UNANTICIPATED MONEY AND THE DEMAND FOR FOREIGN ASSETS - A RATIONAL EXPECTATIONS APPROACH.

by

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Abstract:

This paper is an attempt at empirically investigating one of the building blocks of the foreign exchange market - the demand for foreign financial assets - under two alternative monetary policy rules (the fixed rate of growth of money rule and the feedback monetary policy rule) using time series data for Germany over the period 1974:1 - 1992:4. The approach adopted here offers us the opportunity of investigating the impact of monetary policy shocks on the demand for foreign assets. It turns out that the data-set picks the feedback monetary policy rule as characterising the data generation process better. Based on this feedback policy rule roughly 23% of monetary policy shocks have permanent effects (and the remaining percentage of these shocks exhibit transitory effects) on the demand for foreign assets. Our empirical results indicate, among others, that unanticipated monetary policy shocks and increases in the interest rate differential drive German investors abroad whereas domestic interest rate increases and currency depreciations demonstrate the "safe-haven" effect (that is to say domestic investors prefer having their investments in assets denominated in Deutsche Marks).

Key Words: Unanticipated Money, Foreign Assets, Monetary Policy Rules and Rational Expectations.

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1. INTRODUCTION

The period after the breakdown of Bretton Woods in the early 1970's till now is marked by flexible exchange rate systems in most industrialized countries. The volatility of the exchange rate that came with the breakdown of Bretton Woods gave theoretical as well as empirical international macroeconomists the opportunity to enquire into the nature and functions of the flexible exchange rate system with the aim of understanding the system better in order to be able to predict and control it. The approach utilized by the theorists can be put into three broad categories - the monetary approach, the portfolio balance model and the target zones model. The first two approaches are generally referred to as Asset Based Exchange Rate Models.

Most of the empirical tests of these models however leave much to be desired. Dornbusch (1979) and Frenkel (1984) however are able to find evidence in support of the Flexible Price Monetary model of exchange rates. But once their sample is extended to beyond 1980 the model performs very badly. To rehabilitate the Monetary Model MacDonald and Taylor (1992) used cointegration approach that views the monetary model as a long-run equilibrium condition to which the exchange rate converges. The Portfolio Balance Model (PBM) has been empirically tested by Branson, Halttunen and Masson (1977) using a reduced form exchange rate equation derived from the PBM - the results revealed insignificant coefficients and the persistence of autocorrelation. Empirical tests of the Target Zones Model have not yielded any fruitful results.(See for instance Bertola and Caballero (1992), Flood, Rose and Mathieson (1991) and Lindberg and Söderlind (1991)).

It is our contention that a better understanding of the foreign exchange market can be attained by explicitly specifying the microeconomic decision processes underlying the relationships so specified - be they on exchange rate determination[1] or on the demand for foreign assets[2].

[1]. Woo(1985) suspects that the inability of some "recent studies to find empirical support for the monetary approach to exchange rate determination may be due to an inappropriate specification of the money demand function" and inappropriate estimation techniques.

[2]. There are however, a number of articles in international monetary theory and finance that utilize microeconomic foundations in determining the optimal portfolio of investors. See

Utilizing this microeconomic foundations approach (which necessarily incorporates the Rational Expectations Hypothesis (REH) because expectations are inextricably linked with discussions on the foreign exchange market[3]) we hope our models can capture the effect of preferences on the whole process of aggregate portfolio determination and hence of the determination of the exchange rate. This is because under rational expectations the agents are assumed to know the policy strategies of the authorities and can hence correctly predict effects of policy changes on their portfolios. They accordingly alter the composition of their portfolios in response to the new information even before policy is implemented (i.e. when policy is anticipated). This is one element lacking in most empirical work on the determination of the exchange rate.

Whereas the reduced form approach may yield satisfactory policy-response forecasts within a stable policy regime, the same approach may yield rather unsatisfactory forecasts in an attempt to analyze the effect of alternative policy settings (e.g. fixed versus feedback policy rules and analysis of effects of permanent and transitory shocks) on relevant variables[4]. To correct for this inadequacy requires the specification and estimation of the beliefs and preferences of the agents in the economy - this approach has come to be known as the structural approach to policy analysis. The decision-making agents and policy authorities must be regarded as playing a dynamic game the rules of which are common knowledge. Each player's move is carefully planned subject to the expected move of the opponent and

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Branson and Henderson(1985) and/or Adler and Dumas(1983) and relevant articles cited in these articles. This class of models are very significant for empirical specification of the problem at hand.

[3].The demand for foreign exchange is either for transactionary and/or speculative purposes. Whichever way one views the decision problem in this regard it is a decision taken in anticipation of returns claimable in the future. This element of anticipation is the main reason why expectations play such an imperative role in any analysis on the foreign exchange market. This point was driven home vividly by Isard(1978) when he stated that:
"..... Furthermore, the relative demands for domestic and foreign assets can shift substantially ... in response to a change in expected yield differentials, ... Thus, in a world of floating exchange rates a revision in expectations of future exchange rates can quickly change the balance of supply and demand in foreign exchange markets...." (see pages 24 - 25.)

[4]. See Anderson(1979) for further discourse on this issue.

the expected return of the move. This is what rational expectations is all about - "models should let behaviour change with the rules of the game."[5] This is the approach adopted in this paper.

The main objective of this paper is to single out and empirically test for the significance of the real as well as monetary factors that influence the demand for foreign assets as a first step towards understanding exchange rate fluctuations. Our emphasis here is on the effect of monetary policy on foreign asset demands given specific monetary policy rules - the constant rate of growth of money and the feedback monetary policy rules. This empirical endeavour is carried out within the context of the asset market approach using time series data over the period 1974:1 - 1992:4 for Germany.

The rest of the paper is organized as follows. Section 2 provides the microeconomic foundations to explaining the demand for foreign assets using the mean-variance approach of Branson and Hendersen (1985). The approach adopted in this paper views the demand for foreign assets as determined/influenced by real and nominal factors as well as by expectations. The implications of the rational expectations hypothesis and of monetary policy rules on the structure of the demand for foreign assets are also explored in this section. Section 3 - Econometric Specification, Estimation and Analysis - deals with the problem of appropriate specification of the monetary policy rules utilized as well as of expectations and discusses the economic policy implications of the estimated demand for foreign assets. The final section summarizes the empirical findings comparing these with established international monetary theory, and concludes.

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[5].See chapter 1 - Rational Expectations and the Reconstruction of Macroeconomics - of Sargent(1979).

2. INTERNATIONAL ASSET DEMANDS

The model used here to derive the demand equations for assets is the Branson-Henderson[6] two-asset model with the exchange rate and home price index stochastic. The home investor consumes goods produced in two different countries and priced in the currency of the producing country. His problem is that of choosing a portfolio consisting of these consumption goods and some combination of foreign and domestic securities with fixed nominal values and certain nominal returns. Under the assumptions that percentage changes in prices follow geometric Brownian motion and secondly that the investor's instantaneous utility function exhibits constant relative risk aversion the consumption and investment problems of the investor are separable[7]. In this two-asset model the investor allocates a proportion δ of his nominal wealth to foreign-currency denominated securities F and the remaining $1-\delta$ to domestic currency-denominated securities B so that (dropping the time subscript, t, for convenience)

$$eF = \delta W \qquad \text{and} \qquad B = (1 - \delta)W, \qquad \text{for } \delta \in (0, 1) \qquad (1.1)$$

where F and B are holdings of foreign and domestic securities/assets respectively, e is the exchange rate (i.e. the home currency price of foreign currency) and W is the investor's nominal wealth. The investor's nominal wealth is deflated by the domestic price index to obtain his real wealth, ω, as in (1.2) below. For the domestic price index we adopt the specification $Q = P^{1-\beta}(eP^{*})^{\beta}$ where P (P^{*}) is the domestic (foreign) price level and β is the share of total expenditure expended on foreign goods. Hence the investor's real wealth, ω, is given by

$$\omega = \frac{W}{Q} = \frac{(B + eF)}{Q} \qquad (1.2)$$

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[6]. See Branson and Henderson(1985) as well as Branson and Jaffee(1990) for an excellent exposition of this model. Adler and Dumas(1983) and Fraga(1986) may also be of interest.

[7]. Merton(1971) derives this result.

The investor's problem is to choose the optimum portfolio share, δ*, such that

$$\delta^* = \underset{\{\delta \,\epsilon\, (0,1)\}}{\textbf{argmax}} \; \{ \; E(\frac{d\omega}{\omega}) \; - \; \frac{1}{2} \; R \; Var(\frac{d\omega}{\omega}) \; \} \qquad (1.3)$$

where E is the expectations operator and Var(.) denotes the variance of the percentage change in real wealth. The mean-variance as well as the time-separability forms of the objective function are consistent with a HARA utility function of the form U = $\gamma^1 C^\gamma$ - where C denotes real consumption - such that R (which is the Arrow-Pratt coefficient of relative risk-aversion) = [- CU"(C)/U'(C)] = (1-γ).

The stochastic processes for the rates of return on the respective assets as well as for the exchange rate and the domestic price index are specified in (1.4) below. (We defer the specification of the respective stochastic processes for the domestic and foreign price levels until later). Notice that the first two processes - i.e. (1.4a) indicate that the foreign and domestic securities can be classified as short bonds with certain nominal returns i_f and i_b respectively.

$$\frac{dF}{F} = i_f dt \qquad \frac{dB}{B} = i_b dt \qquad (1.4a)$$

$$\frac{de}{e} = \xi dt + \sigma_e dz_e \qquad \frac{dQ}{Q} = \pi_q dt + \sigma_q dz_q \qquad (1.4b)$$

where i_f and i_b denote respectively the nominal rates of return on foreign and domestic assets, ξ and π_q are the expected percentage depreciation of domestic currency and expected domestic inflation respectively whereas σ_e^2 *and* σ_q^2 are the variances of the stochastic processes. dz_e and dz_q are Wiener processes referred to in the literature as Gaussian white noise.

The stochastic differential of real wealth, ω, obtained using Ito's Lemma[8], is given by

$$\begin{aligned} d\omega &= \frac{1}{Q}dB + \frac{F}{Q}de + \frac{e}{Q}dF - \frac{W}{Q^2}dQ + \frac{1}{2}\{-\frac{2}{Q^2}dBdQ + \frac{2}{Q}dFde - 2\frac{e}{Q^2}dFdQ \\ &\qquad - 2\frac{F}{Q^2}dedQ + 2\frac{W}{Q^3}dQ^2\} \\ &= \frac{1}{Q}dB + \frac{F}{Q}de + \frac{e}{Q}dF - \frac{W}{Q^2}dQ - \frac{1}{Q^2}dBdQ + \frac{1}{Q}dFde - \frac{e}{Q^2}dFdQ \\ &\qquad - \frac{F}{Q^2}dedQ + \frac{W}{Q^3}dQ^2 \end{aligned}$$

Dividing the above equation by ω and utilizing (1.1), (1.2) and (1.4) yields

$$\begin{aligned} \frac{d\omega}{\omega} &= (1-\delta)\frac{dB}{B} + \delta\frac{de}{e} + \delta\frac{dF}{F} - \frac{dQ}{Q} - (1-\delta)\frac{dB}{B}\frac{dQ}{Q} + \delta\frac{dF}{F}\frac{de}{e} \\ &\qquad - \delta\frac{dF}{F}\frac{dQ}{Q} - \delta\frac{de}{e}\frac{dQ}{Q} + (\frac{dQ}{Q})^2 \end{aligned} \qquad (1.5)$$

Ito's lemma is further applied to the products of the stochastic processes in the dω/ω expression above yielding:

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[8]. According to Ito's Lemma given the function $H = J(K_1,...,K_n,t)$ which is twice continuously differentiable and defined over $R^n \times [0, \infty)$ where the K_is follow geometric Brownian motion given by $dK_i/K_i = \pi_i\, dt + \sigma_i\, dz_i$ for i = 1,...,n the stochastic differential of H is given by

$$dH = \sum_i^n (\frac{\delta J}{\delta K_i})\, dK_i + (\frac{\delta J}{\delta t})dt + (\frac{1}{2}) \sum_i^n \sum_j^n (\frac{\delta^2 J}{\delta K_i \delta K_j})\, dK_i dK_j\ .$$

The product $dK_i\, dK_j$ is defined as follows:

$dz_i\, dz_j = r_{ij}\, dt$ and $dz_i\, dt = 0$ for all i, j = 1,...,n where r_{ij} is the correlation between the geometric Brownian motions dz_i and dz_j. For a justification of these results see Merton(1971) and/or Chow(1979).

$$\frac{dB}{B}\frac{dQ}{Q} = 0, \quad \frac{dF}{F}\frac{dQ}{Q} = 0, \quad \frac{dF}{F}\frac{de}{e} = 0, \quad \frac{de}{e}\frac{dQ}{Q} = \rho_{eq}dt \quad and \quad (\frac{dQ}{Q})^2 = \rho_{qq}dt$$

The ρ terms in the expressions above are the covariances, $\rho_{ij} = \sigma_i \sigma_j r_{ij}$ where r_{ij} is the correlation and σs are the covariances of the respective stochastic processes. Utilizing the equations in (1.4a and b) equation (1.5) above can be simplified into

$$\frac{d\omega}{\omega} = [\ (1-\delta)i_b + \delta\xi + \delta i_f - \pi_q - \delta\rho_{eq} + \sigma_q^2\ ]\ dt + \delta\sigma_e dz_e - \sigma_q dz_q$$

Hence finally taking expectations through the above expression and remembering that the expected value of each of the dz_i terms is zero (since they are Gaussian white noise) we obtain

$$E\ (\frac{d\omega}{\omega}) = (1-\delta)i_b + \delta\xi + \delta i_f - \pi_q - \delta\rho_{eq} + \sigma_q^2 \qquad (1.6)$$

Further, using Ito's lemma to evaluate $(d\omega/\omega)^2$ we obtain the variance of $d\omega/\omega$ as the variance of the dz_i - terms in the $d\omega/\omega$ expression or simply as below

$$Var\ (\frac{d\omega}{\omega}) = E\{(\frac{d\omega}{\omega}) - E(\frac{d\omega}{\omega})\}^2 = \delta^2\sigma_e^2 - 2\delta\rho_{eq} + \sigma_q^2 \qquad (1.7)$$

The first order condition to the problem (1.3) - (1.7) is then derived and after some manipulations the optimal share of foreign assets in the investor's portfolio is expressed, as in equation (1.8') below, as the weighted average of the logarithmic utility investor's portfolio and the minimum-variance portfolio:

where

δ^*_L = the logarithmic utility investor's portfolio, and

δ^*_{MIN} = the minimum variance portfolio (i.e. the solution for δ derived by minimising the portfolio variance in equation (1.7). (Adler and Dumas (1983) and Branson and Jaffee (1990) use this formulation).

However given the specification of the domestic price index as discussed above we further specify the stochastic behaviour of the domestic and foreign price levels as below:

$$\frac{dP}{P} = \pi_P dt + \sigma_P dz_P$$
$$\frac{dP^*}{P^*} = \pi_{P^*} dt + \sigma_{P^*} dz_{P^*} \qquad (1.9)$$

Taking account of the above stochastic processes we further utilise Ito's lemma once again to evaluate the stochastic product of dQ/Q and de/e yielding an expression for the covariance term ρ_{eq} as $(1 - \beta)\rho_{Pe} + \beta\sigma_e^2 + \beta\rho_{P*e}$ which is then substituted into equation (1.8') above yielding[9]

$$\delta^* = \frac{1}{R\sigma_e^2}\{ i_f - i_b + \xi \} + \frac{(R-1)}{R\sigma_e^2}\{ (1-\beta)\rho_{Pe} + \beta\sigma_e^2 + \beta\rho_{P*e} \} \qquad (1.8)$$

This optimal δ as derived above is a function of beliefs (represented by the expected percentage change in the exchange rate, ξ), preferences of the investor (as represented by R) and variance-covariances of the set of stochastic variables. From the expression for the optimal portfolio share of foreign financial assets, δ^*, we infer that, given that all other factors remain unchanged, increases in exchange rate volatility - measured by σ^2_e - drive investors home. Furthermore, all other things remaining equal, increases in the interest rate differential either as a result of increases in the nominal return on the foreign (-currency

[9]. The detailed algebraic manipulations involved in the derivation of the expression for ρ_{eq} given the stochastic processes of foreign and domestic prices are presented in Branson and Henderson(1985) and hence we do not repeat them here.

denominated) asset and/or reductions in the nominal returns on the domestic (-currency denominated) asset drive investors abroad. We also infer that the higher the covariance between the exchange rate and the domestic price level (and/or the foreign price level) the higher the share of the foreign asset in the investor's portfolio if R > 1.

Modelling Expected Percentage Depreciation (ξ_t)

Volatility and unpredictability of price changes characterise organised asset markets. Typically, in asset markets current prices reflect expectations as to the future course of events and new information which induces changes in expectations are immediately reflected in price changes. Indeed this element of volatility and unpredictability of asset prices as well as of the relevance of expectations are vividly driven home by Frenkel (1981) - see also footnote 3 on page 3 - where the author affirms that:

> "In [asset markets] current prices reflect expectations .... The strong dependence of current prices on expectations about the future is unique to the determination of durable asset prices .... "

Consequently to capture this view of asset markets consider the following general model of Frenkel (1981), Copeland (1984) and Gros(1989):

$$e_t = z_t + bE\,[\Delta e_{t+1}|I_t\,] + v_t \qquad (1.10)$$

where e_t is the logarithm of the nominal domestic price of foreign currency and z_t is a vector of fundamental factors that affect the current exchange rate. According to Frenkel (1981) these factors "may include domestic and foreign money supplies, incomes, levels of output, etc". $E[\Delta e_{t+1}|I_t] = \xi_t$ is the expected percentage depreciation of the domestic currency between periods t and t+1 based on available information at period t, I_t, and v_t is an iid residual term with zero-mean and a constant variance, and which could be construed as representing all other fundamental variables not included in z_t. In this paper we restrict the elements of the z_t vector to include (logarithms of) domestic money and real output even though we recognise that given the very broad approach adopted in deriving/modelling expected depreciation such foreign variables as the logarithms of foreign money and output as well as an indicator of foreign fiscal policy (e.g. government expenditure and/or the budget deficit) should also be considered[10]. Taking account of this specification a simple algebraic manipulation of (1.10) yields:

$$e_t = \frac{c_0}{1+b} + \frac{c_1}{1+b} \ln m_t + \frac{c_2}{1+b} \ln y_t + \frac{b}{1+b} E[e_{t+1}|I_t] + \frac{v_t}{1+b} \quad (1.11)$$

where b, c_0, c_1, and c_2 are parameters. Taking first differences through equation (1.11) above yields an expression for spot exchange rate changes as a function of monetary and real output growth and of future expectations of changes in the spot rate (or, more appropriately expected depreciation), ξ_t, as specified in (1.12) below.

$$\Delta e_t = k_0 + k_1 \Delta \ln m_t + k_2 \Delta \ln y_t + k_3 E[\Delta e_{t+1}|I_t] \quad (1.12)$$

where

$k_0 = \Delta v_t/(1 + b)$, represents the total influence of the fundamental variables excluded from z_t (k_0 could be positive or negative depending on the total effect of these excluded variables).

$k_1 = c_1/(1 + b)^{-1} > 0$, in accordance with international monetary theory expansive monetary policy depreciates the domestic currency through the interest rate effect,

$k_2 = c_2/(1 + b)^{-1} < 0$, increases in domestic income (or economic growth) have the tendency of appreciating the domestic currency (see Branson (1983, 1985) and Kruger(1983)), and finally

[10]. The residual diagnostic results of the estimates of our optimal demand functions (excluding these foreign variables from the modelled expected depreciation) do not indicate any signs of mis-specification errors or of any errors due to omitted variables. Further, the inclusion of foreign variables does not alter the results significantly. Hence the data accepts this representation of the general approach to modelling expected depreciation adopted in this paper.

$k_3 = b/(1 + b)^{-1} \in (0, 1)$: the future exchange rate elasticity of the current rate is positive but less than one. [11]

Further, assuming rational expectations such that the above equation applies to expectations of all future exchange rates it follows by forward iteration of (1.12), shifted one-period forward, that

$$(1 - k_3 L^{-1})\, E\, [\Delta e_{t+1} | I_t] = k_0 + k_1 E\, [\Delta \ln m_{t+1} | I_t] + k_2 E\, [\Delta \ln y_{t+1} | I_t] \quad (1.13)$$

Since k_3 ($= b/(1 + b)^{-1}$) $\in (0, 1)$, ($1 - k_3 L^{-1}$) is invertible. Hence multiplying through the expression above by the inverse of ($1 - k_3 L^{-1}$) and taking expectations through it yields

$$\begin{aligned} E\left[\Delta e_{t+1} | I_t\right] &= \frac{k_0}{1 - k_3} + k_1\left[1 + k_3 L^{-1} + k_3^2 L^{-2} + \ldots\right] E\left[\Delta \ln m_{t+1} | I_t\right] \\ &\quad + k_2\left[1 + k_3 L^{-1} + k_3^2 L^{-2} + \ldots\right] E\left[\Delta \ln y_{t+1} | I_t\right] \\ &= \frac{k_0}{1 - k_3} + k_1 E\left[\sum_{j=0}^{\infty} k_3^j \Delta \ln m_{t+1+j} | I_t\right] + k_2 E\left[\sum_{j=0}^{\infty} k_3^j \Delta \ln y_{t+1+j} | I_t\right] \end{aligned} \quad (1.14)$$

which expresses the expected depreciation of the domestic currency, ξ_t, as a function of the discounted sum of expected future domestic rates of growth of money and income.

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[11].This is a stationarity condition imposed a priori and it does not by itself alone rule out the possibility of multiple equilibria characteristic of forward solutions of the nature adopted in this paper. See Blanchard(1979) - and the relevant references cited there - for a discussion of the merits of the forward solutions to rational expectations models.

Monetary Policy

To be able to analyze the effect of alternative monetary policy on agents' demand for foreign assets we utilize the familiar feedback monetary policy rule where we allow monetary policy to accommodate the previous period's exchange rate as specified below:

$$\ln m_t = \ln m_{t-1} + g_0 + g_1 e_{t-1} + \eta_t \qquad (1.15)$$

where η_t follows an autoregressive process given by $\eta_t = \theta\eta_{t-1} + \varepsilon_t$, for $|\theta| < 1$, and ε_t is a white-noise random term (with mean zero and a constant variance σ^2). From the specification above we expect g_1 to be negative such that previous period's depreciation of the domestic currency triggers off a contractionary monetary policy. This is the familiar "leaning-against-the-wind" policy in which previous currency over-valuations are followed by expansionary monetary policy[12]. For a similar usage of this rule see for instance Artus(1976) and Branson, Halttunen and Masson(1977). The Bundesbank has been using monetary targeting in its efforts to maintain low inflation, a controlled money supply and a stable currency since the dawn of the breakdown of Bretton Woods in 1974. See Scheide(1989). Hence one could argue for the inclusion of some additional variables on the right hand side of (1.15). η_t as introduced in the policy rule above could be perceived as capturing all these other variables that may be of concern to the Bundesbank in addition to the exchange rate.

Notice that when g_1 is assumed equal to zero (and the variance of η_t assumed to be sufficiently small) we obtain Friedman's x-percent monetary policy rule - a policy of commitment - under which monetary authorities allow the money stock to grow at a constant rate, g_0. The models emanating from these alternative specifications of the monetary policy rule could be said to encompass each other. Thus one could say that model II (i.e given the feedback monetary policy rule) encompasses model I (i.e given the fixed monetary policy

[12] The literature on intervention in the foreign exchange market is very eloquent on this point of letting interventions reflect changes in money supply or must be perceived as doing so. See Klein and Rosengren(1991). In this context therefore non-sterilized interventions seem more relevant to our case.

rule)[13].

Substituting the interventionist policy rule, (1.15) into (1.12) we obtain, after some manipulation using the stochastic lag operator (L) defined as $L^k(\chi_{t+j}) = E_t(\chi_{t+j-k})$, the expression

$$-k_3 \{L^0 + \phi_1 L + \phi_2 L^2\} e_{t+1} = k X_t \qquad (1.16)$$

where k is the coefficient vector $[(k_0 + k_1 g_0)\ k_1\ k_2\ ]$, and X_t denotes the variable vector $[\ 1\ \eta_t\ \Delta \ln y_t\ ]'$. The lag polynomial on the left-hand side of (1.16) can be factorized as [See Sargent (1979)]

$$[\ L^0 + \phi_1 L + \phi_2 L^2] = (L^0 - \varphi_1 L)(L^0 - \varphi_2 L)$$
$$= - \varphi_2 L(L^0 - \varphi^{-1}{}_2 L^{-1})(L^0 - \varphi_1 L)$$

where $\phi_1 = - (1+k_3)/k_3$, $\phi_2 = (1+k_1 g_1)/k_3$, and the roots of the lag polynomial are given as

$$\varphi_1 = \frac{\phi_1}{2}[-1 + \sqrt{1 - 4(\frac{\phi_2}{\phi_1^2})}\ ], \quad \textit{and} \quad \varphi_2 = \frac{\phi_1}{2}[-1 - \sqrt{1 - 4(\frac{\phi_2}{\phi_1^2})}\ ] \qquad (1.17)$$

for values of g_1, k_1 and k_3 for which $(1+k_3)^2 - 4(1+k_1 g_1)k_3 \geq 0$. It can be shown [see Cryer (1989)] that these roots will exceed 1 in absolute value if and only if

$$\phi_1 + \phi_2 < 1 , \quad \phi_2 - \phi_1 < 1, \quad \text{and} \quad |\phi_2| < 1.$$

If this stationarity condition of the AR(2) process holds then $(L^0 - \varphi^{-1}{}_2 L)$ is invertible. Indeed $(L^0 - \varphi^{-1}{}_2 L^{-1})$ is invertible since a simple manipulation of (1.17) reveals that

[13]. In all subsequent references to these models we shall refer to the respective models as Model I(Fixed Policy Rule) and Model II(Feedback Policy Rule).

$$\varphi_1 = \frac{(1 + k_3)}{2k_3}\left[1 - \sqrt{\left(1 - 4\frac{(1 + k_1 g_1)k_3}{(1 + k_3)^2}\right)}\right] \lesseqgtr 1 \qquad \text{and}$$

$$\varphi_2 = \frac{(1 + k_3)}{2k_3}\left[1 + \sqrt{\left(1 - 4\frac{(1 + k_1 g_1)k_3}{(1 + k_3)^2}\right)}\right] > 1$$

which satisfies the condition that φ_2 be greater than one (since $(1 + k_3)/2k_3 > 1$) for (L^0 - $\varphi^{-1}{}_2L^{-1}$) to be invertible. Hence multiplying both sides of (1.16) by the inverse of -$k_3\varphi_2L(L^0 - \varphi^{-1}{}_2L^{-1})$ yields

$$E[e_{t+1}|I_t] = -\frac{k_0+k_1g_0}{k_3\varphi_2(1 - \varphi_2^{-1})} - \frac{k_1}{k_3\varphi_2}E\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^j\eta_{t+1+j}|I_t\right] - \frac{k_2}{k_3\varphi_2}E\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^j\Delta\ln y_{t+1+j}|I_t\right] + \varphi_1 e_t$$

An expression for expected depreciation of the domestic currency is derived after a simple algebraic manipulation of the expression above as:

$$E[\Delta e_{t+1}|I_t] = \frac{k_0+k_1g_0}{k_3(1 - \varphi_2)} - \frac{k_1}{k_3\varphi_2}E\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^j\eta_{t+1+j}|I_t\right] - \frac{k_2}{k_3\varphi_2}E\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^j\Delta\ln y_{t+1+j}|I_t\right] + (\varphi_1 - 1)e_t \qquad (1.18)$$

It is discernible from (1.18) above that the expected change in the exchange rate is determined not only by the current period's exchange rate but also by domestic economic conditions (as represented by domestic expected economic growth and monetary shocks). Notice that since Model I is a restricted version of Model II (the restriction being $g_1 = 0$) all derivations obtained so far are also valid for Model I once we impose the required restriction - this is to say Model II encompasses Model I. More specifically, one can just set g_1 to zero into equation (1.15) above and insert the resulting expression into (1.14) yielding an expression for $E[\Delta e_{t+1}|I_t]$ under the x-percent monetary policy rule as below:

$$E\left[\Delta e_{t+1}|I_t\right] = \frac{k_0 + k_1 g_0}{1 - k_3} + k_1 E\left[\sum_{j=0}^{\infty} k_3{}^j \eta_{t+1+j}|I_t\right] + k_2\ E\left[\sum_{j=0}^{\infty} k_3{}^j \Delta \ln\ y_{t+1+j}|I_t\right] \qquad (1.19)$$

The equations above - (1.18) and (1.19) above - are then substituted in turns into (1.9) yielding the respective optimal share of the foreign assets, δ_t*, as functions of the interest rate differential, i_t^D, expected future rate of growth of income, monetary shocks and the logarithm of the exchange rate, e_t, as in (1.20) and (1.21) below. In both equations below α denotes the inverse of $R\sigma_e^2$ as in (1.8).

Model I (Fixed Policy Rule):

$$\begin{aligned}\delta_t^* = const + \alpha\frac{k_0 + k_1 g_0}{1 - k_3} + \alpha i_t^D + \alpha\ k_1 E\left[\sum_{j=0}^{\infty} k_3^j \eta_{t+1+j}|I_t\right] \\ + \alpha\ k_2 E\left[\sum_{j=0}^{\infty} k_3^j \Delta \ln\ y_{t+j+1}|I_t\right]\end{aligned} \qquad (1.20)$$

Model II (Feedback Policy Rule):

$$\begin{aligned}\delta_t^* = const + \alpha\frac{k_0 + k_1 g_0}{k_3(1 - \varphi_2)} + \alpha i_t^D - \frac{\alpha k_1}{k_3 \varphi_2} E\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^j \eta_{t+1+j}|I_t\right] \\ - \frac{\alpha k_2}{k_3 \varphi_2}\ E\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^j \Delta \ln\ y_{t+1+j}|I_t\right] + \alpha(\varphi_1 - 1)\ e_t\end{aligned} \qquad (1.21)$$

where in both equations '*const*' refers to the second expression on the right hand side of equation (1.8)[14].

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[14]Treating the second expression on the right-hand side of equation (1.8) as a constant implies an underlying assumption of a constant risk premium. This assumption is consistent with the failure of the literature to find empirical evidence in support of the view of a time-varying risk-premium that can explain predictable excess returns on foreign exchange. See for instance Frenkel(1982) and Froot and Thaler(1990).

The information set of the agents includes, in a rational expectations context, the lagged values of all relevant variables in the problem setup as defined so far. However, for simplicity, we shall limit ourselves to a subset, Φ_t, of the information set, I_t, where Φ_t refers to the lag values only of the variable for which we require a forecasted value. Also, we shall let the data determine the appropriate lag length to be incorporated for each variable in this subset. Estimation of the respective unobservable optimal decision equations above is carried out in two stages. The main objective of the first stage estimations is to derive an estimate for the unobservable monetary shock variable, η_t, in equations (1.20) and (1.21). The estimated values of this unobservable variable is then used as an independent variable in the second stage of the estimation procedure. The full models estimated during each estimation stage are presented in the next section which we now turn to.

2. ECONOMETRIC SPECIFICATION, ESTIMATION AND ANALYSIS

Most specifications of econometric models under stochastic decision processes do not follow the usual econometric practice of adding error terms - except where these errors are assumed to emanate from observational errors committed by the researcher - to the reduced form equations since this leads to inconsistency under rational expectations. The main practice has been to denote one of the variables for which it is extremely difficult to obtain reliable data as the error term in the equation. In this paper we add a composite error term to the structural demand equations derived in section I. This composite error term is however assumed to emanate from specification and/or observational error on the part of the econometrician as well as from forecasting the unobservable variables during the first stage of our estimation procedure and not from the agent's decision-making process - hence the agents can be assumed to know the error term so appended on to their optimal decision variable, δ_t*. The error term added to the demand functions, u_{ft}, is assumed to be Gaussian white-noise the validity of which assumption is empirically tested.

Two alternative models are estimated - one model for each monetary policy rule. Model I denotes the estimation of the optimal decision variable, δ_t*, under the fixed policy rule whereas model II shows its estimation under the feedback policy rule (see footnote 13). But before we commence the estimation procedure we discuss some features of our data in the subsection below.

The data[15]

All data utilised in this paper are seasonally adjusted quarterly data over the period 1974:1 -1992:4 obtained from various issues of International Financial Statistics (IFS) - published by the International Monetary Fund (IMF) - and Economic Indicators (Published by the OECD). The variables utilized are described in the Appendix at the end of the paper; and their time series plots (in levels and first differences) are presented in figure 2A. However there seems to exist certain peculiarities of the data that have their roots/origins in the theoretical model used. As presented the theoretical model is very silent as to which assets to include under foreign and domestic assets. A very broad-based approach will be to include both real and financial assets under either of these assets. There are however obvious problems associated with this broad-based approach - these problems include data accessibility and the measurement of nominal returns associated with each of the assets (domestic and foreign). To surmount these problems we restrict our definition of foreign (domestic) assets to include domestic residents' holdings of financial assets denominated in foreign (domestic) currency.

Then there is the issue of what monetary aggregate to use. M_1 plus Quasi-Money is utilised in this paper as a proxy for the monetary instrument of the Bundesbank. Finally, for the nominal returns on the respective assets we use the German (Frankfurt) rate on 3-month loans for the nominal return on domestic assets and the United States Federal Funds Rate is used as a proxy for the nominal return on foreign assets.

Having presented the salient features of our data we proceed to the estimation procedure. To estimate our final models as presented on page 16 we need to obtain estimates for the unobservable variables in equations (1.20) and (1.21) above. To do this estimation is done

[15]. All data utilized are for the erstwhile West Germany before the re-unification.

in two stages[16]. We estimate the respective monetary policy rules and an AR(*p*) specification for the rate of growth as a system during the first stage. The highest lag length, *p*, considered in each case is 8. More specifically, the equation system(s) estimated during this first stage of estimations are depicted by (2.1) below:

$$\begin{aligned} \Delta \ln m_t &= g_0(1-\theta) + \theta \Delta \ln m_{t-1} + g_1 e_{t-1} - g_1 \theta e_{t-2} + \varepsilon_{1t} \\ \Delta \ln y_t &= \rho_0 + \sum_{j=1}^{p} \rho_j \Delta \ln y_{t-j} + \varepsilon_{2t} \end{aligned} \qquad (2.1)$$

where ε_{1t} and ε_{2t} are random disturbance terms with the following characteristics

$$\begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \sim \text{IIDN}(0, \Sigma) \; ; \quad \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

Equation (2.1) above yields the first stage estimates of both models (with the appropriate restrictions imposed in the case of Model I(Fixed Policy Rule)) once the optimal number of lags of y_t are determined. The optimal number of lags of the y_t variable under each model is determined from the results of lag order tests performed using the Akaike Information Criterion (AIC), the Iterated Log Criterion (ILC) of Hannan and Quin and the Log Criterion (BIC) of Schwartz. The results of these respective tests are as presented in table 1a below.

[16]. It has been shown in the literature on two-step estimation procedures such as the one applied here that under certain conditions the standard errors of the estimated coefficients in the second stage of the estimation procedure tend to be underestimated and hence yielding wrong inferences. However Pagan(1984) has demonstrated that if only unanticipated variables appear in the equation estimated in the second stage of the procedure, as we do in this paper, then, without any further assumption(s), the estimated standard errors are consistent estimates of the true standard errors. Indeed, commenting on the results of their first illustrative example, where only estimated residuals appear among a set of regressors, Murphy and Topel(1985) assert that "In general, the estimated coefficients and associated standard errors from FIML are similar to those obtained from the T-S [two-step] procedure. In this case the reduced-form restrictions imposed by the two-step estimator allow efficiency close to FIML."

Table 1a: Lag Order Selection for Models I and II.

Information Criteria.

	Lags	1	2	3	4	5	6	7	8
Model I	AIC	-18.697	-18.911	**-18.993**	-18.166	-18.313	-18.382	-18.731	-17.882
	BIC	**-17.580**	-16.677	-15.643	-14.698	-13.728	-12.681	-11.912	-10.947
	ILC	**-18.316**	-18.149	-17.850	-17.642	-17.407	-17.096	-17.063	-16.833
Model II	AIC	-11.584	-11.669	-11.687	-11.792	-11.841	-11.868	-12.043	**-12.046**
	BIC	**-11.052**	-10.676	-10.198	-9.8060	-0.3591	-8.8899	-8.5681	-8.0752
	ILC	-11.179	-11.330	**-11.339**	-10.994	-10.852	-10.857	-10.691	-10.522

Note: *Figures written in bold text indicate the minimum out of a maximum lag of 8.*

Paulsen (1984) has shown that the BIC and the ILC are consistent in determining the true lag length whereas the AIC tends to overestimate the true lag order - leading to over-parameterisation of the model - and is not consistent. Jacobson(1992) has also shown in a Monte Carlo study that the ILC tends to pick the true lag order with greater accuracy than the other two criteria. Hence based on the ILC the data suggest an optimum lag order of one for model I and of three for model II. However our estimation results for model I using a lag order of one does not only indicate the existence of ARCH effects but also yields insignificant parameter estimates. Hence we utilise a lag length of three for both models under consideration. The estimation results of the first stage of our estimation procedure (more specifically the estimates of (2.1)) are based on this optimal lag order (i.e. p = 3). The first stage estimation results are presented in table 1b below:

Table 1b: First Stage Estimation Results

	Model I (Fixed Policy Rule)		Model II (Feedback Policy Rule)	
Coeff.	$\Delta \ln m_t$	$\Delta \ln y_t$	$\Delta \ln m_t$	$\Delta \ln y_t$
ρ_0		0.0079 (0.0025)		0.0079 (0.0025)
ρ_1		-0.0791 (0.0275)		-0.0791 (0.0275)
ρ_2		-0.0965 (0.0375)		-0.0973 (0.0375)
ρ_3		- 0.0562 (0.0177)		-0.0540 (0.0177)
g_0	0.0086 (0.0014)		0.0082 (0.0014)	
g_1			-0.0181 (0.0066)	
θ	0.01293 (0.0202)		0.1235 (0.0575)	
LOG. LIKELIHOOD	-144.26		-146.88	
		Univariate Residual Analysis		
ARCH(1)	0.1145	0.1022	0.0967	0.0743
ARCH(2)	0.1892	0.1860	0.1915	0.1943
J-B NORM	0.0438	0.1571	0.0390	0.1655

Notes: *The figures in parenthesis are standard errors. The estimates provided for the univariate residual analysis are p-values. ARCH(k) statistics - which has a* $\chi^2(k)$ *distribution -are test statistics for homoskedsticity of the residuals whereas J.B Norm is the Bera-Jarque(1980) test for normally distributed residuals. It has an approximate* $\chi^2(2)$ *distribution. For both statistics a p-value greater than x% indicates that the null hypothesis (of homoskedsticity or normality of the residuals as the case may be) is upheld at the x% significance level.*

From the estimation results as presented above we observe that the estimate of the policy-response parameter, g_1, is of the correct sign and highly significant - a finding that is very encouraging since it indicates empirically that German monetary policy authorities followed the stipulated policy rule over the period of this study. But for illustrative purposes (since we undertake a comparison of the parameter estimates of both models later on in the paper) we could therefore conclude that model II encompasses model I and then continue to estimate only model II and test for the significance of g_1. The univariate residual analysis performed indicate that there exists no hetereoskedastic residuals in either of the models. However the p-values in respect of the Bera-Jarque(1980) normality test indicate to the contrary that the residuals of the first equation in each model may be non-normal. A re-estimation of the models using higher lag orders for the rate of growth of income not only yielded insignificant parameter estimates but also produced residuals with worse statistical performance. Therefore, notwithstanding the residual non-normality problem of the first equations of both models we proceed to the second stage of our estimation procedure where we estimate the coefficients of the respective equations characterising the optimal decision variable, δ_t* under each model. Before we proceed we need to obtain estimates for the monetary shock variable, η_t, and also of the future rate of growth of output. From the estimates of θ obtained during the first stage we obtain the η_t series through repeated substitution of $\eta_t = \theta\eta_{t-1} + \varepsilon_t$ - as[17]

$$\eta_t = \sum_{i=1}^{t} \theta^{t-i} \varepsilon_i + \theta^{t+1} \eta_0 \qquad \text{where} \qquad \eta_0 = 0$$

which is manipulated to obtain the expected future monetary shocks which yield the expressions (2.2a and b) below. (These expressions appear in the estimated equations in the second stage estimations of Models I and II respectively).

$$E\left[\sum_{j=0}^{\infty} k_3^{j} \eta_{t+j+1}\right] = \sum_{j=0}^{\infty}\left[k_3^{j}\theta^{j+1}\right]\eta_{t-1} \qquad (2.2a)$$

$$E\left[\sum_{j=0}^{\infty} (\varphi_2^{-1})^{j} \eta_{t+j+1}\right] = \sum_{j=0}^{\infty}\left[(\varphi_2^{-1})^{j}\theta^{j+1}\right]\eta_{t-1} \qquad (2.2b)$$

E denotes the expectations operator as usual. To obtain the expected future rate of growth of output we rewrite its Markovian process (the second equation of (2.1)) in "companion form" as $Y_t = A\, Y_{t-1} + \varepsilon_t$ where

[17].See Taylor(1979) for a similar approach.

From the above specification we obtain through repeated substitution that $\hat{E}[Y_{t+j}] = A^{j+1} Y_{t-1}$ since $\hat{E}[\varepsilon_{t+l}] = 0$ for $l \geq 1$; where $\hat{E}$ denotes the linear least-squares projection operator[18]. Assuming the eigenvectors of A are linearly independent - so that the inverse of A exists - we can diagonalize[19] A as PDP^{-1} where the columns of P are the eigenvectors of A and D is a diagonal matrix with the diagonal elements being the eigenvalues (λ's) of A. Hence we could express $\Delta\ln y_{t+j}$ as $c_1 (PD^{j+2}P^{-1}) Y_{t-1}$ where c_1 is the first row of the identity matrix I_p. Finally, we derive the expressions for the unobservable growth terms as they appear in (1.18) and (1.18) as (2.3a) and (2.3b) for Models I(Fixed Policy Rule) and II(Feedback Policy Rule) respectively as:

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[18]. It is important to notice that if p = 1, we have an AR(1) process with a constant and hence y_{t+j} is simply

$$y_{t+j} = \rho_0 \frac{(1 + \rho_1^j)}{(1 - \rho_1)} + \rho_1^{j+1} y_{t-1} + \sum_{l=0}^{j} \rho_1^{j-l} \varepsilon_{2t+l}$$

Passing the least-squares projection operator, $\hat{E}$, through this expression yields $\hat{E}[y_{t+j}]$ as the sum of the first two terms of the expression above since ε_{2t} is iid normal with a zero mean and a constant variance.

[19]. See Sydsæter and Øksendal(1989) for the conditions under which a general matrix is diagonalizable.

$$\hat{E}\left[\sum_{j=0}^{\infty} k_3^{\,j}\Delta \ln y_{t+j+1}\right] = c_1 \; P\left[\sum_{j=0}^{\infty} k_3^{\,j} D^{j+2}\right] P^{-1}\; Y_{t-1}$$

$$= c_1 \; P\left[\frac{\lambda_i^2}{(1 - k_3\lambda_i)}\right]_{ii} P^{-1}\; Y_{t-1} \qquad (2.3a)$$

and

$$\hat{E}\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^{j}\Delta \ln y_{t+j+1}\right] = c_1 \; P\left[\sum_{j=0}^{\infty}\left(\frac{1}{\varphi_2}\right)^{j} D^{j+2}\right] P^{-1}\; Y_{t-1}$$

$$= c_1 \; P\left[\frac{\lambda_i^2}{(1 - \varphi_2^{-1}\lambda_i)}\right]_{ii} P^{-1}\; Y_{t-1} \qquad (2.3b)$$

where in both equations $[\ .\ ]_{ii}$ denotes the diagonal elements of the matrix $[\ .\ ]$[20]. In deriving these expressions we make the assumptions that $|k_3\lambda_i| < 1$ and $|\varphi_2^{-1}\lambda_i| < 1$. Substituting (2.2) and (2.3) - (2.2a) and (2.3a) or (2.2b) and (2.3b) as the case may be - into the respective optimal decision vectors (i.e. δ_t*'s, in (1.20) and (1.21) respectively) yields the first equations of the respective equation systems estimated in the second stage as below. In these systems of equations we express Model I as the restricted form of Model II so that the coefficients φ_{10} and φ_{20} in Model I are the restricted equivalents (the restriction being $g_1 = 0$) of φ_1 and φ_2 as they appear in Model II.

[20]. The elements of $P[\ .\ ]P^{-1}$ are not individually identifiable. Hence during estimation they are lumped into the constant a_i, for i = 0,1,2,3 (in both models). See Sargent(1978a and b) for a similar treatment of this non-identifiability problem.

Model I: (Estimated System of Equations given that Monetary Policy follows a Fixed Rule)

$$\delta_t^* = const + \alpha \frac{k_0 + k_1 g_0}{k_3(1 - \varphi_{20})} + \alpha_0 + \alpha i_t^D + \frac{\alpha k_1 \theta^2}{k_3(\theta - \varphi_{20})} \eta_{t-1}$$
$$+ \sum_{j=1}^{3} \alpha_j \Delta \ln y_{t-j} + \alpha(\varphi_{10} - 1)\, e_t + u_{ft} \qquad (2.4)$$
$$\Delta \ln m_t = g_0(1 - \theta) + \theta \Delta \ln m_{t-1} + \epsilon_{1t}$$
$$\Delta \ln y_t = \rho_0 + \sum_{j=1}^{3} \Delta \ln y_{t-j} + \epsilon_{2t}$$

where $(\alpha_1, \alpha_2, \alpha_0) = \{ (\alpha k_2)/(k_3 \varphi_{20}) \} c_1 P\, [\lambda_i^2/(1 - \varphi_{20}^{-1} \lambda_i)]_{ii}\, P^{-1}$.

Model II: (Estimated System of Equations given that Monetary Policy follows a Feedback Rule)

$$\delta_t^* = const + \alpha \frac{k_0 + k_1 g_0}{k_3(1 - \varphi_2)} + \alpha_0 + \alpha i_t^D + \frac{\alpha k_1 \theta^2}{k_3(\theta - \varphi_2)} \eta_{t-1}$$
$$+ \sum_{j=1}^{3} \alpha_j \Delta \ln y_{t-j} + \alpha(\varphi_1 - 1)\, e_t + u_{ft} \qquad (2.5)$$
$$\Delta \ln m_t = g_0(1 - \theta) + \theta \Delta \ln m_{t-1} + g_1 e_{t-1} + g_1 \theta e_{t-2} + \epsilon_{1t}$$
$$\Delta \ln y_t = \rho_0 + \sum_{j=1}^{3} \Delta \ln y_{t-j} + \epsilon_{2t}$$

where $(\alpha_1, \alpha_2, \alpha_0) = \{ (\alpha k_2)/(k_3 \varphi_2) \} c_1 P\, [\lambda_i^2/(1 - \varphi_2^{-1} \lambda_i)]_{ii}\, P^{-1}$.

In both equations we assume conditional homoscedasticity so that we could collapse the sum of the variance-covariance terms in the optimal decision vector given by (1.8) into a constant labelled "*const*", (see also footnote 14 on page 16) and the residuals have the following statistical characteristics:

From the models as specified above the expected future values of the rate of growth variable and the monetary shock variable are obtained by forecasting through A and specified monetary policy rules. Therefore the optimal decision vector, δ_t* in each model is not invariant to changes in specification of, and parameter changes in (2.1)[21]. Hence the residual variance-covariance matrix, $\mathbf{\Sigma}$, as specified above is very appropriate and implies therefore that we estimate the optimal decision vector, the monetary policy rule and the markovian process for the rate of growth as a system.

Several estimation methods have been developed and applied in rational expectations modelling. Most of them are relevant in the estimation of our models. The first method that is relevant in this case where we have future expectational variables is the *extended path method* developed in Fair and Taylor(1983) which is relevant for maximum likelihood estimation of non-linear (as well as linear) rational expectations models with forward-looking expectations. (See also Anderson(1979) and Lipton et al (1982) for a discussion of the extended path method). However since our model is linear in expectations, we could escape the computational costs of this method and instead use the method described by Wallis(1980) for linear rational expectations models. This method imposes stability conditions on the expectational processes for the relevant variables - just as we have done in specifying the information space of agents to include the history of the variables for which we require future forecasts. This is done in order to circumvent the multiple equilibria problem typical in this class of models (see for instance Shiller (1979)). The imposition of these conditions yields a rational expectations model non-linear in parameters (because of the verifiable cross-equation restrictions imposed by the rational expectations hypothesis on the parameters). The most appropriate estimation methods in this case are non-linear (in parameters) optimization methods[22] as utilized during the second stage of our estimation procedure. The estimation results are presented and discussed in the next subsection.

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[21]. This is but the usual policy-invariance characteristic of optimal decision rules the implications of which characteristic is exhaustively discussed by Lucas(1976), Begg(1987) and Wallis(1980).

[22]. See for example Fletcher(1987) for a thorough exposition of the performance of some of these algorithms.

3.1. *Empirical Results*

The estimates of the coefficients of Models I(Fixed Policy Rule) and II(Feedback Policy Rule) as specified in (2.4) and (2.5) respectively are estimated using non-linear least-squares and the Davidon-Fletcher-Powell's optimization algorithm[23]. The estimation results are as presented in table 2 below.

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[23]. The Non-Linear estimator is inconsistent without any further assumptions since the error term appended to the agents' optimal decision variable, δ_t* is in their information set and hence contemporaneously correlated with the independent variables which enter the estimated equation through the expected future values of the rate of depreciation of the exchange rate conditioned on the information set - i.e. $E[\Delta e_{t+1} |I_t]$. However, it is very likely that movements in the interest rate differential dominate the determination of the optimal decision variable such that the bias is small.

Table 2: Second Stage Estimation Results.

	Model I (Fixed Policy Rule)			Model II (Feedback Policy Rule)		
Coeff.	δ_t*	Δln m_t	Δln y_t	δ_t*	Δln m_t	Δln y_t
const	0.2094* (0.0506)			0.2562* (0.0713)		
α	0.0040** (0.0025)			0.0233* (0.0059)		
α_0	0.1194* (0.0506)			0.1662* (0.0713)		
α_1	-0.0149 (0.0703)			0.0124 (0.0106)		
α_2	-0.0336 (0.0682)			0.0081** (0.0060)		
α_3	0.0378 (0.0645)			0.0938 (0.0752)		
k_0	0.0986 (0.0708)			0.0938 (0.0748)		
k_1	0.0115 (0.0709)			0.0064** (0.0045)		
k_2	0.0346 (0.0571)			0.0859* (0.0316)		
k_3	0.1119* (0.0229)			0.2546* (0.0901)		
ρ_0			0.3976* (0.0675)			0.2283 (0.2134)
ρ_1			0.5011* (0.0661)			0.2394* (0.1354)
ρ_2			0.1088** (0.0605)			0.0194 (0.0168)
ρ_3			-0.1058* (0.0444)			-0.0214* (0.0107)
g_0		0.5367* (0.0291)			0.2194* (0.0317)	
g_1					-0.2207* (0.0389)	
θ		0.3469* (0.0143)			0.2325* (0.0132)	
LOG. LIKELIHOOD		-456.5835			-527.8829	
			Univariate Residual Analysis			
ARCH(1)	0.0487	0.7507	0.7344	0.0676	0.0479	0.7473
ARCH(2)	0.0456	0.9483	0.9030	0.0672	0.0485	0.8373
J-B NORM	0.6344	0.6344	0.9066	0.9960	0.4765	0.8934

Notes: *The figures in parenthesis are standard errors. A * (or **) indicates significance of the estimated parameter at 5% (or 10%) level. The estimates provided for the univariate residual analysis are p-values. See Notes under table 1b for descriptions and distributions of the ARCH(k) and J-B Norm statistics.*

To check the relative statistical performance of the models homoscedasticity and normality tests are performed on the residuals using ARCH(k) statistics (with an approximate $\chi^2(k)$ distribution) and Bera-Jarque(1980) normality test statistics (which has an approximate $\chi^2(2)$

distribution) respectively. From the results presented in table 2 above the null hypothesis of normality of the residuals cannot be rejected (at the 5% significance level) for any of the three equations of these models. For the homoscedasticity tests the second equation of model II (Feedback Policy Rule) passed the test marginally at the 5% significant level - the same applies to the δ^* equation of model I (Fixed Policy Rule). Therefore the impression one gets is that these statistics are not informative enough for discriminating between the two models in terms of relative performance in explaining the share of foreign assets in investors' portfolios. Additional empirical evidence (more specifically the significance of g_1) presented in the table however is consistent with the kind of foreign exchange market interventionist policy followed by the Bundesbank during the period under consideration. See also Klein and Rosengren(1989) and Scheide(1989) for a similar conclusion regarding German monetary policy over this period. On the whole given the approach adopted in this paper the empirical evidence so far derived - from the first and second stage estimations - indicate that the feedback policy rule matches the data better than the fixed policy rule. This is however not unexpected since the policy-response parameter, g_1, is of the correct sign and highly statistically significant under both stages of our estimation procedure. To further investigate the significance of this policy-response parameter under model II(Feedback Policy Rule) we performed a likelihood ratio test using the $\chi^2(1)$ statistics given by $-2\log(L^C/L^U)$ where L^C (or L^U) denotes the likelihood of the constrained (or unconstrained) system of equations. Since $\log L^C = -456.5835$ and $\log L^U = -527.8829$ we obtain a χ^2 statistics of 142.5988 with a p-value of zero. Thus the null hypothesis is rejected - thus upholding the assertion that the Bundesbank followed a feedback monetary rule. Hence the interpretations of our empirical results that follow in subsequent paragraphs are conditioned on the feedback monetary policy rule as an element of the agents' information set in the context of rational expectations.

From the estimation results as presented on the table above we infer that about 23% of monetary shocks are permanent under the feedback monetary policy rule. The policy-response parameter, g_1, takes on an estimated statistically significant value of - 0.2207 implying empirically that a one percent depreciation of the Deutsche Mark against the dollar instigates a 22% reduction in the rate of growth of money in the subsequent period. The estimated magnitude, sign and statistically significant value of g_1 are quite encouraging. Further, the estimate of k_1 is of the correct sign and highly statistically significant. Finally the estimated value of k_2 is however of the wrong sign not only in Model I (Fixed Policy Rule) but also in Model II (Feedback Policy Rule). This could be due the fact that the underlying fundamental variables included in our specified general model of the exchange rate do not exhaustively explain the dependent variable. But the main objective of our estimations however does not explicitly include the derivation of an empirically justifiable exchange rate equation although a theory consistent estimate in this case may enhance our chances of attaining the main goal of the paper.

To check for the effects and relative significance of the interest rate differential, the nominal exchange rate and the monetary policy shock variable on German investors' demand for foreign assets we proceed to use the estimates of the coefficients for both models to derive the respective composite coefficients. The composite coefficients are derived and tested for significance using the Wald test statistics which has an approximate χ^2 distribution with one degree of freedom in all cases considered. The composite coefficients (of the first equation in (2.4) and (2.5)) derived from the estimates of the coefficients in table 2 above are presented (in table 3 below) and discussed below. For purposes of comparison we present the results for both Models here even though we emphasise more on Model II (Feedback Policy Rule) results.

Table 3: **Derived Composite Coefficients**.

	Model I (Fixed Policy Rule)			**Model II (Feedback Policy Rule)**		
Variables	Coeff.	W	P-Value	Coeff.	W	P-Value
Constant	0.3292	585.877	0.0000	0.4241	154.298	0.0000
i^D_t	0.0040	2.5052	0.1135	0.0233	15.7907	0.0000
η_{t-1}	0.0011	8.2335	0.0041	0.0022	3.4739	0.0625
$\Delta\ln y_{t-1}$	0.0017	3.4894	0.0618	-0.0038	2.0394	0.1533
$\Delta\ln y_{t-2}$	0.0038	9.6558	0.0019	-0.0025	1.4386	0.2304
$\Delta\ln y_{t-3}$	-0.0042	13.3029	0.0003	-0.0029	3.4209	0.0644
e_t	-0.0041	2.4235	0.1195	-0.0239	15.0948	0.0001

Note: *The Wald-statistics (W) as indicated in the third and sixth columns above have an approximate $\chi^2(1)$ distributions. The P-Values indicate the probability that the derived composite coefficients are jointly zero.*

From the empirical results as presented for model I (Fixed Policy Rule) in the table above we infer that the interest rate differential is insignificant in explaining the demand for foreign assets - this result in addition to the insignificance of the exchange rate could be indications of the rejection by the data of the proposition that German monetary authorities followed a fixed monetary policy rule in their endeavour to stabilise the mark over the period under study. For model II (Feedback Policy Rule), on the contrary, an increase in the interest rate differential (either due to an increase in the foreign nominal interest rate or a fall in the domestic rate) significantly increases the share of foreign assets in investors' portfolios. Specifically, a 1% point increase in the interest rate differential leads to an increase of 2.33 percent in the share of foreign assets in investors' portfolios.

Further, under model II(Feedback Policy Rule), domestic currency depreciations tend to significantly reduce the share of foreign assets in investors' portfolios. More specifically, a 1% depreciation of the domestic currency reduces the share of foreign financial assets in the investors' portfolios by 2.39 percent. This can be viewed as a price effect since the exchange rate is positively correlated with the domestic currency price of foreign financial assets. Further, the empirical results seem to indicate that foreign asset demands are invariant with respect to changes in the rate of growth of output.

Finally, unanticipated monetary policy shocks drive domestic investors abroad. These shocks significantly increase the share of foreign assets in investors' portfolios by 0.22 percent given model II(Feedback Policy Rule) and by 0.11 percent under Model I(Fixed Policy Rule). The differences in the magnitudes of the composite coefficient in respect of the monetary policy shock variable can be explained by the fact that under the fixed policy rule agents make tacit commitments to demand given amounts of foreign assets whereas under the feedback policy rule their demands are but contingent rules and hence are relatively more sensitive to policy shocks. The statistical insignificance of the estimate of the parameter in respect of η_{t-1} in Model I(Fixed Policy Rule) suggests that variations in the monetary shock variable are insignificant in explaining variations in the share of foreign assets in agents' portfolios - exactly as required by the fixed monetary policy rule.

3. SUMMARY AND CONCLUSIONS

This paper is an attempt at modelling the demand for foreign assets without explicitly specifying the preferences of agents. The approach utilized is the usual asset market approach adopted by Merton(1975), Branson and Henderson(1985), and Adler and Dumas(1983). We go a step further in this paper by introducing and empirically analyzing the effect of monetary policy shocks on asset demands under two policy rules (feedback and fixed growth rate of money rules). Though there is substantial evidence in the literature in support of the use of a feedback monetary policy rule in analysing the problem set in this paper we considered an additional policy rule - the fixed monetary policy rule - as a check on our empirical results. Our empirical findings are in consonance with the interventionist exchange rate (or monetary) policies of the Bundesbank - Model II(Feedback Policy Rule) empirically out-performs Model I(Fixed Policy Rule). Thus in a rational expectations context the agents referred to in this paper must have a feedback monetary policy rule of the kind specified in the paper as an element of their information set. Based on this information set of agents (i.e. given the feedback monetary policy rule and other relevant information in a rational expectations context) our empirical results are consistent with international monetary theory even though some of our composite coefficients turned out to be statistically insignificant. Despite this shortcoming, our model offers us the opportunity to disentangle the proportion of monetary shocks that are permanent. One area of research that our models leave uninvestigated is the effect of anticipated monetary policy vis-a-vis that of unanticipated policy[24].

[24]. This approach is adopted in Kumah(1994) but within a different setup.

APPENDIX:

Data Sources and Variable Definitions

The data set utilized (all of which are seasonally adjusted) are derived from various issues of *International Financial Statistics* (IFS) (published by the International Monetary Fund (IMF)) as well as from *Economic Indicators* (published by the OECD) over the period 1974:1 - 1992:4. The variables utilized are as follows:

$f_t = e_t F_t$, domestic agents' holdings of foreign assets denoted in Duetsche Marks, where F_t is denominated in foreign currency.

e_t denotes the end-of-period exchange rate expressed in Duetsche Marks per one US dollar.

B_t denotes domestic agents' holdings of domestic assets (i.e. Net Bearer Bonds Outstanding *plus* Private Sector Long-Term Deposits at Deposit Money Banks *plus* Private Sector Loans on Trust Basis) denoted in billion Deutsche Marks.

m_t denotes the domestic money - i.e. Deposit Banks' Cash Reserves *less* Bankers' Deposits at the Bundesbank *plus* Deposit Money Banks' Deposits at the Bundesbank plus Quasi-Money (i.e. the sum of Public Authorities Time Deposits at Deposit Banks, and Private Sector Savings Deposits at Deposit Money Banks) in billions of Deutsche Marks. Hence $\Delta \ln m_t$ is the rate of growth of money.

y_t denotes domestic GNP at constant 1985 prices (in billions of Deutsche Marks) and hence $\Delta \ln y_t$ is the rate of growth of output.

i_{bt} denotes the nominal return on domestic assets: We used the German rate on 3-month loans (as reported in the OECD Main Economic Indicators) to measure this variable.

i_{ft} is the expected nominal rate of return on foreign assets. The US Federal Funds Rate is used as a measure for this variable.

The time series plots (in logarithmic levels - as the case may be - and first differences) of the respective variables as defined above are shown in the series of plots presented in figure A2.1 on the following pages.

Figure A2.1: Time Series Plots of the Data Utilised (In levels and first differences)

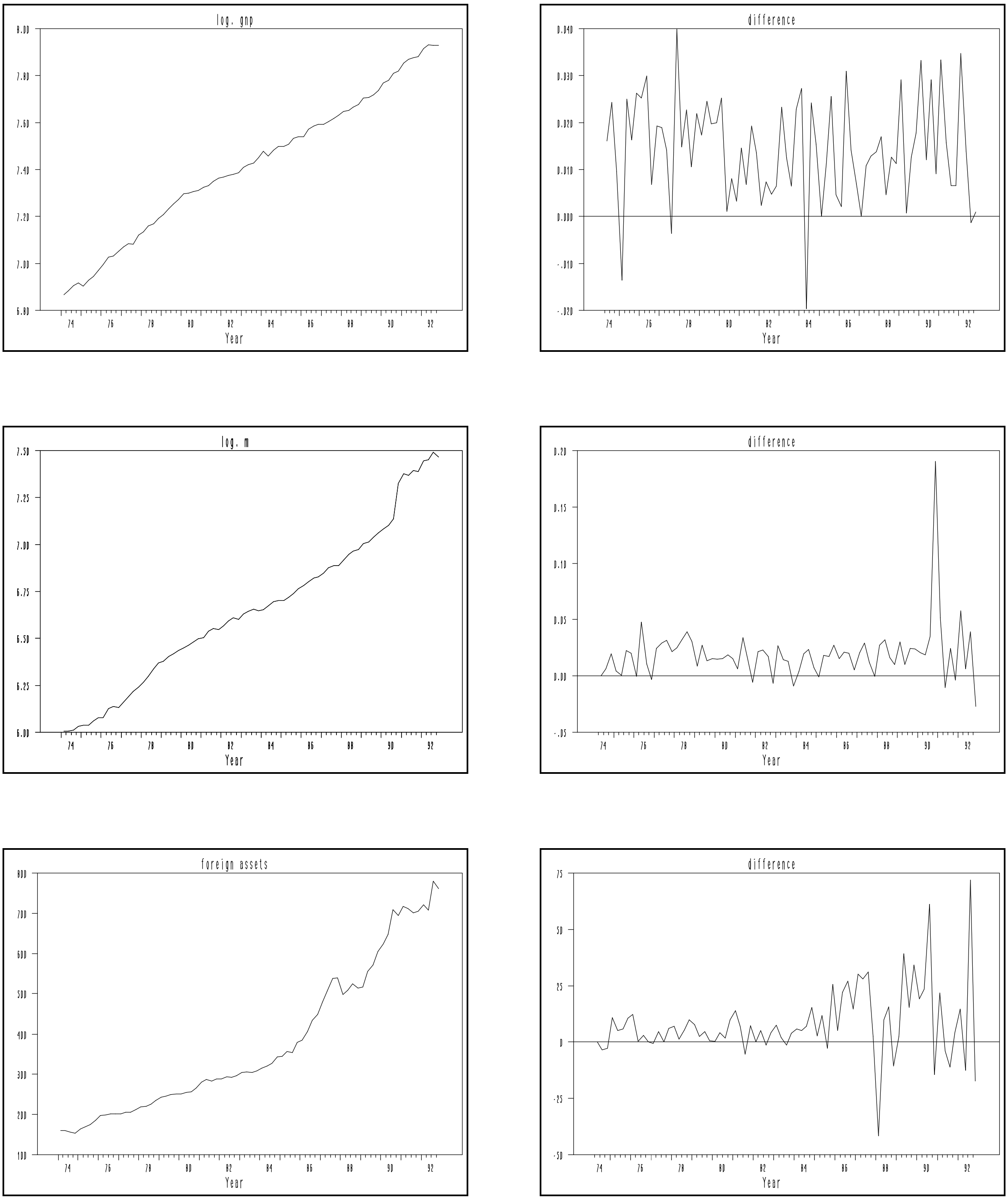


Figure A2.1 *contd*.

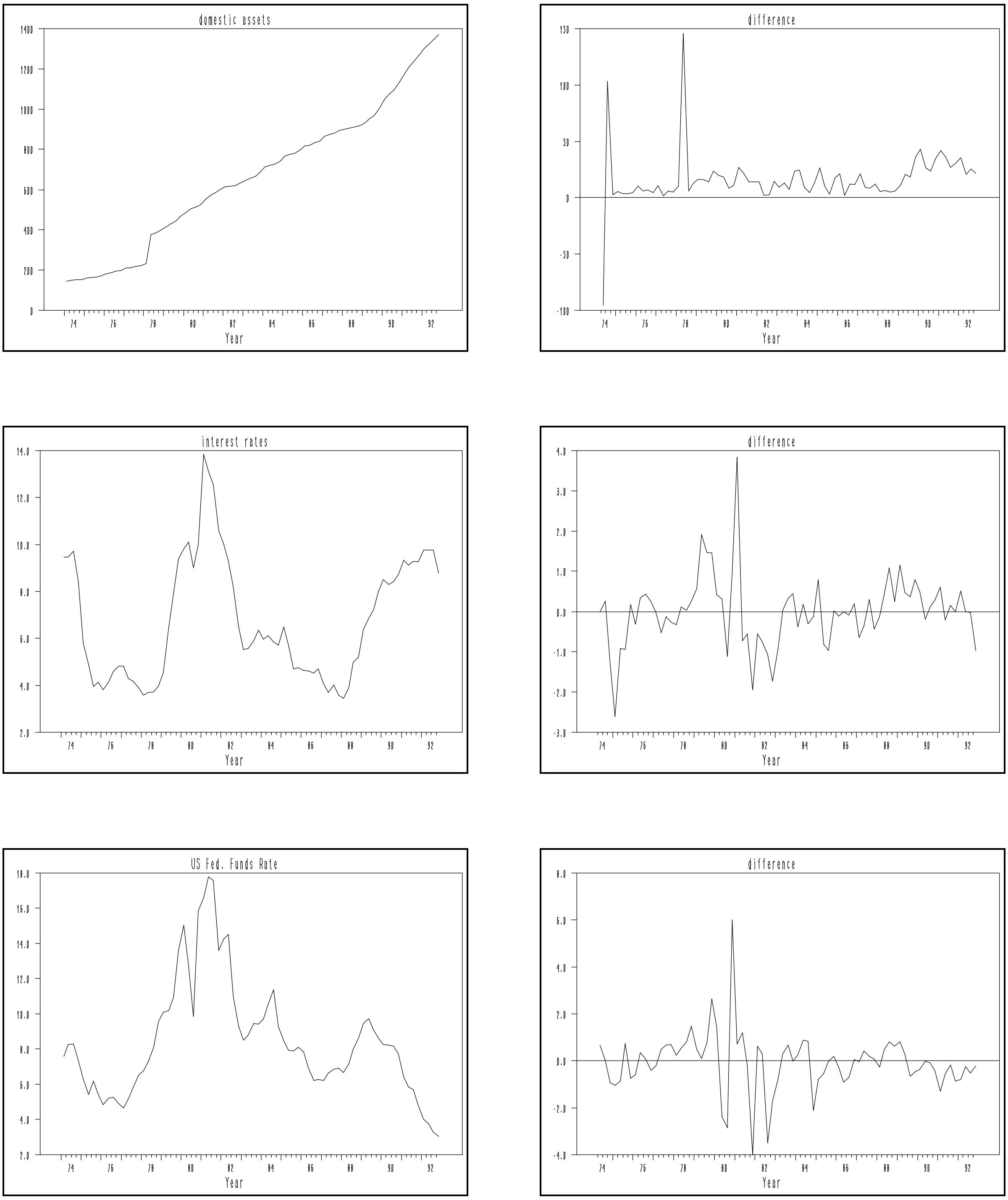
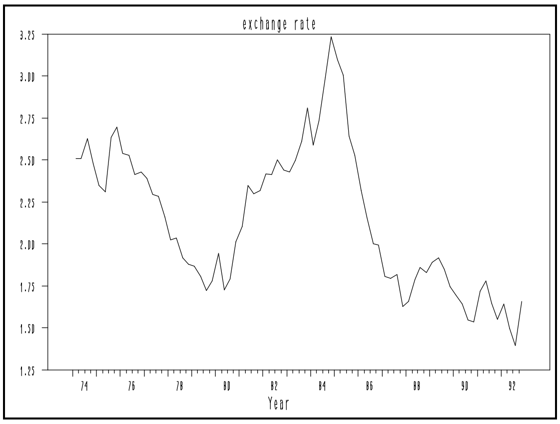
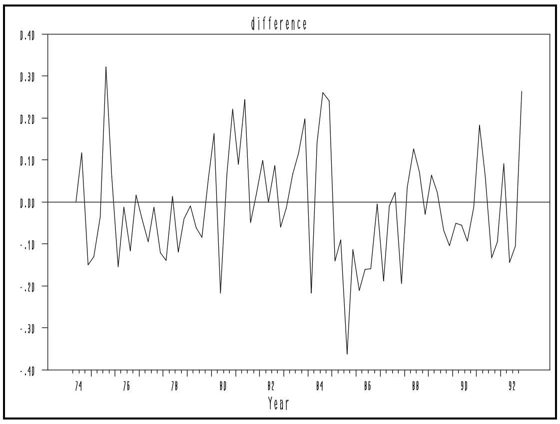


Figure A2.1 *contd.*





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