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Abstract

We propose a new test based on the no-arbitrage condition that compares cross-sectional variation in equity returns to the cross-sectional variation in their conditional covariance with the discount factors. Using the multivariate generalized heteroskedasticity in mean model (MGM) to estimate the 25 portfolios formed on size and book-to-market ratio, together with each with its own arbitrage condition, we find that the no-arbitrage test rejects the consumption-based capital asset pricing model (C-CAPM). Although the conditional covariances of returns with consumption exhibit negative variation across size, they do not vary across the book-to-market ratio. Thus, the C-CAPM can capture size effect, but not value effect. Allowing the coefficients on the consumption covariances to be different largely improves the fit of the C-CAPM, however. The value effect appears to be associated with book-to-market ratio as well as size. Book-to-market ratio separately does not generate information about average returns that cannot be explained by the C-CAPM.

JEL Classification: G12, G14, C32, E44
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1. Introduction

High average returns of small stocks relative to big stocks and high book-to-market equity ratio have been termed size and value effects and long been recognized as "anomalies" within the capital asset pricing model (CAPM) literature (Banz (1981), Fama and French (1992a), and Chan, Hamao, and Lakonishok (1991)), and also the consumption-based CAPM (C-CAPM) with its power utility (standard C-CAPM) framework (Mankiw and Shapiro (1986) and Breeden, Gibbons, and Litzenberger (1989)). This paper examines the relation between the C-CAPM and firm characteristics using the stochastic discount factor (SDF) approach to provide an asset pricing framework. We compare cross-sectional variation in equity returns with the cross-sectional variation in their conditional covariances with the discount factors, while holding the properties of the conditional covariance constant, as the SDF implies that the coefficients on these conditional covariances should be the same across the cross-section of equity returns. This comparison provides testable restrictions on coefficients for the condition covariance over a no-arbitrage condition and, therefore can be interpreted as a no-arbitrage test.

Using the multivariate generalized autoregressive conditional heteroskedasticity in mean (MGM) to estimate various versions of the SDF model proposed by Smith and Wickens (2002) for portfolios sorted by size and book-to-market ratio, we find that the standard C-CAPM is rejected by the no-arbitrage test. Although the consumption covariance with equity return obtained from the estimation of the standard C-CAPM exhibits a negative relation with size, the model appears to miss the value premium completely by not producing dispersion in the consumption covariance across the book-to-market quintiles. In fact, the consumption covariances for portfolios in the highest book-to-market quintile seem to be slightly lower than those in the lowest book-to-market quintiles, indicating lower risk premium is implied by the C-CAPM. Thus, the standard C-CAPM can explain the size effect, but not the value effect. Allowing the coefficients on the conditional covariances of equity returns with consumption to be different improves the fit of the standard C-CAPM sharply. In addition, we add a constant term in the standard C-CAPM to measure variation in excess returns that was left unexplained in the model, and find that it is highly significant. However, the significance of the constant term decreases as the coefficients of consumption covariances are different.

Consequently, the value effect should arise from the fact that there is an additional dimension of risk not captured by the C-CAPM for no arbitrage opportunities in the market. To see this additional risk, double-sorted size and book-to-market portfolios need to be examined, as value effect in the C-CAPM seems to be associated with book-to-market ratio as well as with size. Firm size or book-to-market ratio separately does not generate information about average returns that cannot be explained by the C-CAPM, which is why the standard C-CAPM cannot explain small growth portfolios. On the other hand, the CAPM is not able to explain both size and value effects as in previous studies of the CAPM, implying that consumption contains information about these firm characteristics that is not available through market return.

The evidence found in Fama and French (1993b) casts doubt on the empirical validity of the CAPM, as these authors find that accounting variables, i.e., size and book-to-market ratio, appear to explain the stock returns, and the Fama and French three-factor model, which includes these two characteristics as additional pricing factors from the market return can successfully explain the cross-section of stock returns. However, any anomalies can only be defined relative to a specific pricing model. An alternative pricing model then is the C-CAPM, which takes into account the intertemporal nature of the investor optimization problem.

As Mankiw and Shapiro (1986) and Breeden, Gibbons, and Litzenberger (1989) demonstrate, the standard C-CAPM cannot also explain size and value effects as in the

CAPM context; however, recent studies attempt to explain the cross-section of equity returns with the modified versions of the standard C-CAPM. Lettau and Ludvigson (2001) use the ratio of aggregate consumption to wealth as a conditioning variable in the C-CAPM to allow expected returns to vary over time. Parker and Julliard (2005) measure risk premium by its covariance with consumption growth, as cumulated over many quarters after the return period, to overcome the presence of the slow consumption adjustment process. Yogo (2006) propose a two-factor model that includes nondurable and durable consumption growths and find that the size and value effects are due to the fact that small and value stocks have higher durable consumption betas than do large and growth stocks. These three papers thus assert that there are certain alternative factors missing from the standard C-CAPM, and by taking into account these factors through either conditioning variables or alternative related consumption factors, these modified versions of the standard C-CAPM can explain the cross-section of equity returns as well as the Fama and French three-factor model.

We take a different approach by using the MGM to directly model the joint distribution of equity returns and consumption as in Smith and Wickens (2002). This focus contrasts with most of the time-series econometric models of equity in the literature, which are univariate and do not include conditional covariances, but the MGM allows us to measure directly the sources of aggregate or macroeconomic risk that underlie the behavior of equity return. Smith, Sorensen, and Wickens (2008) follow this approach and employ the SDF model to generate models that involve macroeconomic variables to find that consumption and inflation are significant, while industrial production plays no role in explaining equity returns. We extend their analysis to estimate several portfolios sorted by size and book-to-market ratio together and then test the restrictions implied by the standard C-CAPM, thus linking consumption directly with size and book-to-market ratio.

In Section 2, we discuss the theoretical framework for asset pricing. Section 3 describes the econometric methodology. In Section 4, we report the estimation and testing results for all the models. Section 5 summarizes the findings in this paper.

2. Theoretical Framework

2.1 Stochastic Discount Factor (SDF)

The SDF is based on a proposition that the price of an asset at the beginning of period t (P_t) is determined by the expected discounted value of that asset's payoff in period $t+1$:

$$P_t = E_t[M_{t+1}X_{t+1}] \tag{1}$$

where M_{t+1} is the stochastic discount factor for period $t+1$. For equity, the payoff in real terms is $X_{t+1} = P_{t+1} + D_{t+1}$, where D_{t+1} are dividend payments assumed to be made at the start of period $t+1$. The pricing equation (1) can thus be written as:

$$1 = E_t\left[M_{t+1}(X_{t+1}/P_t)\right] = E_t\left[M_{t+1}R_{t+1}\right] \tag{2}$$

where $R_{t+1} = X_{t+1}/P_t$ is the asset's gross real return. If $m_{t+1} = \ln M_{t+1}$, $r_{t+1} = \ln R_{t+1}$, and the logarithm of the risk free rate (r_t^f) are jointly normally distributed; then the expected excess real return on equity is given by

$$E_t(r_{t+1} - r_t^f) + \frac{1}{2}V_t(r_{t+1}) = -Cov_t(m_{t+1}, r_{t+1})\,. \tag{3}$$

The right-hand side of the equation is the risk premium and the variance term is the Jensen effect.

The no-arbitrage condition (3) can also be expressed in terms of nominal returns. If i_{t+1} is the nominal return on equity, i_t^f is the nominal risk-free rate, P_t^c is the consumer price index, and inflation is given by $1+\pi_{t+1} = P^c_{t+1} / P^c_t$. The pricing equation (1) can then be expressed as

$$1 = E_t\left[M_{t+1}(P^c_t / P^c_{t+1})(1+i_{t+1})\right].$$

The no-arbitrage condition for nominal returns is:

$$E_t(i_{t+1} - i^f_t) + \frac{1}{2}V_t(i_{t+1}) = -Cov_t(m_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1}) . \qquad (4)$$

Upon comparing Equation (4) to Equation (3), the no-arbitrage condition for the nominal return involves one additional term on the right-hand side: The conditional covariance of returns with inflation.

If m_t can be represented as a linear function of $n-1$ factors $z_{i,t}\ \{i=1,...,n-1\}$ so that $m_t = -\sum_{i=1}^{n-1}\alpha_i z_{i,t}$, then a general representation of (3) is

$$E_t(i_{t+1} - i^f_t) = \alpha_0 V_t(i_{t+1}) + \sum_{i=1}^{n}\alpha_i Cov_t(z_{i,t+1}, i_{t+1}) , \qquad (5)$$

where $z_{n,t} = \pi_t$. Different asset pricing models differ, mainly due to their stochastic discount factor, $z_{i,t+1}$, and the restrictions imposed on the coefficients. We consider three pricing models that can be shown to be special cases of the Equation (5): C-CAPM with power utility, CAPM, and General SDF Models.

2.2 C-CAPM

The C-CAPM is a general equilibrium model, which implicitly defines the discount factor as

$$M_{t+1} = \beta\left(U'(C_{t+1})/U'(C_t)\right)$$

where C_t is consumption and $U'(C_t)$ is utility. For the power utility function, $U(C_t) = (C_t^{1-\gamma} - 1)/(1-\gamma)$ with $\gamma =$ constant coefficient of relative risk aversion (CRRA). Thus, the SDF becomes $M_{t+1} = \beta\left(C_{t+1}/C_t\right)^{-\gamma}$. For the nominal return, the relevant no-arbitrage condition can be expressed as

$$E_t(i_{t+1} - i^f_t) + \frac{1}{2}V_t(i_{t+1}) = \gamma Cov_t(\Delta \ln C_{t+1}, i_{t+1}) + Cov_t(\pi_{t+1}, i_{t+1}) , \qquad (6)$$

where $\Delta \ln C_{t+1} \simeq \Delta C_{t+1}/C_t$ is the growth rate of consumption. The C-CAPM with power utility implies that average excess returns differ due to their conditional covariance with consumption, and the CRRA should be the same across equities.

2.3 CAPM

The CAPM implies that the expected return of an asset must be linearly related to the covariance of its return with the return on the market portfolio through

$$E_t(r_{t+1} - r^f_t) = \delta_t Cov_t(r^m_{t+1}, r_{t+1}) ,$$

and, for the market portfolio,

$$E_t(r^m_{t+1} - r^f_t) = \delta_t V_t(r^m_{t+1})$$

where $\delta_t = E_t(r^m_{t+1} - r^f_t)/V_t(r^m_{t+1})$ is the market price of risk and can be interpreted as the CRRA (Merton (1980)). There is no Jensen effect because log-normality is not assumed. The corresponding no-arbitrage condition for nominal returns is

$$E_t(i_{t+1} - i^f_t) = \delta_t Cov_t(i^m_{t+1}, i_{t+1}) \qquad (7)$$

where i^m_{t+1} is the nominal return on the market portfolio.

2.4 The General Stochastic Discount Factor Models

The general SDF models are based on macroeconomic factors and particular versions of the multifactor model, which in Equation (5) has one more discount factor and allows the factors to have unrestricted coefficients. The general SDF model with 2 factors involves any two macroeconomic variables. Similarly, the general SDF model with 3 factors has three macroeconomic variables. Smith, Sorensen, and Wickens (2008) suggest the use of factors that are associated with the business cycle and inflation as financial institutions, which are the main holders of equity and act on behalf of investors at a much more distant point in the future, thus focusing on short-term rather long-term performance. The authors, therefore, use output as an additional source of risk to consumption and inflation, but without seeking to give this model a general equilibrium interpretation.

2.5 The No-Arbitrage Condition Test

In all the SDF models previously discussed, the discount factors are functions of aggregate variables, and thus it is possible to hold the properties of the discount factors constant as one individual asset is compared to another. As the risk premium is represented by the conditional covariance of the returns with the discount factor, we can compare cross-sectional average returns with cross-sectional variation in their conditional covariances with the factors. The implication is that the coefficients on these conditional covariances should be the same across the cross-section of equity returns, as stocks have different returns because they have different conditional covariances with the relevant factors. This relation provides testable restrictions on no-arbitrage conditions, and therefore, it can be interpreted as a no-arbitrage test.

Table 1 provides a summary of restrictions for each asset pricing models implied by its no-arbitrage condition. The C-CAPM with power utility and nominal return (M1) implies that the CRRA is constant and should be the same across the cross-section of expected returns for no arbitrage opportunities in the market. M1 can be interpreted as a restricted version of the C-CAPM. On the other hand, allowing the coefficients of the conditional covariances of returns with consumption to be different generates an unrestricted version of the C-CAPM (M4). The double-sorted, 25 size and book-to-market equity ratio portfolios generate two more versions of the C-CAPM with power utility: 1) restricted book-to-market model (M2) and 2) restricted size model (M3). M2 allows portfolios with different size groups to have different coefficients on the consumption covariances, while M3 allows the coefficients on portfolios with different book-to-market equity ratio groups to be different. Similarly, these restrictions of the C-CAPM are applied for the CAPM, where the market price of risk is expected to be the same across assets, (M5-M8). In addition, the restricted and unrestricted general SDF models, based on two and three macroeconomics variables, are given by M9-M12.

Essentially, all the above asset pricing models can be represented as restricted versions of the SDF model,

$$E_t(i_{t+1}^{sb} - i_t^f) = \alpha_{0,sb} V_t(i_{t+1}^{sb}) + \alpha_{1,sb} Cov_t(\Delta c_{t+1}, i_{t+1}^{sb}) + \alpha_{2,sb} Cov_t(\pi_{t+1}, i_{t+1}^{sb}) + \alpha_{3,sb} Cov_t(\Delta q_{t+1}, i_{t+1}^{sb}) + \alpha_{4,sb} Cov_t(i_{t+1}^{m}, i_{t+1}^{sb}), \quad (8)$$

where s and b indicate size and book-to-market ratio groups that the characteristics portfolios belong to, respectively, and q_t is the industrial production. The different asset models can be obtained by placing different restrictions on α_i, *s*, and *b*.

Table 1
Restrictions on the No-arbitrage Condition

s and *b* indicate size and book-to-market groups for the characteristics portfolios. The numbers are in ascending order of magnitude. The smallest size is denoted by s = 1 while the lowest book-to-market ratio is represented by b = 1. γ denotes constant coefficient of relative risk aversion (CRRA). α_i represents a coefficient for each conditional covariance in Equation 8.

Models	α_0	α_1	α_2	α_3	α_4
M1: C-CAPM with power utility and nominal return	$-\frac{1}{2}$	γ	1	0	0
M2: Restricted book-to-market C-CAPM	$-\frac{1}{2}$	$\alpha_{1,s}$	1	0	0
M3: Restricted size C-CAPM	$-\frac{1}{2}$	$\alpha_{1,b}$	1	0	0
M4: Unrestricted C-CAPM	$-\frac{1}{2}$	$\alpha_{1,sb}$	1	0	0
M5: CAPM	0	0	0	0	δ
M6: Restricted book-to-market CAPM	0	0	0	0	$\alpha_{4,s}$
M7: Restricted size CAPM	0	0	0	0	$\alpha_{4,b}$
M8: Unrestricted CAPM	0	0	0	0	$\alpha_{4,sb}$
M9: Restricted two-factor SDF model	$-\frac{1}{2}$	α_1	α_2	0	0
M10: Unrestricted two-factor SDF model	$-\frac{1}{2}$	$\alpha_{1,sb}$	$\alpha_{2,sb}$	0	0
M11: Restricted three-factor SDF model	$-\frac{1}{2}$	α_1	α_2	α_3	0
M12: Unrestricted three-factor SDF model	$-\frac{1}{2}$	$\alpha_{1,sb}$	$\alpha_{2,sb}$	$\alpha_{3,sb}$	0

3. Econometric Framework

We follow the same econometric approach here as in Smith and Wickens (2002) and Smith, Sorensen, and Wickens (2008) by using the multivariate generalized autoregressive conditional heteroskedasticity in mean model (MGM) to estimate the joint distribution of the excess return on equity with macroeconomic factors in such a way that the return satisfies the no-arbitrage condition under the SDF framework. This approach is achieved by including conditional covariances of the excess equity returns and the macroeconomic factors in the mean of the asset pricing equations and constraining the coefficients on these conditional covariances according to the no-arbitrage condition implied by each asset-pricing model.

Let $\mathbf{x_{t+1}} = (r_{1,t+1} - r_t^f, ..., r_{i,t+1} - r_t^f, \Delta c_{t+1}, \pi_{t+1}, \Delta q_{t+1})'$ and contains n variables and i returns, as several portfolios are estimated at the same time. This parameter is an extension of the MGM in Smith and Wickens (2002). Consumption, inflation, and industrial productions are included, as they give rise to the discount factors in the SDF model, M1-M12 in Table 1, through their conditional covariances with the excess returns. Additional macroeconomic variables can be included in this vector if they improve the estimate of the joint distribution. The MGM model can then be written as

$$\mathbf{x_{t+1}} = \boldsymbol{\alpha} + \boldsymbol{\Gamma}\mathbf{x_t} + \boldsymbol{\Phi}\mathbf{g_{t+1}} + \boldsymbol{\varepsilon}_{\mathbf{t+1}},$$

where

$$\boldsymbol{\varepsilon}_{\mathbf{t+1}} \mid I_t \sim N(0, \mathbf{H_{t+1}}),$$

$$\mathbf{g_{t+1}} = vech(\mathbf{H_{t+1}}).$$

where, $\boldsymbol{\alpha}$ is a $n \times 1$ vector of constant, $\boldsymbol{\Gamma}$ is a $n \times n$ matrix of coefficients in the vector autoregressive (VAR) part (included to obtain better representation of the error terms), $\boldsymbol{\Phi}$ is a $n \times n$ matrix of coefficients of in-mean component, $\boldsymbol{\varepsilon}_{\mathbf{t+1}}$ is an $n \times 1$ vector of errors, and $i =$ number of equity returns. The *vech* operator converts the lower triangle of a symmetric matrix into a vector. The error term, $\boldsymbol{\varepsilon}_{\mathbf{t+1}}$, is conditionally and normally distributed with mean zero and the conditional covariance matrix $\mathbf{H}_{\mathbf{t+1}}$. The first I row of the model is restricted to satisfy the no-arbitrage condition as follows: 1) the first *i* rows of $\boldsymbol{\Gamma}$ must be zero; 2) the first *i* rows of $\boldsymbol{\Phi}$ depends on then specification of each asset pricing model defined in Table 1; 3) the *i+1* to *i+3* rows of $\boldsymbol{\Phi}$ are all zero; and 4) the first *i* elements of $\boldsymbol{\alpha}$ is zero. A likelihood ratio test is used to provide test statistics for the restrictions implied by the no-arbitrage condition in M1-M12 as given in Table 1.

While the MGM model is convenient, it is heavily parameterized, which can create numerical problems in finding the maximum of the likelihood function due to the likelihood of being relatively flat, and hence uninformative. Therefore, to complete the model parameterization for the conditional covariance matrix $\mathbf{H}_{\mathbf{t+1}}$ with the view toward restricting the number of coefficients being estimated, the specification of the conditional covariance matrix is chosen to be the vector diagonal model with variance targeting (Ding and Engle (2001)), which can be written as follows,

$$\mathbf{H}_{\mathbf{t+1}} = \mathbf{H}_{\mathbf{0}}(\mathbf{ii}' - \mathbf{aa}' - \mathbf{bb}') + \mathbf{aa}' \odot (\boldsymbol{\varepsilon}_{\mathbf{t}}\boldsymbol{\varepsilon}_{\mathbf{t}}') + \mathbf{bb}' \odot \mathbf{H}_{\mathbf{t}}$$

where $\odot$ denotes Hadamard product, $\mathbf{H}_{\mathbf{0}}$ is the observed sample covariance matrix, and $\mathbf{a}$ and $\mathbf{b}$ are $n \times 1$ vectors. The number of parameters to be estimated reduces to $2n$. This model is particularly attractive when we estimate several excess returns simultaneously, each with its own arbitrage condition. In addition, the zero restrictions on the coefficients for excess returns in the VAR part of the macroeconomic variables are imposed to further reduce the number of parameters in the MGM model. Estimating the restricted and unrestricted C-CAPM (M1 and M4) for the 25 portfolios sorted by size and book-to-market ratio involve 69 and 93 parameters, respectively, while for the CAPM (M5 and M8) involving 53 and 78 parameters, respectively, we need to include only the market return, instead of the macroeconomic factors, in the joint distribution. We are unable to estimate M10 and M12 for the 25 portfolios, as doing so involves estimating too many parameters for our sample size (118 and 143 parameters, respectively); hence we include the two data sets of the 10 portfolios formed for size and book-to-market ratio separately to test for these general two- and three-factor SDF models, in addition to using these one-sorted portfolios to contrast the estimation results with the two-characteristics-sorted portfolios.

4. Estimation Results

4.1 Data

The data are monthly from 1960.2 to 2004.11 for the U.S. (538 observations). The return on the market portfolio is the value-weighted return on all stocks. The return on a risk-free asset is the one-month Treasury bill rate. Table 2 shows the summary statistics for the 25 value-weighted portfolios, which are the intersections of 5 portfolios formed on size and 5 portfolios formed on the ratio of book-to-market ratio. There are also two sets of ten portfolios sorted by size and book-to-market ratio separately (Table 3). All of the return variables are obtained from Kenneth French's website. Real non-durable growth consumption is from the Federal Reserve Bank of St. Louis. CPI inflation and the volume index of industrial production are both from Datastream.

The descriptive statistics for the excess returns for the 25 portfolios in Table 2 are similar to those in Fama and French (1993) for the period 1963-1991, thus indicating the value effect and relatively weak size effect. In fact, the value effect in our sample seems to be stronger than that in the Fama and French sample as the dispersions in average returns for the first three book-to-market quintiles in our sample are larger. This relatively weak size effect is also seen in Table 3 where one-characteristic sorted portfolios are considered. In general, all excess returns and macroeconomic variables appear to have negative skewness, excess kurtosis, and non-normality, except for the risk-free rate and inflation, which displays positive skewness and show persistent volatility.

Table 2

Summary Statistics: 25 Size and Book-to-Market Portfolios

The table presents descriptive statistics for the excess returns on the 25 portfolios formed as the intersections of the five size and book-to-market ratio groups. Data and full definition of the returns can be found on Kenneth French's webpage. The returns are monthly value-weighted from 1960.2 to 2004.11, 538 observations. t-stat is the test statistics for zero mean hypothesis. ρ (x_t , x_{t-i}) represents the autocorrelation coefficients over the time interval *i* month (s).

Size Quintiles	Book-to-Market Equity Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	Mean					Standard deviation				
Small	-0.07	0.54	0.66	0.90	0.97	8.20	6.98	5.97	5.56	5.85
2	0.10	0.47	0.72	0.82	0.89	7.48	6.07	5.36	5.14	5.73
3	0.18	0.58	0.57	0.73	0.83	6.86	5.44	4.92	4.75	5.36
4	0.34	0.38	0.63	0.75	0.70	6.04	5.15	4.83	4.61	5.35
Big	0.30	0.39	0.46	0.47	0.49	4.80	4.54	4.29	4.19	4.78
	Skewness					Excess Kurtosis				
Small	-0.53	-0.46	-0.60	-0.59	-0.58	2.72	3.38	3.72	4.35	4.20
2	-0.70	-0.89	-0.92	-0.81	-0.76	2.34	4.03	4.56	4.23	4.32
3	-0.65	-0.99	-0.95	-0.59	-0.80	2.07	4.52	3.85	3.12	4.63
4	-0.49	-0.96	-0.75	-0.32	-0.52	1.99	4.93	3.86	1.82	2.72
Big	-0.46	-0.62	-0.53	-0.15	-0.36	1.89	2.60	3.18	1.23	1.17
	Normality					t-statistics for zero mean				
Small	72.7	110.0	111.1	144.2	137.7	-0.18	1.78	2.56	3.74	3.84
2	50.7	90.1	104.7	107.4	117.5	0.29	1.81	3.13	3.72	3.60
3	44.8	94.7	79.9	84.4	124.4	0.60	2.48	2.68	3.58	3.59
4	46.9	112.3	98.4	46.5	73.3	1.29	1.73	3.02	3.77	3.05
Big	44.2	62.0	92.6	27.5	23.8	1.45	2.00	2.50	2.62	2.37
	Average firm size					Average book-to-market ratio				
Small	37	39	38	34	26	0.28	0.57	0.78	1.03	1.85
2	173	175	177	176	172	0.28	0.54	0.76	1.005	1.70
3	413	421	421	424	431	0.27	0.54	0.75	1.004	1.66
4	1068	1063	1070	1079	1075	0.27	0.55	0.75	1.03	1.70
Big	9511	7119	6166	5052	4643	0.26	0.53	0.75	1.004	1.50
	Average percent of market value					Average number of firms				
Small	0.65	0.44	0.43	0.46	0.56	492	312	315	376	603
2	0.94	0.69	0.69	0.63	0.48	152	110	109	99	77
3	1.71	1.27	1.18	1.00	0.71	115	84	78	66	46
4	3.72	2.79	2.38	1.98	1.31	97	73	62	51	34
Big	36.21	16.87	11.29	7.43	4.17	106	66	51	41	25
	$\rho(x_t, x_{t-1})$					$\rho(x_t, x_{t-3})$				
Small	0.20	0.18	0.20	0.20	0.24	-0.06	-0.09	-0.05	-0.04	-0.04
2	0.16	0.16	0.17	0.16	0.15	-0.07	-0.05	-0.05	-0.05	-0.05
3	0.12	0.15	0.16	0.16	0.14	-0.05	-0.01	-0.05	-0.02	-0.04
4	0.11	0.13	0.11	0.08	0.07	-0.04	-0.04	-0.02	0.01	-0.04
Big	0.06	0.04	0.00	-0.02	0.06	0.03	-0.01	-0.02	0.02	-0.01
	$\rho(x_t, x_{t-6})$					$\rho(x_t, x_{t-12})$				
Small	0.02	0.03	0.03	0.02	-0.01	0.00	0.02	0.06	0.08	0.13
2	0.02	0.01	0.02	0.02	-0.01	-0.03	0.03	0.05	0.08	0.10
3	0.02	0.01	0.02	-0.01	-0.01	-0.03	0.03	0.02	0.04	0.08
4	0.02	0.01	-0.03	-0.03	-0.03	-0.03	0.00	0.03	0.06	0.06
Big	-0.03	-0.06	-0.04	-0.06	0.02	0.05	0.01	0.02	0.02	0.02

Table 3

Summary Statistics: 10 Industry Portfolios and Explanatory Variables

The table presents descriptive statistics for the returns on the 10 portfolios and explanatory variables. The returns are monthly value-weighted from 1960.2 to 2004.11, 538 observations. Data and full definition of the 10 portfolios can be found on Kenneth French's webpage. $i_{m,t+1}$ and i_t^f are the returns on the market portfolios and one-month Treasury bill rate respectively. Consumption growth, inflation, and industrial production growth are represented by Δc_{t+1}, $\Delta\pi_{t+1}$, and Δq_{t+1} respectively. Std. Dev is the standard deviation. t-stat is the t-statistic for zero mean hypothesis. t-stat is the test statistics for zero mean hypothesis. ρ (x_t , x_{t-i}) represents the autocorrelation coefficients over the time interval i month(s). BM denotes book-to-market equity ratio. Firm size, book-to-market equity ratio, percent of the market, and number of firms are in average terms.

	Size Deciles									
	Small	2	3	4	5	6	7	8	9	Large
Mean	1.04	0.98	1.03	0.98	1.01	0.92	0.98	0.95	0.91	0.79
Std. Dev.	6.32	6.26	5.99	5.80	5.55	5.25	5.11	4.98	4.54	4.26
Skewness	-0.53	-0.64	-0.78	-0.86	-0.85	-0.84	-0.71	-0.64	-0.56	-0.52
Excess Kurtosis	3.23	3.52	3.22	3.56	3.42	3.20	3.33	2.49	2.45	2.13
Normality	94.89	96.69	72.24	77.02	73.66	68.42	81.48	56.85	52.81	50.40
t-stat	3.81	3.64	3.98	3.91	4.23	4.08	4.45	4.41	4.64	4.29
Firm Size	21	78	139	219	338	512	803	1346	2597	12780
% of Market	1.47	1.37	1.62	2.00	2.59	3.33	4.69	7.39	13.23	62.31
No. of firms	2123	523	347	272	229	194	175	164	152	146
$\rho(x_t, x_{t-1})$	0.24	0.17	0.16	0.16	0.14	0.01	0.12	0.09	0.08	0.01
$\rho(x_t, x_{t-3})$	-0.05	-0.07	-0.07	-0.06	-0.05	-0.04	-0.03	-0.04	-0.03	0.03
$\rho(x_t, x_{t-6})$	0.01	0.02	0.01	0.02	0.02	0.01	0.00	0.00	-0.02	-0.03
$\rho(x_t, x_{t-12})$	0.08	0.04	0.03	0.03	0.01	0.01	0.01	0.01	0.01	0.05
	Book-to-market Deciles									
	Low	2	3	4	5	6	7	8	9	High
Mean	0.67	0.85	0.87	0.86	0.93	0.99	1.05	1.09	1.12	1.21
Std. Dev.	5.24	4.77	4.73	4.66	4.35	4.36	4.26	4.26	4.64	5.33
Skewness	-0.42	-0.69	-0.81	-0.66	-0.70	-0.67	-0.17	-0.28	-0.41	-0.37
Excess Kurtosis	1.67	2.76	3.88	3.12	4.24	3.54	1.80	2.11	2.11	3.26
Normality	38.09	63.18	92.12	78.47	121.46	93.76	50.24	60.74	54.66	112.13
t-stat	2.91	4.14	4.28	4.28	4.96	5.27	5.71	5.93	5.59	5.25
Firm Size	1582	1178	981	813	723	576	527	433	349	177
BM	0.20	0.37	0.49	0.60	0.71	0.82	0.94	1.10	1.35	2.03
% of Market	32.80	15.27	11.46	8.88	7.79	6.20	5.76	4.88	4.29	2.68
No. of firms	592	370	333	312	308	307	312	322	351	433
$\rho(x_t, x_{t-1})$	0.09	0.07	0.07	0.08	0.05	0.03	0.04	0.05	0.09	0.12
$\rho(x_t, x_{t-3})$	0.02	-0.01	-0.02	-0.03	-0.04	0.01	0.02	-0.01	-0.03	-0.03
$\rho(x_t, x_{t-6})$	-0.02	-0.01	-0.04	-0.04	-0.03	0.00	-0.06	-0.03	-0.02	-0.02
$\rho(x_t, x_{t-12})$	0.02	0.02	0.01	0.01	0.02	0.02	0.01	0.07	0.06	0.07

	Explanatory variables										
	$i_{m,t+1}$	i_t^f	Δc_{t+1}	$\Delta\pi_{t+1}$	Δq_{t+1}	Correlation					
Mean	0.94	0.46	0.23	0.35	0.25		$i_{m,t+1}$	i_t^f	Δc_{t+1}	$\Delta\pi_{t+1}$	Δq_{t+1}
Std. Dev.	4.41	0.23	0.73	0.30	0.75	i_t^f	-0.04	1.00			
Skewness	-0.46	1.041	-0.04	0.99	-0.62	Δc_{t+1}	0.15	-0.09	1.00		
Excess Kurtosis	1.90	1.70	1.37	1.68	2.98	$\Delta\pi_{t+1}$	-0.14	0.54	-0.20	1.00	
Normality	44.85	98.95	33.56	82.25	75.70	Δq_{t+1}	-0.03	-0.16	0.14	-0.10	1.00
$\rho(x_t, x_{t-1})$	0.06	0.95	-0.36	0.64	0.36						
$\rho(x_t, x_{t-3})$	0.00	0.90	0.14	0.53	0.27						
$\rho(x_t, x_{t-6})$	-0.02	0.84	0.01	0.52	0.09						
$\rho(x_t, x_{t-12})$	0.02	0.72	-0.07	0.44	-0.04						

4.2 Estimates

4.2.1 C-CAPM

A full set of model estimates with their restricted versions for the C-CAPM with power utility and nominal returns is reported in Table 4. A likelihood ratio test is used to examine the hypothesis implied by each restricted model against the unrestricted model. For M1, the conditional covariance of returns with consumption is highly significant, but the size of the coefficient, 83.25, implies an implausibly large CRRA, which is a common feature of consumption-based models (Campbell (2002), Yogo (2006), Smith, Sorensen, and Wickens (2008)).

Table 4
Estimates of C-CAPM

The table presents the estimates of the C-CAPM (M1-M4): 1960.2-2004.11, 538 observations. γ denotes the coefficient relative risk aversion and α_i represents a coefficient for each conditional covariance in Equation 8. $t(\gamma)$ and $t(\alpha_i)$ are their corresponding t–statistics respectively. The pricing models (M1-M4) are tested against each other using the log-likelihood ratio test. 2Δlog represents the likelihood ratio statistic. The corresponding p-value at 5% significance level is denoted by *p-value*.

Panel A: 25 Size and Book-to-Market Portfolios										
	Panel A1: Restricted C-CAPM (M1)									
	γ	$t(\gamma)$	$2\log$	$p-value$						
	83.25	4.11	89.30	0.0000						
	Panel A2: Restricted Book-to-Market C-CAPM (M2)									
	Size Quintiles									
	Small	2	3	4	Big					
α_{1s}	93.83	81.83	80.50	80.57	87.62					
$t(\alpha_{1s})$	4.13	3.89	3.73	3.58	2.62					
$2\log$	87.27									
$p-value$	0.0000									
	Panel A3: Restricted Size C-CAPM (M3)									
	Book-to-Market Quintiles									
	Low	2	3	4	High					
α_{1b}	11.49	62.46	100.51	130.99	128.32					
$t(\alpha_{1b})$	0.40	2.76	4.31	5.53	5.64					
$2\log$	31.31									
$p-value$	0.0513									
	Panel A4: Unrestricted C-CAPM (M4)									
Size Quintiles	Book-to-Market Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
	α_{sb}					$t(\alpha_{sb})$				
Small	47.79	106.31	145.27	183.46	167.67	0.90	2.55	3.32	4.47	4.66
2	66.32	104.24	155.03	171.43	164.63	1.46	2.76	4.07	4.70	4.61
3	93.06	119.64	146.79	177.21	176.68	1.86	3.55	3.73	4.65	4.45
4	108.03	129.48	146.63	169.83	142.16	2.21	2.82	3.97	4.44	3.83
Big	139.19	171.24	190.97	175.10	247.14	2.16	2.75	3.22	3.35	3.43
Panel B: 10 Size Portfolios										
	Panel B1: Restricted C-CAPM (M1)									
	γ	$t(\gamma)$	$2\log$	$p-value$						
	81.96	3.29	16.71	0.0534						
	Panel B2: Unrestricted C-CAPM (M4)									
	Size Deciles									
	Small	2	3	4	5	6	7	8	9	Large
α_{1s}	141.11	121.66	133.98	124.19	133.63	143.53	137.74	153.18	176.75	181.85
$t(\alpha_{1s})$	3.88	3.68	4.06	3.96	4.32	4.26	4.32	4.31	4.44	4.01
Panel C: 10 Book-to-Market Portfolios										
	Panel C1: Restricted C-CAPM (M1)									
	γ	$t(\gamma)$	$2\log$	$p-value$						
	261.78	7.93	5.96	0.7439						
	Panel C2: Unrestricted C-CAPM (M4)									
	Book-to-Market Deciles									
	Low	2	3	4	5	6	7	8	9	High
α_{1b}	215.24	235.78	249.82	252.10	267.28	270.21	272.45	279.32	254.01	250.89
$t(\alpha_{1b})$	4.18	5.42	5.51	5.88	5.89	6.72	7.44	7.82	6.99	7.33

In M2, all five consumption coefficients are significant, and the likelihood ratio rejects the hypothesis that portfolios within different book-to-market equity ratio quintiles have the same coefficient at any conventional levels. The test statistic is close to that in M1, implying that the differences in the coefficients across size have little weight on the behavior of the estimated returns. On the other hand, the likelihood ratio test marginally accepts M3 ($p - value = 0.0513$); restricting the consumption coefficients for portfolios within the same size quintiles to be the same does not exclude significant information about the excess returns. In other words, size has no or a relatively weak relation to the consumption coefficient. In fact, the coefficients in M2 for each size quintile look very similar, while those in M3 for each book-to-market equity ratio quintiles increase from the lowest to the highest book-to-market quintiles. In addition, the consumption coefficient for the lowest book-to-market equity ratio quintiles in M3 is not significant.

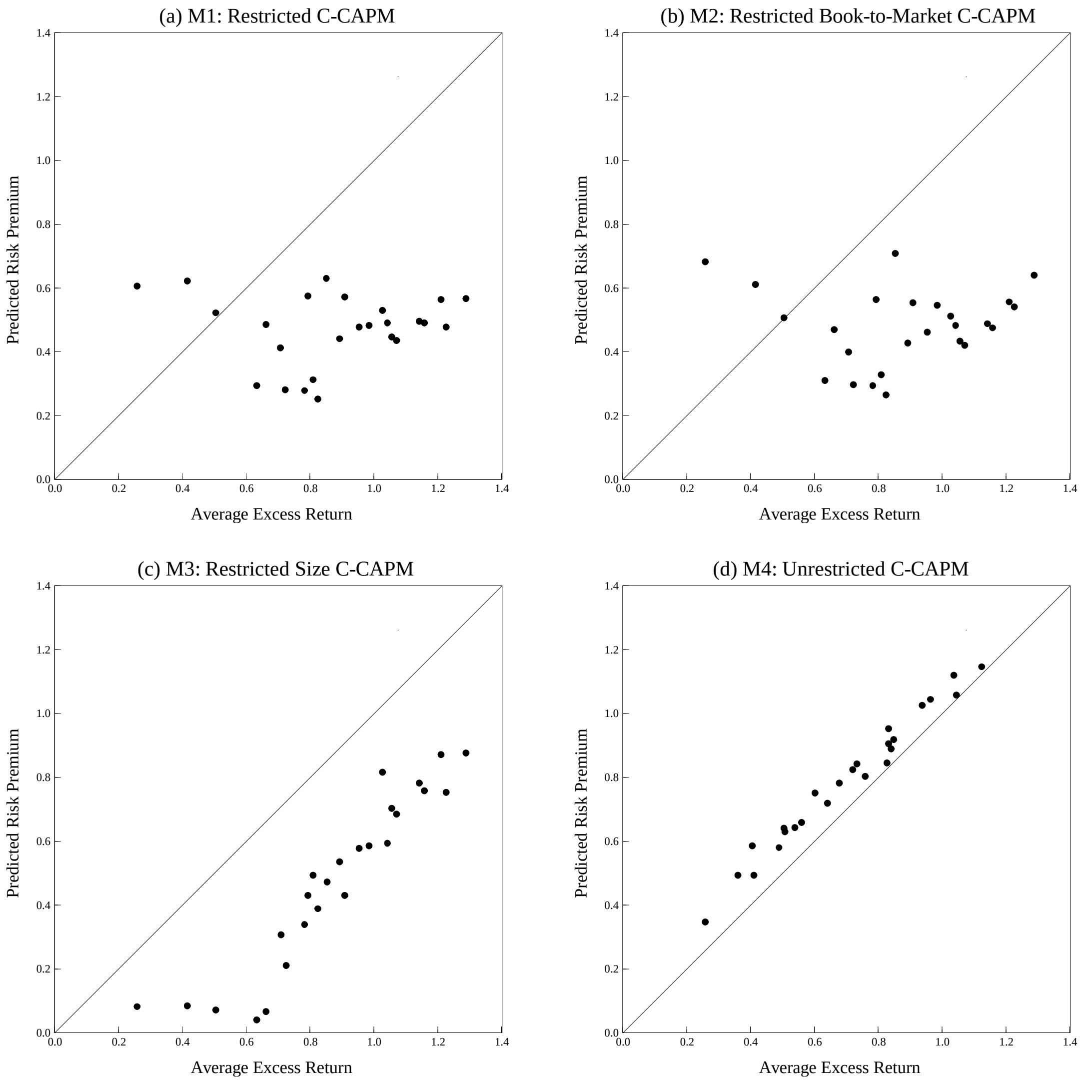
M4 has 23 coefficients that are significant at the conventional level. All coefficients range from 47.79 to 247.14. Looking down each column, there is no clear pattern in the values of the coefficients across the size quintiles when looking across each row; the coefficients tend to rise as the book-to-market ratio increase. Like the insignificance of the consumption coefficient on the lowest book-to-market quintile in M3, the remaining 2 coefficients in the lowest book-to-market quintile and the first 2 smallest size quintiles are not significantly different from zero. The insignificance of the coefficients in the lowest book-to-market quintiles of the 25 portfolios is consistent with the evidence from other empirical asset pricing studies on the 25 portfolios (Fama and French (1993 and 1996), Lettau and Lugvigson (2001), Parker and Julliard (2005), and Yogo (2006)) where the pricing models have difficulty explaining the portfolios in the smallest size and lowest book-to-market quintiles (small growth portfolio). This inability may be due to limits arbitrage from short-sale constraints for these portfolios, and thus frictionless equilibrium models, including C-CAPM, cannot explain the returns on these small growth portfolios (Yogo (2006)).

Figure 1 shows a scatter plot of average actual and estimated excess returns in M1 to M4 for the 25 portfolios. If the pricing model fits the data well, the points should all lie on a 45-degree line. In Figure 1(d), M4 appears to best explain the excess return on these portfolios and is more or less as good as the modified versions of the C-CAPM and the Fama and French three-factor model. The differences in the estimated risk premium and actual excess return range from 0.01% to 0.18% per month, which is lower than those in M1-M4.

Figures 1(a) and 1(b) show that the estimated risk premia from M1 and M2 are similar, implying that imposing the restrictions on size quintiles does not affect the behavior of risk premia. On the other hand, allowing the consumption coefficients to be different as in M3 improves the performance of the model sharply, except for the 5 portfolios in the lowest book-to-market ratio quintiles. This observation suggests that book-to-market equity ratio seems to have additional information about the average excess returns that is not captured by the standard C-CAPM.

Figure 1
Cross-Sectional Fit: C-CAPM for the 25 Size and Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market portfolios. The estimated models are (a) M1, (b) M2, and (c) M3, and (d) M4. The average excess returns are adjusted for the Jensen effect.



We estimate M1 and M4 for the two one-characteristic sorted portfolios to investigate whether these provide information as in the double-sorted portfolios. Panels B and C in Table 4 show that the C-CAPM with power utility performs better, as neither of the M1 is rejected, suggesting that sorting stocks according to size and book-to-market ratio may more accurately distinguish stocks. For the 10 size portfolios, all coefficients on the consumption covariances in M4 are significant, and the model fits the data better than M1 (Figure 2). This is also true for the 10 book-to-market ratio portfolios (Figure 3). In addition, the consumption coefficient for the portfolio in the lowest book-to-market quintile is highly significant, while those for the small growth portfolios in the 25 portfolios are not. The descriptive statistics in Table 2 show that the average book-to-market ratios for these two portfolios are similar, while their average firm sizes are very different. Firms in the smallest size and the lowest book-to-market quintiles seem to be much smaller than other firms in the lowest book-to-market quintiles. Therefore, additional information that is not captured by the C-CAPM may be associated with both book-to-market equity ratio as well as size.

Figure 2
Cross-Sectional Fit: C-CAPM for the 10 Size Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 size portfolios. The estimated models are (a) M1 and (b) M4. The average excess returns are adjusted for the Jensen effect.

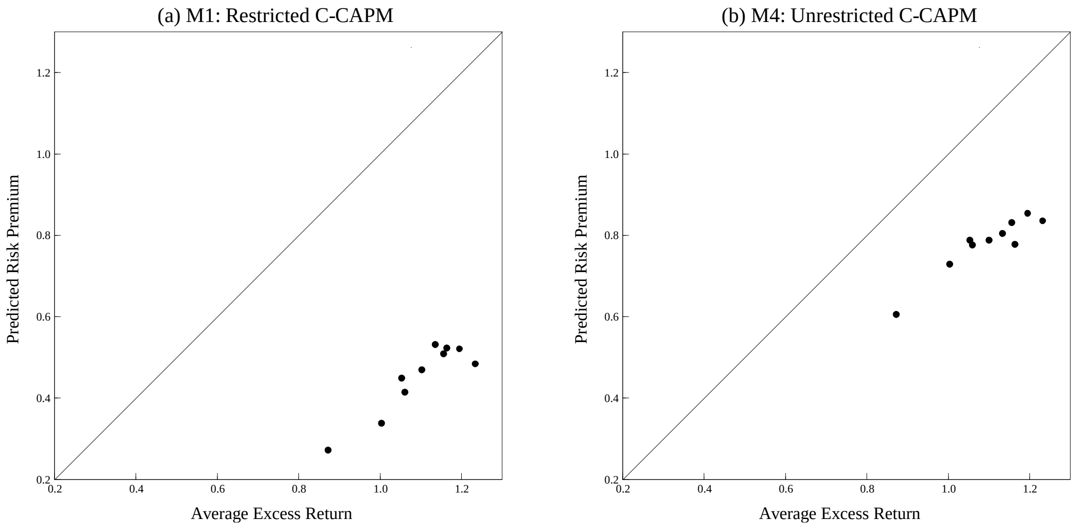
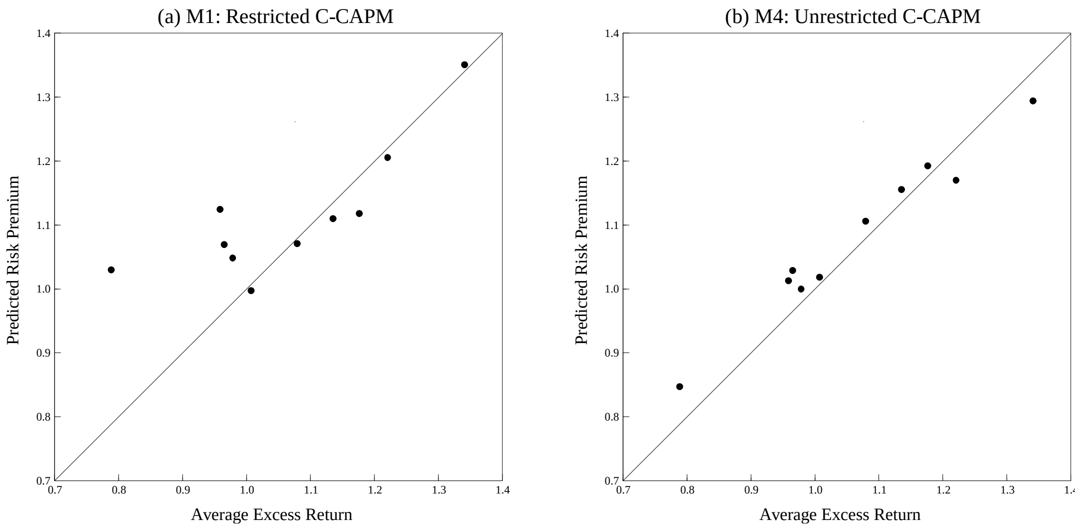


Figure 3
Cross-Sectional Fit: C-CAPM for the 10 Book-to-Market Ratio Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 book-to-market ratio portfolios. The estimated models are (a) M1 and (b) M4. The average excess returns are adjusted for the Jensen effect.



4.2.2 CAPM

Table 5 reports the estimation results for all versions of the CAPM. The market price of risk in M5 is 2.77 with a t-statistics of 2.94 and is lower than those reported in related studies for the U.S. market (Harvey (1989) and Ng (1991)). Comparing M6 to M2, where the coefficients for the first three size quintiles are relatively less significant, suggests that the CAPM cannot price relatively small portfolios well and there is more information in these portfolios related to size left unexplained more by the CAPM than the C-CAPM. This inability to price small portfolios has nothing to do with the book-to-market ratio.

Moreover, as in M3, the coefficient of consumption for the portfolios in the lowest book-to-market ratio in M7 is not significant.

Table 5
Estimates of the CAPM

The table presents the estimates of the CAPM (M5-M8): 1960.2-2004.11, 538 observations. δ denotes the market price of risk and α_i represents a coefficient for each conditional covariance in Equation 8. $t(\delta)$ and $t(\alpha_i)$ are their corresponding t–statistics respectively. The pricing models (M5-M8) are tested against each other using the log-likelihood ratio test. 2Δlog represents the likelihood ratio statistic. The corresponding p-value at 5% significance level is denoted by *p-value*.

Panel A: 25 Size and Book-to-Market Portfolios

Panel A1: Restricted CAPM (M5)

δ	$t(\delta)$	$2\log$	$p-value$
2.77	2.94	153.72	0.0000

Panel A2: Restricted Book-to-Market CAPM (M6)

	Size Quintiles				
	Small	2	3	4	Big
α_{4s}	1.70	1.69	1.74	2.03	2.61
$t(\alpha_{4s})$	1.56	1.68	1.80	2.11	2.79
$2\log$	110.22				
$p-value$	0.0000				

Panel A3: Restricted Size CAPM (M7)

	Book-to-Market Quintiles				
	Low	2	3	4	High
α_{4b}	0.93	2.54	3.51	4.57	4.73
$t(\alpha_{4b})$	0.93	2.68	3.75	4.79	4.89
$2\log$	43.04				
$p-value$	0.0020				

Panel A4: Unrestricted CAPM (M8)

Size Quintiles	Book-to-Market Quintiles									
	Low	2	3	4	High	Low	2	3	4	High
			α_{4sb}					$t(\alpha_{4sb})$		
Small	0.02	2.20	3.02	4.37	4.51	0.02	1.75	2.59	3.78	3.93
2	0.76	2.30	3.55	4.36	4.34	0.67	2.16	3.40	4.10	4.02
3	1.30	3.05	3.14	4.37	4.26	1.15	3.00	3.14	4.26	3.92
4	1.94	2.26	3.59	4.16	3.57	1.74	2.26	3.53	4.04	3.27
Big	1.93	2.22	2.70	3.20	3.21	1.81	2.15	2.53	2.91	2.63

Panel B: 10 Size Portfolios

Panel B1: Restricted CAPM (M5)

δ	$t(\delta)$	$2\log$	$p-value$
-0.47	-0.34	811.30	0.0000

Panel B2: Unrestricted CAPM (M8)

	Size deciles									
	Small	2	3	4	5	6	7	8	9	Large
α_{4s}	3.57	3.07	3.41	3.13	3.44	3.27	3.29	3.26	3.23	2.45
$t(\alpha_{4s})$	3.46	3.27	3.74	3.44	3.93	3.69	3.82	3.80	3.77	2.88

Panel C: 10 Book-to-market Portfolios

Panel C1: Restricted CAPM (M5)

δ	$t(\delta)$	$2\log$	$p-value$
1.98	1.79	404.85	0.0000

Panel C2: Unrestricted CAPM (M8)

	Book-to-market Deciles									
	Low	2	3	4	5	6	7	8	9	High
α_{4b}	3.14	4.29	4.68	4.76	5.50	5.73	6.64	6.96	6.18	6.24
$t(\alpha_{4b})$	3.19	4.73	5.39	5.51	6.06	6.58	7.76	7.97	6.86	7.04

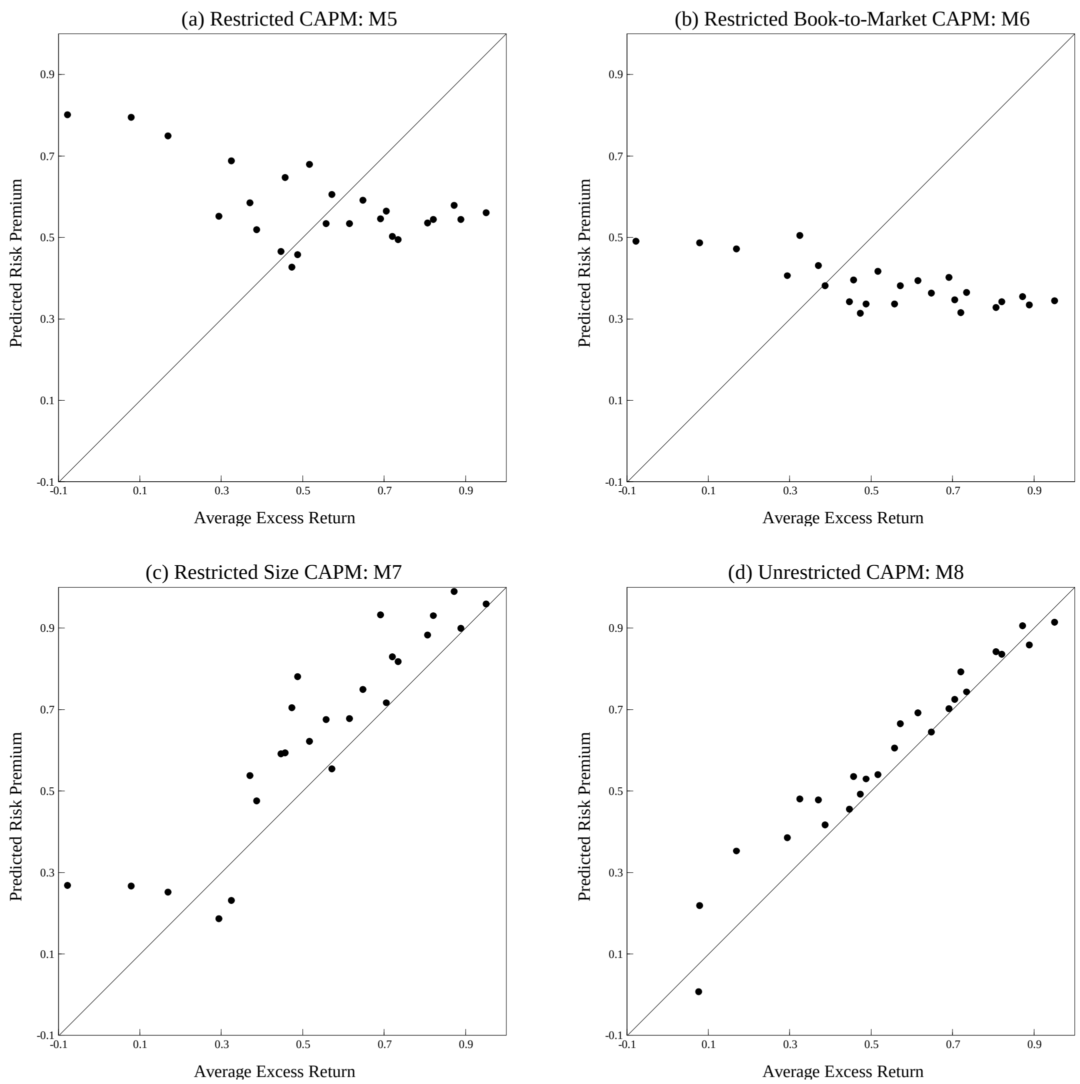
M5, M6, and M7 are rejected based on their likelihood ratio statistics of 153.72, 110.22, and 43.04 respectively. The likelihood ratio statistics for the CAPM are all larger than those for the C-CAPM. As in the C-CAPM, allowing the coefficients on conditional covariances of market return with individual excess return to be different offers extra information about the cross-section of the equity returns. As can be seen from Figure 4, M8 can better explain the variation in the cross section.

In M8, 20 coefficients of conditional covariances of returns with the market return, including one for the market return (its coefficient of the variance), are more than 2 standard errors different from zero, while the other 3 coefficients are significant at a 10% significance level. The coefficients for the first 3 size and lowest book-to-market quintiles are insignificant at any conventional level. These coefficients range from 0.02 to 4.51, exhibiting a clear positive relation to book-to-market ratio. However, the relation between the coefficients and size can be seen only from the portfolios in the last two book-to-market quintiles.

Figure 4

Cross-Sectional Fit: CAPM for the 25 Size and Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market portfolios. The estimated models are (a) M5, (b) M6, and (c) M7, and (d) M8. The average excess returns are adjusted for the Jensen effect.



M5 for the two sets of 10 portfolios does not perform well since the market price of risk is of relatively low significance and in the case of the 10 size portfolios, the market price of risk has a wrong sign. Therefore, the likelihood ratio test rejects M5 for both 10 portfolios. M8 fits the data better than M5 (Figures 5 and 6), but its coefficients exhibit a negative relation with size and a negative relation to book-to-market ratio for both 10 portfolios as in the case of the 25 portfolios. On the other hand, the relation between the consumption coefficients and firm characteristics in the C-CAPM can be seen only in the case of the 25 portfolios. Thus, the C-CAPM can explain size effect, but it has difficulty explaining the value effects; this exposure to the value premium appears to be associated with both the book-to-market ratio and, to some extent, to size.

Figure 5
Cross-Sectional Fit: CAPM for the 10 Size Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 size portfolios. The estimated models are (a) M5 and (b) M8. The average excess returns are adjusted for the Jensen effect.

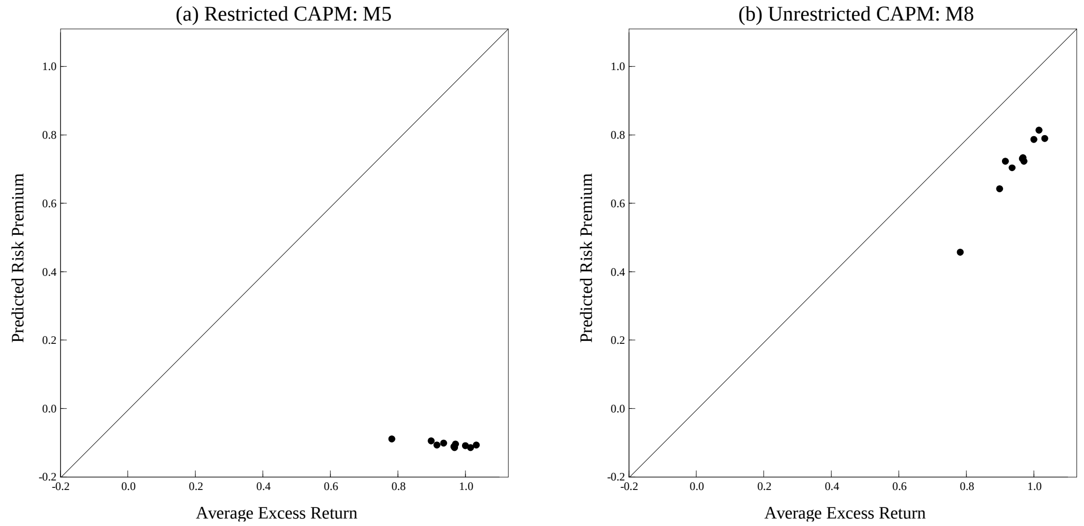
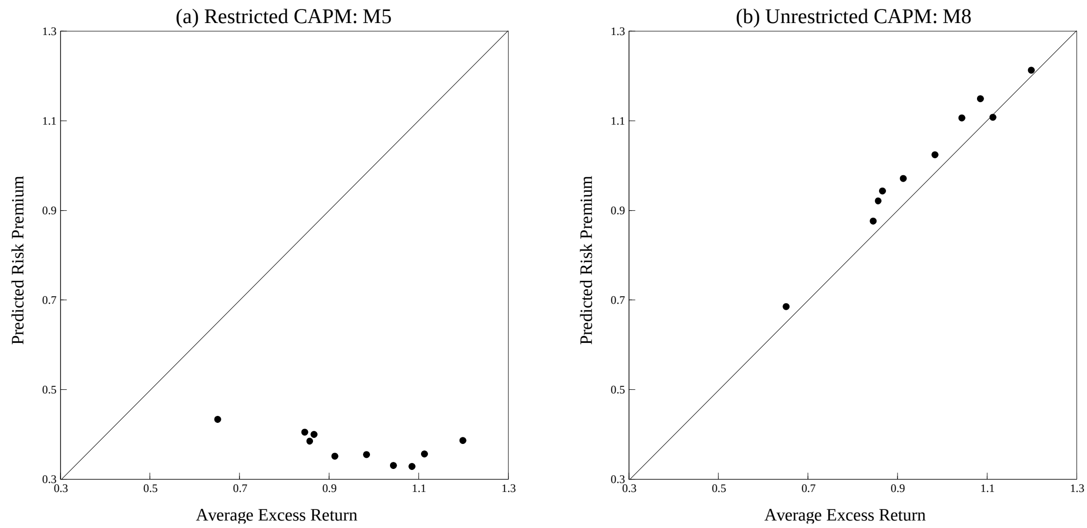


Figure 6
Cross-Sectional Fit: CAPM for the 10 Book-to-Market Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 10 book-to-market ratio portfolios. The estimated models are (a) M5 and (b) M8. The average excess returns are adjusted for the Jensen effect.



4.2.3 C-CAPM and CAPM

We further investigate the ability of the standard C-CAPM (M1) and the CAPM (M4) by comparing the behaviour of the conditional covariances in both models, as given in Table 6 below, with the average excess returns of the 25 portfolios shown in Table 2. The consumption covariance obtained from the estimation of the C-CAPM decrease, as we move down the column, indicates a negative relation with size. Thus, the C-CAPM can capture the size effect. However, in the lowest book-to-market quintiles, average excess returns increase as the size of the portfolios grows larger. This result is consistent with the results in Table 4 where the coefficients for these portfolios in M3 and M4 are not significant from zero.

Table 6
Conditional Covariances of the Returns on the 25 Portfolios in M1 and M5

The table shows the average conditional covariances of the returns with consumption and market returns as implied by the C-CAPM and CAPM, respectively. Each conditional covariance is given in percent per month and estimated by the multivariate GARCH in the mean.

Size Quintile	Book-to-market ratio Quintile				
	Low	2	3	4	High
	Panel A: Conditional covariance of consumption				
Small	0.32218	0.28800	0.31130	0.22016	0.27699
2	0.24066	0.23827	0.23237	0.17594	0.20899
3	0.19671	0.17586	0.16247	0.15597	0.17918
4	-0.00131	-0.00166	-0.00138	0.00593	0.00496
Big	0.00304	0.00001	-0.00133	-0.00127	-0.00040
	Panel B: Conditional covariance of market return				
Small	0.32143	0.35714	0.25829	0.25272	0.28785
2	0.24282	0.22963	0.21088	0.24303	0.17777
3	0.23056	0.18894	0.18875	0.17934	0.16103
4	0.18551	0.21742	0.20223	0.15693	0.20226
Big	0.27074	0.24804	0.19669	0.16809	0.16478

The C-CAPM appears to miss the value premium completely by not producing dispersion in the consumption covariance across the book-to-market quintiles. In fact, the consumption covariances for the 5 portfolios in the highest book-to-market quintile seem to be slightly lower than those in the lowest book-to-market quintiles, indicating lower risk premium is implied by the C-CAPM. On the other hand, the condition covariances of the returns with the market returns in the CAPM, in addition to having a similar behavior across book-to-market quintiles as consumption covariances, appear not to be able to capture the size effect as well. The dispersion of the market covariances is not big enough to explain the difference in the excess returns across size, confirming our previous results where the coefficients for the first two size quintiles in M6 were not highly significant.

We add a constant term in M1-M8 for the estimation of the 25 portfolios to measure variation in excess returns that was left unexplained in each model. In general, we expect the constant term in the C-CAPM to be of more significance than in those in the CAPM because pricing asset returns with market return is expected to be more precise than using aggregate consumption data. However, Table 7 shows that for the magnitude of the constant terms in the CAPM, 1.98, is larger than that for the C-CAPM at 0.86. This larger magnitude of the CAPM is present in every restricted version. The magnitude of the constant is also larger than in Fama and French (1993) with a constant for the CAPM being 0.04 to 0.57 (in absolute terms).

Table 7
Constant term

The table presents the estimates for the constant term, in all versions of the C-CAPM and CAPM, M1-M8 in Table 1, for the 25 portfolios formed based on size and book-to-market ratio. The number in the parentless is the t-statistic associated with each constant term.

Panel A: C-CAPM				
	M1	M2	M3	M4
Constant	0.84	1.14	0.52	0.87
	(5.36)	(6.39)	(2.75)	(1.75)
Panel B: CAPM				
	M5	M6	M7	M8
Constant	1.98	1.75	0.56	0.94
	(9.13)	(6.79)	(1.85)	(1.70)

The information about the cross-section of equity returns left unexplained in the C-CAPM seems to be less than that in the CAPM. Moving from M1 to M4 decreases the significance of the constant terms (except for moving from M1 to M2), suggesting that allowing coefficients of conditional covariances within the same book-to-market ratio to be different is more important than allowing the coefficients to be different across size quintiles; the magnitude and level of significance of the constant terms reduces more when moving from M2 to M3 than when moving from M1 to M2. This argument is also true for the CAPM when moving from M5 to M8.

4.2.4 General SDF Models

Table 8 reports the estimates of the general two- and three-factor SDF models based on consumption, inflation, and industrial production. We are unable to estimate M11 for the 25 portfolios due to the high parameterization of the MGM. As in Smith, Sorensen, and Wickens (2008), we find that industrial production plays no role in evaluating asset returns, but inflation is significant. The coefficient for conditional covariance of inflation for the 10 book-to-market portfolios is positive because the contribution to risk premium by consumption is higher than it is for actual excess return. M11 for both 10 portfolios is rejected by the likelihood ratio test, implying that the coefficients for conditional covariance of consumption and inflation with the returns are similar across size and book-to-market deciles. However, M9 does not explain the data better than the C-CAPM and the CAPM (Figures 7).

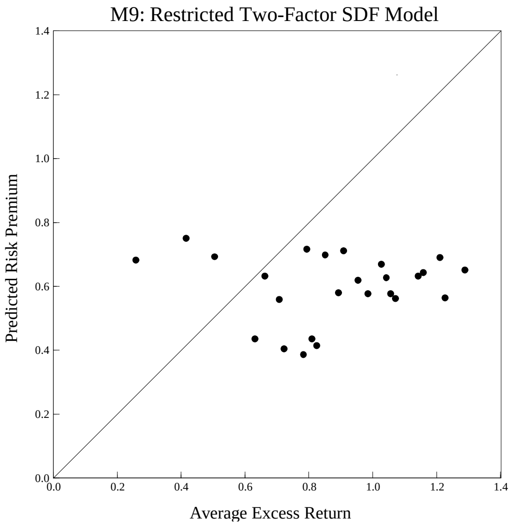
Table 8
Estimates for Restricted Macro SDF Models

The table presents the estimates for the restricted general SDF models (M9 and M11): 1960.2-2004.11, 538 observations. α_i represents a coefficient for each conditional covariance in Equation 8. $t(\alpha_i)$ is its corresponding t–statistics respectively. The pricing models (M9 and M11) are tested against each other using the log-likelihood ratio test. 2Δlog represents the likelihood ratio statistic. The corresponding p-value at 5% significance level is denoted by *p-value*.

Model	α_1	α_2	α_3	2 log	$p-value$
Panel A: 25 Size and Book-to-Market Portfolios					
M9	72.93 (3.40)	-115.72 (-1.74)			
M11	72.82 (3.40)	-116.47 (-1.73)	1.42 (0.06)		
Panel B: 10 size portfolios					
M9	59.29 (2.18)	-130.87 (-2.01)		23.66	0.1666
M11	59.62 (2.17)	-137.76 (-2.08)	16.84 (0.55)	33.43	0.1832
Panel C: 10 Book-to-market portfolios					
M9	307.12 (8.00)	147.29 (1.92)		23.27	0.1805
M11	305.17 (7.87)	141.17 (1.82)	8.58 (0.35)	32.33	0.2201

Figure 7
Cross-Sectional Fit: Two-Factor SDF Model for the 25 Portfolios

The figure plots average actual versus predicted excess returns (% per month) for the 25 size and book-to-market ratio portfolios. The estimated model is M9. The average excess returns are adjusted for the Jensen effect.



5. Conclusion

This paper examines the behavior of the cross-section of equity returns based on the no-arbitrage condition present in the Stochastic Discount Factor approach. We test whether the conditional covariances of the equity returns across portfolios formed on size and book-to-market ratio with discount factors in each asset pricing model can sufficiently explain the excess returns in these portfolios. Our results indicate that the no-arbitrage test rejects the standard C-CAPM as the model can explain the size effects, but not the value effect. Although the consumption covariances exhibit a negative relation with size, but they do not vary across book-to-market ratio. This behavior explains why the likelihood ratio test indicates that the coefficients for the consumption covariances are not similar across the book-to-market ratio.

Allowing the consumption coefficients to be different across portfolios largely improves the fit of the C-CAPM. Without adding any factor to the model, the performance of the unrestricted C-CAPM is comparable to the modified version of the C-CAPM in Lettau and Ludvigson (2001), Parker and Julliard (2005), and Yogo (2006). There appears to be less variation in excess returns that was left unexplained in the unrestricted C-CAPM as the significance of the constant term is lower than that in the standard C-CAPM. Even though the unrestricted C-CAPM cannot explain the small growth portfolios well, this phenomenon is common in most asset pricing models.

For the no-arbitrage opportunities in the market, the relation between the equity return and the book-to-market ratio that cannot be explained by the C-CAPM should arise from the fact that there is an additional dimension of risk left unexplained by the C-CAPM. To see this additional risk, double-sorted size and book-to-market portfolios need to be examined, as the value effect in the C-CAPM seems to be associated with the book-to-market ratio as well as size. There should possibly be one extra dimension of risk associated with both (small) size and (low) book-to-market ratio. Firm size or book-to-market ratio separately does not generate information about average returns that cannot be explained by the C-CAPM. This is why sorting stocks according to size and the book-to-market ratio more accurately distinguishes stocks and why 25 portfolios are widely used in the literature.

For the CAPM, we find that there is little dispersion of conditional covariance for portfolio return with the market return across size and the book-to-market ratio. Therefore, the coefficients of these conditional covariances display systematic relations with both firm characteristics, indicating that both size and book-to-market ratio contain information about cross-section average returns that cannot be explained by the CAPM as in previous studies (Fama and French (1992a and 1993b)). Our results imply that there is more information to be added by the size and book-to-market ratio in the CAPM, which is based on the market portfolio, than in the C-CAPM, wherein consumption contains additional information related to size. In addition, the general SDF models suggest that inflation seems to be significant in determining stock returns, but industrial production plays no role in determining stock returns. However, the pricing models that include inflation do not perform better than the C-CAPM.

References

Banz, R., 1981. The relationship between return and market value of common stock. Journal of Financial Economics 9, 3−18.

Breeden D.T., Gibbons, M.R., Litzenberger, R.H., 1989. Empirical tests of the consumption-oriented CAPM, Journal of Finance 44, 231−262.

Chan, L.K.C., Hamao, Y., Lakonishok, J., 1991. Fundamentals and stock returns in Japan, Journal of Finance 46, 1739-1789.

Campbell, J.Y., 2002. Consumption-based asset pricing, in: Constantinides, G., Harris, M., Stulz, R. (Eds), Handbook of Economic and Finance, Elsevier.

Ding, Z., Engle, R.F., 2001. Large scale conditional covariance matrix modeling, estimation and testing, Academia Economic Papers 29, 157-184.

Fama, E.F., French, K.R., 1992a. The cross-section of expected stock returns, Journal of Finance 47, 427−465.

Fama, E.F., French, K.R., 1993b. Common risk factors in the returns on stocks and bonds, Journal of Financial Economics 33, 3−56.

Harvey, C. R., 1989. Time-varying conditional covariances in tests of asset pricing models, Journal of Financial Economics 24, 289-317.

Lettau, M., Ludvigson, S., 2001. Resurrecting the (C)CAPM: A cross-sectional test when risk premia are time-varying, Journal of Political Economy 109, 1238-1287.

Mankiw, N.G., Shapiro M.D., 1986. Risk and return: Consumption beta versus market beta, Review of Economics and Statistics 68, 452–459.

Merton, R. C., 1980. On estimating the expected returns on the market: An exploratory investigation, Journal of Financial Economics 8, 323−361.

Ng, L., 1991. Tests of the CAPM with time-varying covariances: A multivariate GARCH approach, Journal of Finance 46, 1507−1521.

Parker, J. A., Julliard, C. 2005. Consumption risk and the cross -section of expected returns, Journal of Political Economy 113, 185-222.

Smith, P. N., Wickens, M. R., 2002. Asset pricing with observable stochastic discount factors, Journal of Economic Surveys 16, 397-446.

Smith, P. N., Sorensen S., Wickens M. R., 2008. General equilibrium theories of the equity risk premium: Estimates and Tests, Quantitative and Qualitative Analysis Social Sciences 2, 35-66

Yogo, M., 2006. A consumption-based explanation of expected stock returns, Journal of Finance 61, 539–580.