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INFORMATIONAL CONSTRAINTS AND THE OVERLAPPING GENERATIONS MODEL: FOLK AND ANTI-FOLK THEOREMS

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Abstract
It is well known that inter-generational transfers can be suppported by subgame perfect equilibria in Samuelson's consumption loan model if every generation is perfectly informed about past events. This paper relaxes the perfect information assumption minimally, and finds that transfers cannot be supported by pure-strategy sequential equilibria if the transferable commodity is finitely divisible. Mixed strategies allow transfers to be sustained, so that a version of the Folk theorem holds with informational constraints. However, these equilibria are not robust. If each agent's utility function is subjected to a small random perturbation, these mixed strategy equilibria unravel, and only the zero-transfer allocation survives as the unique rationalizable outcome. These results extend when we allow the commodity to be perfectly divisible, and also apply to a class of repeated games played by overlapping generations of players which includes the prisoners' dilemma. We suggest that money may play an informational role in this context, as a device for overcoming the boundedness of social memory.

Keywords: dynamic games of imperfect information, purification of mixed strategies, information and complexity, money and information.

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1. INTRODUCTION

Samuelson's (1958) overlapping generations model has rightly been described as "one of the most original and stimulating contributions to modern economic theory" (Shell, 1971). Consider the following simple version of this model of an infinitely lived economy. In each period, a single agent is born and lives for two periods. The young agent is endowed with two units of an indivisible and perishable consumption good - fish for example. The old agent is without any endowment. The economy begins at date one with a single old agent and a young agent. In each period, the young agent may consume both fish or she may give one to her mother. The old agent is passive, and has no choices to make. Agents are selfish, and prefer more consumption to less, but they would rather have the same total consumption spread out so as to not starve when old.

This economy has a unique Walrasian equilibrium, where each young agent consumes her endowment when young and starves when old. This equilibrium is autarchic; no trades are possible between generations since the old do not have any endowment to trade with. This equilibrium is also inefficient, and is (strictly) Pareto dominated by the allocation where every young agent gives one fish to her mother. The overlapping generations model hence provides an instance where the first welfare theorem fails to apply. [1]

Samuelson observed that the social contrivance of money could allow society to enforce the Pareto-efficient allocation. Let the old agent in the first period issue "money", and offer this money in exchange for one fish. Let every young agent accept this money in exchange for one fish. If any agent fails to secure this money by giving a fish to her mother, this agent will receive nothing when she is old - should she issue her own money, this will not be accepted. Accepting money and transferring the good is clearly a Nash equilibrium. The overlapping generations model has hence provided an extremely influential theory of money.

From the perspective of non-cooperative game theory, the monetary equilibrium provides what is perhaps the simplest example of "history-dependent" behavior in a dynamic game. Money is not really required for this story, and one can simply consider the dyanamic game where each young agent chooses from the set of possible transfers $\{0,1\}$. It is clear that the monetary equilibrium corresponds to a Nash equilibrium - each agent transfers 1 since by doing so she secures a transfer of 1, whilst she gets 0 if she she transfers 0. If every agent observes the entire history of past actions, the Nash equilibrium supporting the efficient allocation can be made subgame perfect, as Hammond (1975) observed. Although there are infinitely many ways of constructing such a perfect pure strategy equilibrium, perhaps the simplest is the strategy profile GRIM. Each agent transfers one fish to her mother if all previous agents have done so. If any agent fail to provide for her mother, all succeeding agents transfer zero. A more attractive subgame perfect equilibrium is the RESILIENT strategy profile, where agent punishes her mother if and only if she is a "deviant", where a deviant is one who has transferred zero when she should have given one fish to her mother. Each of these strategy profiles can be implemented by a simple two-state automotan, as Fig. 1 shows. The basic idea, that an overlapping generations structure allows finitely lived players to cooperate, has been extended to general repeated games with overlapping generations of players - see Cremer (1986), Kandori (1992), Salant (1991) and Smith (1992).

Although the strategy profiles GRIM and RESILIENT are very simple, and require only two states, this paper will show that they are nevertheless informationally very demanding, and require infinite memory. This may seem surprising since complexity measures (as in Abreu and Rubinstein (1988)) and informational requirements seem closely related - one strategy is defined to be simpler than another strategy if the former is measurable with respect to a coarser information partition, i.e. uses less information. Complexity measures are however controversial - for example, Lipman and Srivastava (1991) suggest that the number of states measure is too simplistic. We shall see in this paper that there is a sharp disjunction between the simplicity of a strategy (as in Abreu-Rubinstein) and the informational requirements of a strategy.

This paper proceeds by incorporating informational constraints directly. These informational limitations arise naturally in the overlapping generations context. The assumption that an agent who is born today has perfect information about all past events is patently unrealistic. Each agent has little direct information about the past, and what information she has is filtered through past generations. One may of course relax the perfect information assumption in a variety of ways. For example, social memory could be uniformly bounded, so that any agent has information only about the actions taken by the last m agents, where m is some natural number. Alternatively, m could be increasing over time (t), as our historians become more adept, although our lack of knowledge about the past, t-m, could also be increasing. The analysis of this paper applies to a very general type of imperfect information, which includes all the above possibilities. Perfect information implies that every agent is omniscient and knows the entire past. It also implies that any agent is ***omnifamous*** - each one of an infinity of future agents is fully informed about her actions. The analysis of this paper applies to any information structure where there are infinitely many agents who are not omnifamous. It also applies to a class of repeated games played by overlapping generations of players which includes the prisoners' dilemma.

The main results of our analysis are as follows. We first analyze pure strategy sequential equilibria, and show that informational constraints have dramatic implications - intergenerational transfers cannot be sustained, and outcomes must be Markovian, i.e. not history dependent. This is illustriated by a simple example, in section 2, and more formally in section 3. In section 4 we consider mixed strategies, with diametrically opposite results - randomized punishments can support the efficient allocation even if information is severely limited. These mixed strategies however turn out to be fragile. In section 5 we perturb the overlapping generations economy in the manner of Harsanyi (1973). All agents are ex ante identical, but each agent's utility function is subject to a small random shock, the realization of which is private information. We show that the randomized punishments which support transfers unravel, and the unique rationalizable outcome which survives is the one where every agent consumes her entire endowment. Such a striking result does not even require us to invoke the equilibrium assumption, and holds even if one considers sequentially rationalizable strategies. To our knowledge, this is the first time that the Harsanyi perturbation has been used to refine equilibria in an extensive form game.

Sections 2-5 assume that the transferable commodity is finitely divisible, so that each agent's action set is finite. Section 6 relaxes this assumption, and shows that the results are qualitatively the same. With perfect divisibility, there are pure strategy equilibria which support transfers, but once again these equilibria do not survive when we perturb the agent's utility function. These negative results lead us to consider alternative information structures. In section 7 we consider the possible informational role of money, as a device for overcoming the boundedness of social memory. We find that money may allow the efficient allocation to be suppported, but the equilibrium which supports this is necessarily GRIM, so that if any agent deviates from this equilibrium, the economy never returns to the efficient path. This provides a theory of crises of confidence based on the informational role of money. The final section concludes.

2. AN EXAMPLE

We present a simple example which illustrates the problem in supporting inter-generational transfers. As in the introduction each young agent is endowed with two fish. She may give one to her mother or none, so that the set of possible transfers she could make is $A = \{0,1\}$ (we assume that preferences are such that transferring 2 and consuming 0 is strictly dominated and may be ruled out). All agents have identical preferences, and the utility $u(a,a')$ where a is the transfer made by the agent when young and a' is the transfer received by her when old, satisfies:

$$u(0,1) > u(1,1) > u(0,0) > u(1,0) \tag{2.1}$$

Let $m = 2$, so that any agent only observes the last two actions taken. Let the first agent transfer 1 and the second agent simply match the action of the first agent. This implies that the first two agents will transfer 1. After t=3, every agent observes the actions of the two previous agents. Hence for $t \geq 2$, the agent's strategy s_t, specifies the action to be taken for every possible pair of actions last observed. We restrict attention to pure strategies, and to strategy profiles where $s_t = s_{t+1} = s$ for $t>1$, i.e. all agents after period 2 adopt the same strategy. Since m=2, there are 4 possible observed histories.

Since we are interested in the possibility of supporting transfers, the strategy must choose 1 after observing (1,1). To sustain this, we must punish a deviator; hence we must choose 0 after (1,0). With these determined, we can fill in the choices after (0,0) and (0,1) in four different ways. These allow four possible strategies, which we label I, II, III and IV. Table 1 shows what happens to a player after any of the four possible observed histories if every agent adopts the same strategy. Given any observed history, the strategy determines the action taken by the agent at date t, and thereby also the information of the agent at date t+1, which we call the the "induced history". The induced history and the strategy determine the "next-period action, i.e. the action taken at t+1. The actions at t and t+1 determine the utility of the agent at t. Table 1 shows why each of these four strategies fails to be sequentially rational, since there is one observed history at which the agent at t can deviate profitably, given that the agent at t+1 is following the strategy. Consider strategy I which is "nice", and chooses zero only after observing (1,0). This is not optimal if the observed history is (0,0), since the agent still gets 1 the next period if she chooses 0 rather than 1. II on the other hand is "grim", and chooses 0 at every state except (1,1). This is too grim; after (0,1), the agent prefers to choose 1 rather than 0. By choosing 1, she ensures that the history next period is (1,1), thereby ensuring a transfer to herself. III and IV are intermediate; they choose 1 after two of the four histories. They too fail, and interestingly, both fail to be optimal after the history (0,0). III calls the player to choose 0, but it is preferable to deviate to 1, since this ensures a transfer of 1 in the next period. IV chooses 1 after (0,0), but the player can deviate to 0 without being punished.

It might be conjectured that the problem arises because we have required every agent to choose the same strategy. However, this is not the case, and removing this restriction does not improve matters. Nor is the case of m=2 particularly special - the point generalizes to m = two million. The problem arises since each agent has better information about the past than her daughter. To support transfers we must reward "altruistic" behavior and punish selfish behavior. This requires that the agent at t+1 must vary her behavior in a non-trivial way depending upon the information she observes. However, the agent at t can manipulate the information that her daughter receives. Any pure strategy profile aimed at supporting transfers either turns out to be too grim or too nice, and any attempt to rectify one problem only brings in the other problem.

We turn now to a formal analysis of the model.

3. PURE STRATEGY EQUILIBRIA

We consider an economy over periods 1,2,... The t-th agent is born in period t, and is YOUNG in period t, and OLD in t+1. Her endowment is e when she is young and 0 when she is old. The young agent chooses an action from a finite set A, where $a \in A$ represents the amount the agent transfers to agent t-1. Given a, agent t's consumption at date t is (e-a). The old agent has no choices to make.

The finiteness of A can be justified since it is physically impossible to have an infinitely divisible commodity. In addition, indivisibilities may be enhanced for informational reasons: subsequent generations may not be able to observe t's transfer as finely as t can.

The agent's utility u, is a function, $u:A \times A \to R$, where u(a,a') is the agent's utility when she transfers a units to her mother and recieves a' units from her daughter, i.e. it is the utility from consuming (e-a) units when young and a' units when old. We assume that u(.) is decreasing in its first argument, the transfer made by the agent. If A has k elements, the agent's utility function can also be identified with a point in R^{2k}.

Although our focus is on Samuelson's consumption-loan model, all our results and analysis apply to a class of repeated games played by overlapping generations of players. Consider a stage game consisting of two roles, YOUNG and OLD; of associated action sets A and C; and payoff functions $v_y:A \times C \to R$, $v_o:C \times A \to R$. There is one player who is born in every period, and who lives for two periods, assuming role YOUNG in the first period, and role OLD in the second. Players seek to maximize the sum of payoffs over their lifetime, possibly disounted by a rate δ. Consider the class of stage games where role OLD has a strictly dominant action, which we label D. This class includes the repeated prisoners' dilemma played by overlapping generations of players, considered for example by Smith (1992). Obviously, every player must choose action D when old in any equilibrium. Given this, a player's lifetime utility depends only upon the action she takes when she is young and the action that the young player takes when she is old. Hence define the payoff function, $u:A \times A \to R$ as follows:

$$u(a,a') = v_y(a,D) + \delta v_o(D,a') \qquad (3.1)$$

Since OLD has a strictly dominant action, this must always be chosen, and we need consider only the action taken by the player when young. Hence all the results of this paper will apply to this class of games as well. Note that in this case (3.1) implies that u(.) is additively separable in its two arguments.

Our first assumption on preferences follows from the preceding discussion.

Assumption P1 $u(a,a')$ is either decreasing in its first argument or additively separable.

For this section and the next we make the following regularity assumption regarding u, that distinct action vectors yield different utility. Since the set of actions is finite this assumption will be satisfied almost always.

Assumption P2 Let $\underline{w}$, $\underline{z} \in A^2$. If $u(\underline{w}) = u(\underline{z})$, then $\underline{w} = \underline{z}$.

Note that P1 and P2 imply that $\underset{a \in A}{\operatorname{argmax}}\, u(a,a')$ is unique and independent of a'; label this action 0 - in the consumption loan model, this corresponds to transferring zero. In the prisoners' dilemma, 0 corresponds to "defect". P1 and P2 imply that this overlapping generations economy has a unique Markov equilibrium, where every agent chooses 0. (A Markov equilibrium is an infinite action sequence $\langle a_t \rangle$, where $u_t(a_t,a_{t+1}) = \max_{a \in A} u_t(a,a_{t+1})$).

Information

The focus of this paper is on relaxing the assumption that social memory is perfect, i.e. that each generation has all the information about the past that its predecessors had. There are a number of plausible ways in which one may introduce imperfect information. For instance, social memory may be uniformly bounded by a natural number m, which could be very large. The agent at date t has perfect information about the actions of all previous agents if $t-1 \leq m$. Otherwise, she is informed about the actions of the last m agents. Alternatively, the bound on social memory may not be uniform, and memory could increase over time. Agent t is informed about actions taken in the last m(t) periods. m(t) could be increasing, although forgetting also takes place, i.e. (t-1) - m(t) also increases sufficiently often. In this case social memory is not bounded above. The results of our paper apply to these examples of imperfect social memory. We generalize as follows.

The history at period t, h_t, is the sequence of preceding actions, $(a_1, a_2, ..., a_{t-1})$. The history at period 1 is the null history, h_1. H_t is the set of all possible histories at t, i.e. $H_t = A^{t-1}$. Consider a pair of agents, i,j with j>i, and define the following:

$a_i(h_j)$ is the i-th component of h_j, i.e. the action taken by player i.

$h_i(h_j)$ is the element of H_i which corresponds to h_j, i.e. it is the first i-1 components of h_j.

h_j/a_i is the history which results when the i-th component of h_j is subsitituted by a_i.

Let B_j be the information partition of agent j, with typical element b_j. B_j is a partition of H_j. If the history at j is h_j, agent j is informed that h_j belongs to $b_j(h_j)$. We call b_j the ***observed history***, and B_j the set of ***observable histories***.

If j>i, let B_{ji} denote the partiton that B_j induces on H_i, with typical element b_{ji}. If $b_j \in B_j$, we define $b_{ji}(b_j,i)$ as follows:

$b_{ji}(b_j,i) = \{h_i : \exists\ h_j \in b_j \text{ and } h_i = h_i(h_j)\}$

B_{ji} defines the information player j has about events prior to date i. Our first assumption says that if player j comes after player i, j has (weakly) less information about events prior to date i than i has.

Assumption M1 If j>i, B_{ji} is (weakly) a coarsening of B_i.

Our second assumption limits the information that players have about the past. Players are assumed to be omniscient and know the entire past if the game has perfect information. It is more fruitful to invert this perspective - under perfect information, for any player i, each one of an infinity of succeeding players is informed about her actions, i.e. i is, to coin a term, ***omnifamous.*** To define this, we first define the notion of being uninformed about the actions of a previous player.

Definition 3.1 Let j>i. Player j is *uninformed about player i* if, $\forall\ h_j \in H_j$, $\forall\ a_i, a_i' \in A$:

$b_j(h_j/a_i) = b_j(h_j/a_i')$

Let $\mathcal{U}(i)$ denote the set of players with index greater than i who are uninformed about player i.

Definition 3.2 Agent i is *not omnifamous* if there exists an agent j, j>i, such that $j \in \mathcal{U}(i)$.

Agent i is ***omnifamous*** if every succeeding agent is informed of her actions. Agent i is ***not omnifamous*** if there is some succeeding agent who is not informed about i's actions. It seems reasonable to assume that every agent is not omnifamous. We need a milder assumption, under which most agents could well be omnifamous.

Assumption M2 There are infinitely many agents who are not omnifamous.

The following equivalence relation on B_j will play an important role in our analysis:

Definition 3.3 Given a set B_j, and $i < j$, $\sim_i$ is an equivalence relation on B_j such that for any b_j, $b'_j \in B_j$:

$b_j \sim_i b'_j$ iff $\exists\, a_i \in A$: $b_j/a_i = b'_j$

b_j and b'_j are i-equivalent if the information regarding the actions of every agent except agent i is the same. Note that for every pair (j,i), with i<j, the equivalence relation $\sim_i$ is defined on the set B_j. If j is uninformed about i, b_j and b'_j are i-equivalent only if they are identical.

Strategies

A pure strategy for agent t is a function $s_t : B_t \longrightarrow A$, i.e. it is a function which is measurable with respect to the partition B_t. Agent t's pure strategy set, S_t, is the set of all such functions. Given any b_t in B_t, we write $s_t(b_t)$ for the element of A which is induced by b_t when s_t is played. A strategy profile, **s**, is a infinite sequence $\langle s_t \rangle$ where $s_t \in S_t$ $\forall t$.

Given any observed history, b_t, an action by agent t, a_t, induces an observed history for t+1, which we write as

$b_{t+1}(a_t,b_t)$, or simply as (a_t,b_t). Any pure strategy also defines a function from the set of observable histories at t, B_t, to the set of observable histories at t+1, B_{t+1}. Write $b_{t+1}(s_t,b_t)$ or simply (s_t,b_t) for $(s_t(b_t),b_t)$. Given a strategy profile **s**, the ***realized history at t***, $b_t(\mathbf{s})$, is the element of B_t which is induced when **s** is played. Similarly, given **s**, $t>\tau$, and an observed history b_τ, the ***realized history at t given b_τ***, $b_t(\mathbf{s},b_\tau)$, is the element of B_t which is induced when **s** is played after b_τ.

Write s_{t+1}/b_t for the map from A to itself which is defined by the pair b_t and s_{t+1}. The interpretation is that if agent t takes action $a \in A$ after history b_t, agent t+1 takes action $s_{t+1}(a,b_t)$. This map defines agent t's utility from action a after history b_t as follows:

$$u_t(a,s_{t+1}/b_t) = u\,[a,\ s_{t+1}(b_{t+1}(a,b_t)] \qquad (3.2)$$

The agent's utility from the strategy s_t, given s_{t+1} and b_t is:

$$u(s_t,s_{t+1}/b_t) = u\,[s_t(b_t),\ s_{t+1}(b_{t+1}(s_t,b_t)] \qquad (3.3)$$

Observe that agent t's utility is affected directly only by her own action and the action of agent t+1. Agent t's utility is affected indirectly by the actions of agents t-i, i=1,2,...t-1, since these actions determine the observed history. Agent t's utility is unaffected by the actions of agents at dates after t+1.

A strategy profile **s** is a sequentially rational equilibrium (abbreviated to equilibrium henceforth) if $\forall t$, $\forall b_t \in B_t$,

$$u(s_t,s_{t+1}/b_t) \geq u(a,s_{t+1}/b_t) \ \forall\ a \in A \qquad (3.4)$$

Remark: Our equilibrium definition is remarkably simple.

We do not have to invoke any beliefs regarding past actions, as is usual in games of imperfect information, since past actions do not directly affect current or future utility. Further, given assumption M1, the information partition of agent t regarding the past is always finer than the information partitions of agent t+k regarding the past (i.e. events before t).

We now define the following notion of measurability of a strategy with respect to a partition, which will play an important role in the proof of our theorem in this section.

Definition 3.4 Given j>i, and the equivalence relation $\sim_i$ on B_j, s_j is ***measurable with respect to*** $\sim_i$ if :

$b_j \sim_i b'_j \Rightarrow s_j(b_j) = s_j(b'_j)$

We now state the main result of this section:

Theorem 1. The overlapping generations game has a unique pure strategy equilibrium where each agent chooses 0.

Proof By assumption M2 we can find an agent with an arbitrarily large index i who is not omnifamous. Hence $\exists$ j>i such that $j \in \mathcal{U}(i)$. We show, by backward induction, that for all t, $i < t \leq j$, that s_t does not condition on agent i's behavior. More precisely, we show that s_t is measurable w.r.t. $\sim_i$, for all t in this range.

i) s_j is measurable w.r.t. $\sim_i$: This follows since $j \in \mathcal{U}(i)$, and hence $\sim_i$ induces the trivial (finest possible) partition of B_j, where each set in the partition is a singleton set.

ii)Let $i < t < j$. If s_{t+1} is measurable w.r.t. $\sim_i$, then s_t is measurable w.r.t. $\sim_i$.

Note first that by M1, if $b_t \sim_i b'_t$ and $a \in A$, then:

$b_{t+1}(a, b_t) \sim_i b_{t+1}(a, b'_t)$

i.e. if player t takes the same action a at two i-equivalent observed histories b_t and b'_t, the resulting observed histories for player t+1 are also i-equivalent.

We claim that if s_{t+1} is measurable w.r.t. $\sim_i$ and $b_t \sim_i b'_t$, then:

$$u_t(s_t, s_{t+1} / b_t) = u_t(s_t, s_{t+1} / b'_t) \quad (3.5)$$

Suppose not. Let $u_t(s_t, s_{t+1} / b_t) > u_t(s_t, s_{t+1} / b'_t)$. Then $s_t(b'_t)$ is not optimal, since by choosing the action $s_t(b_t)$, agent t ensures the history $b_{t+1}(s_t(b_t), b'_t)$. Since $b'_t \sim_i b_t$,

$$b_{t+1}(s_t(b_t), b'_t) \sim_i b_{t+1}(s_t(b_t), b_t)$$
(3.6)

Since s_{t+1} is measurable w.r.t. $\sim_i$, agent t ensures that t+1 takes the same action, and hence the payoff $u_t(s_t, s_{t+1}/b_t)$. Hence if $<s_t>$ is a equilibrium, (3.3) must hold. If (3.3) applies, Assumption P2 implies that $s_t(b_t)$ cannot be distinct from $s_t(b'_t)$. Hence s_t is measurable w.r.t. $\sim_i$.

(i) and (ii) together imply that if agent i is not omnifamous, agent i+1's actions do not depend upon i's actions. From assumptions P1 and P2, agent i must choose 0 irrespective of the observed history. By backward induction it now follows that $\forall\, t<i$, $s_t = 0$ irrespective of the observed history. □

Remark 1: The negative result can be generalized, to the case of extensive form rationalizable strategies, provided that one requires that players have "point-beliefs" about the pure strategies played by succeeding agents. However, the point belief assumption is hardly plausible outside an equilibrium context, and we defer the analysis of rationalizable strategies (to section 5), after analyzing mixed strategies.

We offer the following intuition for Theorem 1. If altruistic behavior is to be supported, agents must vary their behavior depending upon the observed history. Since the strategy profile is pure, this implies that the agent's utility under the strategy profile ***differs*** depending upon the history they have observed. However, each agent has better information about the past than the succeeding agent, and this allows her to manipulate the information that is transmitted. The only way in which this informational advantage can be nullified is if the strategy profile does not condition upon information at all. This intuition suggests that mixed strategies may be able to overcome the problem, and we turn to these.

4. MIXED STRATEGIES

Theorem 1 applies to pure strategies. In this section we ask, is it possible to support efficient outcomes by the use of randomized punishments? We find that the answer is yes, and that in fact one can prove a version of the Folk theorem even for economies where information is severely limited.

Let $\operatorname{argmax}_{a\in A} u(a,a) := 1$ be the ***efficient action***. By assumption P2, this is unique. The following assumption is made for convenience; otherwise the results of this section hold trivially.

Assumption P3 $1 \neq 0$

The following lemma is straight-forward:

Lemma 4.1 $u(0,1) > u(1,1) > u(0,0) > u(1,0)$

Proof The first and last inequalities are implied by P1 and P2. The second inequality is implied by P3. □

In the consumption loan model, if u(.) is increasing in its second argument (i.e. the transfer received by the agent), u(0,0) is the agent's individually rational payoff, and hence the interval [u(0,0),u(1,1] is the set of individually rational and feasible payoffs. The following theorem is therefore similar to a Folk theorem with informational constraints.

Theorem 2. If each agent observes the action taken by the previous agent, the efficient path where every agent chooses 1 can be supported as an equilibrium by the use of randomized punishments. Any payoff between the efficient payoff u(1,1) and u(0,0) is an equilibrium payoff.

Proof We construct a class of equilibrium strategies which randomize between the actions 0 and 1, and the randomization probability depends only upon the action taken by the preceding agent. Let $p^1 \leq 1$ and $p^0 \geq 0$ be numbers satisfying:

$$p^1 = \frac{u(0,0) - u(1,0)}{u(1,1) - u(1,0)} + \frac{u(0,1) - u(0,0)}{u(1,1) - u(0,0)} p^0 \qquad (4.1)$$

Lemma 4.1 ensures that p^0 and p^1 lie in the unit interval. Let p^1 (resp. p^0) represent the local strategy of choosing action 1 with probability p^1 (resp. p^0) and action 0 with probability $(1-p^1)$ (resp. $(1-p^0)$. Define the strategy profile $\langle s_t \rangle$ as follows:

$s_t = p^1$ if $t = 1$ or $a_{t-1} = 1$

$= p^0$ if $t > 1$ and $a_{t-1} \neq 1$

Since s_{t+1} does not condition upon a_{t-1}, u_t is independent of a_{t-1}. Hence for any observed history b_t, we have:

$$u_t(1, s_{t+1}/b_t) = p^1\, u(1,1) + (1-p^1)\, u(1,0) \tag{4.2}$$

If $a \neq 1$,

$$u_t(a, s_{t+1}/b_t) = p^0\, u(a,1) + (1-p^0)\, u(0,0) \tag{4.3}$$

By assumption P1, the expression in (4.3) is maximized at $a = 0$. It is easy to verify that the maximized value, $u_t(0, s_{t+1}/b_t)$, equals $u_t(1, s_{t+1}/b_t)$ given (4.1). This verifies that $<s_t>$ is an equilibrium.

The efficient outcome is supported if we select $p^1 = 1$, with the corresponding p^0, which we label p^{0*}, given by equation (4.1). The payoff to any player is $u(1,1)$. Similarly, the equilibrium with the lowest payoff in this class has $p^0 = 0$, with payoff $u(0,0)$. Since p^1 and p^0 can be continuously varied in this range, we can support any payoff in the interval $[u(0,0), u(1,1]$. □

Note that if $p^1 = 1$, the equilibrium outcome path is pure, as well as efficient. Further, if any player deviates from this path, the economy reverts to the efficient path after a finite number of periods with probability one. This strategy profile, which we call MIXED1, can be implemented by a two-state automaton, as Fig 1. shows. MIXED1 is therefore as complex as GRIM or RESILIENT, but it requires only one period memory.

Any deviant from MIXED1 is punished only weakly, and does not suffer a loss of utility. If every agent observe the actions of the last two agents, we can construct a strategy profile, MIXED2, which makes any deviant strictly worse off. Fig 1 shows that MIXED2 has three states, and is hence more complex than GRIM or RESILIENT, by the number of states measure. Nevertheless, MIXED2 is an equilibrium with two-period memory, whereas GRIM and RESILIENT require infinite memory.

In MIXED1 and MIXED2, and indeed in any informationally economical equilibrium supporting the efficient outcome, players must take different actions at different observed histories, and the equilibrium has to be constructed so that this player is indifferent between these actions. We need to use mixed strategies for this purpose - pure strategies are either too nice or too grim. Since randomized punishments can be fine tuned to be just right, it is possible to induce an agent take different actions at different information sets. Nevertheless, this knife-edge balance is unstable, as we shall see in the next section.

The idea of theorem 2 could be generalized to prove an informationally economical Folk theorem for a class of repeated games played by overlapping generations of players which is more general than that considered in this paper, thus generalizing the results in Kandori (1992). However, we prefer, in this paper, to focus on the robustness of theorem 2.

5. THE PERTURBED GAME

Are the mixed strategy equilibrium which support altruistic behavior robust? In this section we ask whether these equilibria survive when each player's payoff function is perturbed, and this perturbation is private information, in the manner of Harsanyi (1973). We adapt the framework of van Damme (1991, chapter 5) to our set up, which is of an extensive form game.

Index agents by t as before. Recall that each agent's action set, A, has k elements. Let R . Let X_t be a random vector with values in a set Z in R^{2k}.

$Z = \{x \in R^{2k}: -c^i \le x^i \le c^i,\ i = 1,2,..,2k\},\ c^i > 0\ \forall\ i$

Let μ be a probability measure on Z.

The ***disturbed overlapping generations game*** is as follows:

i) Nature chooses an outcome x_t of X_t for each agent t, independently, and by the probability measure μ.

ii) Agent t, t=1,2,..., gets to know the outcome x_t, and nothing else.

iii) Agents 1 chooses an element of ΔA, having observed x_1. Each succeeding agent observes x_t, and the observed history b_t, and chooses an element of ΔA.

iv) If a_t and a_{t+1} are chosen, the payoff to the t-th agent is given by:

$$u_t(a_t, a_{t+1}) = v(a_t, a_{t+1}) + x_t(a_t, a_{t+1}) \quad (5.1)$$

(5.1) shows that the payoff to agent t from any action pair depends upon two components. The first, v(.), is common to all agents, whereas the second, x_t, is private information.

An Informal Argument

Before proceeding with our formal argument, it may be useful to provide an intuitive argument for our main result of this section. Readers who prefer to skip such preliminaries should proceed directly to the sub-section headed "The Formal Analysis".

Our main theorem implies that the disturbed consumption loan model has a unique equilibrium where each agent transfers zero. Some intuition for the main point of the theorem can be gained by considering why the mixed strategy equilibrium of the previous section cannot be approximated in the disturbed game. We simplify the game by allowing only two actions, 0 and 1. The mixed strategy was:

$(1,1) \longrightarrow 1$

$(0,1) \longrightarrow 1$

$(1,0) \longrightarrow 0$

$(0,0) \longrightarrow 1$ with probability p^{0*}, 0 with probability $(1-p^{0*})$

To keep things simple, we perturb only one payoff, the payoff $u(0,1)$, so that the payoffs of agent t are:

$u_t(0,1) = v(0,1) + x_t$

$u_t(a,a') = v(a,a')$ for all other (a,a') in A^2 (5.2)

where x_t is i.i.d. on $[-c,c]$ with a uniform density.

Let the last observed history be $(a,0)$, where a is either 1 or 0. Consider agent t's payoff from the two actions, 1 and 0:

$$u_t(1,s_{t+1}/(a,0) = v(1,1) \quad (5.3)$$

$$u_t(0,s_{t+1}/(a,0) = p^{0*}u_t(0,1) + (1-p^{0*})\, v(1,1)$$

(5.4)

The difference in payoff between the two actions, 1 and 0, is:

$$u_t(1,s_{t+1}/(a,0) - u_t(0,s_{t+1}/(a,0) = x_t\, p^{0*}$$

(5.5)

(5.6) shows that agent t has a unique best response unless $x_t = 0$, i.e. for almost all realizations of x_t. Further, t will choose 1 with probability one if $x_t < 0$, and 0 with probability one if $x_t > 0$. Hence player t-1 should expect t to choose 1 with probability 1/2, and to choose 0 with probability 1/2, if t-1 induces the history (0,0) or the history (1,0). In other words, the aggregate strategy, $s_t(0,0) = s_t(1,0)$, since $s_t(a,0)$ is uniquely determined by (5.5), no matter whether a = 1 or a = 0. However, the strategy requires t to take ***different*** actions at (0,0) and (1,0). Hence, the strategy cannot be an equilibrium.

The basic problem with the mixed strategy equilibrium is that agent t is required to take different (probability distributions over) actions at different information sets. Since future agents cannot distinguish these information sets, agent t must be induced to be indifferent between these actions. Once payoffs are perturbed, these indifferences cannot persist, since for almost all realizations of the private information, the agent has a unique best action. Consequently, the actions of the agent must depend only upon the private information, and not upon the observed history.

We proceed to a formal analysis of the model.

The Formal Analysis

A behavior strategy for agent t is now a Borel measurable function, $\sigma_t : B_t \times Z \longrightarrow \Delta A$. Two behavior strategies of agent t are ***equivalent*** if, for every b_t in B_t, they differ on a subset of Z of μ-measure zero. Let $s_t : B_t \longrightarrow \Delta A$, and let S_t be the set of all such functions s_t. If σ_t is a behavior strategy, σ_t induces an element s_t of S_t, defined by $s_t := \int \sigma_t d\mu$. Call s_t the aggregate of σ_t. If player t plays σ_t, to an outside observer, and to all players $\tau<t$, it seems as though t plays the aggregate s_t of σ_t. Let Σ_t be the set of behavior strategies for player t, and let S_t denote the corresponding aggregates. A behavior strategy profile, $<\sigma>$, is a sequence of behavior strategies, $<\sigma_t>$. Associated with this is the sequence of aggregates, $<s_t>$.

Given σ_t and $b_t \in B_t$, write $\sigma_t(b_t)$ for the restriction of σ_t to $\{b_t\}XZ$, and write $s_t(b_t)$ for the associated aggregate. Given an aggregate for player t+1, s_{t+1}, and b_t, write s_{t+1}/b_t for the map from A to ΔA which is defined by the pair b_t and s_{t+1}. The interpretation is that if agent t takes action $a \in A$ after history b_t, agent t+1 is expected to take actions in A by the probability measure $s_{t+1}(a,b_t)$.

At this point it is convenient to drop time subscripts, since all agents are exante identical. Let $x \in Z$ be a realization of private information for agent at an arbitrary date t. The agent's utility function is hence $u(a,a',x)$, where a is the agent's own action, and a' is the action taken by agent t+1. Let $\rho: A \longrightarrow \Delta A$ be an arbitrary function, and let $\rho(a'/a)$ denote the probability of action a' given a. The interpretation is that ρ could be something similar to s_{t+1}/b_t. Given any $a \in A$, define:

$$u(a,\rho,x) := \sum_{a' \in A} u(a,a'x)\rho(a'/a) \qquad (5.6)$$

$u(a,\rho,x)$ denotes the payoff to t from action a conditional on the realization of private information x, given that ρ is the aggregate strategy adopted by t+1 after some observed history. Define the following:

$$\beta(\rho,x) := \{a'\in A: u(a',\rho,x) = \max_{a\in A} u(a,\rho,x)\}$$

$$\tilde{\beta}(\rho,x) := \{\alpha\in\Delta A: u(\alpha,\rho,x) = \max_{a\in A} u(a,\rho,x)\}$$

$$\beta(\rho) := \{\theta: Z \longrightarrow \Delta A : \theta(x) \in \tilde{\beta}(\rho,x)\}$$

$$\zeta^{a}(\rho) := \{x\in Z : a \in \beta(\rho,x)\}$$

We now define the equilibrium concepts for the disturbed overlapping generations game. The first notion is that of a sequential equilibrium:

Definition 5.1 $<\sigma_t>$ is a sequential equilibrium, if for every t, $\forall\ b_t \in B_t$:

$$\sigma_t(b_t) \in \beta(s_{t+1}/b_t)$$

The equilibrium requirement implies that each player's behavior strategy is a best response to the strategy adopted by the next player, and implies that players have common beliefs about the equilibrium to be played. Our main theorem however can be proved without invoking the equilibrium assumption, and only requires that strategies are ***sequentially rationalizable.*** We define this concept for our specific game; for a detailed discussion of rationalizability in extensive form games, see Pearce (1984).

Let $\Theta_t \subseteq \Sigma_t$ be a set of behavior strategies, and let $<\Theta_t>$ denote a sequence of such sets. Let Q_t be the set of aggregates corresponding to Θ_t, i.e.:

$$Q_t := \{s_t\in S_t : \exists\ \sigma_t\in\Theta_t,\ s_t = \int\sigma_t d\mu\}$$

Definition 5.2 The sequence $<\Theta_t>$, $\Theta_t \subseteq \Sigma_t$, has the *sequential best response property* if $\forall$ t:

$$\sigma_t \in \Theta_t \Rightarrow \exists\, q_{t+1} \in Q_{t+1} : \forall b_t \in B_t,\ \sigma_t(b_t) \in \beta(q_{t+1}/b_t)$$

Definition 5.3 $<\Theta_t>$ is *sequentially rationalizable* if it is the maximal sequence with the sequential best response property, i.e. if $<\Theta_t'>$ has the sequential best reponse property, then $\Theta_t' \subseteq \Theta_t$ $\forall$ t. The set of sequentially rationalizable strategy profiles is the infinite product $\Theta := \overset{\infty}{\underset{t=1}{X}} \Theta_t$.

Henceforth we use the notation Θ_t to denote the set of sequentially rationalizable strategies for player t, and Θ for the set of sequentially rationalizable strategy profiles.

Note that if $<\sigma_t>$ is a sequential equilibrium, $<\sigma_t>$ is sequentially rationalizable, while the converse is not true in general. Rationalizability usually results in a proliferation of outcomes. For example, in the consumption loan model with infinite memory, the outcome path 1,0,0,0... cannot be an equilibrium path whereas it can be rationalized. In a rationalizable path, a player may get less than her individually rational payoff, since expectations may not be fulfilled.

The following assumptions on preferences and the distribution of private information replace assumptions P1 and P2 of section 3.

Assumption D1 u is either decreasing in its first argument or additively separable in its first two arguments for all realizations of x.

Assumption D2 μ is absolutely continuous with respect to

Lebsegue measure.

Our first lemma for this section follows from D2:

Lemma 5.1 Let a, a' $\in$ A. If a $\neq$ a', $\xi^{a}(\rho) \cap \xi^{a'}(\rho)$ has μ-measure zero.

Proof $\xi^{a}(\rho) \cap \xi^{a'}(\rho) = \{x \in Z: u(a,\rho,x) = u(a',\rho,x)\}$. If x $\in \xi^{a}(\rho) \cap \xi^{a'}(\rho)$, then:

$$\sum_{a''\in A} u(a,a'',x)\rho(a''/a) = \sum_{a''\in A} u(a',a'',x)\rho(a''/a') \qquad (5.7)$$

(5.7) defines a hyperplane of Lebesgue measure zero, and hence of μ-measure zero. □

Our second lemma follows from D1 and D2. If $\alpha \in \Delta A$, write $\beta(\alpha)$ to denote $\beta(\rho)$ for the case when ρ is the constant function α.

Lemma 5.2 If α, $\alpha' \in \Delta A$, $\beta(\alpha,x) = \beta(\alpha',x) := \gamma(x)$ for almost all x. $\gamma(x)$ can be chosen to be single element of A.

Proof If u is decreasing in its first argument, then it is strictly decreasing for almost all realizations of x, i.e. $\beta(\alpha,x) = \beta(\alpha',x) = \{0\}$ for almost all x. If u is addivitely separable in its first two arguments, $\beta(\alpha,x) = \beta(\alpha',x)$. Lemma 5.1 implies that these sets are singleton for almost all x. □

Since $\gamma(x)$ is a singleton set, $\gamma: Z \rightarrow A$ defines a Markov strategy. More generally, a ***Markov strategy*** is a function $\omega_t: Z \rightarrow \Delta A$, with aggregate $w_t = \int \omega_t d\mu$. Let Ω denote the set of Markov strategies for any player, a set which is obviously time-invariant. Suppose that every player is restricted to playing a Markov strategy, i.e. the set Ω rather than the set Σ_t. If Ω^* is a subset of Ω, we call Ω^* ***Markov rationalizable*** if the constant sequence $\langle\Omega^*\rangle$ satisfies the best response property, and it is the maximal subset of Ω which does so. The lemmata 5.1 and 5.2 imply that the overlapping generations economy has an essentially unique Markov rationalizable sequence, i.e. all elements in Ω^* are equivalent to γ. It also implies that a Markov rationalizable sequence is also a Markov equilibrium. In the consumption loan model, each player transfers zero after almost every realization of x in any Markov equilibrium. We will extend this notion, and call a behavior strategy profile a Markov equilibrium if it is equivalent to γ. Note that we allow a Markov equilibrium to be "non-Markovian" on sets of measure zero.

Definition 5.4 A behavior strategy profile $\langle\sigma_t\rangle$ is a ***Markov equilibrium*** if $\forall t$, $\forall\, b_t \in B_t$, $\sigma_t(b_t)$ is equivalent to γ.

We make the same informational assumptions as in section 3, viz. M1 and M2. We extend the definitions of section 3 regarding the measurability of strategies in the following manner.

Definition 5.5 A behavior strategy σ_t is ***measurable w.r.t.*** $\sim_i$ if for any b_t, $b_t' \in B_t$ with $b_t \sim_i b_t'$, the set of x_t such that (5.8) does not apply has μ-measure zero:

$$\sigma_t(x_t, b_t) = \sigma_t(x_t, b_t') \qquad (5.8)$$

If σ_t is measurable w.r.t $\sim_i$, it follows that the associated aggregate s_t is likewise measurable w.r.t. $\sim_i$, i.e. if $b_t \sim_i b_t'$, $s_t(b_t) = s_t(b_t')$

The following theorem shows that the overlapping generations game with memory constraints has a unique sequentially rationalizable outcome.

Theorem 3 The disturbed overlapping generations game has an essentially unique sequentially rationalizable outcome which a Markov equilibrium. In the consumption loan model, every agent transfers zero after every observed history.

Proof By assumption M2 we can find an agent with an arbitrarily large index i who is not omnifamous. Hence $\exists$ j>i such that $j \in \mathcal{U}(i)$. We show, by backward induction, that for all t, $i < t \leq j$, that if $\sigma_t \in \Theta_t$, σ_t is measurable w.r.t. $\sim_i$.

i) Every strategy in Σ_j is measurable w.r.t. $\sim_i$, since $j \in \mathcal{U}(i)$, and $\sim_i$ induces the trivial (finest possible) partition of B_j, where each set in the partition is a singleton set. Hence if $\sigma_j \in \Theta_j$, σ_j is measurable w.r.t. $\sim_i$.

ii) Let $i < t < j$. If every s_{t+1} in Q_{t+1} is measurable w.r.t. $\sim_i$, then if $\sigma_t \in \Sigma_t$, σ_t is measurable w.r.t. $\sim_i$.

We now prove (ii). Let s_{t+1} be measurable w.r.t $\sim_i$, let x_t be any realization of X_t. and let $b_t \sim_i b_t'$. We claim that :

$$u_t(\sigma_t, s_{t+1} / b_t, x_t) = u_t(\sigma_t, s_{t+1} / b_t', x_t) \qquad (5.9)$$

Suppose not. Let $u_t(\sigma_t, s_{t+1} / b_t, x_t) > u_t(\sigma_t, s_{t+1} / b_t', x_t)$. Since $b_t' \sim_i b_t$,

$$b_{t+1}(\sigma_t(b_t, x_t), b_t') \sim_i b_{t+1}(\sigma_t(b_t, x_t), b_t)$$
(5.10)

Since s_{t+1} is measurable w.r.t. $\sim_i$,

$$s_{t+1}[b_{t+1}(\sigma_t(b_t', x_t), b_t)] = s_{t+1}[b_{t+1}(\sigma_t(b_t, x_t), b_t)]$$
(5.11)

Hence agent t ensures the payoff $u_t(\sigma_t, s_{t+1}/b_t, x_t)$ by choosing $\sigma_t(b_t, x_t)$, so that $\sigma_t(b_t', x_t)$ is not optimal.

Hence if $\sigma_t \in \beta(s_{t+1})$, (5.9) must hold.

However, by lemma 5.1, for almost all realizations of x_t, agent t has a unique optimal action, so that for almost all realizations of x_t:

$$\sigma_t(b_t, x_t) = \sigma_t(b_t', x_t) \qquad (5.12)$$

Hence σ_t is measurable w.r.t. $\sim_i$, and hence s_t is measurable w.r.t. $\sim_i$.

(i) and (ii) imply that s_{i+1} is measurable w.r.t $\sim_i$. By lemma 5.2, player i's must choose $\gamma(x)$ after every observed history, and for almost all realizations of x.

We can now apply backward induction to players with index less than i. If player i chooses an element $\gamma(x)$ almost everywhere after every history, the corresponding aggregate is measurable w.r.t. $\sim_{i-1}$. Hence i-1 and every preceding player chooses $\gamma(x)$ for almost all x irrespective of the observed history. □

Remark 1: Our proof is based on backward induction, even though the model has an infinite horizon. This is why we are able to get our results with an extremely weak solution concept, such as sequential rationalizability. Note that the argument here is not open to some of the critiques of backward induction, eg. Basu (1991). These critiques consider games such as the centipede game, where although the game has a unique backward induction outcome, a single player moves several times. If such a player deviates from the backward induction prescription, this contradicts the common knowledge of rationality assumption, placing other players in a dilemma. In our model each player moves only once. Should she deviate, this has no implications for future behavior.

Remark 2: The key element in theorem 3 is the use of Harsanyi's (1973) device of perturbing player's payoffs. This device yields powerful results: contrast theorem 2. This is, to our knowledge, the first time that payoff perturbations ala Harsanyi, have been used to refine equilibria significantly in dynamic games.[2] Perturbations, with incomplete information about players types, have indeed been used in dynamic games, and in some cases, such perturbations have refined the set of equilibria. Notable examples are Aumann and Sorin (1989) and Fudenberg and Levine (1989). These models rely upon "large" perturbations, i.e. there is some probability that a player is a "crazy" type, playing a fixed strategy. Our use of perturbations is quite different; in particular we do not require that the set Z, of possible realizations of private information, be large. Indeed, the proof allows Z to be arbitrarily small. A second qualitative difference between our model and the above mentioned papers is that we have no "reputation effects". Remark 1 is relevant here: since each player moves only once, there is no scope for building reputation in our model.

6. PERFECT DIVISIBILITY OF THE TRANSFERABLE COMMODITY

We now examine the implications of allowing the transferable commodity to be perfectly divisible. In our view, perfect divisibility is an unreasonable assumption. The analysis here is mainly in order to demonstrate that the difficulties with sustaining efficient outcomes do not stem from this assumption. In this section, we show that with perfect divisibility one can support the efficient outcome by pure strategies. However, these pure strategies turn out to be non-robust once we perturb the utility function.

Let $A = [0,e]$ and let $u: A \times A \rightarrow R$ be the payoff function, which satisfies C1.

Assumption C1. $u(.)$ is continuous, and is strictly decreasing in its first argument and strictly increasing in its second argument.

As in section 4, label the ***efficient action*** as 1, i.e. $u(1,1) \geq u(a,a)\ \forall\ a \in A$. The interval $[u(0,0),u(1,1)]$ is the set of individually rational and feasible payoffs. If u^* belongs to this interval, by the intermediate value theorem, $\exists$ a^*, $0 \leq a^* \leq 1$, such that $u(a^*,a^*) = u^*$. The following theorem says that any individually rational feasible payoff can be supported by a pure strategy provided that each player observes the action of the previous player.

Theorem 4 If each agent observes the action of the previous agent, any individually rational and feasible payoff can be supported by a pure strategy equilibrium.

Proof Given a u^*, define a^* as above. Define the function $\phi:[0,a^*] \rightarrow [0,a^*]$ by the equation:

$$u(a,\phi(a)) = u(a^*,a^*) \qquad (6.1)$$

We first show that ϕ is well defined. Let $a \in [0,a^*]$. By the definition of a^*:

$$u(a,a) \leq u(a^*,a^*) \qquad (6.2)$$

Further, since $u(.)$ is strictly decreasing in its first argument:

$$u(a,a^*) \geq u(a^*,a^*) \qquad (6.3)$$

Since u(.) is continuous, the intermediate value theorem implies that there exists a $\phi(a)$, $a^* \geq \phi(a) \geq a$, satisfying (6.1). Since u is strictly increasing in its second argument, this solution is unique, so that the function ϕ is well defined. We construct a pure strategy supporting a* which conditions only on the last observed action as follows:

$s_1 = a^*$

If $a_{t-1} \geq a^*$, $s_t = a^*$

If $a_{t-1} < a^*$, $s_t = \phi(s_{t-1})$

It may be verified that this strategy profile constitutes an equilibrium. No matter what the observed history, the strategy ensures a payoff of u(a*,a*). If the agent deviates by choosing any other transfer in [0,a*], she still gets only u(a*,a*). If she deviates by choosing a transfer greater than a*, she only gets a* in the next period and hence her utility is less than u(a*,a*). □

The pure strategies supporting the efficient allocation are infinitely complex - indeed, they have an uncountable number of states. Nevertheless, they require only one period memory. The contrast between the informational requirements of a strategy and the Abreu-Rubinstein number of states measure of complexity could hardly be more stark. If one believes that a strategy should be informationally economical *and* use few states, this compels us to consider automata which output randomized actions. As we noted in section 3, the efficient outcome can be supported by MIXED1 which requires only one-period memory and two states.

We now show that this pure strategy also fails to survive if we perturb the payoff function. This is technically more complicated since u is now infinite dimensional. However, we adopt the procedure of parametrizing u(.) by a single parameter, thereby reducing the question of genericity of u(.) to that of this parameter. Let u_t be given by:

$$u_t(a,a') = v(a,a') - x_t a \qquad (6.4)$$

where x_t is independently and identically distributed by the probability measure μ on the set $Z = [-c,c]$. We assume that c is sufficiently small that u_t is increasing in its first argument for all realizations of x_t.

The following discussion mirrors that of section 5, and as before we drop time subscripts. Let ΔA be the set of probability measures over A. Let $\rho : A \rangle \Delta A$ be Borel measurable, and let $\rho(a)$ denote the element of ΔA that is mapped into by a. Define:

$$u(a,\rho,x) := \int u(a,a',x)\ \rho(a'/a)\ da' \qquad (6.5)$$

$$v(a,\rho) := \int v(a,a')\ \rho(a'/a)\ da' \qquad (6.6)$$

Given (6.4), we have:

$$u(a,\rho,x) = \int v(a,a')\ \rho(a'/a)\ da' - xa \qquad (6.7)$$
$$= v(a,\rho) - xa$$

Define $\beta(\rho,x)$ as in the previous section. Given any realization of private information, x, we define:

$$\beta^*(\rho,x) := [\sup \beta(\rho,x),\ \inf \beta(\rho,x))$$

Given any probability measure over the strategies of player t+1, ρ, and the realization of private information, x, β^* is the half-open interval constructed using the infimum and supremum of best responses of player t. If $\beta(\rho,x)$ is a singleton set, β^* is the null set; otherwise, it is an interval of strictly positive length. Define further the set:

$\chi(\rho) := \{x \in Z : \beta^*(\rho,x) \neq \emptyset\}$

$\chi(\rho)$ is the subset of Z for which player t has multiple best responses to a given mixed strategy of player t+1. We are now in a position to state the following lemma.

Lemma 6.1 $\chi(\rho)$ is at most countable and hence has μ-measure zero.

Proof Suppose x,x' are two realizations of X, with x>x'. Let $a \in \beta(\rho,x)$ and $a' \in \beta(\rho,x')$. From (6.7):

$u(a,\rho,x) - u(a',\rho,x) = v(a,\rho) - v(a',\rho) + x(a-a') \geq 0$

(6.8)

$u(a',\rho,x') - u(a,\rho,x') = v(a',\rho) - v(a,\rho) + x'(a'-a) \geq 0$

(6.9)

Adding these inequalities we have:

$(a-a')(x-x') \leq 0$ (6.10)

Hence $\inf \beta(\rho,x) \leq \sup \beta(\rho,x')$, which implies:

$\beta^*(\rho,x) \cap \beta^*(\rho,x') = \emptyset$ (6.11)

(6.11) implies that for each x in $\chi(\rho)$ we can find a distinct rational number in $\beta^*(\rho,x)$. Hence $\chi(\rho)$ must be at most countable since otherwise we have a one-to-one correspondence between an uncountable set and a subset of the rationals. □

Given lemma 6.1, we can now replicate the proof of theorem 3 to prove the following theorem:

Theorem 5 The disturbed consumption loan model with a perfectly divisible commodity has a unique sequentially rationalizable outcome where each agent transfers zero irrespective of the observed history.

7. THE INFORMATIONAL ROLE OF MONEY

We now consider the possibility that money may play an informational role in the overlapping generations context, by potentially allowing society to retain unbounded memory in some situations. This allows society to support the efficient outcome, even though information about past events is limited.

Consider the simple model of our example, where the set of actions $A = \{0,1\}$. In period one the old agent may issue, (costlessly), a dated piece of paper which we call money. If the young agent in period one transfers 1 to the old agent, she is offered this money in exchange. Every agent can acquire money by one of two means: she may get it from the older agent by transferring 1, or she may simply issue her own money.

Clearly the action sets of the agents are the same as before. However, since money comes with the date of the issuing agent, this gives rise to a different information structure. We assume that in each period, the young agent has no knowledge of the preceding actions. However, she may discern the date of the agent who has issued the money that is offered to her. Agent t's observed history, b_t, is now simply a date, τ with $\tau < t$. The set of possible observed histories, B_t, equals $\{0,1,2,,...,t-1\}$. If agent t observes that the money offered to has date τ, she can infer that agent τ has transferred 0, that every agent after τ has accepted the old's money, and has therefore chosen 1. However, she can make no inference regarding the behavior of agents before τ.

Contrast this information structure with the case of m period social memory, m>1. The information partitions of any agent under these two structures are not ordered, so that neither can be considered more informative than the other. Nevertheless, money allows social memory to be potentially unbounded. If every agent accepts the money issued by the first old agent i.e. of date 0, future agents will have information about all the actions taken by all agents. The potential unboundedness of memory allows us to support efficient allocations in a robust way. For ease of exposition, we consider ***the*** efficient allocation where every agent transfers 1 - the results extend in an obvious way to other efficient allocations where the first k agents transfer 0, and agents with index greater than k+1 transfer 1.

Proposition 1. The efficient allocation can be supported by the strategy profile GRIM, which is the unique efficient pure strategy equilibrium. If any agent deviates and refuses to accept money, money is never accepted subsequently so that the continuation path is inefficient.

Proof Consider the partition of H_t into two sets: the singleton set of consisting of the history with $a_\tau = 1 \; \forall \; \tau < t$, and the set of all other histories where some player has chosen 0. This partition is coarser than B_t, and since GRIM is measurable with respect to this partition, GRIM is a sequentially rational equilibrium.

We now show that GRIM is the unique efficient equilibrium. Let $<s_t>$ be an efficient equilibrium, so that $s_t(b_t{=}0) = 1 \; \forall \; t$. Let $s_{t*+1}(t*) = 1$ for some $t*>1$. This implies $s_{t*}(1) = 0$, contradicting the assumption that $s_t(0) = 1 \; \forall \; t$. If $s_{t*+1}(t*) = 1$, t* gets a transfer of 1 next period by issuing her own money, and hene will not accept money issued at date 0. □

Remark 1. Each agent has strict incentives to use GRIM at every information set, so that the equilibrium is robust to small perturbations in the utility functions of agents as in section 5.

The monetary equilibrium is however fragile in another sense, since it is vulnerable to a "crazy" behavior by any one generation. If any generation were to be foolish enough to deviate, money never regains its value. In other words, the loss in confidence is permanent. This fragility is necessary for the original equilibrium to be self-enforcing. Given informational constraints, crises of confidence must be devastating, and long lasting. The informational role of money hence provides a theory of monetary crises of confidence.

8. CONCLUDING COMMENTS

This paper can be viewed as a contribution to the literature on informational constraints in dynamic games, eg. repeated games with imperfect monitoring as in Green and Porter (1984). The overlapping generations framework provides a natural way of introducing imperfect information, in a way which is different from the repeated games literature.[3] The paper has also a bearing on the issue of strategic complexity. As we have seen, simple strategies may require infinite memory, whereas a strategy which requires only one-period memory may be infinitely complex.

Our substantive results sound a note of caution with regard to the possibility of supporting social cooperation in the context of finitely lived overlapping generations with informational constraints, at least in the absence of mechanisms for preserving infinite memory such as money. The negative result in the arises from the informational assumption is there is always an agent of sufficiently large index who is not omnifamous, so that there is a subsequent agent who is uninformed about i. This creates asymmetric information between the those who are informed and those who are uninformed about this agent. The simplest way of appreciating this point is to consider the case of one period social memory, so that agent t+1 is informed about t's actions, but t+2 is not. Agent t+1 has private information about t's actions, and since these do not directly affect t+1's payoffs, agent t+1 must be indifferent between the actions she takes at different information sets. It is impossible to construct a history-dependent strategy for t+1 which preserves this indifference in a robust way, once we perturb the payoffs. This essential argument extends, via backward induction, to more complex information structures and greater social memory, provided that the asymmetric information between the informed and the uninformed persists.

An alternative way around this negative result is to get rid of the asymmetric information. Suppose that agent t+1 has the same information about the past as agent t, with a high probability (1-ε), but with a small probability, ε, has less information. ε could be a function of m, the size of memory. In such a model, society loses memory, but social forgetting is a stochastic process, and agent t's information partition is stochastic. This generates a model which is formally quite different from that analyzed in this paper, and therefore requires separate analysis. The results in such a model depend upon the assumption one makes about ε. If $\varepsilon(m)$ is sufficiently small no matter how large m is, then one can support inter-generational transfers. This is possible since if $\varepsilon(m)$ is always very small, then any pair of adjacent agents, t and t+1, always have almost the same information. However, if $\varepsilon(m)$ becomes large at very large values of m, the results are similar to those in this paper. The basic point is that in the stochastic model, any generation and its successor have essentially symmetric information about the past - with high probability they have the same information. It is not clear to us that this is more persuasive than the model analyzed in this paper. In our view asymmetric information about the past is an essential difference between generations who are born at different dates. The deterministic model, we would argue, captures this asymmetry better than a stochastic model.

The results of this paper also apply to repeated two-player games played by overlapping generations of players, where where the old player has a dominant strategy. This runs counter to much of the recent literature on dynamic games played by overlapping generations of players - eg. Kandori (1992) and Smith (1992). The anti-folk theorem presented here does not necessarily generalize to the general games played by overlapping generations of players. If there are three or more players in each generation who share the same information about the past, a non-cooperative equilibrium can be constructed where they are induced to reveal this information to future generations. One may therefore be able to prove an informationally economical Folk theorem for such games as Kandori (1992) suggests. Such an equilibrium is however vulnerable to collusion between agents of the same generation. We leave a complete analysis of these games for future work.

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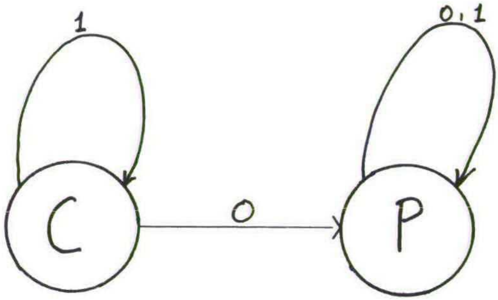
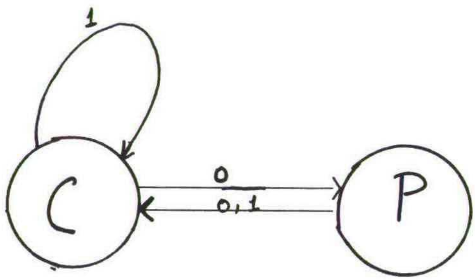
[1]This model also provides an instance where the classical relationships between competitive equilibria and the core fail to apply, as Hendricks et. al. (1980) and Esteban (1986) show.

[2]van Damme (1991, ch.6) notes that the Harsanyi perturbation has very little power in extensive games.

[3]It is possible to introduce imperfect recall into repeated games; however, there are major problems of interpretation which need to be resolved before this can be done in a satisfactory, as Piccione and Rubinstein (1994) note.



GRIM



RESILIENT

STATES: C, P

INITIAL STATE: C

OUTPUT: C ⟶ 1, P ⟶ 0

Fig 1

TABLE 1

	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	1	u(0,1)
I	(0,1)	1	(1,1)	1	u(1,1)
	(0,0)	1*	(0,1)	1	u(1,1)

Action after (0,0) not optimal; choosing 0 induces (0,0) and 1 next period, giving u(0,1)

	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	0	u(0,0)
II	(0,1)	0*	(1,0)	0	u(0,0)
	(0,0)	0	(0,0)	0	u(0,0)

Action after (0,1) not optimal; choosing 1 induces (1,1) and 1 next period, giving u(1,1)

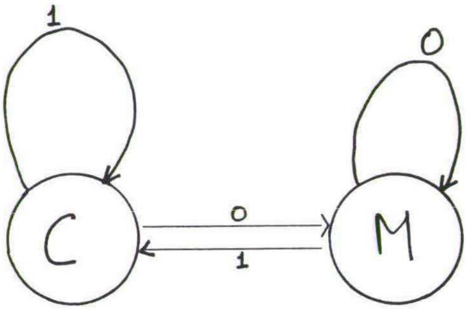
	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	0	u(0,0)
III	(0,1)	1	(1,1)	1	u(1,1)
	(0,0)	0*	(0,0)	0	u(0,0)

Action after (0,0) not optimal; choosing 1 induces (0,1) and 1 next period, giving u(1,1)

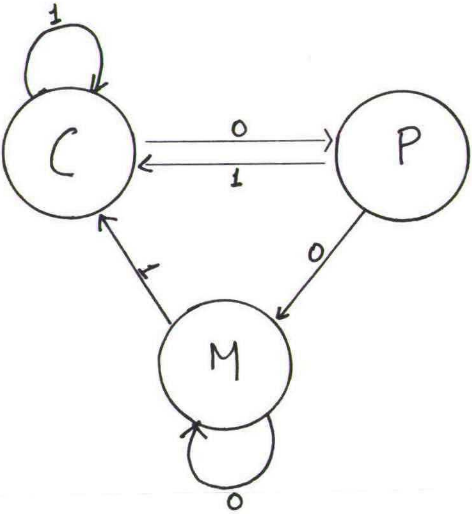
	OBSERVED HISTORY	ACTION THIS PERIOD	INDUCED HISTORY	NEXT-PERIOD ACTION	PAYOFF
	(1,1)	1	(1,1)	1	u(1,1)
	(1,0)	0	(0,0)	1	u(0,1)
IV	(0,1)	0	(1,0)	1	u(0,1)
	(0,0)	1*	(0,1)	0	u(1,0)

Action after (0,0) not optimal; choosing 0 induces (0,0) and 1 next period, giving u(0,1)

* shows sub-optimal action



MIXED1



MIXED2

STATES: C, P, M

INITIAL STATE: C

OUTPUT: C $\rightarrow$ 1, P $\rightarrow$ 0

M $\rightarrow$ 0 with probability p^{0*}, 1 with probability $(1-p^{0*})$

Fig 2

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