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**Linear Production with Transport of  
Products, Resources and Technology**

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# Linear production with transport of products, resources and technology

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## Abstract

This paper considers linear production situations with a finite number of production facilities, each with its own production technology and market prices. The economic agents control resources at the different facilities. Transport of resources, products and technology is restricted. Sufficient conditions for the corresponding TU-game to be balanced are discussed. This result extends the results of Owen (1975), Granot (1986) and Curiel, Derks, and Tijs (1989). An example is presented in some detail.

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# 1 Introduction and preliminaries

Owen (1975), Granot (1986) and Curiel, Derks, and Tijs (1989) analyzed linear production (LP) games. These are transferable utility (TU) games associated with the following type of situation : there is one facility at which a linear production technology is available. A finite number of agents control the resources needed for production. Prices of the products are fixed exogenously. In the corresponding LP-game, the worth of a coalition of agents is the maximal value of a bundle it can produce with the resources it controls. Sufficient conditions were given for the LP-game to be balanced, i.e. to have a non-empty core. Moreover, it was shown that if these conditions are satisfied, there is a core element of the LP-game which can be computed by solving only the dual of the linear program of the grand coalition. Computing the worths of the other coalitions is not necessary.

Koster (1990) analyzed a similar type of situation involving two facilities and transport of products, resources and technology from one facility to the other. The same conditions as above insure the corresponding LP-game is balanced.

This paper generalizes this setup. We consider situations with a finite number of facilities, each with its own linear production technology and exogenously fixed prices on products. We assume the markets are insatiable : every product manufactured or imported at a facility can be sold for its price at that facility. Furthermore, there are no capacity restraints, but at each facility only a finite amount of resources is available, which is controlled by the players. The facilities are public goods : usage of a facility by a coalition does not inhibit its use by another coalition. A linear cost is associated to the use of the technology of a facility. If these production sites were isolated, nothing new would be obtained. However, we allow transport of products, resources and technology between the facilities, along exogenously given routes. The possible transport routes for products, resources and technologies are represented by directed graphs. We assume there are linear losses during transport and linear transport costs. In the corresponding LP-game, each coalition of players tries to produce a bundle of maximal worth with the resources it controls, possibly transporting resources, products and technologies to take advantage of opportunities at every site.

In section 2 an example is presented in some detail to clarify the ideas. In section 3 the model is formally presented. Generalizing the results mentioned above, it is shown that under certain conditions on the control over resources, the LP-game is balanced, and that a core element can be found by solving only the dual of the linear program of the grand coalition.

## Preliminaries

A few concepts from cooperative game theory and graph theory are summarized below. A transferable utility game or *TU-game*  $(N, v)$  consists of a finite set  $N = \{1, \dots, n\}$  of players, and a *characteristic function*  $v : 2^N \rightarrow \mathbf{R}$ , with  $v(\emptyset) = 0$ . For a coalition  $S \subseteq N$ , the worth  $v(S)$  represents the economic revenue  $S$  can generate. The problem how to allocate the worth of the grand coalition  $N$  among the players is then addressed, taking into account that no coalition  $S$  will accept an allocation which allocates less to  $S$  than  $S$  could generate by seceding. This leads to the *core*  $C(v)$  of a TU-game  $(N, v)$ , defined

by

$$C(v) = \{x \in \mathbf{R}^N \mid \sum_{i \in N} x_i = v(N) \text{ and } \sum_{i \in S} x_i \geq v(S) \text{ for all } S \subseteq N\}.$$

A game with a non-empty core is called *balanced*.

A directed graph or *digraph*  $D$  on a set of vertices  $V$  is a subset of  $V \times V$ . An *arc* is an ordered pair  $(v, w) \in V \times V$ . For a vertex  $v \in V$ , we denote  $D(v) := \{w \in V \mid (v, w) \in D\}$  and  $D^{-1}(v) := \{u \in V \mid (u, v) \in D\}$ . If the digraph  $D$  represents transport possibilities,  $w \in D(v)$  is interpreted as ‘ $v$  can export to  $w$ ’, and  $u \in D^{-1}(v)$  is interpreted as ‘ $v$  can import from  $u$ ’. A digraph  $D$  on  $V$  is *reflexive* if  $(v, v) \in D$  for all  $v \in V$ . A digraph  $D$  on  $V$  is *transitive* if  $(u, v) \in D$  and  $(v, w) \in D$  imply  $(u, w) \in D$ .

## 2 An example

In this section, we present an example to clarify the ideas. Consider three facilities,  $f$ ,  $g$  and  $h$ , which allow linear production of five products  $p_1, \dots, p_5$ , using two resources  $r_1$  and  $r_2$ .

At facility  $f$  products  $p_1$  and  $p_2$  can be manufactured. Producing one unit of  $p_1$  requires one unit of  $r_1$  and three units of  $r_2$ , while manufacturing one unit of  $p_2$  requires no units of  $r_1$  and two units of  $r_2$ . We represent these technology constraints by a *technology matrix*  $A^f$  of which the first column corresponds to product  $p_1$ , the second column to  $p_2$  and the rows correspond in a similar way to the resources  $r_1, r_2$ . So,

$$A^f = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix},$$

and production of a bundle  $q = (q_1, q_2)$  of products at facility  $f$  requires the resource bundle  $A^f q = (q_1, 3q_1 + 2q_2)$ .

At facility  $g$  products  $p_3$  and  $p_4$  can be manufactured, so the technology matrix  $A^g$  has columns corresponding to  $p_3$  and  $p_4$  and rows corresponding to  $r_1$  and  $r_2$ . At facility  $h$  product  $p_5$  can be manufactured, so the column of  $A^h$  corresponds to  $p_5$  and the rows correspond to  $r_1$  and  $r_2$ . The technology coefficients in these matrices are as follows.

$$A^g = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}, \quad A^h = \begin{pmatrix} 2 \\ 1 \end{pmatrix}.$$

At each facility there is an exogenously given vector of prices at which products can be sold at that facility. The price vector at facility  $f$  is  $c^f = (4, 1, 4, 1, 4)$ , which means that at  $f$  a bundle  $q = (q_1, \dots, q_5)$  of products has worth  $4q_1 + q_2 + 4q_3 + q_4 + 4q_5$ , which we denote by  $\langle c^f, q \rangle$ . Implicitly we assume the markets are insatiable: everything produced can be sold. Similarly, the price vectors at  $g$  and  $h$  are  $c^g = (1, 3, 1, 3, 1)$  and  $c^h = (2, 1, 2, 1, 2)$  respectively. Note that a price is specified for each product at each facility. The structure of the price vectors and the similarity of the production matrices at  $f$  and  $g$  is due to products  $p_1, p_3$  and  $p_5$  being close substitutes produced at different facilities. A similar argument explains the equality in prices for products  $p_2$  and  $p_4$ . By abuse of notation, if  $P' \subseteq P = \{p_1, \dots, p_5\}$ ,  $q \in \mathbf{R}^{P'}$  and  $c \in \mathbf{R}^P$  is a price vector, we will write  $\langle c, q \rangle$  instead of  $\sum_{p \in P'} c_p q_p$ .

There are two players, called 1 and 2, each of whom owns a bundle of resources at each facility. Player 1 owns the bundles  $b^f(1) = (0, 5)$ ,  $b^g(1) = (3, 0)$  and  $b^h(1) = (2, 0)$  at  $f$ ,  $g$  and  $h$  respectively, while player 2 owns the bundles  $b^f(2) = (0, 3)$ ,  $b^g(2) = (1, 2)$  and  $b^h(2) = (0, 3)$  at  $f$ ,  $g$  and  $h$  respectively. The players can cooperate by pooling their resources.

Players can transport products, resources and technologies according to the following rules. Transport costs are zero. Resource transport is possible from  $g$  to  $f$  and  $h$ , and from  $f$  to  $h$ . Product transport is possible from  $f$  to  $h$  and vice versa and technology transport is possible from  $f$  to  $h$ . These transport possibilities are modeled by means of transport graphs (see figure 1).

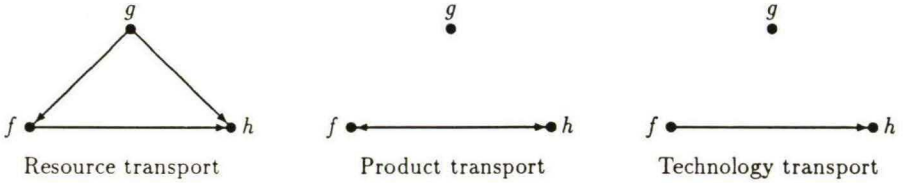


Figure 1: The transport graphs

We will first analyze the situation in which no transport is possible, and then gradually include transport possibilities. With such a LP situation a cooperative transferable utility (TU) game  $(N, v)$  is associated in the following way. The player set is  $N = \{1, 2\}$ , and a coalition  $S \subseteq N$  has value  $v(S)$  equal to the maximal revenue it can obtain through the sale of goods produced with resources the members of  $S$  own.

First, suppose there is no transport at all. In order to know what player 1 can obtain from production at facility  $f$ , we have to solve

$$\max\{c^f, q^f \mid A^f q^f \leq b^f(1), q^f \geq 0, q^f \in \mathbf{R}^{(p_1, p_2)}\} = 2.5.$$

Similarly, at  $g$  she can obtain

$$\max\{c^g, q^g \mid A^g q^g \leq b^g(1), q^g \geq 0, q^g \in \mathbf{R}^{(p_3, p_4)}\} = 0,$$

and at  $h$ ,

$$\max\{c^h, q^h \mid A^h q^h \leq b^h(1), q^h \geq 0, q^h \in \mathbf{R}^{(p_5)}\} = 0.$$

As there is no interaction between the three facilities, we can total these three revenues to obtain  $v(\{1\}) = 2.5$ . Similarly, we compute  $v(\{2\}) = 1.5 + 3 + 0 = 4.5$ , and  $v(\{1, 2\}) = 4 + 3 + 2 = 9$ . Because the resource bundles  $b^f(S)$  vary from coalition to coalition and the prices  $c^f$  are constant, it may be easier to compute the value of the dual programs, which have the same feasible region for all coalitions.

If players can transport *technology* along the routes depicted in figure 1, they can manufacture products at  $h$  using either the production techniques represented in the technology matrix  $A^f$  or the techniques represented in technology matrix  $A^h$ . Accordingly, we replace the technology matrix  $A^h$  by the matrix  $\bar{A}^h = (A^f, A^h)$ . The other

technology matrices are unchanged :  $\bar{A}^f = A^f$ ,  $\bar{A}^g = A^g$ . We are again in the same sort of situation as when there was no transport at all, except that now

$$q^h \in \mathbf{R}^{\{p_1, p_2, p_5\}} \quad \text{and} \quad \bar{A}^h = \begin{pmatrix} 1 & 0 & 2 \\ 3 & 2 & 1 \end{pmatrix}.$$

Denoting the corresponding characteristic function by  $v_T$ , we see that

$$v_T(\{1\}) = 2.5, \quad v_T(\{2\}) = 6, \quad v_T(\{1, 2\}) = 10.$$

Now, suppose players can also transport *products* along the routes depicted in figure 1. Then a good produced at facility  $h$  can be sold at either facility  $f$  or facility  $h$ , hence it can be sold for the maximum of the price at  $f$  and the price at  $h$ . The same goes for products manufactured at  $f$ , hence we replace the price vectors  $c^f$  and  $c^h$  with the (coordinatewise) maximum of  $c^f$  and  $c^h$ . Denote these new price vectors by  $\bar{c}^h = \bar{c}^f$ . The only change from the previous situation is that now  $\bar{c}^h = (4, 1, 4, 1, 4)$ . Denote the corresponding characteristic function by  $v_{TP}$ . Computing the worths of the coalitions yields

$$v_{TP}(\{1\}) = 2.5, \quad v_{TP}(\{2\}) = 6, \quad v_{TP}(\{1, 2\}) = 12.6.$$

Finally, suppose transport of resources is also possible. In contrast to the previous cases, this program cannot be solved by three separate linear programs. Denoting  $t^{f^1 f^2}$  the bundle of resources transported from a facility  $f^1$  to a facility  $f^2$ , one can see that the optimization problem player 1 now faces is

$$\begin{aligned} \max \quad & \langle \bar{c}^f, q^f \rangle + \langle \bar{c}^g, q^g \rangle + \langle \bar{c}^h, q^h \rangle \\ \text{s. t.} \quad & \bar{A}^f q^f \leq b^f(1) + t^{g^f} - t^{f^h}, \\ & \bar{A}^g q^g \leq b^g(1) - t^{g^f} - t^{g^h}, \\ & \bar{A}^h q^h \leq b^h(1) + t^{f^h} + t^{g^h}, \\ & q^f, q^g, q^h \geq 0, \\ & t^{f^h}, t^{g^f}, t^{g^h} \geq 0, \\ & t^{g^f} + t^{g^h} \leq b^g(1), \\ & t^{f^h} \leq b^f(1), \end{aligned}$$

which has the value 12. Hence, denoting the characteristic function corresponding to this situation by  $v_{TPR}$ , we see  $v_{TPR}(1) = 12$ . Player 2 faces a similar linear program with value  $v_{TPR}(2) = 8.5$  and coalition  $\{1, 2\}$  has to solve

$$\begin{aligned} \max \quad & \langle \bar{c}^f, q^f \rangle + \langle \bar{c}^g, q^g \rangle + \langle \bar{c}^h, q^h \rangle \\ \text{s. t.} \quad & \bar{A}^f q^f \leq b^f(1) + b^f(2) + t^{g^f} - t^{f^h}, \\ & \bar{A}^g q^g \leq b^g(1) + b^g(2) - t^{g^f} - t^{g^h}, \\ & \bar{A}^h q^h \leq b^h(1) + b^h(2) + t^{f^h} + t^{g^h}, \\ & q^f, q^g, q^h \geq 0, \\ & t^{f^h}, t^{g^f}, t^{g^h} \geq 0, \\ & t^{g^f} + t^{g^h} \leq b^g(1) + b^g(2), \\ & t^{f^h} \leq b^f(1) + b^f(2), \end{aligned}$$

which yields  $v_{TPR}(\{1, 2\}) = 21.4$ .



### 3 The Main Result

Consider a finite set  $N = \{1, \dots, n\}$  of players, who can make use of a finite set  $F$  of facilities to manufacture products. The (finite) set of resources is denoted by  $R$ , the (finite) set of all products by  $P$  and the subset of those products which a facility  $f \in F$  can manufacture by  $P^f$ . Assume the sets  $P^f$  are disjoint. Production is linear, i.e. there exist nonnegative numbers  $(a_{rp}^f)_{r \in R, p \in P^f}$  for each facility  $f$ , such that production of  $q_p$  units of product  $p \in P^f$  at facility  $f$ , requires  $a_{rp}^f q_p$  units of resource  $r$  as input. Hence, production of a bundle  $q \in \mathbf{R}_+^{P^f}$  of products at facility  $f$  requires the bundle of resources  $A^f q$ , where  $A^f = (a_{rp}^f)_{r \in R, p \in P^f}$  is the technology matrix at  $f$ . Assume that for each facility  $f$ , and for each product  $p \in P^f$ , there is at least one resource  $r$  with  $a_{rp}^f > 0$ . This means that no product can be created out of nothing.

At every facility  $f$ , there is an exogenously given price vector  $c^f \in \mathbf{R}^P$ . We assume the markets in which the products are sold to be insatiable, i.e. every product  $p$  produced at (or transported to) a facility  $f$  can be sold at  $f$  for the price  $c_p^f$ . Hence, with some abuse of notation, a bundle  $q \in \mathbf{R}^{P^f}$  is worth

$$\langle c^f, q \rangle = \sum_{p \in P^f} c_p^f q_p$$

at facility  $f$ .

The resources available are controlled by the players in the following way. For each coalition  $S \in 2^N \setminus \{\emptyset\}$  and each facility  $f$ , there is a bundle  $b^f(S) \in \mathbf{R}_+^R$  of resources  $S$  can use to produce at facility  $f$ . These resource bundles constitute a *resource game*  $(N, b^f)$  for each facility  $f$  and each resource  $r$ . Grouping the resources, one gets the function  $b^f$ . In the example presented in section 2, the resource games were additive :  $b_r^f(S) = \sum_{i \in S} b_r^f(i)$  for all coalitions  $S$ .

The facilities are connected by three transport networks. These are represented by reflexive directed graphs, one for product transport denoted by  $D_P$ , one for resource transport denoted by  $D_R$ , and one for technology transport, denoted by  $D_T$ . We interpret these graphs as follows :  $(f, f') \in D_R$  ( $D_P$ ,  $D_T$ ) denotes that resources, (products, technology) can be transported from facility  $f$  to facility  $f'$ . We assume the digraphs are reflexive because at a facility  $f$ , the technology and resources of  $f$  itself are always available and products produced at  $f$  can always be sold at  $f$ .

In the example, there were neither transport costs nor licence costs. In the general model, linear costs are associated to transport of resources and products. For  $(f, f') \in D_R$ , denote by  $G_r^{ff'}$  the cost of transporting one unit of resource  $r$  from  $f$  to  $f'$ . Similarly, for  $(f, f') \in D_P$  and  $p \in P^f$  the cost of transporting one unit of product  $p$  from  $f$  to  $f'$  is denoted by  $E_p^{ff'}$ . Assume that per unit of product  $p \in P^g$  produced at facility  $f \in D_T(g)$ , a licence fee  $L_p^{fg}$  has to be paid by the producer.

Moreover, suppose not everything sent arrives, and denote by  $\rho_r^{ff'}$  and  $\pi_p^{ff'}$  the fraction which arrives after transport of resource  $r$  and product  $p$  from  $f$  to  $f'$ , respectively.

A *linear production situation with transports*, in short, an *LPT*, is a collection  $N, F, R, P, (P^f, A^f, c^f, b^f)_{f \in F}, D_R, D_P, D_T, (E_p^{fg}, \pi_p^{fg})_{(f,g) \in D_P}, (G_r^{fg}, \rho_r^{fg})_{(f,g) \in D_R}, (L_p^{fg})_{(f,g) \in D_T}$  as described above.

With an LPT, we associate a (TU-)game  $(N, v)$  as follows :  $N$  is the set of agents and the worth  $v(S)$  of a coalition  $S \in 2^N$  is the maximal value of a production plan using the resources  $S$  controls. More precisely, a production plan specifies which products are to be made where, according to which technology and with which resources. A production plan for coalition  $S$  has to satisfy the condition that at no facility more resources are used than the resources available after resource transport. After transport of the manufactured products to markets where they are most profitable, they are sold. The revenue obtained by this sale minus the costs generated by transport, is the value of the production plan.

Taking transport possibilities into account, one can see that at a facility  $f$ , every product  $p$  in

$$\bar{P}^f := \bigcup_{g \in D_T^{-1}(f)} P^g$$

can be produced. Hence we replace  $P^f$  with  $\bar{P}^f$  and  $A^f$  with

$$\bar{A}^f := \bigcup_{g \in D_T^{-1}(f)} A^g.$$

Moreover, suppose  $(h, f) \in D_T$ . Then each unit of product  $p \in P^h \subseteq \bar{P}^f$  produced at facility  $f$  requires a licence fee of  $L_p^{hf}$  to be paid and generates  $c_p^g \pi_p^{fg} - E_p^{fg}$  when sold at a facility  $g \in D_P(f)$ . Denoting  $y * z$  the vector with coordinates

$$(y * z)_k = y_k z_k$$

for two vectors  $y$  and  $z$  of the same size, we see that we can replace  $c^f$  with

$$\bar{c}^f := \max_{g \in D_P(f)} (c^g * \pi^{fg} - E^{fg}) - \sum_{h \in D_T^{-1}(f)} L^{hf} e^{P^h},$$

where  $e^{P^h}$  is the vector defined by

$$e_p^{P^h} = \begin{cases} 1 & \text{if } p \in P^h, \\ 0 & \text{if } p \notin P^h. \end{cases}$$

Hence, production of a bundle  $\bar{q}^f \in \mathbf{R}^{\bar{P}^f}$  at a facility  $f$  requires the resource bundle  $\bar{A}^f \bar{q}^f$  and yields a net payoff of

$$\langle \bar{c}^f, \bar{q}^f \rangle = \sum_{p \in \bar{P}^f} \bar{c}_p^f \bar{q}_p^f.$$

If we denote by  $t^{hf}$  the resource bundle transported from  $h$  to  $f$ , we see that the cost of this transport is  $\langle G^{hf}, t^{hf} \rangle$  and that only the bundle  $t^{hf} * \rho^{hf}$  arrives at  $f$ .

Hence, the worth of a coalition  $S$  is

$$\begin{aligned}
v(S) &= \max \sum_{f \in F} \left[ \langle \bar{c}^f, \bar{q}^f \rangle - \sum_{h \in D_R^{-1}(f)} \langle G^{hf}, t^{hf} \rangle \right] \\
\text{s. t. } & \bar{A}^f \bar{q}^f \leq b^f(S) + \sum_{h \in D_R^{-1}(f)} (t^{hf} * \rho^{hf}) - \sum_{g \in D_R(f)} t^{fg} \text{ for all } f \in F, \\
& \sum_{g \in D_R(f)} t^{fg} \leq b^f(S) \text{ for all } f \in F, \\
& \bar{q}^f \geq 0 \text{ for all } f \in F, \\
& t^{fg} \geq 0 \text{ for all } (f, g) \in D_R.
\end{aligned}$$

**Theorem 1** If all resource games  $(N, b_r^f)$  in a linear production situation with transports are balanced, then the associated game  $(N, v)$  is also balanced and a core element can be computed by solving just one linear program.

**Proof :** The linear program is bounded for each coalition  $S$ . This is ensured by the assumption that no product can be manufactured without using resources. Moreover, producing nothing is feasible for all  $S$ , so the programs are all feasible. Hence the dual programs are bounded and feasible, and the values of the primal and dual program coincide for each  $S$ . The dual program for coalition  $S$  is

$$\begin{aligned}
& \min \sum_{f \in F} (\langle y^f, b^f(S) \rangle + \langle z^f, b^f(S) \rangle) \\
& \text{s. t. } \\
& y^f \bar{A}^f \geq \bar{c}^f \text{ for all } f \in F, \\
& y^f - y^g * \rho^{fg} + z^f \geq -G^{fg} \text{ for all } (f, g) \in D_R, \\
& y^f, z^f \geq 0 \text{ for all } f \in F.
\end{aligned}$$

Note that  $y^f, z^f \in \mathbf{R}^R$  for all  $f \in F$  and that the feasible region of the dual program is independent of  $S$ . For each facility  $f$  and each resource  $r$ ,  $(N, b_r^f)$  is balanced, hence take  $u_r^f \in \mathbf{R}^N$  a core element of  $(N, b_r^f)$ . Let  $(y^f)_{f \in F}, (z^f)_{z \in F}$  be an optimal vector of the dual program of the grand coalition  $N$ . This vector is a feasible vector for the dual programs for all coalitions. Define  $x \in \mathbf{R}^N$  by

$$x = \sum_{f \in F} \sum_{r \in R} (y_r^f + z_r^f) u_r^f.$$

Then for each coalition  $S$ ,

$$\begin{aligned}
\sum_{i \in S} x_i &= \sum_{f \in F} \sum_{r \in R} (y_r^f + z_r^f) \sum_{i \in S} u_{r,i}^f \\
&\geq \sum_{f \in F} \sum_{r \in R} (y_r^f + z_r^f) b_r^f(S) \\
&= \sum_{f \in F} (\langle y^f, b^f(S) \rangle + \langle z^f, b^f(S) \rangle) \\
&\geq v(S).
\end{aligned}$$

The first inequality holds because  $u_r^f$  is a core element of  $(N, b_r^f)$ , the second one because  $(y^f)_{f \in F}, (z^f)_{z \in F}$  is a feasible vector for the dual program of  $S$ . If  $S = N$  then these inequalities are equalities. Hence  $x$  is a core element of  $(N, v)$  and  $(N, v)$  is balanced.  $\square$

The optimal vector  $(y^f)_{f \in F}, (z^f)_{z \in F}$  can be interpreted economically as follows.  $y_r^f$  is a shadow price for resource  $r$  when used at facility  $f$ . The constraints of the dual programs imply that  $y_r^f + z_r^f \geq y_r^g \rho_r^{fg} - G_r^{fg}$  for each  $(f, g) \in D_R$  and each resource  $r$ . The right hand side can be seen as the shadow price for resource  $r$  when transported from facility  $f$  to facility  $g$ . By complementary slackness,  $y_r^f + z_r^f = y_r^g \rho_r^{fg} - G_r^{fg}$  if  $t_r^{fg} > 0$ . So, if any amount of resource  $r$  is exported from facility  $f$ , then  $y_r^f + z_r^f$  is the maximal shadow price for resource  $r$  among the facilities  $g \in D_R(f)$ . Hence  $z_r^f$  is the extra value of resource  $r$  when resource transport is allowed.

The converse of the theorem is not true in general, but if  $N, F, R, P, (P^f, b^f)_{f \in F}, D_R, D_P, D_T, (E^{fg}, \pi^{fg})_{(f,g) \in D_P}, (G^{fg}, \rho^{fg})_{(f,g) \in D_R}, (L^{fg})_{(f,g) \in D_T}$  are given and there exist an  $f \in F$  and an  $r \in R$  such that the resource game  $(N, b_r^f)$  is not balanced, production matrices  $(A^f)_{f \in F}$  and cost vectors  $(c^f)_{f \in F}$  can be constructed such that the LPT-game associated to this LPT is not balanced.

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