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INDIVIDUAL AND COLLECTIVE TIME CONSISTENCY

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Abstract:

This paper reconsiders the Strotz-Pollak problem of consistent planning and argues that a solution to this problem requires a refinement of Subgame-perfectness. Such a refinement is offered through an analysis based on Greenberg's 'theory of social situations'. A unifying framework is presented whereby consistent planning as a requirement for individual time consistency and renegotiation-proofness as a requirement for collective time consistency are captured through the same general concept.

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1. INTRODUCTION

The purpose of this paper is to reconsider the problem of consistent planning which was raised by Strotz (1955-56) and Pollak (1968). It will be argued that a solution to this problem as originally posed requires a refinement of Subgame-perfectness. Through an analysis based on Greenberg's (1990) *theory of social situations* such a refinement will be offered.

Strotz (1955-56) and Pollak (1968) were concerned with an individual decision maker wishing to revise an initially optimal plan if there at some later point in time is a better plan available. The recent literature on renegotiation-proofness in repeated games (see Section 6 for references) is concerned with the grand coalition collectively wishing to revise a Subgame-perfect equilibrium (SPE) if there in some subgame is a Pareto-dominating SPE available. A unifying framework encompassing both individual and collective time consistency will be presented.

Section 2 will motivate the subsequent analysis by arguing conceptually and illustrating through examples that Subgame-perfectness does not solve the Strotz-Pollak problem of consistent planning. Section 3 introduces the general model, while Section 4 applies this model to games with perfect information. It is established under what conditions having at each node the active player suggest a path/profile/SPE in the remaining subgame is equivalent to Greenberg's (1990) notion of optimistic stability in the tree situation. Based on this equivalence result a solution to the Strotz-Pollak problem of consistent planning is presented in Section 5, and compared in Section 7 to a solution to the problem of consistent planning suggested by Kocherlakota (1991). Section 6 relates individual and collective time consistency, while Section 8 asks whether planning by a single individual with time inconsistent preferences differs from a game where different individuals make decisions at different times. The relation of the present analysis – where time consistency requires a refinement of Subgame-perfectness – to the analysis of macroeconomic policy games – where time consistency is a weaker requirement than Subgame-perfectness – is discussed in Section 9. Section 10 contains concluding remarks, while all proofs are relegated to Section 11.

2. THE PROBLEM OF CONSISTENT PLANNING

Consider an individual decision maker facing a decision tree. At the initial node, the individual would like to realize a path through the decision tree that maximizes the individual's payoff as evaluated at the initial node. Such a path is said to be *optimal*. Likewise, a decision rule (defined by the property that it at every node of the decision tree determines an action) is said to be *optimal* if it generates an optimal path. An optimal path (or decision rule) is *time consistent* if, for each node reachable by the optimal path, the path (or decision rule) is still optimal in the sense of maximizing the individual's payoff as evaluated at the reached node.

Strotz (1955-56) and Pollak (1968) are concerned with the case where there is no optimal and time consistent path (or decision rule). In such a case, the preferences of the individual are said to time inconsistent. The following two illustrations are included in order to convince the reader that in real life seemingly rational individual decision makers do in fact face such time inconsistencies.

Procrastination. It is a common experience that people tend to postpone unpleasant tasks, preferring to have them done in the next period (day, week, ...). Yet, when the next period comes along, still further postponement seems preferable. Such "... [p]rocrastination occurs when present costs are unduly salient in comparison with future costs ..." in the words of Akerlof (1991, p.1) who gives the subject an interesting treatment filled with real-life examples. Hence, at any time, the rate of time preference between the present and the first future period is greater than between a future period and its successor. These are the kind of *inconsistent time preferences* which are explicitly analyzed by Strotz (1955-56). Time preferences can be shown to be consistent if and only if the utility function is strongly recursive in the sense of Blackorby et al. (1973, Theorem 3).

Intoxication. The following situation may also seem realistic: After work, some would prefer to go by the local pub and have *one* beer instead of going straight home. At the pub, *after* the first beer, it may, however, seem preferable to consume another three beers. These preferences are time inconsistent if, when leaving work, going straight home is preferable to consuming four beers at the pub. Such *endogenous preferences* are treated by e.g. Hammond (1976) who argues that there is no need in a formal analysis to distinguish between preferences changing exogenously due to the passing of time (i.e. inconsistent time preferences) and preferences changing endogenously due to the actions – e.g. the consumption of alcohol and other intoxicating substances – taken (i.e. endogenous preferences). Both types of time inconsistency are thoroughly reviewed by Elster (1979), whose terminology I have adopted above.

In the case where there is no optimal and time consistent path (or decision rule), Strotz (1955-56) suggests two possibilities: *Precommitment* or *Consistent planning.* Precommitment amounts to nothing less than changing the decision tree and will not be discussed here. The problem of consistent planning is according to Strotz (1955-56, p.173) for the individual "to find the best plan among those that he will actually follow".

If no optimal and time consistent path (or decision rule) exists, this problem of consistent planning requires that the decision tree be turned into an extensive game where the individual at different times corresponds to separate players. The payoff that a player receives from a path through the tree equals the payoff the path yields when evaluated at the reachable node at which this player makes a decision. A decision rule (as defined above) corresponds to a profile of the players' strategies.

Peleg and Yaari (1973) and Goldman (1980) analyze the notion of consistent planning in such a game theoretic context. They claim that a plan is the best that will actually be followed ("optimal in the Strotz-Pollak sense", Peleg and Yaari, 1973, p.345; "a Strotz-Pollak equilibrium", Goldman, 1980, p.534) if and only if it is a SPE of the corresponding extensive game. The following examples will make it clear, though, that any SPE is not a solution to the problem that Strotz (1955-56) posed.

Example 1 (see Figure 1) considers an individual who lives at times 1, 2, and 3. At time 1 he has to decide whether to perform an unpleasant task now (D) or later (U). If he chooses U at time 1 he has to decide between now (D) and later (U) at time 2 as well. Note that the individual at time 2 is indifferent between U and D. Therefore, (U, D) is the ***unique optimal and time consistent path*** since this path is the only one that is optimal at each node that the path reaches. Following (U, D) enables the individual to postpone the unpleasant task from period 1 to period 2. However, the set of (pure strategy) SPEa is $\{(U, D), (D, U)\}$. Hence, there exists a SPE in which the individual at the initial node does not choose the best plan among those that he will actually follow, rather he receives a lower payoff by performing the task immediately. He does so fearing that if he postponed the task at time 1 he would postpone the task at time 2 as well and be worse off as evaluated at the initial node.

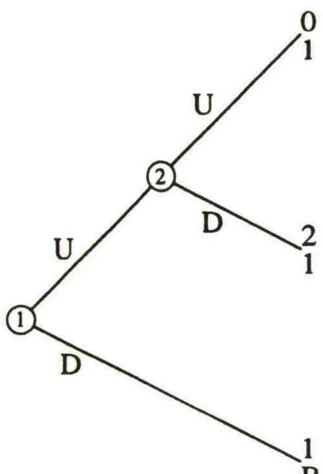

FIGURE 1

Example 2 – which is inspired by an example by Asilis et al. (1991) – has the individual choosing $a_i \in \{0, 1\}$ for each $i \in \mathbb{N}$ with the payoff at time i being given as $min_{j \geq i} a_j$. In this example there also exists a *unique optimal and time consistent path*, viz. (1, 1, 1, ...). Note, however, that (0, 0, 0, ...) is a SPE path since no single player can profitably deviate from the strategy profile determining the action 0 for each player at every node. Then, by using (0, 0, 0, ...) as a punishment, any feasible path can be supported as a SPE path. Hence, even though the problem of consistent planning has a unique (and trivial since the optimal path is time consistent) solution, the concept of a SPE has no bite what so ever in this example. (As example of the same kind, but where the optimal path is time *in*consistent is analyzed in Asheim, 1988a. There the optimal and time *in*consistent path can be supported as a SPE path.)

These examples are peculiar in the sense that the game of Example 1 is non-generic, while the payoff function in the game of Example 2 is not continuous at time infinity. Still, they show that the notion of Subgame-perfectness is not conceptually valid as a solution to the problem of consistent planning. The reason is the following:

The notion of consistent planning is based on the premise that the players (being agents of the individual at different times) are symmetric with respect to their ability to influence later players. In particular, player 1 cannot commit later players to follow his optimal plan. In a SPE, each player can only reconsider his own action; thus, by the symmetry requirement player 1 is not the one suggesting or coordinating on a particular SPE. Hence, by playing according to a SPE the individual is not at any time doing any planning; instead he is following an exogenously given decision rule from which the individual does not wish to do single deviations. (Note that considering an equilibrium as an exogenously recommended course of action is consistent with the classical view of games; see e.g. Kohlberg and Mertens 1986, footnote 3.)

This discussion – as well as Examples 1 and 2 above – suggests that a refinement of Subgame-perfectness is required in order to conceptually solve the problem of consistent planning. Such a refinement will be offered in the subsequent sections through a concept which will turn out to have two equivalent interpretations:

(i) A player can – given that one of his decision nodes has been reached – choose any *path* in the remaining subgame, taking into account that later players can do so in turn.

(ii) A player can – given that one of his decision nodes has been reached – choose any *strategy profile* in the remaining subgame, taking into account that later players can do so in turn.

The concept constructed on this basis respects the symmetry between the decisions that the individual takes at different times, while allowing him to engage in planning by choosing "the best plan among those he will actually follow". It will be established that this concept yields the set of the optimal and time consistent paths when this set is non-empty in every subgame. Examples 1 and 2 satisfy this condition.

3. THE GENERAL MODEL

Consider a multi-stage game G where at each stage a subset of the set of players $N := \{1, \dots, n\}$ (n finite or infinite) are active in the sense of taking part in a simultaneous-move game in that stage. The game is one of *almost perfect information* in the sense that, at each stage, players know all previously taken actions, but not actions taken by other players in the same stage. In order to capture such a game define the set of *histories* inductively as follows: Let the set of histories in (the first) period 1 be given as follows: $H(1) = \{0\}$. Let $H(t)$ denote the set of histories in period t. At $h \in H(t)$, a subset N^h of N are active. For each $i \in N^h$, the action set at h is non-empty and denoted by A_i^h; with A^h denoting the Cartesian product of A_i^h over all $i \in N^h$. Define the set of histories in period $t+1$ as follows:

$$H(t+1) \;:=\; \{(h, a) \mid h \in H(t),\; N^h \neq \emptyset, \text{ and } a \in A^h\}.$$

For convenience, write $(0, a) = a$ such that $H(2) = A^0$. This completes the induction.

The set of histories H is now given by: $H := \cup_{t \in \mathbb{N}} H(t)$. Note that the game may have terminal nodes; in fact, it may be finite horizon. The set of terminal nodes is given by: $H_0 := \{h \in H \mid N^h = \emptyset\}$. The set of *subgames* $H \backslash H_0$ is naturally ordered by $\leq$, i.e. $h \leq k$ means that h equals or precedes k. Note that – by convention – $h \leq k$ is taken to imply that k is not a terminal node.

If $h \in H$, then a feasible *path* at h, denoted $\pi = (a(1), a(2), \dots)$, is either a finite sequence of feasible actions leading up to a terminal node (*i.e.* $k = (h, \pi) \in H_0$) or an infinite sequence of feasible actions. Define Π^h as the set of feasible paths at h. For notational convenience, write $\Pi^h = \{0\}$ if $h \in H_0$, understanding that $(h, 0) = h$. A feasible path π at h yields player i the payoff $U_i(h, \pi)$.

Let $H_i^h := \{k \geq h \mid i \in N^k\}$. Then the set of *strategies* for player $i \in \cup_{k \geq h} N^k$ in the subgame h – consisting of all x_i satisfying for all $k \in H_i^h$, $x_i(k) \in A_i^k$ – is denoted by X_i^h. The set of (*strategy*) *profiles* in the subgame h, X^h, is the Cartesian product of X_i^h over all $i \in \cup_{k \geq h} N^k$. If $h \leq k$ and $x \in X^h$, then denote by x^k the restriction of x to the subgame k. Note that $x^k \in X^k$. If $h \leq k$, then $x \in X^h$ determines (through the restriction of x to k) a path $\pi^k(x)$ in the subgame k, and thereby yields player i the payoff $U_i(k, \pi^k(x))$ given that the subgame k has been reached. Let $u_i^k(x) := U_i(k, \pi^k(x))$. Denote by X_E^h the set of SPEa and by Π_E^h the set of SPE paths of the subgame h. Write $\Pi_E^h = \{0\}$ if $h \in H_0$. Write X, Π, X_E, and Π_E for X^h, Π^h, X_E^h and Π_E^h if $h = 0$.

The situation which will turn out to integrate the problems of individual and collective time consistency is given as follows. In each subgame h the players active at the root of h can choose any SPE in h that is viable when taking into account that the players active at the roots of later subgames can do so in turn. In order to capture this situation, let a *standard of behavior* (SB) for G on SPEa be a correspondence Σ assigning to each subgame $h \in H \backslash H_0$ a subset, $\Sigma(h)$, of X_E^h. A SB Σ is said to be *optimistic internally stable* on X_E if

(IS) For any $h \in H \backslash H_0$ and any $x \in \Sigma(h)$, there do not exist $k \geq h$ and $y \in \Sigma(k)$ such that $u_i^k(x) < u_i^k(y)$ for all $i \in N^k$.

A SB Σ is said to be *optimistic externally stable* on X_E if

(ES) For any $h \in H \backslash H_0$ and any $x \in X_E^h \backslash \Sigma(h)$, there exist $k \geq h$ and $y \in \Sigma(k)$ such that $u_i^k(x) < u_i^k(y)$ for all $i \in N^k$.

A SB is said to be *optimistic stable* on X_E if it is both optimistic internally and optimistic externally stable on X_E. This is a special application of the general notion of optimistic stability, which is due to von Neumann and Morgenstern (1953), and which is a central solution concept in Greenberg's (1990) *theory of social situations*. The term '*optimistic* stability', which has been coined by Greenberg (1990), refers here to the optimistic attitude of the players active at the root of h in the sense of believing that they can choose *any* viable outcome in h. Greenberg (1990) has also a notion of *conservative* stability, which does not correspond to von Neumann and Morgenstern stability, and which will not be treated here.

4. GAMES WITH PERFECT INFORMATION

A game with perfect information G is characterized by the property that, for all $h \in H \backslash H_0$, $|N^h| = 1$; i.e., at each stage, only one player moves. Denote by i^h the single player active at h. A game with perfect information is a *Strotz-Pollak game* if (i) $i^0 = 1$, and (ii) $i^h = i$ and $a \in A^h$ imply $i^{(h,a)} = i+1$ whenever $(h, a) \in H \backslash (\{0\} \cup H_0)$. A game with perfect information is *continuous* if for every $\varepsilon > 0$, there exists an integer κ such that if the first κ nodes of two paths π and ϱ coincide, then for all $i \in N$, $|U_i(\pi) - U_i(\varrho)| < \varepsilon$. A game with perfect information is *finite action* if, for each $h \in H \backslash H_0$, $|A^h|$ is finite.

Motivated by Section 2, consider the situations where in each subgame h, the player active at the root of h can choose

(i) any path at h that is viable when taking into account that the players active at the roots of subgames that is reached by the path can do so in turn.

(ii) any (strategy) profile in h that is viable when taking into account that the players active at the roots of later subgames can do so in turn.

Situation (ii) is identical to the one considered in Section 3 with the one difference that all profiles are considered, not only those that are Subgame-perfect. Hence, let a SB for G on profiles be a correspondence Σ assigning to each subgame $h \in H \backslash H_0$ a subset, $\Sigma(h)$, of X^h. A SB Σ for G is said to be *optimistic internally stable* on X if

(IS) For any $h \in H \backslash H_0$ and any $x \in \Sigma(h)$, there do not exist $k \geq h$ and $y \in \Sigma(k)$ such that $u_i^k(x) < u_i^k(y)$ for $i = i^k$.

A SB Σ for G is said to be *optimistic externally stable* on X if

(ES) For any $h \in H \backslash H_0$ and any $x \in X^h \backslash \Sigma(h)$, there exist $k \geq h$ and $y \in \Sigma(k)$ such that $u_i^k(x) < u_i^k(y)$ for $i = i^k$.

A SB is said to be *optimistic stable* on X if it is both optimistic internally and optimistic externally stable on X.

Situation (i) is different since it is defined in terms of paths, not profiles. Therefore, given some path $\pi = (a(1), \ldots, a(s), \ldots) \in \Pi^h$, say that k is *reachable* from h through π if $k = (h, a(1), \ldots, a(s))$ for some s. Let a SB for G on paths be a correspondence σ assigning to each subgame $h \in H\backslash H_0$ a subset, $\sigma(h)$, of Π^h. An SB σ for G is said to be *optimistic internally stable* on Π if

(IS) For any $h \in H\backslash H_0$ and $\pi \in \sigma(h)$, there does not exist $k \in H\backslash H_0$ reachable from h through π and $\varrho \in \sigma(k)$ such that $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^k$.

An SB σ for G is said to be *optimistic externally stable* on Π if

(ES) For any $h \in H\backslash H_0$ and $\pi \in \Pi^h\backslash\sigma(h)$, there exists $k \in H\backslash H_0$ reachable from h through π and $\varrho \in \sigma(k)$ such that $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^k$.

An SB for G is said to be *optimistic stable* on Π if it is both optimistic internally and externally stable on Π.

Situation (i) will be shown to be closely related to the *tree situation* in Greenberg's (1990, Ch. 8) *theory of social situations*. For each $k \in H\backslash\{0\}$, let $P(k)$ denote the immediate predecessor of the node k. If $\pi \in \Pi^h$, then player i is said to be able to *induce* k from h through π if $i^{P(k)} = i$ and $P(k)$ is reachable from h through π. Let a SB for G in the tree situation be a correspondence σ assigning to each history $h \in H$ a subset, $\sigma(h)$, of Π^h. An SB σ for G is said to be *optimistic internally stable* in the tree situation if

(IS) $h \in H$ and $\pi \in \sigma(h)$ imply that for any $i \in N$ there do not exist $k \in H$ and $\varrho \in \sigma(k)$ such that i can induce k from h through π and $U_i(h, \pi) < U_i(k, \varrho)$.

An SB σ for G is said to be *optimistic externally stable* in the tree situation if

(ES) $h \in H$ and $\pi \in \Pi^h\backslash\sigma(h)$ imply that for some $i \in N$ there exist $k \in H$ and $\varrho \in \sigma(k)$ such that i can induce k from h through π and $U_i(h, \pi) < U_i(k, \varrho)$.

An SB for G is said to be *optimistic stable* in the tree situation if it is both optimistic internally and externally stable in the tree situation. Greenberg's (1990) tree situation has above been introduced without presenting the general structure and terminology of the *theory of social situations.*

Note that a SB on paths / for the tree situation assigns to each subgame / history a set of paths while a SB on SPEa / on profiles assigns to each subgame a set of profiles. In order to show an equivalence between these various optimistic stable SBs, the following definitions are required. Let σ be a SB that is optimistic stable on paths / for the tree situation. Define a SB Σ^{σ} which assigns to each subgame a set of profiles by the property that $\Sigma^{\sigma}(h) = \{x \in X^h \mid \pi^k(x) \in \sigma(k) \text{ for all } k \geq h\}$ if $h \in H \backslash H_0$. Conversely, let Σ be a SB that is optimistic stable on SPEa / on profiles. Define a SB σ^{Σ} which assigns to each subgame a set of paths by the property that $\sigma^{\Sigma}(h) = \{\pi \in \Pi^h \mid \pi = \pi^h(x) \text{ for some } x \in \Sigma(h)\}$ if $h \in H \backslash H_0$ (and, in the case of tree situation, $\sigma^{\Sigma}(h) = \Pi^h = \{0\}$ if $h \in H_0$). Also, say that a SB σ which assigns to each subgame / history a set of paths is *non-empty valued* if, for each $h \in H \backslash H_0$, $\sigma(h) \neq \emptyset$.

The equivalence between the optimistic stable SBs of this section can be established for general extensive games with perfect information subject to the condition that the SB that is optimistic stable for the tree situation be non-empty valued.

PROPOSITION 1. *For a general extensive game with perfect information, the following implications hold.*

(a) *If σ is a non-empty valued SB that is optimistic stable in the tree situation, then σ constrained to $H \backslash H_0$ is a SB that is optimistic stable on Π.*

(b) *If σ is a SB that is optimistic stable on Π, then Σ^{σ} is a SB that is optimistic stable on X.*

(c) *If Σ is a SB that is optimistic stable on X, then σ^{Σ} is a non-empty valued SB that is optimistic stable in the tree situation.*

As the following example makes clear, the assumption that σ in the tree situation be non-empty valued is essential.

Example 3. Consider the one player static game in which the player chooses $a \in [0, 1)$ and receives a payoff $U = a$. Here a SB that optimistic stable on Π does not exist, neither does a SB that is optimistic stable on X. However, σ with $\sigma(0) = \emptyset$ and $\sigma(h) = \{0\}$ for each $h = a \in [0, 1)$ is optimistic stable in the tree situation.

For a game with perfect information that is either a Strotz-Pollak game or a continuous game, a strategy profile is a SPE if and only if no one-shot deviation is profitable. For games in this class that are finite action, the optimistic stable SB of Section 2 can be included in the equivalence.

PROPOSITION 2. *For an extensive game with perfect information that is either a Strotz-Pollak game or a continuous game, the following implication holds.*

(c′) If Σ is a SB that is optimistic stable on X, then Σ is a SB with $\Sigma(0) \neq \emptyset$ that is optimistic stable on X_E.

For an extensive game with perfect information that is either a finite action game or, for each subgame, has a unique SPE, the following implication holds.

(c″) If Σ is a SB with $\Sigma(0) \neq \emptyset$ that is optimistic stable on X_E, then σ^{Σ} is a non-empty valued SB that is optimistic stable in the tree situation.

As the following example shows, if an extensive game is neither of the Strotz-Pollak variety nor continuous, the first implication of Proposition 2 may not hold.

Example 4. (Greenberg, 1990, Example 8.2.5) There are two players, 1 and 2, who each can choose, in his turn, two actions. Player 1 can choose either U or D, and player 2 can choose either L or R. Turns are alternating, with player 1 being the one to start. Hence, $\Pi = \{(a(1), \ldots, a(t), \ldots) \mid a(t) \in \{U, D\}$ if t is odd and $a(t) \in \{L, R\}$ if t is even$\}$. The paths in Π yield the following payoffs: $U(\pi)$ $(:= (U_1(\pi), U_2(\pi))) = (5, 0)$ if $\pi \in \Pi$ and, for all $a \in \{U, D, L, R\}$, a is played infinitely many times. $U(\pi) = (0, 5)$ if $\pi \in \Pi$ and, for some $a \in \{U, D, L, R\}$, a is played finitely many times. Here, $\Pi_E^h = \{\pi | \ (h, \pi) \in \Pi$ and $U(h, \pi) = (0, 5)\}$ since player 2 can individually force the payoff profile (0, 5), e.g. by always playing L; hence $\Pi^h \backslash \Pi_E^h = \{\pi | \ (h, \pi) \in \Pi$ and $U(h, \pi) = (5, 0)\}$. As shown by Greenberg (1990, Example 8.2.5), the SBs σ^1 defined by $\sigma^1(h) = \Pi_E^h$ for all $h \in H$ and σ^2 defined by $\sigma^2(h) = \Pi^h \backslash \Pi_E^h$ for all $h \in H$ are each optimistic stable in the tree situation. By Proposition 1, the SBs Σ^{σ^1} and Σ^{σ^2} are each optimistic stable on profiles. However, Σ^{σ^2} is not even a SB on SPEa since $\Sigma^{\sigma^2}(0)$ is disjoint from the set of SPEa.

As the following example shows, if an extensive game is neither finite action nor has a unique SPE in each subgame, the second implication of Proposition 2 may not hold.

Example 5. Consider a two player game in which player 1 at the initial node 0 can end the game by choosing A (yielding the payoff profile (1, 1)) or continue the game by choosing $B \in [0, 2)$. In the latter circumstance, player 2 has the choice between L (yielding the payoff profile $(B, 0)$) or R (yielding the payoff profile $(-B, 0)$). This game has a unique (pure strategy) SPE in which 1 plays A and 2 – if called – plays R. Hence, the unique Σ that is optimistic stable on SPEa has $\Sigma(0) = X_E \neq \emptyset$ and, for each $B \in [0, 2)$, $\Sigma(B) = X_E^B = \{L, R\}$. However, the unique σ that is optimistic stable in the tree situation has $\sigma(0) = \emptyset$ and, for each $B \in [0, 2)$, $\sigma(B) = \{L, R\}$, $\sigma(B, L) = \{0\}$, and $\sigma(B, R) = \{0\}$.

5. A REVISION-PROOF PLAN

The concept that this paper suggests in order to solve the Strotz-Pollak problem of consistent planning will be referred to as a *revision-proof plan*. When an optimal and time consistent path (or decision rule) does not exist, the individual need know what he will do at the next stage under the different contingencies that his various actions now will give rise to. Hence, interpreting a revision-proof plan as a path is not informationally simpler for the individual than interpreting a revision-proof plan as a strategy profile in the Strotz-Pollak game. For, in order to determine such a time consistent path, he needs to determine the set of time consistent paths in every subgame. Hence, two alternative and (due to Proposition 1) equivalent definitions will be offered.

DEFINITION 1. (Asheim, 1987) In a Strotz-Pollak game G, $\pi \in \Pi$ is a *revision-proof path* if there exists a SB σ, with $\pi \in \sigma(0)$, that is optimistic stable on Π.

DEFINITION 2. In a Strotz-Pollak game G, $x \in X$ is a *revision-proof equilibrium* if there exists a SB Σ, with $x \in \Sigma(0)$, that is optimistic stable on X.

The following two corollaries establish inter alia the equivalence of Definitions 1 and 2 and, moreover, that these definitions yield a refinement of Subgame-perfectness.

COROLLARY 1. *(a) In a Strotz-Pollak game G, if π is a revision-proof path, then there exists a revision-proof equilibrium x with $\pi^0(x) = \pi$. (b) In a Strotz-Pollak game G, π is a revision-proof path if and only if there exist a non-empty valued SB σ, with $\pi \in \sigma(0)$, that is optimistic stable in the tree situation. (c) In a Strotz-Pollak game G, if π is a revision-proof path, then $\pi \in \Pi_E$.*

COROLLARY 2. *(a) In a Strotz-Pollak game G, if x is a revision-proof equilibrium, then $\pi^0(x)$ is a revision-proof path. (b) In a finite action Strotz-Pollak game G, x is a revision-proof equilibrium if and only if there exist a SB Σ, with $x \in \Sigma(0)$, that is optimistic stable on X_E. (c) In a Strotz-Pollak game G, if x is a revision-proof equilibrium, then $x \in X_E$.*

The relation between the concept of Definitions 1 and 2 and the set of optimal and time consistent paths when this set is non-empty is established in the following proposition.

PROPOSITION 3. *(i) If π is a revision-proof path in the Strotz-Pollak game G and there exists an optimal and time consistent path ϱ, then $U_i(\pi) = U_i(\varrho)$ for $i = 1$. (ii) If there exists a non-empty set of optimal and time consistent paths in each subgame of the Strotz-Pollak game G, then π is a revision-proof path if and only if π is optimal and time consistent.*

By part (ii), it follows that in Example 1 (of Section 2) the unique revision-proof path is (U, D), while in Example 2 (of Section 2) the unique revision-proof path is $(1, 1, 1, \ldots)$. Hence, the concept of Definitions 1 and 2 yields the satisfactory solution in these examples. See also Asheim (1987, 1988a) for an analysis of a game where Subgame-perfectness by allowing any feasible path has no bite, but where the application of Definition 1 successfully solves the problem of consistent planning.

In spite of the argument of Section 2, viz. that the Strotz-Pollak problem of consistent planning is not solved by the concept of Subgame-perfectness, Subgame-perfectness is in accordance with Strotz-Pollak planning under the condition of the following proposition.

PROPOSITION 4. *If there exists a unique SPE in each subgame of the Strotz-*

Pollak game G, *then the unique* $\pi \in \Pi_E$ *is the unique revision-proof path, and the unique* $x \in X_E$ *is the unique revision-proof equilibrium.*

Example 1 (of Section 2) would satisfy the condition of Proposition 4 if player 2 were not indifferent between U and D at time 2.

On the question of existence of a revision-proof path/equilibrium, Greenberg (1990, Corollary 8.3.2) combined with Proposition 1 yields the following result.

PROPOSITION 5. *In a finite horizon, finite action Strotz-Pollak game* G, *there exists a unique SB* σ *that is optimistic stable on* Π. *Furthermore,* σ *is non-empty valued.*

Peleg and Yaari (1973, Section III) presents an example for which they claim that no solution to the Strotz-Pollak problem of consistent planning exists. Even though their example is not covered by Proposition 5 above, it is straight-forward to show that the SPE that Goldman (1980, note 4) constructs for this example is in fact a revision-proof equilibrium in the sense of Definition 2. However, the following example illustrates the case of a finite horizon, compact action game with continuous payoffs for which there exists no revision-proof path/equilibrium.

Example 6. (Hellwig and Leininger, 1987, Section III) Consider a three player Strotz-Pollak game, where for each $i \in \{1, 2, 3\}$, $a_i \in [0, 1]$, and where $U_1 = -a_3 - a_1$, $U_2 = a_3 + \frac{1}{2}a_2$, and $U_3 = a_3(\frac{1}{2} - a_1 - a_2) - a_3(1 - a_3)$. Hellwig and Leininger (1987) show that the unique SPE path is $(\frac{1}{2}, 1, 0)$. However, in the subgame $h = a_1 = \frac{1}{2}$, there is a unique revision-proof path $(0, 1)$ which yields player 2 a higher payoff than continuing the SPE path. Hence, the unique SPE path is not revision-proof. Note that if the action sets are turned into discrete grids, the unique revision-proof path is $(\frac{1}{2}+\varepsilon, 1, 0)$ for some $\varepsilon > 0$.

6. RENEGOTIATION-PROOFNESS AS COLLECTIVE TIME CONSISTENCY

In a repeated game, a SPE can be supported by a threat which – if called – is Pareto-inferior to the original SPE. Hence, if the players can coordinate before each stage of the game, they prefer renegotiating back to the original SPE rather than undertaking the threat. However, this undermines the credibility of the threat and questions the viability of the original SPE. The literature on renegotiation-proof equilibria (see e.g. Farrel and Maskin, 1989, Bernheim and Ray, 1989, Asheim, 1991, and Pearce, 1987, as well as Bergin and MacLeod, 1991, for a survey) seeks to answer the following question: What SPEa are not prone to this kind of criticism?

A SPE that is not renegotiation-proof may be looked at as a plan that is not collectively time consistent (see e.g. Bernheim and Ray, 1989): There are subgames where the grand coalition as a collective gains by revising the plan. In these terms there are obvious similarities between the Strotz-Pollak notion of consistent planning as a problem of individual time consistency and the notion of renegotiation-proofness as a problem of collective time consistency. There are also differences, though: In the problem of individual time consistency the revision occurs even on the equilibrium path, while in the problem of collective time consistency the revision (in the interesting cases) occurs only off the equilibrium path after a deviation by one of the players.

Still, it is by now straightforward to show that the notion of a SB that is optimistic stable on SPEa yields a definition of both individual and collective time consistency. By Corollary 2(b) it is already established for (finite action) Strotz-Pollak games that x is a revision-proof equilibrium if and only if there exists a SB Σ, with $x \in \Sigma(0)$, that is optimistic stable on SPEa. What remains is to establish a corresponding result for collective time consistency in repeated games, viz. that x is a renegotiation-proof equilibrium if and only if there exists a SB Σ, with $x \in \Sigma(0)$, that is optimistic stable on SPEa.

First, it is necessary to spell out how repeated games fit into the general model of Section 3. A repeated game G consists of a T-fold play of a n-person history-independent simultaneous-move game, where T is finite or infinite. Hence, $N^h = N$ for all $H \backslash H_0$. Furthermore, for each $i \in N$, $A_i^h = A_i$ for all $H \backslash H_0$. Finally, $H_0 = A^T$ if $T < \infty$ and $H_0 = \emptyset$ if $T = \infty$. The path $\pi = (a(1), \ldots, a(T)) \in A^T$ yields player i the payoff $U_i(\pi) := \frac{1}{T} \cdot \Sigma_{t=1}^{T} v_i(a(t))$ if $T < \infty$ and $U_i(\pi) := (1-\delta) \cdot \Sigma_{t=1}^{\infty} \delta^{t-1} \cdot v_i(a(t))$ with $\delta \in (0, 1)$ if $T = \infty$, where v_i is the stage game payoff function of player i.

Now, the following observations can be made.

PROPOSITION 6. *A SPE x of a repeated game G is a Pareto-perfect equilibrium as defined in Definition 1 of Asheim (1991) if and only if there exists a SB Σ, with $x \in \Sigma(0)$, that is optimistic stable on X_E.*

COROLLARY 3. *A SPE x of a finitely repeated game G where, for each $i \in N$, A_i is compact and v_i is continuous is a Consistent equilibrium as defined by Bernheim and Ray (1989, Section 3) if and only if there exists a SB Σ, with $x \in \Sigma(0)$, that is optimistic stable on X_E.*

Hence, according to the usual and uncontroversial definition of renegotiation-proofness in finitely repeated games and the extension that I (Asheim, 1991) suggest for infinitely repeated games, renegotiation-proofness as a requirement for collective time consistency is closely related to the Strotz-Pollak notion of consistent planning as a requirement for individual time consistency.

In the present paper, the Strotz-Pollak problem of consistent planning has been analyzed by turning the decision tree into an extensive game where the individual at different times corresponds to different players. An alternative would be to treat the decision problem as a one player "game" where the evaluation of a path through the decision tree depends on which node along the path the evaluation takes place. I have in Asheim (1988b) presented a definition of subgame-perfectness in multi-stage games that can be applied to such a one player "game", and for which it is identical to the definition of a revision-proof equilibrium. (See also Asilis et al. (1991) for a similar definition where Roth (1976) semi-stability is used instead of von Neumann and Morgenstern stability). Furthermore, the non-recursive definitions of Pareto-perfectness and Perfectly coalition-proofness in Asheim (1988b) reduce to the definition of revision-proofness in such a one player "game". If the one player "game" has a finite horizon, the recursive definition of a Perfectly coalition-proof equilibrium in Bernheim et al. (1987, p.10) can be shown to be applicable and yielding revision-proofness.

7. CONSISTENT PLANNING IN STATIONARY STROTZ-POLLAK GAMES

A finite horizon Strotz-Pollak game is *stationary* if all subgames are isomorphic to the game itself. Such a stationary structure is shared by infinitely repeated games. Stationary Strotz-Pollak games therefore allow for the application of concepts of renegotiation-proofness as developed for infinitely repeated games. Kocherlakota (1991) has successfully applied Farrell and Maskin's (1989) concepts of *weakly renegotiation-proof* and *strongly renegotiation-proof* equilibria, the analogs being called *symmetric* and *reconsideration-proof* equilibria, respectively. A symmetric equilibrium is a SPE yielding the individual the same payoff in every subgame (as evaluated at the root of the subgame). Kocherlakota (1991) considers such symmetry a necessary condition for a time consistent plan. A symmetric equilibrium is reconsideration-proof if there is no symmetric equilibrium yielding the individual a higher payoff, a concept for which Kocherlakota (1991) establishes general existence. Kocherlakota (1991) also shows that if an optimal and time consistent path exists, then this path is a reconsideration-proof path, a property that – by Proposition 3 (ii) – is shared by revision-proofness in stationary Strotz-Pollak games. The following proposition gives a result on the relation between revision-proof and reconsideration-proof equilibria.

PROPOSITION 7. (Kocherlakota, private communication) In a stationary Strotz-Pollak game, a revision-proof equilibrium is symmetric if and only if it is reconsideration-proof.

As the following example shows, it is possible to construct a stationary Strotz-Pollak game which admits revision-proof equilibria, none of which are symmetric.

Example 7. (Kocherlakota, 1991, Example 1). Consider the stationary Strotz-Pollak game where for each $i \in \mathbb{N}$, $a_i \in [0, 1]$ and $U_i = \Sigma_{j \geq i}^{\infty} \beta^{j-i}(a_j - a_{j+1})$ with $\beta \in (0, 1)$. The set of individually rational and feasible payoffs is $[0, 1]$. The unique stationary SPE determines the action 1 for each player at every node. Any path which at each time is individually rational (including the optimal path $(1, 0, 0, ...)$) can be supported as a SPE by using the stationary SPE path $(1, 1, 1, ...)$ as a punishment. Any symmetric equilibrium yields the payoff 0; hence, they are all reconsideration-proof, including the unique stationary SPE.

CLAIM 1. *For the game of Example 7, (i)* $\pi^0 = (\beta^0, \beta^1, \beta^2, ...)$ *is a revision-proof path, and, furthermore, (ii) no symmetric equilibrium is revision-proof.*

Holden (private communication) has pointed out that Pearce's (1987) concept of renegotiation-proofness is easily applicable to stationary Strotz-Pollak games. In particular, let $\ell(x) := \inf\{u_i^h(x) \mid h \in H \text{ with } i = i^h\}$, and say that the SPE x is a *Pearce time consistent equilibrium* if there exists no SPE y such that $\ell(x) < \ell(y)$. The interpretation is that the individual will accept a punishment if any SPE involves as harsh a punishment in some subgame. In Example 7, $\ell(x) = 0$ for any SPE x, so that Pearce time consistency has no bite. In the following example, though, Pearce time consistency picks out the reasonable paths, all of which are revision-proof.

Example 8. Consider the stationary Strotz-Pollak game where for each $i \in \mathbb{N}$,

$a_i \in \{0, 1\}$ and $U_i = a_i - a_{i+1} - a_{i+2}$. The set of individually rational and feasible payoffs is $\{-1, 0, 1\}$. The unique stationary SPE determines the action 1 for each player at every node. Any path which at each time is individually rational can be supported as a SPE by using the stationary SPE path $(1, 1, 1, \ldots)$ as a punishment. The stationary SPE, yielding the payoff -1, is the unique symmetric equilibrium and, hence, the unique reconsideration-proof equilibrium.

CLAIM 2. *For the game of Example 7, (i)* $\pi^0 = (0, 0, 0, \ldots)$, $\pi^1 = (1, 0, 0, \ldots)$, *and* $\pi^2 = (1, 1, 0, 0, \ldots)$ *are the Pearce time consistent paths; furthermore, (ii)* π^0, π^1, *and* π^2 *are all revision-proof; and finally, (iii) the unique symmetric equilibrium is neither Pearce time consistent nor revision-proof.*

The problem with the concept of reconsideration-proofness in the context of these examples is that it determines as time consistent the symmetric SPEa – which all hold the individual down to his reservation payoff – even though there exist SPEa that for every subgame yields the individual strictly more than his reservation payoff.

Finally, note that Examples 7 and 8 each illustrates the case of a game which allows for multiple SBs that are optimistic stable on paths/profiles.

8. A SINGLE PLANNER OR DIFFERENT INDIVIDUALS

Returning to the game of Example 1, does it make any difference for the solution of the game whether the game models planning by a single individual or strategic interaction between two individuals? One difference between the two interpretations is that a single individual faces no problems of communication, while in multi-person games communication is in general an issue – even in perfect information games – when there is a need to coordinate on one out of several equilibria.

In the game of Example 1, a forward induction argument (see van Damme, 1989) would say that player 1, by choosing U, can indicate his desire to play the SPE (U, D), since his only reason for playing U would be that he expects player 2 to choose D. Hence, by involving forward induction as a vessel of communication from player 1 to player 2, one can argue that (U, D) is the reasonable prediction in this game even if the game is interpreted as modeling strategic interaction between two individuals.

In the present game with $B < 1$ – i.e., player 2 prefers the SPE (U, D) to (D, U) – this prediction is shared by a number of recent papers (Tranæs, 1991; Ponssard, 1991; as well as an earlier and different approach by Leininger, 1986): Following Tranæs (1991), player 2 choosing D is then a *credible promise* that should be anticipated by player 1 and induce him to chose U. However, with $B > 1$ – i.e., player 2 prefers the SPE (D, U) to (U, D) – the above mentioned contributions as well as Bennett and van Damme (1990) would yield the opposite prediction: Following Bennett and van Damme (1990) and Tranæs (1991), player 2 choosing U is then a *credible threat* that should be anticipated by player 1 and induce him to chose D. The invalidity of the forward induction argument in this case is explained by Bennett and van Damme (1990, p. 14) as follows: In the present game with $B > 1$, "... the forward induction logic is not compelling: although player 1 may indicate his desire to play a particular subgame perfect strategy combination, he has no means of enforcing this strategy combination since he has no further moves in the game." Of course this criticism of the forward induction argument is not valid if the game models planning by a single individual since then the individual making the move of player 1 is the same as the one making the move of player 2.

Hence, when the game is interpreted as modeling planning by a single individual, it is easier to defend (U, D) as the unique solution also when $B > 1$.

9. Time Consistency in Macroeconomic Policy Games

Starting with Kydland and Prescott (1977), there is a considerable literature on the topic of consistent planning in the context of macroeconomic *policy games*. In such games a government's (the *leader's*) policy – which at the outset is optimal – may cease to be so at some later time. This may occur since the policy was designed to induce the general public (the *follower*) to take specific actions. When these actions have been irrevocably taken, the leader may wish to revise the originally planned policy since the constraints imposed by the follower's preferences on the leader's choice set, by then, have changed. In this literature the conclusion is that time consistency is a weaker requirement than Subgame-perfectness (see Fershtman, 1989), which is a conclusion seemingly in contrast with the one of the present paper. Happily, there is no conflict since the matters treated in the present paper are different from those of Fershtman (1989) along two dimensions.

The first dimension relates to the difference between time inconsistent *preferences* and time inconsistent *constraints* (see e.g. Persson and Svensson, 1989). While Fershtman (1989) and the literature on policy games are concerned with the time inconsistency of the constraints imposed by the follower's preferences on the leader's choice set, the present paper as well as the Strotz-Pollak literature on consistent planning are concerned with the time inconsistency of the planner's preferences. I.e., the latter literature poses the problem of an individual decision maker who wishes to revise an optimal path when his own initial actions have been irrevocably taken since his preferences, by then, have changed.

Relating to this dimension, there is a major difference on a formal level: In a Strotz-Pollak game, there are as many players as there are points in time at which the individual makes decisions. In a policy game there is one leader and one (Fershtman, 1989) or a continuum (Chari and Kehoe, 1990) of followers, with the leader at different times being the same player. Hence, one should bear in mind that the two kinds of games are quite different when discussing the relation between Subgame-perfectness and time consistency in each of them.

The second dimension along which the present paper differs from Fershtman (1989) is conveyed by letting the follower in Fershtman's (1989) framework be a trivial player which at any of his decision nodes can take only one action. Then Fershtman's (1989) definition of a *time consistent open loop Stackelberg equilibrium* becomes identical to the definition in the introduction to Section 2 of an optimal and time consistent path. Similarly, Fershtman's (1989) definition of a *time consistent Stackelberg equilibrium with decision rule strategies* becomes identical to the definition in the introduction to Section 2 of an optimal and time consistent decision rule. Hence, Fershtman (1989) is the analog – in the case of time inconsistent constraints – to a definition of an optimal and time consistent path (or decision rule): In order to establish whether a path (or decision rule) is optimal and time consistent, it suffices to check *along* the (generated) path.

A definition of time consistency when the optimal (commitment) policy is not time consistent is given by Chari and Kehoe (1990) for the case of an infinite horizon economy. In their model, time consistency constitutes a refinement of Subgame-perfectness for a totally different reason than that of the present paper: The followers are modeled as a continuum of competitive and anonymous private agents, each with his private history, so that the only proper subgame is the original game itself. Therefore, Subgame-perfectness does not imply sequential rationality. Chari and Kehoe's (1990) *sustainable plans* allow for the use of trigger strategies, implying that the government – after a deviation – may wish to let bygones be bygones by announcing a revised policy whereby it escapes the punishment. In a policy game, such trigger strategies are defendable, however, since the general public may not believe the government's pledge that no new deviation will occur subsequently. Such an issue of confidence does not arise when analyzing the Strotz-Pollak problem of consistent planning by an individual with time inconsistent preferences.

10. CONCLUDING REMARKS

In the present paper the Strotz-Pollak notion of consistent planning – viz. that the individual should choose "the best plan among those he will actually follow" – has been given a precise meaning. The concept of a revision-proof path/equilibrium as introduced here is implicitly contained in the work of Greenberg (1990, Ch. 8). The present contribution gives this concept interpretations that are more closely related to the problem that Strotz (1955-56) originally posed.

It is a conclusion of this paper that – when analyzing the problem of consistent planning through an extensive game where the individual at different times corresponds to separate players (a 'Strotz-Pollak game') – a revision-proof equilibrium is a refinement of Subgame-perfectness. This conclusion contrasts the literature on time consistency in policy games (see e.g. Fershtman, 1989); however, as argued in Section 9, the matters treated in this paper are different from those of Fershtman (1989).

A general model of an extensive game of almost perfect information has been introduced in Section 3, yielding the Strotz-Pollak game and the repeated game as special cases. Through this framework it has been possible to capture consistent planning as a requirement for individual time consistency and renegotiation-proofness as a requirement for collective time consistency through the same general concept, viz. a SB that is optimistic stable on SPEa.

Extensive games that are neither repeated nor of the Strotz-Pollak variety have not been explicitly analyzed. Applying the concept of a SB that is optimistic stable on SPEa to, e.g., Example 1 of van Damme (1989) would convince the reader that this concept in some instances captures a notion of forward induction. A discussion of forward induction in such games is, however, outside the scope of the present paper. Note, though, Al-Najjar (1991) for an analysis of forward induction in a framework that is related to the present one.

11. PROOFS

The observations of the following two lemmas will turn out to be useful.

LEMMA 1. *Let G be a game with perfect information, and let Σ be a SB for G that is optimistic stable on X_E / on X. Then $x \in \Sigma(h)$ implies $x^k \in \Sigma(k)$ for any $k \geq h$.*

Proof. Let $y \in X^h$. Suppose $y^k \notin \Sigma(k)$ for some $k \geq h$. By (ES), there exist $k' \geq k$ and $x \in \Sigma(k')$ such that $u_i^{k'}(x) > u_i^{k'}(y)$ for $i = i^{k'}$. By (IS), $y \notin \Sigma(h)$ since $k' \geq h$. □

Lemma 1 implies the following corollary.

COROLLARY 4. *Let G be a game with perfect information, and let Σ be a SB for G that is optimistic stable on X_E / on X. Then $\pi \in \sigma^{\Sigma}(h)$ implies $\pi' \in \sigma^{\Sigma}(k)$ if k is reachable from h through π and $(h, \pi) = (k, \pi')$.*

LEMMA 2. *Let G be a game with perfect information, and let σ be a SB for G that is optimistic stable on Π / in the tree situation. Then $\pi \in \sigma(h)$ implies $\pi' \in \sigma(k)$ if k is reachable from h through π and $(h, \pi) = (k, \pi')$.*

Proof. Let $\varrho \in \Pi^h$. Suppose $\varrho' \notin \sigma(k)$ where k is reachable from h through π and $(h, \varrho) = (k, \varrho')$. By (ES), there exists k' and $\pi \in \sigma(k')$ such that k' / $P(k')$ is reachable from k through ϱ' and $U_i(k, \varrho') < U_i(k', \pi)$ for $i = i^{k'}$ / for $i = i^{P(k')}$. By (IS), $\varrho \notin \sigma(h)$ since k' / $P(k')$ is reachable from h through ϱ. □

Lemma 2 implies the following corollary.

COROLLARY 5. *Let G be a game with perfect information, and let σ be a non-empty valued SB for G that is optimistic stable on Π / in the tree situation. Then, for any $h \in H \backslash H_0$, $\sigma(h) = \{\pi^h(x) \mid x \in \Sigma^{\sigma}(h)\}$.*

Proof of Proposition 1. (a) (σ constrained to $H\backslash H_0$ is optimistic internally stable on Π.) By (IS) and Lemma 2, for any $h \in H\backslash H_0$, all paths in $\sigma(h)$ yields i^h the same payoff. The optimistic internal stability on Π follows from Lemma 2. (σ constrained to $H\backslash H_0$ is optimistic externally stable on Π.) By (ES), $\pi \in \Pi^h\backslash\sigma(h)$ implies that there exist $k \in H$ and $\varrho \in \sigma(k)$ such that $P(k)$ is reachable from h through π and $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^{P(k)}$. Since σ is non-empty valued, by (IS), there exists $\varrho' \in \sigma(P(k))$ such that $U_i(k, \varrho) \leqq U_i(P(k), \varrho')$ for $i = i^{P(k)}$. The optimistic external stability on Π follows since $U_i(h, \pi) < U_i(P(k), \varrho')$ for $i = i^{P(k)}$.
(b) (σ is non-empty valued.) Suppose $\sigma(h) = \emptyset$. Consider any $\pi \in \Pi^h$. By (ES), there exists $k \in H\backslash H_0$ reachable from h through π such that $\sigma(k) \neq \emptyset$. Let k be the first such node, and let $\varrho \in \sigma(k)$. Then, by (ES), $\varrho' \in \sigma(h)$ where $(h, \varrho') = (k, \varrho)$. ($\Sigma^\sigma$ is optimistic internally stable on X.) By (IS), for any $h \in H\backslash H_0$, all paths in $\sigma(h)$ yields i^h the same payoff. The optimistic internal stability on X follows from the definition of Σ^σ. (Σ^σ is optimistic externally stable on X.) If $x \in X^h\backslash\Sigma^\sigma(h)$, then for some $k \geq h$, $\pi^k(x) \notin \sigma(k)$. By (ES), there exists $k' \in H\backslash H_0$ reachable from k through $\pi^k(x)$ and $\varrho \in \sigma(k')$ such that $(u_i^{k'}(x) =)\ U_i(k, \pi^k(x)) < U_i(k', \varrho)$ for $i = i^{k'}$. Since σ is non-empty valued, by Corollary 5, there exists $y \in \Sigma^\sigma(k')$ with $\pi^{k'}(y) = \varrho$. The optimistic external stability on X follows since $k' \geq h$.
(c) (σ^Σ is non-empty valued.) Suppose $\Sigma(h) = \emptyset$. By Lemma 1, there exists $x \in X^h$ with $x^k \in \Sigma(k)$ for all $k \geq h$ such that $\Sigma(k) \neq \emptyset$. Then, by (ES), $x \in \Sigma(h)$. (σ^Σ is optimistic internally stable in the tree situation.) By (IS), for any $h \in H\backslash H_0$, all paths in $\sigma^\Sigma(h)$ yields i^h the same payoff. Suppose, for some $h \in H\backslash H_0$, $\pi \in \sigma^\Sigma(h)$, and there exist $k \in H$ and $\varrho \in \sigma^\Sigma(k)$ such that $P(k)$ is reachable from h through π and $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^{P(k)}$. Construct $y \in X^{P(k)}$ such that $(P(k), \pi^{P(k)}(y)) = (k, \varrho)$ and $y^{(P(k),\, a)} \in \Sigma(P(k), a)$ for all $a \in A^{P(k)}$. Since, by Corollary 4, for all $x \in \Sigma(P(k))$, $u_i^{P(k)}(x) = U_i(h, \pi) < U_i(k, \varrho) = u_i^{P(k)}(y)$ for $i = i^{P(k)}$, by (ES), $y \in \Sigma(P(k))$. However, this contradicts (IS). (σ^Σ is optimistic externally stable in the tree situation.) If $\pi \in \Pi^h\backslash\sigma^\Sigma(h)$, there exists $k \in H$ and $\varrho \in \sigma^\Sigma(k)$ such

that $P(k)$ is reachable from h through π and $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^{P(k)}$. Because otherwise, by (ES) and Lemma 1, there exists $x \in \Sigma(h)$ – with $\pi^h(x) = \pi$ and $x^{k'}$ chosen arbitrarily in $\Sigma(k')$ for all $k' \in H\backslash H_0$ such that k' is not, but $P(k')$ is reachable from h though π – implying that $\pi \in \sigma^{\Sigma}(h)$. □

Proof of Proposition 2. (c') ($\Sigma(0) \neq \emptyset$.) Follows from the proof of Proposition 1(c). (Σ is optimistic stable on X_E.) Suppose $x \in \Sigma(h)$ is not a SPE. For the class of games considered, there exists a one-shot deviation (w.l.o.g.) at h to $y \in X^h$ (where $y_i = x_i$ for $i \neq i^h$) that is profitable for i^h. By (IS), $y \notin \Sigma(h)$, but $y^k = x^k \in \Sigma(k)$ for all $k > h$. By (ES), there exists $z \in \Sigma(h)$ with $(u_i^h(x) <)\ u_i^h(y) < u_i^h(z)$ for $i = i^h$. However, this contradicts (IS) since $x \in \Sigma(h)$. Hence, for each $h \in H\backslash H_0$, $\Sigma(h) \subseteq X_E^h$, and Σ is optimistic stable on X_E.
(c") (σ^{Σ} is non-empty valued.) Follows from the definition of σ^{Σ} since $\Sigma(0) \neq \emptyset$. (σ^{Σ} is optimistic internally stable in the tree situation.) By (IS), for any $h \in H\backslash H_0$, all paths in $\sigma^{\Sigma}(h)$ yields i^h the same payoff. Suppose, for some $h \in H\backslash H_0$, $\pi \in \sigma^{\Sigma}(h)$, and there exist $k \in H$ and $\varrho \in \sigma^{\Sigma}(k)$ such that $P(k)$ is reachable from h through π and $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^{P(k)}$. For the class of games considered, there exists $y \in X_E^{P(k)}$ maximizing $u_i^{P(k)}(y')$, $i = i^{P(k)}$, over all $y' \in X^{P(k)}$ with $\pi^k(y') = \varrho$ and $y'^{(P(k),\, a)} = z(a) \in \Sigma(P(k), a)$ for all $a \in A^{P(k)}$. Since, by Corollary 4, for all $x \in \Sigma(P(k))$, $u_i^{P(k)}(x) = U_i(h, \pi) < U_i(k, \varrho) \leq u_i^{P(k)}(y)$ for $i = i^{P(k)}$, by (ES), $y \in \Sigma(P(k))$. However, this contradicts (IS). (σ^{Σ} is optimistic externally stable in the tree situation.) If $\pi \in \Pi_E^h\backslash\sigma^{\Sigma}(h)$, there exists $k \in H$ and $\varrho \in \sigma^{\Sigma}(k)$ such that $P(k)$ is reachable from h through π and $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^{P(k)}$. Because otherwise, by (ES) and Lemma 1, there exists $x \in \Sigma(h)$ – with $\pi^h(x) = \pi$ and $x^{k'}$ chosen arbitrarily in $\Sigma(k')$ for all $k' \in H\backslash H_0$ such that k' is not, but $P(k')$ is reachable from h though π – implying that $\pi \in \sigma^{\Sigma}(h)$. If $\pi \in \Pi^h\backslash\Pi_E^h$, there exists $k \in H$ such that $P(k)$ is reachable from h through π and, for all $\varrho \in \Pi_E^k$, $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^{P(k)}$. Because otherwise, there exists $x \in X_E^h$ – with

$\pi^h(x) = \pi$ and $x^{k'} \in X_E^{k'}$ satisfying $U_i(h, \pi) \gtreqless U_i(k', \pi^{k'}(x))$ for $i = i^{P(k')}$ for all $k' \in H\backslash H_0$ such that k' is not, but $P(k')$ is reachable from h though π – implying that $\pi \in \Pi_E^h$. The optimistic external stability in the tree situation follows since, for each $h \in H$, $\emptyset \neq \sigma^{\Sigma}(h) \subseteq \Pi_E^h$. □

Proof of Corollary 1. (a) Proposition 1. *(b)* Proposition 1. *(c)* Proposition 1(b) and 2(c′). □

Proof of Corollary 2. (a) Proposition 1. *(b)* Propositions 1 and 2. *(c)* Proposition 2(c′). □

Proof of Proposition 3. (i) Since a Strotz-Pollak path exists, there exists a SB σ that is optimistic stable on Π. By (ES), $\varrho \in \sigma(0)$ since there do not exist $h \in H\backslash H_0$ reachable from 0 through π and $\varrho' \in \Pi^h \supseteq \sigma(h)$ such that $U_i(\varrho) < U_i(h, \varrho')$ for $i = i^h$. By (IS), $U_i(\pi) = U_i(\varrho)$ for $i = i^0 = 1$. *(ii)* Define the SB σ on Π by, for each $h \in H\backslash H_0$, $\sigma(h) = \{\pi \in \Pi^h | \ \pi$ is optimal and time consistent$\}$. Let σ' be any SB that is optimistic stable on Π. If $\pi \in \sigma(h)$, there do not exist $k \in H\backslash H_0$ reachable from h through π and $\varrho \in \Pi^k \supseteq \sigma(k)$ such that $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^k$. Hence, σ satisfies (IS), and by (ES) of σ', for each $h \in H\backslash H_0$, $\sigma(h) \subseteq \sigma'(h)$. If $\pi \in \Pi^h \backslash \sigma(h)$, then there exist $k \in H\backslash H_0$ reachable from h through π and $\varrho \in \sigma(k) \subseteq \sigma'(k)$ such that $U_i(h, \pi) < U_i(k, \varrho)$ for $i = i^k$. Hence, σ satisfies (ES), and by (IS) of σ', for each $h \in H\backslash H_0$, $\sigma(h) \supseteq \sigma'(h)$. □

Proof of Proposition 4. Trivially, Σ defined by, for each $h \in H\backslash H_0$, $\Sigma(h) = X_E^h$, is the unique SB that is optimistic stable on X_E. The result follows from Propositions 1 and 2. □

Proof of Proposition 6. Follows directly from Definition 1 of Asheim (1990). □

Proof of Corollary 3. Proposition 1 of Asheim (1990). □

Proof of Proposition 7. Suppose that some symmetric revision-proof equilibrium y is not reconsideration-proof. Then there exists a symmetric equilibrium x' such that $u_1^0(y) < u_1^0(x')$. Since y is revision-proof, there exists a SB Σ, with $y \in \Sigma(0)$, that is optimistic stable on X. By (IS) of Σ, $x' \in X\backslash\Sigma(0)$. By (ES) of Σ, there exist $h \in H$ and $x \in \Sigma(h)$ such that $(u_i^h(y)) < u_i^h(x') < u_i^h(x)$ for $i = i^h$. This contradicts (IS) of Σ. Hence, if a revision-proof equilibrium is symmetric, then it is reconsideration-proof. The converse is trivial. □

Proof of Claim 1. (i) Consider the class of paths $\{\pi^j\}$, $j \in \mathbb{N}$, where π^j consists of the play of 1 j times, followed by π^0. The first j plays of π^j is called the stationary phase; i.e., the last play of $1 = \beta^0$ is part of the non-stationary phase. Construct the following strategy profile $\hat{x}$: Start with π^0. Deviation from the stationary phase of π^j is not punished; deviation from π^j when π^j prescribes the play of $\beta^{j'}$, $j' \in \{0, 1, 2, ...\}$, triggers $\pi^{j'}$. This determines a unique path $\hat{\pi}^h = \pi^h(\hat{x})$ in each subgame $h \in H$. Let σ be defined by $\sigma(h) = \{\hat{\pi}^h\}$ for all $h \in H$. Part (i) follows if σ is optimistic stable on Π. The (IS) of σ is trivial. In order to establish (ES), suppose $\varrho \in \Pi^h\backslash\sigma(h) = \Pi^h\backslash\{\hat{\pi}^h\}$ and there does not exist $k \in H$ reachable from h through ϱ such that $U_i(h, \varrho) < U_i(k, \hat{\pi}^k)$ for $i = i^k$. If ϱ differs from $\hat{\pi}^h$ only during the stationary phase of $\hat{\pi}^h$, then $U_i(h, \varrho) < U_i(h, \hat{\pi}^h) = U_i(k, \hat{\pi}^k)$ for $i = i^k$ where k is the last node at which ϱ differs from $\hat{\pi}^h$. Therefore, ϱ differs from $\hat{\pi}^h$ during the non-stationary phase of $\hat{\pi}^h$. Let the first deviation during the non-stationary phase occur when $\hat{\pi}^h$ prescribes the play of β^j. Let h' denote the node following this deviation such that the deviation occurs at $P(h')$. Let $(h, \varrho) = (h', \varrho')$. Then $\varrho' \in \Pi^{h'}\backslash\{\hat{\pi}^{h'}\}$ because otherwise $U_i(h', \varrho') < \beta \cdot U_i(P(h'), \hat{\pi}^{P(h')})$ for $i = i^{P(h')}$. Furthermore, there exists k' reachable from h' through ϱ' such that $U_i(k', \hat{\pi}^{k'}) = 1/(1+\beta)$ for $i = i^{k'}$. By induction, this leads to a contradiction since $\Sigma_{i'=i}^{\infty} U_{i'}(h, \varrho) \leqq 1/(1-\beta)$, where $i = i^h$, and establishes (ES) of σ.

(ii) Let y be a symmetric equilibrium. Suppose there exist a SB Σ, with

$y \in \Sigma(0)$, that is optimistic stable on X. By (IS) of Σ, $\hat{x} \in X \backslash \Sigma(0)$. By (ES) of Σ, there exist $h \in H$ and $x \in \Sigma(h)$ such that $(u_i^h(y)) < u_i^h(\hat{x}) < u_i^h(x)$ for $i = i^h$. This contradicts (IS) of Σ, establishing that no such optimistic stable Σ exists. □

Proof of Claim 2. (i) Construct the strategy profiles x^j, $j = 0, 1, 2$, as follows: Start with π^j. Deviation from $\pi^{j'}$ when $\pi^{j'}$ prescribes the play of 1 is not punished; deviation from $\pi^{j'}$ when $\pi^{j'}$ prescribes the play of 0 triggers π^2. Note that x^0, x^1, and x^2 are all SPEa, and that π^0, π^1, and π^2 are the only feasible paths that yield all players along the paths a payoff of at least 0.

(ii) Let σ^j, $j = 0, 1, 2$, be defined by $\sigma^j(h) = \{\pi^h(x^j)\}$ for all $h \in H$. Part (ii) follows if σ^j, $j = 0, 1, 2$, are optimistic stable on Π. The (IS) of σ^j is trivial. In order to establish (ES), assume $\varrho \in \Pi^h$ and there does not exist $k \in H$ reachable from h through ϱ such that $U_i(h, \varrho) < U_i(k, \pi^k(x^j))$ for $i = i^k$. Let h' be reachable from h through ϱ with $\pi^{h'}(x^j) = \pi^1$. Let $(h, \varrho) = (h', \varrho')$. Then $\varrho' = (1, 0, 0, a, a, \ldots)$ since π^1 yields player $i^{h'}$ the payoff 1. If $h' > h$, this in turn implies that $(h, \varrho) = (P(h'), (1, \varrho'))$ since $\pi^{P(h')}(x^j) = \pi^2$ yielding player $i^{P(h')}$ the payoff 0. What remains is to show that $\varrho'' = \pi^0$ if $\pi^{h''}(x^j) = x^0$ and $(h, \varrho) = (h'', \varrho'')$. Suppose k' is the first node reachable from h'' through ϱ'' at which ϱ'' differs from π^0; i.e., $i = i^{k'}$ plays 1 instead of 0. Then it follows from the above that $-1 = U_i(h'', \varrho'') < U_i(k', \pi^{k'}(x^j)) = 0$ for $i = i^{k'}$, which establishes (ES) of σ^j, $j = 0, 1, 2$.

(iii) Proof of Claim 1 (ii) with $\hat{x} = x^j$, $j \in \{0, 1, 2\}$. □

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