) and (5.7) results in

$$(5.9) \qquad H = (\hat{w} - ww')$$

$$(5.10) \qquad h = -w + (\hat{w} - ww')Aw$$

which can also be expressed as

$$(5.11) \qquad \begin{pmatrix} H & h \\ h' & 0 \end{pmatrix} = \begin{pmatrix} \hat{w}-ww' & -w \\ w' & 1 \end{pmatrix} \begin{pmatrix} A-(2+w'Aw)\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} \begin{pmatrix} \hat{w}-ww' & w \\ w' & 1 \end{pmatrix}$$

or equivalently as

$$(5.12) \qquad \begin{pmatrix} A-(2+w'Aw)\iota\iota' & \iota \\ \iota' & 0 \end{pmatrix} = \begin{pmatrix} \hat{w}^{-1} & \iota \\ -\iota' & 0 \end{pmatrix} \begin{pmatrix} H & h \\ h' & 0 \end{pmatrix} \begin{pmatrix} \hat{w}^{-1} & -\iota \\ \iota' & 0 \end{pmatrix}$$

As is evident from the last expression, given structures of H and h and values of w the Allais matrix is determined apart from an additive constant. By selecting a standard pair of goods r and s and assigning to the corresponding a_{rs} a value, zero, say, one can solve this lack of determination. The resulting values for the other a_{ij} can then be used to evaluate the extent to which the measured interaction corresponds with one's prior ideas.

Relation (5.11) is useful to trace the consequences of special structures of A for the specification of H. A particular type of special structure is the subject of the next section.

6. Seperability of preferences

The separability of the stucture of preferences is a source of restrictions on the Allais coefficients and hence on the demand function. Separability assumes a partition of the set of all n goods into N non-overlapping subsets of goods such that the preference order defined on a subset is independent of the consumption levels of goods not in the subset.

Write

(6.1) $\quad q' = (q_A', q_B', \ldots, q_N')$

for the partition of the quantity vector q into N subvectors. Let n_F be the number of goods in subset F and let S_F be the index set of the goods of subset F. Separability implies that the utility function can be written as

(6.2) $\quad u(q) = z(u_A(q_A), u_B(q_B), \ldots, u_n(q_N))$

One has then

(6.3) $\quad u_i(q) = \dfrac{\partial z}{\partial u_F}\dfrac{\partial u_F}{\partial q_i} \qquad i \in S_F$

and for $i \in S_F$, $j \in S_G$, $F \neq G$

(6.4) $\quad u_{ij} = \dfrac{\partial^2 z}{\partial u_F \partial u_G}\dfrac{\partial u_F}{\partial q_i}\dfrac{\partial u_G}{\partial q_j} = \zeta_{FG} u_i u_j$

with

(6.5) $\quad \zeta_{FG} = \dfrac{\partial^2 z}{\partial u_F \partial u_G} \Big/ \left[\dfrac{\partial z}{\partial u_F}\dfrac{\partial z}{\partial u_G}\right]$

Use (6.4) in (3.5) to obtain

(6.6) $\quad a_{ij} = \mu(q)\zeta_{FG} - \alpha(q) = \tau_{FG} = \tau_{GF}$

All a_{ij} corresponding to $i \in S_F$ and $j \in S_G$, $F \neq G$, are equal and the value does not depend on the nature of i or j but on the characteristics of subsets F and G.

In the special case of strong separability or additive preferences (6.3) specialises to

(6.7) $u(q) = z(\Sigma_F u_F(q_F))$

Then $\partial z/\partial u_F$ is independent of F and $\partial^2 z/(\partial u_F du_G)$ is independent of F and G. Otherwise said ζ_{FG} is a constant, say ζ. Let the standard pair of good, r and s, be also from different subsets. Then $a(q) = \mu(q)\zeta$ and according to (6.6) one has

(6.8) $a_{ij} = 0$ or $\tau_{FG} = 0 \qquad i \in S_F, j \in S_G, F \neq G$

The matrix A is then a block-diagonal matrix with the diagonal blocks corresponding to the various subsets.

An extreme case is that of complete preference independence, where each subset consists of one good only. Then

(6.9) $a_{ij} = 0 \quad \forall\ i,j,i \neq j$

and the matrix A is a diagonal matrix. Note that the a_{ij} are invariant under monotone transformations of u(q). Complete preference independence is an ordinal property and not a cardinal one as Frisch (1959) once stated.

To trace the consequences of separability for demand it is useful to write

(6.10) $A = A_D + JTJ'$

for the full matrix of Allais coefficients. Here A_D is a block diagonal matrix with A_F as a typical block. The typical element of A_F is a_{hi} with $h,i \in S_F$. It is assumed that A_F is nonsingular and thus that A_D is nonsingular. In (6.10) T is the NxN matrix of τ_{FG}. Its diagonal is zero. The nxN matrix J is defined by

(6.11) $$J = \begin{bmatrix} j_A & 0 & \dots & 0 \\ 0 & j_B & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & j_N \end{bmatrix}$$

where j_F is the n_F-vector of all elements equal to unity. Note that $J\iota_N=\iota_n$ where ι_N is the N-vector and ι_n is the n-vector of all elements equal to unity, respectively.

The case of strong separability corresponds to T = 0, that of commodity-wise strong separability to diagonal A_D and A.

The particular structure (6.1)) for A finds its counterpart in one for S. Expression (4.7) gives S as a function of A^{-1}. On the basis of (6.10) one can write

$$(6.12) \qquad A^{-1} = (A_D+JTJ')^{-1} = A_D^{-1} - R\dot{S}R' + RQ^{-1}R'$$

with

$$R = A_D^{-1}J(J'A_D^{-1}J)^{-1} \qquad \dot{S} = J'A_D^{-1}J \qquad Q = \dot{S}^{-1}+T$$

Here $R\dot{S}R'$ is a block diagonal matrix of the same form as A_D or A_D^{-1}. The nxN matrix R is like J as defined by (6.11) with the j_F replaced by

$$(6.13) \qquad r_F=A_F^{-1}j_F(j_F'A_F^{-1}j_F)^{-1}$$

Clearly, J'R = I and $\iota_n'R = \iota_N'J'R = \iota_N'$. The NxN matrix Q is the matrix T with the zero diagonal elements replaced by $(j_F'A_F^{-1}j_F)^{-1}$.

It follows from (6.12) that

$$(6.14) \qquad A^{-1}\iota_n=A_D^{-1}\iota_n-R\dot{S}\iota_N+RQ^{-1}\iota_N = A_D^{-1}\iota_n-A_D^{-1}\iota_n+RQ^{-1}\iota_N=RQ^{-1}\iota_N$$

while

$$(6.15) \qquad \varphi =\iota_n'A^{-1}\iota_n=i_n'RQ^{-1}\iota_N=\iota_N'Q^{-1}\iota_N$$

Using (6.12), (6.14) and (6.15) in (4.7) gives

$$S=A_D^{-1}-R\vartheta R'+R[Q^{-1}-Q^{-1}\iota_N(\iota_N'Q^{-1}\iota_N)^{-1}\iota_N'O^{-1}]R'$$

Now $S_D=A_D^{-1}-R\vartheta R'$ is a block diagonal matrix. Let

$$\Sigma=Q^{-1}-Q^{-1}\iota_N(\iota_N'Q^{-1}\iota_N)^{-1}Q^{-1}\iota_N'Q^{-1} \tag{6.16}$$

then

$$S=S_D+R\Sigma R' \tag{6.17}$$

which expresses clearly the formal similarity with (6.10). It is evident from (6.16) that Σ has the same relation to Q as S has to A. Given an estimate of Σ one can go back to Q and T to evaluate its proper meaning.

On the basis of (4.6), (6.14) and (6.15) one may write

$$b = RQ^{-1}\iota_N(\iota_N'Q^{-1}\iota_N)^{-1} = (1/\varphi)RQ^{-1}\iota_N \tag{6.17}$$

Because of the special nature of R one has that

$$b_F = (1/\varphi)r_F e_F'Q^{-1}\iota_N \tag{6.18}$$

i.e. b_F is proportional to r_F with $e_F'Q^{-1}\iota_N/\varphi$ as the factor of proportionality. b_F is n_F-vector of marginal propensities to spend on the commodities of subset F out of the total budget in. Then β_F is the n_F-vector of the marginal propensities to spend on all goods of subset F together. It follows from (6.18) that

$$\beta_F = j_F'b_F = e_F'Q^{-1}\iota_N/\varphi \tag{6.19}$$

because (6.13) implies that $j_F'r_F = 1$. Consequently

$$b_F = \beta_F r_F$$

or

$$B = R\hat{\beta} \tag{6.19}$$

where B has the same structure as R and J with the b_F as the diagonal arrays. It cannot be guaranteed that all β_F are nonzero. Assuming this to be the case, however, one can express (6.17) also as

$$S = S_D + B\hat{\beta}^{-1}\Sigma\hat{\beta}^{-1}B' = S_D + B\Phi B' \tag{6.20}$$

For a particular pair of goods i and j belonging to different subsets, F and G respectively, one has with $\varphi_{FG} = \sigma_{FG}/(\beta_F\beta_G)$

$$s_{ij} = \varphi_{FG} b_i b_j \tag{6.21}$$

which is the usual representation of groupwise separable demand.

Under strong separability T = 0. Then $Q = \hat{s}^{-1}$ and (6.16) simplifies to

$$\Sigma = \hat{s} - s(\iota's)^{-1}s'$$

Here $\iota's = \varphi$. It follows from (6.19) that then $\beta_F = s_F/\varphi$ amd $\beta = (1/\varphi)s$. Consequently

$$\Sigma = \varphi(\hat{\beta} - \beta\beta')$$

and

$$\Phi = \hat{\beta}^{-1}\Sigma\hat{\beta}^{-1} = \varphi(\hat{\beta}^{-1} - \iota_N\iota_N') \tag{6.22}$$

This means that under strong separability the φ_{FG} in (6.21) becomes φ, i.e. independent of the nature of the subsets F and G.

In the case of complete preference independence $s_{ij} = \varphi b_i b_j$ for all $i \neq j$.

It is evident that specification (6.2) can be very useful for estimation. It can also be used in constructing commodity aggregates such that the interaction between the demand for the aggregates are characterised by the elements of the Φ matrix. These issues will not be pursued further here. We will rather turn to an extension of the Allais approach to the representation of the impact of other determinants than prices and the budget on demand.

7. Allais-type of coefficients for other determinants

The preference order may depend on factors that are in principle observable like age, health, sex, weather conditions, advertising and so on. In empirical research it is useful to be able to control for these, i.e. to include these factors in the explanation of demand. The changes in demand caused by variation in these other determinants have to fit in the budget. Their measurable impact on demand reflects this, causing a problem in evaluating the pure preference shifting effect of such other determinants. A way out of this dilemma is offered by an approach similar to that of the Allais coefficients.

Given the consumer equilibrium condition (3.9) the other determinants affect demand by way of their changing u_q, the vector of marginal utilities. Let x be the vector of quantifiable other determinants and let $u(q,x)$ be twice differentiable in x. One has for x_k being a typical element of the x vector:

$$(7.1) \qquad \frac{\partial u_i}{\partial x_k} = \frac{\partial^2 u}{\partial q_i \, \partial x_k}$$

which like u_{ij} is not invariant under monotone increasing transformation of the utility function. Analogous to (3.1) one has for $v = F(u)$

$$(7.2) \qquad \frac{\partial v_i}{\partial x_k} = F' \frac{\partial^2 u}{\partial q_i \, \partial x_k} + F'' \frac{\partial u}{\partial q_i} \frac{\partial u}{\partial x_k}$$

Analogy with (3.4) then leads to the following invariant interaction coefficient

$$(7.3) \qquad e_{ik} = \kappa(q,x) \left[\frac{\partial u_i/\partial x_k}{u_i \partial u/\partial x_k} - \frac{\partial u_j/\partial x_k}{u_j \partial u/\partial x_k}\right]$$

where $\kappa(q,x) = \Sigma g x_g \partial u/\partial x_g$ and j refers to a good j on which x_k has a 'standard' type of impact, say a neutral one. The second term in (7.3) is taken to be a constant for all i. It is denoted by $\varepsilon_k(q,x)$.

One can use (7.3) to express (7.1) as

$$(7.4) \qquad \partial u_i/\partial x_k = (u_i/k(q,x))e_{ik}\partial u/\partial x_k + \varepsilon_k(q,x)u_i\partial u/\partial x_k$$

For constant q and changing x one then has

$$du_i = \Sigma_k(\partial u_i/\partial x_k)\, dx_k$$

or

$$(7.5) \qquad d\ln u_i = \Sigma_k e_{ik}\vartheta_k dx_k/x_k + \Sigma_k \varepsilon_k(q,x)(\partial u/\partial x_k)dx_k$$

with $\vartheta_k = x_k\partial/\partial x_k/ \Sigma_g x_g \partial u/\partial x_g$. The ϑ_k represent the relative importance of x_k among all the x-variables. The last term in (7.5) is independent of i. The first term on the right-hand side shows the role of the e_{ik}. These coefficients measure the extent to which x_k specifically changes the desirability of good i in comparison to its impact on good j. The sign of e_{ik} indicates whether this desirability increases, stays the same or decreases.

The x_k can take on negative or zero values and to replace in (7.5) dx_k/x_k by dlnx is not in general permissible. Still we will use dlnx simply as a notational shorthand for dx_k/x_k. Then (7.5) can be rewritten as

$$(7.6) \qquad d\ln u_i = \Sigma_k e_{ik}\vartheta_k d\ln x_k + z$$

with $z = \Sigma_k\varepsilon_k(q,x)(du/\partial x_k)dx_k$. Let E^* be the matrix with as typical element $e_{ik}\vartheta_k$. Then the vector expression of (7.6) reads as

$$(7.7) \qquad dlnu_q = E^* dlnx + z\iota$$

The impact on demand of the x variables can be easily traced. The impact on inverse demand is fairly straightforward. That on regular or direct demand is derived in what follows.

One starts off again from (4.1) but (4.2) now becomes

$$(7.8) \qquad dlnu_q = A\hat{w}dlnq + E^* dlnx + (\alpha du_1 + z)\iota$$

where du_1 refers to the change in utility associated with dq. Combining (7.8) with (4.1) gives

$$A\hat{w}dlnq = (dln\lambda - \alpha du_1 - z)\iota + dlnp - E^* dlnx$$

or

$$\hat{w}dlnq = A^{-1}\iota(dln\lambda - \alpha du_i - z) + A^{-1}dlnp - A^{-1}E^* dlnx$$

Using (4.4) results in a way analogous to (4.5) in

$$(7.9) \quad \hat{w}dlnq = b(dlnm - w'dlnp) + Sdlnp - SE^* dlnx$$

where b and S are defined by (4.6) and (4.7), respectively. The effect of the x variables is a rather complicated function of S and E^*. It is not such an easy matter to formulate prior ideas about that effect.

Let $Z = SE^*$ be in principle directly measurable. Can one retrieve E^* from that? Realising that $S\iota = 0$ and $b'\iota = 0$ one has

$$\begin{bmatrix} S & b \\ b' & 0 \end{bmatrix} \begin{bmatrix} (I-\iota b')E^* \\ 0 \end{bmatrix} = \begin{bmatrix} Z \\ 0 \end{bmatrix}$$

Use (4.13) and the property that $\iota Z = 0$ to obtain

$$\begin{bmatrix} (I-\iota b')E^* \\ 0 \end{bmatrix} = \begin{bmatrix} A & \iota \\ \iota' & 0 \end{bmatrix} \begin{bmatrix} Z \\ 0 \end{bmatrix}$$

or

$$E^* = AZ + \iota b'E^* \tag{7.10}$$

In scala terms one has

$$e_i'E^*e_k = e_i'AZe_k + b'E^*e_k$$

Set $e_\iota'E^*e_k$ equal to zero. Then $b'E^*e_K = -e_j'AZe_k$ and

$$e_i'E^*e_k = (e_i-e_j)'AZe_k \tag{7.11}$$

is the final result.

Note that per additional x variable one has one degree of freedom which is fixed by the choice of the good with the standard response to x_k. Also note that it is possible to retrieve $e_{ik}\theta_k$ but not so easily e_{ik}. As is clear from (7.6) the $e_{ik}\theta_k$ are a kind of elasticities. Strictly speaking, the e_{ik} are analogous to the a_{ij} and the e_{ik} to the $a_{ij}w_j$.

8. A numerical example

To illustrate the relation between the Slutsky coefficients on the one hand and the Allais coefficients on the other hand we will use a set of s_{ij} and b_i values based on a regular Rotterdam demand system for food, Belgium, estimated with annual national accounts data for the period 1954-1984 - see Barten (1987).

The original exercise covered nine food items. Some of these had very small budget shares. These have been integrated with each other in the case of Coffee and tea, Sugar and sweets and Other food which constitute here the category Other food, while Fish has been combined with Meat.

The resulting six items are given in Table 1, together with their share in the budget, taken as an average over the sample period.

Table 1 gives the b_i. The budget elasticities can be calculated from b_i/w_i. It appears that Meat, fish and Vegetables, fruit are elastic. Other food has an elasticity of virtually one. The other three items are inelastic.

Table 1. Budget shares, estimated values of b_i and s_{ij} for food, Belgium 1954-1984

Commodity	w_i	b_i	s_{ij} x 100					
			1	2	3	4	5	6
1. Bread, pastry	0.12	0.03	-4.22					
2. Meat, fish	0.39	0.57	1.26	-11.48				
3. Dairy products	0.12	0.06	-0.81	3.32	-2.35			
4. Oils, fats	0.09	0.03	-0.28	0.94	-0.44	-1.10		
5. Vegatables, fruit	0.15	0.18	-0.10	5.01	0.00	0.45	-6.49	
6. Other food	0.13	0.14	4.15	0.95	2.80	0.43	1.12	-6.93

Table 1 also displays the s_{ij}. The S matrix is symmetric. Therefore only its lower triangular part is given. The row (and columns) of S add up to zero as can be verified. Of the 15 possible interactions 10 have a positive sign corresponding with substitution in the Hicks-Allen sense. Meat, fish is a substitute for all other items as is the case for Other food.

The s_{ij} values have been multiplied by 100 because of convenience of presentation. The estimates s_{ij} values tend to decrease with n, the number of commodities taken into account (here six), and with the degree of aggregation. Responses of demand to price changes tend then to be minor because of the absence of close substitutes.

It should be realised that the b_i and s_{ij} are point estimates with a varying but not overly high precision. This increases the need for a plausibility test. At the same time, though, our results as a representation of the actual state of affairs should be taken with the proverbial grain of salt.

The next step consists in constructing the matrix S bordered by the b vectors and with a zero in the SE corner, like it appears in (4.13). This matrix is inverted to yield the matrix $M = A - \frac{1}{\varphi}\iota\iota'$, which is given in Table 2. The small order of magnitude of the s_{ij} causes the m_{ij} to be fairly large in absolute value. Note that in Table 2 their values are divided by 10.

To construct the Allais coefficients from the m_{ij} one needs to select a standard pair. We took this to be 2. Meat, fish and 6. Other food, with $m_{2,6} = 4.47$. Subtracting this value from all elements of the matrix M yields the matrix of Allais coefficients given in Table 3. Here the minus sign indicates substitution, the plus sign complementarity. Of the 15 interactions 10 are substitutes, the same in number as in the case of the S matrix but there are differences in the pairs which are mutually substitutes or complements. Meat, fish is again a substitute of almost all other items.

Table 2 Elements of matrix M

Commodity	m_{ij} / 10					
	1	2	3	4	5	6
1. Bread, pastry	-5.49					
2. Meat, fish	0.45	-0.32				
3. Dairy products	2.45	-0.33	-5.56			
4. Oils, fats	0.00	0.21	2.36	-9.86		
5. Vegatables, fruit	0.21	0.12	0.43	-0.31	-1.17	
6. Other food	-2.89	0.45	1.61	-0.33	0.15	-2.84

Table 3 Allais coefficients

Commodity	a_{ij} / 10					
	1	2	3	4	5	6
1. Bread, pastry	-5.94					
2. Meat, fish	0.01	-0.76				
3. Dairy products	2.00	-0.78	-6.01			
4. Oils, fats	-0.44	-0.24	1.91	-10.31		
5. Vegatables, fruit	0.23	0.33	-0.02	0.75	-1.62	
6. Other food	-3.34	0	1.16	-0.78	-0.29	-3.29

Other food is the exception, by construction. Other food is now a complement of Dairy products. This last item is a complement of Oils, fats, which is somewhat counterintuitive and of Bread, pastry, which makes sense. Vegetables, fruit appear to be a substitute of all other items.

The values of the a_{ij} are rather high. One can turn them into elasticities by multiplying the a_{ij} by w_j - see (4.2). This does not help very much. The diagonal elasticities range from -9.3 for Oils, fats to -2.4 for Vegetables, fruits. A relatively high value of a_{ii} can be seen to reflect a high sensivity of the preference order for good i. It would correspond with the nature of i as a basic need or necessity. In a relative sense, Meat, fish and Vegetables, fruit would then be more of a luxury. This is also reflected in these budget elasticities being larger then one.

Another way to analyse the resulting a_{ij} values is to express them in the form of interaction intensities, given in Table 4. It appears that only a very few interactions are of substance. Bread, pastry and Dairy products are rather strong complements which makes sense. Dairy products are also complementary to Oils, fats and Other food. The latter is highly substitutable by Bread, pastry, which is somewhat puzzling. Meat, fish is a rather strong substitute of Dairy products, another source of animal protein, and of Vegetables, fruit.

Table 4 Allais interaction intensities a^{*}_{ij}

	1	2	3	4	5	6
1. Bread, pastry	-1					
2. Meat, fish	0.00	-1				
3. Dairy products	0.33	-0.36	-1			
4. Oils, fats	-0.06	-0.09	0.24	-1		
5. Vegatables, fruit	-0.08	-0.30	-0.01	-0.18	-1	
6. Other food	-0.75	0	0.26	-0.13	-0.13	-1

This example has demonstrated that one can retrieve Allais coefficients from estimates of S and b and that their relative values make sense in some cases and are difficult to understand in other cases. Their high absolute values may be due to the degree of aggregation of elementary goods into agglomerates or to a systematic underestimation of the elements of the matrix S. Since the matrix A is in a certain sense a generalised inverse of S, low values for the s_{ij} produce high values of the a_{ij} and vice versa. Further research is needed to clarify this issue.

8. Concluding remarks

The formal expression given by Allais to the notion of complementarity, substitution and independence is invariant under monotone increasing transformations of the utility function. In other words, it reflects properties of the preference order. At the same time it is rather close to one's intuition about these concepts.

The Allais coefficients are reflected in the coefficients of estimable regular or inverse demand systems. They can also be retrieved from estimates of these systems. These calculated values can be compared with prior ideas based on introspection. The plausibility of the estimates can then be judged. The Allais coefficients also reflect in a natural way the eventual separability of preferences. The effects of preference shifting variables can be given an interpretation similar to the Allais coefficients.

Until now most of the time only separability of preferences has been used to specify demand relations. It is of interest to take into account also other aspects of the preference order. The Allais coefficients provide a useful tool for this purpose.

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