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Credit, Profitability and Instability: A Strictly Structural Approach

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Paulo L dos Santos, Address: Department of Economics, Soas, Thornhaugh Street, Russell Square, London, WC1H 0XG, Britain. Email: ps45@soas.ac.uk. Phone: +44 20 7898 4726 . This working paper and the broader work on which it draws have benefited immeasurably from ongoing discussions with Costas Lapavitsas and Duncan Foley. The have also benefited from more recent exchanges with Deepankar Basu. The usual disclaimers apply.

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Abstract

This paper offers a purely structural characterisation of the content, limits and contradictions of credit relations in capitalist accumulation. Considering steady-state evolutions and step-change perturbations in a dynamic model of the Marxian circuit of capital, it establishes that sustained paces of net credit extension may boost aggregate profitability, the rate of accumulation, and the aggregate financial robustness of capitalist enterprises. These gains are limited by the economy's dynamic productive capacities, and tempered by the risks of credit and monetary disruptions created payment obligations established by credit. Economies with higher paces of net credit extension are shown to be more vulnerable to the disruptions to accumulation variously emphasised by Marxian, Keynesian and Post-Keynesian contributions.

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1. Introduction

Any theorisation of contemporary capitalism, its instabilities and contradictions requires deliberate and systematic attention to credit relations. In recent decades credit flows have exhibited unprecedented magnitude, volatility and, arguably, macroeconomic significance. In the US economy, the period between 1993 and 2007 saw the largest sustained post-war cycle of net credit extension to the private non-financial sector, increasing its indebtedness by more than 60 percentage points of GDP.[1] Much of this lending doubtlessly inflated successive bubbles in financial and real-estate asset prices across this period.[2] But these flows also contributed to aggregate demand and thus to profitability, both directly and possibly through "wealth effects" associated with asset-price inflation. At the same time it was the "credit-crunch" of 2007 and the near freezing of credit markets following the Panic of October 2008 that drove much of the world into recession. The anaemic recovery that followed may be understood to be conditioned by the ongoing process of "deleveraging", which has seen negative credit flows for the entire private non-financial sector of the US economy for the first time since World War II.

This paper advances a dynamic, integrated analytical approach to the macroeconomic content, limits and contradictions of credit relations. It does so by advancing a structural model of accumulation and credit based on Marx's (1885) circuit-of-capital conceptualisation of accumulation, its mathematical formalisation offered by Foley (1982, 1986), and on a more recently developed approach to the monetary foundations and contradictions of credit relations.[3] In the resulting framework the driver of economic life is investment undertaken in the pursuit of monetary profits whose capitalisation is the fundamental source of growth. Capitalist profitability, in turn, is determined dynamically, in the sphere of commodity production by the generation of *surplus value*, and in the sphere of exchange by sales flows allowing the timely realisation of monetary profits.

Credit relations are defined in the sphere of exchange, and their macroeconomic content arises in the first instance from their role in conditioning timely sales of commodity output that realise monetary profits. They have systematic foundation on hoards of value in monetary form that develop in the circuit of capital and in the circuits of revenue of workers and capitalist consumers.

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[1] From a cycle low of 115 percent of GDP in early 1994 to a peak of 177 percent of GDP in early 2009. Calculated from seasonably adjusted data in Flow of Funds of the United States.

[2] Such processes and the transfers of value associated with them are another defining characteristic of contemporary capitalism and require separate macroeconomic analysis and characterisation.

[3] Deployed to the analysis of credit allocation in dos Santos (2011a) and more fully developed in dos Santos (2011b).

Money hoards are understood to support the *ex nihilo* issuance of bank liabilities by providing dynamic scopes for their circulation, so long as those are accepted as stores of value. The acceptability of bank liabilities is most generally grounded on claims held in bank portfolios. The magnitude and distribution of existing money hoards relative to production and consumption plans conditions the demand for credit. The composition in which holders of money are willing to maintain their holdings conditions the ability of the credit system to accommodate any given level of net credit extension demanded.

<Figure 1 here>

The resulting structural framework yields a number of findings, which are formally presented and demonstrated as theorems within the model's framework. Like sustained own investment, sustained paces of net credit extension are shown to boost the rate of accumulation and the aggregate profitability of total social capital. This ability is ultimately constrained by the economy's dynamic productive capacity. It is also uniquely tempered by the inherent credit and monetary risks posed by credit relations and the payment obligations they create. Those obligations render the profit realised by enterprises net of debt servicing costs into the concrete imperative governing decisions over investment, credit demand, and its accommodation by the banking system. This residual *profit of enterprise* also becomes the fulcrum of capitalist crises, as it generally defines bankruptcy, bank losses and the resulting monetary instability.

The paper characterises the risks posed by credit relations in relation to aggregate flows of profit of enterprise, considering them along exponential steady states, and across step-change perturbations to parameter values. It shows that aggregate increases in indebtedness do not in themselves increase financial fragility. In fact, sustained paces of net credit extension, and of own investment, are shown to boost profit of enterprise, and to *strengthen* the aggregate financial robustness of capitalist enterprises.

Interruptions to the pace of net credit extension are shown to be sufficient to disrupt the profitability of capitalist enterprises, and to increase the likelihood of credit and corollary monetary crises. The framework offered by the paper also allows consideration of the interaction between credit disruptions and broader disruptions to mark-up rates, rates of interest, and the pace of own investment. As such, it allows a structural examination of the credit moments of crisis processes emphasised respectively by Marxian, Post-Keynesian and Keynesian contributions.

In line with this purpose, the paper advances a distinctive measure of aggregate financial vulnerability based on the fact that for any given disruption to accumulation, there will be a minimum *ex post* pace of net credit extension that ensures positive aggregate flows of profit of enterprise. This minimum pace offers a useful measure of aggregate financial vulnerability, as during any disruption it will be generally difficult to sustain higher levels of net credit extension. On this basis, the paper formally demonstrates that while economies with high paces of borrowing and own investment are more profitable and robust while lending is sustained, they are more prone to credit crises when faced with any given perturbation involving a fall in the pace of own investment.

The real structural contradiction of credit relations lies in that their very ability to boost profitability and the pace of accumulation, and to improve aggregate financial robustness, simultaneously encourages greater paces of net credit extension and renders the economy more vulnerable to any given negative disruption. The capitalist process is recurrently and endogenously driven to credit crises and attendant monetary disruptions.

The discussion proceeds as follows. Section 2 outlines the structural macroeconomic framework on which the paper's model is founded, including explicit consideration of the monetary foundations, requirements and contradictions of capitalist credit relations. Section 3 considers steady-state results concerning the impact of credit on growth, profitability and aggregate financial robustness. Section 4 considers a general step perturbation to steady-state paths. Section 5 concludes with a brief discussion of the implication of these findings for analysis of the credit moments of the capitalist business cycle.

2. The Continuous-Time Model of the Circuit of Capital

Marxian political economy conceptualises capitalist social reproduction at the aggregate level on the basis of the circuit of capital, a schematisation of the moments and forms taken by capital value as it seeks self-expansion. For a single pulse of value this process takes the form,

$$M - C(lp, mp) - P - C' - M' > M \tag{2.1}$$

Capitalists advance value in monetary form M as investment in input commodities labour power and means of production (*lp* and *mp* respectively). These are deployed and used in the process of production P which yields output commodities C' that are subsequently sold for a quantity of money M'. The entire process is driven by the expectation that, on the sale of output, $M' > M$, which hinges on the creation of surplus value from the employment of labour power in production and its realisation in the form of monetary profits in commodity exchange. Positive monetary profits allow the recommitment of value to the circuit in growing scales, and are the fundamental bases for positive rates of accumulation.

Process (2.1) is fundamentally dynamic. Aggregate social capital is composed of myriad pulses of value transversing the circuit, so that at any given time a certain measure of value is engaged in each stage of the process. In the model presented here, the demand and supply flows ensuring commodity sales at any point in time are located in relation to process (2.1). Further, each phase of the process in understood to pose time lags, which delay the flow of value through investment, production and sale. Those lags collectively define the turnover time of capital, an average measure of the time necessary for a quantum of value to complete a cycle, which is an obvious determinant of the dynamic profitability of capital.

This section offers a formalisation of this process based on Foley (1982, 1986), including explicit consideration of the monetary and balance-sheet foundations of credit extension. While primarily laying the basis for the paper's findings, the section also establishes as theorems structural results concerning credit, growth and the evolution of capitalist net worth.

2.1 Flows of Value in Capitalist Reproduction: Production and Sale

At the centre of the formalisation offered by Foley (1982, 1986) are *lag functions* describing the flow of value through each of the phases in process (2.1), and through circuits of revenue. At any time t, the lag function $x_i(t-\tau;t)$ describes the proportion of value that entered phase i of the circuit at time $\tau \leq t$ currently exiting that phase. In this formulation it is possible to describe the evolution of the lag process over time. The preservation of value only requires that,

$$\int_{\tau}^{\infty} x_i(t-\tau;t)\,dt = 1, \tag{2.1}$$

ensuring all value entering all phases of the circuit eventually exits. This formalisation allows a dynamic depiction of process (2.1) for aggregate social capital.

Let $C(t)$ denote the aggregate flow of investment, with a fraction $\kappa(t)C(t)$ invested in labour-power and $(1-\kappa(t))C(t)$ invested in means of production. Once the decision to invest value into production is taken, its transformation into outputs over time may be represented mathematically by a lag process function, $x_p(t')$. In this case the flow of output commodities $P(t)$ may be expressed as

$$P(t) = \int_{-\infty}^{t} C(\tau)x_p(t-\tau;t)\,d\tau \tag{2.2}$$

Once commodity output is produced, the monetary realisation of profits through sales is also subject to a lag. This *realisation* lag is considered to be an endogenous function of the relative paces of commodity supplies and aggregate demand $D_A(t)$. Supposing for simplicity it takes the form of a simple time delay $T_r(t)$ at any point in time,[4] and assuming commodities follow a simple first-in-first-out progression through inventories, it can be shown that,[5]

$$\dot{T}_r(t) = 1 - \frac{D_A(t)}{(1+q)P(t-T_r)} \tag{2.3}$$

So that enhanced demand flows in relation to output flows will tend to shorten the realisation delay period $T_r(t)$, and relatively slack demand lengthen it.

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[4] Formally $x_r(t) = \delta(T_r)$, the Dirac delta function, taking the value of zero for all $t \neq T_r$, and infinity for $t = T_r$.

[5] By the Mean Value Theorem, Foley (1986).

Commodity sales generate total revenues $S(t)$, giving surplus value its embodiment as a monetary mark-up $q(t)$ over production costs. In line with conventional Marxian analysis, the view taken here is that $q(t) = \varepsilon(t)\kappa(t)$, the product of the share of labour power in total productive capital $\kappa(t)$, and $\varepsilon(t)$, the rate of exploitation, both of which are determined prior to the sale of commodities.

It is useful to divide $S(t)$ between sales representing the recovery of production costs, and those representing profits, respectively,

$$S'(t) = \frac{S(t)}{1+q(t)} \tag{2.4}$$

$$S''(t) = \frac{q(t)S(t)}{1+q(t)} \tag{2.5}$$

The flow $S'(t)$ defines sales that make simple, zero-growth reproduction possible, and is termed here the *scale of simple reproduction*. Suppose that capitalists are in a position to make full reinvestment of $S'(t)$ and of a proportion $p(t)$ of profits $S''(t)$, so that value in the form of money amounting to $S'(t) + p(t)S''(t)$ is set aside for reinvestment, and $(1 - p(t))S''(t)$ is allocated to fund capitalist consumption.

The flow back into investment of capitalist revenues set aside for investment is subject to an investment lag process, denoted here by $x_v(t')$. The relative severity of this lag describes the pace of investment, as well as the relative measure of capital value existing in monetary form. It thus also offers a measure of the liquidity of aggregate social capital.

Money also flows through the circuits of revenue of workers and capitalists *qua* consumers. Those circuits commence with the receipt, respectively, of wage payments $\kappa(t)C(t)$ and of capitalist revenues allocated to funding of consumption, $(1 - p(t))S''(t)$. The expenditure of these revenues is also subject to lag processes, which represent dynamic generalisations of the respective marginal propensities to consume.[6] Those processes are respectively denoted by $x_w(t')$ and $x_c(t')$.

The movement of money, and thus its velocity, is fully characterised by the three lag processes $x_v(t')$, $x_w(t')$ and $x_c(t')$ describing its flow through the circuit of capital and the circuits of revenue. A unit of revenue entering any one of these processes at time τ will on average be spent after a lapse of time given by,

$$T_i(\tau) = \int_{\tau}^{\infty} (t-\tau) x_i(t-\tau;t)\,dt \tag{2.6}$$

The reciprocal of this quantity, $T_i(\tau)^{-1}$, provides an *ex post* measure of the velocity of money across each lag process. Its average value across all three processes, weighted by the respective size of the corresponding money stocks, provides a similar *ex post* measure of money velocity in the economy.

2.2 Flows of Value in Capitalist Reproduction: Investment, Demand and Realisation

Capitalist investment and aggregate demand may now be characterised. Total investment will be given by the own funds emerging from this lag process plus net credit extension to capitalists, $\dot{B}(t)$,

$$C(t) = \int_{-\infty}^{t} \left[S'(\tau) + p(\tau) S''(\tau) \right] x_v(t-\tau;t)\,d\tau + \dot{B}(t) \tag{2.6}$$

Investment draws on past unspent capitalist revenues and net credit extension. It also constitutes the most general conditioner of demand. It immediately funds purchases of means of production. It also funds wage payments and capitalist profits that will in turn appear as commodity demand. Demand for consumption goods by workers and capitalists will be given by,

$$D_w(t) = \int_{-\infty}^{t} \kappa(\tau) C(\tau) x_w(t-\tau;t)\,d\tau \tag{2.8}$$

[6] Foley (1986).

$$D_c(t) = \int_{-\infty}^{t} (1 - p(\tau))S''(\tau)x_c(t - \tau; t)\, d\tau \qquad (2.9)$$

Aggregate demand, or total value seeking commodities will thus be,

$$S(t) = D(t) = (1 - \kappa(t))C(t) + D_w(t) + D_c(t) \qquad (2.10)$$

From (2.8) and (2.9) it is clear that without net extension of consumption loans, demand for consumer goods will only be sustainable if it does not exceed current income flows, or $D_w(t) \leq \kappa(\tau)C(\tau)$, and $D_c(t) \leq (1 - p(\tau))S''(\tau)$. In that case, is is evident from (2.10) that,

$$S(t) \leq C(t) + (1 - p(t))S''(t) \qquad (2.11)$$

Assuming for simplicity that the mark-up and capitalisation rates are constant over time, manipulation of (2.10) and the explicit expression of investment flows yields,

$$S'(t) \leq \int_{-\infty}^{t} S'(\tau)x_v(t - \tau; t)\, d\tau + \frac{1}{1 + pq}\dot{B}(t) \qquad (2.12)$$

Inequality (2.11) points to the constraints bearing upon the accumulation of capital in the sphere of circulation, which relate to its endogenous ability to generate demand flows capable of realising value mark-ups as monetary profits.

Theorem 2.1: *If the investment lag process $x_v(t')$ is constant over time and non-trivial (ie, investment is not instantaneous) growth in the scale of simple reproduction $S'(t)$ requires positive flows of net credit extension $\dot{B}(t)$.*

Proof: If the investment lag process is constant over time then,

$$\int_{-\infty}^{t} x_v(t - \tau; t)\, d\tau = 1 \qquad (2.13)$$

In this case the term under the integral in (2.12) is a weighted average of past values of $S'(t)$. But growth in $S'(t)$ by definition requires $S'(t) > S'(\tau), \forall \tau < t$. This will only be possible if $\dot{B}(t) > 0$. □

It is of course also possible for the present scale of simple reproduction to exceed all of its past values if the investment lag process becomes less severe. In this case, growth and profits arise from an increase in the pace of investment, which also amounts to an increase in the pace of dishoarding, the level of illiquidity borne by social capital, and to an increase in the average velocity of money. While these processes may lead to temporary spurts of growth, they are not sustainable basis for accumulation as capitalist money hoards cannot become arbitrarily small, social capital cannot become arbitrarily illiquid, and the velocity of money cannot rise indefinitely. Expanding investment plans will systematically include a demand for credit, which in contemporary monetary systems is accommodated in the first instance through the *ex nihilo* issuance of new bank liabilities enjoying at least some acceptance as a form of money.[7]

2.3 Stocks of Value in Capitalist Accumulation

The lag processes in the economy give rise to five stocks of idle value: unfinished output, inventories, and money hoards in the circuit of capital; and the saved personal income of workers and capitalists. The evolution of these stocks is defined by the flows above. In the circuit of capital, stocks of unfinished commodities, inventories and money dynamically obey, respectively,

$$\dot{\Pi}(t) = C(t) - P(t) \tag{2.14}$$

$$\dot{\mathrm{N}}(t) = P(t) - S'(t) \tag{2.15}$$

$$\dot{\mathrm{M}}_K(t) = S'(t) + pS''(t) - \left(C(t) - \dot{B}(t)\right) \tag{2.16}$$

Stocks of unfinished output evolve as invested value $C(t)$ becomes input commodities and as value $P(t)$ emerges as finished output. Inventories are fed by new output and depleted by sales (net of mark-ups). Money hoards in the circuit are formed as capitalists set aside their own unconsumed revenues for future investment and sink their own capital into investment, given by total investment $C(t)$ net of new borrowing, which draws on newly created (and thus not previously held) money.

[7] See dos Santos (2011b).

The lags in the expenditure of personal revenues also give rise to money holdings, given respectively for workers and capitalists *qua* consumers,

$$\dot{\mathrm{M}}_w(t) = \kappa(t)C(t) - D_w(t) \tag{2.17}$$

$$\dot{\mathrm{M}}_c(t) = (1-p)S''(t) - D_c(t) \tag{2.18}$$

The total stock of money in the economy, $M(t) \equiv M_K(t) + M_c(t) + M_w(t)$, will evolve in line with (2.16)-(2.16), which by (2.10) will follow,

$$\dot{\mathrm{M}}(t) = \dot{B}(t) \tag{2.19}$$

It is now possible to provide a dynamic characterisation of investment, savings and the sources of profit realisation. Adding (2.18) and (2.19) and substitution into (2.10) yield,

$$C(t) = \left[S'(t) + pS''(t)\right] + \dot{\mathrm{M}}_c(t) + \dot{\mathrm{M}}_c(t) \tag{2.20}$$

Present investment is understood here to fund present savings by capitalist producers, consumers, and by workers. At the same time, the dynamic formulation in (2.7) ensures that present investment is in turn funded from past savings by capitalist producers, and present net borrowing, itself shown below to be conditioned by existing holdings of previously unspent money. Equation (2.20) may be manipulated to yield an explicit expression for the sources of realisation of profits,

$$pS''(t) = \int_{-\infty}^{t} \left[S'(\tau) + pS''(\tau)\right] x_v(t-\tau;t)\,d\tau + \dot{B}(t) - S'(t) - \dot{\mathrm{M}}_c(t) - \dot{\mathrm{M}}_c(t) \tag{2.21}$$

Present capitalised profits arise as a result of capitalist own investment, which itself draws from past revenues including profits, plus net borrowing, minus recovered past capital outlays and savings by workers and capitalist consumers. Clearly the scale of net borrowing will help drive the realisation of profits whose capitalisation helps drive the accumulation of capital.

In the present framework total social capital made up of all value engaged by capitalist producers, that is, $K(t) \equiv \Pi(t) + \mathrm{N}(t) + \mathrm{M}_K(t)$. Its evolution is driven by investment, which is funded by capitalised profits and net borrowing, and motivates Theorem 2.2.

$$\dot{K}(t) \equiv pS''(t) + \dot{B}(t) \tag{2.22}$$

Theorem 2.2: *As long as a fraction of profits is capitalised, net credit extension makes a greater contribution to total social capital value than it does to total stocks of debt.*

Proof: By (2.21) and (2.20) it is evident that $\dot{K}(t)$ is proportional to $(1+p)\dot{B}(t)$. □

Theorem 2.2 and equation (2.19) highlight the analytical advantages of the dynamic framework developed here in relation to traditional Circuitist approaches to credit and accumulation. First, dynamic consideration of net credit extension makes possible monetary profits, the pursuit of which is the fundamental imperative of capitalist relations. Second, positive profits and their partial recommitment to investment gives rise to positive net worth for capitalist producers, which arises as the result of the expansion of own investment through the capitalisation of profits, and of the pace of net credit extension. On these bases it is possible to characterise the level of leverage or gearing taken on by aggregate social capital, and allows explicit consideration of the consequences, risks and contradictions posed by capitalist credit relations.

In order to pursue analysis of these, it is necessary first to inquire into the relationship between credit extension and the stocks of money arising in the accumulation of capital.

2.4 Money Hoards and Credit Relations

Money hoards in the circuit of capital and circuits of revenue fundamentally condition the pace of net credit extension in capitalist accumulation. They support credit relations by providing dynamic scopes for the circulation of bank liabilities issued *ex nihilo*, so long as the latter are accepted as embodiments of capital value.[8] Their magnitude and distribution relative to current production and consumption plans conditions the net demand for credit. The willingness of money holders to

[8] As argued in dos Santos (2011a, 2011b).

maintain a fraction of their holdings in the form of bank liabilities conditions the ability of the credit system to accommodate any given level of net credit demanded.

Credit relations give rise to a hierarchy of money forms in the capitalist economy. As argued by Lapavitsas (2003), in the resulting monetary structure different forms of credit money posses different market and institutional foundations for their circulation. The circulation of bank liabilities is supported by assets in bank portfolios and reserve holdings of more social forms of money. Through the development of the interbank market bank assets come to include claims on all borrowers, and banks become able to draw on the banking system's collective reserves. The main domestic reserve money are liabilities of the Central Monetary Authority (CMA). Their circulation is established as the money of settlement of the interbank market, and is later boosted by being deemed legal tender by the state.

The domestic legal-tender status of CMA liabilities does not have any purchase outside the national economy. Their circulation among capitalists in a position to withdraw value from the national economy has to be supported on market bases, which require the sound functioning of the interbank market, and CMA reserve holdings of a world money that settles international obligations. Although this need is most clearly felt at times of crisis, the CMA can ill afford to neglect rate at which its liabilities exchange for world money, which conditions domestic profitability the smooth functioning of the interbank market.

Considering for simplicity a closed economy, this structure may be represented formally. Suppose money holders are willing to hold a fraction $d(t)$ of their total holdings $M(t)$ in the form of bank liabilities $D(t)$, with the balance held as CMA liabilities $H^C(t)$. Banks hold CMA reserves $H^R(t)$, ensuring they bear relative illiquidity levels,

$$l(t) \equiv \frac{D(t) - H^R(t)}{D(t)} \tag{2.23}$$

Aggregate illiquidity is set by the balance-sheet choices of money holders and banks,

$$y(t) \equiv d(t)l(t) = \frac{D(t) - H^R(t)}{M(t)} \tag{2.24}$$

Total money in circulation, understood to be the result of endogenous money creation may be structurally measured in relation to total CMA reserves $H(t)$,

$$M(t) = \frac{1}{1-y(t)} H(t) \tag{2.25}$$

This relationship evolves dynamically in line with,

$$\dot{M}(t) = \dot{B}(t) = \frac{1}{1-y(t)} \left\{ \dot{H}(t) + \dot{y}(t) M(t) \right\} \tag{2.26}$$

Credit extension throws new money into circulation. New bank liabilities instantaneously appear in the hands of workers hired by the borrower, and capitalists selling inputs to the borrower. These holders will adjust their money portfolios in line with $d(t)$. Bank lending not only creates holdings of bank liabilities, but also creates demand on banks for CMA reserves.

In line with an endogenous understanding of money creation, equation (2.26) is taken to describe the various manners in which the banking system may accommodate any given level of credit demand $\dot{B}(t)$. The balance of methods used will hinge bank attempts to maximise profits. Bank profits will be conditioned by the costs of ensuring levels of bank liability circulation $d(t)$, and the costs and terms on which CMA money may be obtained. Their ability to accommodate credit demand is thus constrained by the general perceptions of the quality of bank assets, capital levels, and holdings of CMA reserves, and the costs and terms at which the CMA makes its own liabilities available. The latter depend on policy goals, which in settings of convertibility are constrained by the need to ensure the circulation of CMA liabilities.

3. The Contributions, Limits and Contradictions of Credit Relations in Accumulation

The general dynamic structure of accumulation, credit and portfolio behaviour outlined above above may be used to describe a wealth of dynamic macroeconomic interactions and behaviour. For instance, its explicit treatment of money hoards in accumulation and the balance-sheet requirements of any given pace of net credit extension leads to a distinctive appreciation of Steindl's (1952) "paradox of debt". Equation (2.21) may be manipulated to yield an expression relating own investment and net borrowing to savings by capitalist enterprises, workers and capitalist consumers,

$$\left[1+p(t)q(t)\right]S'(t)+\dot{\mathrm{M}}_c(t)+\dot{\mathrm{M}}_w(t)=\int_{-\infty}^{t}\left[1+p(\tau)q(\tau)\right]S'(\tau)x_v(t-\tau;t)\,d\tau+\dot{B}(t) \qquad (3.1)$$

Investment levels on the right-hand side fund "savings", consisting of additions to money hoards held respectively by enterprises, capitalist consumers and workers. These savings condition borrowing, but only indirectly, as total money hoards (including those held by enterprises) support given paces of net credit extension as described by the balance-sheet identity in (2.26).

Suppose capitalists facing bad prospects wish to reduce their pace of own investment and net borrowing in proportions that target a new, lower level of leverage. If the resulting fall in aggregate investment is met with an inelastic response in flows of savings by workers and capitalist consumers, (3.1) ensures capitalist revenues and retained earnings will fall disproportionately. This will dynamically frustrate original individual plans to lower levels of leverage, as capitalist revenues and retained earnings flows are reduced. The only way to reach the original target leverage involves lowering levels of liquidity, with enterprises increasing the pace at which retained earnings are recommitted to investment, which also runs counter to original plans.

At the same time, if demand for bank liabilities and the degree of CMA accommodation of bank demand for reserves remain unchanged, banks will experience increases in their levels of liquidity as inelastic savings fall less severely than credit demand. In this case interest rates may fall, which may help stabilise the situation as the pace of savings from personal revenues falls and as capitalist borrowing plans are revised. Alternatively, if demand for bank liabilities is also falling, or if the CMA's ability to accommodate net lending is compromised, as may be the case in a general crisis, vicious cycles of frustrated attempts to reduce leverage fuelling further decreases in investment and leverage may follow.

A wealth of thought experiments along these lines are possible. In order to derive tractable results concerning the most general features of the relationship between credit relations and capitalist accumulation, this section considers simple dynamic evolutions in which the model's parameters are held constant. The resulting exponential steady state paths afford general insights into the impact of sustained net credit extension on rates of growth and different aggregate measures of profitability. They also cast light into the inherent limits and contradictions of capitalist credit relations, which are pursued further in the next section through quasi-dynamic consideration of a discrete step changes to the economy's parameters.

3.1 Credit, Its General Impact on Growth and Its Limits

Abstracting from lags in consumption expenditures, the economy's structure is fully described by the system of integral and differential equations given by (2.2) - (2.7), (2.14) - (2.16), (2.22), and the definition of total capital in circulation.[9]

The resulting system contains an autonomous sub-system describing the economy's evolution in the sphere of circulation, where accumulation is constrained by its endogenous ability to generate demand flows allowing the dynamic realisation of profits. This sub-system may be identified from the definition of investment flows in (2.6) and the definition of sales flows given by (2.10) considered for instantaneous consumption expenditures. Normalising the pace of lending to the scale of simple reproduction, so that $\dot{B}(t) = h(t)S'(t)$, those two equations yield,

$$(1+p(t)q(t))S'(t) = \int_{-\infty}^{t} (1+p(\tau)q(\tau))S'(\tau)x_v(t-\tau;t)d\tau + h(t)S'(t) \tag{3.2}$$

Any given paths for $h(t)$, $p(t)$, $q(t)$ and $x_v(t-\tau;t)$ fully specify a path for $S'(t)$ as defined by (3.2). Put differently, the pace of own investment, the pace of credit and money creation, and the paths for the mark-up and capitalisation rates fully specify the structural evolution of this economy, when it is considered exclusively from the sphere of commodity circulation. So long as inventories are positive, productive constraints do not bind, and the specification of $S'(t)$ in (3.2) fully describes the economy's evolution, which can be recovered from the system's remaining equations.

While the dynamic interactions between the pace of own investment and net borrowing are potentially complex and manifold, in seeking to characterise the broad structural relationship

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[9] The system, recapped in Appendix A, is well-specified, has ten equations and ten endogenous unknown functions (five flows, four stocks, and one endogenous realisation time delay). The solutions for all ten unknowns are determined by three parameter paths $h(t)$, $p(t)$, and $q(t)$, and the evolution of the investment and production lag processes, $x_v(t-\tau;t)$ and $x_p(t-\tau;t)$.

between credit and accumulation it is useful to consider the most general, abstract case in which all parameters and lag processes are constant. In such a case, equation (3.1) simplifies to,

$$S'(t) = \frac{1+pq}{1+pq-h} \int_{-\infty}^{t} S'(\tau) x_v(t-\tau) d\tau \tag{3.3}$$

This is a Fredholm integral equation of the first kind with a difference Kernel, whose exponential solution is well known.[10] The resulting solutions for all stocks and flows in the economy are given in Appendix A, and are characterised by the Laplace Transforms of the lag processes for the system's complex-valued rate of growth g,

$$x_i^*(g) \equiv \mathscr{L}\left\{x_i(t), g\right\} = \int_0^\infty e^{-gt} x_i(t) dt, \tag{3.4}$$

An endogenous characterisation of the system's rate of growth can be obtained from the solution of (3.3), which requires that,

$$\frac{1}{x_v^*(g)} = \frac{1+pq}{1+pq-h} \tag{3.5}$$

This equation describes the determination in the sphere of exchange of the the system's steady-state rate of growth $g = g(h, p, q; x_v(t'))$. It is trivial to establish form (3.4) and (3.5) that when the pace of net credit extension is zero, so will the economy's rate of accumulation. The broader shape of $g = g(h, p, q; x_v(t'))$ is established in the next three results.

Theorem 3.1: *The economy's endogenous rate of accumulation $g = g(h, p, q; x_v(t'))$ as specified in (3.5) is a monotonically increasing, convex function on the pace of net credit extension, h.*

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[10] See Polyanin and Manzhirov (2008), p 115.

Proof: The derivative of both sides of (3.5) with respect to h may be identified using the chain rule and the fact that the derivative with respect to g of the Laplace Transform of a function $f(t)$ will be given by $-\mathscr{L}\{tf(t),g\}$. This yields,

$$g_h(h,p,q;x_v(t')) = \frac{1}{(1+pq)\mathscr{L}\{tx_v(t),g\}} \tag{3.6}$$

Since $\mathscr{L}\{tx_v(t),g\} > 0$, for the lag process functions considered here, this derivative is positive. Taking the second partial derivative,

$$g_{hh}(h,p,q;x_v(t')) = \frac{\mathscr{L}\{t^2x_v(t),g\}}{(1+pq)\mathscr{L}\{tx_v(t),g\}^2} \geq 0 \qquad \square$$

Seen exclusively from the sphere of circulation, steady states with higher sustained paces of net credit extension will experience higher sales flows and thue quicker paces of accumulation. The convexity on h of $g(h,p,q;x_v(t'))$ follows from the fact that by helping expand the scale of reproduction, net lending contributes to the expansion of credit demand and its accommodation.

Equation (3.5) also affords characterisations of the relationship between the pace of own investment, net credit extension and the rate of accumulation rate of accumulation.

Proposition 3.2: *Steady states with quicker paces of own investment, measured by the present value of the investment lag function $x_v(t')$ discounted by any positive rate, will exhibit higher rates of growth for any given set of parameters p, q, and h.*

Proof: By (3.4), $x_v^*(g) = \int_0^\infty e^{-gt}x_v(t)dt$, so that by the very definition of the pace of investment, an investment lag process ${}^1x_v(t)$ will correspond to a quicker pace of investment than another lag process ${}^2x_v(t)$ if and only if ${}^1x_v^*(g) < {}^2x_v^*(g)$ for any real g. Consequently, for any p, q, and h, the first lag process will satisfy equation (3.5) with $g_1 > g_2$, which satisfies it for the second. □

Explicit consideration of exponential-decay and discrete-delay specifications for the investment lag process allows a more precise demonstration of the result expressed in Proposition 3.2. Under those specifications the lag processes take the respective forms $x_v(t) = ve^{-vt}$, and $x_v(t) = \delta(v^{-1})$, where v thus provides a positive measure of the pace of investment in each setting. The Laplace Transforms of the processes are $x_v^*(g) = e^{-g/v}$, and $x_v^*(g) = \dfrac{v}{v+g}$. This leads to the following result,

Theorem 3.3: *Under exponential and discrete-delay lag processes, the endogenous steady-state rate of accumulation is rising on the respective positive measures of the pace of own investment.*

Proof: The expressions for the Laplace Transforms of the investment lag processes and Equation (3.5) allow explicit expressions for the economy's endogenous rate of accumulation. In the case of exponential decays it is given by,

$$g(h,p,q,v) = \frac{hv}{(1+pq-h)} \tag{3.7}$$

For discrete delays,

$$g(h,p,q,v) = v\ln\left\{\frac{1+pq}{1+pq-h}\right\} \tag{3.8}$$

And it is trivially obvious that in both specifications $\dfrac{\partial}{\partial v}g(h,p,q,v) > 0$. □

It is also evident that higher paces of own investment enhance the impact of net credit extension on the rate of accumulation.

Corollary 3.4: *Under exponential decay and discrete-delay lag processes, the impact of the pace of net credit extension on the endogenous steady-state rate of accumulation is rising on the respective positive measures of the pace of own investment.*

Proof: From (3.7) it is evident that for the respective lag processes,

$$\frac{\partial^2 g(h,p,q,v)}{\partial h \partial v} = \frac{1+pq}{(1+pq-h)^2} > 0,$$

$$\frac{\partial^2 g(h,p,q,v)}{\partial h \partial v} = \frac{1}{(1+pq-h)} > 0 \qquad \square$$

Higher paces of sustained net credit extension and of own investment will individually and jointly yield steady states with quicker paces of accumulation. But their ability to boost demand, profit flows and rates of growth faces important constraints in production. In line with Foley (1982), those may be understood by considering the endogenous evolution of the realisation lag in (2.3). Along steady state paths, T_r is constant, requiring (2.3) is equal to zero, which yields,

$$\frac{1}{x_r^*(g(h))x_p^*(g(h))} = 1+pq \tag{3.9}$$

As demand starts to exhaust inventories, T_r approaches zero, and demand-pull inflationary pressures build up. At such levels of demand, additional increases in the pace of credit extension or in the pace of own investment no longer ensure higher paces of real accumulation, but bid up money prices instead. Increases in the pace net credit extension will only contribute to higher paces of accumulation if $T_r < 0.$ This requirement turns (3.7) into,

$$\frac{1}{x_p^*(g(h))} \leq 1+pq \tag{3.10}$$

Inequality (3.10) defines a maximum rate of real growth g_m, for which it holds with equality. Here this maximum rate of growth corresponds to a maximum sustained pace of net credit extension, h_m, for which $g_m = g(h_m)$. The existence of a unique h_m is ensured by the fact that when $h = 0$, $g(h) = 0$, and $x_p^*(0)=1$, ensuring (3.10) holds strictly as long as $pq>0$. As h increases, the left hand side of (3.10) increases monotonically, while its right hand side stays constant.

This maximum pace is conditioned by the mark-up and capitalisation rates, the production lag process, as well as by the investment lag process, since $g=g(h,p,q;x_v(t'))$. Put differently, h_m is conditioned by the relationship between the velocity of money, which sets the pace at which money thrown to accumulation returns to markets as demand for commodities, the time productivity of labour, which sets the pace at which value thrown into investment returns to markets as supplies of commodities, and the rate of profit capitalisation, which measures the pace at which profits augment productive capacities through their reinvestment.

It is possible to solve explicitly for h_m for discrete-delay and exponential decay lag processes. As above let, in the first case v and π denote the reciprocal of the investment and production time delays respectively. In this case it follows from (3.10) that,

$$\frac{1}{x_p^*(g)}=e^{\frac{v}{\pi}\ln\left\{\frac{1+pq}{1+pq-h}\right\}}=\left\{\frac{1+pq}{1+pq-h}\right\}^{\frac{v}{\pi}}\leq 1+pq \tag{3.11}$$

Which may be solved to express the constraint on h_m,

$$h_m=(1+pq)-(1+pq)^{1-\frac{\pi}{v}} \tag{3.12}$$

It is trivial to establish that this maximum pace of steady-state net credit extension is rising on both pq and on π/v, as intuitively motivated above.

The same reasoning can be followed for exponential decays, where v and π will denote the respective rates of decay describing the dynamic evolution of own investment and commodity output production. From (3.7) and (3.10) it follows that,

$$h_m = \frac{(1+pq)\pi pq}{\pi pq + v} \tag{3.13}$$

Which ensure that h_m is also rising both on the rate of profit capitalisation and the time-productivity of labour relative to the pace of investment, measured here by π/υ.

3.2 Credit, Profitability and Profit of Enterprise

The model's steady-state solutions also afford general insights into the structural relationship between net lending, own investment, aggregate demand and capitalist revenues. Credit relations can be shown directly to condition aggregate profitability on total capital in circulation, as well as the relative weight of the aggregate debt burden. As such they condition aggregate flows of *profit of enterprise*, that is gross profits net of interest payments, and aggregate measures of credit risk. These relationships have important implications for investment and funding behaviour, the business cycle and the crises.

Consider first the effect of the pace of net credit extension on the steady-state rate of profit on total capital in circulation. This will be given by aggregate profit flows relative to total capital in circulation,

$$\rho_s(h) \equiv \frac{S''(t)}{K(t)} = \frac{qg(h)}{(pq+h)} \tag{3.14}$$

Theorem 3.4: *Exponential steady-states with higher paces of net credit extension will,* ceteris paribus*, enjoy higher rates of profit on aggregate social capital.*

Proof: From (3.14) it follows that,

$$\frac{d}{dh}\rho_s(h) = \frac{q}{(pq+h)^2}\{g'(h)(pq+h) - g(h)\} \tag{3.15}$$

By Theorem 3.1, $g'(h)pq > 0,\ \forall pq > 0$. Also, $g(h)$ is convex, which since $g(0) = 0$ ensures that the other terms inside the brackets in (3.18) obey $g'(h)h - g(h) > 0$, all of which establishes that,

$$\frac{d}{dh}\rho_s(h) > 0 \qquad \square$$

Two important points follow from (3.14). First, as shown by Proposition 3.2 and Theorem 3.3, higher paces of own investment will lead to higher rates of accumulation for any given levels of h, p, and q. By (3.14) they are shown also to lead to higher rates of aggregate profit. Both quicker paces of own investment and quicker paces of net lending are in themselves sufficient to heighten the dynamic profitability of social capital.

Second, following Foley (1982), it is possible to use (3.14) to characterise the relationship between profitability and the economy's rate of growth, which in this framework is explicitly mediated by the pace of net credit extension. This yields a modified version of the Cambridge Equation,

$$g(h) = \rho_s(h)\left\{p + \frac{h}{q}\right\} \tag{3.16}$$

Sustained net credit extension thus contributes to growth through two distinct channels. It supports the realisation of profits on existing capital value, some of which are capitalised and recommitted to investment, and it directly finances new investment undertakings. At the broadest level it is on the basis of these potential effects on aggregate profitability and growth that banking capital stakes a claim on a share of aggregate capitalist profits.

Yet it is that very claim that ensures credit relations also contribute to the possibility of credit and monetary crises. In similar fashion to own investment, sustained net credit flows boost demand, profits and accumulation. But they uniquely establish obligatory claims on future profits, which will be divided between interest payments, and residual profit of enterprise.[11] This division creates

[11] In the monetary structure considered in section 2.4 interest payments on loans fund interest payments on bank liabilities, interest payments by banks on borrowings of CMA liabilities, and bank profits. As neither bank capital nor the fiscal operations of the capitalist state are considered here, bank profits and the distinctive seigniorage flows accruing to the CMA are not considered. Interest revenues on bank liabilities are treated as all other capitalist revenues, and are subject to the same allocation and delays.

opportunities for additional gains in the profitability of borrowing enterprises through leverage. But it also gives rise to potential instability and crises.

To inquire into the relationship between credit relations, investment and aggregate credit risk it is thus necessary to consider flows of profit of enterprise, and the rate of return on own capital they represent. The rate of profit of enterprise offers a more concrete measure of the imperative governing capitalist investment than the rate of profit on total social capital. Profit of enterprise flows also offer an aggregate measure of credit risk. As those fall towards zero it is increasingly likely that large number of individual enterprises are experiencing negative profits net of debt service payments.

In such situations, distressed enterprises will seek to meet debt service obligations and avoid bankruptcy by drawing on their existing money holdings, securing new credit, or liquidating capital value in commodity form. Credit relations will here be destructive of capital value. Interest payments will effect appropriations by the banking system of capital value previously held by enterprises. Further, if sufficient numbers of enterprises are unable to meet debt payments, significant bank asset losses follow. As claims by banks on capitalist enterprises sustain the circulation of claims on banks as monetary embodiments of capital, such crises prove destructive of capital value in monetary form.[12]

Formally, profit of enterprise is given by $E(t) \equiv S''(t) - i(t)B(t)$. Its rate is measured in relation to the stock of own capital, $K(t) - B(t)$, yielding, along exponential steady states,

$$\rho_e(h) \equiv \frac{S''(t) - iB(t)}{K(t) - B(t)} = \frac{1}{p}\left\{ g(h) - \frac{h}{q} i \right\} \tag{3.18}$$

Using (3.17) to express this rate in relation to the social rate of profit yields,

$$\rho_e(h) = \rho_s(h) + \frac{h}{pq}\left\{ \rho_s(h) - i \right\} \tag{3.19}$$

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[12] dos Santos (2011b).

Equation (3.19) identifies two distinct contributions sustained net credit extension makes to the aggregate profitability of capitalist enterprise. First, sustained net credit extension boosts the profitability of all capital in circulation, as demonstrated in Theorem 3.3. Second, it additionally presents the possibility of gains from leverage commensurate to, as shown by the second term of (3.21), the overall quantity of leverage multiplied by the gains (or losses) made possible by the different returns earned on capital in circulation and loans. Here credit relations effect distributions of profit flows that, so long as the rate of interest is lower than the rate of return on circulating capital, benefit enterprises.

The relationship between net credit extension and flows of profit of enterprise may be measured by considering the ratio of gross profits to debt service obligations for any given rate of interest, which provides an aggregate measure of financial robustness of capitalist enterprises,

$$R(h) \equiv \frac{S''(t)}{iB(t)} = \frac{qg(h)}{ih} \tag{3.20}$$

Note first that for any given pace of net credit extension, this measure will be higher in economies with quicker paces of own investment, by Proposition 3.2 and Theorem 3.3. Similarly, higher sustained paces of net credit extension will boost steady-state financial robustness,

Theorem 3.5: *Exponential steady-states with higher paces of net credit extension will,* ceteris paribus*, enjoy higher levels of financial robustness, as measured by profit flows relative to debt service flows. The same holds for steady-states with quicker paces of own investment.*
Proof: Taking the derivative of both sides of (3.20),

$$\frac{d}{dh}R(h) = \frac{q}{ih^2}\{g'(h)h - g(h)\} \tag{3.21}$$

Since $g(0) = 0$ and, by Theorem 3.1, $g(h)$ is convex, this derivative is positive. □

The discussion in this section yields two important structural findings on the role of credit relations in capitalist instability and crises. First and most generally, the dynamic profitability and financial robustness of capitalist enterprises have determinations in the sphere of production and in the sphere of exchange. In the former they are conditioned by the composition of capital, and the rate of exploitation, and constrained by the time-productivity of labour. In the latter, they are conditioned by the overall pace of investment, including own and leveraged undertakings.[13] This result represents but a dynamic generalisation and application to profit of enterprise of the dependence of profits on investment, well understood since its explicit discovery by Michal Kalecki.[14]

Notably, this structural result points to potentially destabilising accelerator effects if investment, credit-demand and credit-accommodation behaviour are driven by the profitability of capitalist enterprises. If sufficient numbers of individual capitalists boost their pace of overall investment, increasing their relative illiquidity and indebtedness, aggregate sales flows, profitability and financial robustness will be boosted. Significantly, the same self-fulfilling logical may play out in the opposite direction. Despite these potential collective gains, the individual and competitive character of appropriation ensures the levels of coordination necessary to sustain such paths elude capitalists as a class, particularly during times of downturn and distress. As Kalecki (1967) noted, "capitalists do many things as a class, but they certainly do not invest as a class".

Second, these findings have important implications for analysis of the distinctive contribution of credit to profitability and instability. Greater sustained paces of net credit extension and higher aggregate levels of leverage are not in themselves capable of increasing the likelihood of crisis. In fact, as with own investment, they contribute positively to aggregate profitability and financial robustness. But credit relations do make a distinctive contribution to instability in accumulation, by rendering the economy more vulnerable to any endogenous or exogenous disruption to mark-up rates, rates of interest, paces of own investment, or to the pace of net credit extension itself. The next section turns precisely to those issues.

4. The Contribution of Credit Relations to Financial Vulnerability and Crises

Despite the collective gains sustained net credit extension generates for social capital, disruptions to credit flows arise recurrently in competitive capitalist accumulation. The portfolio behaviour

[13] They will also generally be conditioned by paces of consumption, assumed to be instantaneous here.

[14] See, for instance, Kalecki (1954).

underpinning credit extension is governed by the imperative of individual appropriation. Expectations of impending financial problems, just or just speculative, are themselves sufficient to trigger individual actions resulting in falls in the pace of credit extension, which in turn create or exacerbate financial distress in enterprises and banks. In such a setting, competitive appropriation regularly ensures that it is individually sensible to abandon positions in the domestic economy, reduce holdings of bank liabilities, and taper the pace of net credit extension, even when the opposite actions could collectively moderate the impact or wholly avert widespread financial distress. Even more so than investing, capitalists do not hold liabilities as a class.

Falling paths in the net pace of credit extension create problems not posed by other developments reducing demand flows, such as falls in the pace of own investment or consumption. Past lending bequeaths to the future payment obligations broadly proportional to itself, so that any demand disruptions raise the spectre of falls in aggregate profit of enterprise. In an economy with credit, profit of enterprise is the defining measure and fulcrum of crises, irrespective of their original causes. Under negative or small positive levels of aggregate profit of enterprise it is very likely that many enterprises will be unable to make debt payments. In such situations, enterprises will seek to avoid bankruptcy by securing and holding on to *means of payment*, originating in their earlier money holdings, the liquidation of capital in commodity form, or in new credit.[15] Particularly in a setting of falling net credit extension and aggregate demand, those measures will in many cases fail to prevent bankruptcy. The resulting deterioration of bank assets may erode confidence in the robustness of banks (and of the national economy), and trigger monetary disruptions that feed further contractions in credit.

This section analyses the contributions of credit extension and broader disruptions to capitalist credit crises. It does so by inquiring into the impact of changes in the parameters of accumulation on aggregate profit of enterprise. Instrumentally, it considers step-change perturbations to exponential steady-state evolutions.[16] On these bases it helps identify the credit moments of disruptions to accumulation traditionally emphasised by existing Marxian, Keynesian and Post-Keynesian analyses.

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[15] See de Brunhoff's (1979) synthesis and development of Marx's (1894) account of the credit moments of crises.

[16] Along the lines of Foley (1986), § 15.

The section also offers a new, aggregate measure of financial vulnerability: the minimum *ex post* pace of credit extension an economy needs to sustain in order ensure positive aggregate profit of enterprise for any given disruption. On this basis it shows that while economies following exponential steady states with higher paces of credit extension are more robust while lending is sustained, they are simultaneously more vulnerable to the possibility that any given disruption will trigger generalised bankruptcies, credit and monetary distress.

4.1 The Contribution of Credit Relations to Crisis

Equations (3.14) and (3.20) show that falls in aggregate profitability and financial robustness require either falls in the pace of own investment, falls in the mark-up rate, or increases in the effective rate of interest facing capitalist enterprises. The contribution of each of these developments to crisis has been variously emphasised by heterodox analyses. Some Post- and New-Keynesian contributions have pointed to endogenous processes ensuring the rate of interest rises as the pace of lending picks up during the cyclical upswing.[17] Classical Keynesian contributions have emphasised the impact of low paces of investment (as well as the pace of consumption) in extending crises. Traditional Marxian accounts of capitalist crises distinctively focus on the sphere of production, where falling mark-up rates may arise as a result of endogenous and competitive technical innovation that lower the relative weight of labour power in the composition of capital.

Contractions in the pace of credit extension will typically be an integral part of those disruptions. It is easy to see from the account offered above that those contractions will in themselves adversely affect aggregate profitability, profit of enterprise and financial robustness. Any fall in leveraged or own investment immediately reduces contemporaneous profit flows by the same amount.[18] At the same time, only the rate of change of stocks of total capital in circulation and debt change instantaneously. The social rate of profit, flows of profit of enterprise, and financial robustness as measured in (4.15), (4.18), and (4.20) respectively, will thus immediately deteriorate as a result.

To examine the impact of disruptions of credit flows more formally, in a manner conducive to their integration into analyses of capitalist crises along Keynesian, Post-Keynesian or Marxian lines, consider the following perturbation. Suppose an economy is characterised by a constant parameter

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[17] Ultimately drawing on Kalecki's (1937) original "principle of increasing risk".

[18] Lags in workers' consumption may partially delay the transmission of this fall to current profits.

vector $\vec{x}_0 = \left[p_0, q_0, x_p(t'), x_v^0(t'), h_0, i_0 \right]$, with $h_0, p_0, q_0 > 0$, and lag processes ensuring positive inventories. This economy will follow steady-state paths with exponential stocks and flows $\left\{ S_0(t), S_0'(t), S_0''(t), C_0(t), P_0(t), B_0(t), K_0(t) \right\}$. If unperturbed, at $t = \theta$ aggregate profit of enterprise will be given by,

$$E_0(\theta) = S_0''(\theta) - i_0 B_0(\theta) \tag{4.22}$$

Suppose instead that at $t = \theta$ the economy experiences a step change in its parameters to new vector $\vec{x}_1 = \left[p_1, q_1, x_p(t'), x_v^1(t'), h_1, i_1 \right]$, under which it is assumed inventories are always positive. From this time onwards, own investment will be funded from retained earnings accrued before $t = \theta$, which evolved exponentially, and those accrued after the step change. Own investment funds present sales and profits alongside new net borrowing $h_1 S'(t)$. Expressing this relationship in relation to gross profit flows $G(t)$ yields,

$$G(t) = \frac{q_1}{(1 + p_1 q_1 - h_1)} \left\{ \int_{-\infty}^{\theta} \left\{ \frac{(1 + p_0 q_0)}{q_0} S_0''(\tau) x_v^1(t-\tau) d\tau \right\} + \int_{\theta}^{t} \frac{(1 + p_1 q_1)}{q_1} S''(\tau) x_v^1(t-\tau) d\tau \right\} \tag{4.23}$$

Interest obligations will be given by payments on debt acquired prior to the step change, which is taken to command an interest rate i_v, and payments on debt acquired since the step change,

$$Y(t) = i_v B_0(\theta) + \frac{i_1 h_1}{q_1} \int_{\theta}^{t} S''(\tau) d\tau \tag{4.24}$$

Profit of enterprise after the step change will thus be given by,

$$E(t) = G(t) - Y(t) \tag{4.25}$$

Equations (4.23) - (4.25) offer a full description of the non-exponential evolution of flows of profit of enterprise after the step change. Note that as the time since the step change increases the first terms in both (4.23) and (4.24) tend to vanish, and the evolution of profit of enterprise approximates a steady-state evolution defined by $\vec{x}_1$.

But it is the early evolution of profit of enterprise that is most germane to present purposes, as it will determine the likelihood of enterprise distress and credit crisis immediately following the perturbation. Consider its level at the exact time of the step change, $t = \theta$. In that case the second terms in both $G(t)$ and $Y(t)$ vanish, yielding,

$$E(\theta) = \frac{(1+p_0 q_0)}{(1+p_1 q_1 - h_1)} \frac{q_1}{q_0} \int_{-\infty}^{\theta} S_0''(\tau) x_v^1(\theta - \tau) d\tau - i_v B_0(\theta) \tag{4.26}$$

Expressing $S_0''(t')$ in its full exponential form, and expressing the integral in relation to $t'' = \theta - \tau$ renders it equal to $S_0''(\theta)\mathscr{L}\left\{x_v^1, g(h_0)\right\}$. Using the exponential definition of $B_0(\theta)$ and equation (4.5) to express $(1+p_0 q_0)$, we have,

$$E(\theta) = S_0''(\theta)\left\{\frac{q_1}{q_0}\frac{(1+p_0 q_0 - h_0)}{(1+p_1 q_1 - h_1)}\frac{\mathscr{L}\left\{x_v^1, g(h_0)\right\}}{\mathscr{L}\left\{x_v^0, g(h_0)\right\}} - \frac{i_v h_0}{q_0 g(h_0)}\right\} \tag{4.27}$$

It is now possible to consider the impact of the step change in the economy's parameters on aggregate profit of enterprise, which will change in relation to that of an unperturbed economy as,

$$E(\theta) - E_0(\theta) = S_0''(\theta)\left\{\frac{q_1}{q_0}\frac{(1+p_0 q_0 - h_0)}{(1+p_1 q_1 - h_1)}\frac{\mathscr{L}\left\{x_v^1, g(h_0)\right\}}{\mathscr{L}\left\{x_v^0, g(h_0)\right\}} + (i_0 - i_v)\frac{h_0}{q_0 g(h_0)} - 1\right\} \tag{4.28}$$

Equation (4.28) makes clear that all developments emphasised in Marxian, Keynesian and Post-Keynesian approaches to crises can contribute in this setting to falls in aggregate profit of enterprise, consequently contributing to heightened financial instability. A fall in the mark-up rate, ensuring $q_1 < q_0$, is sufficient to make (4.28) negative, *ceteris paribus*. The same holds for falls in the pace of own investment, under which $\mathscr{L}\left\{x_v^1, g(h_0)\right\} < \mathscr{L}\left\{x_v^0, g(h_0)\right\}$, and increases in the rate of interest applicable to previously acquired debt.

The equation also points to reductions in the pace of credit extension, which ensure $(1+p_0q_0-h_0)<(1+p_1q_1-h_1)$, as an additional source of instability. It is possible to conclude that purely speculative concerns about prospects for enterprises or banks leading to falls in the pace of credit extension will be in themselves sufficient to lower aggregate profit of enterprise, adversely affecting enterprises and banks. Reductions in the pace of lending may also of course occur as an integral part of broader disruptions affecting the pace of own investment, mark-up or interest rates. In those crises, such reductions make their own contribution to falling profits of enterprise.

4.2 A Measure of Aggregate Financial Vulnerability

This examination of the evolution of aggregate profit of enterprise across a step change in the economy's parameters leads to a distinctive measure of aggregate financial vulnerability. Any such perturbation defines a minimum new pace of credit extension below which aggregate profit of enterprise will be negative. This minimum value is a useful measure of aggregate financial vulnerability as sustaining levels of credit extension is generally difficult under general crisis conditions. The higher the levels of lending required to keep aggregate profit of enterprise from approaching zero, the more vulnerable the economy is to the corresponding disruptions to mark up rates, interest rates, or to the pace of own investment.

This minimum pace h_1^* is determined by all other parameter values in $\vec{x}_0$ and $\vec{x}_1$. Significantly, this includes the *ex ante* pace of credit extension h_0. On this basis it is possible to identify the level of financial vulnerability faced by an economy evolving along exponential steady states to any arbitrary shock as a function of its present pace of credit extension.

Formally, equation (4.27) can be simplified by using equation (4.5), yielding,

$$E(\theta)=S'_0(\theta)\left\{q_1\frac{(1+p_0q_0)}{(1+p_1q_1-h_1)}\mathscr{L}\left\{x_v^1,g(h_0)\right\}-\frac{i_vh_0}{g(h_0)}\right\} \tag{4.29}$$

Aggregate profit of enterprise will be positive across a step parameter change only if the term inside the brackets is positive. The minimum level of the new pace of credit extension h_1 that ensures this will be given by,

$$h_1^* = (1+p_1q_1) - (1+p_0q_0)\left\{\frac{q_1}{i_v}\frac{g(h_0)}{h_0}\mathscr{L}\left\{x_v^1, g(h_0)\right\}\right\} \tag{4.30}$$

A number of significant results regarding the financial vulnerability of an economy to any given disruption follow. Consider first the relative severity of the disruption. Perturbations taking the economy to states with a low mark-up rates q_1 relative to the applicable rate of interest i_v will require higher ongoing paces of credit extension to remain stable.[19] Perturbations involving stronger reductions in the pace of own investment, which yield lower values for $\mathscr{L}\left\{x_v^1, g(h_0)\right\}$, will also demand higher paces of credit extension to ensure aggregate solvency. As expected, stronger disruptions require greater reticence in credit extension to ensure financial stability.

Equation (4.30) also allows characterisation of an economy's financial vulnerability in relation to its *ex ante* pace of net credit extension. The impact on h_1^* of h_0 involves two effects. Mathematically, the first effect involves the ratio between the economy's original rate of growth $g(h_0)$ and the original pace of credit extension h_0. The second effect involves the impact of original levels of net credit extension on the Laplace Transform of the new lag process taken with respect to the original rate of growth, $\mathscr{L}\left\{x_v^1, g(h_0)\right\}$.

Economically, the first effect was already evident under Theorem 3.5. An economy with high sustained paces of credit extension experiences high profit flows relative to all stocks in the economy, including debt. As a result such an economy enjoys greater financial robustness *ex ante*, which supports its robustness immediately following the disturbance. This will push h_1^* towards lower values. But this is countered by the second effect, which arises from the dynamic relationship between present and past sales flows. High paces of net lending sustain high rates of growth, particularly in economies with quick paces of own investment. Higher growth implies all past flows are smaller relative to present flows than in slower-growing economies. Past retained earnings, which fund present own investment, will thus be smaller relative to the present scale of

[19] Notably, in a situation of distress the higher credit extension requirement may itself lead to increases in the rate of interest, compounding the original problem.

reproduction. The more leveraged an economy is, the more reliant it is on future credit flows to sustain sales and profits, and the more vulnerable it is to any disruptions to aggregate demand.

It is possible to characterise the net result of both effects for perturbations involving falls in the pace of own investment when lag processes take the form of exponential decays and discrete delays. This establishes that the second effect dominates the first one across perturbations involving falls in the pace of own investment.

Theorem 4.1: *Under exponential-decay and discrete-delay lag processes, economies evolving with higher steady-state paces of credit extension are more vulnerable to perturbations involving reductions in the pace of own investment, as they will require higher minimum paces of credit extension after such perturbations to maintain positive aggregate profit of enterprise.*

Proof: Under exponential decays equation (4.30) becomes,

$$h_1^* = (1 + p_1 q_1) - (1 + p_0 q_0)\frac{q_1}{i_v} v_0 \left[1 + p_0 q_0 - h_0 (1 - v_0/v_1)\right]^{-1} \tag{4.31}$$

From which it is evident that,

$$\frac{\partial}{\partial h_0} h_1^* = (1 + p_0 q_0)\frac{q_1}{i_v}\left[1 + p_0 q_0 - h_0 (1 - v_0/v_1)\right]^{-2}\left[v_0 (v_0/v_1 - 1)\right] \tag{4.32}$$

Which will be positive whenever $v_1 < v_0$, as will be the case for any perturbation that lowers the pace of own investment. As shown in Appendix B, the same line of reasoning may be followed for discrete delays, under which $\frac{\partial}{\partial h_0} h_1^* \geq 0$ for the same type of perturbation. □

Economies evolving along exponential steady states with high paces of credit extension are more financially robust while lending is sustained. They are simultaneously more vulnerable to negative disruptions to accumulation, under which they need to maintain higher levels of credit extension to avoid generalised credit crises. Significantly, this vulnerability does not follow from movements in the interest rate or the mark-up rate. It follows from the sheer structural dependence of highly leveraged systems on ongoing high paces of credit extension to sustain sales and profit flows.

5. Conclusions

The paper offered a dynamic, structural characterisation of the contributions, limits and contradictions of credit relations in the accumulation of capital. The resulting framework makes distinctive contributions to existing heterodox work that has offered deliberate analyses of money and credit in the process of accumulation.

In relation to Circuitist approaches,[20] the framework advanced by the paper can account for capitalist monetary profits. The capitalisation of those profits lays the basis for positive net worth and, consequently, aggregate levels of leverage arising from borrowing and own investment decisions. As such the framework allows analysis of the risks posed by credit as inherent features of credit relations. Explicit and dynamic consideration of the process of production and supply also allows characterisations of the limits of the ability of net credit extension to boost real growth. Finally, the framework is based on explicit consideration of the monetary limits and constraints bearing on the *ex nihilo* issuance of bank liabilities.

In relation to traditional Marxian approaches, the framework locates credit relations in relation to stocks and flows of money in the circuit of capital. It offers explicit consideration of the creation and circulation of credit money advanced *ex nihilo* in the process of accumulation. It also enables analysis of credit relations as one of the key determinants of dynamic profitability, instability and crisis within the sphere of circulation. Most broadly, the discussion outlined a framework for integrated analyses of the credit moments of crises originating in overproduction, as well as those involving endogenously rising interest rates, or falling paces of own investment.

On the basis of the comparative-static properties of exponential steady states and consideration of simple step-change perturbations to such states, a number of structural results were established. Sustained net credit extension boosts growth, aggregate profitability and the aggregate financial robustness of capitalist enterprises. These benefits are constrained by the economy's dynamic productive capacities, and tempered by the risks of credit crises posed by potential disruptions to accumulation. Paradoxically, economies with high paces of borrowing and own investment, while more profitable and robust as long as the pace of lending is sustained, are more prone to credit crises when faced with any given perturbation involving a fall in the pace of own investment. Credit

[20] See Lavoie (1992) and Graziani (2003) for the canonical expositions of French and Italian circuitism, respectively.

relations are fundamentally contradictory in that their very ability to boost accumulation and present financial robustness inherently renders the economy more financially vulnerable.

These structural findings have important implications for analysis of the credit moments of the business cycle. The impact of paces of own and leveraged investment on the aggregate profitability of social capital and of capitalist enterprises points to destabilising accelerator effects. Individual expectations about profitability may drive investment, credit-demand and credit-accommodation decisions that are sufficient in themselves to boost aggregate profitability and the financial robustness of enterprises. On the upswing this ensures market signals thus drive the boom along, while simultaneously rendering the economy more vulnerable to any given disruption that tapers the pace of own investment.

Endogenous tendencies for falling mark-up rates or rising rates of interest will ensure such disruptions are recurrent, as will purely speculative falls in credit demand, its accommodation, or in holdings of bank liabilities. Once aggregate profit of enterprise is disrupted, the same self-fulfilling logic generates self-reinforcing falls in the profits of capitalist enterprises. But while the sky may have seemed the limit for profitability and robustness during the upswing, bankruptcy enforces a hard floor against which accumulation soon crashes. Credit relations come to destroy capital values through payment obligations that exceed gross profit flows, and through monetary disruptions that destroy capital in the form of claims on banks.

So long as investment and borrowing are guided by enterprise profits, swings along these lines are structurally inherent to capitalist credit. Because of their significant capacity to boost profitability while simultaneously rendering accumulation more fragile, credit relations force capitalism constantly to straddle between expanding and curbing the scale of credit. Critical analyses of the cyclical and historical limits of contemporary capitalism will be well served by explicit consideration of the attendant contradictions.

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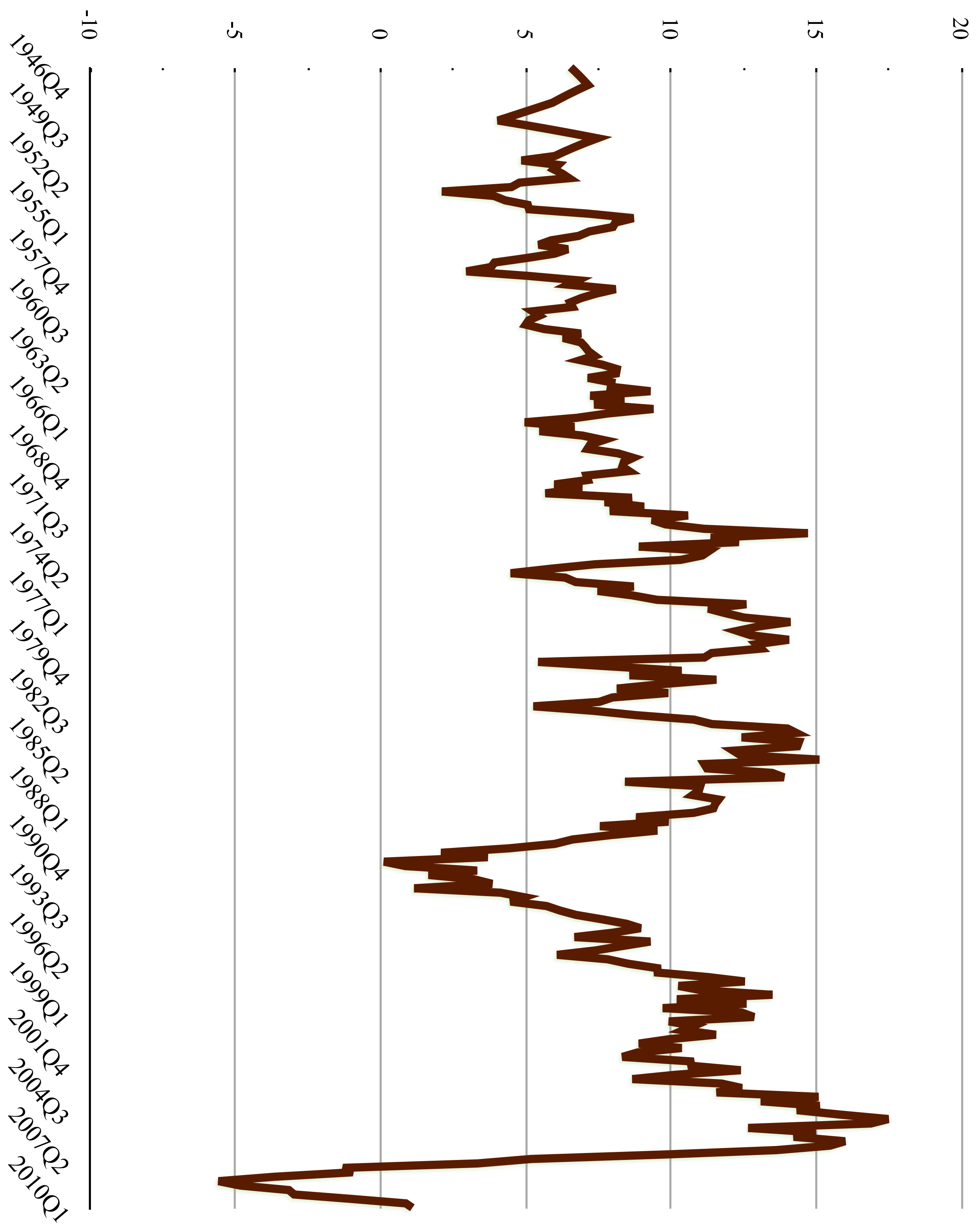
Figure 1

Net Borrowing by US Households and Nonfinancial Business to GDP, Seasonably Adjusted

Calculated from Flow of Funds of the United States, 1946Q4 - 2011Q1

(values until 1951 estimated from annual data)



Appendix A | The Full Model Postulated and Its Steady-State Solutions

Flows

$$P(t) = \int_{-\infty}^{t} C(t')x_p(t-t')dt' \tag{A.1}$$

$$S'(t) = \frac{S(t)}{1+q} \tag{A.2}$$

$$S''(t) = \frac{qS(t)}{1+q} \tag{A.3}$$

$$C(t) = \int_{-\infty}^{t} \left[S'(\tau) + pS''(\tau)\right]x_v(t-\tau)d\tau + \dot{B}(t) \tag{A.4}$$

$$S(t) = C(t) + (1-p)S''(t) \tag{A.5}$$

Stocks

$$\dot{\Pi}(t) = C(t) - P(t) \tag{A.6}$$

$$\dot{\mathrm{N}}(t) = P(t) - S'(t) \tag{A.7}$$

$$\dot{\mathrm{M}}(t) = S'(t) + pS''(t) - \left(C(t) - \dot{B}(t)\right) = \dot{B}(t) \tag{A.8}$$

Which add up to,

$$\dot{\mathrm{K}}(t) = pS''(t) + \dot{B}(t) \tag{A.9}$$

The endogenous realisation lag

$$\dot{T}_r(t) = 1 - \frac{D_A(t)}{(1+q)P(t-T_r)} \tag{A.10}$$

Full Exponential Solutions

Exponential growth paths for all stocks and flows in the model can be normalised to the scale of simple reproduction $S'(t)$, yielding

$$S'(t) = S'(0)e^{g(h)t} \tag{B.1}$$

Total sales

$$S(t) = S'(t)(1+q) \tag{B.2}$$

Profits

$$S''(t) = qS'(t) \tag{B.3}$$

Investment

$$C(t) = [1+pq]S'(t) \tag{B.4}$$

Output

$$P(t) = [1+pq]x_p^*(g)S'(t) \tag{B.5}$$

Money hoards and debt outstanding

$$M(t) = \frac{[1+pq]}{g(h)}\left[1-x_v^*(g)\right]S'(t) = h\frac{S'(t)}{g(h)} = B(t) \tag{B.6}$$

Unfinished outputs

$$\Pi(t) = \frac{[1+pq]}{g(h)}\left[1-x_p^*(g)\right]S'(t) \tag{B.7}$$

Inventories,

$$\mathrm{N}(t) = \frac{\left[(1+pq)x_p^*(g)-1\right]}{g(h)}S'(t) \tag{B.8}$$

Total capital in circulation,

$$K(t) = \frac{[h+pq]}{g(h)}S'(t) \tag{B.9}$$

Appendix B

Proof of Theorem 4.1 for Discrete-Delay Lag Processes

Under discrete delay lag processes equation (4.30) becomes,

$$h_1^* = \left(1 + p_1 q_1\right) - \frac{q_1}{i_v}\left\{\frac{v_0 \ln\left(\frac{\left(1 + p_0 q_0\right)}{\left(1 + p_0 q_0 - h_0\right)}\right)}{h_0\left(\frac{\left(1 + p_0 q_0\right)}{\left(1 + p_0 q_0 - h_0\right)}\right)^{\frac{v_0}{v_1}}}\right\} \tag{B.1}$$

Which leads to,

$$\frac{\partial}{\partial h_0} h_1^* = \left(1 + p_0 q_0\right)\frac{q_1}{i_v}\left\{\frac{v_0\left[\left(h_0 v_0 + v_1\left(1 + p_0 q_0 - h_0\right)\right)\ln\left(\frac{\left(1 + p_0 q_0\right)}{\left(1 + p_0 q_0 - h_0\right)}\right) - h_0 v_1\right]}{h_0^2 v_1\left(1 + p_0 q_0 - h_0\right)\left(\frac{\left(1 + p_0 q_0\right)}{\left(1 + p_0 q_0 - h_0\right)}\right)^{\frac{v_0}{v_1}}}\right\} \tag{B.2}$$

The sign of (B.2) hinges on the sign of,

$$\left(h_0 v_0 + v_1\left(1 + p_0 q_0 - h_0\right)\right)\ln\left(\frac{\left(1 + p_0 q_0\right)}{\left(1 + p_0 q_0 - h_0\right)}\right) - h_0 v_1 \tag{B.3}$$

Which will be positive when,

$$\frac{v_0}{v_1} > \left[\left(\ln\left(\frac{1 + p_0 q_0}{1 + p_0 q_0 - h_0}\right)\right)^{-1} - \frac{1 + p_0 q_0 - h_0}{h_0}\right] \tag{B.4}$$

Since by hypothesis $v_0/v_1 > 1$, and the term inside the brackets in (B.4) can be numerically shown to be smaller than 1 whenever $h >> 0$, the theorem holds as postulated for exponential-decay lag processes.