



Forest  
Conservation  
and CO2  
emissions: A  
Viable  
Approach

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GERAD,  
Montreal ;  
CREA, SDFi-  
DAUPHINE ;  
GERAD,  
Montreal  
Natural Risks,  
climate change,  
natural and  
renewables  
resources, LEF

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# Forest Conservation and CO2 emissions: A Viable Approach

Pablo Domenech, Patrick Saint-Pierre & Georges Zaccour

GERAD, Montreal ; CREA, SDFi-DAUPHINE ; GERAD, Montreal

Natural Risks, climate change, natural and renewables  
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Nancy, 7 septembre 2010



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## 1 Sustainable Development

## 2 Applications

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## 3 Recovering Sustainability

- Viability Multipliers
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# Sustainable Development

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The concept « **Sustainable Development** » is widely adopted, however finding explicit « **Sustainable** » strategies is difficult.

What means :

- to be sustainable ?
- to satisfy or to reach an objective ?
- to satisfy, to manage, to impose constraints ?
- to preserve resources ?
- to guarantee the welfare of different generations ?
- to preserve equity between successive generations ?



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# Sustainable Development

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Mathematical concepts are needed for analysing complex dynamical systems

- define a **state** :  $\mathbf{x} \in \mathbf{X} = \mathbb{R}^n$  ,
- define state **constraints** :  $K$  ,
- precise **parameters** :  $p$  ,
- consider the **dynamics** of the state evolution :  
 $\mathbf{x}'(t) \in F(\mathbf{x}(t), p)$  ,
- consider **controls** or **regulons** that formalise possible actions on the evolutionary system :  
 $F(\mathbf{x}, p) = \{f(\mathbf{x}, p, u), u \in U(\mathbf{x})\}$  ,
- consider non controllable **perturbations** :  
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# A fundamental ethic concept

## Sustainability and Equity

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La **Sustainability** is a notion which refers to different “qualitative criteria ” such as :

- the non decrease of ...
- the maintenance of ...
- the return back to ...
- the compatibility between ...





# Mathematical translation of ethic concepts

## Sustainability / Equity / Viability

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- Viability Theory was developed in order to answer to these challenges.
- It addresses Sustainability in term of respecting constraints at any time and Equity in term of respecting thresholds on some state variable variation at any time and finding decisions on time.
- Depending on context one can construct “Viability Tools” which answer to Sustainability problems as soon as they are well identified.
- Aiming at measuring impact or at optimizing a criteria, it is possible, in return for higher algorithmic complexity, to give answer for problematics where sustainability and optimality interfere.

# A fundamental mathematical concept :

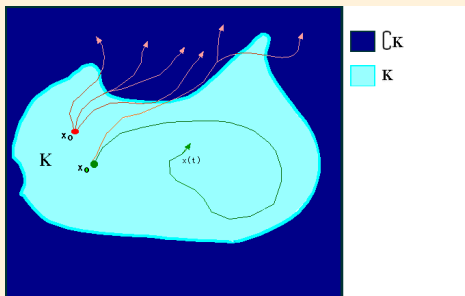
## Dynamics and Constrains

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An environment is described by a set of constraints  $K$ . The evolution is governed by a law  $f$  dependent of a parameter variable  $u$ .

# A fundamental mathematical concept :

## The Viability Kernel

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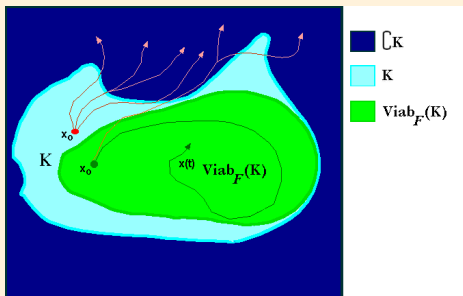
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It is the set of the states in the environment  $K$  from which starts *at least an evolution that remains always in  $K$* . All viable evolution remains necessarily always in  *$Viab_F(K)$* .

# A fundamental mathematical concept :

## Dynamics and Constrains

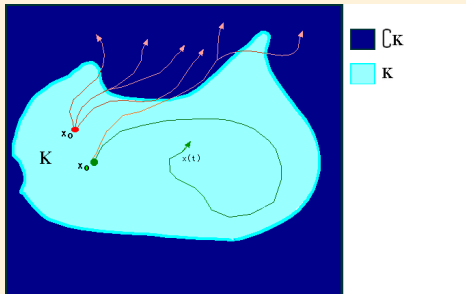
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Now the evolution is submitted to unpredictable uncertainty described by a law  $f$  depending on a parameter variable  $v$ .

# A fundamental mathematical concept :

## The Invariance Kernel

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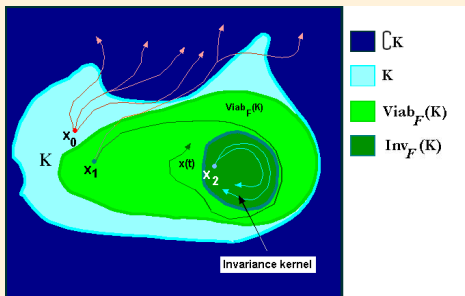
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It is the set of the states in the environment  $K$  from which *all* evolution starts that remains always in  $K$ .

# A fundamental mathematical concept : ... And To Reach a Target in Finite Time

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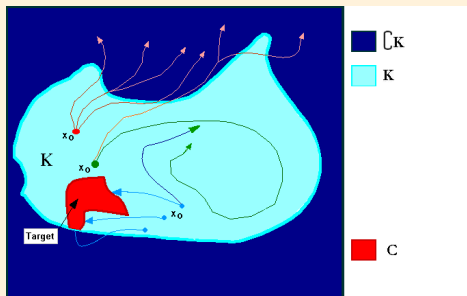
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Evolutions must remain in  $K$  until they reach a target at some fixed time or in finite time.

# A fundamental mathematical concept :

## The Capture Basin

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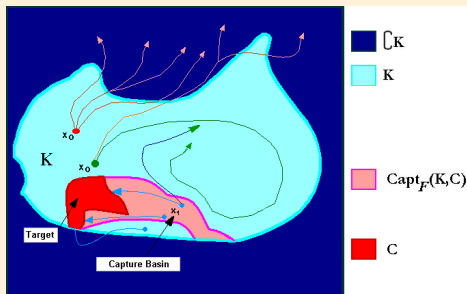
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The capture basin of the target, viable in an environment, is the subset (possibly empty) of the states of the environment  $K$  from which *at least* one evolution remains *viable* in the environment until it reaches the target *in finite time*..



# Constraint Management and Objective Realization

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Viability Theory is devoted to fulfilling various objectives while keeping physical or normative constraints but new set of problems emerge, for instance :

- What normative constraints adjustments are necessary to fulfill sustainability and equity of at least one evolution starting from the actual situation ?
- What decision or policy must be taken so as to ensure "Sustainability" and "Equity" ? What cost ?
- In more general case, what can be done if "Sustainability" and "Equity" seems not to be achievable in the actual context ?





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# Questions Read Through Applications

## Some Environmental Modeling

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- Tropical Forest : (with Sophie Martin (LISC, Clermont-Ferrand) and Claire Bernard)
  - A problem of biodiversity conservation
  - A local and global model
  - Coupling economy, society and environment
  
- World Forest :(with Pablo Domenech and Georges Zaccour (GERAD, HEC Montreal))
  - A environmental problem
  - A global model
  - Coupling economy and environment

*(DEDUCTION - ANR Project)*



# The Humid Tropical Forest

## Preservation of Biodiversity in Fianarantsoa Corridor

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- **The Sustainability Problem :**

An increasing and lacking in revenue population look for lands for cultivating in a context where soil property rights do exist only for cleared lands.

- there is not necessarily incompatibility between biodiversity protection and subsistence agriculture maintenance.
- consider the poverty problem face to numerous legal, cultural, environmental , agronomic, biologic, economic or social questions at stake.

- **The Model to be consider :**

- is a coexistence model between population activity and environment preservation



# The Humid Tropical Forest : A Coexistence Population/Environnement Model

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This coexistence model aims at

- determining conditions for preserving a certain amount of forest surface in the Fianarantsoa corridor,
- characterizing appropriate regulations of population,
- evaluating necessary monetary transfers,
- suggesting economical development and employment strategies,

which allows biodiversity preservation of the forest and autochthon population welfare.



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# The Humid Tropical Forest :

## The model

- **S** the surface dedicated to agriculture (ha) ;
- **N** the population level ;
- **K** the capital).

$$\begin{aligned}
 S'(t) &= d(t) \\
 N'(t) &= r(t)N(t) - v(t)N(t) \\
 K'(t) &= -cN(t) - \beta_S(t)d(t) + \\
 &\quad \mu \min(S(t), \gamma(1 - v(t))N(t)) + \\
 &\quad \omega v(t)N(t) + \tau(t)
 \end{aligned}$$



# The Humid Tropical Forest : Coexistence Parameters

- The different parameters in the model :

$F_0$  : forest surface in 1900 (ha)

$c$  : minimal level of consumption (US\$)

$d_{max}$  : maximal surface can be converted into culture (ha per capita)

$\gamma$  : maximal cultivable surface (ha per capita)

$\omega$  : individual wages (US\$ per capita)

$\beta_S(t)$  : land use planning cost (US\$ per ha)

$\mu$  : net revenue per ha (US\$)

$F_{min}$  : minimal surface of primary forest one want to preserve (ha)

- The population growth rate is let "free" :  $r(t) \in [r_{min}; r_{max}]$  playing the role of a regulon ; Viability analysis will reveal population regulations that are compatibles with **Sustainability**.

# The Humid Tropical Forest : Coexistence

## Economical Mechanisms

- Four controls or “regulons” :

**$d(t)$**  : **land use planning effort** (ha per year) : additional surface to be converted (clearing, irrigation network,...) :

$$0 \leq d(t) \leq N(t)d_{max}$$

where  $d_{max}$  is the maximal convertible surface per capita and year ;

**$v(t)$**  : **Proportion of wage-earners** who may work somewhere else or apply for another activities for a while :

$$0 \leq v(t) \leq 1$$

**$r(t)$**  : **Population growth rate.**

$$r_{min} \leq r(t) \leq r_{max}$$

**$\tau(t)$**  : **Monetary transfer** (US\$ per year) : support that the international community is ready to pay to local institutions on biodiversity conservation account.

$$0 \leq \tau \leq \tau_{max}$$

# The Humid Tropical Forest : Coexistence

## What Sustainability ?

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Maintain biodiversity of primary forests and ensure an acceptable living standard to local populations, that is :

- an ecological constraint : preserve a minimal surface of primary forest :  $0 \leq S \leq F_0 - F_{min}$

- economical constraints,

- ✱ secure a global and individual capitals :

$$K_{current} \leq K \quad \text{and} \quad \frac{K_{current}}{N_{current}} \leq \frac{K}{N}$$

- ✱ ensure a non decreasing revenue per capita :

$$\left( \frac{K}{N} \right)' \geq 0$$



# The Humid Tropical Forest : Coexistence

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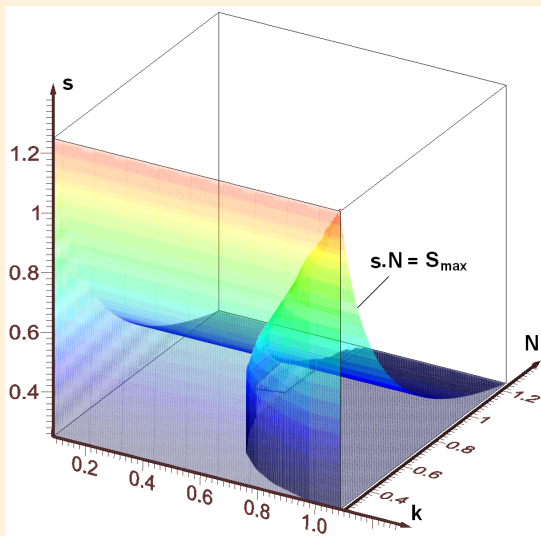
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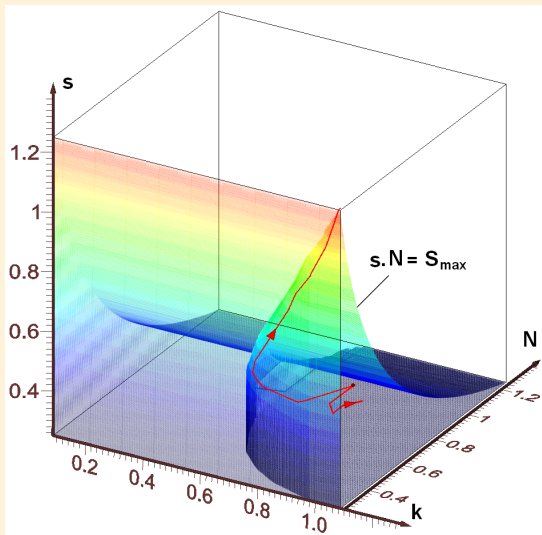
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# Global Forest :

## Managing Forest as Sink of Carbon

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- **The Challenge** : Manage world forest as a world public good so as to
  - keep minimal surface necessary to maintain its role of sink of carbon,
  - contain the cumulated **CO<sub>2</sub>** up to reasonable level,
  - maintain the standard of living of populations who get revenues from forest exploitation,
  - supply the industrial and firewood world demand.
  - determine monetary transfers.



# Global Forest : Managing Forest as Sink of Carbon

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# Global Forest :

## The Van Soest and Lensink Model (2000)

Variables :

- **$F$**  The **World Forest Stock**,  $\rho_t$  and  **$D_t$**  the reforestation and deforestation efforts.
- **$E$**  The **Global  $CO_2$  Emissions**.
- **$S$**  The **cumulated stock of  $CO_2$**  which increases with emissions and decrease with  **$CO_2$**  forest sequestration.

$$\begin{aligned}
 F'(t) &= \rho_t + \eta \left( 1 - \frac{F_t}{F_0} \right) F_t - D_t, \\
 E'(t) &= V_t \cdot E_t, \\
 S'(t) &= E_t - \phi F_t.
 \end{aligned}$$

where  $V_t \in [V_{\min}, V_{\max}]$  is a decision variable (emissions adjustment).



# Global Forest : Sink of Carbon

## Economical Mechanisms

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- Global wood supply :

$$q(t) = n \cdot D(t) + m\gamma\delta F(t)$$

- Wood market price

$$P(t) = \bar{P} - \theta q(t),$$

- Analysis of revenues drawn from forest, cleared and cultivated lands :

$$R(t) = P(t)q(t) + P_A \left( \bar{Z} + \frac{\psi \bar{Z} D_t}{F_0 - F(t)} + \frac{\beta F(t)}{F_0} \right) [F_0 - F(t)] + T(t)$$

$T(t)$  : Possible monetary transfer "North/South", control variable.

# Global Forest : Sink of Carbon Sustainability ?

Forest Conservation and CO2 emissions: A Viable Approach

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In this context, **Sustainability** includes both economical et environmental components :

- Ensure a minimal production :  $q(t) \geq \underline{q}$
- Maintain minimal revenues :  $R(t) \geq \underline{R}$ ,
- Preserve a world forest surface and contain carbon emissions and stock under "normal", "consensual" limits :  
 $\forall t \geq 0, (F(t), E(t), S(t)) \in K$

where

$$K = \{(F, E, S) \in [F_{\min}, F_{\max}] \times [E_{\min}, E_{\max}] \times [S_{\min}, S_{\max}]\}$$

# Global Forest : Sink of Carbon

## Constraints and Data

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Constraints

$$F_{\min} = 0 \text{ million hectares}$$

$$F_{\text{current}} = 3925 \text{ million hectares}$$

$$F_{\max} = 5500 \text{ million hectares}$$

$$E_{\min} = 21.2 \text{ gigatons of CO}_2$$

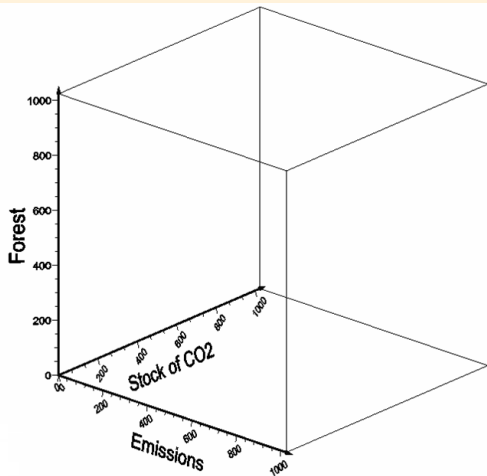
$$E_{\text{current}} = 28 \text{ gigatons of CO}_2$$

$$E_{\max} = 42.4 \text{ gigatons of CO}_2$$

$$S_{\min} = 0 \text{ gigatons of CO}_2$$

$$S_{\text{current}} = 800 \text{ gigatons of CO}_2$$

$$S_{\max} = 1653 \text{ gigatons of CO}_2$$





# Global Forest : Sink of Carbon

## State and Control Variables

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- $F$**  : Global Stock of Forest
- $E$**  : World annual  $CO_2$  emissions
- $S$**  : Accumulated  $CO_2$  in the atmosphere
- $\rho$**  : Annual reforestation
- $D$**  : annual deforestation



# Global Forest : Sink of Carbon

## Other Parameters

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**$V$**  : Possible adjustment velocities of  $CO_2$  emissions

**$T$**  : Monetary Transfers in US\$.

**$\eta$**  : Natural forest growth

**$\varphi$**  : Absorption rate of  $CO_2$

**$R$**  : Actual minimal revenue of landholders

**$q$**  : Minimal wood supply

**$n$**  : Forest exploitation



# Global Forest : Sink of Carbon

## Other Parameters

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$\beta$  : Floor productivity of the forest

$\gamma$  : Selective forest exploitation

$\delta$  : Fraction of reforested forest

$\theta$  : Derivative of the demand function

$\bar{P}$  : Floor price for wood

$\psi$  : Extra productivity of deforested land

$P_A$  : Mean Price of good production

$\bar{Z}$  : Mean land productivity

# Global Forest : Sink of Carbon

## Numerical Results and Decision Policy

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One can visualize domains where **Sustainability** is effective or not

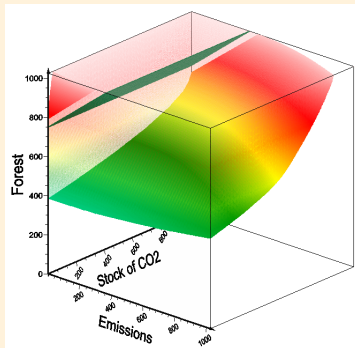


FIGURE: Viability in the lack of economical constraints

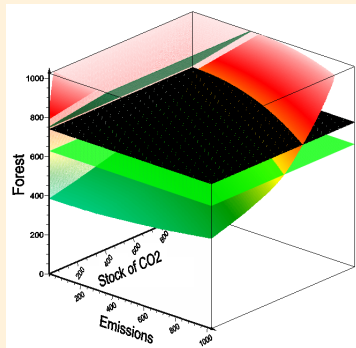


FIGURE: Introduction of economical constraints

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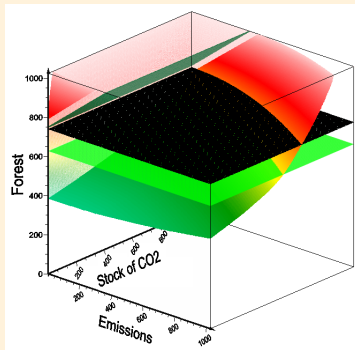


FIGURE: Introduction of  
economical constraints

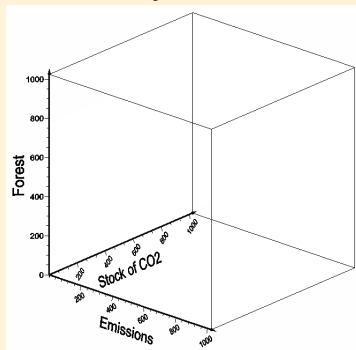


FIGURE: The Viability Kernel is  
empty : "Sustainability" fails





# Sommaire

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**3 Recovering Sustainability**

- Viability Multipliers
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# Viability Tools for recovering Sustainability

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Viability Tools for recovering Sustainability :

- Modify the Dynamics : Viability Multipliers.
- Change Normative Constraints,
- Construct Upper Threshold functions,
- Relax Control Set,



# Viability Tools for recovering Sustainability

## Modify the Dynamics : Viability Multipliers

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Constraints

$$X'(t) \in F(X(t), u(t))$$



$$X'(t) \in F(X(t), u(t)) + m(t)$$

where  $m(t) \in M$ .

Problem : Determine  $M$  such that

$$Viab_{F,M}(K) \neq \emptyset$$

# Exemple : Maintaining Biodiversity through monetary transfers

In the Humid Forest Model :

$$\begin{aligned}
 S'(t) &= d(t) \\
 N'(t) &= r(t)N(t) - v(t)N(t) \\
 K'(t) &= -\underline{c}N(t) - \beta_S(t)d(t) + \\
 &\quad \mu \min(S(t), \gamma(1 - v(t))N(t)) + \\
 &\quad \omega v(t)N(t) + \\
 &\quad \tau(t)
 \end{aligned}$$

$\tau(t)$  is the monetary flow transfer.

Questions :

- Determine the rule  $t \rightarrow \tau(t)$  such that biodiversity can be maintained.
- If one change  $\tau(t)$  in  $\tau(t) + 1$  , What more forest surface it will be possible to preserve.

# Exemple : Maintaining Biodiversity through monetary transfers

Forest Conservation and CO2 emissions: A Viable Approach

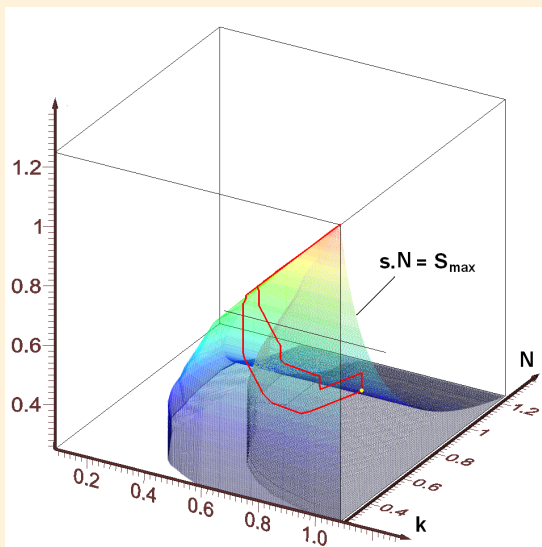
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# Exemple : Maintaining Biomass through monetary transfers

We modify the revenue constraint by introducing a control variable

$T(t)$  : Possible monetary transfer "North/South"

$$R(t) = P(t)q(t) + P_A \left( \bar{Z} + \frac{\psi \bar{Z} D_t}{F_0 - F(t)} + \frac{\beta F(t)}{F_0} \right) [F_0 - F(t)] + T(t)$$

There exists a time dependent threshold  $T^*(t)$  such that  $\forall t, \forall T(t) \geq T^*(t)$ , both economical and environmental sustainability recover.

# Global Forest : Sink of Carbon

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One can visualize domains where **Sustainability** is effective or not

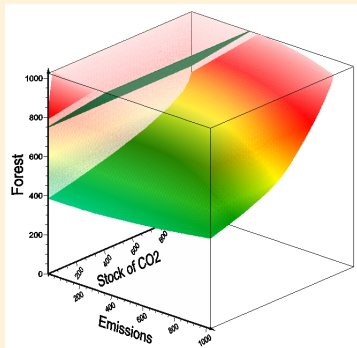


FIGURE: Viability in the lack of economical constraints

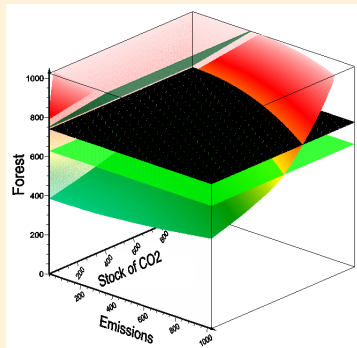


FIGURE: Introduction of economical constraints

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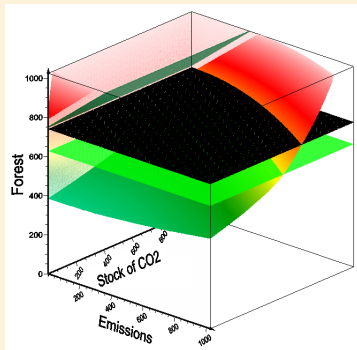


FIGURE: Introduction of  
economical constraints

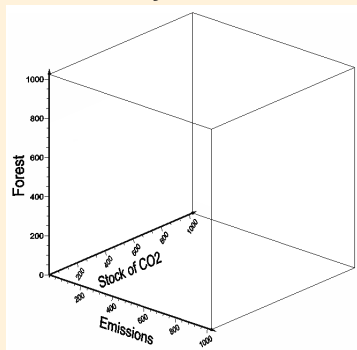


FIGURE: The Viability Kernel is  
empty : "Sustainability" fails



# Global Forest : Sink of Carbon

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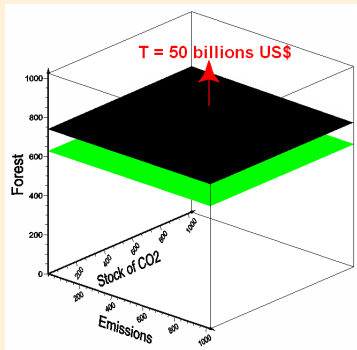


FIGURE: lack of monetary transfer

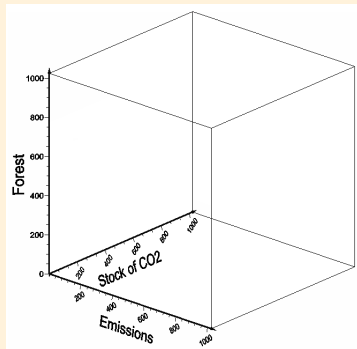


FIGURE: The Viability Kernel is empty : "Sustainability" fails

# Global Forest : Sink of Carbon

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One can visualize domains where **Sustainability** is effective or not

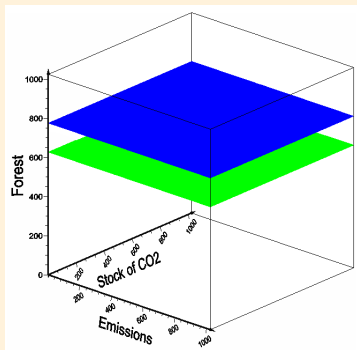


FIGURE: Introduction of monetary transfer

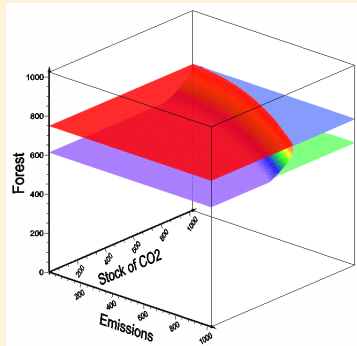


FIGURE: The Viability Kernel is non empty : "Sustainability" become realizable

# Global Forest : Sink of Carbon

## Numerical Results and Decision Policy

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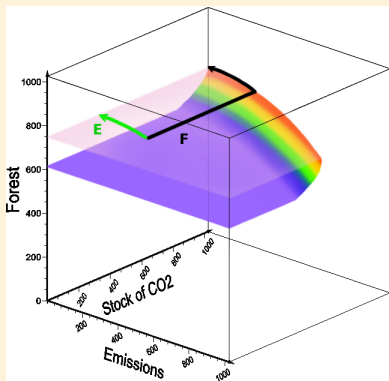
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One can visualize decision "Sustainable" rules and select "Sustainable" evolutions.





# Conclusion

## Applied Viability

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- it is a new way to study a dynamical controlled, constrained and stochastic system
- to manage objective
- to measure impact
- in the presence of uncertainty
- but even if the set valued numerical techniques needed to implement viability algorithm require a huge numerical support, the spectrum of potential applications is really very wide

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# Merci

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## POUR VOTRE ATTENTION