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A NOTE ON GAMES CORRESPONDING TO SEQUENCING SITUATIONS WITH DUE DATES

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A NOTE ON GAMES CORRESPONDING TO SEQUENCING SITUATIONS WITH DUE DATES

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Abstract

It is shown that sequencing situations in which all jobs have equal processing times, the due date date of each job is a multiple of its processing time and the cost of each job is linear in its completion time, yield the same class of convex games as the sequencing situations in in which all jobs have equal processing times, the ready time of each job is a multiple of its processing time and the cost of each job is linear in its completion time.

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Convex cooperative games, one-machine sequencing situations, due dates, ready times.

In a one-machine sequencing situation there is a queue of agents, each with one job, before a machine. Each job has to be processed on the machine. The finite set of agents is denoted by N and $\mid N \mid = n$. By a bijection $\sigma : N \rightarrow \{1, ..., n\}$ we can describe the position of the agents in the queue. Specifically, $\sigma(i) = j$ means that player i is in position j. The due date d_i of the job of agent i is the latest time the processing of this job has to be completed. The processing time p_i of the job of agent i is the time the machine takes to handle this job. We assume that every agent has a linear cost function $c_i : [0, \infty) \rightarrow I\!R$ defined by $c_i(t) = \alpha_i t$ with $\alpha_i > 0$ the cost coefficient of player i. The completion time $C(\sigma, i)$ of the job of agent i if processed according to a bijection σ (in a semi-active way) is the sum of the earliest time the job can start w.r.t. σ and its processing time. In this note we concentrate on sequencing situations that satisfy

$(A1)$ $\quad d_i \in \{1, ..., n\}$ and $p_i = 1$ for all $i \in N$

Further, it is assumed that there is an initial bijection $\sigma_0 : N \rightarrow \{1, ..., n\}$ on the jobs of the players before the processing of the machine starts with the properties

$(A2)$ $\quad d_i \leq d_j$ for all $i, j \in N$ with $\sigma_0(i) < \sigma_0(j)$, and $C(\sigma_0, i) \leq d_i$ for all $i \in N$

and

$(A3)$ $\quad \sigma_0(i) = C(\sigma_0, i)$ for all $i \in N$.

Note that the assumptions $(A1) - (A2)$ imply that in the initial bijection there is no time gap in the job processing and that in particular the last job that is processed

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according to σ_0 is completed at time n. In spite of the conclusion that assumption $(A3)$ is superfluous, we have added it here for the sake of convenience and symmetry with ready time sequencing situations discussed later on. A sequencing situation as described above is denoted by $(N, \sigma_0, d, p, \alpha)$ and will be refered to as a d-sequencing situation.

The total costs $c_\sigma(S)$ of a coalition $S \subseteq N$ w.r.t. a bijection σ are given by

$$c_\sigma(S) := \sum_{i \in S} \alpha_i(C(\sigma, i)).$$

The (maximal) cost savings of a coalition S depend on the set of admissible rearrangements of this coalition. Since each job has to be completed before its due date, we will consider only those $\sigma : N \to \{1, ..., n\}$ that satisfy $C(\sigma, i) \leq d_i$. Such a bijection $\sigma : N \to \{1, ..., n\}$ will be called *admissible* for S if it satisfies $P(\sigma, i) = P(\sigma_0, i)$ for all $i \in N \backslash S$, where $P(\sigma, i) = \{j \in N \mid \sigma(j) < \sigma(i)\}$. Hence, we consider an order to be admissible for S if each agent outside S has the same starting time as in the initial order. Moreover, the agents of S are not allowed to jump over players outside S. The set of all admissible rearrangements for a coalition S is denoted by Σ_S . Note that by the assumptions on the initial and admissible bijections we have for any $\sigma \in \Sigma_S$ that $\sigma(i) = C(\sigma, i)$ for all $i \in N$.

Given a sequencing situation $(N, \sigma_0, d, p, \alpha)$ the corresponding sequencing game is defined in such a way that the the worth of a coalition S is equal to the maximal cost savings the coalition can achieve by means of admissible rearrangements. Formally we have

$$v(S) \quad = \quad \max_{\sigma \in \Sigma_S} \{\sum_{i \in S} \alpha_i C(\sigma_0, i) - \sum_{i \in S} \alpha_i C(\sigma, i)\} \tag{1}$$

From the definition of admissible rearrangements it follows that the essential coalitions for sequencing games are the connected ones. A coalition S is called *connected* with respect to σ_0 if for all $i, j \in S$ and $k \in N$, $\sigma_0(i) < \sigma_0(k) < \sigma_0(j)$ implies $k \in S$.

Next, we describe the special class of one-machine sequencing situations, in which all jobs have equal processing times and the ready time of each job is a multiple of the processing time and the corresponding class of games. The description of these sequencing games is identical to the sequencing situations corresponding to due dates. The only difference is that there is no due date imposed on a player but a ready time. The ready time r_i of the job of agent i is the earliest time that the job can be processed on the machine. We will concentrate on sequencing situations that satisfy

$(B1)$ $\quad r_i \in \{0, ..., n-1\}$ and $p_i = 1$ for all $i \in N$.

The initial order σ_0 has the properties

$(B2)$ $\quad r_i \leq r_j$ for all $i, j \in N$ with $\sigma_0(i) < \sigma_0(j)$ and $C(\sigma_0, i) \geq r_i + 1$ for all $i \in N$

and

$(B3)$ $\quad \sigma_0(i) = C(\sigma_0, i)$ for all $i \in N$.

Note that the assumptions $(B1) - (B3)$ imply that in the initial bijection σ_0 there are no time gaps in the job processing and that the job that is processed last is completed at time n. A sequencing situation as described above is denoted by $(N, \sigma_0, r, p, \alpha)$ and will be refered to as an r-sequencing situation.

In r-sequencing situations we will only consider those bijections $\sigma : N \to \{1, ..., n\}$ that satisfy $C(\sigma, i) \geq r_i + 1$ for all $i \in N$. The set of admissible rearrangements, denoted by

$\mathcal{A}_S$, has the same restrictions with respect to interchanging positions between players of a coalition S as before. Hence, we may again conclude that for any $\sigma \in \mathcal{A}_S$ we have that $\sigma(i) = C(\sigma, i)$. The corresponding sequencing game is defined by

$$v(S) = \max_{\sigma \in \mathcal{A}_S}\{\sum_{i \in S} \alpha_i C(\sigma_0, i) - \sum_{i \in S} \alpha_i C(\sigma, i)\} \tag{2}$$

Hamers, Borm and Tijs (1995) show that sequencing games arising from r-sequencing situations are convex by establishing relations between optimal orders of subcoalitions. These relations are obtained by analysing the procedure described in *Rinnooy Kan (1976)* that provides an optimal order. For the optimal order in d-sequencing situations we can use the procedure of *Smith (1956)*, which operates similar to the procedure of *Rinnooy Kan (1976)*. Both procedures aim for having the jobs with the largest cost coefficient α_i as far as possible at the front of the queue. The Smith-procedure has to take into account the due dates, whereas the Rinnooy Kan-procedure has to take into account the ready times. For this reason the Smith-procedure starts at the end of the queue, whereas the the Rinnooy Kan-procedure starts at the front of the queue. In spite of this difference it is possible for d-sequencing situations to establish similar relations between optimal orders of various subcoalitions as for r-sequencing situations. However, where in the Rinnooy Kan-procedure these relations are established if a player is added at the end of a (sub)queue, in the Smith-procedure these relations can be established if a player is added at the front of a (sub)queue. Following exactly the same line of argument it can be infered that sequencing games arising from d-sequencing situations are convex games.

In fact, we will show even a stronger result: both classes of sequencing situations generate the same class of sequencing games.

Theorem 1 *Let $R(N)$ and $D(N)$ be the class of sequencing games that arise from r-sequencing situations and d-sequencing situations, respectively. Then $R(N) = D(N)$.*

PROOF: We show that $R(N) \subseteq D(N)$. Let $(N, v) \in R(N)$. Let $(N, \sigma_0, r, p, \alpha)$ be an r-sequencing situation that generates the game (N, v). W.l.o.g. we can take $\sigma_0(i) = i$ for all $i \in N$. Let $S = \{i, i+1, ..., j\}$, be a connected set w.r.t. σ_0. Then

$$v(S) = \max\{\sum_{k=i}^{j} \alpha_k k - \sum_{k=i}^{j} \alpha_k x_k \mid x_k \geq r_k + 1 \ \forall k \in S, \{x_i, ..., x_j\} = \{i, ..., j\}\}. \tag{3}$$

Consider the d-sequencing situation (N, τ_0, d, p, β) in which for all $i \in N$ we define $\tau_0(i) = n + 1 - i$, $d_i = n - r_i$ and $\beta_i = c + (\alpha_n - \alpha_i)$ with $c = \max_{i \in N} \alpha_i$.
We first show that (N, τ_0, d, p, β) satisfies the assumptions $(A1) - (A3)$. Obviously, $(A3)$ is a consequence of $(B1)$, while $(A1)$ follows immediately from the definition of d and $(B1)$. If $\tau_0(l) < \tau_0(m)$ then $m < l$ which implies that $r_m \leq r_l$. The definition of d yields immediately that $d_l \leq d_m$. Further, we have for any $l \in N$ that $\sigma_0(l) = l \geq r_l + 1 = n + 1 - d_l$. This implies that $d_l \geq n + 1 - l = \tau_0(l) = C(\tau_0, l)$. Hence $(A2)$ is satisfied.
Note that from the definition of τ_0 it follows that S is also connected w.r.t. τ_0. Then for the game (N, w) corresponding to (N, τ_0, d, p, β) it holds that

$$\begin{aligned} w(S) \; = \; \max\{\sum_{k=i}^{j} \beta_k(n+1-k) - \sum_{k=i}^{j} \beta_k y_k \mid y_k \leq d_k \ \forall k \in S, \\ \{y_i, ..., y_j\} = \{n+1-j, ..., n+1-i\}\} \end{aligned} \tag{4}$$

Let $\hat{y}$ be an optimal solution of (4). By defining $\hat{x}$ by $\hat{x}_k = n+1-\hat{y}_k$ for all $k \in \{i,...j\}$ we have

$$\begin{aligned}
w(S) &= \sum_{k=i}^{j} \beta_k(n+1-k) - \sum_{k=i}^{j} \beta_k \hat{y}_k \\
&= \sum_{k=i}^{j} (c+\alpha_n-\alpha_k)(n+1-k) - \sum_{k=i}^{j} (c+\alpha_n-\alpha_k)(n+1-\hat{x}_k) \\
&= (c+\alpha_n)\sum_{k=i}^{j}(\hat{x}_k - k) + \sum_{k=i}^{j} \alpha_k(k-\hat{x}_k) \\
&= \sum_{k=i}^{j} \alpha_k(k-\hat{x}_k) \\
&\leq v(S),
\end{aligned}$$

where the first equality holds since $\hat{y}$ is optimal, the second equality by the definition of τ_0, β and $\hat{x}$, the third equality and fourth equality by straightforward calculations. The inequality holds by (3) since $\hat{x}_k = n+1-\hat{y}_k \geq n+1-d_k = n+1-(n-r_k) = r_k+1$ and $\{\hat{x}_i, ..., \hat{x}_j\} = \{i, ..., j\}$.
Let $\hat{x}$ be an optimal solution of (3). By defining $\hat{y}$ by $\hat{y}_k = n+1-\hat{x}_k$ for all $k \in S$ we can show in the same way as above that $v(S) \leq w(S)$, which completes the first part of this proof.
Obviously, the second part, $D(N) \subseteq R(N)$, can be dealt with in an analogous way. □

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