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# ON DURABLE GOODS MONOPOLIES AND THE (ANTI-) COASE-CONJECTURE

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# On Durable Goods Monopolies and the (Anti-) Coase-Conjecture

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**Abstract.** Consider the durable goods monopoly game with uniformly distributed consumers' valuations. If the time horizon is finite, this game generically has a unique subgame perfect equilibrium outcome. When, following Coase's original idea, the finite time horizon is divided into successively more subperiods, the limiting solution still displays positive profits of the monopolist. When the time horizon is infinite, then the relative patience of the consumers versus the monopolist determines whether perfect price discrimination or competitive outcomes result. If all players are very patient, everything is possible, even with stationary strategies.

## 1. INTRODUCTION

Suppose an exhibition of the fine arts is on in a large town. It will be open for several month and there is a single agency selling tickets to the exhibition. This agency has the option to post a new price for tickets every day, say. Residents in town may derive different utilities from seeing the exhibits, but no potential visitor would consider seeing the exhibition twice. In the latter sense a visit to the exhibition is a durable good, because once you have seen the exhibits you are happier (to a certain extend) for the rest of your life. If the town is sufficiently isolated, there will be no new residents moving in during the months in question. Thus, although the monopolistic ticket-agency can offer its supply at a new price every day, there is only a single demand facing it over the whole time horizon. Since the experience of seeing the exhibits cannot be resold, the ticket-agency does not have to fear competing suppliers.

For such a situation it has been suggested by economic theory [**Coase**, 1972] that, it the ticket-agency could change its price every minute, say,

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then admission to the exhibition would be nearly free. In particular, if price changes can be made almost instantaneously, the agency's profit would be driven to zero, its competitive level.

The latter has come to be known as the *Coase-Conjecture* on durable goods monopolies. It holds that to achieve a competitive outcome it does not necessarily take many suppliers. Alternatively many pricing decisions, each based on different incentives, may suffice: Since in later periods the supplier does not care anymore about profits already made, its incentive structure changes over time such that the supplier's agents of different periods compete against each other. Agents of later periods will try to steal away customers from earlier agents by cutting prices which will induce potential customers, rationally anticipating this, to hold out. The more frequent price changes can be made, the closer to zero (or: marginal cost) this will drive prices and overall profits.

On the other hand: How far can the monopolist's agents of later periods *credibly* cut their prices, given that, if the manouvre succeeded, they would be in a position very much like the earlier agents, *except* that they do not have as many competitors in later periods? There seems to be a puzzle here, the resolution of which requires formal analysis. Accordingly the literature has addressed durable goods monopolies ever since the original Coase-Conjecture.

Some variants of the Coase-Conjecture have become established as theorems in the meantime. In particular, **Gul, Sonnenschein, and Wilson** [1986; quoted as Gul et.al. in the sequel] have shown that on durable monopoly markets with an infinite time horizon there exist subgame perfect equilibria in which the monopolist earns essentially nothing, while customers achieve almost all of their reservation value as the equilibrium payoff, provided that the uniform discount factor is sufficiently high. As will be shown below, two assumptions are vital for this result: A uniform discount factor for the seller and all potential customers *and* infinitely many, short sales periods. In an earlier paper **Stokey** [1981] has analysed a more specific durable monopoly market. But, since Stokey did not model consumers explicitly as players, her results received a proper game-theoretic foundation only by the later results of Gul et.al.. For stationary equilibria the validity of the Coase-Conjecture has been established even in models, where the durable good depriciates [**Bond and Samuelson**, 1984]. **Kahn** [1986] generalises Stokey's results by allowing for quadratic cost functions. The latter can be viewed as a commitment device, because increasing marginal costs provide an incentive to sell constant amounts each period. Hence Kahn can show that the monopolist makes a positive profit. Assuming commitment power on the part of the monopolist and allowing for rationing,

**Van Cayseele** [1991] derives intertemporal price discrimination. For the case, where marginal costs depend on market penetration **Olsen** [1992] shows that, although prices may converge to marginal costs in every period (when the period length becomes small), it still takes considerable time until the market is saturated. **Bulow** [1982] studies two-period markets, where customers may rent a unit of the durable each period and the commodity does not depreciate. Equivalently the monopolistic seller may face a resale supply, the size of which depends on the monopolist's previous sales. This setup differs from the one employed in the present paper, because a customer's utility from holding a unit of the commodity depends on the length of the remaining time horizon. **Ausubel and Deneckere** [1989; see also **Ausubel and Deneckere**, 1987] prove a Folk Theorem for durable monopoly games with an infinite time horizon by relaxing the restriction, used by Gul et.al., to Markov strategies. Although they rely on subgame perfection, the infinite horizon is essential for their results, as it always is for Folk Theorems [**Fudenberg and Maskin**, 1986; for a critical evaluation see: **Güth, Leininger, and Stephan**, 1991]. More recently **von der Fehr and Kühn** [1990] have shown that the results of **Ausubel and Deneckere** [1989] depend crucially on the continuous type space of consumers and the possibility to vary sales prices continuously. The Coase-Conjecture does not hold true, when the monopolist also supplies non-durables which are demand related to the durable [**Kühn and Padilla**, 1991]. Other related studies are **Riley and Zeckhauser** [1983], who allow for individual sales offers in a possibly long sequence of successive bilateral bargaining rounds; **Conlisk, Gerstner, and Sobel** [1984] do not rely on constant demand, but assume new cohorts of potential customers in every period. New cohorts of consumers each period are also allowed into the market in **Sobel** [1991]. In general there seems to be some dissatisfaction in the literature with the Coase-Conjecture as proved by Gul et.al.. And this is despite the fact that, to the best of our knowledge, there seems to be no attempt to study the two most obvious generalizations: Different discount factors for the monopolist and the consumers, and a finite time horizon with a large number of short subperiods as originally suggested by **Coase** [1972].

The papers most closely related to the present one are the paper by **Gul et.al.** [1986] and, although it deals with a quite different economic problem, the paper by **Sobel and Takahashi** [1983]. These two papers and their (dis-)similarities to the present approach will now be discussed in more detail.

**Gul et.al.** allow for fairly general demand functions [1986, p.158], but on the other hand restrict themselves to an infinite time horizon and the case of a uniform discount factor for both the monopolist and all consumers. By contrast, in the present paper we use a very special linear demand function, but study both finite and infinite horizon models and allow for a different time preference of the monopolist as compared to potential customers. Gul et.al. [1986, p.173] claim that the main restriction of their setup is that all potential customers use the same discount factor. We agree that this is a major restriction, but argue that also the assumption of the same discount factor for the monopolist and all potential customers is restrictive in the sense that it already rules out many interesting economic phenomena. In particular the assumption of a uniform discount factor obscures the fact that in general everything between perfect price discrimination and a Coasian outcome is supportable as a subgame perfect equilibrium (in stationary strategies) on an infinite durable monopoly market. (For all what follows we will refer to the infinite durable monopoly game, whenever the overall time horizon is infinite; otherwise we refer to finite durable monopoly games.)

Because of the infinite time horizon, Gul et.al. [1986, p.162] have to rely on stationarity assumptions imposed onto the players' strategies and they have to rule out strategies which condition on the behavior of very small sets of consumers. By contrast we follow Coase for a large part of the paper by approaching the situation with infinitely many sales periods via dividing a given time interval of finite length into successively more subperiods of equal length. This enables us to *derive* restrictions on the players' strategies from generic properties of the game (and desirable game-theoretic robustness properties of behavior strategies) for all equilibria. For the remaining part of the paper, where we investigate the infinite game, we rely on *asymptotic convergence* [**Güth, Leininger, and Stephan**, 1991]: Rather than imposing stationarity, we study the limit of the solution to finite games, when the time horizon approaches infinity. This allows us to carry over properties of equilibrium strategies derived in the finite context to the infinite horizon. Again, asymptotic convergence seems to us a more convincing approach - from a purely game-theoretic perspective - than ad-hoc assumptions on the players' equilibrium strategies.

Summarizing, the model presented below is much more special than the one considered by Gul et.al. with respect to demand functions, but more general with respect to the time horizon and time preferences. However, the validity of our approach via asymptotic convergence finds extra support in the results generated in the present paper: We identify a unique equilibrium for all finite games and are able to derive the limiting solution, when the number of sales periods approaches infinity. This allows us to prove an Anti-Coase-Conjecture stating that, as the number of subperiods within a given finite time interval approaches infinity, the monopolist's profit does *not* converge to the competitive level, but stays positive. With fixed discount factors and an infinite number of possible sales periods (the case corresponding to Gul et.al.) still unique solutions can be identified via asymptotic convergence for all interior discount factors. But now the possible outcomes range from perfect price discrimination to Coasian outcomes, depending on the *relative* patience of the monopolist as compared to consumers. With very patient players on both sides of the market almost everything is possible. Since both the latter Folk-Theorem-like result and the Anti-Coase-Conjecture are negative results in spirit, the non-degenerate example with the simple linear demand side suffices to make the point.

The other paper closely related to our research is **Sobel and Takahashi** [1983]. Although the economic context is rather different, there is a conceptual similarity. Sobel and Takahashi study bilateral bargaining between a potential buyer and a seller, who only has probabilistic beliefs over the buyer's true valuation of the (single) commodity. Each period the seller makes a price offer which the buyer can either accept or reject. If an offer is accepted (or rejected in the last period), the game ends, while it continues otherwise with a new price offer by the seller. Future payoffs are discounted.

The similarity to the present model results, because in Sobel and Takahashi's model the distribution function of the buyer's valuation plays a similar role as the demand function in the present model. Although, due to incomplete information on the part of the seller, Sobel and Takahashi's game does not have proper subgames, they also derive the solution by backward induction. But, as Gul et.al. and the more closely related later work by **Fudenberg, Levine, and Tirole** [1985], they approach the infinite horizon, when the number of possible bargaining rounds goes to infinity. This explains, why with infinitely many possible bargaining rounds the seller cannot sell at all at positive prices [**Sobel and Takahashi**, 1983, p.424]. As discussed above, this may not hold true, if one approaches the infinite case by chopping up a given finite time interval into successively more periods.

Durable monopolies with finitely many sales periods are interesting beyond their purpose to approximate the infinite game. To test for the Coase-Conjecture empirically by econometric estimation will generate enormous identification problems. Thus an empirical test of the Coase-Conjecture will require experiments [which were, after all, the original motivation of our research, cf. **Güth and Ritzberger**, forthcoming]. But experiments cannot be performed for markets with infinitely many sales periods. It takes an insight into finite analogues to the price competition envisaged by **Coase** [1972] to experimentally test its empirical relevance.

The remainder of the paper is organized as follows: Section 2 formally defines the durable monopoly game. The core Section 3 analyses finite horizon games and contains the Anti-Coase-Conjecture. Section 4 considers the infinite game with fixed discount factors and contains the Folk-Theorem-like result. Section 5 presents some concluding remarks.

## 2. The Model

The game to be studied in the sequel is a $T$-stage game, $1 \leq T \leq \infty$. To begin with the number of stages or periods, $T$, is taken finite and is thought of as the number of subperiods, say, "market days", within a finite interval of time. To solve for the game with $T = \infty$ this number of market days within a finite time interval will be increased and the limit of the solutions, when $T$ approaches infinity, will be studied.

The game is one between a single monopolistic supplier of a durable and indivisible commodity, and a large number of different potential customers, say, consumers. For the monopolist the marginal cost of production is assumed to be constant. Let $p_t$ denote the unit profit in period $t$, $t = 1, \ldots, T$, that is: period $t$'s sales price minus the constant marginal cost of production. In the sequel $p_t$ will simply be called the price in period $t$, because marginal costs can be assumed to be zero without loss of generality.

All the potential consumers have a "willingness to pay" or reservation value $v$ for one unit of the durable good, i.e. a customer wants either one unit of the commodity or none. As with prices $p_t$ the redemption values are normalized in the sense that they measure the difference between a consumer's willingness to pay and the monopolist's marginal costs of production. By neglecting consumers, whose willingness to pay is smaller than the monopolist's marginal cost of production, it can be assumed that 1 is the highest and 0 the lowest such value, i.e. redemption values $v$ for all consumers satisfy $0 \leq v \leq 1$. It is assumed that for every number $v \in [0, 1]$ there exists exactly one potential customer with reservation value $v$ and that this uniform distribution of reservation values over the unit interval is common knowledge.

If $x_t$ denotes the number of units sold in period $t$, the monopolist's profit, $\pi_t$, in period $t$ is given by $\pi_t = p_t x_t$. The monopolist's time preference is given by a constant discount factor $\rho \in (0, 1)$, such that the monopolist's payoff function for the game can be described by

$$\Pi_1 = \sum_{t=1}^{T} \rho^{t-1} p_t x_t \, .$$

In a subgame starting in period $t > 1$ the payoffs $p_\tau x_\tau$ of periods $\tau < t$ are, of course, given such that the monopolist will attempt to maximize

$$\Pi_t = \sum_{\tau=t}^{T} \rho^{\tau-t} p_\tau x_\tau \, .$$

Since a consumer with reservation value $v$ will buy at most one unit, her payoff in the case, where she never buys, can be normalized to zero. If she does buy one unit in period $t$ at the price $p_t$, the payoff to the consumer with reservation value $v$ (referred to as consumer $v$) is given by $\delta^{t-1}(v - p_t)$, where $\delta \in (0, 1)$ is the common discount factor of consumers. The reservation value $v$ can be thought of as the discounted stream of benefits to consumer $v$ from enjoying the durable good from period $t$ onwards. At the price $p_t = v$ consumer $v$ is indifferent between buying and not buying. The payoff function $u_v$ of the potential customer $v$ is thus given by

$$u_v = \begin{cases} \delta^{t-1}(v - p_t), & \text{if } v \text{ buys in } t, \\ 0, & \text{otherwise.} \end{cases}$$

Observe that a consumer can buy only once and after having bought, she can neither resell, nor participate actively in the game in any other way.

To complete the definition of the extensive form game it remains to specify the market process together with the information requirements. In every period $t$, $t = 1, \ldots, T$, the monopolist moves first by setting a price $p_t \in [0, 1]$. Then, observing the price, all potential customers, who are still active (have not bought yet in any previous period) choose simultaneously whether to buy or to continue waiting. At the end of the period all decisions in period $t$ become publicly known. Since perfect recall is assumed, any player in any period knows all the previous moves leading to the current subgame.

This completes the description of the durable monopoly game. The situation envisaged by **Coase** [1972] can be represented by setting $\delta = \exp(-r\Delta)$, $\rho = \exp(-R\Delta)$, $r > 0$, $R > 0$, where $\Delta = 1/T$ is the length of market days, and considering the limit as $T$ approaches infinity, such that the overall time horizon stays $T\Delta = 1$.

## 3. Finite Time Horizon

The focus of this section is to derive two major results on the durable monopoly game with a finite time horizon. First it will be shown that generically (or under a certain refinement of subgame perfection) there exists a unique equilibrium path of the durable monopoly game. Second the limiting behavior of the equilibrium outcome for $\Delta = 1/T \to 0$ will be studied.

The technique of the proofs rests on two cornerstones. The first, Lemma 1, describes a property of all subgame perfect equilibria of the game: It says that, under very weak conditions, for each period $t$ the set of still active consumers along the equilibrium path is an interval with lower bound zero and an upper bound which is decreasing over time. The second cornerstone is a sequence of recursively defined coefficients, the properties of which will be described in Lemmas 2 and 3. Since the proofs of these two Lemmas require lengthy induction arguments, the proofs are relegated to the Appendix.

Concerning the first lemma, there will be two notions used which require clarification. Recall that the strongest notion of Nash-refinements is a *strict* equilibrium, i.e. a Nash equilibrium, where the equilibrium strategy of each player is the unique best response to the equilibrium strategies of the other players. In games with a continuum of players this is perhaps too much to ask for. Under perfect information an infinite action space of a single player already results in infinitely many subgames and thus in infinitely many pure strategies for all players. Combined with a continuum of players this can result in unavoidable indifferences: Even in the one-shot monopoly game with a continuum of potential customers, there will always be one customer, who is indifferent between buying and not buying. For this class of games it is, therefore, natural to weaken the notion of a strict equilibrium and to require that equilibrium strategies are unique best responses along the equilibrium path for all players except, possibly, for a null set of players. For the situation at hand we call a Nash equilibrium *strict along the equilibrium path with respect to almost all players* (SPAAP), if the equilibrium strategy combination assigns a unique (payoff-) maximal move at each information set reached along the equilibrium path, given the equilibrium strategies of the other players, to all players except for a closed set of potential customers with (Lebesgue-) measure zero. Observe that this definition implicitely assigns positive mass to the monopolist.

The second notion in Lemma 1 which requires specification is that the Lemma states that its conclusion applies "generically" to all subgame perfect equilibria of the durable monopoly game. Such a notion is usually best defined by stating, when a property is "non-generic". We will say that some property of the game is *non-generic*, if it is possible to find a continuous (cumulative) distribution function on the unit interval (the set of possible consumer valuations) which is arbitrary close to the uniform distribution (say, in the supremum-norm), but which destroys the property (if it is substituted for the uniform distribution).

Most of the notation will be developed, where it is needed. For the time being, let, for any given subgame perfect equilibrium $\sigma$, the set of consumers $v \in [0, 1]$, who are still active in period $t$ along the equilibrium path induced by $\sigma$ (because $\sigma$ prescribes that they do not buy before period $t$, unless a deviation occurs), be denoted by $\mathcal{V}_t(\sigma) \subset [0, 1]$. (The symbol "int" will assign the interior of a set.)

LEMMA 1. *For any subgame perfect equilibrium $\sigma$ which is strict along the equilibrium path with respect to almost all players (SPAAP) and generically for all subgame perfect equilibria $\sigma$ of the durable monopoly game,*

$$int\, \mathcal{V}_t(\sigma) = (0,\, v_t(\sigma)),$$

*with $v_1(\sigma) = 1$, and $0 \le v_{t+1}(\sigma) \le v_t(\sigma) \le 1$, for all $t = 1, \ldots, T-1$.*

PROOF: For some given subgame perfect equilibrium $\sigma$ let $p(\sigma) \in \Re^T_{++}$, $p(\sigma) = (p_1(\sigma), \ldots, p_T(\sigma))$ denote the sequence of prices induced along the equilibrium path. Denote by $(\sigma_{-v},\, w_t)$ the strategy combination induced by $\sigma$ in the subgame after in period $t$ a consumer $v$, who was supposed to buy in $t$ under $\sigma$, has deviated to waiting, $v \in \mathcal{V}_t(\sigma)$.

Consider some $v \in \mathcal{V}_t(\sigma)$, who in equilibrium does not buy in period $t$ (at the price $p_t(\sigma)$). For this $v \in \mathcal{V}_t(\sigma)$ it must either be true that $v \le p_{t+k}(\sigma), \forall\, k = 0, \ldots, T-t$, or there must exist some $k = 1, \ldots, T-t$, such that

$$v - p_t(\sigma) \le \delta^k (v - p_{t+k}(\sigma))$$
$$\iff v \le \frac{p_t(\sigma) - \delta^k p_{t+k}(\sigma)}{1 - \delta^k}.$$

Consequently, for all $v' < v$, $v' \in \mathcal{V}_t(\sigma)$, either $v' < p_{t+k}(\sigma)$, $\forall\, k = 0, \ldots, T-t$, or

$$v' < \frac{p_t(\sigma) - \delta^k p_{t+k}(\sigma)}{1 - \delta^k}$$
$$\iff v' - p_t(\sigma) < \delta^k (v' - p_{t+k}(\sigma)),$$

for at least one $k = 1, \ldots, T-t$. We wish to show that in period $t$ all $v' < v$ will not buy in equilibrium either. Suppose some $v' < v$ does buy in period $t$ according to $\sigma$. Then $v' < p_t(\sigma)$ is impossible, and it must be true that there exists some $k = 1, \ldots, T-t$ such that

$$0 \le v' - p_t(\sigma) < \delta^k (v' - p_{t+k}(\sigma)),$$
$$\text{and} \quad v' - p_t(\sigma) \ge \delta^l (v' - p_{t+l}((\sigma_{-v'},\, w_t))),$$

for all $l = 1, \ldots, T-t$. But these two inequalities imply

$$p_{t+k}((\sigma_{-v'}, w_t)) > p_{t+k}(\sigma)$$

for at least one $k = 1, \ldots, T-t$. Since $(\sigma_{-v'}, w_t)$ differs from $\sigma$ only by an individual deviation (of mass zero), the sequence of residual demands $(\int_{\mathcal{V}_\tau(\sigma)} dv)_{\tau=t}^{T}$, viz. the mass of all still active consumers, on which the monopolist's pricing decisions are based (by subgame perfection), remains unchanged by the deviation. Hence, that for some $k$ *two* prices can be subgame perfect equilibrium choices (in the subgame), implies that in period $t+k$ the monopolist has two distinct best responses to the consumers' equilibrium strategies. This certainly contradicts the requirement that the equilibrium satisfies SPAAP. Thus consider subgame perfection without the strictness requirement: Since the monopolist's profit function of period $t+k$ always contains a term of the form

$$p_{t+k} \int_{\mathcal{V}_{t+k}(\sigma)\cap(p_{t+k},\,1]} dv\,,$$

the property that the monopolist has two distinct best responses can always be destroyed by choosing a slightly different distribution function of consumers close to the uniform (twist in on $\mathcal{V}_{t+k}(\sigma)$, where the one-period profit is quadratic in $p_{t+k}$ under the uniform distribution).

Iterating this argument backwards from $t = T-1$ to $t = 1$ shows that, if $v \in \mathcal{V}_t(\sigma)$ does not buy in period $t$, then any $v' < v$ will not buy either for all $t = 1, \ldots, T$, under the two alternative hypotheses of the Lemma. It follows that $\mathcal{V}_{t+1}(\sigma)$ is an interval, if $\mathcal{V}_t(\sigma)$ is an interval, such that

$$\operatorname{int} \mathcal{V}_{t+1}(\sigma) = (\inf \mathcal{V}_t(\sigma),\ \max_{k=1,\ldots,T-t} \frac{p_t(\sigma) - \delta^k p_{t+k}(\sigma)}{1-\delta^k}),$$

for all $t = 1, \ldots, T-1$. Now clearly, $\mathcal{V}_1(\sigma) = [0,\,1]$, $v_1(\sigma) = 1$, $\inf \mathcal{V}_{t+1}(\sigma) = \inf \mathcal{V}_t(\sigma)$, imply $\inf \mathcal{V}_t(\sigma) = 0$, for all $t = 1, \ldots, T$, and $v_{t+1}(\sigma) = \sup \mathcal{V}_{t+1}(\sigma) \leq \sup \mathcal{V}_t(\sigma)$ implies $0 \leq v_{t+1}(\sigma) \leq v_t(\sigma) \leq 1$, as required. Of course, this was derived under the assumption that not all $v \in \mathcal{V}_t(\sigma)$ buy in period $t$ in equilibrium. If they do, then $\mathcal{V}_{t+1}(\sigma) = \emptyset$, $v_{t+1}(\sigma) = 0$, verifies the claim of the Lemma. ∎

Subgame perfection enters the proof of Lemma 1 in an essential way, because it is required to keep the monopolist from threatening with "punishment" price choices which are not or not generically best responses at the corresponding information sets. A short comment on the notion of a generic game, as used above, may be appropriate here also:

The reader may wonder, why "non-generic" was defined in terms of the distribution rather than, say, in terms of discount factors. The reason for this is simply that discount factors have no role to play in the final period $t = T$. And although one could presumably apply similar arguments in terms of discount factors for any period $t < T$, this does not rule out the possibility that the monopolist has two best responses in the final period. (Strictness along the equilibrium path with respect to almost all players, SPAAP, of course, rules this out immediately.)

The next ingredient to the analysis is an, at this stage, purely formal definition of a sequence of coefficients. Fix some $T$, $1 \leq T < \infty$, and define recursively the sequence $(a_t)_{t=1}^{T}$ of coefficients by

$$a_t = \frac{(1-\delta+\frac{\delta}{2}a_{t+1})^2}{1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_{t+1}}, \quad a_T = 1,$$

for $t = 1, \ldots, T-1$ and for $\delta \in (0, 1)$ and $\rho \in (0, 1)$. The properties of these coefficients are given in the next two lemmas, the proofs of which can be found in the Appendix.

LEMMA 2. *The coefficients defined above satisfy*

$$(i) \quad 1-\delta+\frac{\delta}{2}a_t > \frac{\rho}{2}a_t > \frac{\rho}{4}a_t, \ \forall t = 1, \ldots, T;$$

$$(ii) \quad 0 < a_t < 2, \ \forall t = 1, \ldots, T.$$

(PROOF: see Appendix.)

Using these coefficients now define the sequence of linear functions $(F_t)_{t=1}^{T}$, $F_t\colon \Re_{++} \to \Re_{++}$, by

$$F_t(p) = \frac{\frac{1}{2}a_t}{1-\delta+\frac{\delta}{2}a_t}p \, .$$

LEMMA 3. *If* $p = (p_1, \ldots, p_T) \in \Re_{++}^{T}$ *satisfies* $p_t = F_t(p_{t-1})$, $\forall t = 2, \ldots, T$, *then*

$$(i) \quad \frac{p_{t-1}-\delta p_t}{1-\delta} < \frac{p_{t-2}-\delta p_{t-1}}{1-\delta}, \ \forall t = 3, \ldots, T;$$

$$(ii) \quad \frac{p_t-\delta^k p_{t+k}}{1-\delta^k} > \frac{p_t-\delta^{k+1}p_{t+k+1}}{1-\delta^{k+1}}, \ \forall k = 1, \ldots, T-t-1, \quad \forall t = 1, \ldots, T-1.$$

(PROOF: see Appendix.)

This completes the preparations and allows us to state the main Theorem.

**THEOREM 1.** *The durable monopoly game has, for any fixed, finite $T$, a unique subgame perfect equilibrium outcome which is strict along the equilibrium path with respect to almost all players (SPAAP). The SPAAP-equilibrium outcome generically coincides with the only subgame perfect equilibrium outcome of the game. This unique outcome can be characterized as follows:*
*(i) In any period $t = 1, \ldots, T-1$ all consumers, who have not yet bought in any previous period $\tau < t$ and have a reservation value $v \in [0, 1]$ satisfying*

$$v > \frac{p_t - \delta F_{t+1}(p_t)}{1-\delta}$$

*will buy in period $t$, and all consumers, for whom the above strict inequality is reversed ($<$) will wait. In period $t = T$ all consumers, who have not yet bought and have a reservation value $v > p_T$ will buy, and all consumers $v < p_T$ will not buy.*
*(ii) The monopolist will in any period $t = 2, \ldots, T$ set a price $p_t = F_t(p_{t-1})$ and will in period $t = 1$ set the price $p_1 = \frac{1}{2}a_1$.*

PROOF: The idea of the proof is to invoke Lemma 1 which generically resp. under SPAAP allows us to use $v_t(\sigma) = \sup \mathcal{V}_t(\sigma)$ as a state variable, and calculate the equilibrium by backward induction. Thus the proof is constructive and uniqueness is guaranteed by Lemma 1.

Let $t = T$. Clearly by subgame perfection applied to the last subgame of period $T$, any $v > p_T$, $v \in \mathcal{V}_T(\sigma)$, will buy and any $v < p_T$ will not buy. Thus the monopolist's problem in the first subgame of period $T$ is to choose $p_T$ such as to maximize

$$\pi_T = p_T \max\{0,\, v_T(\sigma) - p_T\}.$$

The first order condition of this problem reads for $v_T(\sigma) \geq p_T$

$$\begin{aligned} & v_T(\sigma) - 2p_T = 0 \\ \iff\ & p_T = \frac{1}{2}v_T(\sigma) = \frac{1}{2}a_T v_T(\sigma) \leq v_T(\sigma), \end{aligned}$$

with second order condition $-2 < 0$. Such a choice yields a profit of

$$\pi_T^*(v_T(\sigma)) = \frac{1}{4}v_T(\sigma)^2 = \frac{1}{4}a_T v_T(\sigma)^2 \geq 0$$

which is strictly larger than zero, if $v_T(\sigma) > 0$, and thus a unique best response. In period $t = T-1$ for all consumers $v \in \mathcal{V}_{T-1}(\sigma)$, who satisfy

$$v > \frac{p_{T-1} - \delta p_T}{1-\delta},$$

it is the unique best response to buy in period $t = T - 1$, and for all $v \in \mathcal{V}_{T-1}(\sigma)$, who satisfy

$$v < \frac{p_{T-1} - \delta p_T}{1 - \delta},$$

it is the unique best response to wait. Due to the rational expectations of all consumers one must have,

$$v_T(\sigma) = \frac{p_{T-1} - \delta p_T}{1 - \delta} = \frac{p_{T-1} - \frac{\delta}{2} v_T(\sigma)}{1 - \delta} = \frac{p_{T-1}}{1 - \delta/2} > 0$$

which in turn implies

$$p_T = \frac{\frac{1}{2} p_{T-1}}{1 - \delta/2} = \frac{\frac{1}{2} a_T}{1 - \delta + \frac{\delta}{2} a_T} p_{T-1} = F_T(p_{T-1}).$$

This verifies the monopolist's pricing rule for $t = T$ and $v_T(\sigma) > 0$.

The above implicitely defines the value function of the monopolist for period $T$ by

$$\pi_T^*(p_{T-1}) = \frac{\frac{1}{4} p_{T-1}^2}{(1 - \delta/2)^2} = \frac{\frac{1}{4} a_T}{(1 - \delta + \frac{\delta}{2} a_T)^2} p_{T-1}^2 .$$

Now assume that for some $t + 1 < T$ one has along the equilibrium path $p_{t+k}(\sigma) = F_{t+k}(p_{t+k-1}), \forall k = 1, \ldots, T - t$, and

$$\pi_{t+1}^*(p_t) = \frac{\frac{1}{4} a_{t+1}}{(1 - \delta + \frac{\delta}{2} a_{t+1})^2} p_t^2 = \frac{1}{4} a_{t+1} v_{t+1}(\sigma)^2$$

as the value function for the monopolist's profits from period $t + 1$ onwards. Consider the last subgame of period $t$, where consumers $v \in \mathcal{V}_t(\sigma)$ decide, given $p_t$. From the induction hypothesis $p_{t+1} = F_{t+1}(p_t)$ are rational expectations, and it follows that for any $v \in \mathcal{V}_t(\sigma)$, who satisfies

$$v > \frac{p_t - \delta F_{t+1}(p_t)}{1 - \delta}$$

it is the unique best response to buy in period $t$ at $p_t$, because

$$\frac{p_t - \delta F_{t+1}(p_t)}{1 - \delta} > \frac{p_t - \delta^k p_{t+k}}{1 - \delta^k}$$

for all $k = 2, \ldots, T-t$, from Lemma 3, (ii), and the induction hypothesis. For any $v \in \mathcal{V}_t(\sigma)$, who satisfies

$$v < \frac{p_t - \delta F_{t+1}(p_t)}{1-\delta},$$

it is a best choice not to buy at the price $p_t$, but rather wait for at least $t+1$. Thus

$$v_{t+1}(\sigma) = \frac{p_t - \delta F_{t+1}(p_t)}{1-\delta} = \frac{p_t}{1-\delta+\frac{\delta}{2}a_{t+1}}.$$

Consequently, the monopolist's problem in the first subgame of period $t$ is to choose $p_t$ such as to maximize

$$\begin{aligned}\pi_t &= p_t[v_t(\sigma) - \frac{p_t}{1-\delta+\frac{\delta}{2}a_{t+1}}] + \frac{\frac{\rho}{4}a_{t+1}}{(1-\delta+\frac{\delta}{2}a_{t+1})^2}p_t^2 = \\ &= p_t v_t(\sigma) - \frac{1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_{t+1}}{(1-\delta+\frac{\delta}{2}a_{t+1})^2}p_t^2 = p_t v_t(\sigma) - \frac{p_t^2}{a_t},\end{aligned}$$

if this quantity is larger than what can be obtained from $p_t \geq v_t(\sigma)$. The first order condition for this problem is

$$v_t(\sigma) - \frac{2p_t}{a_t} = 0$$

with second order condition $-2/a_t < 0$ by Lemma 2, (ii). Such a choice yields a profit of

$$\pi_t^*(v_t(\sigma)) = \frac{1}{4}a_t v_t(\sigma)^2$$

and a price $p_t$ of

$$p_t = \frac{1}{2}a_t v_t(\sigma) < v_t(\sigma)$$

as the unique best response, by Lemma 2, (ii). In order to see that the monopolist does not have an incentive to set $p_t \geq v_t(\sigma)$, observe that, if he does, then $v_{t+1}(\sigma) = v_t(\sigma)$ implies

$$\pi_t(v_t(\sigma)) = \rho\pi_{t+1}^*(v_{t+1}(\sigma)) = \rho\pi_{t+1}^*(v_t(\sigma)) = \frac{\rho}{4}a_{t+1}v_t(\sigma)^2$$

such that

$$\frac{\rho}{4}a_{t+1}v_t(\sigma)^2 < \frac{(1-\delta+\frac{\delta}{2}a_{t+1})^2 v_t(\sigma)^2}{4(1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_{t+1})}$$

$$\iff \rho a_{t+1}[1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_{t+1}] < (1-\delta+\frac{\delta}{2}a_{t+1})^2$$

$$\iff \rho(1-\delta)a_{t+1}+\rho(\frac{\delta}{2}-\frac{\rho}{4})a_{t+1}^2 < (1-\delta)^2+\delta(1-\delta)a_{t+1}+\frac{\delta^2}{4}a_{t+1}^2$$

$$\iff 0 < (1-\delta)^2+(\delta-\rho)(1-\delta)a_{t+1}+(\frac{\delta^2}{4}-\frac{\rho\delta}{2}+\frac{\rho^2}{4})a_{t+1}^2$$

$$\iff 0 < [1-\delta+(\frac{\delta}{2}-\frac{\rho}{2})a_{t+1}]^2\,,$$

by Lemma 2, (i). By the same argument as above, the consumers' optimal behavior in the last subgame of period $t-1$ yields

$$v_t(\sigma) = \frac{p_{t-1}-\frac{\delta}{2}a_t v_t(\sigma)}{1-\delta} = \frac{p_{t-1}}{1-\delta+\frac{\delta}{2}a_t}$$

such that

$$\pi_t^*(p_{t-1}) = \frac{\frac{1}{4}a_t}{(1-\delta+\frac{\delta}{2}a_t)^2}p_{t-1}^2$$

verifies the value function for period $t$. Also this verifies

$$p_t = \frac{1}{2}a_t v_t(\sigma) = \frac{\frac{1}{2}a_t}{1-\delta+\frac{\delta}{2}a_t}p_{t-1} = F_t(p_{t-1})\,.$$

This completes the induction argument. All that remains to be shown is $v_t(\sigma) \geq v_{t+1}(\sigma)$. But this follows from

$$v_t(\sigma) = \frac{p_{t-1}-\delta p_t}{1-\delta} > \frac{p_t-\delta p_{t+1}}{1-\delta} = v_{t+1}(\sigma)\,,$$

$p_t = F_t(p_{t-1})$, and $p_{t+1} = F_{t+1}(p_t)$, and Lemma 3, (i). This completes the verification of the subgame perfect equilibrium strategies.

Observe that the equilibrium constructed above is one, where all players play their unique best choices against the equilibrium strategies of the other players at all information sets reached by the equilibrium path, except for the finitely many consumers, who satisfy $v = v_t(\sigma)$, for some $t = 1, \ldots, T$. Since finitely many points in the unit interval have (Lebesgue-) measure zero, this equilibrium satisfies SPAAP and thus its outcome (up to the behavior of the finitely many indifferent consumers) is unique by Lemma 1, as required by the Theorem. ∎

The equilibrium constructed in the proof of Theorem 1 displays the following recursive properties:

$$(1)\quad p_t(\sigma) = \frac{1}{2}a_t v_t(\sigma) = F_t(p_{t-1}(\sigma)), \quad p_1(\sigma) = \frac{1}{2}a_1\,;$$

$$(2)\quad v_t(\sigma) = \frac{p_{t-1}}{1-\delta+\frac{\delta}{2}a_t} = \frac{(1-\delta+\frac{\delta}{2}a_t)v_{t-1}(\sigma)}{2(1-\delta+(\frac{\delta}{2}-\frac{\varrho}{4})a_t)}, \quad v_1(\sigma) = 1\,;$$

$$(3)\quad \pi_t^*(\sigma) = \frac{1}{4}a_t v_t(\sigma)^2 = \frac{\frac{1}{4}a_t\pi_{t-1}^*(\sigma)}{1-\delta+(\frac{\delta}{2}-\frac{\varrho}{4})a_t}, \quad \pi_1^*(\sigma) = \frac{a_1}{4}\,.$$

Once the sequence of coefficients $(a_t)_{t=1}^T$ is calculated, equations (1)-(3) allow a simple calculation of the three important variables of the game: prices, active consumers, and profits (along the equilibrium path). Since the $a_t$'s only depend on $T$, it is possible to write $p_t(T)$, $v_t(T)$, and $\pi_t^*(T)$ instead of using $\sigma$ as the argument. Observe from (1) that prices always decrease over time, because

$$\frac{1}{2}a_t < 1-\delta+\frac{\delta}{2}a_t \iff a_t < 2,$$

from Lemma 2, (ii).

Theorem 1 reveals, why the traditional wisdom that the monopolist has to price competitively, because his own agents of different periods compete against each other, does not hold in the present setting: The Bertrand-type competition of the monopolist's agents across periods is an artefact of stating the problem in an infinite horizon in the first place, rather than considering the limit of the solution for finite $T$ (where $T$ is the number of subperiods in a time interval of fixed unit length), when the grid on a finite time interval becomes infinitely fine, as assumed by the original Coase-Conjecture. When the limiting solution for finite $T$ is studied, $T \to \infty$, agents do not compete as fiercly anymore, because the possibilities of a precommitment increase over time: The agent in the last period has, by subgame perfection, no other option than behaving as a one-shot monopolist against the remaining set of still active consumers. Because rational expectations obtain in equilibrium, the monopolist's agents of period $T-1$ know that the agents of period $T$ are commited to do so, and can, therefore, determine the optimal residual demand in $T$ by choosing a higher price $p_{T-1}$ than $p_T$. This structure unravels backwards. The consequences of this are that, as there are more subperiods, the monopolist's overall profit more and more approaches the profit he gains, if he waits for the last period, where he behaves monopolistically.

The next proposition pins down, what these remarks suggest. Let now the time horizon be normalized to 1 and consider the durable monopoly game in $T$ subperiods of length $\Delta = 1/T$ such that $\Delta T = 1$ is the overall time horizon. The appropriate discount factors are now given by

$$\delta = e^{-r\Delta}, \quad \rho = e^{-R\Delta}, \quad r > 0,\ R > 0.$$

Although players do almost not discount from one subperiod to the next, they do discount over the entire time horizon from $t = 0$ to $t = 1$, when $\Delta \to 0$, and their time preference may differ.

PROPOSITION 1 (ANTI-COASE-CONJECTURE). *If $\Delta \to 0$, i.e. $T \to \infty$, then*

$$\pi^*(0) = lim_{\Delta\to 0}\pi_1^*(1/\Delta) = e^{-R}/4 > 0.$$

PROOF: Rewrite the definition of the sequence of coefficients $(a_t)_{t=1}^T$ as

$$a_{1-\tau\Delta} = \frac{(1 - e^{-r\Delta} + \frac{1}{2}e^{-r\Delta}a_{1-\tau\Delta+\Delta})^2}{1 - e^{-r\Delta} + (\frac{1}{2}e^{-r\Delta} - \frac{1}{4}e^{-R\Delta})a_{1-\tau\Delta+\Delta}}, \quad a_1 = 1,$$

for all $\tau = 1, \ldots, T-1$. Then

$$\frac{a_{1-\tau\Delta+\Delta} - a_{1-\tau\Delta}}{\Delta} = \frac{1}{\Delta}[1 - e^{-r\Delta} + (\frac{e^{-r\Delta}}{2} - \frac{e^{-R\Delta}}{4})a_{1-\tau\Delta+\Delta}]^{-1} \times$$
$$\times [(1 - e^{-r\Delta} + (\frac{e^{-r\Delta}}{2} - \frac{e^{-R\Delta}}{4})a_{1-\tau\Delta+\Delta})a_{1-\tau\Delta+\Delta} -$$
$$- (1 - e^{-r\Delta} + \frac{e^{-r\Delta}}{2}a_{1-\tau\Delta+\Delta})^2].$$

Taking limits, using L'Hospital, yields

$$\lim_{\Delta\to 0}\frac{(1 - e^{-r\Delta})a + (\frac{e^{-r\Delta}}{2} - \frac{e^{-R\Delta}}{4})a^2 - (1 - e^{-r\Delta} + \frac{e^{-r\Delta}}{2}a)^2}{\Delta(1 - e^{-r\Delta} + (\frac{e^{-r\Delta}}{2} - \frac{e^{-R\Delta}}{4})a)} =$$
$$= \lim_{\Delta\to 0}[re^{-r\Delta}a - (\frac{re^{-r\Delta}}{2} - \frac{Re^{-R\Delta}}{4})a^2 - 2(1 - e^{-r\Delta} +$$
$$+ \frac{e^{-r\Delta}}{2}a)re^{-r\Delta}(1 - \frac{a}{2})][1 - e^{-r\Delta} + (\frac{e^{-r\Delta}}{2} - \frac{e^{-R\Delta}}{4})a +$$
$$+ \Delta(re^{-r\Delta} - (\frac{re^{-r\Delta}}{2} - \frac{Re^{-R\Delta}}{4})a)]^{-1} = Ra,$$

such that $\dot{a}(t) = R\,a(t)$, $a(1) = 1$. This differential equation has the solution $a(t) = \exp\{Rt - R\}$. This implies from (3) that $\pi^*(0) = \lim_{\Delta\to 0}\pi_1^*(1/\Delta) = a(0)/4 = e^{-R}/4$, as required. ∎

It is of some interest to calculate the limiting paths for prices and the sets of active consumers. From (1) the pricing equation can be written

$$p_{t\Delta+\Delta} = \frac{\frac{1}{2}a_{t\Delta+\Delta}}{1 - e^{-r\Delta} + \frac{1}{2}e^{-r\Delta}a_{t\Delta+\Delta}}\,p_{t\Delta}, \quad p_\Delta = \frac{e^{R\Delta-R}}{2},$$

such that

$$\frac{p_{t\Delta+\Delta} - p_{t\Delta}}{\Delta} = \frac{\frac{1}{2}a_{t\Delta+\Delta} - (1 - e^{-r\Delta} + \frac{1}{2}e^{-r\Delta}a_{t\Delta+\Delta})}{\Delta(1 - e^{-r\Delta} + \frac{1}{2}e^{-r\Delta}a_{t\Delta+\Delta})}\,p_{t\Delta}\,.$$

This yields, again using l'Hospital, from

$$\lim_{\Delta\to 0}\frac{\frac{a}{2} - 1 + e^{-r\Delta} - \frac{e^{-r\Delta}}{2}a}{\Delta(1 - e^{-r\Delta} + \frac{e^{-r\Delta}}{2}a)} = \lim_{\Delta\to 0}\frac{-re^{-r\Delta} + \frac{re^{-r\Delta}}{2}a}{1 - e^{-r\Delta} + \frac{e^{-r\Delta}}{2}a} =$$
$$= r(1 - 2/a) < 0, \quad a \in (0,\, 2),$$

the differential equation $\dot{p}(t) = r[1 - 2\exp\{R - Rt\}]p(t)$, with initial condition $p(0) = \exp\{-R\}/2$, which has the solution

$$p(t) = \frac{1}{2}\exp\{rt - \frac{2r}{R}e^{R}(1 - e^{-Rt}) - R\}. \tag{4}$$

Observe that the more patient the monopolist is (the smaller $R$ is) the higher prices are, as intuition suggests. But also, the more patient consumers are (the smaller $r$ is), the *higher* prices are, because $Rt < 2e^R(1 - e^{-Rt})$, $\forall t \in (0,\, 1]$. Thus consumers, who do not mind waiting, make it easier for the monopolist to let prices drop more slowly.

By an analogous procedure as for prices, one obtains from (2) that

$$v_{t\Delta+\Delta} = \frac{(1 - e^{-r\Delta} + \frac{1}{2}e^{-r\Delta}a_{t\Delta+\Delta})v_{t\Delta}}{2(1 - e^{-r\Delta} + (e^{-r\Delta}/2 - e^{-R\Delta}/4)a_{t\Delta+\Delta})}, \quad v_\Delta = 1,$$

implies

$$\frac{v_{t\Delta+\Delta} - v_{t\Delta}}{\Delta} = \frac{\frac{1}{2}e^{-r\Delta} - \frac{1}{2} - \frac{1}{4}e^{-r\Delta}a_{t\Delta+\Delta} + \frac{1}{4}e^{-R\Delta}a_{t\Delta+\Delta}}{\Delta(1 - e^{-r\Delta} + (e^{-r\Delta}/2 - e^{-R\Delta}/4)a_{t\Delta+\Delta})}\,v_{t\Delta}\,,$$

yielding again with the help of l'Hospital

$$\lim_{\Delta\to 0}\frac{\frac{1}{2}e^{-r\Delta} - \frac{1}{2} - \frac{1}{4}e^{-r\Delta}a + \frac{1}{4}e^{-R\Delta}a}{\Delta(1 - e^{-r\Delta} + (e^{-r\Delta}/2 - e^{-R\Delta}/4)a)} = r - R - 2r/a < -R,$$

if $a \in (0,\ 2)$, such that $\dot{v}(t) = [r - R - 2r\exp\{R - Rt\}]v(t)$, with initial condition $v(0) = 1$ yields

$$v(t) = \exp\{(r - R)t - \frac{2r}{R}e^{R}(1 - e^{-Rt})\}. \tag{5}$$

Note that $v(t)$ will always satisfy $v(t) \in (0,\ 1)$, $\forall t \in (0,\ 1]$, because $v(0) = 1$ and $\dot{v}(t) < 0$. Since $v(1) > 0$ there will always remain a non-empty set of unsatisfied consumers, and - depending on $r$ and $R$ - this set may even be larger than in the one-shot game (this will for example be the case, if $\ln(2) > R$ and $r < [2(e^R - 1) - R]^{-1}[R(\ln(2) - R)]$ holds).

A comparison of equations (4) and (5) shows that prices are always lower than they would be in a sequence of one-shot monopoly games against the remaining set of consumers:

$$p(t) = \frac{1}{2}e^{R(t-1)}v(t), \quad \forall\, t \in (0,\ 1].$$

From this it follows that, if the monopolist becomes very impatient, $R \to \infty$, then the Coase-Conjecture obtains, because not only do all prices converge to zero, but also $\pi^*(0)$, the monopolist's overall profit, approaches zero. This is so, because, only if the monopolist is very impatient, there will be true competitive pressure from his own agents of future periods, who contribute - in this special case - nearly nothing to the payoff of the monopolist's agent in the initial period $t = 0$.

If only $r \to +\infty$, then consumers are so impatient that the monopolist's agent of the initial period $t = 0$ plays against the vast majority of consumers like a one-shot monopolist. Since for all $t \in (0,\ 1]$

$$t - \frac{2}{R}e^{R}(1 - e^{-Rt}) < 0 \iff Rt < 2e^{R}(1 - e^{-Rt})$$

is implied by

$$Rt = 0 = 2e^{R}(1 - e^{-Rt}) \iff t = 0$$

and the derivative inequalities (with respect to $t$)

$$R < 2Re^{R(1-t)} \iff 1 < 2e^{R(1-t)},\ \forall t \in [0,\ 1],$$

one obtains from (4) for all $t \in (0,\ 1]$ that

$$\lim_{r \to +\infty} p(t) = \lim_{r \to +\infty} \frac{1}{2}e^{r(t - \frac{2}{R}e^{R}(1 - e^{-Rt})) - R} = 0,$$

with $p(0) = e^{-R}/2 > 0$. Thus very impatient consumers make the monopolist set strictly positive prices initially which will rapidly drop towards zero, but not sufficiently rapid to leave a noticeable part of consumers holding out for later stages of the game.

Equation (4) also implies that

$$\lim_{r\to 0} p(t) = \frac{e^{-R}}{2}, \quad \forall t \in [0, 1],$$

such that very patient consumers, $r \to 0$, correspond to the case, where the monopolist manouvers himself really into the position of a one-shot monopolist in the terminal period by insisting on the same price for all periods. Still, however, equation (5) shows that $v(t) = e^{-Rt}$, such that consumers buy sequentially.

The final case that can be inferred from equation (4) is the case of a very patient monopolist, $R \to 0$. In this case

$$\lim_{R\to 0} \frac{1 - e^{-Rt}}{R} = \lim_{R\to 0} te^{-Rt} = t$$

implies that $p(t) = \frac{1}{2}e^{-rt}$ and equation (5) implies for $R \to 0$ that $v(t) = e^{-rt}$, and one has $\pi^*(0) = 1/4$, the one-shot monopoly profit. Thus the case of a very patient monopolist corresponds to a "sequential monopoly" in the sense that each period $t$ looks like a one-shot monopoly game with a set of consumers $\mathcal{V}_t = [0, e^{-rt}]$.

As general conclusions one obtains that very patient consumers induce nearly constant prices, while a very patient monopolist induces $p(t) = \frac{1}{2}v(t)$, a "sequential monopoly".

## 4. Infinite Time Horizon

In this section we will study the resulting equilibria of the infinite game which are approximated by solutions to finite games, keeping time preference parameters and the length of sales periods fixed. This is the case, for linear demand functions, covered by **Gul et.al.** [1986]. The more general approach with respect to $\delta$ and $\rho$ presented here sheds a new and surprising light onto the analysis by Gul et.al..

To study the case $T = \infty$ is interesting beyond the purpose of illustrating that it makes a difference how limits are taken. The case $T = \infty$ is one, where two different specifications of the demand side become indistinguishable. We have assumed that, independently of the period $t$ in which consumer $v$ buys the durable, she always derives utility $v$ (in terms of our initial example: she is happier by $v$ for the rest of her life after having seen the exhibits once). If the consumer would rent the durable at price $p_t$ for one period and renting is only possible as long as the monopolist is active, then her utility from renting the durable from period $t$ onwards depends on the remaining time horizon [as, for example, in **Bulow**, 1982]. When in such a world with a finite overall time horizon the length of subperiods becomes arbitrarily small, then the economy's total surplus is driven to zero towards the end of the time horizon (even if nobody has bought yet). Thus such an economy operating within a finite time interval has a built-in bias towards Coasian outcomes, because its total surplus shrinks over time independently of time preferences. When $T = \infty$, this cannot happen and the rental durable goods market becomes equivalent to the scenario with a fixed $v$ independent of $t$. Thus we now turn to exploiting equations (1)-(3) for the case, where $T$ approaches infinity, by applying *asymptotic convergence*, i.e. by selecting as a solution to the infinite game the limit of the solution to finite games.

What drives the solutions to the finite games are the sequences of coefficients $a_t$, described in Lemmas 2 and 3. As a first step towards the analysis of the game with $T = \infty$ it will now be shown that for each given pair of discount factors $(\delta, \rho)$ the $a_t$'s have a unique limit as $T \to \infty$ for all $(\delta, \rho) \in [0, 1]^2$.

LEMMA 4. *When $T$ approaches infinity, then for any finite $t$ the value of the coefficient $a_t$ approaches*

$$a(\delta, \rho) = \frac{2(1-\delta)}{1-\delta+\sqrt{1-\rho}}\,.$$

*The mapping $(\delta, \rho) \mapsto a(\delta, \rho)$ is a continuous function on $[0, 1]^2 \setminus \{(1, 1)\}$, but has a discontinuity at $(\delta, \rho) = (1, 1)$, where $a(1, 1) = 1$.*

PROOF: Define the function $\varphi$: $[0, 2] \to [0, 2]$ by

$$\varphi(a) = \frac{(1-\delta+\frac{\delta}{2}a)^2}{1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a}\,,$$

such that the coefficients $a_t$ from the finite games are implicitely defined by a difference equation of the form

$$a_k = \varphi(a_{k-1}), \quad a_0 = 1\,,$$

where $k$ is just the reverse indexing as the one with $t$, taking $t = T$ as the starting value, $k = 0$. Observe that $\varphi(0) = 1 - \delta \in [0, 1]$ and $\varphi(2) = 2/(2-\rho) \in [1, 2]$. In the proof of Lemma 2 it has been shown that the derivative $\varphi'(a) > 0$, $\forall a \in (0, 2)$. Since $\varphi$ is continuous on

[0, 2], it has at least one fixed point by Brouwer's fixed point theorem. Any fixed point of $\varphi$ must satisfy $a = \varphi(a)$ which results in

$$[\delta(2-\delta)-\rho]a^2 + 4(1-\delta)^2 a - 4(1-\delta)^2 = 0$$
$$\iff a = \frac{2(1-\delta)}{1-\delta \pm \sqrt{1-\rho}}\,.$$

Now assume $a = 2(1-\delta)/[1-\delta-\sqrt{1-\rho}]$ and $a \in [0, 2]$ for $(\delta, \rho) \in [0, 1]^2 \setminus \{(1, 1)\}$. Then one must have $1-\delta > \sqrt{1-\rho}$ and simultaneously $1-\delta \leq 1-\delta-\sqrt{1-\rho}$ which, however, is impossible. Thus $\varphi$ has only a single fixed point on [0, 2] which satisfies

$$a = a(\delta, \rho) = \frac{2(1-\delta)}{1-\delta+\sqrt{1-\rho}}\,.$$

Since the fixed point is unique for all $(\delta, \rho) \in [0, 1]^2 \setminus \{(1, 1)\}$, the derivative of $\varphi$ at the fixed point must satisfy $\varphi'(a(\delta, \rho)) < 1$. Then a simple phase diagram argument shows that any sequence $a_k = \varphi(a_{k-1})$, $a_0 = 1$, converges to $a(\delta, \rho)$ as $k \to \infty$. This proves the first part of the Lemma.

Clearly $a(\delta, \rho)$ is a continuous function in $(\delta, \rho)$ for all $(\delta, \rho) \in [0, 1]^2 \setminus \{(1, 1)\}$, such that it remains to study the behavior of $a(\delta, \rho)$ at $(\delta, \rho) = (1, 1)$. If $(\delta, \rho) = (1, 1)$, then the function $\varphi$ becomes the identity. Thus, although any value $a \in [0, 2]$ now yields a fixed point of $\varphi$, the only fixed point which satisfies the initial condition is $a = 1$. ∎

Lemma 4 sheds considerable doubt on the claim by **Gul et.al.** [1986, p.173] that a uniform discount factor for both the monopolist *and* the consumers is *not* an essential restriction. If the point $(\delta, \rho) = (1, 1)$ is approached along the diagonal, $\rho(\delta) = \delta$, then the limit of $a(\delta, \rho)$ as $\delta \uparrow 1$ is $a(1, 1) = 0$. And this special way of approaching the point (1, 1) in fact explains, why Gul et.al. obtain a Coase-Conjecture, as can be seen from the proof of Theorem 2 below. On the other hand, the only solution at $(\delta, \rho) = (1, 1)$ which satisfies asymptotic convergence has $a(1, 1) = 1$ and, therefore, a positive overall profit of $\pi_1^*(\sigma) = \lim_{T\to\infty} \pi_1^*(T) = 1/4$ equal to the one-shot monopoly profit. This happens, because the mapping assigning equilibrium outcomes which satisfy asymptotic convergence has a discontinuity at this point. However, in the course of proving Theorem 2 below we will show that much more can be sustained at $(\delta, \rho) = (1, 1)$ by other equilibria. Let again $\pi_1^*(\sigma)$ denote the monopolist's overall profit in an equilibrium $\sigma$.

THEOREM 2 ("FOLK THEOREM"). *Consider the infinite horizon game with $T = \infty$ and fixed discount factors $(\delta, \rho) \in [0, 1]^2$. Pick any number*

*$\bar{\pi} \in (0, 1/2)$. Then there exists a continuous path $\{\delta(s), \rho(s)\}_{s\in\Re_+}$, with $\delta\colon \Re_+ \to (0, 1)$, $\rho\colon \Re_+ \to (0, 1)$, and with $\lim_{s\to\infty}\delta(s) = 1 = \lim_{s\to\infty}\rho(s)$ such that for any pair of discount factors $\delta$ and $\rho$ in the closure of this path, $(\delta, \rho) \in (\bigcup_{s\in\Re_+}(\delta(s), \rho(s))) \cup \{(1, 1)\}$, there exists an equilibrium which yields the monopolist a payoff*

$$\pi_1^*(\sigma) = \bar{\pi}.$$

*In particular, any payoff $\bar{\pi} \in (0, 1/2)$ is an equilibrium payoff at $(\delta, \rho) = (1, 1)$.*

PROOF: First assume $(\delta, \rho) = (1, 1)$. By restating the proof of Theorem 1 within an infinite time horizon it can be shown that any $\bar{\pi} \in (0, 1/2)$ can be supported by a suitably chosen equilibrium as the monopolist's overall profit: Since under $\delta = 1$ any consumer $v \geq p_t$ will be indifferent between buying now or later, given that she expects constant prices across all future periods, the value of $v_t$ can be chosen freely within an interval with lower bound $p_t = p$. Since under $\rho = 1$ the monopolist does not care in which period profits are made, a constant price sequence can freely be chosen from an interval with upper bound $v_t = v$. Since the ratio of (constant) prices to the (also constant) $v_t$'s is arbitrary (within the unit interval), this construction allows to generate any profit $\pi_1^*(\sigma) \in (0, 1/2)$.

Observing that for all $(\delta, \rho) \in [0, 1]^2 \setminus \{(1, 1)\}$ Lemma 4 yields an equilibrium with $\pi_1^*(\sigma) = a(\delta, \rho)/4$ by equation (3), one can construct an equilibrium payoff correspondence

$$\pi_1^*(\delta, \rho) = \begin{cases} \frac{\frac{1}{2}(1-\delta)}{1-\delta+\sqrt{1-\rho}}, & \text{if } \rho\delta < 1, \\ [0, 1/2], & \text{if } \rho\delta = 1, \end{cases} \tag{6}$$

which is continuous on $[0, 1]^2$. Now choose a path $(\delta, \rho)\colon \Re_+ \to (0, 1)^2$ such that for all $s \in \Re_+$

$$\rho(s) = 1 - [(\frac{1}{2\bar{\pi}} - 1)(1 - \delta(s))]^2$$

for some $\bar{\pi} \in (0, 1/2)$ and $\lim_{s\to\infty}\delta(s) = 1 = \lim_{s\to\infty}\rho(s)$. Clearly the above equation defines a contour set (iso-profit-curve) of the correspondence $\pi_1^*(\delta, \rho)$ such that

$$\pi_1^*(\delta(s), \rho(s)) = \bar{\pi}, \quad \forall s \in \Re_+ \cup \{\infty\},$$

because

$$\lim_{s\to\infty}\frac{\partial\rho(s)}{\partial\delta(s)} = 0 \quad \text{and} \quad \frac{\partial^2\rho(s)}{\partial\delta(s)^2} = -2(\frac{1}{2\bar{\pi}} - 1)^2 < 0.$$

From the definition the limit of $\pi_1^*(\delta(s), \rho(s))$, when $s \to \infty$, also equals $\bar{\pi}$. ∎

What Theorem 2 says is that, for any equilibrium profit of the monopolist between almost nothing and almost all the surplus of the economy, there is a one-dimensional manifold in the plane of discount factors along which this profit will be sustained by the equilibrium outcome satisfying asymptotic convergence. Moreover, each of these iso-(equilibrium)-profit curves ends at the point $(\delta, \rho) = (1, 1)$, where everything is possible. This resembles a Folk Theorem, despite the fact that the durable monopoly with an infinite time horizon is not a supergame. On the other hand Theorem 2 differs from a Folk Theorem, because for any interior pair of discount factors it assigns a unique equilibrium payoff. Its main message is that everything in the infinite game depends on the *relative* patience of the monopolist versus potential customers. If $\rho = 1$ and $\delta \in [0, 1)$, then $\pi_1^*(\sigma) = 1/2$ and by perfectly price-discriminating against consumers the monopolist extracts all the surplus. If $\delta = 1$ and $\rho \in [0, 1)$, then the opposite is true, $\pi_1^*(\sigma) = 0$. This is the case of the Coase-Conjecture, where the consumers' greater patience forces the monopolist's agents to compete against each other. If $(\delta, \rho) = (0, 0)$, then, $\pi_1^*(\sigma) = 1/4$, the one-shot monopoly profit results. At $(\delta, \rho) = (1, 1)$ any number $\bar{\pi} \in (0, 1/2)$ can be supported as an equilibrium payoff to the monopolist. Although the equilibrium correspondence (6), used in the proof of Theorem 2, is constructed such as to be continuous, the reaction of equilibrium payoffs to slight parameter variations in the vincity of $(\delta, \rho) = (1, 1)$ is dramatic.

An outside observer of different durable monopoly markets with very patient players may see radically different profits of the monopolist across markets, even if the discount factors look almost the same. But, if this observer takes finitely many observations of prices on each market, she will also see almost constant price sequences on each market, although at very different levels. This is so, because Lemma 4 yields for any finite $t$ that $a_t = a(\delta, \rho)$ such that equation (1) yields

$$p_t(\sigma) = \frac{\frac{1}{2}a(\delta, \rho)}{1 - \delta + \frac{\delta}{2}a(\delta, \rho)}\, p_{t-1}(\sigma) = \frac{p_{t-1}(\sigma)}{1 + \sqrt{1 - \rho}}$$

such that

$$\lim_{\rho \to 1} p_t(\sigma) = p_{t-1}(\sigma).$$

Thus, for $\rho$ sufficiently close to 1, the equilibrium price sequence will be almost constant for all finite $t$ and will, therefore, depend on $p_1(\sigma) = a_1/2 = a(\delta, \rho)/2$. Consequently, the choice of $\delta$ (in a neighbourhood of 1) will determine the level of the almost constant sequence $\{p_t(\sigma)\}_t$ by determining the level of $p_1(\sigma) = a(\delta, \rho)/2$. This observation shows that close to $(\delta, \rho) = (1, 1)$ there is no tendency whatsoever for prices to drop quickly - contrary to what a Coasian intuition would suggest.

How can these findings be reconciled with the results by **Gul et.al.** [1986]? It was already mentioned that, when the point $(\delta, \rho) = (1, 1)$ is approached along the diagonal, $\delta = \rho$, then the monopolist's equilibrium profit approaches zero. On the other hand there is this set-valuedness at this point in (6) (where the function assigning asymptotically covergent equilibrium payoffs had a discontinuity), close to which payoffs vary dramatically with parameters. Still there is a sense in which the operation of assigning the Coase-Conjecture to the limiting point $(\delta, \rho) = (1, 1)$ can be viewed as robust. The reason for this robustness is a fundamental asymmetry in the durable monopoly game which, at least around the point $(\delta, \rho) = (1, 1)$, manifests itself in the result stated next:

PROPOSITION 2 (LOCAL COASE-CONJECTURE). *Consider any twice continuously differentiable function* $\rho\colon (-\infty, 1] \rightarrow [0, 1]$ *which satisfies* $\rho(1) = 1$. *Then*

$$lim_{\delta \rightarrow 1} \pi_1^*(\delta, \rho(\delta)) > 0$$

*implies that the derivative of* $\rho$ *with respect to* $\delta$ *at the point* $\delta = 1$ *is zero,* $\rho'(1) = 0$.

PROOF: From $\rho(\delta) \in [0, 1]$ and $\rho(1) = 1$ it follows that $\rho'(1) \geq 0$. Since $\rho(1) = 1$, l'Hospital has to be used to calculate the limit of equation (6),

$$\lim_{\delta \rightarrow 1} \frac{\frac{1}{2}(1-\delta)}{1 - \delta + \sqrt{1 - \rho(\delta)}} = \lim_{\delta \rightarrow 1} \frac{\sqrt{1 - \rho(\delta)}}{\rho'(\delta) + 2\sqrt{1 - \rho(\delta)}}.$$

Clearly, if this limit is strictly positive, then $\rho'(1) > 0$ is impossible, such that $\rho'(1) = 0$ follows. ∎

The significance of Proposition 2 is the following: Whenever the point $(\delta, \rho) = (1, 1)$ is approached along a path $(\delta, \rho(\delta))_{\delta \uparrow 1}$, $\rho(1) = 1$, and the limit of $\pi_1^*(\delta, \rho(\delta))$ is taken along this path, then any path which has $\rho'(1) > 0$ will yield $\pi_1^*(1, 1) = 0$, the Coasian outcome. Only paths $(\delta, \rho(\delta))_{\delta \uparrow 1}$ which *locally*, in first order approximation, look like the "edge" $\bigcup_{\delta \in (0, 1]} (\delta, 1)$ of $(\delta, \rho)$-space will yield a positive limiting profit to the monopolist.

This does *not* mean that any sequence $\{(\delta_h, \rho_h)\}_{h=1}^{\infty}$, with $(\delta_h, \rho_h) \in (0, 1)^2$, $(\delta_h, \rho_h) \longrightarrow_{h \rightarrow \infty} (1, 1)$, which is close to the diagonal in some metric (on the space of sequences) will generate the Coasian outcome. A metric on the space of sequences will simply not be able to discriminate between the slopes $\rho'(1)$ which are necessary to apply l'Hospital. What Proposition 2 states is much weaker: It says that in a topology which discriminates between slopes $\rho'(1)$ of differentiable paths the operation of assigning a unique value to $\pi_1^*(1, 1)$ is continuous, only if the value assigned is zero. In this sense Proposition 2 is indeed a "very" local result. The limit operation which assigns the Coasian outcome is robust only in a topology which is of an order finer than information on how the values of $\rho$ and $\delta$ relate to each other. If data is only available on the values of $\rho$ and $\delta$, then such a limit result requires information that cannot possibly be obtained by the analyst.

Still this puts the Coase-Conjecture as demonstrated by **Gul et.al.** [1986] into the correct theoretical perspective: Locally in first order approximation it looks *as if* the monopolist would have to be infinitely more patient than consumers to be able to extract a positive share of the surplus. Of course, globally the picture is much less extreme: Since the contour set of $\pi_1^*(\delta, \rho) = 0$ from equation (6) is the "edge" $\bigcup_{\rho \in [0, 1]}(1, \rho)$ of the space of discount factors, it globally takes consumers, who are infinitely more patient than the monopolist, to establish the Coase-Conjecture. Proposition 2 only shows that there is a weak sense in which the assignment of the Coasian outcome to the extreme point $(\delta, \rho) = (1, 1)$ can be viewed as a robust limit operation (in a sufficiently fine topology).

The intuitive reason that allows for a "Local Coase-Conjecture" in the spirit of Proposition 2 is the following asymmetry in the durable monopoly game: When all players are very patient, then each potential customer looses very little by holding out to take advantage of future price cuts. If the monopolist, on the other hand, tries to hold out by letting prices drop only very slowly, then this will cost her more, because to be able to credibly do so, she will have to start out with low prices in the first place.

The general conclusion on infinite durable monopoly games is, however, that the *relative* patience of consumers versus the monopolist determines "who eats whom in the durable goods monopoly" [to quote from **von der Fehr and Kühn**, 1990]. Extremely patient consumers will enforce almost competitive outcomes. A very patient monopolist will perfectly price discriminate. If both parties are very myopic, then prices start out high and drop fast, approximating the outcome of the one-shot monopoly. If both parties are very patient, prices drop very slowly, but how they start out is basically indeterminate - with a local tendency in favor of the Coase-Conjecture.

## 5. Concluding Remarks

The present paper has explored the generic equilibria of a durable monopoly market with a very special (linear) demand function, but with both finite and infinite time horizon *and* with possibly asymmetric time preferences for the monopolist and potential customers.

The Coase-Conjecture within its original setting [**Coase**, 1972], i.e. within a finite time interval divided into infinitely many small subperiods, does not hold true. It takes an extremely myopic monopolist to establish that the monopolist's equilibrium profit will be almost zero. The main reason for this result is that with bounded time preference parameters and a finite horizon the monopolist's ability to control the incentive structure of his own future agents increases over time, even if the length of sales periods is very small.

The picture is altered, once the finite time horizon is given up. If the number of possible sales periods is infinite, but discount factors (and the length of sales periods) remain fixed, then for very patient players a result which resembles a Folk Theorem holds: Although asymptotic convergence will still select a unique equilibrium for the boundary case $(\delta, \rho) = (1, 1)$ (with a payoff to the monopolist equal to the one-shot monopoly profit), there are other equilibria in the limit in which everything is possible. The *relative* time preference in fact determines whether a Coase-Conjecture will hold in the infinite game or whether the monopolist will almost perfectly price discriminate against consumers, thus extracting all the surplus. With possibly asymmetric time preferences the Coase-Conjecture, as demonstrated by **Gul et.al.** [1986], thus turns out to be a special case, generated by assuming symmetric time preferences. Still there is a local argument in favor of the conclusion drawn by Gul et.al.: With extremely fine information the Coasian outcome can be made a robust limiting outcome. The only drawback which this limit has is that it does not satisfy asymptotic convergence.

These results suggest that, if the monopolist has the opportunity to commit to closing down her operation after some finite time horizon, then she has an incentive to do so. Also, if the monopolist would be able to commit to longer sales periods, by having an employee doing the transactions after having set the price for the week herself, say, then such a commitment will make the monopolist better off. This, after all, may be the reason, why in our initial example we saw an agency selling the tickets to the exhibition, rather than the company that provided the exhibits.

It is unlikely that the technique of the present analysis can easily be generalised to more general forms of demand functions like the ones used by **Gul et.al.** [1986]. Still the arguments of Lemma 1 suggest that *qualitatively* our conclusions will hold true for all distributions of the consumers' valuations in a neighbourhood of the uniform distribution. Given the negative flavor that comes with an Anti-Coase-Conjecture and a Folk Theorem, a non-degenerate simple example should be sufficient to make the point. Beyond this, the richness of phenomena generated by our model may serve as a guidance for future research using more general demand functions, but also more structure concerning time preferences.

## APPENDIX

PROOF OF LEMMA 2: Let $t = T$; then $a_T = 1$, such that $1 - \delta + \delta/2 = 1 - \delta/2 > \rho/2 > \rho/4$ and $0 < a_T = 1 < 2$ hold. Then, in order to apply induction, assume that

$$(i') \quad 1 - \delta + \frac{\delta}{2} a_{t+1} > \frac{\rho}{4} a_{t+1},$$

$$(ii') \quad 1 - \delta + \frac{\delta}{2} a_{t+1} > \frac{\rho}{2} a_{t+1},$$

$$(iii') \quad 0 < a_{t+1} < 2.$$

Then $(i')$ implies $a_t > 0$. From

$$\frac{\partial}{\partial a} \frac{(1 - \delta + \frac{\delta}{2} a)^2}{1 - \delta + (\frac{\delta}{2} - \frac{\rho}{4})a} = [(1 - \delta + \frac{\delta}{2} a)[\frac{\delta}{2}(1 - \delta + (\frac{\delta}{2} - \frac{\rho}{4})a) +$$

$$+ \frac{\rho}{4}(1 - \delta)]][1 - \delta + (\frac{\delta}{2} - \frac{\rho}{4})a]^{-2}\Big|_{0<a<2} >$$

$$> \frac{(1 - \delta + \frac{\delta}{2} a)[\frac{\delta}{4}(1 - \frac{\rho}{2})a + \frac{\rho}{4}(1 - \delta)]}{[1 - \delta + (\frac{\delta}{2} - \frac{\rho}{4})a]^2} > 0,$$

(which follows by using $\frac{\delta}{2}(1 - \delta + (\frac{\delta}{2} - \frac{\rho}{4})a) > \frac{\delta}{4}(1 - \frac{\rho}{2})a \iff 2 > a > 0$) it follows that $(iii')$ implies

$$a_t < \frac{1}{1 - \rho/2} < 2,$$

where the second inequality follows from $\rho < 1$. Thus $(i')$ and $(iii')$ imply (ii).

Finally observe that

$$\frac{\partial}{\partial \delta}[1 - \delta + (\frac{\delta}{2} - \frac{\rho}{4})a_t] = \frac{1}{2} a_t - 1 < 0,$$

from the above, which implies

$$1-\delta+(\frac{\delta}{2}-\frac{\rho}{2})a_t>\frac{1}{2}(1-\rho)a_t>0 \Longrightarrow$$
$$\Longrightarrow 1-\delta+\frac{\delta}{2}a_t>\frac{\rho}{2}a_t>\frac{\rho}{4}a_t\,,$$

as required in (i). ∎

Proof of Lemma 3: (i) From $p_t=F_t(p_{t-1})$ one has

$$\frac{p_{t-1}-\delta p_t}{1-\delta}=(1-\delta)^{-1}[1-\frac{\frac{\delta}{2}a_t}{1-\delta+\frac{\delta}{2}a_t}]p_{t-1}=$$
$$=\frac{p_{t-1}}{1-\delta+\frac{\delta}{2}a_t}\,.$$

Consequently, the inequality (i) holds, if and only if

$$\frac{p_{t-2}}{1-\delta+\frac{\delta}{2}a_{t-1}}>\frac{p_{t-1}}{1-\delta+\frac{\delta}{2}a_t}=\frac{\frac{1}{2}a_{t-1}p_{t-2}}{(1-\delta+\frac{\delta}{2}a_t)(1-\delta+\frac{\delta}{2}a_{t-1})}$$
$$\Longleftrightarrow 1-\delta+\frac{\delta}{2}a_t>\frac{1}{2}a_{t-1}\,.$$

Using the definition of $a_{t-1}$ the latter inequality reads

$$1-\delta+\frac{\delta}{2}a_t>\frac{1}{2}\frac{(1-\delta+\frac{\delta}{2}a_t)^2}{1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_t} \Longleftrightarrow$$
$$1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_t>\frac{1}{2}(1-\delta+\frac{\delta}{2}a_t) \Longleftrightarrow$$
$$1-\delta+\frac{\delta}{2}a_t>\frac{\rho}{2}a_t\,,$$

which is the first part of Lemma 2, (i).

(ii) The definition of $F_t$ implies that

$$p_{t+k}=\prod_{j=0}^{k-1}\frac{\frac{1}{2}a_{t+k-j}}{1-\delta+\frac{\delta}{2}a_{t+k-j}}p_t\,,\quad \forall k=1,\ldots,T-t,$$

such that

$$\frac{p_t-\delta^k p_{t+k}}{1-\delta^k}>\frac{p_t-\delta^{k+1}p_{t+k+1}}{1-\delta^{k+1}}$$

can equivalently be written as

$$\Big(\frac{1-\delta^k 2^{-k}\prod_{i=0}^{k-1}(1-\delta+\frac{\delta}{2}a_{t+k-i})^{-1}a_{t+k-i}}{1-\delta^k}\Big)p_t >$$
$$> \Big(\frac{1-\delta^{k+1}2^{-k-1}\prod_{i=0}^{k}(1-\delta+\frac{\delta}{2}a_{t+k+1-i})^{-1}a_{t+k+1-i}}{1-\delta^{k+1}}\Big)p_t$$

which is equivalent to

$$1-\delta^{k+1}-\delta^k(1-\delta^{k+1})2^{-k}\prod_{i=0}^{k-1}(1-\delta+\frac{\delta}{2}a_{t+k-i})^{-1}a_{t+k-i} >$$
$$> 1-\delta^k-\delta^{k+1}(1-\delta^k)2^{-k-1}(1-\delta+\frac{\delta}{2}a_{t+k+1})^{-1}a_{t+k+1}\times$$
$$\times\prod_{i=0}^{k-1}(1-\delta+\frac{\delta}{2}a_{t+k-i})^{-1}a_{t+k-i}$$

which, finally, is equivalent to

$$2^k\prod_{i=0}^{k-1}\frac{1-\delta+\frac{\delta}{2}a_{t+k-i}}{a_{t+k-i}} > \frac{1-\delta^{k+1}}{1-\delta}-$$
$$-\delta\frac{1-\delta^k}{1-\delta}\,\frac{\frac{1}{2}a_{t+k+1}}{1-\delta+\frac{\delta}{2}a_{t+k+1}} = \frac{1-\delta^{k+1}+\frac{\delta^{k+1}}{2}a_{t+k+1}}{1-\delta+\frac{\delta}{2}a_{t+k+1}}.$$

The latter inequality will now be demonstrated by induction. Let $k=1$; then

$$2\frac{1-\delta+\frac{\delta}{2}a_{t+1}}{a_{t+1}} > \frac{1-\delta^2+\frac{\delta^2}{2}a_{t+2}}{1-\delta+\frac{\delta}{2}a_{t+2}} \iff$$
$$\frac{2(1-\delta)[1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_{t+2}]+\delta[1-\delta+\frac{\delta}{2}a_{t+2}]^2}{1-\delta+\frac{\delta}{2}a_{t+2}} > 1-\delta^2+\frac{\delta^2}{2}a_{t+2}$$
$$\iff 1-\delta+\frac{\delta}{2}a_{t+2} > \frac{\rho}{2}a_{t+2},$$

which is the first part of Lemma 2, (i). Next assume

$$2^{k-1}\prod_{i=0}^{k-2}\frac{1-\delta+\frac{\delta}{2}a_{t+k-1-i}}{a_{t+k-1-i}} > \frac{1-\delta^k+\frac{\delta^k}{2}a_{t+k}}{1-\delta+\frac{\delta}{2}a_{t+k}}.$$

This implies

$$2^k \prod_{i=0}^{k-1} \frac{1-\delta+\frac{\delta}{2}a_{t+k-i}}{a_{t+k-i}} > \frac{2(1-\delta^k)+\delta^k a_{t+k}}{a_{t+k}} =$$

$$= \frac{2(1-\delta^k)[1-\delta+(\frac{\delta}{2}-\frac{\rho}{4})a_{t+k+1}]+\delta^k[1-\delta+\frac{\delta}{2}a_{t+k+1}]^2}{[1-\delta+\frac{\delta}{2}a_{t+k+1}]^2} =$$

$$= [(1-\delta+\frac{\delta}{2}a_{t+k+1})[2(1-\delta^k)+\delta^k(1-\delta+\frac{\delta}{2}a_{t+k+1})]-$$

$$-\frac{\rho}{2}(1-\delta^k)a_{t+k+1}][1-\delta+\frac{\delta}{2}a_{t+k+1}]^{-2} \geq$$

$$\geq \frac{1-\delta^{k+1}+\frac{\delta^{k+1}}{2}a_{t+k+1}}{1-\delta+\frac{\delta}{2}a_{t+k+1}},$$

because the second (weak) inequality is equivalent to

$$2(1-\delta^k)+\delta^k(1-\delta+\frac{\delta}{2}a_{t+k+1})-\frac{\frac{\rho}{2}(1-\delta^k)a_{t+k+1}}{1-\delta+\frac{\delta}{2}a_{t+k+1}} \geq$$

$$\geq 1-\delta^{k+1}+\frac{\delta^{k+1}}{2}a_{t+k+1} \iff$$

$$1-\delta+\frac{\delta}{2}a_{t+k+1} \geq \frac{\rho}{2}a_{t+k+1}$$

which, again, follows from the first part of Lemma 2, (i). ∎

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