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Abstract

Whilst joint ventures offer a potentially attractive form of corporate and industrial organisation, they also experience high rate of break-up within ten years from their initial formation. In this paper, we model this process not as an uncertain random event, but rather as the predictable outcome of underlying economic variables, with break-up within a finite time resulting even under conditions of complete certainty. Given the prevalence of joint venture break-ups, it is in the interests of both partners in an equity joint venture to be fully aware of their own optimal durations of the joint venture in their initial negotiations for the formation of the equity joint venture. Where the underlying economic parameters imply differences in their individual optimal durations of the joint venture, there is therefore scope for mutually beneficial agreements on a binding date for the break-up of the joint venture, and for side payments to enable this binding agreement to be reached, either as cash payments or in terms of their relative shareholdings in the jointly-owned separate company that will manage the equity joint venture. In addition, there is scope for a differential corporate tax rate on the joint venture, compared to that on the go-it-alone businesses of the two partners, in order bring the two partners' privately optimal durations into line with the socially optimal duration of the joint venture.

Keywords: Joint ventures, equity shareholdings, optimal duration, corporate tax rates.

JEL Classification: D21, H25, L22, L24.

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1. Introduction

Whilst joint ventures offer a potentially attractive form of corporate and industrial organisation, their high rate of break-up within ten years from their initial formation has been noted by Hewitt (2008), Kogat (1989) and others. In this paper, we model this process not as an uncertain random event, but rather as the predictable outcome of underlying economic variables, with break-up within a finite time resulting even under conditions of complete certainty. Given the prevalence of joint venture break-ups, it is in the interests of both partners in an equity joint venture to be fully aware of their own optimal durations of the joint venture in their initial negotiations for the formation of the equity joint venture. Where the underlying economic parameters imply differences in their individual optimal durations of the joint venture, there is nevertheless scope for mutually beneficial agreements on a binding date for the break-up of the joint venture, and for side payments to enable this binding agreement to be reached, either as cash payments or in terms of their relative shareholdings in the jointly-owned separate company that will manage the equity joint venture. From the viewpoint of economic policy, it is also of interest to determine how far the privately determined duration of the equity joint venture will diverge from a socially optimal duration, and the associated scope for corporate taxes and subsidies on the joint venture to remedy this divergence. We will examine these questions in more detail in the following sections.

2. The Individual Optimal Durations

We will investigate the consequences of firms A and B considering the formation of an equity joint venture company J through which to pursue all of their interests in country Z, and in which they will individually own shares in the equity joint venture. The two firms are aware, however, that there are conflicting pressures and potential instabilities involved in joint ventures and wish to determine the optimal duration of their equity joint venture. After the agreed break-up of the joint venture at time t = T, firms A and B will pursue their own individual interests in country Z. To determine the optimal length of duration of the joint venture, firms A and B therefore need to evaluate how they would fare on a stand-alone basic compared to their participation in the equity joint venture. In the following analysis, the notation i = A, B will denote firms A and B pursuing their own interests as stand-alone businesses in country Z, while the notation i = J will denote A and B acting as partners together in the operation of the equity joint venture.

In order to relate the optimal duration of the equity joint venture to underlying economic variables, we will assume a demand curve facing each firm i = A, B, J at each time t = 0,...., ∞ of the form:

$$q_{it} = \theta_{it} p_{it}^{-\varepsilon_i} \tag{2.1}$$

where p_{it} is the price which firm i charges for its differentiated output at time t and $\varepsilon_i > 1$ is the price elasticity of demand which firm i faces. Its total cost function is given by:

$$C_{it} = c_{it}.q_{it} \tag{2.2}$$

where $c_{it} > 0$ is a constant unit cost per unit of output at time t that includes the annual cost of leasing capital equipment used in the production process for firm i. Firm i's total after-tax profit at time t is therefore equal to:

$$\pi_{it} = (1-\tau_i)(p_{it}q_{it} - C_{it}) = (1-\tau_i)\theta_{it}(p_{it}^{1-\varepsilon_i} - c_{it}p_{it}^{-\varepsilon_i}) \tag{2.3}$$

where τ_i is the corporate profit tax facing firm i, with the first order condition for profit maximisation:

$$(\partial\pi_{it} / \partial p_{it}) = (1-\tau_i)\theta_{it}[(1-\varepsilon_i)p_{it}^{-\varepsilon_i} + c_{it}\varepsilon_i p_{it}^{-\varepsilon_i -1}] = 0 \tag{2.4}$$

and hence the profit maximising price for firm i at time t of:

$$p_{it}^* = \varepsilon_i c_{it} / (\varepsilon_i - 1) \tag{2.5}$$

and using (2.1) – (2.4) the net revenue of firm i at time t is:

$$\pi_{it} = (1-\tau_i)\theta_{it}\chi_i c_{it}^{1-\varepsilon_i} \quad \text{where} \quad \chi_i \equiv \varepsilon_i^{-\varepsilon_i}(\varepsilon_i - 1)^{\varepsilon_i - 1} \tag{2.6}$$

with $(\partial\pi_{it} / \partial\tau) < 0, (\partial\pi_{it} / \partial c_{it}) < 0, (\partial\pi_{it} / \partial\varepsilon_i) < 0$ in (2.6) and $(\partial^2\pi_{it} / \partial p_{it}^2) < 0$ from (2.4) and (2.5).

We will assume that for the joint venture for t = 0,…,T:

$$\theta_{Jt} = \theta_{J0}\exp(g_{J\theta}t) \ \ and \ \ c_{Jt} = c_{J0}\exp(g_{Jc}t) \tag{2.7}$$

so that the demand facing the joint venture during its life in itself grows at the instantaneous rate of $g_{J\theta}$, and that its unit costs change at the instantaneous rate of g_{Jc}. The net revenue for the equity joint venture at time t = 0,…,T is therefore given by:

$$\pi_{Jt} = a_{J0}\exp(g_J t) \ \text{ where } \ a_{J0} \equiv (1-\tau_J)b_{J0}, b_{J0} \equiv \theta_{J0}\chi_J c_{J0}^{1-\varepsilon_J} \ \ and \ \ g_J \equiv g_{J\theta} - (\varepsilon_J - 1)g_{Jc} \tag{2.8}$$

When firms i = A, B are outside the joint venture, we similarly assume that they experience their own instantaneous rates of change of $g_{i\theta}$ and g_{ic} in their demand parameter θ_{it} and in their unit costs c_{it}. However, if firms A and B form the joint venture at time t = 0, they subsequently gain valuable knowledge transfer and learning experience whilst in the joint venture that may further boost their potential demand if they were each later to go-it-alone and reduces their unit costs as stand-alone firms, compared to what would have prevailed without the knowledge transfer and learning that they gain from their participation in the equity joint venture (EJV). We then have:

$$\theta_{it} = \theta_{i0}\exp((g_{i\theta} + \ell_{i\theta})t) \ \ and \ \ c_{it} = c_{i0}\exp((g_{ic} - \ell_{ic})t) \qquad \text{for i = A, B and t = 0,….,T} \tag{2.9}$$

where $\ell_{i\theta} \geq 0$ and $\ell_{ic} \geq 0$ are the respective rates of improvements in the demand parameter θ_{it} and the unit costs due to the process of learning within the joint venture. From (2.6) and (2.9), we have the potential net revenue of firm i at time t if it broke away from the joint venture at time t equal to:

$$\pi_{it} = a_{i0} \exp((g_i + \ell_i)t) \text{ where } a_{i0} \equiv (1-\tau_i)b_{i0},\, b_{i0} \equiv \theta_{i0}\chi_i c_{io}^{1-\varepsilon_i},\, g_i \equiv g_{i\theta} - (\varepsilon_i - 1)g_{ic},\, \ell_i \equiv \ell_{i\theta} + (\varepsilon_i - 1)\ell_{ic} \quad (2.10)$$

Let ℓ_i denote the rate of l earning for firm i on its go-it-alone venture whilst in the EJV, and let g_a denote the rate of growth in i's go-it-alone venture once started. We then have the present value at time t = 0 of firm i's net revenue as a go-it-alone venture if it is started at time T after firm i has been in the EJV from time 0 to time T to be given by:

$$V_{iT} = \int_{t=T}^{\infty} a_{i0} e^{(g_i+\ell_i)T-} e^{g_i(t-T)} e^{-rt} dt = a_{i0} e^{(g_i+\ell_i-r)T} / (r - g_i) \qquad \text{for i = A, B} \qquad (2.11)$$

where a_0 is a positive constant and the interest rate $r > g_i$ for a finite value to (2.11).

If the EJV when started at time t = 0 then grows at the overall rate g_J up until its break-up at time T, it has a present value of:

$$V_{JT} = \int_{t=0}^{T} a_{Jo} e^{g_J t} e^{-rt} dt = a_{J0}(1 - e^{(g_J - r)T}) / (r - g_J) \qquad (2.12)$$

If firm i owns a fraction α_i of the shares in the equity joint venture, firm i's optimal choice of the break-up date T is that which solves:

$$\max_T W_{iT} \equiv \alpha_i V_{JT} + V_{iT} \qquad \text{for i = A, B} \qquad (2.13)$$

generating the first-order condition:

$$W_{iT}' = \alpha_i V_{JT}' - V_{iT}' = 0 \text{ where } W_{iT}' \equiv (\partial W_{iT} / \partial T), V_{JT}' \equiv (\partial V_{JT} / \partial T), V_{iT}' \equiv -(\partial V_{iT} / \partial T) \qquad (2.14)$$

$$\text{with } V_{JT}' = a_{J0} \exp((g_J - r)T),\; V_{iT}' = ((r - g_i - \ell_i)/(r - g_i)) a_{i0} \exp((g_i + \ell_i - r)T) \qquad (2.15)$$

implying that at the optimal break-up date T_i^* for the EJV for firm i:

$$\exp((g_i + \ell_i - g_J)T_i^*) = (r - g_i)\alpha_i a_{J0} / (r - g_i - \ell_i) a_{i0} \qquad (2.16)$$

where we require

$$g_i + \ell_i < r \qquad (2.17)$$

given $r > g_i$, for an interior solution to (2.16). We have from (2.16):

$$T_i^* = (1/(g_i + \ell_i - g_J)) \ln((r - g_i)\alpha_i a_{J0} / (r - g_i - \ell_i) a_{i0}) \qquad (2.18)$$

Differentiation of (2.18) using (2.6), (2.8) and (2.10) yields the following comparative static results:

$$(\partial T_i^* / \partial g_J) > 0, (\partial T_i^* / \partial \alpha_i) > 0, (\partial T_i^* / \partial \theta_{J0}) > 0, (\partial T_i^* / \partial \tau_i) > 0, (\partial T_i^* / \partial c_{i0}) > 0, (\partial T_i^* / \partial \varepsilon_i) > 0 \qquad (2.19)$$

$$(\partial T_i^* / \partial r) < 0, (\partial T_i^* / \partial \theta_{i0}) < 0, (\partial T_i^* / \partial c_{J0}) < 0, (\partial T_i^* / \partial \varepsilon_J) < 0, (\partial T_i^* / \partial \tau_J) < 0 \quad (2.20)$$

so that partner i's optimal length of stay in the equity joint venture is an increasing function of the growth rate of the EJV, their percentage shareholding in the equity joint venture, the EJV's initial level of demand, the initial unit costs of firm i as a stand-alone business, and the price elasticity of demand that it would face as a stand-alone business. However, partner i's desired length of participation in the joint venture is lower the higher is the interest rate, the higher is the level of initial demand it would face as a stand-alone venture, and the higher are the initial unit costs, the price elasticity of demand, and the corporate tax rate that the joint venture faces. Such differences in the price elasticity of demand facing the joint venture and partner i operating as a stand-alone business give concrete economic expression to the concept of differences in their relative market power, here to increase their net revenue in equation (2.6) through pursuing profit-maximising pricing policies.

In addition, we may show that:

$$(\partial T_i^* / \partial \ell_i) \, (>,=,<) \, 0 \; as \; T_i^* \, (<,=,>) \, 1/(r - g_i - \ell_i) \quad (2.21)$$

so that, if T_i^* is initially less than the critical value on the RHS of (2.21), a higher rate of learning whilst in the joint venture would raise their optimal duration of the joint venture. However, if T_i^* already exceeds this critical value, a higher rate of learning for partner i whilst in the joint venture implies a reduction in their optimal length of participation in the joint venture, with more to be lost from not realising sooner the greater benefits to going-it-alone from the higher rate of learning which has already taken place within the joint venture. Similarly, we have:

$$(\partial T_i^* / \partial g_i) \, (>,=,<) \, 0 \; as \; T_i^* \, (<,=,>) \, \ell_i / [(r - g_i - \ell_i)(r - g_i)] \quad (2.22)$$

though with a lower critical value, given $r > g_i + \ell_i$, in (2.22) for the responsiveness of partner i's optimal duration of the joint venture to increases in the rate of growth (aside from learning) of the profitability of going-it-alone than is the critical value in (2.21).

The second-order condition for (2.13) is that at $T = T_i^*$:

$$\partial^2 W_{iT} / \partial T^2 = \partial [\alpha_i V_{JT}^{'} - V_{iT}^{'}] / \partial T = [(g_i + \ell_i - r)(g_i + \ell_i - g_J) a_{io} \exp((g_i + \ell_i - r)T)] / (r - g_i) < 0 \quad (2.23)$$

which will be satisfied if

$g_i + \ell_i > g_J$ and hence $(\alpha_i a_{J0} / (r - g_i - \ell_i)) > (a_{i0} / (r - g_i))$ in (2.16) for $T_i^* > 0$ (2.24)

given (2.17) and $r > g_i$. Condition (2.24) ensures that Case I in Figure 1 prevails, in which the curve $V_{iT}^{'}$ as a function of the duration time T of the joint venture cuts the corresponding $\alpha_i V_{JT}^{'}$ curve from below.

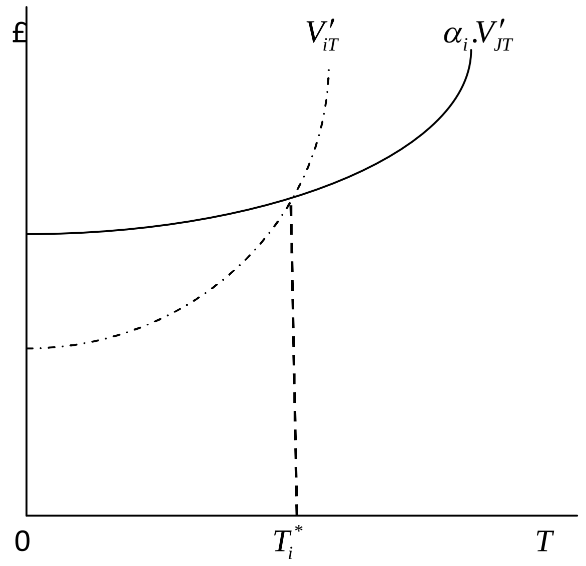

FIGURE 1: Case 1

The curve V'_{iT} represents here the marginal benefit of partner i breaking away slightly sooner from the joint venture to run their own stand-alone business in country Z, whereas the curve $\alpha_i V'_{JT}$ involves the present value of partner i's share in the additional net revenue of the equity joint venture which remaining in it for slightly longer would produce. The optimal duration of the joint venture for partner i in (2.14) occurs here at the value of T where the two curves intersect, with (2.23) ensuring that the marginal benefit of firm i pursuing their stand-alone business is also rising faster at this point than is the present value of partner i's share in the additional net revenue of the equity joint venture.

In order for firm i to be willing to join the joint venture at time t = 0 if it does last for i's optimal duration time, it is necessary and sufficient if condition (2.17) holds that the present value of their profits from doing so are at least as great as would result from going-it-alone from time t = 0 onwards, i.e.

$$(W_{iT_i^*}/V_{i0}) = (\alpha_i a_{J0}(r-g_i)/a_{i0}(r-g_J))[1+((g_i+\ell_i-g_J)\exp((g_J-r)T_i^*)/(r-g_i-\ell_i))] \geq 1 \qquad (2.25)$$

using (2.11) – (2.16). Given (2.24), the sum of the terms in the square brackets in (2.25) is greater than one, so that it will be sufficient for (2.25) to hold that:

$$(\alpha_i a_{J0}/(r-g_J)) \geq (a_{i0}/(r-g_i)) \qquad (2.26)$$

implying that the present value of joining the joint venture at time t = 0 and remaining in it for the infinite future is at least as great as that of going it alone throughout. If (2.26) does not hold, so that staying in the joint venture for ever is not preferable to going-it-alone throughout, the necessary and sufficient condition for partner i to be willing to enter the joint venture at time t = 0, given (2.24), is that their optimal duration is no greater than the critical value given by the condition:

$$T_i^* \leq (1/(r-g_J))\ln((r-g_i-\ell_i)([a_{i0}(r-g_J)/\alpha_i a_{J0}(r-g_i)]-1)/(g_i+\ell_i-g_J)) \qquad (2.27)$$

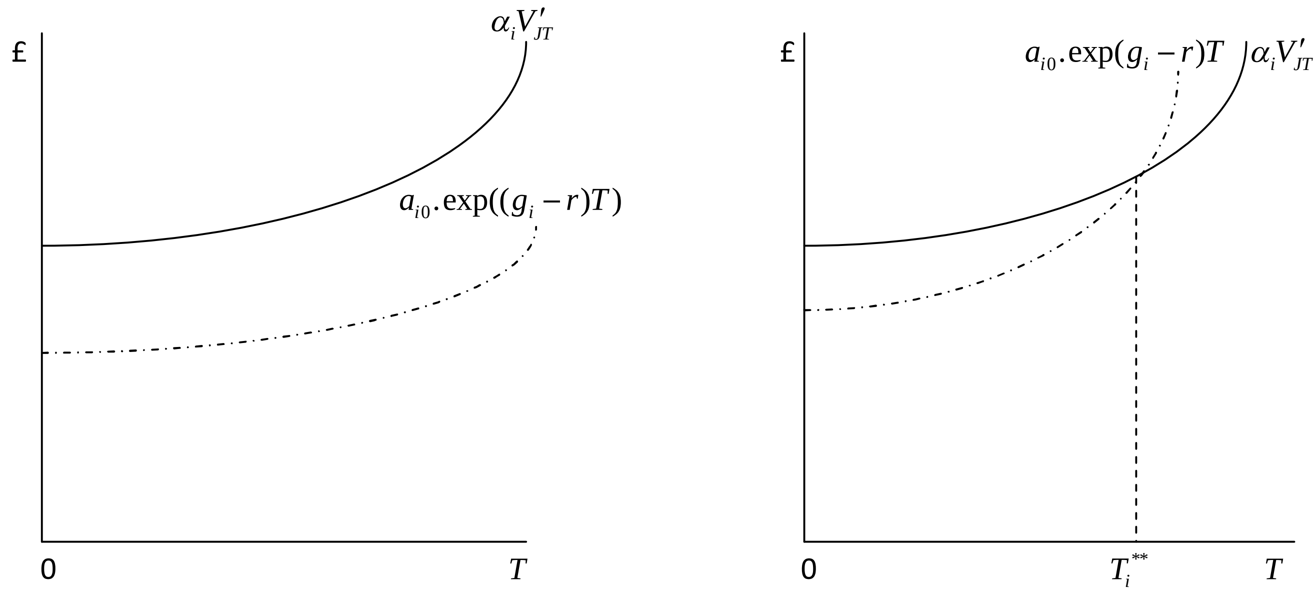

FIGURE 2A: Case II

FIGURE 2B: Case III

However, if condition (2.24) does not hold, there are two other cases of interest in which partner i may be willing to join the equity joint joint venture. The first is where:

$$g_J > g_i \text{ and } \alpha_i a_{J0} \geq a_{i0} \tag{2.28}$$

as in Case II in Figure 2A above. Here, rather than having a finite positive value to T_i^*, firm i achieves a higher present value of net revenue by joining the joint venture at time t = 0 and remaining in it for the infinite future. The second additional case of interest involves:

$$g_J > g_i \text{ and } (\alpha_i a_{J0} / (r - g_J)) < (a_{i0} / (r - g_i)) \tag{2.29}$$

as in Case III in Figure 2A above, where firm i here achieves the highest present value of net revenue by not joining the joint venture at time t = 0, but instead going-it-alone until an optimal time:

$$T_i^{**} = (1/(g_J - g_i)) \ln(a_{i0} / \alpha_i a_{J0}) \tag{2.30}$$

to switch to participation in the joint venture on a permanent basis.

3. The Joint Optimum

If we focus upon Case I in which there is a positive finite value to each partner's optimal duration of the joint venture, we may note from (2.18) that in general there will be disagreement amongst the partners as to their desired duration times. Unless there is a binding contract at the formation of the joint venture to prevent it from occurring, the partner with the smallest value to their respective T_i^* has an incentive to break up the joint venture before the other partner's optimal duration time. From (2.18) differences in the two partners' optimal duration times may arise because of differences in their rates of learning ℓ_i within the joint venture, in their go-it-alone growth rates g_i, in their initial profit levels a_{i0}, and in their equity shares α_i in the EJV. If there is a common overall rate of increase $\gamma = g_i + \ell_i$ in their go-it-alone profitability for both partners $i = A, B$, from (2.18) there will be differences between the two partners' desired durations of the equity joint venture, according to:

$$T_A^* \ (>,=,<) \ T_B^* \ \text{ as } \ \alpha_A \ (>,=,<) \ \alpha_A^{"} \ \text{ where } \alpha_i^{"} \equiv G_{i0} / (G_{A0} + G_{B0}) \text{ and } G_{io} \equiv a_{i0} / (r - g_i) \text{ for } i = A, B \qquad (3.1)$$

given that $\alpha_A + \alpha_B = 1$. Partner A's optimal duration of the joint venture will therefore here exceed, equal or fall short of partner B's optimal duration according to whether partner A's equity share in the equity joint venture exceeds, equals or is less than firm A's share of the total present value of A's and B's go-it alone businesses were they to operate from time t = 0 onwards.

In order to examine the scope for advantageous bargaining and contracting between the two partners on an agreed duration time for the joint venture, we will consider next the optimal duration time, T^*, which maximises the total value of the two partners' wealth, given by:

$$\max_T W_T \equiv V_{JT} + V_{AT} + V_{BT} \qquad (3.2)$$

The first-order condition for (3.2) yields:

$$\partial W_T / \partial T = (S_T + L_{AT} + L_{BT}) e^{-rT} = 0 \qquad (3.3)$$

where $$S_T \equiv a_{J0} \exp(g_J T) - a_{A0} \exp((g_A + \ell_A)T) - a_{B0} \exp((g_B + \ell_B)T) \qquad (3.4)$$

is the degree of *synergy* that results from the joint venture, measured in terms of the excess of the net revenue at time T that the joint venture generates over the sum of the net revenue that would result from partners A and B starting their own go-it-alone ventures at time T. However, we also have:

$$L_{AT} \equiv \ell_A a_{A0} \exp((g_A + \ell_A)T) / (r - g_A) = \ell_A V_{AT} e^{rT} \qquad (3.5)$$

$$L_{BT} \equiv \ell_B a_{B0} \exp((g_B + \ell_B)T) / (r - g_B) = \ell_B V_{BT} e^{rT} \qquad (3.6)$$

as the value at time T of the additional learning in running their own go-it-alone ventures which partners A and B respectively receive from continuing in the joint venture for a further small unit of time. Condition (3.3) thus implies that the optimal break-up date of the joint venture to maximise the total partners' wealth is not when the synergy from the joint venture drops to zero, but rather when the total value of the synergy and the additional learning which the joint venture generates per unit of time drops to zero.

The second-order condition for (3.2) given (3.3) is that:

$$(\partial^2 W_T / \partial T^2) = \sum_{i=A,B} (r - g_i - \ell_i) a_{J0}((g_J - g_i - \ell_i)/(r - g_i)) \exp((g_i + \ell_i - r)T) < 0 \tag{3.7}$$

which will be satisfied if (2.17), (2.24) and $r > g_i$ for i = A, B hold.

For the case where there is a common overall rate of increase

$$\gamma = g_i + \ell_i \text{ for } i = A, B \tag{3.8}$$

in their go-it-alone options for both partners, we obtain their jointly optimal duration of the EJV to be given explicitly by:

$$T^* = (1/(\gamma - g_J)) \ln(a_{J0} / [((a_{A0}/(r - g_A)) + (a_{B0}/(r - g_B)))(r - \gamma)] \tag{3.9}$$

which will be a positive and finite under the above conditions if:

$$S_0 + L_{A0} + L_{B0} > 0 \tag{3.10}$$

requiring that the sum of the initial value of the joint venture's synergy plus the initial value of the learning effects is positive. From (2.18) and (3.9) we have here:

$$T_i^* \ (>,=,<)\ T^* \text{ as } \alpha_i \ (>,=,<)\ \alpha_i^{''} \ \text{ for } i = A, B \text{ with } T_A^* = T_B^* = T^* \text{ if } T_A^* = T_B^*, \ \ T_A^* < T^* < T_B^* \text{ if } T_A^* < T_B^* \tag{3.11}$$

using (3.1). Moreover, even when $g_A + \ell_A$ differs from $g_B + \ell_B$, we may show from (2.18) and (3.3) – (3.6) that if $T_A^* = T_B^*$, i.e. both partners have the same individually optimal duration for the joint venture, we will also have $T_A^* = T_B^* = T^*$, so their individual optimal durations are also the same as the jointly optimal duration for the joint venture. However, if $T_i^* \neq T^*$ for any $i = A, B$, then we must have $T_A^* \neq T_B^*$, so that if any individual partner's optimal duration differs from the joint optimum, it also differs from the other partner's optimal duration of the joint venture. One of the partners, such as partner A, will then prefer an earlier termination of the joint venture than the other partner, and, in the absence of a binding agreement to the contrary, will break away from the joint venture before the other partner's desired termination date for the equity joint venture, so that without such an agreement, the payoff to the two partners is given by the 'disagreement pair':

$$d = (d_A, d_B) \text{ where } d_A = W_{AT_A^*}, \ d_B = W_{BT_A^*} \tag{3.12}$$

As a result, from (3.13) below, the other partner, here partner B, suffers an economic loss from a lower resultant total return, W_{AT}, from their participation in the joint venture than if the termination date had been closer to their individual optimal duration T_B^*.

From (2.14) – (2.17) and (2.24), we have for $\ell_i^{'} \equiv \ell_i / (r - g_i)$:

$$W_{iT}^{'} = (1 - \ell_i^{'}) a_{i0} \exp((g_i + \ell_i) - r)T)[\exp((g_i + \ell_i - g_J)(T_i^* - T)) - 1] \ (<,=,>)\ 0 \text{ as } T \ (>,=,<)\ T_i^* \tag{3.13}$$

with $(\partial^2 W_{iT} / \partial T^2) = [(g_J - r)W_{iT}^{'} + (1-\ell_i^{'})a_{i0}(g_J - g_i - \ell_i)\exp((g_i + \ell_i - r)T)] < 0 \text{ for all } T \le T_i^*$ (3.14)

$$= \exp((g_J - r)T)a_{i0}(1-\ell_i^{'})\exp((g_i + \ell_i - r)T_i^*[(g_J - r) + (r - g_i - \ell_i)\exp((g_i + \ell_i - r)(T - T_i^*))] \quad (3.15)$$

$< 0 \text{ for all } T \ge T_i^*$ using (2.17) and (2.24) (3.16)

If a legally enforceable agreement between the two firms, with appropriate sanctions for non-fulfilment, is feasible, there is then scope for mutually beneficial bargaining between the two firms to reach an agreed duration of the joint venture. However, since from (3.13) any duration greater than T_A^* that is closer to T_B^* will in itself reduce firm A's return from the joint venture, firm A will require compensation for agreeing to any such increased duration of the joint venture. If this compensation takes the form of a cash payment from firm B to firm A of an amount m and their underlying utility functions are linear in their respective net financial wealth, the set of their net payoffs that may result from a binding agreement on the duration of the joint venture is given by:

$$S = \{(s_A, s_B) \in \mathbb{R}^2 : s_A + s_B \le W_{JT^*}, s_A = W_{AT^{'}} + m \ge d_A, s_B = W_{BT^{'}} - m \ge d_B\} \quad (3.17)$$

which is a closed and bounded convex set that contains also the disagreement point d . Solving

$$\max_{s_A, s_B} (s_A - d_A)(s_B - d_B) \quad (3.18)$$

over the set S yields the Nash bargaining solution (see Gravelle and Rees, 1992, pp 380-386) :

$$s_A + s_B = W_{JT^*} \text{ and } m = 0.5[(W_{BT^*} - W_{BT_A^*}) + (W_{AT_A^*} - W_{AT^*})] \quad (3.19)$$

in which both sides agree to the efficient and jointly optimal duration T^* and partner B gives up half of their gain from the increased duration of the project beyond A's individually optimal duration T_A^* to partner A, and additionally compensates partner A for half of the loss which A suffers from this increased duration of the joint venture beyond T_A^* . An alternative possibility to a cash-time bargaining solution is for firm B to give firm A an increase of ψ in A's share of the equity joint venture in return for firm A agreeing to a longer duration for the joint venture than T_A^*. The set of possible agreements now becomes:

$$X = \{(\psi, T^{'}) \in \mathbb{R}^2 : 0 \le \psi \le \alpha_B, T_A^* \le T^{'} \le T_B^*\} \quad (3.20)$$

with an associated set of feasible payoffs given by:

$$S^o = \{(s_A^o, s_B^o) \in \mathbb{R}^2 : s_A^o = (\alpha_A + \psi)V_{JT^{'}} + V_{AT^{'}}, s_B^o = (\alpha_B - \psi)V_{JT^{'}} + V_{BT^{'}}, T_A^* \le T^{'} \le T_B^*, 0 \le \psi \le \alpha_B\} \quad (3.21)$$

that is again a closed and bounded convex set that contains the disagreement point d . Solving

$$\max_{s_A^o, s_B^o} (s_A^o - d_A)(s_B^o - d_B) \quad (3.22)$$

over the set S^o yields the Nash bargaining solution:

$$T^{'} = T^{*} \text{ and } \psi = 0.5[(W_{BT^*} - W_{BT_A^*}) + (W_{AT_A^*} - W_{AT^*})] / V_{JT^*} \tag{3.23}$$

which attains the same efficient pair of outcomes for the two partners as the cash-time agreement. From (2.18) and (3.9), we will then obtain an equality between the individually optimal durations T_A^* and T_B^* and the jointly optimal duration T^* for the case where (3.8) holds under the new ratio of their shareholdings in the equity joint venture:

$$(\alpha'_A / \alpha'_B) = (a_{A0} / (r - g_A)) / (a_{B0} / (r - g_B)) \quad for\ \alpha'_A = \alpha_A + \psi, \alpha'_B = \alpha_B - \psi \tag{3.24}$$

4. The Socially Optimal Duration

From the viewpoint of economic policy, it is also of interest to determine the socially optimal duration of the joint venture, and the nature and extent of any deviation from the social optimum which a privately determined duration will produce. The social optimal duration T^{**} of the joint venture will be defined here as

$$\arg\max_{T} \ F(T) \ \ s.t. \ R(T) \geq \bar{R} \ \text{ where } \ F(T) \equiv w_1 CS(T) + w_2 PS(T) \tag{4.1}$$

and $R(T)$ is the present value of the tax revenue generated by the joint venture and by firms A and B going it alone when the duration of the joint venture is T years, and $\bar{R}$ is the government's target level of this tax revenue From (2.8) – (2.12) we have:

$$R(T) = \sum_{i=J,A,B} \tau_i Z_i(T) \text{ for } Z_J(T) \equiv b_{J0}(1 - e^{(g_J - r)T}) / (r - g_J), Z_i(T) \equiv b_{i0} e^{(g_i + \ell_i - r)T} / (r - g_i) \text{ for } i = A, B \tag{4.2}$$

$CS(T)$ in (4.1) is the present value of the consumer surplus generated by the joint venture and by firms A and B going it alone when the duration of the joint venture is T years, and w_1 in (4.1) is the weight placed upon consumer surplus in the social evaluation. From (2.1), (2.5) and (2.6), we have the consumer surplus that is generated by firm i at time t to be given by

$$CS_{it} \equiv \int_{p_{it} = p_{it}^*}^{\infty} q_{it} dp_{it} = \theta_{it} c_{it}^{1-\varepsilon_i} \varepsilon_i^{1-\varepsilon_i} (\varepsilon_i - 1)^{\varepsilon_i - 2} \ \text{ for } i = J, A, B \text{ and } t = 0, \ldots, \infty \tag{4.3}$$

and hence from (2.8) – (2.20) and (4.2):

$$CS(T) = \sum_{i=J,A,B} \varepsilon_i (\varepsilon_i - 1)^{-1} Z_i(T) \tag{4.4}$$

$PS(T)$ in (4.1) is the present value of the total producers' surplus generated by the joint venture and by firms A and B going it alone when the duration of the joint venture is T years, and w_2 in (4.1) is the weight placed upon producers' surplus in the social evaluation. From (2.8) – (2.12) and (3.2),

$$PS(T) = \sum_{i=J,A,B} (1-\tau_i) Z_i(T) = W_T \tag{4.5}$$

(4.1) may now be written as:

$$\arg\max_T \; L(T) \;\text{ where }\; L(T) \equiv \sum_{i=J,A,B} \phi_i Z_i(T) \text{ and } \phi_i \equiv w_1 \varepsilon_i (\varepsilon_i - 1)^{-1} + w_2 (1-\tau_i) + \lambda \tau_i \tag{4.6}$$

where λ is the Lagrangean multiplier associated with the net tax revenue constraint in (4.1). (4.6) involves the solution to the first order condition:

$$\phi_J b_{J0}\, \mathrm{e}^{(g_J - r)T} + \sum_{i=A,B} \phi_i b_{i0} (g_i + \ell_i - r) e^{(g_i + \ell_i - r)T} / (r - g_i) = 0 \tag{4.7}$$

The case where there is a common overall rate of increase (3.8) in their go-it-alone options for both partners is again of particular interest in yielding an explicit analytical solution, here given by:

$$T^{**} = (1/(\gamma - g_J)) \ln(\phi_J b_{J0} / [((\phi_A b_{A0} / (r - g_A)) + (\phi_B b_{B0} / (r - g_B)))(r - \gamma)] \tag{4.8}$$

From (3.9), (3.24) and (4.8), we also then have:

$$T^{**} = T^* + (1/(\gamma - g_J)) \ln(\phi'_J / [\alpha'_A \phi'_A + \alpha'_B \phi'_B]) \;\text{ where } \phi'_i \equiv \phi_i / (1-\tau_i) \text{ for } i = J, A, B \tag{4.9}$$

so that the socially optimal duration of the joint venture exceeds, equals, or is less than the privately joint optimal duration as given by:

$$T^{**} (>,=,<)\, T^* \text{as} \sum_{i=A,B} ((\phi_i / \phi_J) - ((1-\tau_i)/(1-\tau_J))) b_{i0} / (r - g_i)\; (<,=,>)\; 0 \tag{4.10}$$

It also follows from (4.6) and (4.10) that the socially optimal duration T^{**} of the joint venture exceeds equals, or is less than the privately joint optimal duration T^* if for each partners i = A,B, their stand-alone corporate tax rates τ_i compared to the corporate tax rate τ_J which the joint venture faces are such that their corresponding ratio $(1-\tau_i)/(1-\tau_J)$ exceeds, equals, or is less than the critical value:

$$\varphi_i / \varphi_J \text{ where } \varphi_j \equiv w_1 \varepsilon_j (\varepsilon_j - 1)^{-1} + \lambda \text{ for } j = J, A, B \tag{4.11}$$

that in turn varies with the relative price elasticities ε_j of demand that they face. In particular, it follows from (4.10) and (4.11) that:

$$\tau_J > \tau_i \text{ and } \varepsilon_i > \varepsilon_J \text{ for } i = A, B \text{ implies } T^{**} > T^*, \text{ whilst } \tau_i > \tau_i \text{ and } \varepsilon_J > \varepsilon_i \text{ for } i = A, B \text{ implies } T^{**} < T^* \tag{4.12}$$

Securing the alignment of the privately joint optimal duration T^* of the joint venture with the socially optimal duration T^{**} in contrast requires corporate tax rates to be set such $(1-\tau_i)/(1-\tau_J)$ is equal to the ratio φ_i / φ_J for $i = A, B$ in (4.11), and hence $\tau_J < \tau_i$ if $\varepsilon_J > \varepsilon_i$ for each $i = A, B$ and $\tau_J > \tau_i$ if $\varepsilon_J < \varepsilon_i$ for each $i = A, B$. Thus, a corporate tax break in the form of a lower corporate tax rate on the joint venture than would prevail on the go-it-alone businesses of the two partners is justified here to bring the privately joint optimal duration of the joint venture into line with the socially optimal duration, whenever the joint venture faces a higher price elasticity of demand than the partners would going it alone. However, the converse applies when the joint venture faces a lower price elasticity of demand than the partners would going it alone, with equality between the private and social optimal durations of the joint venture instead requiring a higher corporate tax rate on the joint venture than on the go-it alone businesses of the two partners.

5. Conclusion

Rather than being a random unpredictable event, the break-up of an equity joint venture after a finite time can be modelled as the predictable consequence of underlying economic parameters under conditions of complete certainty. There is then scope for both partners in the equity joint venture to gain from bargaining in the initial formation of the equity joint venture to achieve an agreed termination date of the joint venture, with side payments in the form of cash or equity transfers whenever the underlying economic parameters result in differences in the initial individual optimal durations of the joint venture between the two partners. In addition, there is scope for a differential corporate tax rate on the joint venture, compared to that on the go-it-alone businesses of the two partners, in order bring the two partners' privately optimal durations into line with the socially optimal duration of the joint venture.

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