



No. E2007003

2007-07

## **Development Strategy, Optimal Industrial Structure and Economic Growth in Less Developed Countries**

Justin Yifu Lin<sup>□</sup>

China Center for Economic Research, Peking University  
jlin@ccer.pku.edu.cn

Pengfei Zhang<sup>#</sup>

CCER, Peking University; CID Harvard University; and NBER  
pengfei\_zhang@ksg.harvard.edu  
zhangpengfei@pku.org.cn

**NO. E2007003 July 2007**

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<sup>□</sup> China Center for Economic Research, Peking University, 100871, Beijing, P. R. China. Tel: 86-10-6275-7375, Fax: 86-10-6275-4719. Email: jlin@ccer.edu.cn.

<sup>#</sup> Room 327, NBER, 1050 Massachusetts Ave, Cambridge, MA 02138. Tel: 617-588-0358, Fax: 617-868-2742. Email: pengfei\_zhang@ksg.harvard.edu, or zhangpengfei@pku.org.cn.

# Development Strategy, Optimal Industrial Structure and Economic Growth in Less Developed Countries\*

Justin Yifu Lin

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Pengfei Zhang

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**Abstract:** In this paper, we develop an endogenous growth model that combines structural change with repeated product improvements. There are two sectors in the present paper, one is traditional sector, and the other is modern sector. The technological progress in the traditional sector takes the form of horizontal innovation based on expanding variety, while the technologies in the modern sector become not only increasingly capital-intensive but also progressively productive over time. The application of the basic model to the less developed economies show that the optimal industrial structure in the less developed countries (LDCs) is endogenously determined by its factor endowments; the firm in the LDCs that enters the capital-intensive, advanced industry in the developed countries (DCs) would be nonviable owing to the relative scarcity of capital in the LDCs' factor endowments; whether the industrial structure matches with the factor endowment structure or not is the fundamental cause to explain differences in economic performance among the LDCs.

**Keywords:** Capital Intensity, Development Strategy, Factor Endowments, Endogenous Growth, Industrial Structure, Productivity, Technology, Viability

**JEL Classification:** D24, O11, O14, O30, O40, O41

(July, 2007)

(Comments Welcome)

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\* We thank Binkai Chen, Zhaoyang Xu, as well as all members of CCER Development Workshop for their helpful comments. Much of Zhang's work for this paper was completed at CID Harvard University and NBER. Zhang would like to thank Martin Feldstein, Ricardo Hausman and Dani Rodrik as well as these two organizations for their kind hospitality. Of course, the usual disclaimer applies.

# 1. Introduction

Since the Industrial Revolution in the eighteenth century, the world's countries have evolved into two groups. The first group includes rich, industrialized, developed countries (DCs), while the second group includes poor, agrarian, less developed countries (LDCs) (Lin, 2003). Nevertheless, prior to the World War II, only a few governments (most notably the Soviet Union) regarded economic growth as their direct responsibility and adopted policies for which economic growth was the primary stated objective, nor was the development economics a separate field of study (Krueger, 1995). In the great revival of interest in economic development that has marked the past decade, attention has centered on two main questions: first, what determines the over-all rate of economic advance?; second, what is the optimal allocation of given resources to promote growth (Chenery, 1961)? There are two different and occasionally controversial approaches to tackle the questions above respectively. Analysis of the determinants of the growth rate is the main purpose of the modern growth theory, i.e., neoclassical growth theory and recently endogenous growth theory, while providing solutions to the second question has relied mainly on the principles, e.g., comparative advantage, from trade theory.<sup>1</sup>

According to neoclassical growth theory (e.g., Ramsey, 1928; Solow, 1956; Swan, 1956; Cass, 1965; Koopmans, 1965) which focuses its attention on the process of capital formation with the assumption of the same given technology between LDCs and DCs, LDCs would grow faster than DCs and that the gap in per capita income between LDCs and DCs would narrow because of the diminishing returns to capital. Furthermore, if the marginal returns to capital continue to fall, the economy will enter a steady state with unchanging standard of livings. These unsatisfying conclusions of the neoclassical growth theory have led the current generation of new growth theorists to formulate models in which per capita income grows indefinitely (e.g., Arrow, 1962; Shell, 1967; Romer, 1986, 1990; Lucas, 1988; Jones and Manuelli, 1990; King and Rebelo, 1990; Segerstrom, et al. 1990; Grossman and Helpman 1991a; Rebelo, 1991; Aghion and Howitt, 1992).<sup>2</sup> Though new growth theory is insightful for explaining the continuous growth in DCs; however, it can not satisfactorily explain the tremendous differences in economic performance among the LDCs, as well as the extraordinary growth and convergence during the last three decades of the twentieth century of the newly industrialized economies in Asia, and recently China. As pointed out by Aghion (2004), the quality improvement paradigm, and new growth theories in general, remain of little help for development policy, without regard to specifics such as a country's current stage of development.

A main development in growth economics in the recent years has been to point at the fundamental role of institutions in the growth process (e.g., Acemoglu, et al. 2005), although few studies have led so far to precise policy recommendations beyond the general claims about the importance of property right enforcement (Aghion, 2004). Appropriate institution theory (Acemoglu, et al. 2006) can explain why the organization or institutions that maximize growth, or that are actually chosen by societies, vary with distance to the frontier, and they definitely reveal some intrinsic relationship between institution and economic growth, but they can not succeed in analyzing these institutions' formalization, change and abandonment.

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<sup>1</sup> The chief criticism is that comparative advantage is essentially a static concept which ignores a variety of dynamic elements (Chenery, 1961).

<sup>2</sup> Please refer to Grossman and Helpman (1994) for the details of three approaches to formulate models in which per capita income grows indefinitely.

Regardless of the great contribution whether the modern growth theory or the trade theory has made, neither of them could successfully explain the following economic phenomenon alone: after World War II, although many LDCs' governments adopted various policy measures to industrialize their economies, only a small number of economies in East Asia have actually succeeded in raising their level of per capita income to the level in DCs. Lin (2003) met this problem by providing a reasonable explanation with intrinsic logic consistency, which argues that the tremendous differences in economic performance among the LDCs can be explained largely by their governments' development strategies. Motivated by the dream of national building, most the LDC governments, both socialist and non-socialist alike, pursued Catch-up type comparative-advantage-defying (CAD) strategy to accelerate the development of the then advanced capital-intensive industries after World War II (Lin, 2003). However, an economy's (optimal) industrial structure is endogenously determined by that economy's factor endowment structure. The firms in the government's priority industries are not viable in an open, competitive market because these industries do not match the comparative advantage of their particular economy (Lin and Tan, 1999; Lin, 2003).<sup>3</sup> As such, it is imperative for the government to introduce a series of regulations and interventions in the international trade, financial sector, labor market, and so on so as to mobilize resources for setting up and supporting the continuous operation of the non-viable firms (Lin, et al. 2003; Lin and Zhang, 2007). This kind of development mode might be good at mobilizing the scarce resources and concentrating on a few clear, well-defined priority sector (Ericson, 1991), but the economy of this type becomes very inefficient as the result of misallocation of resources, rampant rent seeking, macro instability, and so forth (Lin, 2003). On the contrary, if the government in the LDCs, e.g. the newly industrialized economies in Asia, and recently China, pursues the comparative-advantage-following (CAF) strategy, which attempts to induce the firm's entry of industry according to the economy's exiting comparative advantage and facilitate the firm's adoption of appropriate technology by borrowing at low costs from the more advanced countries, the economy will enjoy rapid growth and the economic growth rate in these LDCs could be greater than that in the DCs owing to the advantage of the latter-comers and the faster upgrades in factor endowments in this LDC (Lin, 2003; Lin and Zhang, 2006; and Zhang, 2006), thus, the convergence of these LDCs with DCs would also come true.

The main purpose of the present paper is to develop an endogenous growth model that combines structural change with repeated product improvements to discuss the issues of development strategy, optimal industrial structure and viability in the LDC in a dynamic general-equilibrium framework.<sup>4</sup> There are two sectors in the present model, one is traditional sector, and the other is modern sector. The technological change in traditional sector takes the form of horizontal innovation based on expanding variety, while the technological progress in

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<sup>3</sup> A normally managed firm is viable if this firm earns a socially acceptable expected profit without external subsidies or protections. Please refer to Lin and Tan (1999) and Lin (2003) for the definition of viability.

<sup>4</sup> The government's economic policies toward industrial development could be grouped into two different and mutually exclusive development strategies: the CAD strategy, which attempts to encourage firms that ignore the existing comparative advantage of the economy in their entry/choice of industry/technology; and the CAF strategy, which attempts to facilitate the firms' entry/choice of industry/technology according to the economy's existing comparative advantage (Lin, 2003). Furthermore, there are two types of CAD strategy, one is Catch-up type CAD strategy in the LDCs, and the other is adopted mostly in DCs to support certain obsolete labor-intensive or resource-intensive industries for the purpose of protecting jobs. The Catch-up type CAD strategy can be referred as CAD strategy-type I, whereas the job-protecting CAD strategy can be referred as CAD strategy-type II. In the present paper, we focus on Catch-up type CAD strategy in the LDCs only.

modern sector is accompanied by incessantly creating advanced capital-intensive industry to replace backward labor-intensive one. The results of our model show that an economy's optimal industrial structure is endogenously determined by that economy's endowment structure; the firm in the LDCs that enters the capital-intensive, advanced industry in the DCs would be nonviable owing to the relative scarcity of capital in the LDCs' factor endowments; and whether the industrial structure matches with the factor endowment structure or not is the fundamental cause to explain differences in economic performance among the LDCs.

The rest of the paper is organized as follows. Section 2 discusses the related literatures in details. Section 3 constructs a specific model of endogenous economic growth which combines industrial structural upgrading with creative destruction. In section 4, we describe the equilibrium of the economy described above in infinite horizon firstly, and then characterize the dynamic trajectory of the present economy. The dynamic trajectory could be used to characterize the evolution of the (optimal) industrial structure in the LDCs. Based on the actual (intentioned) industrial structure in the LDC deviating from the optimal industrial structure in this country or not; we investigate the issues of development strategy and economic performance in the LDCs in section 5. Section 6 contains some brief concluding remarks. Finally, some details of the model that do not appear in the text are provided in the appendix.

## 2. Related Literature Review

Before introducing the basic model, it is rewardful for us to discuss some related literature in detail. The present paper related to a great number of different literatures. Our paper pertains to work on structural change, i.e., the systematic change in the relative importance of various sectors (e.g., Kuznets, 1957, 1973; Chenery, 1960; Baumol, 1967; Laitner, 2000; Kongsamut, et al. 2001; and Ngai and Pissarides, 2007). In Kongsamut, et al. (2001), the production function of the different sectors, i.e., agriculture, services, and manufacturing sector, are proportional, while in Ngai and Pissarides (2007) which focuses on exogenous Total Factor Productivity differences across different sectors, all sectors have identical Cobb-Douglas production functions. More closely related to our paper are Acemoglu and Guerrieri (2005 and 2006), Zhang (2006), Lin and Xu (2007) as well as Zuleta and Young (2007). Acemoglu and Guerrieri (2005 and 2006) first illustrates, when the elasticity of substitution of different products with different capital intensities in the aggregate production function of the final good is not equal to unity, the inevitable outcome of directed technical change is the non-balanced growth between different sectors. However, Acemoglu and Guerrieri (2005 and 2006) only analyzed the transitional dynamics of non-balanced growth in the case of exogenous technical change, but did not analyzes the transitional dynamics of non-balanced growth of an economy with endogenous technical change. Based on the model set-up in Acemoglu and Guerrieri (2005), Zhang (2006) extends it to analyze the transitional dynamics of non-balanced growth of an economy under endogenous technical change.<sup>5</sup> Zuleta and Young (2007) developed a two sector model of non-balanced economic growth with induced innovation, in which one sector ("goods" production) with technologies differentiated by the elasticity of output with respect to capital and becoming increasingly capital-intensive over time.<sup>6</sup> Zuleta and Young (2007) further assumes that though every technology is available at any instant,

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<sup>5</sup> Zhang would like to thank Prof. Acemoglu's face-to-face talk with him at Yale about the transitional dynamics in Zhang (2006).

<sup>6</sup> Seater (2005) developed a one sector exogenous growth model with the similar technical change as in Zuleta and Young (2007).

the adoption of a technology (i.e., innovation) is costly and the cost to innovation is increasing in its capital, thus creates a tradeoff between investment in capital and capital-intensity. In Zuleta and Young (2007), however, both the investment to capital deepening, which is tantamount to the upgrading of endowment structure in Lin (2003), and the investment to adopt more capital intensive production function are the results of optimal decisions by the identical firm at any instant, and there is no creative destruction either, i.e., more advanced products render previous ones obsolete. Moreover, the real intentions of all these papers mentioned above are to construct a model of non-balanced economic growth which is consistent with structural change. Though some of the above models are consistent with Kuznets facts (Kuznets, 1957, 1973) as well as the well-known Kaldor facts (Kaldor, 1961), none of these papers investigate or are suitable to discuss the issues of development strategy, optimal industrial structure, viability, and economic performance in the LDCs, which is the main purpose of the present paper.

There are two sectors in the present paper, one is the traditional sector with horizontal innovation as that in Romer (1990), i.e., innovation based on expanding variety which is called process innovation in the present paper, the other is the modern sector, whose technologies are differentiated by the capital intensities and will become not only increasingly capital-intensive over time as that in Seater (2005) as well as Zuleta and Young (2007), but also progressively productive over time as creative destruction or vertical innovation in Segerstrom, et al. (1990), Grossman and Helpman (1991a), as well as Aghion and Howitt (1992), i.e., innovation based on quality improving. In order to make a distinction between the horizontal innovation, i.e., process innovation in the traditional sector in this paper, and the innovation in the modern sector with increasingly capital-intensive and progressively productive, we denote the innovation in the modern sector in the present paper as product innovation.<sup>7</sup> There is much likeness in form but difference in essence between the product innovation in the present paper and the creative destruction or vertical innovation in Aghion and Howitt (1992), etc. Aghion and Howitt (1992), the pioneer paper of economic growth through creative destruction, provides a model of endogenous growth in which vertical innovations, generated by a competitive research sector, constitute the underlying source of growth. All these papers embody the vertical innovations in the repeated potential quality improvements, where each new generation of product or input performs proportionately better than the last,<sup>8</sup> and therefore the direct conclusion is more advanced technologies will be more beneficial to all countries, no matter they are DCs or LDCs. Thus, they all lose sight of the specifics such as a country's current stage of development which is also emphasized by Aghion (2004). And none of these papers take accounts the fact that adopting more productive technologies also requires higher capital-ratio usage, i.e., the issue of appropriate technology in the present paper.

Appropriate technology was first introduced by Atkinson and Stiglitz (1969), and has been revived recently by Diwan and Rodrik (1991) as well as Basu and Weil (1998). Basu and Weil (1998) is the first paper that provides the formal model to discuss appropriate technology in the economic growth framework, and they argue that the technologies are specific to particular combinations of inputs, i.e., capital-labor ratio in their paper. Nevertheless, the technological

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<sup>7</sup> Although the technologies in the modern sectors of the present paper resemble those in Zuleta and Young (2007), which are differentiated by the capital intensities and will become increasingly capital-intensive over time, the fact that technologies with higher capital intensities would be more productive in the present paper is totally neglected in Zuleta and Young (2007).

<sup>8</sup> In fact, the vertical innovations in these papers identified with the cost-reducing innovations.

progress in Basu and Wei (1998) is the by-product of “localized learning by doing”, as introduced by Atkinson and Stiglitz (1969), while in the present paper the technological progress in modern sector accompanied by the increase of capital intensity within the generations of product requires an intentional investment of resources by profit-seeking firms or entrepreneurs which is emphasized in Grossman and Helpman (1994).<sup>9</sup> Based on the endogenous growth model with expanding variety, Acemoglu and Zilibotti (2001) argues that many technologies used by the LDCs are developed in the OECD economies and are designed to make optimal use of the skills of these richer countries’ workerforces, thus, the necessary outcome is low productivity in the LDCs owing to the skill scarce in these countries. However, Acemoglu and Zilibotti (2001) cannot satisfactorily explain the extraordinary growth and convergence rates during the last three decades of the twentieth century for the NIEs in Asia, including Hong Kong, Singapore, South Korea, Taiwan, and recently China (Lin and Zhang, 2006; Zhang, 2006).<sup>10</sup> By extending the model in Acemoglu and Zilibotti (2001), Lin and Zhang (2006) concludes that the economic growth rate in the LDCs which choose the most appropriate (optimal) technologies that match the endowment structure of those countries may be greater than those in DCs and convergence can take place in these LDCs to DCs owing to the lower costs of technical progress in these LDCs. The endowment structure, whether in Acemoglu and Zilibotti (2001) or in Lin and Zhang (2006), nevertheless, is skill level, not the ratio of capital to labor which could be upgraded owing to capital deepening in the present paper.<sup>11</sup>

Finally, as will be illustrated in the following model, the capital intensity in the modern sector will approach the exogenously given parameter  $\tilde{\delta}$  asymptotically. Thus, when  $\tilde{\delta} < 1$ , the engine of economic growth in the long run is innovations in the present paper, i.e., horizontal innovation based on expanding variety (Romer, 1990) in traditional sector and vertical innovation (Segerstrom, et al. 1990; Grossman and Helpman, 1991a; as well as Aghion and Howitt, 1992) in modern sector; while if  $\tilde{\delta} = 1$ , in the present paper, there would be two different forces to fuel economic growth in the long run. The economic impetuses of the modern sectors come from the capital deepening, i.e., upgrading of endowment structure in Lin (2003), as that in Jones and Manuelli (1990), King and Rebelo (1990), as well as Rebelo (1991), and the engine of economic growth in the traditional sector is process innovation, i.e., horizontal innovation in Romer (1990).

### 3. The Basic Model

#### 3.1 Consumer Behavior

We consider an economy with  $L(t)$  workers at time  $t$ , supplying their labor without any disutility. The population has a constant exponential growth rate  $\tilde{g}$ . We also assume that all

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<sup>9</sup> In the model proposed by Basu and Weil (1998), there is never a problem of countries using technologies that do not match their level of development. However, Hsieh and Klenow (2007) argues that both China and India would get big TFP (Total Factor Productivity) gains from rationalizing allocation of capital and labor in both countries (TFP would double), while China appears to have benefited from recent reform efforts, but India shows little gain.

<sup>10</sup> As a matter of fact, the LDCs (South) and the DCs (North) share the same economic growth rates in Acemoglu and Zilibotti (2001).

<sup>11</sup> In Acemoglu and Zilibotti (2001), there is no change of skill level either in DCs or in LDCs, while Lin and Zhang (2006) only takes account of exogenous skill accumulation both in DCs and in LDCs.

households share identical constant relative risk aversion (CRRA) preferences over total household consumption index  $C(t, j)$ , and all population growth takes place within exiting households, which implies that the economy admits a representative agent with CRRA preferences:

$$U_\tau = \int_\tau^\infty \frac{C(t, j)^{1-\theta} - 1}{1-\theta} \exp[-\rho(t-\tau)] dt \quad (1)$$

where  $\rho$  is a subjective discount rate,  $\theta \geq 0$  is the coefficient of relative risk aversion, and  $C(t, j)$  represents an index of consumption (sub-utility function) of  $j$ th generation goods at time  $t$ . To reflect household's tastes for diversity in consumption, we adopt for  $C(t, j)$  a specification that impose a constant elasticity substitution (CES) between consumption of traditional goods, denoted by  $C_1$ , and consumption of modern goods of  $j$ th generation, denoted by  $C_2(j)$ .<sup>12</sup> Specifically, we have

$$C(j) = \left[ \gamma C_1^\varepsilon + (1-\gamma)(C_2(j))^\varepsilon \right]^{1/\varepsilon}, \quad 0 < \varepsilon < 1 \quad (2)$$

where  $\gamma \in (0, 1)$  is the share parameter of the two goods above, and  $\varepsilon \in (0, 1)$  determines the elasticity of substitution between consumption goods in traditional sector and that in modern sector. It is convenient for us to choose traditional goods as numeraire and denote the price for modern goods of  $j$ th generation to be  $p_j$ .

The representative consumer maximizes (1) subject to an intertemporal budget constraint. The consumption optimization problem can be solved in two stages. First, the representative consumer takes price  $p_j$  as given and chooses  $C_1$  and  $C_2(j)$  to maximize static utility in (2) for a given level of expenditure at time  $t$ , denoted by  $E(t)$ .

$$\max_{C_1, C_2(j)} \left[ \gamma C_1^\varepsilon + (1-\gamma)(C_2(j))^\varepsilon \right]^{1/\varepsilon}$$

subject to statistic budget constrain:

$$C_1(t) + p_j(t)C_2(j, t) = E(t) \quad (3)$$

The first-order conditions of the above maximization problem yield the following demand functions for  $C_1$  and  $C_2(j)$ :

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<sup>12</sup> We omit time arguments to simplify the notations whenever this causes no confusion from now on.



$$C_1 = \frac{E(t)}{1 + p_j \left[ \frac{(1-\gamma)}{\gamma p_j} \right]^{\frac{-1}{\varepsilon-1}}} \quad (4)$$

and

$$C_2(j) = \frac{E(t)}{\left[ \frac{(1-\gamma)}{\gamma p_j} \right]^{\frac{1}{\varepsilon-1}} + p_j} \quad (5)$$

Substituting (4) and (5) into (2) yields

$$C(t, j) = E(t)\aleph(p_j)$$

where  $\aleph(p_j)$  amounts to

$$\left\{ \gamma \left[ 1 + p_j \left( \frac{(1-\gamma)}{\gamma p_j} \right)^{\frac{-1}{\varepsilon-1}} \right]^{-\varepsilon} + (1-\gamma) \left[ \left( \frac{(1-\gamma)}{\gamma p_j} \right)^{\frac{1}{\varepsilon-1}} + p_j \right]^{-\varepsilon} \right\}^{1/\varepsilon}$$

Substituting  $C(t, j) = E(t)\aleph(p_j)$  into (1), the representative agent's utility function becomes

$$U_\tau = \int_\tau^\infty \frac{\left[ E(t)\aleph(p_j) \right]^{1-\theta} - 1}{1-\theta} \exp[-\rho(t-\tau)] dt \quad (6)$$

The second-stage consumption optimization problem involves choosing the time pattern of expenditures  $E(t)$  to maximize (6) subject to the representative consumer's intertemporal budget constraint:

$$\int_\tau^\infty \exp[R(t) - R(x)] E(x) dx = B(t) \quad (7)$$

where  $R(t)$  is the cumulative nominal interest factor from time 0 to time  $t$ , i.e.,

$R(t) \equiv \int_0^t \exp[r(x)] dx$  with  $R(0) \equiv 1$ , and  $B(t)$  is the representative agent's present value of the stream of factor incomes plus the value of initial asset holding at time  $t$ .

We write the current value Hamiltonian

$$\mathbf{H} = \frac{\left[ E(t)\aleph(p_j) \right]^{1-\theta} - 1}{1-\theta} + \mathcal{G}(t) [r(t)B(t) - E(t)]$$

where  $\mathcal{G}(t)$  is the (current value) costate variable associated with the representative consumer's intertemporal budget constraint (7).

The intertemporal optimization problem of the above representative agent implies the

following Euler equation

$$\frac{\dot{E}(t)}{E(t)} = \frac{r(t) - \rho}{\theta} \quad (8)$$

and the transversality condition

$$\lim_{t \rightarrow \infty} \exp(-\rho t) \mathcal{G}(t) B(t) = 0$$

where  $r(t)$  is the nominal interest rate at time  $t$ .

Before leaving consumption side of the economy, it will be useful for our later analysis to consider the relationship of the representative consumer's spending allocated to traditional goods with respect to modern goods. Differentiating (3) with respect to time  $t$  yields

$$\frac{\dot{C}_1(t)}{C_1(t)} \frac{C_1(t)}{E(t)} + \frac{p_j(t) C_2(j,t)}{E(t)} \frac{\dot{C}_2(j,t)}{C_2(j,t)} + \frac{p_j(t) C_2(j,t)}{E(t)} \frac{\dot{p}_j(t)}{p_j(t)} = \frac{\dot{E}(t)}{E(t)}$$

Denote the share of the representative consumer's spending allocated to traditional goods by

$$s_1 \equiv \frac{C_1(t)}{E(t)}, \text{ and it is obvious that we have } 1 - s_1 = \frac{p_j(t) C_2(j,t)}{E(t)}, \text{ which is the share of the}$$

representative consumer's spending allocated to modern goods. Then we have

$$s_1 \frac{\dot{C}_1(t)}{C_1(t)} + (1 - s_1) \frac{\dot{C}_2(j,t)}{C_2(j,t)} + (1 - s_1) \frac{\dot{p}_j(t)}{p_j(t)} = \frac{\dot{E}(t)}{E(t)} \quad (9)$$

### 3.2 Producer Behavior and Static Equilibrium

Turning to the production side, there are only two primary factors of production, capital  $K(t)$  and labor  $L(t)$ , and two sectors in the economy. One is traditional sector and the other is modern sector. We assume the product in traditional sector, denoted by  $Y_1$ , can be used as consumption goods only, while the product in modern sector, denoted by  $Y_2(j)$  which is the product of  $j$ th generation, can be consumed by households, installed by firms as capital, or invested by entrepreneurs as R&D expenditures.

#### 3.2.1 Producer in the Traditional Sector

As that in Funkel and Strulik (2000), we assume that the production of the homogenous goods  $Y_1$  in the competitive traditional sector requires the variable inputs capital  $K(t)$ , labor  $L(t)$  and an index of intermediates  $D$ , where the production function of traditional goods is

$$Y_1 = F_1(K, D, L) = A_1 K^{\eta_K} D^{\eta_D} L^{1-\eta_K-\eta_D}, \quad 0 \leq \eta_K, \eta_D, \eta_K + \eta_D \leq 1, \quad A_1 > 0$$

We assume  $A_1 \equiv 1$  for simplicity without any loss of generality, and we also assume that

$\eta_D \equiv 1$  to avoid unnecessary complexity in the present paper, thus, the production function of

traditional goods becomes<sup>13</sup>

$$Y_1 = F_1(K, D, L) = D$$

Following Grossman and Helpman (1991b), the index of intermediates  $D$  is represented by

$$D = \left[ \int_0^N z(i)^\alpha di \right]^{1/\alpha}, \quad 0 < \alpha < 1 \quad (10)$$

where  $z(i)$  denotes the input of intermediate good  $i$ ,  $N$  is the number (measure) of available intermediated goods, i.e., the technology in traditional sector, and  $\alpha$  is the elasticity of substitution between different intermediated inputs. At every moment in time, the existing producers of intermediate goods engage in oligopolistic price competition, and intermediate good  $z(i)$  is produced with the following Cobb-Douglas production function:

$$z(i) = l(i)^\beta k(i)^{1-\beta}, \quad 0 < \beta < 1 \quad (11)$$

where  $l(i)$  and  $k(i)$  is labor and capital employed in the production of the existing intermediate good  $z(i)$ .

Facing the given price of the existing intermediate good  $z(i)$ , which is denoted by  $q(i)$ , and the price for the product of the traditional sector, which is normalized to be 1, the inverse demanding function for the existing intermediate good  $z(i)$  by the competitive firm in the traditional sector is given by:

$$q(i) = (Y_1)^{1-\alpha} z(i)^{\alpha-1} \quad (12)$$

And the profit maximization problem of the existing intermediate firm  $i$  can be equivalently written as

$$\text{Max}_{l(i), k(i)} (Y_1)^{1-\alpha} l(i)^{\alpha\beta} k(i)^{\alpha(1-\beta)} - w.l(i) - r.k(i) \quad (13)$$

The first-order conditions in (13) are

$$\alpha\beta(Y_1)^{1-\alpha} l(i)^{\alpha\beta-1} k(i)^{\alpha(1-\beta)} = w \quad (14)$$

$$\alpha(1-\beta)(Y_1)^{1-\alpha} l(i)^{\alpha\beta} k(i)^{\alpha(1-\beta)-1} = r \quad (15)$$

Combining (14) with (15) yields the existing intermediate firm  $i$ 's factor demand functions

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<sup>13</sup> Grossman and Helpman (1994) summarizes that so far there are three approaches to formulate models in which per capita income grows indefinitely. If we follow Funkel and Strulik (2000) and assume  $0 < \eta_K, \eta_D < 1$  as well as interpret labor  $L$  as knowledge in Romer (1986) or human capital in Lucas (1988) with spillover effect or external effects, then when  $\tilde{\delta} = 1$ , all three approaches in endogenous growth theory will be unified in a single model. To our knowledge, by doing so, it would be the first model that unifies three approaches Grossman and Helpman (1994) in a single model.

$$l(i) = \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)} (Y_1)^{(1-\alpha)} \right]^{\frac{1}{1-\alpha}} \quad (16)$$

$$k(i) = \frac{w(1-\beta)}{r\beta} \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)} (Y_1)^{(1-\alpha)} \right]^{\frac{1}{1-\alpha}} \quad (17)$$

Substituting the existing intermediate firm  $i$ 's factor demand functions in (16) and (17) into (12), then  $q(i)$ , i.e., the price of the existing intermediate good  $z(i)$ , satisfies

$$q(i) = \alpha^{-1} (1-\beta)^{-(1-\beta)} \beta^{-\beta} w^\beta r^{1-\beta} \quad (18)$$

Thus, in a symmetric equilibrium, all the existing intermediate firms in traditional sector would charge the same price and share identical factor demand functions, which implies

$$l(i) = \frac{L_1}{N} \quad \text{and} \quad k(i) = \frac{K_1}{N} \quad (19)$$

where  $L_1$  and  $K_1$  is the total amount of labor and capital used in the traditional sector respectively.

$$L_1 = N \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)} \right]^{\frac{1}{1-\alpha}} Y_1 \quad (20)$$

$$K_1 = N \frac{w(1-\beta)}{r\beta} \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)} \right]^{\frac{1}{1-\alpha}} Y_1 \quad (21)$$

Now the production function of the existing intermediate good  $z(i)$  in (11) becomes

$$z(i) = \frac{1}{N} (L_1)^\beta (K_1)^{1-\beta} \quad (22)$$

and the production function of the traditional sector in (10) could be rewritten as

$$Y_1 = D = N^{\frac{1-\alpha}{\alpha}} (L_1)^\beta (K_1)^{(1-\beta)} \quad (23)$$

Combining (19) and (23) with (14) and (15) implies the wage rate and interest rate satisfy

$$w = \alpha \beta N^{\frac{1-\alpha}{\alpha}} (K_1/L_1)^{1-\beta} \quad (24)$$

$$r = \alpha(1-\beta) N^{\frac{1-\alpha}{\alpha}} (K_1/L_1)^{-\beta} \quad (25)$$

Substituting (24) and (25) into (13), the profit function of the existing intermediate firm  $i$  in traditional sector can be obtained by

$$\pi_1(i) = (1-\alpha) N^{\frac{1-2\alpha}{\alpha}} (L_1)^\beta (K_1)^{1-\beta} \quad (26)$$

As that in Judd (1985), Romer (1990) as well as Grossman and Helpman (1990), we also assume that production of a new intermediate good require R&D expenditures  $X_1$  in terms of the modern goods devoted to the invention of a new blueprint, moreover, we also assume that process innovation outlays are made by private, profit entrepreneurs, who receive indefinite patent protection and will appropriate some of the benefits from a new process innovation in the form of

oligopoly profits. The oligopolistic entrepreneur of intermediate firm  $i$  in traditional sector's present value of future operating profits from producing  $z(i)$  discounted to time  $t$  is given by

$$V_1(i, t) = \int_t^{\infty} \exp[R(t) - R(x)] \pi_1(i, x) dx$$

where  $\pi_1(i, x)$  is the flow profits of firm  $i$  from producing intermediate good  $z(i)$  in traditional sector which is expressed by (26) at time  $x$ .

Differentiating  $V_1(i, t)$  with respect to time  $t$  yields

$$\frac{\dot{V}_1(i, t)}{V_1(i, t)} = r(t) - \frac{\pi_1(i, t)}{V_1(i, t)} \quad (27)$$

With the spillover effect from the current stock of knowledge in traditional sector to future process innovations emphasized in Romer (1990) in mind, we assume that if  $X_1$  units of modern goods engage in research in traditional sector, they generate a flow of new products  $\dot{N}$  given by

$$\dot{N} = b_1 N^{-\varphi_1} X_1$$

where  $b_1$  is a strictly positive constant measuring the technical difficulty of creating new blueprints in traditional sector, and  $\varphi_1 \in (-1, \infty)$  measures the degree of spillovers in technology creation.<sup>14</sup>

Then, with free entry by intermediate firm  $i$ , if there are positive but finite resources devoted to R&D in traditional sector at time  $t$ , we must have the zero-profit condition for firm  $i$  as

$$V_1(i, t) = p_j(t) \frac{N^{\varphi_1}}{b_1} \quad (28)$$

### 3.2.2 Producer in the Modern Sector

Producing the product in the modern sector also requires the variable inputs, capital  $K(t)$  and labor  $L(t)$ , but not intermediates. The production function of the product of  $j$ th generation in the modern sector is given by

$$Y_2(j) = F_2[A_2(j), K, L] = A_2(j) K_2^{\delta_j} L_2^{1-\delta_j}, \quad 1 - \beta < \delta_j \leq \tilde{\delta}$$

where  $\tilde{\delta}$  is an exogenously given parameter which satisfies  $\tilde{\delta} \leq 1$ ,  $K_2$ ,  $L_2$  and  $\delta_j$  is capital and labor used, as well as the capital intensity in the modern sector for the product of  $j$ th

<sup>14</sup> Please see Jones (1995), Young (1999), or footnote 21 in Acemoglu and Guerrieri (2006) for the detailed discussion of the range of value of  $\varphi_1$ .

generation respectively, and  $A_2(j)$  is the productivity of the  $j = 1, 2, \dots$ th generation product.

The parameters in the present paper which satisfy  $1 - \beta < \delta_j < \tilde{\delta}$  imply that the modern sector is more capital-intensive than traditional sector at any moment.

Following the literatures on horizontal innovation or creative destruction (e.g., Segerstrom, et al., 1990; Grossman and Helpman, 1991a; as well as Aghion and Howitt, 1992), we assume the productivity of the  $j = 1, 2, \dots$ th generation in the modern sector, denoted by  $A_2(j)$ , is exactly  $f(\tilde{\delta} - \delta_{j-1})$  times as that of the generation before it. That is, we have

$$A_2(j) = f(\tilde{\delta} - \delta_{j-1})A_2(j-1)$$

where  $f(\cdot)$  is an exogenously given function which satisfies  $f(x) \geq 1$ .<sup>15</sup> We choose units so that the productivity of the lowest generation, i.e., the one available at time  $t = 0$  (the starting point of the analysis), be equal to unity; that is we assume  $A_2(0) = 1$ .<sup>16</sup>

In contrast with the literatures on horizontal innovation or creative destruction mentioned above which focus on productivity (or product quality) rising only, the present paper embodies product innovation in technological progress with incessantly capital intensive and progressively productive over time. We assume that the relationship between capital intensity  $\delta_j$  of the generation  $j = 1, 2, \dots$  with that of generation  $j-1$  satisfies the following condition for simplicity

$$\delta_j = \delta_{j-1} + b_2(\tilde{\delta} - \delta_{j-1})^{\varphi_3} \quad (29)$$

where  $b_2 > 0$  and  $\varphi_3 > 0$ .

Equation (29) implies in infinite horizon we have

$$\lim_{t \rightarrow \infty} \delta(t) = \tilde{\delta}$$

When  $\tilde{\delta} = 1$ , the production function in the modern sector takes the form of

$$\lim_{t \rightarrow \infty, j \rightarrow \infty} F_2[A_2(j), K, L] = A_2^* K \quad \text{in infinite horizon, where } A_2^* = \lim_{t \rightarrow \infty, j \rightarrow \infty} A_2(j, t).$$

In infinite horizon, the interest rate would be a constant in the constant growth equilibrium (CGE), which will be shown in the next section, requires the productivity in the modern sector to be also a

<sup>15</sup> The function  $f(\cdot)$  in the present paper always equals a positive constant in Aghion and Howitt (1992) as well as other paper on vertical innovations.

<sup>16</sup> As the starting point of the analysis, we assume that the modern sector begins at time  $t = 0$  with one firm which has access to a universally known backstop technology in the perfectly competitive output market and factor market until the first generation product is invented.

constant in the long-run, i.e.,  $A_2^*$  to be a constant in this case. A sufficient condition that could guarantee  $A_2^*$  to be a constant is that  $f(\cdot)$  is an increasing function which satisfies

$$\lim_{x \rightarrow 0^+} f(x) = 1. \text{ }^{17}$$

In this paper, we focus here the case of  $\tilde{\delta} < 1$ . When  $\tilde{\delta} < 1$ , the production function in modern sector takes the form of  $\lim_{t \rightarrow \infty, j \rightarrow \infty} F_2[A_2(j), K, L] = \lim_{t \rightarrow \infty, j \rightarrow \infty} A_2(j) K_2^{\tilde{\delta}} L_2^{1-\tilde{\delta}}$  in the long-run. The necessary condition that guarantees sustained growth in modern sector is  $\lim_{x \rightarrow 0^+} f(x) > 1$ , and we assume  $f(x) \equiv \lambda$  for simplicity, where  $\lambda$  is an exogenously given constant and satisfies  $\lambda > 1$ .<sup>18</sup> Now the production function in the modern sector could be rewritten as

$$Y_2(j) = F_2[A_2(j), K, L] = \lambda^j K_2^{\tilde{\delta}_j} L_2^{1-\tilde{\delta}_j} \quad (30)$$

Of course, more advanced products, i.e., the product with higher productivity and rising capital intensity, could not be produced until they have been invented. We follow the approaches taken in Aghion and Howitt (1992) as well as in chapter 4 of Grossman and Helpman (1991b), and assume that research in modern sector produces a random sequence of product innovations. Any firm in the modern sector that carries out R&D at intensity  $\iota$  for a time interval of length  $dt$  will succeed in its attempt to develop the product of generation  $j = 1, 2, \dots$  based on the existing generation before it with probability  $\iota dt$  which follows a Poisson distribution, and R&D expenditures per unit of time in this activity are  $X_2 \equiv (\lambda^j)^{\varphi_2} \iota$  in terms of the modern goods when the entrepreneur attempts to develop that product, where  $\varphi_2 > 1/(1-\tilde{\delta}) > 1$  reflects the fact that the more advanced the technology in modern sector, the more R&D expenditures needed for further product innovation in this sector. Furthermore, we assume the parameters of the present model satisfy  $\alpha\beta(1+\varphi_1) = \varphi_2(1-\tilde{\delta})(1-\alpha)$  to guarantee the balanced growth between the traditional sector and the modern sector in infinite horizon.

Once the product of generation  $j = 1, 2, \dots$  has been invented in the research lab, successful innovator obtains a patent which is assumed to last forever on condition that no new generation has been invented, or else, the present generation product in the modern sector will be replaced by the next generation/vintage. And the producers with the requisite know-how and patent rights can manufacture the product of  $j$ th generation in the modern sector according the production

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<sup>17</sup> Aghion and Howitt (1992) concludes their paper with several directions to generalize and extend the analysis of the model, such as assuming that technology is ultimately bounded, thereby requiring the size of innovations eventually to fall.

<sup>18</sup> As that in Aghion and Howitt (1992), the present model can be extended to allow firms to choose the optimal innovation size  $\lambda$ .

function in (30). We assume that all firms in modern sector engage in price competition at output market and are price-taker at factor market, and we also assume that only one leader-firm, e.g., firm  $j$ , in the modern sector has access to the state-of-the-art technology, while another one follower-firm, i.e., firm  $j-1$ , masters the technology that is one step behind it.

For the moment we assume innovations are always drastic, which means the successful innovator is unconstrained by potential competition from the previous patent.<sup>19</sup>

From (4) and (5), the inverse demand function faced by a monopolistic firm  $j$  in modern sector charging price  $p_j$  can be solved out as follows:

$$p_j = \frac{(1-\gamma)(C_2(j))^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}}$$

Let us denote the fraction of modern goods of the  $j$ th generation consumed by the households by  $\mu_j \equiv \frac{C_2(j)}{Y_2(j)}$ ,<sup>20</sup> then we have

$$p_j = \frac{(1-\gamma)(\mu_j Y_2(j))^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}} \quad (31)$$

Facing the given inverse demand function in (31), given factor prices  $w_j$  and  $r_j$ , the firm  $j$  in the modern sector will choose  $K_2$  and  $L_2$  to maximize profit, given by:

$$\pi_2(j) = \frac{(1-\gamma)(\mu_j)^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}} (\lambda^j K_2^{\delta_j} L_2^{1-\delta_j})^\varepsilon - r_j K_2 - w_j L_2 \quad (32)$$

The first-order conditions in (32) are

$$\frac{(1-\gamma)(\mu_j)^{\varepsilon-1} \lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}} \varepsilon \delta_j K_2^{\varepsilon\delta_j-1} L_2^{\varepsilon(1-\delta_j)} = r_j \quad (33)$$

$$\frac{(1-\gamma)(\mu_j)^{\varepsilon-1} \lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}} \varepsilon (1-\delta_j) K_2^{\varepsilon\delta_j} L_2^{\varepsilon(1-\delta_j)-1} = w_j \quad (34)$$

Combining (33) with (34) implies that the factor demand functions of the firm  $j$  in modern sector should satisfy

$$L_2 = \left\{ \frac{(1-\gamma)(\mu_j)^{\varepsilon-1} \lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon-1}} \varepsilon (1-\delta_j)^{(1-\varepsilon\delta_j)} (\delta_j)^{\varepsilon\delta_j} (w_j)^{-(1-\varepsilon\delta_j)} (r_j)^{-\varepsilon\delta_j} \right\}^{\frac{1}{1-\varepsilon}} \quad (35)$$

<sup>19</sup> Please see the appendix for the details of the case of nondrastic innovations.

<sup>20</sup> In the present model,  $(1-\mu_j)$  denotes the savings rate.



$$K_2 = \frac{\delta_j w_j}{(1 - \delta_j) r_j} L_2 \quad (36)$$

Substituting (35) and (36) into (32), we can solve out the profit function of the firm  $j$  in modern sector as follows

$$\pi_2(j) = (1 - \varepsilon) \frac{(1 - \gamma) (\mu_j)^{\varepsilon - 1} \lambda^{j\varepsilon}}{\gamma C_1^{\varepsilon - 1}} \left( K_2^{\delta_j} L_2^{1 - \delta_j} \right)^\varepsilon \quad (37)$$

And the price of the product of the  $j$ th generation in the modern sector is determined by

$$p_j = \lambda^{-j} \varepsilon^{-1} (\delta_j)^{-\delta_j} (1 - \delta_j)^{-(1 - \delta_j)} (w_j)^{(1 - \delta_j)} (r_j)^{\delta_j} \quad (38)$$

At time  $t$ , the value to an outside research firm  $j$  that aims to develop a product whose productivity is  $\lambda$  times as many as the state of the art and carries out R&D at intensity  $\iota$  when this firm is successful in the  $j$ th product innovation, which is denoted by  $V_2(j, t)$ , is the expected present value of the flow of monopoly profits  $\pi_2(j, x)$  discounted to time  $t$ , where the duration of  $\pi_2(j, x)$  follows the exponential distribution with parameter  $\iota x$ :

$$V_2(j, t) = \int_t^\infty \exp[R(t) - R(x)] \pi_2(j, x) \prod(j, x) dx$$

where  $\prod(j, x)$  equals the probability that there will be exactly  $j$  innovations from the starting point to time  $x$ , thus, we have

$$\prod(j, x) = \frac{(\iota x)^j e^{-\iota x}}{j!}$$

The newcomer firm  $j$  in modern sector would choose research intensity  $\iota$  for a time interval of length  $dt$  to maximize

$$\max_\iota V_2(j, t) \iota dt - p_j(t) (\lambda^j)^{\rho_2} \iota dt \quad (39)$$

The maximization problem in (39) implies

$$p_j(t) (\lambda^j)^{\rho_2} \geq V_2(j, t), \quad \iota \geq 0 \text{ and } \left[ p_j(t) (\lambda^j)^{\rho_2} - V_2(j, t) \right] \iota = 0$$

Thus, as long as the R&D operates at a positive but finite scale, we must have  $\iota > 0$ , and  $p_j(t) (\lambda^j)^{\rho_2} = V_2(j, t)$ . And the variation of the value to an outside research firm  $j$  discounted to time  $t$ , denoted by  $\dot{V}_2(j, t)$ , can be expressed as

$$\frac{\dot{V}_2(j, t)}{V_2(j, t)} = r(t) + \iota(t) - \frac{\pi_2(j, t)}{V_2(j, t)} \quad (40)$$

### 3.3 Market Clearing Conditions

We close the model by describing market clearing conditions. The output market clearing

condition in traditional sector implies

$$C_1 = Y_1$$

If we neglect the capital depreciation in our model for simplicity, then the output market clearing condition in modern sector is:

$$C_2(j) + \dot{K} + X_1 + X_2 = Y_2(j)$$

According to the analysis above, the factor market clearing conditions can be expressed as:

$$L_1 + L_2 = L$$

$$K_1 + K_2 = K$$

where  $L_1$  ( $K_1$ ) and  $L_2$  ( $K_2$ ) denotes the levels of labor (capital) used in traditional sector and modern sector respectively. It is convenient for the analysis below to denote the fraction of labor

and capital used in traditional sector by  $\kappa_L = \frac{L_1}{L_1 + L_2}$  and  $\kappa_K = \frac{K_1}{K_1 + K_2}$ . From (20), (21),

(35), and (36), we find

$$\kappa_L = \frac{1}{1 + \frac{\left[ \frac{1}{\gamma} (1-\gamma) (\mu_j)^{\varepsilon-1} \lambda^{j\varepsilon} \varepsilon (1-\delta_j)^{(1-\varepsilon\delta_j)} (\delta_j)^{\varepsilon\delta_j} (w_j)^{-(1-\varepsilon\delta_j)} (r_j)^{-\varepsilon\delta_j} \right]^{\frac{1}{1-\varepsilon}}}{N \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)} \right]^{\frac{1}{1-\alpha}}}} \quad (41)$$

and

$$\kappa_K = \frac{1}{1 + \frac{\left[ \frac{1}{\gamma} (1-\gamma) (\mu_j)^{\varepsilon-1} \lambda^{j\varepsilon} \varepsilon (1-\delta_j)^{(1-\varepsilon\delta_j)} (\delta_j)^{\varepsilon\delta_j} (w_j)^{-(1-\varepsilon\delta_j)} (r_j)^{-\varepsilon\delta_j} \right]^{\frac{1}{1-\varepsilon}}}{\frac{w(1-\beta)}{r\beta} \frac{(1-\delta_j)r_j}{\delta_j w_j} N \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha w^{-(1-\alpha+\alpha\beta)} r^{-(\alpha-\alpha\beta)} \right]^{\frac{1}{1-\alpha}}}}$$

Since the traditional sector and the modern sector share the identical factor prices in equilibrium, thus we have

$$\kappa_K = \frac{\kappa_L}{\kappa_L + \xi_j (1 - \kappa_L)} \quad (42)$$

where  $\xi_j \equiv \frac{\delta_j \beta}{(1-\delta_j)(1-\beta)}$ .

## 4. Dynamic Equilibrium

The dynamic equilibrium in this economy is given by paths for prices of factors, intermediates and modern goods  $w$ ,  $r$ ,  $[q(i)]_{i=1}^N$ ,  $p$ , allocations of factors  $L_1$ ,  $L_2$ ,  $K_1$ ,

$K_2$ , as well as R&D expenditures  $X_1$ ,  $X_2$  such that producers maximize profits, and the representative consumer choose consumption and savings decision  $C_1$ ,  $C_2$  and  $E$  to maximize his utility under the market clearing conditions.

#### 4.1 Equilibrium in Infinite Horizon

It is convenient for us to study the equilibrium in infinite horizon firstly, and then turn to characterizing the dynamic trajectory of the present economy. We guess-and-verify the existence of a constant growth equilibrium (CGE) in infinite horizon, i.e.,  $t \rightarrow \infty$ , such that consumer expenditures  $E(t)$  grow at a constant rate  $g_E^*$

$$\lim_{t \rightarrow \infty} \frac{\dot{E}(t)}{E(t)} = g_E^* \quad (43)$$

Substituting (8) into (43) implies

$$\lim_{t \rightarrow \infty} r(t) \equiv r^* = \theta g_E^* + \rho$$

which means the interest rate in CGE is also a constant.

We focus here the special case of CGE, i.e., balanced growth equilibrium (BGE), such that modern sector and traditional sector grow at the same constant rate in infinite horizon for simplicity. The conditions that guarantee the existence of a BGE in the present model is

$$\alpha\beta(1 + \varphi_1) = \varphi_2(1 - \tilde{\delta})(1 - \alpha)$$

which will be proved in the analysis below.<sup>21</sup>

In BGE, the fraction of modern goods of the  $j$ th generation consumed by the households is constant, i.e.,  $\lim_{t \rightarrow \infty, j \rightarrow \infty} \mu_j(t) = \mu^*$ , thus, we have:

$$\lim_{t \rightarrow \infty, j \rightarrow \infty} \frac{\dot{C}_2(j, t)}{C_2(j, t)} \equiv g_{C_2}^* = \lim_{t \rightarrow \infty} \frac{\dot{Y}_2}{Y_2} \equiv g_{Y_2}^*$$

Differentiating  $p_j = \frac{(1 - \gamma)(\mu_j Y_2(j))^{\varepsilon - 1}}{\gamma Y_1^{\varepsilon - 1}}$  with respect to time  $t$  implies in BGE

$$g_p^* \equiv \lim_{t \rightarrow \infty} \frac{\dot{p}}{p} = 0$$

Equation (9) and  $g_p^* = 0$  imply that in BGE the share of the representative consumer's

spending allocated to traditional sector, denoted by  $s_1^*$ , is a constant and

<sup>21</sup> The BGE in infinite horizon that we characterize here can be proved to be unique. Moreover, the model described here could feature the non-balanced growth between modern sector and traditional sector in infinite horizon under the condition that the technical change is biased when  $\alpha\beta(1 + \varphi_1) \neq \varphi_2(1 - \tilde{\delta})(1 - \alpha)$ .

$$g_{C_1}^* = g_{Y_1}^* = g_{C_2}^* = g_{Y_2}^* = g_E^*$$

where  $\lim_{t \rightarrow \infty} s_1(t) \equiv s_1^*$ ,  $\lim_{t \rightarrow \infty} \frac{\dot{C}_1(t)}{C_1(t)} \equiv g_{C_1}^*$ , and  $\lim_{t \rightarrow \infty} \frac{\dot{Y}_1(t)}{Y_1(t)} \equiv g_{Y_1}^*$ .

Let us first derive the growth rates of the key objects in traditional sector in BGE. Differentiating (23) with respect to time  $t$  yields

$$\frac{\dot{Y}_1}{Y_1} = \frac{1-\alpha}{\alpha} \frac{\dot{N}}{N} + \beta \frac{\dot{L}_1}{L_1} + (1-\beta) \frac{\dot{K}_1}{K_1}$$

Thus, the growth rate of traditional sector in BGE is given by

$$g_{Y_1}^* = \frac{1-\alpha}{\alpha} g_N^* + \beta g_{L_1}^* + (1-\beta) g_{K_1}^* \quad (44)$$

where  $\lim_{t \rightarrow \infty} \frac{\dot{L}_1(t)}{L_1(t)} = g_{L_1}^*$  and  $\lim_{t \rightarrow \infty} \frac{\dot{K}_1(t)}{K_1(t)} = g_{K_1}^*$ .

Differentiating the interest rate in (25) with respect to time  $t$  implies that we have

$$g_{K_1}^* = \frac{1-\alpha}{\alpha\beta} g_N^* + g_{L_1}^* \quad (45)$$

where  $\lim_{t \rightarrow \infty} \frac{\dot{N}}{N} \equiv g_N^*$ .

Combining (44) with (45), we find

$$g_{Y_1}^* = \frac{1-\alpha}{\alpha\beta} g_N^* + g_{L_1}^* \quad (46)$$

Differentiating the zero-profit condition for firm  $i$  in traditional sector which is expressed by (28) with respect to time  $t$  yields

$$\frac{\dot{V}_1(i,t)}{V_1(i,t)} = \frac{\dot{p}}{p} + \varphi_1 \frac{\dot{N}}{N} \quad (47)$$

Combining (27) with (47) yields

$$\frac{\dot{p}}{p} + \varphi_1 \frac{\dot{N}}{N} = r(t) - \frac{\pi_1(i,t)}{V_1(i,t)} \quad (48)$$

In BGE, we have show that  $\lim_{t \rightarrow \infty} \frac{\dot{N}}{N} \equiv g_N^*$  is a constant, thus we have

$$\lim_{t \rightarrow \infty} \frac{\dot{V}_1(i,t)}{V_1(i,t)} = \lim_{t \rightarrow \infty} \frac{\dot{\pi}_1(i,t)}{\pi_1(i,t)} = \varphi_1 g_N^*$$

Differentiating the profit function of the existing immediate firm  $i$  in traditional sector which is expressed by (26) with respect to time  $t$ , we obtain

$$\lim_{t \rightarrow \infty} \frac{\dot{\pi}_1(i, t)}{\pi_1(i, t)} = \varphi_1 g_N^* = \frac{1-2\alpha}{\alpha} g_N^* + \beta g_{L_1}^* + (1-\beta) g_{K_1}^* \quad (49)$$

Combining (45) and (46) with (49) yields

$$g_{Y_1}^* = \frac{\alpha\beta(1+\varphi_1)}{\alpha\beta(1+\varphi_1) - (1-\alpha)} g_{L_1}^* \quad (50)$$

Now we turn to the growth rate of the key objects in modern sector in BGE. The properties of the Poisson distribution imply that in infinite horizon, the expected number of product innovations in a time interval of length  $t$  is  $t^* t$  (Feller, 1968). Thus, in infinite horizon, the production function in modern sector becomes

$$Y_2 = \lim_{t \rightarrow \infty, j \rightarrow \infty} F_2[A_2(j), K, L] = \lambda^{t^* t} K_2^{\tilde{\delta}} L_2^{1-\tilde{\delta}} \quad (51)$$

where  $t^*$  is the optimal rate of innovations in the long-run.

Differentiating (51) with respect to time  $t$  yields

$$\frac{\dot{Y}_2}{Y_2} = t^* \ln \lambda + \tilde{\delta} \frac{\dot{K}_2}{K_2} + (1-\tilde{\delta}) \frac{\dot{L}_2}{L_2}$$

Therefore, the growth rate of modern sector in BGE is given by

$$g_{Y_2}^* = t^* \ln \lambda + \tilde{\delta} g_{K_2}^* + (1-\tilde{\delta}) g_{L_2}^* \quad (52)$$

where  $g_{K_2}^* \equiv \lim_{t \rightarrow \infty} \frac{\dot{K}_2}{K_2}$ , and  $g_{L_2}^* \equiv \lim_{t \rightarrow \infty} \frac{\dot{L}_2}{L_2}$ .

From (33), the interest rate in BGE can be expressed by

$$r^* = \frac{(1-\gamma)(\mu^*)^{\varepsilon-1} \lambda^{\varepsilon t^*}}{\gamma Y_1^{\varepsilon-1}} \varepsilon \tilde{\delta} K_2^{\varepsilon \tilde{\delta}-1} L_2^{\varepsilon(1-\tilde{\delta})} \quad (53)$$

Differentiating (53) with respect to time  $t$  yields

$$(\varepsilon-1)g_{Y_1}^* = \varepsilon t^* \ln \lambda + (\varepsilon \tilde{\delta}-1)g_{K_2}^* + \varepsilon(1-\tilde{\delta})g_{L_2}^* \quad (54)$$

Combining (52) with (54) yields

$$g_{Y_1}^* = g_{K_1}^* = g_{Y_2}^* = g_{K_2}^*$$

The fact that  $g_{K_1}^* = g_{K_2}^*$  implies the fraction of capital used in traditional sector is a constant in BGE, i.e.,  $\kappa_K^*$  is a constant, where  $\kappa_K^* = \lim_{t \rightarrow \infty} \kappa_K(t)$ . Thus, from (42) and

$\xi^* \equiv \lim_{j \rightarrow \infty} \xi_j = \frac{\tilde{\delta}\beta}{(1-\tilde{\delta})(1-\beta)}$ , we know the fraction of labor used in traditional sector is also a

constant in BGE, i.e.,  $\kappa_L^*$  is a constant, where  $\kappa_L^* \equiv \lim_{t \rightarrow \infty} \kappa_L(t)$ , which implies that

$$g_{L_1}^* = g_{L_2}^* = \tilde{g}$$

Substituting  $g_{L_1}^* = \tilde{g}$  into (50) yields

$$g_{Y_1}^* = g_{K_1}^* = g_{Y_2}^* = g_{K_2}^* = \frac{\alpha\beta(1+\varphi_1)}{\alpha\beta(1+\varphi_1)-(1-\alpha)} \tilde{g}$$

Because the product innovations occur in the modern sector according to a time-varying Poisson process with instantaneous arrival rate  $\iota(x)$  and the expected number of success before time  $t$  equals to  $\psi(t) \equiv \int_0^t \iota(x) dx$ , thus, the properties of the Poisson distribution imply that  $\lambda^j$  amounts to  $\lambda^{\psi(t)}$  at time  $t$  (Feller, 1968). From the analysis above, we must have  $p_j(t) \lambda^{\varphi_2 \int_0^t \iota(x) dx} = V_2(j, t)$  as long as there is positive and bounded growth in modern sector, which implies

$$\varphi_2 t^* \ln \lambda = \lim_{t \rightarrow \infty} \frac{\dot{V}_2(t)}{V_2(t)}$$

Thus, substituting  $t^*$  into (40) implies

$$r^* + t^* (1 - \varphi_2 \ln \lambda) = \lim_{t \rightarrow \infty} \frac{\pi_2(t)}{V_2(t)}$$

and

$$\lim_{t \rightarrow \infty} \frac{\dot{\pi}_2(t)}{\pi_2(t)} = \varphi_2 t^* \ln \lambda$$

In BGE, the profit function of the firm  $j$  in modern sector which is expressed by (37) reduces to

$$\lim_{t \rightarrow \infty, j \rightarrow \infty} \pi_2(j, t) = (1 - \varepsilon) \frac{(1 - \gamma) (\mu^*)^{\varepsilon-1} \lambda^{\varepsilon t^*}}{\gamma Y_1^{\varepsilon-1}} \left( K_2^\delta L_2^{1-\delta} \right)^\varepsilon \quad (55)$$

Differentiating (55) with respect to time  $t$  yields

$$\varphi_2 t^* \ln \lambda + (\varepsilon - 1) g_{Y_1}^* = \varepsilon t^* \ln \lambda^* + \varepsilon \tilde{\delta} g_{K_2}^* + \varepsilon (1 - \tilde{\delta}) g_{L_2}^* \quad (56)$$

Combining (54) with (56) yields

$$g_{K_2}^* = \varphi_2 t^* \ln \lambda \quad (57)$$

Substituting (57) into (52), we obtain

$$g_{Y_2}^* = \frac{\varphi_2 (1 - \tilde{\delta})}{\varphi_2 (1 - \tilde{\delta}) - 1} \tilde{g}$$

And the optimal intensity of innovations in the long-run, denoted by  $t^*$ , is determined by

$$t^* = \frac{(1-\tilde{\delta})}{\ln \lambda[\varphi_2(1-\tilde{\delta})-1]} \tilde{g}$$

From the analysis above and comparing the growth rate in modern sector with that in traditional sector in BGE, we know the parameters of our model which satisfy

$$\alpha\beta(1+\varphi_1) = \varphi_2(1-\tilde{\delta})(1-\alpha)$$

could indeed guarantee the existence and uniqueness of the BGE in the present paper.

From (20) and (35), the fraction of labor used in traditional sector in BGE can be expressed as  $\kappa_L^* = \frac{1}{1+\ell^*}$ , where

$$\ell^* = \lim_{t \rightarrow \infty} \frac{\left[ \frac{1-\gamma}{\gamma} \varepsilon(1-\tilde{\delta})^{(1-\varepsilon\tilde{\delta})} \tilde{\delta}^{\varepsilon\tilde{\delta}} \right]^{\frac{1}{1-\varepsilon}} (r^*)^{\frac{(\alpha-\alpha\beta)-\varepsilon\tilde{\delta}}{1-\alpha-\frac{\varepsilon\tilde{\delta}}{1-\varepsilon}}}}{\mu^* \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha \right]^{\frac{1}{1-\alpha}}} \lambda^{\frac{\varepsilon t^*}{1-\varepsilon}} w^{\frac{(1-\alpha+\alpha\beta)(1-\varepsilon\tilde{\delta})}{1-\alpha-\frac{\varepsilon\tilde{\delta}}{1-\varepsilon}}} \frac{1}{N}$$

and

$$\lim_{t \rightarrow \infty} \frac{\dot{w}}{w} = \frac{(1-\alpha)}{\alpha\beta(1+\varphi_1)-(1-\alpha)} \tilde{g}$$

Finally, the fraction of modern goods consumed by households in BGE, denoted by  $\mu^*$ , can be solved by

$$\dot{K} + \frac{\dot{N}N^{\varphi_1}}{b_1} + \lambda^{\varphi_2 t^*} t^* = (1-\mu^*) \lambda^{t^*} \left[ \frac{\xi^*(1-\kappa_L^*)K}{\kappa_L^* + \xi^*(1-\kappa_L^*)} \right]^{\tilde{\delta}} [(1-\kappa_L^*)L]^{1-\tilde{\delta}}$$

as well as the initial conditions of capital  $K_0$ , labor  $L_0$ , and technology in traditional sector and modern sector, i.e.,  $N_0$  and  $j_0$ .

## 4.2 Dynamic Trajectory

Before turning the issues of development strategy, optimal industrial structure, viability, and economic performance in the LDCs, we need to study the dynamic trajectories of the economy described in the present paper. The dynamic trajectories of this economy can be characterized by an autonomous system of nonlinear differential equations which contains three control variables,

$$e \equiv \frac{E}{LN^{\frac{1-\alpha}{\alpha\beta}}}, X_1, \text{ and } X_2 \text{ as well as seven state variables, } \delta, \kappa_L, p, \mu, k \equiv \frac{K}{LN^{\frac{1-\alpha}{\alpha\beta}}},$$

$$n \equiv \frac{N^{\frac{(\varphi_1+1)\alpha\beta-(1-\alpha)}{\alpha\beta}}}{L}, \text{ and } I \equiv \frac{\lambda^{(\varepsilon-\varphi_2) \int_0^t i(x) dx} L}{N^{\frac{(1-\alpha)(\varepsilon-\delta\varepsilon-1)}{\alpha\beta}}}.$$

First and foremost, we need to solve out the equilibrium interest rate  $r(t)$  in dynamic trajectories. From (25), we know that the equilibrium interest rate in the dynamic trajectories is determined by:

$$r(t) = \alpha(1 - \beta) \left[ \frac{\xi + (1 - \xi)\kappa_L}{k} \right]^\beta$$

where  $\xi \equiv \frac{\delta\beta}{(1 - \delta)(1 - \beta)} > 1$ .

Secondly, we need to calculate the dynamics of capital intensity in the modern sector. Once more, we could invoke the property of Poisson distribution to argue that the expected time of a firm in the modern sector that carries out R&D at intensity  $\iota(t)$  to develop the product of generation  $j = 1, 2, \dots$  based on the existing generation before it is  $dt = 1/\iota(t)$ . Therefore, from

(29), the dynamics of capital intensity in the modern sector, denoted by  $\delta$ , is given by

$$\dot{\delta}(t) \equiv \lim_{dt \rightarrow 0} \frac{\delta(t + dt) - \delta(t)}{dt} = b_2[\tilde{\delta} - \delta(t)]^{\varphi_3} \iota(t) \quad (58)$$

Thirdly, we should discover the evolution of the optimal R&D intensity in the dynamic paths.

Differentiating  $p\lambda^{\varphi_2 \int_0^t \iota(x) dx} = V_2(t)$  with respect to time  $t$  implies

$$\frac{\dot{p}}{p} + \varphi_2 \ln \lambda \iota(t) = \frac{\dot{V}_2(t)}{V_2(t)} \quad (59)$$

Combining (40) with (59), we obtain

$$(\varphi_2 \ln \lambda - 1)\iota(t) = r(t) - \frac{\dot{p}}{p} - \frac{\pi_2(t)}{V_2(t)}$$

where  $\frac{\pi_2(t)}{V_2(t)} = (1 - \varepsilon) \frac{(1 - \gamma)\mu^{\varepsilon-1} (1 - \kappa_K)^{\delta\varepsilon} (1 - \kappa_L)^{(1-\delta)\varepsilon}}{\gamma P (\kappa_L)^{\beta(\varepsilon-1)} (\kappa_K)^{(1-\beta)(\varepsilon-1)}} \mathbf{I} k^{\delta\varepsilon - (1-\beta)(\varepsilon-1)}$ .

Thus, the evolution of normalized accumulative R&D intensity, denoted by  $\mathbf{I}$ , should satisfy

$$\mathbf{I} = \frac{r(t) - \frac{\dot{p}}{p} - (1 - \varepsilon) \frac{(1 - \gamma)\mu^{\varepsilon-1} (1 - \kappa_K)^{\delta\varepsilon} (1 - \kappa_L)^{(1-\delta)\varepsilon}}{\gamma P (\kappa_L)^{\beta(\varepsilon-1)} (\kappa_K)^{(1-\beta)(\varepsilon-1)}} \mathbf{I} k^{\delta\varepsilon - (1-\beta)(\varepsilon-1)}}{(\varphi_2 \ln \lambda - 1)} \quad (60)$$

where  $\kappa_K = \frac{\kappa_L}{\kappa_L + \xi(1 - \kappa_L)}$ .

Fourthly, we need to characterize the evolution of the technology in the traditional sector.



Substituting  $\frac{\pi_1(i,t)}{V_1(i,t)} = b_1 \frac{(1-\alpha)(\kappa_L)^\beta (\kappa_K)^{1-\beta} k^{1-\beta}}{p}$  and  $\frac{\dot{N}}{N} = \frac{\alpha\beta \left(\frac{\dot{n}}{n} + \tilde{g}\right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)}$  into

(48) implies the dynamics of normalized technology in traditional sector, denoted by  $\frac{\dot{n}}{n}$ , is determined by

$$\frac{\dot{p}}{p} + \varphi_1 \frac{\alpha\beta \left(\frac{\dot{n}}{n} + \tilde{g}\right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)} = r(t) - b_1 \frac{(1-\alpha)(\kappa_L)^\beta (\kappa_K)^{1-\beta} k^{1-\beta}}{p} \quad (61)$$

Fifthly, from (41), we obtain the fraction of labor used in traditional sector  $\kappa_L$  as

$$\frac{1}{\kappa_L} = 1 + \frac{\left[\frac{(1-\gamma)}{\gamma} \varepsilon\right]^{\frac{1}{1-\varepsilon}} \mu^{-1} (1-\delta)^{\frac{1-\varepsilon\delta}{1-\varepsilon}} \delta^{\frac{\varepsilon\delta}{1-\varepsilon}} \left(\frac{k}{\xi + (1-\xi)\kappa_L}\right)^{\frac{\varepsilon(\delta-1+\beta)}{1-\varepsilon}} (nI)^{\frac{\varepsilon}{(\varepsilon-\varphi_2)(1-\varepsilon)}}}{\left[(1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha\right]^{\frac{1}{1-\alpha}} (\alpha\beta)^{\frac{\varepsilon-\alpha}{(1-\varepsilon)(1-\alpha)}} \left(\frac{1-\beta}{\beta}\right)^{\frac{\varepsilon\delta + \alpha(\beta-1)}{1-\varepsilon + 1-\alpha}}} \quad (62)$$

Sixthly, it is time for us to understand the law of price change in the present model. From (31), the price of the product in the modern sector can be rewritten as

$$p = \frac{(1-\gamma)}{\gamma} \left( \frac{\mu \lambda^{y(t)} K_2^\delta L_2^{1-\delta}}{N^{\frac{1-\alpha}{\alpha}} L_1^\beta K_1^{1-\beta}} \right)^{\varepsilon-1} \quad (63)$$

Differentiating (63) with respect to time  $t$ , we obtain the law of the price of the modern goods, denoted by  $\frac{\dot{p}}{p}$ , as

$$\frac{\dot{p}}{p} = (\varepsilon-1) \left[ \frac{\dot{\mu}}{\mu} + \iota \ln \lambda + (\delta + \beta - 1) \frac{\dot{k}}{k} + \frac{(1-\alpha)(\delta-1)\alpha\beta}{\alpha\beta(\varphi_1+1)\alpha\beta - (1-\alpha)} \left(\frac{\dot{n}}{n} + \tilde{g}\right) \right] - \delta \frac{\dot{\kappa}_K}{1-\kappa_K} - (1-\delta) \frac{\dot{\kappa}_L}{1-\kappa_L} - \beta \frac{\dot{\kappa}_L}{\kappa_L} - (1-\beta) \frac{\dot{\kappa}_K}{\kappa_K} \quad (64)$$

where  $\delta \ln \left( \frac{(1-\kappa_K)K}{(1-\kappa_L)L} \right)$  is an infinitesimal and we neglect it in (64).

Seventhly, the Euler equation of the representative agent requires the optimal path for the normalized consumption expenditure  $e$  must satisfy

$$\frac{\dot{e}}{e} = \frac{r-\rho}{\theta} - \tilde{g} - \frac{(1-\alpha) \left(\frac{\dot{n}}{n} + \tilde{g}\right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)} \quad (65)$$

Eighthly, from (9), the dynamics of the fraction of modern goods consumed by households

$\mu$  could be determined by

$$\begin{aligned} \frac{r(t) - \rho}{\theta} = s_1 & \left[ \frac{2(1-\alpha)\beta \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)} + \beta \left( \frac{\dot{k}}{k} + \frac{\dot{\kappa}_K}{\kappa_K} \right) + \tilde{g} + (1-\beta) \frac{\dot{\kappa}_L}{\kappa_L} \right] + (1-s_1) \frac{\dot{p}}{p} \\ & + (1-s_1) \left[ \frac{\dot{\mu}}{\mu} + \iota \ln \lambda + \frac{\delta \dot{k}}{k} + \frac{\delta(1-\alpha) \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)} - \frac{\delta \dot{\kappa}_K}{1-\kappa_K} + \tilde{g} - \frac{(1-\delta)\dot{\kappa}_L}{1-\kappa_L} \right] \end{aligned} \quad (66)$$

where  $s_1 = \frac{(\kappa_L)^\beta (\kappa_K)^{1-\beta}}{e} k^{1-\beta}$  and again we neglect  $b_2 [\tilde{\delta} - \delta]^{\varphi_3} \iota \ln \frac{(1-\kappa_K)K}{(1-\kappa_L)L}$ , which is an infinitesimal, in (66).

Ninthly, the market clearing condition in modern sector implies the dynamics of normalized

capital, denoted by  $\frac{\dot{k}}{k}$ , follows

$$\begin{aligned} \frac{\dot{k}}{k} = (1-\mu)(1-\kappa_K)^\delta (1-\kappa_L)^{1-\delta} (\ln)^{\frac{1}{\varepsilon-\varphi_2}} k^{\delta-1} \\ - \frac{\frac{\varphi_2}{\Gamma^{\varepsilon-\varphi_2}} n^{\frac{\varepsilon}{\varepsilon-\varphi_2}} \iota}{k} \tilde{g} - \frac{\left( \frac{\alpha\beta n}{b_1 k} + 1 - \alpha \right) \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1 + 1)\alpha\beta - (1-\alpha)} \end{aligned} \quad (67)$$

Finally, the other two control variables  $X_1$  and  $X_2$  can be computed as  $X_1 = \frac{1}{b_1} \dot{N} N^{\varphi_1}$

and  $X_2 = \lambda^{\varphi_2} \int_0^t t^{(x)} dx$  with the initial value of technology in traditional sector at the starting point of the analysis, i.e., the exact value of  $N(t)$  at time  $t=0$ , denoted by  $N(0)$ , as well as  $A_2(0) = 1$  assumed above.

Summarizing the results above, we can characterize the dynamics of the present (decentralized) economy in the following proposition.

**Proposition 1:** The dynamic equilibrium of the present economy could be characterized by an autonomous system of nonlinear differential equations which contains eight variables,  $\delta$ ,  $\Gamma$ ,  $n$ ,  $\kappa_L$ ,  $p$ ,  $e$ ,  $\mu$ , and  $k$ , given by eight equations (58), (60), (61), (62), (64), (65), (66), and (67). Moreover, in infinite horizon, there exists a unique BGE in the present economy.

## 5. Optimal Industrial Structure, Viability and Economic

### Performance in LDCs

In this section, we will use the above basic model to explore the issues of development strategy, optimal industrial structure, viability, and economic performance in the LDCs. We consider a theoretical world consisting of a DC and a LDC that shares the identical demographics and all households in these countries have the same CRRA preferences.<sup>22</sup> What distinguishes the DC and the LDC exogenously is their factor endowment structures, i.e., the relative abundance of capital in the DC, denoted by  $\bar{K}$ , and the relative scarcity of capital in the LDC, denoted by  $\underline{K}$ , while (optimal) industrial structure in the DC and the LDC will be endogenously determined. In the present model, the ratio of the capital in the DC to that in the LDC, denoted by  $\bar{K}/\underline{K}$ , could be interpreted broadly, as a metaphor for the LDC's current stage of development. The larger  $\bar{K}/\underline{K}$  is, the more backward economy in this LDC, and  $\bar{K}/\underline{K}$  will decrease and approach unity eventually as the LDC converges to the DC. Moreover, the ratio of technologies in the DC to those used in the LDC denotes the distance to technology frontiers in the LDC, where  $\bar{N}/\underline{N}$  denotes the distance to technology frontier of the traditional sector, and  $\bar{j}/\underline{j}$  denotes the distance to technology frontier of the modern sector. When these terms are large, the LDC is far from the world technology frontier.

The facts that technologies adopted by the DC are in the new frontier imply that technological progress in the DC could be obtained only through R&D as described in the subsection 3.2 of the present paper,<sup>23</sup> thus, the industrial structure of a decentralized economy in the DC could be characterized by the autonomous system of nonlinear differential equations which contains endogenous variables  $\bar{\delta}$ ,  $\bar{I}$ ,  $\bar{n}$ ,  $\bar{\kappa}_L$ ,  $\bar{p}$ ,  $\bar{e}$ ,  $\bar{\mu}$ , and  $\bar{k}$  in the proposition 1.<sup>24</sup>

For the LDC, however, technological innovation may be the result of technology transfer or the imitation of existing technology held by DC. Thus, it is futile, when attempting to understand technology choices, (optimal) industrial upgrading and economic performance in the LDCs, to focus primarily on mechanisms that generate new technology in the DC. The technological gap between DC and LDC is filled with a whole spectrum of different technologies (Lin, 2003), providing the actual technologies used in the LDC lie inside the technology frontier of the DC's, and therefore the LDC is faced with the question of which technology in the spectrum is appropriate to adopt, i.e., the LDC is required to determine the (optimal) value of technology in the traditional sector as well as the (optimal) value of technology in the modern sector, which is denoted by  $\hat{N}$ ,  $\hat{\lambda}$ , and  $\hat{\delta}$  respectively.<sup>25</sup> On the other hand, facing the given technologies, the LDC is also confronted with the decision of what is the optimal fraction of factors, denoted by

<sup>22</sup> In the present paper, the upper bar is used as a superscript to indicate the variables in the DC, while the lower bar is an index of the LDC. To simplify the analysis, we also assume there are no international trade as well as no capital mobility in the present theoretical world.

<sup>23</sup> Over 90 percent of the R&D expenditure in the world is carried on in the OECD, and over 35 percent is in the United States (Acemoglu and Zilibotti, 2001).

<sup>24</sup> Please see appendix for endogenous industrial upgrading of a planned economy in the DC.

<sup>25</sup> We use a variable with a hat to denote its optimal value.

$\hat{\kappa}_L$  ( $\hat{\kappa}_K$ ), that can be used in the traditional sector.<sup>26</sup> Now it is time for us to provide the exact definition of the industrial structure in the present economy.

**Definition 1:** In the present paper, we summarize the precise technologies, denoted by  $N$ ,  $\lambda^j$ , and  $\delta_j$ , as well as the active fraction of labor used in the traditional sector, denoted by  $\kappa_L$ , as the industrial structure in a country.

And the optimal industrial structure in the LDC could be defined as:

**Definition 2:** The optimal industrial structure in the LDC, denoted by  $\hat{N}(t)$ ,  $\hat{j}(t)$ , and  $\hat{\kappa}_L(t)$ , are the paths for  $\underline{N}(t)$ ,  $\underline{j}(t)$ , and  $\underline{\kappa}_L(t)$  in the dynamic equilibrium which maximize the social welfare.<sup>27</sup>

In the following part of this section, building on the model developed in section 4, we begin with the dynamic trajectories of optimal industrial structure in the LDC firstly, and then we will investigate the issues of development strategy, viability, and economic performance in the LDC based on the LDC's deviating from the optimal industrial structure in this country or not. In consideration of the enforcement of Intellectual Property Rights (IPR) internationally would make an impact on optimal industrial structure in the LDC, it is convenient for us to investigate the optimal industrial structure in the LDC without IPR protection internationally firstly, then turn to the case in the LDC under IPR protection internationally.

### 5.1 The Optimal Industrial Structure in the LDC without IPR Protection Internationally

To start with, we assume IPR are not enforced internationally for simplicity, thus, both the DC and the LDC have access to the same set of technologies, and the LDC needs not to pay royalties to the DC. In consideration of the zero R&D expenditures in the LDC, free-entry condition and zero profit condition imply that we could assume the firms in the LDC are price takers in product markets as well as in factor markets.<sup>28</sup>

The analysis in subsection 3.2 implies that given the technologies in the LDC, the aggregate production function of the traditional goods in the LDC is  $\underline{Y}_1 = \underline{N}^{\frac{1-\alpha}{\alpha}} (\underline{L}_1)^\beta (\underline{K}_1)^{1-\beta}$  and the production function of the  $\underline{j}$  th generation goods of the modern sector in the LDC is

$\underline{Y}_2(\underline{j}) = \lambda^{\underline{j}} (\underline{K}_2)^{\delta_{\underline{j}}} (\underline{L}_2)^{1-\delta_{\underline{j}}}$ . The paths for optimal industrial structure in the LDC without IPR protection internationally are determined by the solutions of the following social welfare maximization problem.

$$\max_{\underline{N}(t), \underline{j}(t), \underline{\kappa}_L(t)} \underline{U}_\tau = \int_\tau^\infty \frac{\left[ \gamma \underline{C}_1^\varepsilon + (1-\gamma) (\underline{C}_2(\underline{j}))^\varepsilon \right]^{(1-\theta)/\varepsilon} - 1}{1-\theta} \exp[-\rho(t-\tau)] dt$$

where

<sup>26</sup> Given the technologies used in a country, the fraction of factors used in traditional sector will determine the output of the traditional sector as well as the output of the modern sector in this country.

<sup>27</sup> We take the utility of the representative household as our measure of social welfare.

<sup>28</sup> In this case, the dynamic equilibrium in the LDC also represents a Pareto-optimal outcome.

$$\underline{C}_1 = \underline{N}^{\frac{1-\alpha}{\alpha}} (\underline{\kappa}_L L)^\beta (\underline{\kappa}_K \underline{K})^{1-\beta}$$

and

$$\underline{C}_2(\underline{j}) + \dot{\underline{K}} = \lambda^{\underline{j}} [(1 - \underline{\kappa}_K) \underline{K}]^{\delta_{\underline{j}}} [(1 - \underline{\kappa}_L) L]^{1-\delta_{\underline{j}}}$$

with  $\underline{N}(t) \leq \bar{N}(t)$  and  $\underline{j}(t) \leq \bar{j}(t)$ .

Under the condition without IPR protection internationally, the assumptions of the economy of the LDC in the present paper also satisfy all assumptions of the First and Second Welfare Theorems, thus the set of competitive equilibrium allocations and the set of Pareto-optimal allocations coincide exactly, and the latter are more simple solutions to the appropriate constrained optimization problem. Without IPR protection internationally, the social welfare maximization problem of the representative household in the LDC, which is also a Pareto-optimal allocation problem can be conveniently decomposed into two component parts. One is a static problem, which determines the optimal allocation of capital and labor to the traditional sector and the modern sector as well as the optimal technologies used in these two sectors; and the other is a dynamic problem which determines the optimal capital accumulation, i.e., the saving rate, in the course of economic development.

In the static allocation problem, given the production function in traditional sector which is  $\underline{Y}_1 = \underline{N}^{\frac{1-\alpha}{\alpha}} (\underline{L}_1)^\beta (\underline{K}_1)^{1-\beta}$  and the production function in the modern sector which is  $\underline{Y}_2(\underline{j}) = \lambda^{\underline{j}} (\underline{K}_2)^{\delta_{\underline{j}}} (\underline{L}_2)^{1-\delta_{\underline{j}}}$  as well as the given price of the modern product, denoted by  $\underline{p}_j$ , the optimal employment of factors in the LDC implies that

$$\underline{K}_1 = \frac{\left[ \frac{(1-\beta)(1-\delta_{\underline{j}})}{\beta\delta_{\underline{j}}} \right]^{\frac{\delta_{\underline{j}}}{\delta_{\underline{j}}-1+\beta}} \left[ \frac{\beta \underline{N}^{\frac{1-\alpha}{\alpha}}}{(1-\delta_{\underline{j}}) \underline{p}_j \lambda^{\underline{j}}} \right]^{\frac{1}{\delta_{\underline{j}}-1+\beta}} \beta \delta_{\underline{j}} L - (1-\beta)(1-\delta_{\underline{j}}) \underline{K}}{\delta_{\underline{j}} - (1-\beta)} \quad (68)$$

and

$$\underline{L}_1 = \frac{L \delta_{\underline{j}} \beta \underline{K}_1}{(1-\beta)(1-\delta_{\underline{j}})(\underline{K} - \underline{K}_1) + \delta_{\underline{j}} \beta \underline{K}_1} \quad (69)$$

From (68) and (69), we could solve out the optimal fraction of labor, denoted by  $\hat{\underline{\kappa}}_L$ , that can be used in the traditional sector as the function of  $\underline{K}(t)$ ,  $\underline{p}_j$ ,  $\underline{N}(t)$ , and  $\underline{j}(t)$ .

It is well known that the gross domestic product (GDP) in the LDC, denoted by  $\underline{Y}_1 + \underline{p}_j \underline{Y}_2(\underline{j})$ , could be written as

$$\underline{Y}_1 + \underline{p}_j \underline{Y}_2(\underline{j}) = \underline{N}^{\frac{1-\alpha}{\alpha}} (\underline{L}_1)^\beta (\underline{K}_1)^{1-\beta} + \underline{p}_j \lambda^{\underline{j}} (\underline{K} - \underline{K}_1)^{\delta_{\underline{j}}} (L - \underline{L}_1)^{1-\delta_{\underline{j}}} \quad (70)$$

Substituting (68) and (69) as well as  $\underline{p}_j = \frac{(1-\gamma)\left[\underline{\mu}Y_2(j)\right]^{\varepsilon-1}}{\gamma Y_1^{\varepsilon-1}}$  into (70) implies that the maximand for the GDP in the LDC are the function of  $\underline{K}$ ,  $\underline{N}(t)$ ,  $\underline{j}(t)$ , and  $\underline{\kappa}_L(t)$  for the given  $\underline{\mu}$ . From the FOCs in (70), we could solve out the optimal value of technology in the traditional sector as well as the optimal value of technology in the modern sector, denoted by  $\hat{N}$  and  $\hat{j}$ , as the function of  $\underline{K}$  and  $\underline{\mu}$ .

The dynamic optimum implies that the above static allocation must be efficient at all times. Substituting  $\hat{\kappa}_L$ ,  $\hat{N}$  and  $\hat{j}$ , all of which are functions of  $\underline{K}$  and  $\underline{\mu}$ , into social welfare function in (1), we can reformulate the dynamic allocation problem as the following social welfare maximization problem.

$$\max_{\underline{\mu}} \underline{U}_\tau = \int_\tau^\infty \frac{\left[ \gamma \underline{C}_1^\varepsilon + (1-\gamma) \left( \underline{C}_2(j) \right)^\varepsilon \right]^{(1-\theta)/\varepsilon} - 1}{1-\theta} \exp[-\rho(t-\tau)] dt$$

where

$$\begin{aligned} \underline{C}_1 &= \hat{N}^{\frac{1-\alpha}{\alpha}} (\hat{\kappa}_L L)^\beta (\hat{\kappa}_K \underline{K})^{1-\beta} \\ \underline{C}_2 &= \underline{\mu} \lambda^j \left[ (1-\hat{\kappa}_K) \underline{K} \right]^{\delta_j} \left[ (1-\hat{\kappa}_L) L \right]^{1-\delta_j} \end{aligned}$$

subject to the resource constraint

$$\dot{\underline{K}} = (1-\underline{\mu}) \lambda^j \left[ (1-\hat{\kappa}_K) \underline{K} \right]^{\delta_j} \left[ (1-\hat{\kappa}_L) L \right]^{1-\delta_j}$$

as well as  $\hat{N}(t) \leq \bar{N}(t)$  and  $\hat{j}(t) \leq \bar{j}(t)$ .

This is a familiar problem of optimal control, and we could use Pontryagin's minimum principle to obtain the optimal trajectories for the industrial structure in the LDC, denoted by  $\hat{N}$ ,  $\hat{j}$ , and  $\hat{\kappa}_L$ , from the necessary and sufficient conditions for the above optimal program. It is obvious that the paths of optimal industrial structure in the LDC without IPR protection internationally would depend on the capital  $\underline{K}$  in this country. Moreover, when the capital scarcity in the LDC is severe enough compared that in the DC, i.e.,  $\underline{K}(t) \ll \bar{K}(t)$ , from (68) and (69), we must have  $\hat{j}(x) < \bar{j}(x)$  and  $\delta_{\hat{j}}(x) < \delta_{\bar{j}}(x)$  for all  $x < t$ .

The above dynamic equilibrium in the LDC which also represents a Pareto-optimal outcome implies that the interest rate in the LDC reaches its maximum value at any moment. The Euler equation in (8) implies that the economic growth rate in the LDC is positively correlated with the

interest rate in this country, and interest rate is a decreasing function of the capital in the LDC. Thus, the LDC which follows the optimal industrial structure that maximize social welfare would achieve a high rate of growth, i.e., growth miracle, at the preliminary development stage. Moreover, because the LDC and the DC share the identical BGE in the long-run, thus, without external intervention, the convergence of the economy in the LDC to that in the DC would come true ultimately. Summarizing the analysis above, we could establish the following result.

**Proposition 2:** (1). Under the circumstance that IPR not enforced internationally, the optimal industrial structure in the LDC, denoted by  $\hat{N}$ ,  $\lambda^j$ ,  $\delta_j$ , and  $\hat{\kappa}_L$ , are endogenously determined by the factor endowment structure  $\underline{K}(t)$  in this LDC; (2). When the capital scarcity in the LDC is severe enough compared that in the DC, i.e.,  $\underline{K}(t) \ll \bar{K}(t)$ , we must have  $\hat{j}(x) < \bar{j}(x)$  and  $\delta_j(x) < \bar{\delta}_j(x)$  for all  $x < t$ , which means, at the preliminary stage, the LDC should adopt technologies that are inside the technology frontier of the DC; (3). Before a LDC catches up with the DC, as long as the LDC follows the optimal industrial structure characterized above, the LDC could always experience the most rapid economic growth. Moreover, at its preliminary development stage, the economic growth rate in the LDC might be greater than that in the DC; (4). Without external intervention, the convergence of the economy in the LDC to that in the DC would come true ultimately.

## 5.2 The Optimal Industrial Structure of a Decentralized Economy in the LDC under IPR Protection Internationally

The assumption that that IPR not enforced internationally is, of course, unrealistic. Now we turn to the case with IPR enforced internationally. In this case, the LDC could not have free access to the frontier technology in the DC, new technologies in the LDC are developed as the result of the LDC's own R&D, imitation, technology imports from the DC or some other forms, and the R&D expenditures are indispensable in the LDC no matter of what forms are.<sup>29</sup> Thus, at time  $\tau$ , the LDC whose capital stock equals  $\underline{K}(\tau)$  still faces which is the optimal industrial structure she should follow, i.e., the LDC needs to choose  $\hat{N}(\tau)$ ,  $\hat{j}(\tau)$ , and  $\hat{\kappa}_L(\tau)$ .<sup>30</sup> We denote the (endogenous) industrial structure in the DC whose capital stock equals  $\bar{K}(\tau)$  at time  $\tau$  to be  $\bar{N}(\tau)$ ,  $\bar{j}(\tau)$ , and  $\bar{\kappa}_L(\tau)$ , which are characterized by the autonomous system of differential equations in proposition 1 with initial values of technology in traditional sector at time  $t = 0$ , denoted by  $\bar{N}(0)$ , and the initial value of technology in modern sector at time  $t = 0$ , which is  $j(0) = 1$  as assumed previously.

<sup>29</sup> We sum up all the expenditures on technical progress in the LDC as R&D expenditures for simplicity.

<sup>30</sup> From the analysis below, we know that once  $\hat{N}(\tau)$ ,  $\hat{j}(\tau)$ , and  $\hat{\kappa}_L(\tau)$  are determined at time  $\tau$ , the optimal industrial structure in the LDC would be characterized by an autonomous system of nonlinear differential equations resembles to that in proposition 1.

Before investigating the issues of optimal industrial structure in the LDC under IPR internationally, we need to specify the R&D expenditure equation in the LDC explicitly. As regards the R&D expenditures in the traditional sector in the LDC, we assume that if  $\underline{X}_1$  units of modern goods engage in research in this sector, they would generate a flow of new products in the LDC given by

$$\dot{\underline{N}} = b_1 \left( \bar{N}/\underline{N} \right)^{-\phi_1} \underline{N}^{-\phi_1} \underline{X}_1$$

where  $\phi_1$  is a exogenously given positive parameter, and parameters  $b_1$  and  $\phi_1$  are the same as those in the DC. And we also assume that research in modern sector in the LDC produces a random sequence of product innovations. Any firm in the modern sector in the LDC that carries out R&D at intensity  $\underline{l}$  for a time interval of length  $dt$  will succeed in its attempt to develop the product of generation  $\underline{j} = 1, 2, \dots$  based on the existing generation before it with probability  $\underline{l}dt$  which follows a Poisson distribution, and R&D expenditures per unit of time in this activity are  $\underline{X}_2 \equiv \left( \lambda^{\bar{j}}/\lambda^{\underline{j}} \right)^{\phi_2} (\lambda^{\underline{j}})^{\phi_2} \underline{l}$  in terms of the modern goods when the entrepreneur attempts to develop that product, where  $\phi_2$  is a exogenously given negative parameter, and the parameter  $\phi_2$  is the same as that in the DC. The properties of the Poisson distribution imply that

$$\underline{X}_2 = \left( \lambda^{\int_0^t \bar{l}(x)dx} / \lambda^{\underline{j}(\tau) + \int_{\tau}^t \underline{l}(x)dx} \right)^{\phi_2} \lambda^{\phi_2 [\underline{j}(\tau) + \int_{\tau}^t \underline{l}(x)dx]} \underline{l} \text{ at time } t, \text{ where } \underline{j}(\tau) \text{ denotes the (initial)}$$

technology of the modern sector that could (should) be chosen by the LDC at time  $\tau$ .

The R&D expenditure equations above in the LDC reflect the following stylized facts: when  $\bar{N}/\underline{N}$  and  $\bar{j}/\underline{j}$  are large, the LDC is far from the world technology frontier, the LDC will have more opportunities than the DC in upgrading its technologies, thus, the cost for the LDC to upgrade its technologies can be lower than that in the DC; Moreover, as the technologies in the LDC approach the technology frontiers in the DC, i.e.,  $\bar{N}/\underline{N}$  and  $\bar{j}/\underline{j}$  become close to unity, the R&D expenditures in the LDC will increase and will be the same as those in the DC ultimately.

The presence of intertemporal-spillover effect implies that the trajectory of (endogenous) industrial upgrading in a decentralized economy may not coincide with that in a planned economy. Thus, it is necessary for us to draw a distinction between the optimal industrial structure of a decentralized economy and the counterpart of a planned economy in the LDC. In the main text, we only explore the optimal industrial structure of a decentralized economy in the LDC, while leave the case of a planned economy in the appendix.

Similarly, we could characterize the dynamic trajectories of the decentralized economy in the



LDC by an autonomous system of nonlinear differential equations which contains three control variables,  $\underline{e} \equiv \frac{\underline{E}}{LN^{\alpha\beta}}$ ,  $\underline{X}_1$ , and  $\underline{X}_2$  as well as seven state variables,  $\underline{\delta}$ ,  $\underline{\kappa}_L$ ,  $\underline{p}$ ,  $\underline{\mu}$ ,

$$\underline{k} \equiv \frac{\underline{K}}{LN^{\alpha\beta}}, \quad \underline{n} \equiv \frac{N^{\frac{(\varphi_1+1)\alpha\beta-(1-\alpha)}{\alpha\beta}}}{L}, \quad \text{and} \quad \underline{I} \equiv \frac{\lambda^{(\varepsilon-\varphi_2)} \int_{\tau}^t \underline{l}(x) dx}{N^{\frac{(1-\alpha)(\varepsilon-\underline{\delta}\varepsilon-1)}{\alpha\beta}}}.$$

Firstly, the equilibrium interest rate in the LDC, denoted by  $\underline{r}(t)$ , in the dynamic trajectory is given by:

$$\underline{r}(t) = \alpha(1-\beta) \left[ \frac{\underline{\xi} + (1-\underline{\xi})\underline{\kappa}_L}{\underline{k}} \right]^\beta$$

where  $\underline{\xi} \equiv \frac{\delta\beta}{(1-\delta)(1-\beta)} > 1$ .

Secondly, the dynamics of capital intensity of the modern sector in the LDC, denoted by  $\dot{\underline{\delta}}(t)$ , could be attained by:

$$\dot{\underline{\delta}}(t) = b_2 [\tilde{\delta} - \underline{\delta}(t)]^{\varphi_3} \underline{l}(t) \quad (71)$$

Thirdly, the dynamics of normalized accumulative R&D intensity in the LDC, denoted by  $\underline{I}$ , should satisfy

$$\begin{aligned} \underline{l} = & \frac{\underline{r} - \phi_2 \ln \lambda \bar{t}}{[(\varphi_2 - \phi_2) \ln \lambda - 1]} - \frac{\dot{\underline{p}}}{\underline{p}[(\varphi_2 - \phi_2) \ln \lambda - 1]} \\ & \frac{(1-\varepsilon)(1-\gamma)\underline{\mu}^{\varepsilon-1}(1-\underline{\kappa}_K)^{\delta\varepsilon}(1-\underline{\kappa}_L)^{(1-\delta)\varepsilon} \lambda^{(\varepsilon-\varphi_2)\underline{j}(\tau)} \underline{I} \underline{k}^{\delta\varepsilon-(1-\beta)(\varepsilon-1)}}{\gamma \underline{p} (\underline{\kappa}_L)^{\beta(\varepsilon-1)} (\underline{\kappa}_K)^{(1-\beta)(\varepsilon-1)} \left( \lambda^{\bar{\psi}(t)} / \lambda^{\underline{j}(\tau)+\underline{\psi}(t)} \right)^{\phi_2} [(\varphi_2 - \phi_2) \ln \lambda - 1]} \end{aligned} \quad (72)$$

where  $\underline{\kappa}_K = \frac{\underline{\kappa}_L}{\underline{\kappa}_L + \underline{\xi}(1-\underline{\kappa}_L)}$  and  $\underline{\psi}(t) = \int_{\tau}^t \underline{l}(x) dx$ .

Fourthly, the evolution of the normalized technology in traditional sector in the LDC, denoted by  $\frac{\dot{\underline{n}}}{\underline{n}}$ , requires

$$\begin{aligned} \underline{r}(t) - b_1 \frac{(1-\alpha)(\underline{\kappa}_L)^\beta (\underline{\kappa}_K)^{1-\beta} \underline{k}^{1-\beta}}{\underline{p} (\bar{N}/N)^\phi} \frac{\underline{k}^{1-\beta}}{\underline{n}} = \\ \frac{\dot{\underline{p}}}{\underline{p}} + \frac{\phi_1 \alpha \beta \left( \frac{\dot{\underline{n}}}{\underline{n}} - \frac{\dot{\underline{n}}}{\underline{n}} \right)}{(\varphi_1+1)\alpha\beta - (1-\alpha)} + \frac{\phi_1 \alpha \beta \left( \frac{\dot{\underline{n}}}{\underline{n}} + \tilde{g} \right)}{(\varphi_1+1)\alpha\beta - (1-\alpha)} \end{aligned} \quad (73)$$

Fifthly, we could derive the fraction of labor used in traditional sector in the LDC, denoted by  $\underline{\kappa}_L$ , as

$$\frac{1}{\underline{\kappa}_L} = 1 + \frac{\left(\frac{1-\gamma}{\gamma\varepsilon^{-1}}\right)^{\frac{1}{1-\varepsilon}} (1-\underline{\delta})^{\frac{1-\varepsilon\underline{\delta}}{1-\varepsilon}} \underline{\delta}^{\frac{\varepsilon\underline{\delta}}{1-\varepsilon}} \left(\frac{\beta}{1-\beta}\right)^{\frac{\varepsilon\underline{\delta} + \alpha(\beta-1)}{1-\varepsilon + \frac{\alpha(\beta-1)}{1-\alpha}}} \underline{k}^{\frac{\varepsilon(\underline{\delta}-1+\beta)}{1-\varepsilon}} (n\underline{I})^{\frac{\varepsilon}{(\varepsilon-\varphi_2)(1-\varepsilon)}}}{\lambda^{\frac{\varepsilon j(\tau)}{\varepsilon-1}} \underline{\mu} \left[ (1-\beta)^{\alpha(1-\beta)} \beta^{(1-\alpha+\alpha\beta)} \alpha \right]^{\frac{1}{1-\alpha}} (\alpha\beta)^{\frac{\varepsilon-\alpha}{(1-\varepsilon)(1-\alpha)}} \left[ \xi + (1-\xi)\underline{\kappa}_L \right]^{\frac{\varepsilon(\underline{\delta}-1+\beta)}{1-\varepsilon}}} \quad (74)$$

Sixthly, the law of price change in the LDC, denoted by  $\frac{\dot{p}}{p}$ , is determined by

$$\frac{\dot{p}}{p} = (\varepsilon-1) \left[ \begin{array}{l} \frac{\dot{\underline{\mu}}}{\underline{\mu}} + \underline{\iota} \ln \lambda + (\delta + \beta - 1) \frac{\dot{\underline{k}}}{\underline{k}} + \frac{(1-\alpha)(\delta-1) \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1+1)\alpha\beta - (1-\alpha)} \\ -\delta \frac{\dot{\underline{\kappa}}_K}{1-\underline{\kappa}_K} - (1-\delta) \frac{\dot{\underline{\kappa}}_L}{1-\underline{\kappa}_L} - \beta \frac{\dot{\underline{\kappa}}_L}{\underline{\kappa}_L} - (1-\beta) \frac{\dot{\underline{\kappa}}_K}{\underline{\kappa}_K} \end{array} \right] \quad (75)$$

where  $\dot{\underline{\delta}} \ln \left( \frac{(1-\underline{\kappa}_K)\underline{K}}{(1-\underline{\kappa}_L)\underline{L}} \right)$  is an infinitesimal and neglected in (75).

Seventhly, the Euler equation of the representative agent requires the optimal path for the normalized consumption expenditure in the LDC, denoted by  $\frac{\dot{e}}{e}$ , must satisfy

$$\frac{\dot{e}}{e} = \frac{\underline{r} - \rho}{\theta} - \tilde{g} - \frac{(1-\alpha) \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1+1)\alpha\beta - (1-\alpha)} \quad (76)$$

Eighthly, the dynamics of the fraction of modern goods consumed by households in the LDC, denoted by  $\frac{\dot{\underline{\mu}}}{\underline{\mu}}$ , could be determined by

$$\begin{aligned} \frac{\underline{r} - \rho}{\theta} = \underline{s}_1 & \left[ \frac{2(1-\alpha)\beta \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1+1)\alpha\beta - (1-\alpha)} + \beta \left( \frac{\dot{\underline{k}}}{\underline{k}} + \frac{\dot{\underline{\kappa}}_K}{\underline{\kappa}_K} \right) + \tilde{g} + \frac{(1-\beta)\dot{\underline{\kappa}}_L}{\underline{\kappa}_L} \right] + (1-\underline{s}_1) \frac{\dot{p}}{p} \\ + (1-\underline{s}_1) & \left[ \frac{\dot{\underline{\mu}}}{\underline{\mu}} + \underline{\iota} \ln \lambda + \frac{\delta \dot{\underline{k}}}{\underline{k}} + \frac{\underline{\delta}(1-\alpha) \left( \frac{\dot{n}}{n} + \tilde{g} \right)}{(\varphi_1+1)\alpha\beta - (1-\alpha)} - \frac{\delta \dot{\underline{\kappa}}_K}{1-\underline{\kappa}_K} + \tilde{g} - \frac{(1-\delta)\dot{\underline{\kappa}}_L}{1-\underline{\kappa}_L} \right] \end{aligned} \quad (77)$$

where  $\underline{s}_1 = \frac{(\underline{\kappa}_L)^\beta (\underline{\kappa}_K)^{1-\beta}}{\underline{e}} \underline{k}^{1-\beta}$  and again we neglect  $b_2[\tilde{\delta} - \underline{\delta}]^{\phi_3} \underline{l} \ln \frac{(1 - \underline{\kappa}_K)\underline{K}}{(1 - \underline{\kappa}_L)\underline{L}}$  in (77),

which is an infinitesimal.

Ninthly, the market clearing condition in the modern sector in the LDC implies the dynamics of normalized capital in the LDC, denoted by  $\frac{\dot{\underline{k}}}{\underline{k}}$ , should follow

$$\begin{aligned} \frac{\dot{\underline{k}}}{\underline{k}} = & (1 - \underline{\mu}) \lambda^{\underline{j}(\tau)} (1 - \underline{\kappa}_K)^{\underline{\delta}} (1 - \underline{\kappa}_L)^{1 - \underline{\delta}} (\underline{In})^{\frac{1}{\varepsilon - \phi_2}} \underline{k}^{\underline{\delta} - 1} - \\ & \left( \frac{\lambda^{\underline{\psi}(\tau)}}{\lambda^{\underline{j}(\tau) + \underline{\psi}(\tau)}} \right)^{\phi_2} \frac{\underline{l}^{\frac{\phi_2}{\varepsilon - \phi_2}} \underline{n}^{\frac{\varepsilon}{\varepsilon - \phi_2}} \underline{l}}{\underline{k}} - \tilde{g} - \frac{\left( \frac{\alpha \beta \underline{n}}{b_1 \underline{k}} (\bar{N}/\underline{N})^{\phi_1} + 1 - \alpha \right) \left( \frac{\dot{\underline{n}}}{\underline{n}} + \tilde{g} \right)}{(\phi_1 + 1)\alpha\beta - (1 - \alpha)} \end{aligned} \quad (78)$$

Finally, the other two control variables  $\underline{X}_1$  and  $\underline{X}_2$  in the LDC can be calculated as

$$\dot{\underline{N}} = b_1 (\bar{N}/\underline{N})^{-\phi_1} \underline{N}^{-\phi_1} \underline{X}_1 \quad \text{and} \quad \underline{X}_2 = \left( \lambda^{\int_0^\tau \bar{r}(x) dx} / \lambda^{\underline{j}(\tau) + \int_\tau^t \underline{l}(x) dx} \right)^{\phi_2} \lambda^{\phi_2 \left[ \underline{j}(\tau) + \int_\tau^t \underline{l}(x) dx \right]} \underline{l} \quad \text{with the}$$

initial condition of technology in the traditional sector in the LDC at time  $\tau$ , denoted by  $\underline{N}(\tau)$

and  $\underline{j}(\tau)$ , where  $\underline{N}(\tau)$  and  $\underline{j}(\tau)$  could (should) be chosen by the LDC at time  $\tau$ .

From the results above, we could establish the following proposition.

**Proposition 3:** The dynamic equilibrium of a decentralized economy in the LDC under IPR protection internationally could be characterized by an autonomous system of nonlinear differential equations which contains eight variables,  $\underline{\delta}$ ,  $\underline{l}$ ,  $\underline{n}$ ,  $\underline{\kappa}_L$ ,  $\underline{p}$ ,  $\underline{e}$ ,  $\underline{\mu}$ , and  $\underline{k}$ , determined by eight equations (71), (72), (73), (74), (75), (76), (77), and (78). Moreover, in the present (decentralized) economy of the LDC, there also exists a unique BGE in infinite horizon, which is the same as that in the DC.

Since the dynamic equilibrium of a decentralized economy in the LDC under IPR protection internationally has been solved out in proposition 3, now we could reformulate the optimal industrial structure in the LDC as the following social welfare maximization problem at time  $\tau$ :

$$\max_{\underline{N}(\tau), \underline{j}(\tau), \underline{\kappa}_L(\tau)} \underline{U}_{-\tau} = \int_{\tau}^{\infty} \frac{\left[ \gamma \underline{C}_1^\varepsilon + (1 - \gamma) \left( \underline{C}_2(\underline{j}) \right)^\varepsilon \right]^{(1 - \theta)/\varepsilon} - 1}{1 - \theta} \exp[-\rho(t - \tau)] dt$$

subject to the autonomous system of nonlinear differential equations in proposition 3, and

$$\underline{N}(\tau) \leq \bar{N}(\tau) \quad \text{as well as} \quad \underline{j}(\tau) \leq \bar{j}(\tau).$$

Once  $\hat{\underline{N}}(\tau)$ ,  $\hat{\underline{j}}(\tau)$ , and  $\hat{\underline{\kappa}}_L(\tau)$  have been determined, the optimal industrial structure in the LDC after time  $\tau$ , i.e.,  $\hat{\underline{N}}(t)$ ,  $\hat{\underline{j}}(t)$ , and  $\hat{\underline{\kappa}}_L(t)$  at time  $t > \tau$ , could also be solved out

by proposition 3. Thus, we could establish the following result by the same logic as that in proposition 2.

**Proposition 4:** (1). Under the circumstance that IPR protection internationally, the optimal industrial structure in the LDC, denoted by  $\hat{N}(t)$ ,  $\hat{\psi}(t) + \hat{j}(\tau)$ ,  $\hat{\delta}(t)$ , and  $\hat{k}_L(t)$ , are endogenously determined by the factor endowment structure  $\underline{K}(t)$  in this LDC; (2). When the capital scarcity in the LDC is severe enough compared that in the DC, i.e.,  $\underline{K}(t) \ll \bar{K}(t)$ , we must have  $\hat{N}(x) < \bar{N}(x)$ ,  $\hat{\psi}(x) + \hat{j}(\tau) < \bar{\psi}(x)$ , and  $\hat{\delta}(x) < \bar{\delta}(x)$  for all  $x < t$ , which means, at the preliminary stage, the LDC should adopt technologies that are inside the technology frontier of the DC; (3). Before a LDC catches up with the DC, as long as the LDC follows the optimal industrial structure characterized above, the LDC could always experience the most rapid economic growth. Moreover, at its preliminary development stage, the economic growth rate in the LDC might be greater than that in the DC; (4). Without external intervention, the convergence of the economy in the LDC to that in the DC would come true ultimately, provided the LDC follows the optimal industrial structure characterized above.<sup>31</sup>

Summarizing the results in proposition 2 and proposition 4 implies that the optimal (appropriate) industry of the modern sector in the LDC at its primary development stage, should not be the most advanced and capital intensive industry that adopts the frontier technologies in the DC, even there is absence of any barriers for the technology transfer from DC to LDC, because LDC takes time to accumulate capital and upgrade its factor endowment structures to achieve the level in the DC.<sup>32</sup>

### 5.3 Development Strategy, Viability and Economic Performance in the LDC

The government is the most important institution in any economy, especially in LDCs, whose economic policies could shape the macro incentive structure that firms in the economy face. The government's economic policies toward industrial development could be grouped into two different and mutually exclusive development strategies: the CAD strategy and the CAF strategy.<sup>33</sup> Based on whether the actual (or intentioned) industrial structure in the LDC coincides with the optimal one characterized in subsection 5.1 or in subsection 5.2 or not, in the present paper, we could define CAD strategy and CAF strategy rigorously as follows.

**Definition 3:** The development strategy pursued by the government in the LDC whose actual

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<sup>31</sup> It is obvious that, when  $\underline{k} = \bar{k}$ ,  $\underline{n} = \bar{n}$ , and  $\underline{I} = \bar{I}$ , the autonomous system of nonlinear differential equations in proposition 3, which characterizes the endogenous evolutions of industrial structure of a decentralized economy in the LDC, reduces to that in proposition 1 which characterizes the endogenous evolutions of industrial structure of a decentralized economy in the DC.

<sup>32</sup> The intuition for the result stems from the feature of our model that each technology in the modern sector, denoted by  $\lambda^j$  is appropriate for one and only one capital-labor ratio, denoted by  $\delta_j$ , thus, the technologies in the modern sector are specific to particular factor endowment structure. As pointed out above, the model developed by Basu and Weil (1998) has the same feature as that in our paper; however, there is never a problem of countries using technologies that do not match their level of development in Basu and Weil (1998).

<sup>33</sup> Please refer to Lin (2003), Lin, et al. (2003), as well as Lin and Zhang (2007) for the details of development strategies in the LDCs.

(or intended) industrial structure coincides with the optimal industrial structure in this LDC is denoted to be CAF strategy; while the development strategy pursued by the government in the LDC whose actual (or intended) industrial structure deviates from the optimal one in this country is entitled by CAD strategy.

Now we could provide a new line of thoughts to analyze the root cause of the differences of economic performance in the LDCs, which emphasize the fundamental role of match/mismatch of the actual (or intended) industrial structure with the factor endowment structures, therefore determines the optimal industrial structure of an economy, in determining the economic performance in LDCs.

From the proposition 2 in subsection 5.1, it is obvious that the GDP in the LDC at time  $t$ , denoted by  $\underline{Y}(t) + \underline{p}_j(t)\underline{Y}_2(\underline{j}, t)$  would reach its maximal value when actual industrial structure chosen by the LDC, denoted by denoted by  $\underline{N}(t)$ ,  $\lambda^{\underline{j}(t)}$ ,  $\underline{\delta}_j(t)$ , and  $\underline{\kappa}_L(t)$ , coincides with the optimal industrial structure, denoted by  $\hat{N}(t)$ ,  $\lambda^{\hat{j}(t)}$ ,  $\hat{\delta}_j(t)$ , and  $\hat{\kappa}_L(t)$ , in this LDC. This result is summarized in the following corollary.

**Corollary 1:** When IPR are not enforced internationally, if a LDC wants to close the gap of output per labor with that in DC as well as to accelerate economic growth in this LDC, she has no choice but pursue CAF strategy and follows the optimal paths of industrial upgrading characterized in subsection 5.1.

As pointed out previously in the present paper, the assumption that IPR not enforced internationally is unrealistic. Thus, it is necessary to discuss development strategy, viability and economic performance of a decentralized economy in the LDC under IPR protection internationally. The results in proposition 4 imply that the actual industrial structure in the LDC whose capital stock equals to  $\underline{K}(\tau)$  that pursues CAF strategy under IPR protection internationally should coincide with the optimal industrial structure in this country and satisfy  $\hat{N}(\tau) < \bar{N}(\tau)$ ,  $\hat{j}(\tau) < \bar{j}(\tau)$ , and  $\hat{\delta}(\tau) < \bar{\delta}(\tau)$  when  $\underline{K}(\tau) \ll \bar{K}(\tau)$  at time  $\tau$ . However, motivated by the dream of national building, most the LDC governments, pursued Catch-up type CAD strategy to accelerate the development of the then advanced capital-intensive industries after World War II, thus the actual industrial structure deviated from the optimal one in these LDCs. When a LDC government pursues Catch-up type CAD strategy, in point of the viability of the firm in this government's priority industries, we could obtain the following proposition (Please see appendix for the proof).

**Proposition 5:** When a LDC government pursues Catch-up type CAD strategy, the firm in this LDC that enters the capital-intensive, advanced industry in the DCs would be nonviable owing to the relative scarcity of capital in the LDCs' factor endowments.

Therefore, it is imperative for the government to introduce a series of regulations and interventions to mobilize resources for setting up and supporting the continuous operation of the non-viable firms and the economy of this type becomes very inefficient as the result of misallocation of resources, rampant rent seeking, macro instability, and so forth.

Finally, based on the analysis in section 5, we could sum up the main result of this paper in

the following theorem.

**Theorem 1:** Whether the industrial structure matches with the factor endowment structure or not is the fundamental cause to explain differences in economic performance among the LDCs. And the most important task for the government in the LDC wishing to improve economic performance is to get its development strategy right.

## 6. Concluding Remarks

In the present paper, we have developed an endogenous growth model that combines structural change with repeated product improvements. The distinctive characteristic of our model originates from the technology in the modern sector, which becomes not only increasingly capital-intensive, but also progressively productive over time as the result of innovation by the profit-seeking firms. Owing to each technology in the modern sector is appropriate for one and only one capital-labor ratio, i.e., the technologies in the modern sector are specific to particular factor endowment structure, we could draw the conclusion that a LDC's optimal industrial structure is endogenously determined by that economy's endowment structure. Based on whether the actual (or intentioned) industrial structure in the LDC coincides with the optimal one which is endogenously determined by the factor endowment structure in this economy, the government's economic policies toward industrial development in the LDC could be divided into two different and mutually exclusive development strategies: the CAD strategy and CAF strategy.

If the government in the LDCs, e.g. the newly industrialized economies in Asia, and recently China, pursues the CAF strategy, the economy will enjoy rapid growth and the economic growth rate in these LDCs could be greater than that in the DCs, thus, the convergence of these LDCs with DCs could come true ultimately. On the contrary, most the LDC governments, both socialist and non-socialist alike, pursued Catch-up type CAD strategy to accelerate the development of the then advanced capital-intensive industries after World War II. However, the firms in the government's priority industries are not viable as shown in the present paper, it is imperative for the government to introduce a series of regulations and interventions in the international trade, financial sector, labor market, and so on so as to mobilize resources for setting up and supporting the continuous operation of the non-viable firms.<sup>34</sup> This kind of development mode might be good at mobilizing the scarce resources and concentrating on a few clear, well-defined priority sector (Ericson, 1991), but the economy of this type becomes very inefficient as the result of misallocation of resources, rampant rent seeking, macro instability, and so forth (Lin, 2003).

Building on the framework developed in the present paper, we have provided a new unified line of thoughts to analyze the root cause of the differences of economic performance in the LDCs, which argue that whether the industrial structure matches with the factor endowment structure or not is the fundamental cause to explain diversity in economic performance among the LDCs. We believe the framework developed here is powerful in explaining economic phenomena in LDCs. Moreover, it might be useful to generalize and extend the analysis in the present paper, e.g., to include financial institutions and investigate the optimal financial structure in LDCs.<sup>35</sup>

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<sup>34</sup> Lin and Zhang (2007) has constructed a model to explore the intrinsic logic of government intervention policies in less developed countries (LDCs) and argued that many interventionist and distorted institutional arrangements in LDCs after the World War II can be largely explained by their governments' adoption of Catch-up type CAD strategy.

<sup>35</sup> Please refer to Lin, et al. (2006) for the detailed analysis of optimal financial structures in the LDCs.

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## Appendix

### A.1 Nondrastic Innovations

Until this point we have assumed that innovations are drastic. Now we turn to nondrastic innovations. As pointed out by Aghion and Howitt (1992), innovations are nondrastic if and only if the previous incumbent (follower) could make a positive profit when the current one (leader)

was charging the price  $p_j = \frac{(1-\gamma)(\mu_j Y_2(j))^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}}$  which yields an unconstrained maximum to

the current incumbent's profit. The follower could not make a positive profit if and only if the leader-firm charging a price that falls epsilon below the average cost of this nearest competitor/follower, i.e., if and only if the condition

$$p_j \leq \lambda^{-(j-1)} (\delta_{j-1})^{-\delta_{j-1}} (1-\delta_{j-1})^{-(1-\delta_{j-1})} (r_j)^{\delta_{j-1}} (w_j)^{1-\delta_{j-1}} \quad (79)$$

were to hold with strict inequality.

Comparing the RHS in (79) with that in (38) implies that innovations are nondrastic if and only if

$$\lambda \varepsilon (\delta_j)^{\delta_j} (\delta_{j-1})^{-\delta_{j-1}} (1-\delta_j)^{(1-\delta_j)} (1-\delta_{j-1})^{-(1-\delta_{j-1})} (r_j)^{\delta_{j-1}-\delta_j} (w_j)^{\delta_j-\delta_{j-1}} < 1$$

From the analysis in section 4, it is obvious that we have  $\lim_{j \rightarrow \infty} \delta_j = \tilde{\delta}$ , thus, innovations are drastic in infinite horizon equilibrium if and only if  $\lambda \varepsilon \geq 1$ .

Like the case of drastic innovation, in equilibrium of nondrastic innovations, all the capital and labor employed in modern sector are combined with the leader-firm. Facing the given inverse demand function in (38), given factor prices  $w_j$  and  $r_j$ , the leader-firm  $j$  in the modern sector will choose  $K_2$  and  $L_2$  to maximize profit

$$\pi_2(j) = \frac{(1-\gamma)(\mu_j)^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}} \left( \lambda^j K_2^{\delta_j} L_2^{1-\delta_j} \right)^{\varepsilon} - r_j K_2 - w_j L_2$$

subject to peak price faced by this leader firm

$$\frac{(1-\gamma)(\mu_j \lambda^j K_2^{\delta_j} L_2^{1-\delta_j})^{\varepsilon-1}}{\gamma C_1^{\varepsilon-1}} \leq \lambda^{-(j-1)} (\delta_{j-1})^{-\delta_{j-1}} (1-\delta_{j-1})^{-(1-\delta_{j-1})} (r_j)^{\delta_{j-1}} (w_j)^{1-\delta_{j-1}}$$

From the above profit maximization problem, we could obtain the factor demand functions of the firm  $j$  in the modern sector which are similar to (35) and (36). And the remaining unresolved details of dynamic equilibrium in the case of nondrastic innovations also closely resemble those of drastic innovations, thus, we focus on the drastic innovations only in the present paper for simplicity.

### A.2 Endogenous Industrial Upgrading of a Planned Economy in the DC

Like that in LDC in subsection 5.1, the social welfare maximization problem in the DC can be decomposed into separate static and dynamic resource allocation problems. The presence of intertemporal-spillover effect does not alter the static allocation problem of the social planner in

the DC. And in the static allocation problem of a planned economy in the DC, given the production function in traditional sector which is  $\bar{Y}_1 = \bar{N}^{\frac{1-\alpha}{\alpha}} (\bar{L}_1)^\beta (\bar{K}_1)^{1-\beta}$  and the production function in the modern sector which is  $\bar{Y}_2(\bar{j}) = \lambda^{\bar{\psi}(t)} (\bar{K}_2)^{\delta_j} (\bar{L}_2)^{1-\delta_j}$  as well as the given price of the modern product, denoted by  $\bar{p}_{\bar{j}}$ , the optimal employment of factors in the DC implies that we could calculate  $\bar{\kappa}_L$  as the function of  $\bar{K}$ ,  $\bar{N}$ , and  $\bar{\psi}(t)$  or  $\bar{j}$  by

$$\bar{K}_1 = \frac{\left[ \frac{(1-\beta)(1-\delta_j)}{\beta\delta_j} \right]^{\frac{\delta_j}{\delta_j-1+\beta}} \left[ \frac{\beta\bar{N}^{\frac{1-\alpha}{\alpha}}}{(1-\delta_j)\bar{p}_{\bar{j}}\lambda^{\bar{\psi}(t)}} \right]^{\frac{1}{\delta_j-1+\beta}} \beta\delta_j L - (1-\beta)(1-\delta_j)\bar{K}}{\delta_j - (1-\beta)}$$

and

$$\bar{L}_1 = \frac{L\delta_j\beta\bar{K}_1}{(1-\beta)(1-\delta_j)(\bar{K} - \bar{K}_1) + \delta_j\beta\bar{K}_1}$$

Then the dynamic allocation problem in a planned economy of the DC could be reformulate as the maximization of

$$\bar{U}_\tau = \int_\tau^\infty \frac{\left[ \gamma\bar{C}_1^\varepsilon + (1-\gamma)(\bar{C}_2(\bar{j}))^\varepsilon \right]^{(1-\theta)/\varepsilon} - 1}{1-\theta} \exp[-\rho(t-\tau)] dt$$

subject to the resource constraints

$$\dot{\bar{\psi}}(x) = \bar{v}(x)$$

$$\bar{C}_1 = \bar{N}^{\frac{1-\alpha}{\alpha}} (\bar{\kappa}_L L)^\beta (\bar{\kappa}_K \bar{K})^{1-\beta}$$

$$\bar{C}_2 = \bar{\mu}\lambda^{\bar{j}} \left[ (1-\bar{\kappa}_K)\bar{K} \right]^{\delta_j} \left[ (1-\bar{\kappa}_L)L \right]^{1-\delta_j}$$

$$\dot{\bar{K}} + \frac{\dot{\bar{N}}\bar{N}^{\varphi_1}}{b_1} + \lambda^{\varphi_2\bar{\psi}(t)}\bar{v} = (1-\bar{\mu})\lambda^{\bar{j}} \left[ (1-\bar{\kappa}_K)\bar{K} \right]^{\delta_j} \left[ (1-\bar{\kappa}_L)L \right]^{1-\delta_j}$$

The above social welfare maximization problem is a standard problem of optimal control, and Pontryagin's minimum principle could be employed to calculate the (optimal) endogenous industrial upgrading of a planned economy in the DC. From the Hamiltonian we could derive the necessary and sufficient conditions for the above maximization problem of the social planner in the DC. Given the initial value of technology in traditional sector at the starting point of the analysis, i.e., the exact value of  $\bar{N}(0)$ , and the initial value of technology in modern sector at the starting point of the analysis, which is  $A_2(0) = 1$  as assumed previously, these necessary and sufficient conditions would characterize the (endogenous) industrial upgrading of a planned

economy in the DC.

### A.3 Optimal Industrial Structure of a Planned Economy in the LDC under IPR Protection Internationally

The optimal industrial structure of a planned economy in the LDC under IPR protection internationally could also be decomposed into separate static and dynamic resource allocation problems.

At time  $\tau$ , in the static allocation problem, given the production function in traditional sector in the LDC which is  $\underline{Y}_1 = \underline{N}(\tau)^{\frac{1-\alpha}{\alpha}} (\underline{L}_1)^\beta (\underline{K}_1)^{1-\beta}$  and the production function in the modern sector which is  $\underline{Y}_2(j) = \lambda^{j(\tau)+\underline{\psi}(t)} (\underline{K}_2)^{\delta_j} (\underline{L}_2)^{1-\delta_j}$  as well as the given price of the modern product, denoted by  $\underline{p}_j$ , the optimal employment of factors in the LDC implies that we could calculate  $\underline{\kappa}_L(\tau)$  as the function of  $\underline{K}(\tau)$ ,  $\underline{N}(\tau)$ , and  $\underline{j}(\tau)$  by

$$\underline{K}_1 = \frac{\left[ \frac{(1-\beta)(1-\delta_j)}{\beta\delta_j} \right]^{\frac{\delta_j}{\delta_j-1+\beta}} \left[ \frac{\beta \underline{N}^{\frac{1-\alpha}{\alpha}}}{(1-\delta_j)\underline{p}_j \lambda^{j(\tau)+\underline{\psi}(t)}} \right]^{\frac{1}{\delta_j-1+\beta}} \beta\delta_j \underline{L} - (1-\beta)(1-\delta_j)\underline{K}}{\delta_j - (1-\beta)}$$

and

$$\underline{L}_1 = \frac{L\delta_j\beta\underline{K}_1}{(1-\beta)(1-\delta_j)(\underline{K} - \underline{K}_1) + \delta_j\beta\underline{K}_1}$$

Then the dynamic allocation problem of a planned economy in the LDC could be reformulated as

$$\max_{\underline{\mu}, \underline{L}, \underline{N}(\tau), \underline{j}(\tau)} \underline{U}_\tau = \int_\tau^\infty \frac{\left[ \gamma \underline{C}_1^\varepsilon + (1-\gamma)(\underline{C}_2(j))^\varepsilon \right]^{(1-\theta)/\varepsilon} - 1}{1-\theta} \exp[-\rho(t-\tau)] dt$$

subject to the resource constraints

$$\dot{\underline{\psi}}(x) = \underline{l}(x)$$

$$\underline{C}_1 = \underline{N}^{\frac{1-\alpha}{\alpha}} (\underline{\kappa}_L \underline{L})^\beta (\underline{\kappa}_K \underline{K})^{1-\beta}$$

$$\underline{C}_2 = \underline{\mu} \lambda^j [(1-\underline{\kappa}_K)\underline{K}]^{\delta_j} [(1-\underline{\kappa}_L)\underline{L}]^{1-\delta_j}$$

$$\dot{\underline{K}} + \frac{1}{b_1} \dot{\underline{N}} \left( \frac{\underline{N}}{\underline{N}} \right)^{\phi_1} \underline{N}^{\phi_1} + \lambda^{\phi_2 \bar{\psi}(t) + (\phi_2 - \phi_2) \underline{\psi}(t)} \underline{l} = (1-\underline{\mu}) \lambda^j [(1-\underline{\kappa}_K)\underline{K}]^{\delta_j} [(1-\underline{\kappa}_L)\underline{L}]^{1-\delta_j}$$

where  $\underline{j}(t) = \underline{j}(\tau) + \underline{\psi}(t)$ .

Once again, the above maximization problem is a standard problem of optimal control, and

Pontryagin's minimum principle could be employed to solve out the optimal evolution of  $\underline{K}(t)$ ,  $\underline{N}(t)$ ,  $\underline{\kappa}_L(t)$ , and  $\underline{\psi}(t)$  or  $\underline{j}(t)$  at time  $t > \tau$ , by the necessary and sufficient conditions for the above social welfare maximization problem in the LDC derived from the Hamiltonian. Once  $\hat{N}(\tau)$ ,  $\hat{\kappa}_L(\tau)$ , and  $\hat{j}(\tau)$  have been chosen at time  $\tau$ , the optimal industrial structure of a planned economy in the LDC at time  $t > \tau$ , denoted by  $\hat{N}(t)$ ,  $\hat{\kappa}_L(t)$ , and  $\hat{j}(t)$  could also be determined.

Comparing the analysis in subsection A.2 with that in subsection A.3, it is obvious that the qualitative results in section 5 still holds for the present planned economy in the LDC.

#### A.4 Proof of Proposition 5

We will prove proposition 5 based on an extreme assumption firstly, then extend it to the general case. The extreme assumption is that we model the Catch-up type CAD strategy in the LDC whose capital stock equals to  $\underline{K}(\tau)$  by assuming that the actual industrial structure chosen by the government in the LDC coincides with that in the DC whose capital stock equals to  $\bar{K}(\tau)$  for tractability, i.e.,  $\underline{N}(\tau) = \bar{N}(\tau)$ ,  $\underline{j}(\tau) = \bar{j}(\tau)$ ,  $\underline{\delta}(\tau) = \bar{\delta}(\tau)$ , and  $\underline{\kappa}_L(\tau) = \bar{\kappa}_L(\tau)$  at time  $\tau$ , when  $\underline{K}(\tau) \ll \bar{K}(\tau)$ .

From the analysis in subsection 3.2, we know that, without external subsidies, as long as the R&D operates at a positive but finite scale, the present value of firm  $j$  in any country discounted to time  $\tau$ , denoted by  $V_2(j, \tau)$ , must satisfy free-entry condition, i.e.,  $p_j(\tau)(\lambda^j)^{\phi_2} = V_2(j, \tau)$ . Naturally, the present value of the firm in the DC, whose capital stock equals to  $\bar{K}(\tau)$ , which enters into industry  $\bar{j}$  discounted to time  $\tau$ , denoted by  $\bar{V}_2(\bar{j}, \tau)$ , meets the above free-entry condition precisely, owing to there is a positive and bounded R&D intensity in the DC, which implies that we have

$$p_{\bar{j}}(\tau)(\lambda^{\bar{j}})^{\phi_2} = \bar{V}_2(\bar{j}, \tau) \quad (80)$$

Equation (40) could be reformulated as

$$\frac{V_2(j, t)}{p_j(t)} = \frac{\dot{V}_2(j, t)}{[r(t) + i(t)] p_j(t)} + \frac{\pi_2(j, t)}{[r(t) + i(t)] p_j(t)} \quad (81)$$

Substituting  $\frac{\pi_2(j)}{p_j} = (1 - \varepsilon)Y_2(j)$  into (81) yields

$$\frac{V_2(j, t)}{p_j(t)} = \frac{\dot{V}_2(j, t)}{[r(t) + i(t)] p_j(t)} + \frac{(1 - \varepsilon)Y_2(j, t)}{[r(t) + i(t)]} \quad (82)$$

where  $p_j(t) = \frac{(1-\gamma)}{\gamma} \left( \frac{\lambda^{\psi(t)} [(1-\kappa_K)K]^\delta [(1-\kappa_L)L]^{1-\delta} - (X_1 + X_2)}{N^{\frac{1-\alpha}{\alpha}} (\kappa_L L)^\beta (\kappa_K K)^{1-\beta}} \right)^{\varepsilon-1}$  and satisfies

$$\frac{\partial p_j(t)}{\partial K} < 0.$$

Differentiating (82) with respect to  $K$  obtains

$$\begin{aligned} \frac{\partial}{\partial K} \left[ \frac{V_2(j,t)}{p_j(t)} \right] &= \frac{\frac{\partial}{\partial K} \dot{V}_2(j,t)}{[r(t)+\iota(t)] p_j(t)} - \frac{\dot{V}_2(j,t)}{[r(t)+\iota(t)]^2 p_j(t)} \frac{\partial r(t)}{\partial K} \\ &\quad - \frac{\dot{V}_2(j,t)}{[r(t)+\iota(t)] (p_j(t))^2} \frac{\partial p_j(t)}{\partial K} + \frac{(1-\varepsilon)}{[r(t)+\iota(t)]} \frac{\partial Y_2(j,t)}{\partial K} - \frac{(1-\varepsilon) Y_2(j,t)}{[r(t)+\iota(t)]^2} \frac{\partial r(t)}{\partial K} \end{aligned} \quad (83)$$

It is obvious that  $\dot{V}_2(j,t) > 0$ ,  $\frac{\partial Y_2(j,t)}{\partial K} > 0$ ,  $\frac{\partial r(t)}{\partial K} < 0$ , which imply that we have

$\frac{\partial}{\partial K} \left[ \frac{V_2(j,t)}{p_j(t)} \right] > 0$ , owing to  $\frac{\partial}{\partial K} \dot{V}_2(j,t)$  is a higher-order infinitesimal term which could be

neglected in (83). Therefore, given  $\underline{N}(\tau) = \bar{N}(\tau)$ ,  $\underline{j}(\tau) = \bar{j}(\tau)$ ,  $\underline{\delta}(\tau) = \bar{\delta}(\tau)$ , and  $\underline{\kappa}_L(\tau) = \bar{\kappa}_L(\tau)$  at time  $\tau$ ,  $\underline{p}_j(\tau)(\lambda^{\underline{j}})^{\varrho_2} > \underline{V}_2(\underline{j}, \tau)$  is a direct conclusion from (80) when  $\underline{K}(\tau) < \bar{K}(\tau)$ , thereby we could obtain proposition 5.

Furthermore, as a matter of fact,  $\frac{V_2(j,t)}{p_j(t)}$  is a continuous function of its all arguments, thus,

when there is a severe scarcity of capital in the LDC, i.e.,  $\underline{K}(\tau) \ll \bar{K}(\tau)$ , the condition that  $\underline{N}(\tau) = \bar{N}(\tau) - \Delta_N$ ,  $\underline{j}(\tau) = \bar{j}(\tau) - \Delta_j$ , and  $\underline{\kappa}_L(\tau) = \bar{\kappa}_L(\tau) + \Delta_\kappa$  when  $\Delta_N > 0$ ,  $\Delta_j > 0$ , and  $\Delta_\kappa$  are all small enough, could still suffice for  $\underline{p}_j(\tau)(\lambda^{\underline{j}})^{\varrho_2} > \underline{V}_2(\underline{j}, \tau)$ , where  $\Delta_N$ ,  $\Delta_j$ , and  $\Delta_\delta$  could be construed as the extent of Catch-Up in the LDC in the present paper.

*Q.E.D.*