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March 2005

Paper No. 05-2005

Household Demand, Network Externality Effects and Intertemporal Price Discrimination*

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Forthcoming in *Journal of Economics*

Abstract

This paper examines the optimality of intertemporal price discrimination when network externality effects are present in the consumption of a durable good. We conduct our study in two settings. In a model with two household types, utilities are dependent on the cumulative proportion of households that have purchased the durable good. Next, in a model with a continuum of household types, we extend the analysis to the case where households consume both a durable good and a stream of non-durable goods. We show that in both settings, the presence of network externalities facilitates a sales strategy with intertemporal price discrimination.

Key Words: intertemporal price discrimination, durable good, household demand, network externality

JEL Classification: D40

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* I would like to thank two anonymous referees for their perceptive comments and constructive advice in the revision of the paper. I am also grateful to the Managing Editor, Giacomo Corneo, for their encouragement; and to Hing-Man Leung and John Quah for discussions on the subject. The usual disclaimer applies here. Research support from the Wharton-SMU Research Centre, Singapore Management University, is gratefully acknowledged.

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1. Introduction

This paper studies optimal monopoly pricing of a newly-introduced durable good when network externality effects are present in its consumption. The rationale for a sales strategy that involves intertemporal price discrimination (IPD, for short) is straightforward: charge different prices over time – typically a high price when the durable good is first introduced, and then progressively lower prices – in order to sort consumers such that those with higher valuations would decide to purchase the durable good earlier, while those with lower valuations would make purchases later. When successfully implemented, an IPD sales strategy allows firms with monopoly power to generate higher profits compared with a constant-price policy.

In practice, IPD often goes hand in hand with other sales practices. For instance, it is a common marketing strategy to create different versions of a product sequentially or sell limited quantities of a product over time. These two sales strategies allow firms to price-discriminate over time. Price discrimination strategies are also often related to quality improvements or cost reduction effects (arising from economies of scale, learning-by-doing, etc), as well as changes in product specification or customization to suit different demographic segments or customer categories. In general, these sales practices increase the scope for price discrimination.

Beginning with the analysis by Coase (1972) and Stokey (1979, 1981), a vast literature has developed that studied the optimality of IPD sales strategies in different settings. It is well established that the monopoly's ability to credibly commit to its pricing policy is critical for the success of an IPD strategy. Credible commitment depends on several factors, including the monopoly's reputation, industry practices and whether the monopoly is able to undertake actions to make it costly to renege on the commitment and adversely affect its reputation and future profitability. For instance, if the monopoly continuously launches new products, the credibility of its future sales strategies would be dependent on its reputation. In the case of the publishing industry, it is a common practice to wait for at least six months to a year between the hard-cover and the soft-cover edition. Hence, the commitment to "hard-back' pricing is easily made credible (see Clerides, 2003). However, in the film industry, the time gap between initial theatrical screenings and video release has shortened considerably due to rampant piracy. This indirectly puts a cap on the price of a movie ticket for theatrical screenings. In the retail sector, departmental stores may achieve commitment by promising rebates if prices are lowered within a specified period.

The basic point here is that if the monopoly's commitment is not credible, this limits its ability to set differential prices over time and reduces the potential profits that it can earn from an IPD sales strategy. More specifically, as the period over which a monopolist is able to make its sales commitment diminishes, its ability to exert market power is reduced correspondingly. In the limit, the monopoly's market power vanishes, and it is not possible for the monopolist to price-discriminate. The competitive market pricing is the only outcome. This conjecture was first raised by Coase (1972) and subsequently proven in different settings (see, for instance, Stokey (1981) and Bulow (1982)). When the monopoly can credibly commit to its pricing policy, Stokey (1979) established that IPD is feasible for several classes of utility functions, when restrictive conditions regarding the consumers' rate of time preference and their reservation prices hold.

Subsequently, other authors have analyzed different settings under which the optimal sales strategy entails an IPD pricing policy. For instance, Conlisk, Gerstner and Sobel (1984) and Sobel (1984) have demonstrated that an IPD sales strategy is optimal when there is continual entry of new customers. The monopoly optimally chooses to sell to high-valuation consumers first, and then hold periodic sales to clear the market. Landsberger and Meilijson (1985) also extended Stokey's analysis and showed that an IPD sales strategy is optimal for a durable good monopolist if it possesses discount rates different from those of consumers. The early literature is surveyed in Varian (1989). Recent literature on the subject includes Cayseele (1991), which provided a rationing perspective to explain the optimality of IPD; Png (1991), which considered the use of a most-favored customer strategy to engage in IPD; Van Ackere and Reyniers (1995), which studied the use of trade-ins and introductory offers to create IPD opportunities; Ault, Randolph, Laband and Saba (2000), which studied the use of rebates for an IPD strategy; as well as Locay and Rodriguez (2002), which considered the optimality of IPD when demand is stochastic and consumers are uncertain if the product would be available in the future. They also considered the existence of a "snob effect" that may prompt some consumers to buy early when the product is more exclusive. The implications of random price discrimination are studied in Colombo (2003).

In this paper, our focus is on the presence of network externalities and its impact on the optimality of an IPD sales strategy. For a wide range of electronic durable goods (such as karaoke sets, personal computers, laptops, DVD players, MP3 players, etc), positive network externality effects are present, as the benefits that a household can derive from their consumption may increase over time. This may be due to greater availability of complementary goods (such as software and music and movie titles of compatible format) that are consumed with the durable good or the existence of a greater community of users.

We formalize these ideas in a simple model with two household types (High and Low) in Section 2. Our analysis focuses on the case where the monopoly is able to commit to its pricing strategy. The situation where the monopoly is unable to credibly commit to its pricing strategy has been studied in Stokey (1981), Karp (1996) and Mason (2000). In particular, Mason (2000) demonstrates that the Coase conjecture fails when network externality benefits are present.

In our analysis, we demonstrate that the presence of network externality effects increases the scope for IPD. Following the standard approach in the literature on network externalities, we model the impact of network externality effects on consumption by allowing the utility derived by both types of households to depend on the proportion of households that have purchased the durable good. This modeling approach is similar to that used in Cabral, Salant and Woroch (1999), which also studied the effects of network externality on monopoly pricing. In their model, the focus was to delineate the circumstances under which introductory pricing is an optimal sales strategy.

Next, in Section 3, we extend the analysis to the case of a continuum of household types that consume both a durable good and a stream of non-durable goods, subject to an intertemporal budget constraint. The monopoly sells the durable good at two dates and is able to credibly commit to the prices it sets for both dates. Households differ in the utility they derive from the durable good when network externality effects are absent. However, in the presence of network externalities, the flow utility that households enjoy from the consumption of the durable good now depends on the cumulative proportion of households that have already purchased the durable good. In this model, households must decide on the optimal date to purchase the durable good as it affects how much they can afford to spend on the consumption of the non-durable good and the marginal utility they derive from its consumption. Section 4 concludes the paper.

2. A Model with Two Household Types

Suppose there are two types of households, *H* and *L*, who demands at most one unit of a durable good, which if acquired, provides a flow of utility of X_k (k = *H* and *L*). For our analysis, let $X_L = \theta X_H$, where $\theta \in (0, 1)$. Let *r* denote the common intertemporal discount rate for the monopoly and the households. The discounted present value of the stream of utilities for household of type *k* is $\frac{X_k}{r}e^{-rt}$, if the durable good is acquired at time *t*.

Let the measure of households be unity, with μ being the measure of households of type *H*, and $(1-\mu)$ being the measure of households of type *L*. We assume that if a household is indifferent between buying the durable good or doing without it, the durable good would be purchased. Consider a new durable good to be supplied by a monopoly who announces that sales will take place on two dates only: t = 0 and $t = T$. The monopoly must decide on the optimal prices to sell at t = 0 and $t = T$, as well as the optimal *T*. The new durable good is produced at a constant marginal cost, which we shall assume, without loss of generality, to be zero. Thus, the monopoly's objective is to maximize its revenue and its sales strategy is characterized by $\{P_0, P_T, T\}$. We assume that the monopoly's commitment to its announced pricing policy $\{P_0, P_T, T\}$ is credible, so that households only need to decide whether to buy the durable good at t = 0 or t = T, or never.

2.1 No Network Externality Effects

When network externality effects are absent, the decision by a household whether to purchase the durable good, or not, depends simply on the present value of the flow of utility net of purchase price, which is denoted $V_k(t) \equiv \left[\frac{X_k}{r} - P_t\right]e^{-rt}$, and evaluated at t = 0 and t = T. Consider an IPD sales strategy for the monopoly, defined as one where type-H households buy at t = 0 while type-L households buy at t = T. The following conditions must hold for the two types of households if the sales strategy is successful:

$$\frac{X_H}{r} - P_0 \geq \text{Max}\left\{0,\ \left[\frac{X_H}{r} - P_T\right]e^{-rT}\right\} \tag{1}$$

$$\text{Max}\left\{0,\ \frac{\theta X_H}{r} - P_0\right\} < \left[\frac{\theta X_H}{r} - P_T\right]e^{-rT} \tag{2}$$

The monopoly's revenue under the IPD sales strategy (denoted strategy D), is $\Pi^D \equiv \Pi(P_0, P_T, T) = \mu P_0 + (1-\mu)P_T e^{-rT}$. The monopoly can optimally select $P_0^D = \frac{X_H}{r}\left[1-(1-\theta)e^{-rT}\right]$, $P_T^D = \frac{\theta X_H}{r}$, and an optimal T, denoted T^*, to ensure the conditions in (1) and (2) are satisfied. In this case, the monopoly earns a revenue of $\Pi^D \equiv \frac{X_H}{r}\left[\mu + (\theta - \mu)e^{-rT^*}\right]$.

There are two other alternative sales strategies for the monopoly: sell to both types of households at t = 0, or just sell to type-H households at t = 0. If the monopoly sells to both types of households, this non-discrimination sales strategy (denoted strategy ND), generates a revenue of $\Pi^{ND} = P_0$. The optimal price to set in this case is $P_0^{ND} = \frac{\theta X_H}{r}$, so that the monopoly earns a revenue of $\Pi^{ND} = \theta \frac{X_H}{r}$. Finally, if the monopoly decides to sell only to type-H households (denoted strategy H), the optimal price to set here is $P_0^H = \frac{X_H}{r}$, and the monopoly earns a revenue of $\Pi^H = \mu \frac{X_H}{r}$.

Comparing the profitability of the three types of sales strategies, it is straightforward to show that an IPD sales strategy is always dominated by one of the other two strategies. Specifically, when $\mu > \theta$, we have $\Pi^H > \Pi^D$, so that the monopoly earns a higher revenue by just selling to type-H households. When $\mu < \theta$, we have $\Pi^{ND} > \Pi^D$; again, the monopoly can do better by selling to both types of households at a single price at $t = 0$.

This set of results shows that an IPD pricing strategy is never an optimal sales strategy for all combinations of the parameters, μ and θ, even though households differ in their utilities derived from the durable good. In this next sub-section, we show that in the presence of network externality effects, an IPD sales strategy can be optimal under certain conditions in our model.

2.2 IPD Sales Strategy with Network Externalities

Suppose the consumption of the durable good creates network externality effects on the utility enjoyed by both types of households. The utility that a type-k household derives from the durable good at time t is now given by

$$U_t^k \equiv X_k \left[1 + \phi_t (1 - e^{-\delta t}) \right], \qquad k = H, L \tag{3}$$

where ϕ_t denotes the cumulative proportion of households that have already purchased the durable good at time t. In this setting, the flow utility that a type-k household derives from the consumption of the durable good increases with ϕ_t. Since sales take place in two discrete periods in our model, this formulation suffices to capture the impact of positive network externality effects on the consumption of the durable good. We further allow the network externality effects to grow over time, with the rate of growth proportional to δ. As δ tends to infinity, network externality effects become almost instantaneous.

In our model, sales of the durable good takes place at two dates. If sales take place continuously, the actual history of purchase will also matter in determining the magnitude of the externality effects, besides the cumulative proportion of households that have purchased the durable good. For instance, a sales path where purchases occurred almost immediately after the new durable good is launched will likely generate greater network externality effects compared with another sales path with the same cumulative proportion of households purchasing the durable good, but where sales take place continuously over the same period. In our two-period model, the cumulative proportion of households purchasing the good in the second period coincides with the sales history.

Since the monopoly sells the durable good at possibly only two dates, t = 0 and t = T, the discounted present value of the stream of utilities if a type-k household buys the durable good at t = 0 and at t = T, are given by, respectively,

$$V_0^k(\phi_0,\phi_T) \equiv X_k \int_0^T \left[1+\phi_0(1-e^{-\delta t})\right] e^{-rt}dt + X_k \int_T^\infty \left[1+\phi_T(1-e^{-\delta t})\right] e^{-rt}dt \tag{4}$$

$$= \frac{X_k}{r}\left\{1+\frac{\phi_0\delta}{r+\delta}+\frac{1}{r+\delta}(\phi_T-\phi_0)e^{-rT}\left[\delta+r\left(1-e^{-\delta T}\right)\right]\right\}$$

$$V_T^k(\phi_T) \equiv X_k \int_T^\infty \left[1+\phi_T(1-e^{-\delta t})\right] e^{-rt}dt \tag{5}$$

$$= \frac{X_k}{r}e^{-rT}\left\{1+\frac{1}{r+\delta}\phi_T\left[\delta+r\left(1-e^{-\delta T}\right)\right]\right\}$$

If the monopoly successfully employs an IPD sales strategy, we will have $\phi_0 = \mu$ and ϕ_T = 1, since type-H households buy the durable good at t = 0 and type-L households buy at t = T.

Note that in this setting, even though type-H households buy the durable good at t = 0, the utility they derive from the durable good at t = T will also be higher, since more households have bought the durable good at t = T. An alternative setting that has also been considered in the literature is the case where the network externality effects on a household's utility depends only on the current network size. This type of network externality effects is referred to as 'excluded' by Bensaid and Lesne (1996). An example of 'excluded' network externality effects is the case of computer software. Early buyers of software do not benefit from the externalities they generate for later purchasers (and users), such as the discovery of bugs and problems, unless they buy updated versions or are given free upgrades by the software developer.

In our model, the positive impact on type-*H* household's utility is given by the second term in (4); which is $\frac{1}{r(r+\delta)} X_k \left(1-\mu\right) e^{-rT}\left[\delta + r\left(1-e^{-\delta T}\right)\right]$ under a successful IPD sales strategy. In order for type-*H* households to prefer to buy the durable good at t = 0, rather than at t = T, the following optimality condition must hold: $V_0^H(\mu,1) - P_0 > V_T^H(1) - P_T e^{-rT}$. Similarly, for type-*L* households to buy at t = T rather than t = 0, the following optimality condition must hold: $V_0^L(1,1) - P_0 < V_T^L(1) - P_T e^{-rT}$. Together, these two conditions impose the following constraints on the monopoly's pricing policy:

$$P_0 - P_T e^{-rT} < \frac{X_H}{r}\left\{\left(1-e^{-rT}\right) + \frac{\mu}{r+\delta}\left[\delta\left(1-e^{-rT}\right) - re^{-rT}\left(1-e^{-\delta T}\right)\right]\right\} \tag{6}$$

$$P_0 - P_T e^{-rT} > \frac{\theta X_H}{r}\left\{\left(1-e^{-rT}\right) + \frac{1}{r+\delta}\left[\delta\left(1-e^{-rT}\right) - re^{-rT}\left(1-e^{-\delta T}\right)\right]\right\} \tag{7}$$

From (6) and (7), a feasible range for $\left(P_0 - P_T e^{-rT}\right)$ exists if

$$1-\theta+\left(\mu-\theta\right)\frac{\delta}{r+\delta} \geq e^{-rT}\left[1-\theta+\left(\mu-\theta\right)\frac{\delta}{r+\delta}+\left(\mu-\theta\right)\frac{r}{r+\delta}\left(1-e^{-rT}\right)\right] \tag{8}$$

If $\mu > \theta - \left(1-\theta\right)\frac{r+\delta}{r}$, the sign of the left-hand side of (8) is positive, so that the constraint in (8) simplifies to the condition, $e^{rT} \geq \left[1+\frac{r(\mu-\theta)\left(1-e^{-\delta T}\right)}{(1-\theta)(r+\delta)+\delta(\mu-\theta)}\right]$. By choosing a sufficiently large T, say $T \geq \frac{1}{r}\ln\left[1+\frac{r(\mu-\theta)}{(1-\theta)(r+\delta)+\delta(\mu-\theta)}\right]$, a feasible range for $\left(P_0 - P_T e^{-rT}\right)$ exists.

However, if instead, $\mu < \theta - (1-\theta)\frac{r+\delta}{r}$, the sign of the term on the left-hand side of (8) is negative. The constraint in (8) simplifies instead to $e^{rT} \le \left[1 + \frac{r(\mu-\theta)\left(1-e^{-\delta T}\right)}{(1-\theta)(r+\delta)+\delta(\mu-\theta)}\right]$. It is straightforward to verify that this constraint can only be satisfied with equality if T is set equal to zero. For $T > 0$, the left-hand of the above inequality grows faster than the right-hand side. Therefore, for the case when $\mu < \theta - (1-\theta)\frac{r+\delta}{r}$, an IPD sales strategy is not feasible.

Combing the above results, if the proportion of type-H households, μ, is greater than $\theta - (1-\theta)\frac{r+\delta}{r}$, there exists an optimal IPD sales strategy $\left\{P_0^D, P_T^D, T^*\right\}$ that maximizes the monopoly's revenue. It comprises the following:

$$P_0^D = \frac{X_H}{r}\left\{1-(1-\theta)e^{-rT^*} + \frac{\delta}{r+\delta}\left[\mu\left(1-e^{-rT^*}\right)+\theta e^{-rT^*}\right] - \frac{r(\mu-\theta)e^{-rT^*}}{r+\delta}\left(1-e^{-\delta T^*}\right)\right\} \tag{9}$$

$$P_T^D = \frac{\theta X_H}{r}\left\{1+\frac{1}{r+\delta}\left[\delta + r\left(1-e^{-\delta T^*}\right)\right]\right\} \tag{10}$$

$$e^{rT^*} > \left[1+\frac{r(\mu-\theta)}{(1-\theta)(r+\delta)+\delta(\mu-\theta)}\left(1-e^{-\delta T^*}\right)\right] \quad \text{(to satisfy constraint 7)} \tag{11}$$

For type-H households, we can easily verify that $V_0^H(\mu,1) - P_0^D = 0 > V_T^H(1) - P_T^D e^{-rT^*}$, so that they prefer to purchase at $t = 0$. Similarly, for type-L households, $V_0^L(1,1) - P_0^D < 0 = V_T^L(1) - P_T^D e^{-rT^*}$, so that they buy at $t = T^*$. The monopoly revenue under an IPD sales strategy is given by $\Pi^D \equiv \mu P_0^D + (1-\mu)P_T^D e^{-rT^*}$, which is

$$\Pi^D = \frac{X_H}{r}\left\{\mu + (\theta-\mu)e^{-rT^*} + \frac{\mu^2\delta}{r+\delta} + \frac{(\theta-\mu^2)}{r+\delta}e^{-rT^*}\left[\delta + \left(1-e^{-\delta T^*}\right)\right]\right\} \tag{12}$$

We are ready to determine the circumstances under which the IPD sales strategy yields the highest revenue compared with selling to both types of households or just type-H households. If the monopoly decides to sell to both types of households, it can do so by setting

$P_0^{ND} = \frac{\theta X_H}{r}\left[1 + \frac{\delta}{r+\delta}\right]$ to sell at t = 0, or setting $P_T^{ND} = \frac{\theta X_H}{r}\left\{1 + \frac{1}{r+\delta}\left[\delta + r\left(1 - e^{-\delta T}\right)\right]\right\}$ to sell at $t = T$. In the first case, it earns a revenue of $\Pi_0^{ND} = P_0^{ND}$, while in the second case, it earns a discounted revenue of $\Pi_T^{ND} = P_T^{ND} e^{-rT}$. However, we can show that

$$\frac{d\left(\Pi_0^{ND} - \Pi_T^{ND}\right)}{dT} = \theta X_H e^{-rT}\left[\frac{\delta}{r+\delta} + r\left(1 - e^{-\delta T}\right) + 1 - \delta e^{-\delta T}\right] > 0 \quad \text{for all } T \tag{13}$$

Since $\Pi_0^{ND} = \Pi_T^{ND}$ if T is set equal to zero, the above result implies that even if network externality effects are present, the monopoly should not delay selling the durable good if the sales strategy is to sell to both types of households.

Finally, if the monopoly decides to sell only to type-*H* households, it can set $P_0^H = \frac{X_H}{r}\left[1 + \frac{\mu\delta}{r+\delta}\right]$ to sell at t = 0 to earn $\Pi_0^H = \mu P_0^H$. Alternatively, it can set $P_T^H = \frac{X_H}{r}\left\{1 + \frac{\mu}{r+\delta}\left[\delta + r\left(1 - e^{-\delta T}\right)\right]\right\}$ to sell at $t = T$ and earn $\Pi_T^H = \mu P_T^H e^{-rT}$. Again, we note that $\Pi_0^H = \Pi_T^H$ if T is set equal to zero, and that

$$\frac{d\left(\Pi_0^H - \Pi_T^H\right)}{dT} = \mu X_H e^{-rT}\left[\frac{\mu\delta}{r+\delta} + \mu r\left(1 - e^{-\delta T}\right) + 1 - \mu\delta e^{-\delta T}\right] > 0 \quad \text{for all } T \tag{14}$$

Hence, as in the case of selling to both types of households, the presence of network externality effects will not lead to a delay in the sale of the durable good if the monopoly intends to sell only to type-*H* households.

The monopoly' optimal sales strategy for each configuration of the system parameters is determined by a comparison of the revenue generated under the three sales strategies:

$$\Pi^D = \frac{X_H}{r}\left\{\mu + (\theta - \mu)e^{-rT^*} + \frac{\mu^2\delta}{r+\delta} + \frac{(\theta - \mu^2)}{r+\delta} e^{-rT^*}\left[\delta + r\left(1 - e^{-\delta T^*}\right)\right]\right\}$$

$$\Pi_0^{ND} = \frac{\theta X_H}{r}\left[1 + \frac{\delta}{r+\delta}\right]$$

$$\Pi_0^H = \frac{\mu X_H}{r}\left[1 + \frac{\mu\delta}{r+\delta}\right]$$

Let $\mu^* \equiv \dfrac{-(r+\delta)+\sqrt{(r+\delta)^2+4\theta\delta(r+2\delta)}}{2\delta}$. It is routine to verify that Π_0^{ND} > (<) Π_0^H if μ < (>) μ^*. Moreover, the following inequality holds: $\mu^* > \theta > \theta-(1-\theta)\dfrac{r+\delta}{r}$.

We noted earlier that if the proportion of type-*H* households, μ, is less than $\theta-(1-\theta)\dfrac{r+\delta}{r}$, an IPD sales strategy is not feasible. In this case, setting $P_0^{ND}=\dfrac{\theta X_H}{r}\left[1+\dfrac{\delta}{r+\delta}\right]$ to sell to both types of households is the optimal sales strategy. For μ > $\theta-(1-\theta)\dfrac{r+\delta}{r}$, we need to compare Π^D against Π_0^{ND} when $\mu \in \left[\theta-(1-\theta)\dfrac{r+\delta}{r},\mu^*\right]$, and Π^D against Π_0^H when $\mu \in \left[\mu^*, 1\right]$. Let

$$\mu^+(T)\equiv\frac{-(r+\delta)+\sqrt{(r+\delta)^2+4\theta\left[2(r+\delta)^2-3re^{-\delta T}(r+\delta)+\left(re^{-\delta T}\right)^2\right]}}{2\left[\delta+r\left(1-e^{-\delta T}\right)\right]} \tag{15}$$

$$\mu^-(T)\equiv\frac{-(r+\delta)+\sqrt{(r+\delta)^2+4\theta\left(\delta-\chi(T)\right)\left(r+2\delta-\chi(T)\right)}}{2\left(\delta-\chi(T)\right)},\ \chi(T)\equiv re^{-rT}\left[\frac{1-e^{-\delta T}}{1-e^{-rT}}\right] \tag{16}$$

It is routine to show that Π^D > (<) Π_0^{ND} if μ > (<) $\mu^-(T)$, and that Π^D > (<) Π_0^H if μ < (>) $\mu^+(T)$. We prove the following lemma (proof provided in Appendix A):

Lemma 1: For all $T > 0$, [a] $\mu^* < \mu^+(T) < 1$, [b] $\theta < \mu^-(T) < \mu^*$.

Utilizing Lemma 1, we obtain the following result:

Proposition 1: [a] If $\mu \in \left(\mu^*, 1\right)$, the monopoly can choose the optimal T^* (> 0) satisfying the condition in (11), to ensure that $\mu \in \left(\mu^*,\mu^+(T)\right)$ and engage in an IPD sales strategy that yields higher revenue compared with just selling to type-*H* households at *t* = 0;

[b] If $\mu \in \left(\theta, \mu^*\right)$, the monopoly can choose the optimal T^* (> 0) satisfying the condition in (11), to ensure that $\mu \in \left(\mu^-(T),\mu^*\right)$ and engage in an IPD sales strategy that yields higher revenue compared with selling to both types of households at *t* = 0.

Proposition 1 indicates that provided the monopoly selects a sufficiently long interval between the sale of the durable good to type-*H* households and to type-*L* households, an IPD sales strategy with sales to type-*H* households at $t = 0$ and sales to type-*L* households at $t = T$ will be optimal under certain configurations of the system parameters, when it was previously not optimal in the absence of network externality effects (as discussed in Section 2.1).

3. A Continuum of Household Types

We extend our analysis now to consider a setting with a continuum of infinitely-lived household types. In this model, a household demands at most one unit of a durable good, but also consumes an infinite stream of a non-durable good continuously, subject to an intertemporal budget constraint of *Y*. Let the measure of households be unity. The density function and distribution function of household types are given by $g(X)$ and $G(X)$, respectively. The durable good is costlessly produced by a monopoly, who announces a pricing policy $\{P_0, P_T, T\}$ at $t = 0$, and is able to commit to selling only at two dates: at a price of P_0 at $t = 0$, and at P_T at $t = T$.

As before, households are fully informed about the monopoly's pricing policy. Our approach to solve for the monopoly's optimal sales strategy is as follows: first, for a given pricing policy, we solve for the optimal consumption strategy of each household, and the optimal date to purchase (if at all) of the durable good. The monopoly's pricing policy therefore induces a purchase schedule by the households: those that purchase at $t = 0$, those that purchase at $t = T$, and those that do not purchase at all. This in turn allows us to solve for the monopoly's optimal sales strategy and the associated optimal pricing policy to maximize its revenue. In Koh (2004), we consider the case where sales takes place continuously between $t = 0$ and $t = T$.

Let $X \in [m, M]$ denote the flow utility that a household derives from the consumption of the durable good with no network externality effects. The utility enjoyed by a type-*X* household at time *t* in the presence of network externality effects is modeled as follows:

$$U_t(s_t, q_t) = s_t X\left[1 + \phi_t(1 - e^{-\delta t})\right] + \ln q_t \qquad (17)$$

where $s_t = 0$ if the durable good is not yet purchased at time *t*, $s_t = 1$ if it has been purchased at *t* or earlier; q_t is the amount of the non-durable good consumed at date *t* and $\ln q_t$ is the utility function for the consumption of the non-durable good. As before, ϕ_t denotes the cumulative proportion of households who had purchased the durable good at time *t*, while δ is a measure of the rate of expansion of the network externality effects over time.

Let *z* (= 0, *T*) denote the date of purchase of the durable good and $\{q_t\}$ denote the consumption plan of the non-durable good. The common intertemporal rate of discount for both the monopoly and the households is *r*. For our analysis, we assume a constant unit price for the non-durable good whose supply is perfectly elastic, so that the intertemporal budget constraint for each household is given by

$$P_z e^{-rz} + \int_0^\infty e^{-rt} q_t dt \ \le \ Y \ , \ z \ = 0 \text{ or } T \qquad (18)$$

The present discounted value of the stream of utilities for type-*X* household if it decides to purchase the durable good at *z* = 0, and at *z* = *T*, are given as follows:

$$W_0\left(\{q_t\}, \phi_0, \phi_T, X\right) = \frac{X}{r}\left\{1 + \frac{\phi_0 \delta}{r+\delta} + \frac{1}{r+\delta}\left(\phi_T - \phi_0\right)e^{-rT}\left[\delta + r\left(1 - e^{-\delta T}\right)\right]\right\} + \int_0^\infty e^{-rt} \ln q_t dt \qquad (19)$$

$$W_T\left(\{q_t\}, \phi_0, \phi_T, X\right) = \frac{X}{r}e^{-rT}\left\{1 + \frac{1}{r+\delta}\phi_T\left[\delta + r\left(1 - e^{-\delta T}\right)\right]\right\} + \int_0^\infty e^{-rt} \ln q_t dt \qquad (20)$$

With the monopoly committing credibly to its pricing policy, each household's maximization problem is given by the following Lagrangian:

$$L\left(z,\{q_t\},\lambda,\phi_0,\phi_T,X\right)=W_z\left(\{q_t\},\phi_0,\phi_T,X\right)+\lambda\left\{Y-P_z e^{-rz}-\int_0^\infty e^{-rt}q_t dt\right\},\ z\ =0\ ,T \tag{21}$$

The differentiation of $L\left(z,\{q_t\},\lambda,\phi_0,\phi_T,X\right)$ with respect to q_t yields $\lambda=1/q_t$, so that the optimal consumption plan of the non-durable good is a constant stream. Let $q(X,z)$ denote the optimal stream of consumption of the non-durable good. Utilizing (18), we obtain $q(X,z)=\ln r\left[Y-P_z e^{-rz}\right]$. Under the household's optimal consumption plan $\{z,q(X,z)\}$, the household's optimized utility, denoted $V_z\left(\phi_0,\phi_T,X\right)$, is

$$V_0\left(\phi_0,\phi_T,X\right)=\frac{X}{r}\left\{1+\frac{\phi_0\delta}{r+\delta}+\frac{1}{r+\delta}\left(\phi_T-\phi_0\right)e^{-rT}\left[\delta+r\left(1-e^{-\delta T}\right)\right]\right\}++\frac{1}{r}\ln r\left[Y-P_0\right] \tag{22}$$

$$V_T\left(\phi_0,\phi_T,X\right)=\frac{X}{r}e^{-rT}\left\{1+\frac{1}{r+\delta}\phi_T\left[\delta+r\left(1-e^{-\delta T}\right)\right]\right\}+\frac{1}{r}\ln r\left[Y-P_T e^{-rT}\right] \tag{23}$$

For a IPD sales strategy to be feasible, the pricing policy must be such that households with higher valuations of the durable good would purchase it earlier compared with households who value it less. If a household decides to purchase the durable good at a later date, it must weigh the marginal loss of the utility from its consumption against the marginal gain in utility from consuming a larger stream of the non-durable good.

Let X_M denote the household type that is indifferent between purchasing the durable good at $t\ =\ T$ or not at all. Similarly, let X_I denote the household type that is indifferent between buying the durable good at $t\ =0$ or $t=T$. The optimal timing of purchase for the households is as follows:

$$\begin{array}{lll} t=0, & X\ \in\ \left[X_I,\ M\right], & (24)\\ t=T, & X\ \in\ \left[X_M,\ X_I\right) & \\ t=\infty\ \text{(i.e. no purchase)} & X\ \in\ \left[m,\ X_M\right) & \end{array}$$

By the definition of X_M, $V_T\left(\phi_0,\phi_T,X_M\right)\ =\ \frac{1}{r}\ln rY$, and $\phi_T\ =\ 1-G(X_M)$. This allows us to solve for P_T as a function of X_M and T.

$$P_T = e^{rT} Y \left[1 - \exp\left\{ -X_M e^{-rT} \left(2 - G(X_M)\right) \left[1 - \frac{r}{r+\delta} e^{-\delta T} \right] \right\} \right] \tag{25}$$

Similarly, by the definition of X_I, we have $V_0\left(\phi_0, \phi_T, X_I\right) = V_T\left(\phi_0, \phi_T, X_I\right)$, and ϕ_0 = $1 - G(X_I)$, so P_0 can be written as a function of X_I, X_M and *T*:

$$P_0 = Y \left[1 - \exp\left\{ \begin{array}{l} -X_M e^{-rT} \left(2 - G(X_M)\right) \left[1 - \dfrac{r}{r+\delta} e^{-\delta T} \right] \\ -X_I \left[\left(1 - e^{-rT}\right) + \left(2 - G(X_I)\right) \left\{ \dfrac{\delta}{r+\delta} - e^{-rT} \left[1 - \dfrac{r}{r+\delta} e^{-\delta T} \right] \right\} \right] \end{array} \right\} \right] \tag{26}$$

From (25) and (26), it is clear that in selecting the pricing policy $\left\{P_0, P_T, T\right\}$, the monopoly is effectively choosing the two marginal household types, X_I and X_M, and the time-gap (T) between the sales dates. The monopoly's sales strategy $\{X_I, X_M, T\}$ determines the prices P_0 and P_T, as well as the magnitude of the network externality effects over time.

It is easy to verify that $P_0 > P_T e^{-rT}$, so that the discounted price at $t = T$ is lower, even though it is possible that $P_T > P_0$, as network externality effects increase the utilities that households derive from the durable good and the price that they are willing to pay for the durable good. Next, the partial derivatives of P_0 and P_T with respect to *T* are:

$$\frac{\partial P_0}{\partial T} = -r\left[Y - P_0\right] e^{-rT} \left\{ \left[X_M \left(2 - G(X_M)\right) - X_I \left(2 - G(X_I)\right) \right] \left[1 - e^{-\delta T} \right] - X_I \right\} \tag{27}$$

$$\frac{\partial P_T}{\partial T} = rP_T - r\left[Y - P_T e^{-rT} \right] X_M \left(2 - G(X_M)\right) \left[1 - e^{-\delta T} \right] \tag{28}$$

In general, the signs of the partial derivatives of P_0 and P_T with respect to *T* are ambiguous. Thus, in the presence of network externality effects, the monopoly may be able to charge a higher price at $t = T$ if the time gap between sales is increased, since the utilities that households derive from the durable good will be higher when the purchase is made at a later date. However, we can verify that the discounted price $P_T e^{-rT}$ is still decreasing in *T*.

Similarly, it is straightforward to show that the monopoly may be able to charge a higher P_0 when the time gap, *T*, is lengthened. This is because, by paying a lower discounted price for the durable good and having more budget to consume the non-durable good, some household types may derive higher discounted utilities if they decide to purchase the durable good later at $t = T$. Depending on the distribution of household types, $G(X)$, the monopoly may be able to increase P_0, such that the household-type X_I is indifferent between purchasing at $t = 0$ and at $t = T$. In fact, if X is uniformly distributed over [0, 1], so that $X\left(2-G(X)\right)$ = $X(2 - X)$ is increasing in X, then we will have $\dfrac{\partial P_0}{\partial T} > 0$.

3.1 The Optimal Sales Strategy

In this sub-section, we characterize the monopoly's optimal sales strategy. The present discounted value of the monopoly's revenue function is given by

$$\Pi\left(X_I, X_M, T\right) = P_0\left[1-G(X_I)\right] + P_T e^{-rT}\left[G(X_I)-G(X_M)\right] \tag{29}$$

We differentiate $\Pi\left(X_I, X_M, T\right)$ with respect to X_I, X_M and T in turn to yield a set of first-order conditions to characterize the monopoly's optimal sales strategy $\left\{X_I^*, X_M^*, T^*\right\}$. First, we show that the optimal sales strategy entails selling the durable good at two dates. The partial derivative of $\Pi\left(X_I, X_M, T\right)$ with respect to *T* yields

$$\frac{\partial\Pi\left(X_I, X_M, T\right)}{\partial T} = \left[1-G(X_I)\right]\frac{\partial P_0}{\partial T} + e^{-rT}\left[\frac{\partial P_T}{\partial T} - rP_T\right]\left[G(X_I)-G(X_M)\right] \tag{30}$$

Utilizing (27) and (28), we have

$$\left.\frac{\partial\Pi\left(X_I, X_M, T\right)}{\partial T}\right|_{T=0} = r\left[Y-P_0\right]X_I\left[1-G(X_I)\right] > 0 \tag{31}$$

This implies that the optimal time-gap, T^*, is positive, so that sales of the durable good take place at two dates, at $t = 0$ and at $t = T^*$. The first-order condition for T^* is given below:

$$\left[Y-P_0^*\right]\left[1-G(X_I^*)\right]\left\{\left[X_I^*\left(2-G(X_I^*)\right)-X_M^*\left(2-G(X_M^*)\right)\right]\left[1-e^{-\delta T^*}\right]+X_I^*\right\} \qquad (32)$$

$$=\left[Y-P_T^*e^{-rT^*}\right]\left[G(X_I^*)-G(X_M^*)\right]X_M^*\left(2-G(X_M^*)\right)\left[1-e^{-\delta T^*}\right]$$

Next, the first-order condition for X_M^*, evaluated at $\left\{X_I^*, X_M^*, T^*\right\}$, yields

$$\left[1-G(X_I^*)\right]\left[Y-P_0^*\right]\Omega+\left[G(X_I^*)-G(X_M^*)\right]\left[Y-P_T^*e^{-rT^*}\right]\Omega=g(X_M^*)P_T^* \qquad (33)$$

where $\Omega \equiv \left[1-\frac{r}{r+\delta}e^{-\delta T^*}\right]\left(2-G(X_M^*)-X_M^*g(X_M^*)\right)$ and P_0^* and P_T^* denote the optimal prices under $\left\{X_I^*, X_M^*, T^*\right\}$. Similarly, the first-order condition for X_I^* is

$$\left[1-G(X_I^*)\right]\left[Y-P_0^*\right]\Phi=g(X_I^*)\left[P_0^*-P_T^*e^{-rT^*}\right] \qquad (34)$$

where $\Phi \equiv 1-e^{-rT^*}+\left(2-G(X_I^*)-X_I^*g(X_I^*)\right)\left\{\frac{\delta}{r+\delta}-e^{-rT^*}\left[1-\frac{r}{r+\delta}e^{-\delta T^*}\right]\right\}$.

Finally, we can derive the monopoly's optimized revenue by substituting the first-order condition in (33) into the monopoly's revenue function given in (29). In general, P_0^* is not equal to P_T^* under the optimal sales strategy $\left\{X_I^*, X_M^*, T^*\right\}$, which is characterized in (32), (33) and (34). We summarize our results in the following Proposition:

Proposition 2: The optimal sales strategy of the monopoly involves IPD, with sales taking place at two dates. The present discounted revenue of the durable-good monopoly under the optimal sales strategy $\left\{X_I^*, X_M^*, T^*\right\}$ is given by

$$\Pi\left(X_I^*, X_M^*, T^*\right)=\left[1-G(X_M^*)\right]Y-\frac{g(X_M^*)e^{rT^*}Y\left[1-\exp\left\{-X_M^*e^{-rT^*}\left(2-G(X_M^*)\right)\left[1-\frac{r}{r+\delta}e^{-\delta T^*}\right]\right\}\right]}{\left(2-G(X_M^*)-X_M^*g(X_M^*)\right)\left[1-\frac{r}{r+\delta}e^{-\delta T^*}\right]} \qquad (35)$$

4. Conclusion

In this paper, we have presented two models to study the optimal sales strategies for a durable-good monopoly in the presence of network externality effects. Our analysis formalized the general belief that the presence of positive network externalities increases the scope for profitable intertemporal price discrimination by a durable-good monopoly. Specifically, in the first model where there are two household types, we have shown that while intertemporal price discrimination is suboptimal in the absence of network externality effects, it is part of an optimal sales strategy when network externality effects are present. In the second model where a continuum of household types that consume a durable good and an infinite stream of non-durable goods, our analysis demonstrated that intertemporal price discrimination is generally optimal when network externality effects are present. Furthermore, a higher price may be charged for future sales under the optimal sales strategy. This is because the presence of network externality effects raises the utility of the durable good over time, so that households who purchase it later are willing to pay a higher price for it. At the same time, by delaying the purchase, households also benefit by devoting a bigger share of the budget to the consumption of the non-durable good.

Appendix A

Proof of Lemma 1:

[a] We write $\mu^{+}(T)-\mu^{*}=\dfrac{-r(r+\delta)\left(1-e^{-\delta T}\right)+\delta\xi(T)-\left[\delta+r\left(1-e^{-\delta T}\right)\right]K}{2\delta\left[\delta+r\left(1-e^{-\delta T}\right)\right]}$ where

$\xi(T)\equiv\sqrt{(r+\delta)^2+4\theta\left[2(r+\delta)^2-3re^{-\delta T}(r+\delta)+\left(re^{-\delta T}\right)^2\right]}$ and $K\equiv\sqrt{(r+\delta)^2+4\theta\delta(r+2\delta)}$.

Let $N(T)\equiv-r(r+\delta)\left(1-e^{-\delta T}\right)+\delta\xi(T)-\left[\delta+r\left(1-e^{-\delta T}\right)\right]K$. We first note that $N(0)=0$ and that $N(\infty)=(r+\delta)\left[r+\delta\sqrt{1+8\theta}-\sqrt{(r+\delta)^2+4\theta\delta(r+2\delta)}\right]$. Since $\theta<1$, it follows that $\left(r+\delta\sqrt{1+8\theta}\right)^2-\left[(r+\delta)^2+4\theta\delta(r+2\delta)\right]=\sqrt{1+8\theta}-(1+2\theta)>0$. Therefore, $N(\infty)>0$. These results in turn imply that $\mu^{+}(0)=\mu^{*}$. In fact, it is routine to show that $\mu^{+}(T)<1$, and that $\mu^{+}(\infty)=1$. Next, we obtain

$$\frac{\partial N(T)}{\partial T}\equiv 2\delta^2 r\theta e^{-\delta T}\left[\lambda(T)-\frac{\sqrt{(r+\delta)^2+4\delta\theta(r+2\delta)}-(r+\delta)}{2\delta\theta}\right]$$

where $\lambda(T)\equiv\dfrac{3(r+\delta)-2re^{-\delta T}}{\Phi(T)}$. It is straightforward to show that

$$\left.\frac{\partial N(T)}{\partial T}\right|_{T=0}\equiv\delta r(r+\delta)\left[1-\frac{2\theta\delta+r+\delta}{\sqrt{(r+\delta)^2+4\theta\delta(r+2\delta)}}\right]>0\ ;\quad \left.\frac{\partial N(T)}{\partial T}\right|_{T\to\infty}=0$$

It follows from $\left.\dfrac{\partial N(T)}{\partial T}\right|_{T=0}>0$ that $\xi(0)>\dfrac{\sqrt{(r+\delta)^2+4\delta\theta(r+2\delta)}-(r+\delta)}{2\delta\theta}$. Next, since $\dfrac{\partial\lambda(T)}{\partial T}=\dfrac{2\delta r(r+\delta)^2e^{-\delta T}(1-\theta)}{\left[\xi(T)\right]^3}>0$, it follows that $\dfrac{\partial N(T)}{\partial T}>0$ for $T>0$. Together with the results that $N(0)=0$ and $N(\infty)>0$, this proves that $\mu^{+}(T)-\mu^{*}>0$ for $T>0$.

[b] First, we can show that $\mu^{-}(0) = \theta$ (using L'Hopital's rule) and that $\mu^{-}(\infty) = \mu^{*}$.

Differentiating $\mu^{-}(T)$ with respect to T yields

$$\frac{d\mu^{-}(T)}{dT} = \frac{d\chi(T)}{dT}\left\{\frac{2\delta\theta\left(-r-3\delta+2\chi(T)\right)\left(\delta-\chi(T)\right)\left[\Psi(T)\right]^{-1}-\delta(r+\delta)+\delta\Psi(T)}{2\delta\left(\delta-\chi(T)\right)^{2}}\right\}$$

where $\Psi(T) \equiv \sqrt{(r+\delta)^{2}+4\theta\left(\delta-\chi(T)\right)\left(r+2\delta-\chi(T)\right)}$, and

$$\frac{d\chi(T)}{dT} = \frac{re^{-rT}}{\left[1-e^{-rT}\right]^{2}}\left\{-r\left[1-e^{-\delta T}\right]+\delta e^{-\delta T}\left[1-e^{-rT}\right]\right\}$$

Since $e^{rT} > 1+rT$, and $rT > 1-e^{-rT}$, we have $-\left[1-e^{-\delta T}\right] < -\frac{\delta T}{1+\delta T}$ and $e^{-\delta T} < \frac{1}{1+\delta T}$, so that

$$\frac{d\chi(T)}{dT} < \frac{\delta re^{-rT}\left[1-e^{-rT}-rT\right]}{\left[1-e^{-rT}\right]^{2}\left(1+\delta T\right)} < 0$$

It is routine to show that for $T > 0$, $\frac{2\delta\theta\left(-r-3\delta+2\chi(T)\right)\left(\delta-\chi(T)\right)}{\Psi(T)}-\delta(r+\delta)+\delta\Psi(T) < 0$.

Hence, it follows that $\frac{d\mu^{-}(T)}{dT} > 0$, so that $\mu^{-}(T)$ is strictly increasing. This completes the proof of Lemma 1[b]. *Q.E.D.*

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