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Abstract

This paper concerns the joint modeling, estimation and testing for local and global spatial externalities. Spatial externalities have become in recent years a standard notion of economic research activities in relation to social interactions, spatial spillovers and dependence, etc., and have received an increasing attention by econometricians and applied researchers. While conceptually the principle underlying the spatial dependence is straightforward, the precise way in which this dependence should be included in a regression model is complex. Following the taxonomy of Anselin (2003, International Regional Science Review 26, 153-166), a general model is proposed, which takes into account jointly local and global externalities in both modelled and unmodelled effects. The proposed model encompasses all the models discussed in Anselin (2003). Robust methods of estimation and testing are developed based on Gaussian quasi-likelihood. Large and small sample properties of the proposed methods are investigated.

Key words and phrases: Asymptotic property, Finite sample property, Quasi-likelihood, Spatial regression models, Robustness, Tests of spatial externalities.

JEL Classification: C1, C2, C5

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1 Introduction

Spatial dependence or social interaction among the economic or social actors has recently received a greatly increased attention (Anselin 2003; Goodchild et al. 2000; Glaeser et al. 1996; Akerlof 1997; Abbot 1997; Sampson et al. 1999). Spatial econometric models and methods have been applied not only in specialized fields such as regional science, urban economics, real estate and economic geography, but also increasingly in more traditional fields of economics as well, including demand analysis, labor economics, public economics, international economics, and agricultural and environmental economics (see reviews in Anselin and Bera 1998; Anselin 2001; and Elhorst 2003).

While conceptually it is straightforward to see the principle underlying the resulting spatial dependence, the precise way in which this dependence should be included in a regression model is rather complex. Very recently, the notions of local and global externalities or short range and long range spatial dependence were brought up by Anselin (2003), which since then has caught the attention of many econometricians and applied researchers. Anselin provided a comprehensive taxonomy of spatial econometric models according to different kinds of spatial externalities in an effort to better reconcile econometric practice with theoretical developments. However, the problems of model estimation and testing for some models are not considered; joint modeling and testing of local and global spatial externalities is not discussed; and consistency and asymptotic normality of the parameter estimates for certain models are not formally treated. Thus, it is highly desirable to"unify" all the available models and develop general methods of inference, allowing flexible spatial patterns in the model so that an appropriate one can be identified by the data through testing.

In this article, I propose a general model that takes into account of local and global externalities jointly, in both modelled effects as well as unmodelled effects. The proposed model contains the models discussed in Anselin (2003) and other models available in the literature as special cases. I propose using the quasi-maximum likelihood method (QMLE) for model estimation. QMLE is advantageous over the traditional maximum likelihood estimation (MLE) method in that it is robust against misspecification in error distribution, and is advantageous over the IV or GMM in that it is applicable to a pure spatial process (a model of no covariates), see Lee (2004a). The problem of parameter identifiability, and the consistency and asymptotic normality of the QMLE are formally treated, to set foundations for formal statistical inferences. Tests (joint or marginal) for local and global externalities are developed to facilitate the practitioners to choose the model. These tests all possess simple analytical expressions, and are robust against nonnormality of the error distributions. Monte Carlo simulation shows that both the QMLEs and the tests perform very well in finite samples.

The rest of the paper is organized as follows. Section 2 presents the general model and the quasi-maximum likelihood estimation (QMLE) procedure. Section 3 treats the problems of parameter identifiability, and the consistency and asymptotic normality of the QMLE. Section 4 presents various tests for spatial externalities. Section 5 presents Monte Carlo results for finite sample performance of the proposed methods. Section 6 concludes the paper.

2 A General Spatial Regression Model

In this section, I present a general spatial regression model that takes into account of local and global externalities in the modelled effects as well as the local and global externalities in the unmodelled effects, focusing more on the practical issues of model estimation and covariance estimation to facilitate the practical applications.

2.1 The model

For an $n \times n$ spatial contiguity weights matrix W_n, multiplication of $I_n + \rho W_n$ on a variable generates a local spatial externality, and multiplication of $(I_n - \rho W_n)^{-1}$ on a variable generates a global spatial externality, where I_n is an $n \times n$ identity matrix and ρ is a spatial parameter. See Anselin (2003, Sec. 2) for detailed explanations. A natural generalization of these ideas is to multiply $(I_n + \rho_1 W_n^{\ell})(I_n - \rho_2 W_n^{g})^{-1}$ on a variable to generate simultaneously local and global spatial externalities, where W_n^{ℓ} and W_n^{g}

are, respectively, the local and global spatial weights matrices. Loosely speaking, local spatial externality means that spatial dependence is limited to among the "neighbors", whereas the global spatial externalities means that the spatial dependence exists among the spatial units that may be "far" away from each other. Spatial externalities may exist in the modeled effects (the regressors) as well as in the unmodelled effects (the errors). To give a maximum generality, I consider both local and global externalities in both modelled as well as unmodeled effects.[2] Generically, let $A(W_{1n}^{\ell}, W_{1n}^{g}, \rho)$ be an $n \times n$ matrix function of the $n \times n$ spatial weights matrices W_{1n}^{ℓ} and W_{1n}^{g}, indexed by a $k_1 \times 1$ spatial parameter vector ρ, and $B(W_{2n}^{\ell}, W_{2n}^{g}, \gamma)$ be an $n \times n$ matrix function of the $n \times n$ spatial weights matrices W_{2n}^{ℓ} and W_{2n}^{g}, indexed by a $k_2 \times 1$ spatial parameter vector γ. The proposed model takes the following general form:

$$Y_n = A(W_{1n}^{\ell}, W_{1n}^{g}, \rho)X_n\beta + B(W_{2n}^{\ell}, W_{2n}^{g}, \gamma)u_n \tag{1}$$

where the matrices $A(W_{1n}^{\ell}, W_{1n}^{g}, \rho) \equiv A_n(\rho)$ and $B(W_{2n}^{\ell}, W_{2n}^{g}, \gamma) \equiv B_n(\gamma)$ capture, respectively, the spatial externalities in the covariates X_n and in the error vector u_n, β is a $p \times 1$ vector of model parameters, and u_n is a vector of independent and identically distributed (iid) errors of mean zero and variance σ^2. All W matrices are normalized to have unity row sums. Clearly, it must be that $A_n(0) = I_n$ and $B_n(0) = I_n$, i.e., $\rho = 0$ or $\gamma = 0$ or both indicates the lack of spatial externality in X_n or in u_n or in both.

The model given in (1) is very general, covering most of the models available in the literature. From the above discussions, we see that the local spatial externality corresponds to a spatial moving average (SMA) process, the global spatial externality corresponds to a spatial autoregressive (SAR) process, and the local and global spatial externalities together correspond to a spatial autoregressive moving average (SARMA) process.[3] Most of the models appeared in the literature apply one or more of the these

[2]Spatial effects in Y_n can be converted to the spatial effects in X_n and error terms, see Anselin(2003).

[3]This term is originated from Huang (1984), with the original meaning being a SAR(p) for the response together with a SMA(q) for the error. However, we see no reason why we can not apply a SAR(p) and a SMA(q) to the same variable to produce a SARMA(p, q) error, or a SARMA(p, q) response, or SARMA(p, q) regressors. See also Bera and Anselin (1998) and Anselin (2003) for discussions on SARMA processes.

processes (first order or higher)[4] to one or more of the model components: the response, the regressors, and the disturbance. These can all be reduced to the form specified in Model (1) defined above, with certain constrains (when necessary) being put on ρ and γ, and on the weights matrices. For example, in their popular forms, we have,

- $Y_n = X_n\beta + \varepsilon_n$, with $\varepsilon_n = \gamma W_n \varepsilon_n + u_n$. This is a model with a SAR(1) error or global externality on u_n, which can be written in the form of (1) with $A_n(\rho) = I_n$ and $B_n(\gamma) = (I_n - \gamma W_n)^{-1}$ (see, e.g., Anselin and Bera, 1998; Benirschka and Binkley, 1994; Kelejian and Prucha, 1999);
- $Y_n = X_n\beta + \varepsilon_n$, with $\varepsilon_n = \gamma W_n u_n + u_n$, a model with a SMA(1) error or local externality on u_n. In the form of (1), $A_n(\rho) = I_n$ and $B_n(\gamma) = (I_n + \gamma W_n)$ (see, e.g., Cliff and Ord 1981; Haining 1990; Anselin and Bera 1998).
- $Y_n = \rho W_n Y_n + X_n\beta + u_n$, a model with only a SAR(1) on Y_n, which can be translated into a model with global externality in both X_n and u_n, with $A_n(\rho) = (I_n - \rho W_n)^{-1}, B_n(\gamma) = (I_n - \gamma W_n)^{-1}$, and $\rho = \gamma$ (see, e.g., Anselin 1988; Case et al. 1993; Besley and Case 1995; Lee 2002, 2004a);
- $Y_n = X_n\beta + \rho W_{1n} X_n \beta + \varepsilon_n$ with $\varepsilon_n = \gamma W_n \varepsilon_n + u_n$. This is a model with a SMA(1) on X_n and a SAR(1) on u_n, called the *hybrid* model by Anselin (2003). For this model, $A_n(\rho) = I_n + \rho W_n$ and $B_n(\gamma) = (I_n - \gamma W_n)^{-1}$. It has not been formally studied so far. Alternatively, one can apply SAR(1) on X_n and SMA(1) on u_n;
- $Y_n = \rho W_{1n} Y_n + X_n\beta + \varepsilon_n$ with $\varepsilon_n = \gamma W_{2n} \varepsilon_n + u_n$, a model with SAR(1) on both Y_n and ε_n (see Anselin 1988, p. 60-65). It has been applied by, among others, Case (1991, 1992), Case et al. (1993), and Besley and Case (1995). It is called the spatial ARAR(1,1) model by Kelejian and Prucha (1998, 2001, 2006), who studied generalized spatial 2SLS procedure, asymptotic distribution of Moran I test, and GM estimation of the model with heteroscedastic errors. Using our notation, we have $A_n(\rho) = (I_n - \rho W_{1n})^{-1}$ and $B_n(\gamma) = (I_n - \gamma_1 W_{2n}^g)^{-1}(I_n - \gamma_2 W_{2n}^\ell)^{-1}$, with $\gamma_1 = \rho$, $\gamma_2 = \gamma$, $W_{2n}^g = W_{1n}$, and $W_{2n}^\ell = W_{2n}$.
- $Y_n = X_n\beta + \varepsilon_n$ with $\varepsilon_n = \gamma_1 W_n^g \varepsilon_n + \gamma_2 W_n^\ell u_n + u_n$, a model with SARMA(1,1) (or joint local and global spatial externalities) on errors. In this case, $A_n(\rho) = I_n$ and $B_n(\gamma) = (I_n - \gamma_1 W_n^g)^{-1}(I_n + \gamma_2 W_n^\ell)$;
- $Y_n = Z_n\beta + \varepsilon_n$, with $Z_n = \rho_1 W_{1n}^g Z_n + \rho_2 W_n^\ell X_n + X_n$, and $\varepsilon_n = \gamma_1 W_{2n}^g \varepsilon_n + \gamma_2 W_{2n}^\ell u_n + u_n$, a model with a SARMA(1,1) on u_n and a SARMA(1,1) on X_n. In this case, $A_n(\rho) = (I_n - \rho_1 W_{1n}^g)^{-1}(I_n + \rho_2 W_{1n}^\ell)$ and $B_n(\gamma) = (I_n - \gamma_1 W_{2n}^g)^{-1}(I_n + \gamma_2 W_{2n}^\ell)$.

[4]Higher-order spatial lag operators are defined by applying the spatial weights matrix to a lower-order lagged variable, e.g., a second-order spatial lag in Y_n is obtained as $W_n(W_nY_n) = W_n^2 Y_n$. However, higher-order spatial operators yield redundant and circular neighbor relations, which must be eliminated to ensure proper estimation and inference (Anselin and Bera, 1998, p. 247).

Clearly, the model can be more complicated than any of them listed above. For example, one may use $(I_n + \gamma_1 W_n + \gamma_2 W_n^2 + \gamma_3 W_n^3)$ to generate local effects that extend to several layers of neighbors. Also, the general specification given in (1) can be easily extended to include covariates that are not associated with any spatial effects, and to add heteroscedasticity structure onto the model.

2.2 Model estimation

I now outline the quasi-maximum likelihood estimation (QMLE) procedure based on Gaussian likelihood. Let $\Omega_n(\gamma) = B_n(\gamma)B_n'(\gamma)$. Let $\theta = (\rho', \gamma')'$, and $\xi = (\beta', \theta', \sigma^2)'$. The quasi-loglikelihood, using normal distribution as an approximation to the error distribution, has the form

$$\ell_n(\xi) = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{1}{2\sigma^2}\varepsilon_n(\beta,\rho)'\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho) \tag{2}$$

where $\varepsilon_n(\beta,\rho) = Y_n - A_n(\rho)X_n\beta$. Given θ, the constrained QMLEs of β_0 and σ_0^2 are

$$\hat{\beta}_n(\theta) = [X_n(\rho)'\Omega_n^{-1}(\gamma)X_n(\rho)]^{-1}X_n(\rho)\Omega_n^{-1}(\gamma)Y_n \tag{3}$$

$$\hat{\sigma}_n^2(\theta) = \frac{1}{n}[Y_n - X_n(\rho)\hat{\beta}_n(\theta)]'\Omega_n^{-1}(\gamma)[Y_n - X_n(\rho)\hat{\beta}_n(\theta)], \tag{4}$$

where $X_n(\rho) = A_n(\rho)X_n$.

Substituting $\hat{\beta}_n(\theta)$ and $\hat{\sigma}_n^2(\theta)$ back into (2) for β and σ^2, we obtain the concentrated quasi-loglikelihood function for θ.

$$\ell_n^c(\theta) = -\frac{n}{2}[1+\ln(2\pi)] - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{n}{2}\ln[\hat{\sigma}_n^2(\theta)]. \tag{5}$$

Maximizing $\ell_n^c(\theta)$ gives the QMLE $\hat{\theta}_n$ of θ, which in turn gives the QMLEs of β and σ^2 as $\hat{\beta}_n = \hat{\beta}_n(\hat{\theta}_n)$ and $\hat{\sigma}_n^2 = \hat{\sigma}_n^2(\hat{\theta}_n)$. Maximization of $\ell_n^c(\theta)$ can be conveniently realized using `GAUSS CO` procedure (see the footnote to Assumption I in Section 3 for the issue of parameter space). In cases where computing speed is an issue, one may consider providing the analytical gradient

$$\frac{\partial \ell_n^c(\theta)}{\partial \rho_i} = \frac{[X_{n,\rho_i}(\rho)\hat{\beta}_n(\theta)]'\Omega_n^{-1}(\gamma)\varepsilon_n(\hat{\beta}_n(\theta),\rho)}{\varepsilon_n'(\hat{\beta}_n(\theta),\rho)\Omega_n^{-1}(\gamma)\varepsilon_n(\hat{\beta}_n(\theta),\rho)/n}, \tag{6}$$

$$\frac{\partial \ell_n^c(\theta)}{\partial \gamma_j} = \frac{\varepsilon_n'(\hat{\beta}_n(\theta),\rho)\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_j}(\gamma)\Omega_n^{-1}(\gamma)\varepsilon_n(\hat{\beta}_n(\theta),\rho)}{2\varepsilon_n'(\hat{\beta}_n(\theta),\rho)\Omega_n^{-1}(\gamma)\varepsilon_n(\hat{\beta}_n(\theta),\rho)/n} - \frac{1}{2}\text{tr}[\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_j}(\gamma)], \tag{7}$$

where $i = 1, \cdots, k_1$, $j = 1, \cdots, k_2$, $X_{n,\rho_i}(\rho) = \frac{\partial}{\partial \rho_i} X_n(\rho)$ and $\Omega_{n,\gamma_j}(\gamma) = \frac{\partial}{\partial \gamma_j}\Omega_n(\gamma)$. For large data, repeated calculation of $|\Omega_n(\gamma)|$ as required in the process of maximizing $\ell_n^c(\theta)$ can be a burden. However, often the special form of the $\Omega_n(\gamma)$ matrix allows for a considerable amount of simplifications. For example, in a model with a spatial AR error, $B_n(\gamma) = (I_n - \gamma W_{2n})^{-1}$. Thus $\Omega_n(\gamma) = [(I_n - \gamma W_{2n}')(I_n - \gamma W_{2n})]^{-1}$ and $|\Omega_n(\gamma)| = \prod_{i=1}^n (1 - \gamma w_i)^{-2}$, where w_i are the eigenvalues of W_{2n}. As W_{2n} is a fixed matrix, its eigenvalues only need to be calculated once and be used subsequently.[5]

2.3 Covariance estimation

The previous subsection describes a simple procedure for model estimation. Formal statistical analysis needs the standard errors of the parameter estimates, or more generally the variance-covariance estimate of the QMLE to facilitate more advanced statistical inferences such as confidence interval construction for quantiles. To provide a simple expression for such a covariance estimate, some notation and conventions are necessary, and these notation and conventions will be followed through the rest of the article.

Notation and conventions. Let ξ_0 (and accordingly β_0, θ_0, ρ_0, γ_0 and σ_0^2) represent the true parameter values. Let $G_n(\xi) = \frac{\partial}{\partial \xi}\ell_n(\xi)$ be the gradient vector and $H_n(\xi) = \frac{\partial}{\partial \xi'} G_n(\xi)$ be the Hessian matrix with their detailed expressions given in Appendix A. Let $K_n(\xi_0) = \mathtt{Var}[G_n(\xi_0)]$ and $I_n(\xi_0) = -\mathtt{E}[H_n(\xi_0)]$, with the expectation and variance operators '$\mathtt{E}$' and '$\mathtt{Var}$' corresponding to the true parameters. Specifically, $\mathtt{E}(Y_n) = A_n(\rho_0)X_n\beta_0$ and $\mathtt{Var}(Y_n) = \sigma_0^2\Omega(\gamma_0)$. For a vector v_n and a matrix M_n, $v_{n,i}$ is the ith element of v_n, $m_{n,ij}$ is the ijth element of M_n, $\|v_n\|$ is the Euclidean norm of v_n, $\mathtt{tr}(M_n)$ is the trace of M_n, $\mathtt{diagv}(M_n)$ is a column vector formed by the diagonal elements of M_n, $|M_n|$ is the determinant, M_n' is the transpose, and M_n^{-1} is the inverse of M_n. The partial derivatives of the matrix function $A_n(\rho)$ with respect to the ith element of ρ is denoted as $A_{n,\rho_i}(\rho)$. Similar notation is used for the partial derivatives of $B_n(\gamma)$, $X_n(\rho)$

[5] Accuracy issue may arise when n is large (Kelejian and Prucha, 1998), and in this case sparce matrix technique should be employed (LeSage, 1999). See Griffith, 1988; Anselin, 1988; Magnus, 1982; and Magnus and Neudecker, 1999, for more on matrix calculations.

and $\Omega_n(\gamma)$. Let 1_n be the $n \times 1$ vector of ones. Define

$$\begin{aligned}
Z_{1n}(\theta) &= B_n^{-1}(\gamma)X_n(\rho), \\
Z_{2n}(\theta) &= \left\{B_n^{-1}(\gamma)X_{n,\rho_i}(\rho)\beta,\ \ i = 1, \cdots, k_1\right\}_{n\times k_1}, \\
\Phi_n(\gamma) &= \left\{\texttt{diagv}\left(\Omega^*_{n,\gamma_i}(\gamma)\right),\ \ i = 1, \cdots, k_2\right\}_{n\times k_2}, \\
\Lambda_n(\gamma) &= \left\{\texttt{tr}\left(\Omega^*_{n,\gamma_i}(\gamma)\Omega^*_{n,\gamma_j}(\gamma)\right),\ \ i, j = 1, \cdots, k_2\right\}_{k_2\times k_2},
\end{aligned}$$

where $\Omega^*_{n,\gamma_i}(\gamma) = B_n^{-1}(\gamma)\Omega_{n,\gamma_i}(\gamma)B_n'^{-1}(\gamma),\ i = 1, \cdots, k_2$. When a function is evaluated at ξ_0, the bracketed part will be suppressed, e.g., $Z_{1n} = Z_{1n}(\theta_0)$, $\Phi_n = \Phi_n(\gamma_0)$. Put $Z_n = \{Z_{1n}, Z_{2n}\}$. Let α_0 and $\kappa_0 + 3$ be, respectively, the skewness and kurtosis of $u_{n,i}$.

Using the above notation, the asymptotic variance ($\texttt{AVar}$) of the QMLE $\hat{\xi}_n$ is

$$\texttt{AVar}(\hat{\xi}_n) = I_n^{-1}(\xi_0)K_n(\xi_0)I_n^{-1}(\xi_0),$$

with the expected information matrix and the variance of the gradient being, respectively,

$$I_n(\xi_0) = \begin{pmatrix} \frac{1}{\sigma_0^2}Z_n'Z_n, & 0, & 0 \\ \sim, & \frac{1}{2}\Lambda_n, & \frac{1}{2\sigma_0^2}\Phi_n'1_n \\ \sim, & \sim, & \frac{n}{2\sigma_0^4} \end{pmatrix}, \tag{8}$$

and

$$K_n(\xi_0) = \begin{pmatrix} \frac{1}{\sigma_0^2}Z_n'Z_n, & \frac{\alpha_0}{2\sigma_0}Z_n'\Phi_n, & \frac{\alpha_0}{2\sigma_0^3}Z_n'1_n \\ \sim, & \frac{\kappa_0}{4}\Phi_n'\Phi_n + \frac{1}{2}\Lambda_n, & \frac{\kappa_0+2}{4\sigma_0^2}\Phi_n'1_n \\ \sim, & \sim, & \frac{n(\kappa_0+2)}{4\sigma_0^4} \end{pmatrix}. \tag{9}$$

Note that when the errors are exactly normal, $\alpha_0 = \kappa_0 = 0$, thus $K_n(\xi_0) = I_n(\xi_0)$, and $\texttt{AVar}(\hat{\xi}_n) = I_n^{-1}(\xi_0)$. The detailed derivations for $K_n(\xi_0)$ and $I_n(\xi_0)$ are given in the Appendix A. With these explicit expressions, we obtain an estimate of $\texttt{Var}(\hat{\xi}_n)$ as:

$$\widehat{\texttt{Var}}(\hat{\xi}_n) = I_n^{-1}(\hat{\xi}_n)K_n(\hat{\xi}_n)I_n^{-1}(\hat{\xi}_n),$$

Note that in the above variance estimate, α_0 is estimated by the sample skewness of $B_n^{-1}(\hat{\gamma})\varepsilon_n(\hat{\beta}_n, \hat{\rho}_n)$, and $\kappa_0 + 3$ is estimated by the sample kurtosis of $B_n^{-1}(\hat{\gamma})\varepsilon_n(\hat{\beta}_n, \hat{\rho}_n)$.

Clearly, use of QMLE standard error makes the inferences robust against the excess skewness and kurtosis of the data. When the focus of statistical inference is on the regular regression parameters β as is the case for the empirical applications, a simple inferential statistic is presented in Section 4.

3 Large Sample Properties

In this section, I consider the problems of parameter identifiability, and consistency and asymptotic normality of the QMLEs. These asymptotic theories are essential for statistical inferences for the regression coefficients, and for testing the local and global spatial effects. Let Θ_1 be the parameter space containing the values of ρ, Θ_2 be the space of γ values, and $\Theta = \Theta_1 \times \Theta_2$ be the product space containing the values of θ. The following is a set of regularity conditions that are sufficient for the parameter identifiability and consistency of the QMLEs.

Assumption 1. *The space Θ is compact with θ_0 being an interior point of it.*[6]

Assumption 2. *$\{u_{n,i}\}$ are iid with mean zero, variance σ_0^2, and finite moment $\mathrm{E}(|u_{n,i}|^{4+\epsilon})$ for $\epsilon > 0$.*

Assumption 3. *The elements of X_n are uniformly bounded, and $\lim_{n\to\infty} \frac{1}{n}[Z_{1n}'(\theta)Z_{1n}(\theta)]$ exists and is nonsingular, uniformly in $\theta \in \Theta$.*

Assumption 4. *The sequences of matrices $A_n(\rho)$ and $A_n^{-1}(\rho)$ are uniformly bounded in both absolute row or column sums, uniformly in $\rho \in \Theta_1$.*

Assumption 5. *The sequences of matrices $B_n(\gamma)$ and $B_n^{-1}(\gamma)$ are uniformly bounded in both absolute row and column sums, uniformly in $\gamma \in \Theta_2$,*

Assumption 6. *$Z_{1n}(\theta)$ and $Z_{2n}(\theta)$ are not asymptotically multicolinear, uniformly in $\theta \in \Theta$; and $\lim_{n\to\infty} \frac{1}{n}[Z_{2n}'(\theta)Z_{2n}(\theta)]$ exists and is nonsingular, uniformly in $\theta \in \Theta$.*

[6]Kelejian and Prucha (2006) address an important issue on parameter space when spatial weights matrices are not row-normalized, leading to a practical definition of the parameter space that is typically n-dependent.

Assumption 7. *The elements of* $A_{n,\rho_i}(\rho), i = 1, \cdots, k_1$, *are uniformly bounded, uniformly in* $\rho \in \Theta_1$; *and the elements of* $B_{n,\gamma_j}(\gamma), j = 1, \cdots, k_2$, *are uniformly bounded, uniformly in* $\gamma \in \Theta_2$.

Assumptions 1-3 are standard assumptions that provide essential features on the parameter space, the disturbances and the design matrix. Assumption 2 sets up the basic requirements for the error vector u_n so that the central limit theorems for linear-quadratic forms of Kelejian and Prucha (2001) can be applied. Assumptions 4 and 5 are essential requirements for keeping the spatial dependence to within a manageable degree (see Lee, 2004). Assumption 6 ensures that the additional regressors generated by the spatial externalities in the modelled effect are not asymptotically multicolinear with the regular regressors, and are not asymptotically multicolinear among themselves. Assumption 7 ensures that the two spatial-matrix functions are smooth enough.

3.1 Parameter identifiability and consistency of the QMLE

Define $\tilde{\ell}_n(\xi) = \mathrm{E}\ell_n(\xi)$, where the expectation operator corresponds to the true parameter vector ξ_0. This **expected loglikelihood** is the key function for proving the parameter identifiability and consistency of the QMLEs. It is easy to show that

$$\begin{aligned}\tilde{\ell}_n(\xi) &= -\frac{n}{2}\ln(\pi\sigma^2) - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{\sigma_0^2}{2\sigma^2}\mathrm{tr}[\Omega_n(\gamma_0)\Omega_n^{-1}(\gamma)], \\ &\quad -\frac{1}{2\sigma^2}[X_n(\rho)\beta - X_n(\rho_0)\beta_0]'\Omega_n^{-1}(\gamma)[X_n(\rho)\beta - X_n(\rho_0)\beta_0]. \qquad (10)\end{aligned}$$

Note that $\tilde{\ell}_n(\xi)$ is strictly concave in β and σ^2. Thus, for a given θ, it can be shown that $\tilde{\ell}_n(\xi)$ is partially maximized at

$$\tilde{\beta}_n(\theta) = [X_n'(\rho)\Omega_n^{-1}(\gamma)X_n(\rho)]^{-1}X_n'(\rho)\Omega_n^{-1}(\gamma)X_n(\rho_0)\beta_0, \qquad (11)$$

$$\tilde{\sigma}_n^2(\theta) = \frac{\sigma_0^2}{n}\mathrm{tr}[\Omega_n(\gamma_0)\Omega_n^{-1}(\gamma)] + \frac{1}{n}\beta_0'X_n'(\rho_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)X_n(\rho_0)\beta_0, \qquad (12)$$

where $M_{1n}(\theta) = I_n - Z_{1n}(\theta)[Z_{1n}(\theta)Z_{1n}'(\theta)]^{-1}Z_{1n}'(\theta)$, resulting in a **concentrated expected loglikelihood**

$$\tilde{\ell}_n^c(\theta) = -\frac{n}{2}[1 + \ln(2\pi)] - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{n}{2}\ln[\tilde{\sigma}_n^2(\theta)]. \qquad (13)$$

The parameter identifiability is based on the (asymptotic) behavior of $\tilde{\ell}_n^c(\theta)$ and the consistency of $\hat{\xi}_n$ is based on the (asymptotic) behavior of the difference $\ell_n^c(\theta) - \tilde{\ell}_n^c(\theta)$.

Theorem 1. (Identifiability.) *Under Assumptions 1–7, ξ_0 is globally identifiable.*

Proof: A sketch of the proof is given below. The details are supplemented in Appendix B under Lemmas B.1 – B.3. Under Assumption 3, β_0 and σ_0^2 are identifiable once θ_0 is identified. Thus, the problem of global identifiability of ξ_0 reduces to the problem of global identifiability of θ_0. Following White (1996, Definition 3.3), one needs to show that

$$\limsup_{n\to\infty}\left[\max_{\theta\in\bar{N}_\epsilon(\theta_0)}\frac{1}{n}\tilde{\ell}_n^c(\theta) - \frac{1}{n}\tilde{\ell}_n^c(\theta_0)\right] < 0, \tag{14}$$

where $\bar{N}_\epsilon(\theta_0)$ is the compact complement of an open sphere in Θ centered at θ_0 with fixed radius $\epsilon > 0$.

Given in Appendix B, Lemma B.1 shows that $\frac{1}{n}\ln|\Omega(\gamma)|$ is uniformly equicontinuous on Θ_2, Lemma B.2 shows that $\tilde{\sigma}_n^2(\theta)$ is uniformly equicontinuous on Θ, and Lemma B.3 proves that $\tilde{\sigma}_n^2(\theta)$ is uniformly bounded away from zero on Θ. Thus, $\frac{1}{n}\tilde{\ell}_n^c(\theta)$ is uniformly equicontinuous on Θ.

Now, using the auxiliary quantities $\tilde{\ell}_{n,a}^c(\theta)$ and $\tilde{\sigma}_{n,a}^2(\gamma)$ defined in the proof for Lemma B.3, we have, $\tilde{\ell}_n^c(\theta) = \tilde{\ell}_{n,a}^c(\theta) - \frac{n}{2}[\ln\tilde{\sigma}_n^2(\theta) - \ln\tilde{\sigma}_{n,a}^2(\gamma)]$, $\tilde{\ell}_n^c(\theta_0) = \tilde{\ell}_{n,a}^c(\gamma_0)$, and

$$\frac{1}{n}\tilde{\ell}_n^c(\theta) - \frac{1}{n}\tilde{\ell}_n^c(\theta_0) = \frac{1}{n}[\tilde{\ell}_{n,a}^c(\gamma) - \tilde{\ell}_{n,a}^c(\gamma_0)] - \frac{1}{2}[\ln\tilde{\sigma}_n^2(\theta) - \ln\tilde{\sigma}_{n,a}^2(\gamma)].$$

From the proof of Lemma B.3, we have concluded that $\frac{1}{n}[\tilde{\ell}_{n,a}^c(\gamma) - \tilde{\ell}_{n,a}^c(\gamma_0)] \le 0$, and that $\tilde{\sigma}_{n,a}^2(\gamma)$ is bounded away from zero uniformly on Θ_2. From (12), $\tilde{\sigma}_{n,a}^2(\gamma) \le \tilde{\sigma}_n^2(\theta)$, and thus $\frac{1}{n}\tilde{\ell}_n^c(\theta) - \frac{1}{n}\tilde{\ell}_n^c(\theta_0) \le 0$. If the global identifiability condition were not satisfied, there would exist a sequence $\theta_n \in \bar{N}_\epsilon(\theta_0)$ that would converge to $\theta_+ = \{\rho_+', \gamma_+'\}' \neq \theta_0$ such that $\lim_{n\to\infty}[\frac{1}{n}\tilde{\ell}_n^c(\theta_n) - \frac{1}{n}\tilde{\ell}_n^c(\theta_0)] = 0$. As $\frac{1}{n}\tilde{\ell}_n^c(\theta)$ is uniformly equicontinuous on Θ, this would be possible only if $\lim_{n\to\infty}\frac{1}{n}[\tilde{\ell}_{n,a}^c(\gamma_+) - \tilde{\ell}_{n,a}^c(\gamma_0)] = 0$ and $\lim_{n\to\infty}[\tilde{\sigma}_n^2(\theta_+) - \tilde{\sigma}_{n,a}^2(\gamma_+)] = 0$. The latter requirement is a contradiction to Assumption 6, which guarantees that $\forall\,\theta \in \bar{N}_\epsilon(\theta_0)$, $\frac{1}{n}\beta_0'X_n'(\rho_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)X_n(\rho_0)\beta_0 > 0$. Therefore, θ_0 and hence ξ_0 must be globally identifiable.

Theorem 2. (Consistency.) *Under Assumptions 1–7, we have,* $\hat{\xi}_n \xrightarrow{p} \xi_0$.

Proof: Following the global identifiability proved in Theorem 1, it suffices to show that $\frac{1}{n}[\ell_n^c(\theta) - \tilde{\ell}_n^c(\theta)] \xrightarrow{p} 0$, uniformly in $\theta \in \Theta$ (White, 1996, Theorem 3.4). From (5) and (13), we have $\frac{1}{n}[\ell_n^c(\theta) - \tilde{\ell}_n^c(\theta)] = -\frac{1}{2}[\ln \hat{\sigma}_n^2(\theta) - \ln \tilde{\sigma}_n^2(\theta)]$. By a Taylor expansion of $\ln \hat{\sigma}_n^2(\theta)$ at $\tilde{\sigma}_n^2(\theta)$, we obtain $|\ln \hat{\sigma}_n^2(\theta) - \ln \tilde{\sigma}_n^2(\theta)| = |\hat{\sigma}_n^2(\theta) - \tilde{\sigma}_n^2(\theta)|/\bar{\sigma}_n^2(\theta)$, where $\bar{\sigma}_n^2(\theta)$ lies between $\hat{\sigma}_n^2(\theta)$ and $\tilde{\sigma}_n^2(\theta)$. As $\tilde{\sigma}_n^2(\theta)$ is uniformly bounded away from zero on Θ_2 from Lemma B.3, it follows that $\bar{\sigma}_n^2(\theta)$ will be bounded away from zero uniformly on Θ_2 in probability. So, the problem reduces to proving that $\hat{\sigma}_n^2(\theta) - \tilde{\sigma}_n^2(\theta) \xrightarrow{p} 0$, uniformly in $\theta \in \Theta$, which is given in Lemma B.4 in Appendix B.

3.2 Asymptotic normality of the QMLE

Some additional regularity assumptions are necessary for the asymptotic normality of the QMLEs to hold. These are essentially the conditions to ensure the existence of the inverse of the expected information matrix, and the smoothness of the Hessian matrix in a small neighborhood of θ_0.

Assumption 8. $\lim_{n\to\infty} \frac{1}{n}\Lambda_n$ *exists and is nonsingular.*

Assumption 9. $\frac{\partial}{\partial \gamma_i}\Omega_n^{-1}(\gamma)$ *is uniformly bounded in row and column sums, uniformly in a neighborhood of* γ_0.

Assumption 10. *The elements of* $A_{n,\rho_i\rho_j}(\rho)$ *and their derivatives are uniformly bounded, uniformly in a neighborhood of* ρ_0*; the elements of* $B_{n,\gamma_i\gamma_j}(\gamma)$ *and their derivatives are uniformly bounded, uniformly in a neighborhood of* γ_0.

Theorem 3. (Asymptotic Normality.) *Under Assumptions 1-10, we have*

$$\sqrt{n}(\hat{\xi}_n - \xi_0) \xrightarrow{D} N\left[0,\ I^{-1}(\xi_0)K(\xi_0)I^{-1}(\xi_0)\right]$$

where $I(\xi_0) = \lim_{n\to\infty} \frac{1}{n} I_n(\xi_0)$ *and* $K(\xi_0) = \lim_{n\to\infty} \frac{1}{n} K_n(\xi_0)$.

Proof: An outline is given here and the detail is given in Appendix B under Lemmas B.5 and B.6. A Taylor series expansion of $G_n(\hat{\xi}_n) = 0$ at ξ_0 gives

$$\sqrt{n}(\hat{\xi}_n - \xi_0) = -\left(\frac{1}{n}H_n(\bar{\xi}_n)\right)^{-1} \frac{1}{\sqrt{n}}G_n(\xi_0),$$

where $\bar{\xi}_n$ lies between $\hat{\xi}_n$ and ξ_0. As $\hat{\xi}_n \xrightarrow{p} \xi_0$, $\bar{\xi}_n \xrightarrow{p} \xi_0$. The expressions for the gradient $G_n(\xi)$ and Hessian $H_n(\xi)$ are given in Appendix A.

From Appendix A, we have the elements of $G_n(\xi_0)$: $\frac{1}{\sigma^2} Z_n' u_n$, $\frac{1}{2\sigma_0^2} u_n' \Omega^*_{n,\gamma_i} u_n - \frac{1}{2}\mathtt{tr}(\Omega^*_{n,\gamma_i})$, $i = 1, \cdots, k_2$, and $\frac{1}{2\sigma_0^4} u_n' u_n - \frac{n}{2\sigma_0^2}$. These are either linear or quadratic forms of u_n with iid elements. Thus, the central limit theorems for linear and linear-quadratic forms of Kelejian and Prucha (2001) can be used to prove that

$$\frac{1}{\sqrt{n}} G_n(\xi_0) \xrightarrow{D} N[0,\ K(\xi_0)],$$

where $K(\xi_0) = \lim_{n\to\infty} \frac{1}{n} K_n(\xi_0)$.

Lemma B.5 shows that $\frac{1}{n}[H_n(\bar{\xi}_n) - H_n(\xi_0)] = o_p(1)$, and Lemma B.6 shows that $\frac{1}{n}[H_n(\xi_0) + I_n(\xi_0)] = o_p(1)$. Finally, Assumptions 6 and 8 guarantee the existence of $I_n^{-1}(\xi_0)$. The result of the theorem follows.

4 Tests for Spatial Externalities

With the variance estimate and the large sample properties given in the previous two sections, one can carry out various types of inferences, concerning the regression coefficients β_0, the spatial parameters ρ_0 related to regressors, and the spatial parameters γ_0 related to errors. However, one is often interested in testing the existence/nonexistence of the spatial effects in the model, i.e., testing for ρ_0 or $\gamma_0 = 0$, or both. The special structure of the $I_n(\xi_0)$ and $K_n(\xi_0)$ matrices given in Section 2.3 allow great deal simplifications, resulting in simple analytical forms of inferential statistics for β_0, ρ_0, γ_0 and θ_0, respectively. In particular, we have the asymptotic variances,

$$\mathtt{AVar}(\hat{\beta}_n) = \sigma_0^2 (Z_{1n}' M_{2n} Z_{1n})^{-1} \tag{15}$$

$$\mathtt{AVar}(\hat{\rho}_n) = \sigma_0^2 (Z_{2n}' M_{1n} Z_{2n})^{-1} \tag{16}$$

$$\mathtt{AVar}(\hat{\gamma}_n) = 2\Sigma_n^{-1} + \kappa_0 \Pi_n' \Pi_n, \tag{17}$$

where $M_{1n} = I_n - Z_{1n}(Z_{1n}' Z_{1n})^{-1} Z_{1n}'$, $M_{2n} = I_n - Z_{2n}(Z_{2n}' Z_{2n})^{-1} Z_{2n}'$, $\Sigma_n = \Lambda_n - \frac{1}{n}\Phi_n' 1_n 1_n' \Phi_n$, $\Pi_n = \Phi_n \Sigma_n^{-1} - \tau_n^{-1} 1_n 1_n' \Phi_n \Lambda_n^{-1}$, and $\tau_n = n - 1_n' \Phi_n \Lambda_n^{-1} \Phi_n' 1_n$. Further, it should be interesting to conduct joint inferences for ρ_0 and γ_0. To do this, the asymptotic covariance (ACov) between $\hat{\rho}_n$ and $\hat{\gamma}_n$ is needed. We obtain, after some algebra,

$$\mathtt{ACov}(\hat{\rho}_n, \hat{\gamma}_n) = \alpha_0\sigma_0(Z'_{2n}M_{1n}Z_{2n})^{-1}Z'_{2n}M_{1n}\Pi_n, \tag{18}$$

Thus, the expressions given in (16)-(18) together give the asymptotic variance for $\hat{\theta}_n = (\hat{\rho}_n, \hat{\gamma}_n)'$, which can be used for joint inferences for ρ_0 and γ_0. The detailed derivations for (15)-(18) are given in Appendix A.

The results of (15)-(18) are interesting. They show that estimating γ_0 and σ_0 has no impact asymptotically on the inferences for β_0 and ρ_0. In other words, whether γ_0 and σ_0 are known or estimated does not change the expressions for $\mathtt{Avar}(\hat{\beta}_n)$ and $\mathtt{Avar}(\hat{\rho}_n)$. Similarly, estimating β_0 and ρ_0 has no impact asymptotically on the inferences for γ_0 and σ_0. When $\kappa_0 = 0$, i.e., the kurtosis of the error distribution is the same as that of a normal distribution, $\mathtt{AVar}(\hat{\gamma}) = 2\Sigma_n^{-1}$, which is the same as when errors are exactly normal. When $\alpha_0 = 0$, i.e., the error distribution is symmetric, $\mathtt{ACov}(\hat{\rho}_n, \hat{\gamma}_n) = 0$, which says that $\hat{\rho}_n$ and $\hat{\gamma}_n$ are asymptotically independent.

Inference can be jointly on a parameter vector, or individually on a contrast of the parameter vector to see, e.g., whether the components of the parameter vector are the same or not. Let c be a column vector representing generically a linear contrast of the parameters involved in the inference. The statistics are presented below.

Inference for β_0. Using (15) a simple Wald-type of inferential statistic, which can easily be used for testing on or constructing confidence interval for $c'\beta_0$, takes the following form

$$t_{1n}(\beta_0) = \frac{c'(\hat{\beta}_n - \beta_0)}{\hat{\sigma}_n\{c'(\hat{Z}'_{1n}\hat{M}_{2n}\hat{Z}_{1n})^{-1}c\}^{\frac{1}{2}}}, \tag{19}$$

where $\hat{Z}_{1n} = Z_{1n}(\hat{\theta}_n)$ and $\hat{M}_{2n} = M_{2n}(\hat{\theta}_n)$. From the asymptotic results presented in Section 3, we see that $t_{1n}(\beta_0)$ follows asymptotically the standard normal distribution. To conduct inference on β_0 jointly, the statistic has the form

$$T_{1n}(\beta_0) = \hat{\sigma}_n^{-2}(\hat{\beta}_n - \beta_0)'\hat{Z}'_{1n}\hat{M}_{2n}\hat{Z}_{1n}(\hat{\beta}_n - \beta_0), \tag{20}$$

which follows asymptotically a chi-squared distribution with p degrees of freedom. The statistics $t_{1n}(\beta_0)$ and $T_{1n}(\beta_0)$ allow the presence of the spatial effects in both the regressors and the errors, locally and globally. However, only the estimation of the regressor-related spatial parameters ρ_0 has impact (through the presence of Z_{2n} in the statistics) on the asymptotic distributions of these statistics.

Inference for ρ_0. Statistical inferences for the spatial effects in the regressors can be carried out individually or jointly as well. The statistics are

$$t_{2n}(\rho_0) = \frac{c'(\hat{\rho}_n - \rho_0)}{\hat{\sigma}_n\{c'(\hat{Z}_{2n}'\hat{M}_{1n}\hat{Z}_{2n})^{-1}c\}^{\frac{1}{2}}}, \tag{21}$$

an asymptotic $N(0, 1)$ random variate, where $\hat{Z}_{2n} = Z_{2n}(\hat{\theta}_n)$ and $\hat{M}_{1n} = M_{1n}(\hat{\theta}_n)$, and

$$T_{2n}(\rho_0) = \hat{\sigma}_n^{-2}(\hat{\rho}_n - \rho_0)'\hat{Z}_{2n}'\hat{M}_{1n}\hat{Z}_{2n}(\hat{\rho}_n - \rho_0), \tag{22}$$

an asymptotic chi-squared random variate with k_1 degrees of freedom. The statistics $t_{2n}(\rho_0)$ and $T_{2n}(\rho_0)$ account for the estimation of β_0, γ_0 and σ_0^2. However, only the estimation of β_0 has impact (through the presence of Z_{1n}) on the asymptotic distributions of these statistics.

Inference for γ_0. Again, when inferences concern the spatial effects in the errors, they can be carried out individually or jointly. The statistics are

$$t_{3n}(\gamma_0) = \frac{c'(\hat{\gamma}_n - \gamma_0)}{\{c'(2\hat{\Sigma}_n^{-1} + \hat{\kappa}_0\hat{\Pi}_n'\hat{\Pi}_n)c\}^{\frac{1}{2}}}, \tag{23}$$

which is asymptotically $N(0, 1)$ distributed, and

$$T_{3n}(\gamma_0) = n(\hat{\gamma}_n - \gamma_0)'(2\hat{\Sigma}_n^{-1} + \hat{\kappa}_0\hat{\Pi}_n'\hat{\Pi}_n)^{-1}(\hat{\gamma}_n - \gamma_0), \tag{24}$$

which follows asymptotically a chi-squared distribution with k_2 degrees of freedom. All the estimated (hat) quantities are evaluated at the QMLE $\hat{\xi}_n$. The statistics $t_{3n}(\gamma_0)$ and $T_{3n}(\gamma_0)$ account for the estimation of β_0, ρ_0, and σ_0^2. However, only the estimation of σ_0^2 has impact on the asymptotic distributions of these statistics.

Inference for ρ_0 and γ_0. Finally, it is of interest in seeing whether there are spatial effects at all. In this case, one may use (16)-(18) to construct a statistic to test this overall spatial effect. The statistic takes the form

$$T_{4n}(\theta_0) = \left(\hat{\theta}_n - \theta_0\right)' \begin{pmatrix} \hat{\sigma}_n^2 \hat{\Psi}_n^{-1}, & \hat{\alpha}_0 \hat{\sigma}_n \hat{\Psi}_n^{-1} \hat{Z}'_{2n} \hat{M}_{1n} \hat{\Pi}_n \\ \sim, & 2\hat{\Sigma}_n^{-1} + \hat{\kappa}_0 \hat{\Pi}'_n \hat{\Pi}_n \end{pmatrix}^{-1} \left(\hat{\theta}_n - \theta_0\right), \tag{25}$$

where $\hat{\Psi}_n = \hat{Z}'_{2n} \hat{M}_{1n} \hat{Z}_{2n}$. The statistic $T_{4n}(\theta_0)$ follows an asymptotic chi-squared distribution of $k_1 + k_2$ degrees of freedom. It is sometimes of interest to test a linear contrast of θ_0, e.g., $\rho_0 = \gamma_0$, to see whether a spatial lag model is appropriate or not. In this case, a general test statistic is of the form

$$t_{4n}(\theta_0) = \frac{c'(\hat{\theta}_n - \theta_0)}{\left\{\hat{\sigma}_n^2 c_1' \hat{\Psi}_n^{-1} c_1 + 2\hat{\alpha}_0 \hat{\sigma}_n c_1' \hat{\Psi}_n^{-1} \hat{Z}'_{2n} \hat{M}_{1n} \hat{\Pi}_n c_2 + c_2'(2\hat{\Sigma}_n^{-1} + \hat{\kappa}_0 \hat{\Pi}'_n \hat{\Pi}_n) c_2\right\}^{1/2}}, \tag{26}$$

where $(c_1', c_2')' = c$. The statistic $t_{4n}(\theta)$ follows asymptotically the $N(0,1)$ distribution.

We note that all the estimated (hat) quantities in the above test statistics are evaluated at the QMLE $\hat{\xi}_n$. The statistics given in (19)-(26) are all of Wald-type, and all possess very simple analytical forms. Thus, they can easily be applied by the empirical researchers. Their large sample behavior is governed by the asymptotic normality of the QMLE. Of particular interest is the last one, which allows us to test the appropriateness of the popular spatial lag model where a SAR(1) process is applied only to the responses. In this case the null hypothesis is $H_0 : c'\theta = 0$ with $c\prime = (1, -1)$ and $\theta = (\rho, \gamma)'$. A rejection of H_0 indicates that the spatial lag model is not appropriate. More importantly, the statistics are robust against nonnormality of the errors. This is important as in real empirical applications, there is often little indication *a priori* that the data are normal.

5 Finite Sample Properties

In this section, we investigate the finite sample properties of the regression estimates (the estimates of the regression coefficients), and the finite sample properties of the tests for spatial externalities, using Monte Carlo simulation. Two data generating processes (DGP) are considered. One corresponds to a hybrid model with local spatial externality in X_n and global spatial externality in the errors (Anselin, 2003), and the other is a generalized spatial lag model which reduces to the standard spatial lag model when $\rho_0 = \gamma_0$ and $W_{1n} = W_{2n}$.

$$\text{DGP 1}: \quad Y_n = (I_n + \rho_0 W_{1n})X_n\beta_0 + (I_n - \gamma_0 W_{2n})^{-1}u_n,$$

$$\text{DGP 2}: \quad Y_n = \rho_0 W_{1n}Y_n + X_n\beta_0 + (I_n - \rho_0 W_{1n})(I_n - \gamma_0 W_{2n})^{-1}u_n.$$

The errors $u_{n,i} = \sigma_0 u^0_{n,i}$, with $\{u^0_{n,i}, i = 1, \cdots, n\}$ being generated from (i) the standard normal distribution, (ii) a normal mixture, and (iii) a normal-gamma mixture. In the cases (ii) and (iii), a 70%-30% mixing strategy is followed, i.e., 70% of the errors are from the standard normal distribution, and the remaining 30% from either a normal distribution with mean zero and standard deviation 2, or an exponential distribution with mean one. The mixture distributions are standardized to have mean zero and variance one to be conformable with the model assumptions. Their skewness and kurtosis of $u_{n,i}$ are (0, 4.57) for the normal mixture and (.6, 4.8) for the normal-gamma mixture, compared with (0, 3) for the case of pure standard normal errors.

The spatial weighting matrices are generated according to Rook contiguity, by randomly allocating the n spatial units on a lattice of $k \times m$ $(\geq n)$ squares. In our case, k is chosen to be 5. The two spatial weight matrices in DGP1 and DGP2 can be the same or different, which does not affect much on the simulation results.

I consider DGPs with two regressors X_1 and X_2, where $X_1 \sim U(0, 10)$ and $X_2 \sim N(0, 4)$. The regression coefficients and the error standard deviation are chosen to be $\beta_0 = (5, 2, 2)$ and $\sigma_0 = 1$. The spatial parameters ρ_0 and γ_0 vary from the set {-0.8, -0.5, -0.2, 0.0, 0.2, 0.5, 0.8}. The sample size n varies from the set {50, 100, 200}. For finite sample performance of the QMLEs, I report the Monte Carlo means and the root mean squared errors (RMSE), and for the finite sample performance of the tests, I report the empirical sizes at the 5% nominal level. Each set of Monte Carlo results (corresponding to a combination of values of n, ρ and γ) is based on 2000 samples.

Tables 1-3 present the Monte Carlo means and RMSEs for the parameter estimates based on DGP1 corresponding to the cases of normal error, normal mixture, and normal-gamma mixture, respectively. To save space, only a part of the results are reported. From the tables we see that the QMLEs generally perform very well. The QMLEs of β, σ, and

ρ are almost unbiased with small RMSEs. The QMLE of γ under estimates γ_0 slightly when $\gamma_0 > 0$. The unreported results show that it may over estimates γ_0 slightly when $\gamma_0 < 0$. The bias of $\hat{\gamma}_n$ reduces when sample size increases. Also, the $\hat{\gamma}_n$ is more variable than $\hat{\rho}_n$, and thus a much larger RMSE than that of $\hat{\rho}$. These conclusions are quite robust with respect to the error distributions as seen from the results of Tables 2 and 3. One exception is that the RMSE of $\hat{\sigma}_n$ is larger when errors are nonnormal than when the errors are normal.

Tables 4-6 present the full Monte Carlo results for the sizes of the four tests introduced in Section 4 based on DGP 1 with the three types of errors. From the results we see that all the four tests have a reasonable finite sample performance. Although they over-reject the null hypothesis when the sample size is not large (50, say), but improve quickly when sample size n is increased from 50 to 100, and then to 200. A striking phenomenon is that these tests are robust against nonnormality of the error distributions, as seen by comparing the results in Tables 5 and 6 with those in Table 4.

The whole Monte Carlo experiment with DGP 1 is repeated using DGP2. One difference is that under DGP2, we are interested in, besides the other things, seeing whether ρ and γ are the same, i.e., testing whether a pure spatial lag model suffices for a given data. Thus, T_{4n} is replaced by t_{4n} in the Monte Carlo experiment with $c = (1, -1)'$. The Monte Carlo results are generally consistent with those based on DGP1. To save space, we report only the empirical sizes in Tables 7-9, with full results available from the author upon request. From the results we see that the four tests perform reasonably well in finite samples. When $n = 50$, there could be a large size distortion depending on the values of ρ and γ, in particular T_{1n}, the test for the regression coefficients β. The size distortion worsens when the errors are nonnormal, from the comparison of the results in Table 7 with the results in Tables 8 and 9. However, when n increases, the sizes quickly converge to their normal level. The test of particular interest in this case, t_{4n}, performs reasonably well with empirical sizes very close to their normal level when n reaches 200. The results given in Tables 8 and 9 show that these tests are robust against nonnormality. A special note is that when $\rho = \gamma$ in Tables 7-9, the empirical sizes correspond to the test for a pure spatial lag model.

6 Conclusions and Discussions

A general model jointly incorporating the local and global spatial externalities in both modelled and unmodelled effects is introduced. Robust methods of inferences procedures are developed based on quasi-maximum likelihood estimation method. Simple analytical forms for the inferential statistics are provided. Large sample properties of the QMLE are studied. Extensive Monte Carlo simulation shows that the QMLEs of the model parameters and the tests possess good finite sample properties. The proposed model is very flexible. The methods of inferences are easy to implement and the tests of spatial externalities can be easily carried out.

The model can be extended to include regressors of no spatial dependence, and to allow u_n to be heteroscedastic. Furthermore, the QMLE is efficient only when the likelihood is correctly specified. In the absence of knowledge about the error distribution, it may be possible to extend the adaptive estimation procedure of Robinson (2006) to improve the efficiency of the QMLEs considered in this paper.

Appendix A: Gradient, Hessian and Related Quantities

The **gradient function** $G_n(\xi) = \frac{\partial}{\partial \xi}\ell(\xi)$ has the elements:

$$
\begin{aligned}
G_{n\beta}(\xi) &= \tfrac{1}{\sigma^2} X_n'(\rho)\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho), \\
G_{n\rho_i}(\xi) &= \tfrac{1}{\sigma^2}[X_{n,\rho_i}(\rho)\beta]'\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho), \quad i = 1,\cdots,k_1, \\
G_{n\gamma_i}(\xi) &= \tfrac{1}{2\sigma^2}\varepsilon_n'(\beta,\rho)\Omega_n^{-1}(\gamma)\Omega_{n\gamma_i}(\gamma)\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho) - \tfrac{1}{2}\mathrm{tr}[\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_i}(\gamma)], \\
&\quad\; i = 1,\cdots,k_2, \\
G_{n\sigma^2}(\xi) &= \tfrac{1}{2\sigma^4}\varepsilon_n'\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho) - \tfrac{n}{2\sigma^2}.
\end{aligned}
$$

Note that in the above derivation, we have used the formulas: $\frac{\partial}{\partial\gamma}\ln|\Omega_n| = \mathrm{tr}(\Omega_n^{-1}\frac{\partial\Omega_n}{\partial\gamma})$ and $\frac{\partial}{\partial\gamma}\Omega_n^{-1} = -\Omega_n^{-1}\frac{\partial\Omega_n}{\partial\gamma}\Omega_n^{-1}$.

To derive the expression for $K_n(\xi_0)$, the **variance of** $G_n(\xi_0)$, recall the notation Z_n and Ω^*_{n,γ_i} defined in Section 2.3, and use the relations $\Omega_n = B_nB_n'$ and $\varepsilon_n(\beta_0,\rho_0) = B_nu_n$. The gradient function at ξ_0 can be written as

$$
G_n(\xi_0) = \begin{cases}
\frac{1}{\sigma_0^2}Z_n'u_n, \\
\frac{1}{2\sigma_0^2}u_n'\Omega^*_{n,\gamma_i}u_n - \frac{1}{2}\mathtt{tr}(\Omega^*_{n,\gamma_i}), \quad i = 1,\cdots,k_1, \\
\frac{1}{2\sigma_0^4}u_n'u_n - \frac{n}{2\sigma_0^2}.
\end{cases}
$$

As the elements of u_n are iid with mean zero, variance one, skewness α_0, and kurtosis $\kappa_0 + 3$, the following formulas for conformable matrices Z, Φ_1 and Φ_2 can easily be established,

$$
\begin{aligned}
\mathrm{E}[(Z'u_n)\cdot(Z'u_n)'] &= \sigma_0^2 Z'Z, \\
\mathrm{E}[u_n\cdot(u_n'\Phi_iu_n)] &= \sigma_0^3\alpha_0\ \mathtt{daigv}(\Phi_i), \quad i = 1,2, \\
\mathrm{Cov}(u_n'\Phi_iu_n, u_n'\Phi_ju_n) &= \sigma_0^4\kappa_0\ \mathtt{diagv}(\Phi_i)'\ \mathtt{diagv}(\Phi_j) + \sigma_0^4\mathtt{tr}(\Phi_i\Phi_j + \Phi_i\Phi_j'),
\end{aligned}
$$

for $i, j = 1, 2$, some simple algebra leads to the expression for $K_n(\xi_0)$.

Let $X_{n,\rho_i\rho_j}(\rho) = \frac{\partial^2}{\partial\rho_i\partial\rho_j}X_n(\rho)$, and $\Omega_{n,\gamma_i\gamma_j}(\gamma) = \frac{\partial^2}{\partial\gamma_i\partial\gamma_j}\Omega_n(\gamma)$. The **Hessian matrix function** $H_n(\xi) = \frac{\partial}{\partial\xi'}G_n(\xi)$ has the elements,

$$
\begin{aligned}
H_{n\beta\beta}(\xi) &= -\tfrac{1}{\sigma^2}X_n'(\rho)\Omega_n^{-1}(\gamma)X_n(\rho)\\
H_{n\beta\rho_i}(\xi) &= \tfrac{1}{\sigma^2}X_{n,\rho_i}'(\rho)\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho) - \tfrac{1}{\sigma^2}X_n'(\rho)\Omega_n^{-1}(\gamma)X_{n,\rho_i}(\rho)\beta\\
H_{n\beta\gamma_i}(\xi) &= -\tfrac{1}{\sigma^2}X_n'(\rho)\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_i}(\gamma)\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho)\\
H_{n\beta\sigma^2}(\xi) &= -\tfrac{1}{\sigma^4}X_n'(\rho)\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho)\\
H_{n\rho_i\rho_j}(\xi) &= \tfrac{1}{\sigma^2}[X_{n,\rho_i\rho_j}(\rho)\beta]'\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho) - \tfrac{1}{\sigma^2}[X_{n,\rho_i}(\rho)\beta]'\Omega_n^{-1}(\gamma)X_{n,\rho_j}(\rho)\beta\\
H_{n\rho_i\gamma_j}(\xi) &= -\tfrac{1}{\sigma^2}[X_{n,\rho_i}(\rho)\beta]'\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_j}(\gamma)\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho)\\
H_{n\rho_i\sigma^2}(\xi) &= -\tfrac{1}{\sigma^4}[X_{n,\rho_i}(\rho)\beta]'\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho)\\
H_{n\gamma_i\gamma_j}(\xi) &= \tfrac{1}{2}\text{tr}\left[\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_j}(\gamma)\Omega_n^{-1}(\gamma)\Omega_{n\gamma_i}(\gamma) - \Omega_n^{-1}(\gamma)\Omega_{n,\gamma_i\gamma_j}(\gamma)\right] - \tfrac{1}{2\sigma^2}\varepsilon_n(\beta,\rho)'\\
&\quad \Omega_n^{-1}(\gamma)\left[2\Omega_{n,\gamma_j}(\gamma)\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_i}(\gamma) - \Omega_{n,\gamma_i\gamma_j}(\gamma)\right]\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho)\\
H_{n\gamma_i\sigma^2}(\xi) &= -\tfrac{1}{2\sigma^4}\varepsilon_n(\beta,\rho)'\left[\Omega_n^{-1}(\gamma)\Omega_{n,\gamma_i}(\gamma)\Omega_n^{-1}(\gamma)\right]\varepsilon_n(\beta,\rho)\\
H_{n\sigma^2\sigma^2}(\xi) &= \tfrac{n}{2\sigma^4} - \tfrac{1}{\sigma^6}\varepsilon_n(\beta,\rho)'\Omega_n^{-1}(\gamma)\varepsilon_n(\beta,\rho).
\end{aligned}
$$

The **expected information matrix** $I(\xi_0) = -\text{E}[H(\xi_0)]$ has the elements,

$$
\begin{aligned}
I_{n,\beta\beta}(\xi_0) &= \tfrac{1}{\sigma_0^2}X_n'(\rho_0)\Omega_n^{-1}(\gamma)X_n(\rho_0) = \tfrac{1}{\sigma_0^2}Z_{1n}'Z_{1n},\\
I_{n,\beta\rho}(\xi_0) &= \tfrac{1}{\sigma_0^2}\left\{X_n'(\rho_0)\Omega_n^{-1}(\gamma_0)X_{n,\rho_i}(\rho_0)\beta_0\right\} = \tfrac{1}{\sigma_0^2}Z_{1n}'Z_{2n},\\
I_{n,\rho\rho}(\xi_0) &= \tfrac{1}{\sigma_0^2}\left\{[X_{n,\rho_i}(\rho_0)\beta_0]'\Omega_n^{-1}(\gamma_0)X_{n,\rho_j}(\rho_0)\beta_0\right\} = \tfrac{1}{\sigma_0^2}Z_{2n}'Z_{2n},\\
I_{n,\gamma\gamma}(\xi_0) &= \tfrac{1}{2}\left\{\texttt{tr}\left[\Omega_n^{-1}(\gamma_0)\Omega_{n,\gamma_j}(\gamma_0)\Omega_n^{-1}(\gamma_0)\Omega_{n,\gamma_i}(\gamma_0)\right]\right\} = \tfrac{1}{2}\Lambda_n,\\
I_{n,\gamma\sigma^2}(\xi_0) &= \tfrac{1}{2\sigma_0^2}\text{tr}\left[\Omega_n^{-1}(\gamma_0)\Omega_{n,\gamma_i}(\gamma_0)\right] = \tfrac{1}{2\sigma_0^2}\Phi_n'1_n,\\
I_{n,\sigma^2\sigma^2}(\xi_0) &= \tfrac{n}{2\sigma_0^4},
\end{aligned}
$$

with the remaining elements being null vectors or matrices.

To derive $\texttt{AVar}(\hat{\beta}_n)$, $\texttt{AVar}(\hat{\rho}_n)$, $\texttt{AVar}(\hat{\gamma}_n)$, and $\texttt{ACov}(\hat{\rho}_n, \hat{\gamma}_n)$, given in (15)-(18), note that $K_n(\xi_0) = I_n(\xi_0) + K_n^0$, where

$$
K_n^0 = \begin{pmatrix} 0, & \frac{\alpha_0}{2\sigma_0}Z_n'\Phi_n, & \frac{\alpha_0}{2\sigma_0^3}Z_n'1_n \\ \sim, & \frac{\kappa_0}{4}\Phi_n'\Phi_n, & \frac{\kappa_0}{4\sigma_0^2}\Phi_n'1_n \\ \sim, & \sim, & \frac{n\kappa_0}{4\sigma_0^4} \end{pmatrix}.
$$

Partition $I_n(\xi_0)$ and K_n^0 according to $(\beta_0', \rho_0')'$ and $(\gamma_0', \sigma_0^2)'$, and denote the elements of the partitioned $I_n(\xi_0)$ by I_{11}, I_{12}, I_{21} and I_{22}, and the elements of the partitioned K_n^0 by K_{11}, K_{12}, K_{21} and K_{22}. As $I_{12} = 0, I_{21} = 0$, and $K_{11} = 0$, we have

$$\begin{aligned} \text{AVar}(\hat{\xi}_n) &= I_n^{-1}(\xi_0) K_n(\xi_0) I_n^{-1}(\xi_0) \\ &= \begin{pmatrix} I_{11}^{-1}, & 0 \\ 0, & I_{22}^{-1} \end{pmatrix} + \begin{pmatrix} 0, & I_{11}^{-1} K_{12} I_{22}^{-1} \\ I_{22}^{-1} K_{21} I_{11}^{-1}, & I_{22}^{-1} K_{22} I_{22}^{-1} \end{pmatrix} \end{aligned}$$

which leads immediately to $\text{AVar}[(\hat{\beta}_n', \hat{\rho}_n')'] = I_{11}^{-1} = \sigma_0^2 (Z_n' Z_n)^{-1}$, and thus the expressions $\texttt{AVar}(\hat{\beta}_n)$ and $\texttt{AVar}(\hat{\rho}_n)$ in (15) and (16).

To derive $\texttt{AVar}(\hat{\gamma}_n)$ given in (17), one needs the upper-left corner submatrix of $I_{22}^{-1} K_{22} I_{22}^{-1}$. We have,

$$I_{22}^{-1} = 2\sigma_0^2 \begin{pmatrix} \sigma_0^2 \Lambda_n, & \Phi_n' 1_n \\ 1_n' \Phi_n, & \frac{n}{\sigma_0^2} \end{pmatrix}^{-1} = \begin{pmatrix} \frac{1}{\sigma_0^2} \Sigma_n^{-1}, & -\frac{1}{\tau_n} \Lambda_n^{-1} \Phi_n' 1_n \\ -\frac{1}{\tau_n} 1_n' \Phi_n \Lambda_n^{-1}, & \frac{\sigma_0^2}{\tau_n} \end{pmatrix}.$$

With

$$K_{22} = \begin{pmatrix} \frac{\kappa_0}{4} \Phi_n' \Phi_n, & \frac{\kappa_0}{4\sigma_0^2} \Phi_n' 1_n \\ \sim, & \frac{n\kappa_0}{4\sigma_0^4} \end{pmatrix},$$

some simple algebra leads to the expression for $\texttt{AVar}(\hat{\gamma}_n)$.

Finally, to derive $\texttt{ACov}(\hat{\rho}_n, \hat{\gamma}_n)$ given in (18), one needs the lower-left corner submatrix of $I_{11}^{-1} K_{12} I_{22}^{-1}$. As $I_{11}^{-1} = \sigma_0^2 (Z_n^{-1} Z_n)^{-1}$ where $Z_n = \{Z_{1n}, Z_{2n}\}$, we obtain,

$$I_{11}^{-1} = \sigma_0^2 \begin{pmatrix} (Z_{1n}' M_{2n} Z_{1n})^{-1}, & (Z_{1n}' Z_{1n})^{-1} Z_{1n}' Z_{2n} (Z_{2n}' M_{1n} Z_{2n})^{-1} \\ \sim, & (Z_{2n}' M_{1n} Z_{2n})^{-1} \end{pmatrix}$$

Now, $K_{12} = (\frac{\alpha_0}{2\sigma_0} Z_n' \Phi_n, \frac{\alpha_0}{2\sigma_0^3} Z_n' 1_n)$, which can be written as

$$K_{12} = \frac{\alpha_0}{2\sigma_0^3} \begin{pmatrix} \sigma_0^2 Z_{1n}' \Phi_n, & Z_{1n}' 1_n \\ \sigma_0^2 Z_{2n}' \Phi_n, & Z_{2n}' 1_n \end{pmatrix}.$$

After matrix multiplications, some tedious algebra leads to the expression for $\texttt{ACov}(\hat{\rho}_n, \hat{\gamma}_n)$.

Appendix B: Detailed Proofs of the Theorems

This appendix presents six lemmas. Lemmas B.1 – B.3 fill in the details for the proof of Theorem 1, Lemma B.4 gives additional details for proving Theorem 2, and Lemmas B.5 and B.6 provide details for the proof of Theorem 3. To simplify the proofs of these lemmas, assume without loss of generality that ρ and γ are both scalars.

Lemma B.1. *Under the Assumption 5 and Assumption 7, $\frac{1}{n}\ln|\Omega(\gamma)|$ is uniformly equicontinuous in $\gamma \in \Theta_2$.*

Proof: By the mean value theorem, we have

$$\frac{1}{n}\left(\ln|\Omega_n(\gamma_1)| - \ln|\Omega_n(\gamma_2)|\right) = \frac{1}{n}\mathrm{tr}\left(\Omega_n^{-1}(\bar{\gamma})\Omega_{n\gamma}(\bar{\gamma})\right)(\gamma_1 - \gamma_2),$$

where $\bar{\gamma}$ lies between γ_1 and γ_2. As $\Omega_n(\gamma) = B_n(\gamma)B_n'(\gamma)$, $\Omega_{n,\gamma}(\gamma) = B_{n,\gamma}(\gamma)B_n'(\gamma) + B_n(\gamma)B_{n,\gamma}'(\gamma)$. As $B_n(\gamma)$ is uniformly bounded in absolute row sums, uniformly in $\gamma \in \Theta_2$ (Assumption 5), and the elements of $B_{n,\gamma}(\gamma)$ are uniformly bounded, uniformly in $\gamma \in \Theta_2$ (Assumption 7), it follows that the elements of $\Omega_{n\gamma}(\bar{\gamma})$ are uniformly bounded, uniformly in $\bar{\gamma} \in \Theta_2$. Further, as $B_n^{-1}(\gamma)$ is uniformly bounded in absolute row and column sums, uniformly in $\gamma \in \Theta_2$ (Assumption 5), $\Omega_n^{-1}(\gamma) = B_n^{-1}(\gamma)B_n'^{-1}(\gamma)$ is also uniformly bounded in absolute row and column sums, uniformly in $\gamma \in \Theta_2$.[7] It follows that $\frac{1}{n}\mathrm{tr}\left[\Omega_n^{-1}(\bar{\gamma})\Omega_{n\gamma}(\bar{\gamma})\right] = O(1)$. Thus, $\frac{1}{n}\ln|\Omega(\gamma)|$ is uniformly equicontinuous in $\gamma \in \Theta_2$. As Θ_2 is a compact set, $\frac{1}{n}\left[\ln|\Omega_n(\gamma_1)| - \ln|\Omega_n(\gamma_2)|\right] = O(1)$.

Lemma B.2. *Under the Assumption 3–8, the $\tilde{\sigma}_n^2(\theta)$ defined in (12) is uniformly equicontinuous in $\theta \in \Theta$.*

Proof: By the mean value theorem:

$$\tilde{\sigma}_n^2(\theta_1) - \tilde{\sigma}_n^2(\theta_2) = \tilde{\sigma}_{n\rho}^2(\bar{\theta})(\rho_1 - \rho_2) + \tilde{\sigma}_{n\gamma}^2(\bar{\theta})(\gamma_1 - \gamma_2),$$

where $\theta_1 = (\rho_1, \gamma_1)'$, $\theta_2 = (\rho_2, \gamma_2)'$, and $\bar{\theta}$ lies between θ_1 and θ_2. The partial derivatives

[7] This follows from a property of the matrix norm as the maximum of the absolute row sums is a matrix norm. See Horn and Johnson (1985).

can be shown, after a lengthy algebra, to have the forms,

$$\begin{aligned}\tilde{\sigma}^2_{n\rho}(\theta) &= \frac{1}{n}\beta_0' X'(\rho_0)D_n(\theta)X(\rho_0)\beta_0, \text{ and} \\ \tilde{\sigma}^2_{n\gamma}(\theta) &= -\frac{\sigma_0^2}{n}\mathrm{tr}[\Omega_n(\rho_0)\Omega_n^{-1}(\gamma)\Omega_{n,\gamma}(\gamma)\Omega_n^{-1}(\gamma)] + \frac{1}{n}\beta_0' X'(\rho_0)F_n(\theta)X(\rho_0)\beta_0\end{aligned}$$

where $D_n(\theta) = -B_n'^{-1}(\gamma)[R_n'(\theta)M_{1n}(\theta) + M_{1n}(\theta)R_n(\theta)]B_n^{-1}(\gamma)$,

$R_n(\theta) = B_n^{-1}(\gamma)A_{n,\rho}(\rho)A_n^{-1}(\rho)B_n(\gamma)[I_n - M_{1n}(\theta)]$,

$F_n(\theta) = -B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)\Omega_{n,\gamma}(\gamma)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)$.

As the elements of X_n are uniformly bounded (Assumption 3) and the absolute row sums of $A(\rho)$ are uniformly bounded (Assumption 4), uniformly in $\rho \in \Theta_1$, the elements of $X_n(\rho)$ are uniformly bounded, uniformly in $\rho \in \Theta_2$. The matrices $B_n(\gamma)$ and $B_n^{-1}(\gamma)$ are uniformly bounded in absolute row and column sums, uniformly in $\gamma \in \Theta_2$ (Assumption 5), so are the matrices $\Omega_n(\gamma)$ and $\Omega_n^{-1}(\gamma)$. It follows that the elements of $B_n^{-1}(\gamma)X_n(\rho)$ are uniformly bounded, uniformly in $\theta \in \Theta$. This together with the Assumption 3 ensure that the projection matrices $M_{1n}(\theta)$ and $I_n - M_{1n}(\theta)$ are uniformly bounded in absolute row and column sums, uniformly in $\theta \in \Theta$.[8] Thus, the matrices $D_n(\theta)$, $R_n(\theta)$, and $F_n(\theta)$ are all uniformly bounded in their elements, uniformly in θ in Θ, which leads to $\tilde{\sigma}^2_{n\rho}(\theta) = O(1)$ and $\tilde{\sigma}^2_{n\gamma}(\theta) = O(1)$. Thus, $\tilde{\sigma}^2_n(\theta)$ is uniformly equicontinuous in θ in Θ. As Θ is compact, it follows that $\tilde{\sigma}^2_n(\theta_1) - \tilde{\sigma}^2_n(\theta_2) = O(1)$, uniformly in θ_1 and θ_2 in Θ.

Lemma B.3. *Under the Assumption 3–7, the $\tilde{\sigma}^2_n(\theta)$ defined in (12) is uniformly bounded away from zero on Θ.*

Proof: To prove $\tilde{\sigma}^2_n(\theta)$ is uniformly bounded away from zero on Θ, and to finally show the global identifiability of θ_0, we employ a similar trick as did Lee (2004b, Appendix B). Consider an auxiliary model $Y_n = B_n(\gamma)u_n$, i.e., a pure spatial error process. We have the loglikelihood function $\ell_{n,a}(\gamma, \sigma^2) = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{1}{2\sigma^2}Y_n'\Omega_n^{-1}(\gamma)Y_n$, and its expectation $\tilde{\ell}_{n,a}(\gamma, \sigma^2) = -\frac{n}{2}\ln(2\pi\sigma^2) - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{\sigma_0^2}{2\sigma^2}\mathrm{tr}\,(\Omega_n(\gamma_0)\Omega^{-1}(\gamma))$. The latter is maximized at $\tilde{\sigma}^2_{n,a}(\gamma) = \frac{\sigma_0^2}{n}\mathrm{tr}\,(\Omega_n(\gamma_0)\Omega^{-1}(\gamma))$, resulting in the concentrated function

[8] See Lee (2004b, Appendix A) for the proof of a simpler version of this result.

$\tilde{\ell}^c_{n,a}(\gamma) = -\frac{n}{2}[1+\ln(2\pi)] - \frac{1}{2}\ln|\Omega_n(\gamma)| - \frac{n}{2}\ln\tilde{\sigma}^2_{n,a}(\gamma)$. We have $\tilde{\sigma}^2_{n,a}(\gamma_0) = \sigma_0^2$, and hence $\tilde{\ell}^c_{n,a}(\gamma_0) = -\frac{n}{2}[1+\ln(2\pi)] - \frac{1}{2}\ln|\Omega_n(\gamma_0)| - \frac{n}{2}\ln\sigma_0^2$. By Jensen's inequality, $\tilde{\ell}^c_{n,a}(\gamma) = \max_{\sigma^2}\mathrm{E}[\ell_{n,a}(\gamma,\sigma^2)] \leq \mathrm{E}[\ell_{n,a}(\gamma_0,\sigma_0^2)] = -\frac{n}{2}\ln(2\pi\sigma_0^2) - \frac{1}{2}\ln|\Omega_n(\gamma_0)| - \frac{n}{2}$. It follows that $\tilde{\ell}^c_{n,a}(\gamma) \leq \tilde{\ell}^c_{n,a}(\gamma_0)$, showing that $\ln\tilde{\sigma}^2_{n,a}(\gamma) \geq \frac{1}{n}[\ln|\Omega_n(\gamma_0)| + \ln|\Omega_n(\gamma)|] - \ln\sigma_0^2$. Lemma B.1 shows that $\frac{1}{n}[\ln|\Omega_n(\gamma_0)| + \ln|\Omega_n(\gamma)|] = O(1)$, hence $\ln\tilde{\sigma}^2_{n,a}(\gamma)$ is bounded from below uniformly in $\gamma \in \Theta_2$. Therefore, $\tilde{\sigma}^2_{n,a}(\gamma)$ is bounded away from zero, uniformly in $\gamma \in \Theta_2$. It follows from (12) that $\tilde{\sigma}^2_n(\theta)$ is also bounded away from zero, uniformly in θ in Θ.

Lemma B.4. *Under Assumptions 1–7, $\hat{\sigma}^2_n(\theta) - \tilde{\sigma}^2_n(\theta) \xrightarrow{p} 0$, uniformly in $\theta \in \Theta$.*

Proof: First, $\hat{\sigma}^2_n(\theta)$ can be rewritten as $\hat{\sigma}^2_n(\theta) = \frac{1}{n}Y_n' B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)Y_n$. With the true model $Y_n = X_n(\rho_0)\beta_0 + B_n(\gamma_0)u_n$, we have

$$
\begin{aligned}
\hat{\sigma}^2_n(\theta) &= \frac{1}{n}\beta_0' X_n'(\rho_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)X_n(\rho_0)\beta_0 \\
&\quad + \frac{1}{n}u_n' B_n'(\gamma_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)B_n(\gamma_0)u_n \\
&\quad + \frac{2}{n}\beta_0' X_n'(\rho_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)B_n(\gamma_0)u_n,
\end{aligned}
$$

and referring to the expression for $\tilde{\sigma}^2_n(\theta)$ given in (12), we obtain,

$$
\begin{aligned}
\hat{\sigma}^2_n(\theta) - \tilde{\sigma}^2_n(\theta) &= \frac{1}{n}u_n' B_n'(\gamma_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)B_n(\gamma_0)u_n - \frac{\sigma_0^2}{n}\mathrm{tr}[\Omega_n(\gamma_0)\Omega_n^{-1}(\gamma)] \\
&\quad + \frac{2}{n}\beta_0' X_n'(\rho_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)B_n(\gamma_0)u_n.
\end{aligned}
$$

We show that the last term above is $o_p(1)$, uniformly in $\theta \in \Theta$. Assumptions 3 and 4 guarantee that the elements of $\beta_0' X_n(\rho_0)$ are uniformly bounded. As $B_n^{-1}(\gamma)$ and $M_{1n}(\theta)$ are both uniformly bounded in absolute row and column sums, uniformly in $\gamma \in \Theta_2$, or in $\theta \in \Theta$, the Assumption 1 and an extension of a result of Lee (2004a, Appendix A) to the case of matrix functions lead to

$$
\frac{2}{n}\beta_0' X_n'(\rho_0)B_n'^{-1}(\gamma)M_{1n}(\theta)B_n^{-1}(\gamma)B_n(\gamma_0)u_n = o_p(1), \quad \text{uniformly in } \theta \in \Theta.
$$

Now we show that the difference of the first two terms is $o_p(1)$. Since $B_n^{-1}(\gamma)B_n(\gamma_0)$ is uniformly bounded in both absolute row and column sums, it follows from Assumption

1 and an extended result of Lee (2004a, Appendix A) that

$$\begin{aligned}
& \mathrm{E}\{u_n' B_n'(\gamma_0) B_n'^{-1}(\gamma) M_{1n}(\theta) B_n^{-1}(\gamma) B_n(\gamma_0) u_n\} \\
= \; & \sigma_0^2 \mathrm{tr}[B'(\gamma_0) B_n'^{-1}(\gamma) M_{1n}(\theta) B_n^{-1}(\gamma) B_n(\gamma_0)] \\
= \; & \sigma_0^2 \mathrm{tr}[B_n'(\gamma_0) B_n'^{-1}(\gamma) B_n^{-1}(\gamma) B_n(\gamma_0)] + O(1) \\
= \; & \sigma_0^2 \mathrm{tr}[\Omega_n(\gamma_0) \Omega_n^{-1}(\gamma)] + O(1)
\end{aligned}$$

and that

$$\begin{aligned}
& \mathrm{Var}\{u_n' B_n'(\gamma_0) B_n'^{-1}(\gamma) M_{1n}(\theta) B_n^{-1}(\gamma) B_n(\gamma_0) u_n\} \\
= \; & \sigma_0^4 \kappa_0 \, \mathtt{diagv}[R_n(\theta)]' \mathtt{diagv}[R_n(\theta)] + 2\sigma_0^4 \mathrm{tr}[R_n^2(\theta)],
\end{aligned}$$

where $R_n(\theta) = B_n'(\gamma_0) B_n'^{-1}(\gamma) M_{1n}(\theta) B_n^{-1}(\gamma) B_n(\gamma_0)$. Now, it is easy to show that $R_n(\theta)$ is uniformly bounded in absolute row and column sums, uniformly in $\theta \in \Theta$. Hence, by a matrix norm property, $R_n(\theta) R_n(\theta)$ is also uniformly bounded in absolute row and column sums, uniformly in $\theta \in \Theta$. It follows that the elements of R_n^2 are uniformly bounded, uniformly in $\theta \in \Theta$. Hence,

$$\mathrm{Var}\{u_n' B_n'(\gamma_0) B_n'^{-1}(\gamma) M_{1n}(\theta) B_n^{-1}(\gamma) B_n(\gamma_0) u_n\} = O(n),$$

uniformly in $\theta \in \Theta$. Finally, Chebyshev's inequality leads to

$$\frac{1}{n} u_n' B_n'(\gamma_0) B_n'^{-1}(\gamma) M_{1n}(\theta) B_n^{-1}(\gamma) B_n(\gamma_0) u_n - \frac{\sigma_0^2}{n} \mathrm{tr}[\Omega_n(\gamma_0) \Omega_n^{-1}(\gamma)] = o_p(1),$$

which gives $\hat{\sigma}_n^2(\theta) - \tilde{\sigma}_n^2(\theta) = o_p(1)$ and hence the consistency of the QMLE $\hat{\xi}_n$ of ξ_0.

Lemma B.5. *Under the Assumptions 1-10, we have* $\frac{1}{n}[H_n(\bar{\xi}_n) - H_n(\xi_0)] = o_p(1)$.

Proof: As $\hat{\xi}_n \longrightarrow \xi_0$, $\bar{\xi}_n \longrightarrow \xi_0$. As $H_n(\bar{\xi}_n)$ is either linear or quadratic in $\bar{\beta}_n$, and is linear in $\bar{\sigma}_n^{-k}, k = 2, 4,$ or 6. As $\bar{\beta}_n = \beta_0 + o_p(1)$ and $\bar{\sigma}_n^{-k} = \sigma_0^{-k} + o_p(1)$, we have,

$$\begin{aligned}
& \frac{1}{n} H_n(\bar{\xi}_n) = \frac{1}{n} H_n(\beta_0, \bar{\theta}_n, \sigma_0^2) + o_p(1) \\
& = \frac{1}{n} H_n(\xi_0) + \frac{1}{n} \frac{\partial}{\partial \bar{\rho}_n} H_n(\beta_0, \tilde{\theta}_n, \sigma_0^2)(\bar{\rho}_n - \rho_0) + \frac{1}{n} \frac{\partial}{\partial \bar{\gamma}_n} H_n(\beta_0, \tilde{\theta}_n, \sigma_0^2)(\bar{\gamma}_n - \gamma_0) + o_p(1),
\end{aligned}$$

where $\tilde{\theta}_n$ lies between $\bar{\theta}_n$ and θ_0, and the second equation follows from the mean value theorem. Under the Assumptions 9 and 10, it is easy to show that $\frac{1}{n} \frac{\partial}{\partial \bar{\rho}_n} H_n(\beta_0, \tilde{\theta}_n, \sigma_0^2) = O_p(1)$ and $\frac{1}{n} \frac{\partial}{\partial \bar{\gamma}_n} H_n(\beta_0, \tilde{\theta}_n, \sigma_0^2) = O_p(1)$. The result of Lemma 5 thus follows.

Lemma B.6. *Under the Assumptions 1-10, we have* $\frac{1}{n}[H_n(\xi_0) + I_n(\xi_0)] = o_p(1)$.

Proof: From Appendix A, we have,

$$H_n(\xi_0) + I_n(\xi_0) = \begin{pmatrix} 0, & \frac{1}{\sigma_0^2}(B_n^{-1}X_{n\rho})'u_n, & -\frac{1}{\sigma_0^2}Z_{1n}'\Omega_{n\gamma}^*u_n, & -\frac{1}{\sigma_0^4}Z_{1n}'u_n \\ \sim, & \frac{1}{\sigma_0^2}(B_n^{-1}X_{n\rho\rho}')'u_n, & -\frac{1}{\sigma_0^2}Z_{2n}'\Omega_{n\gamma}^*u_n, & -\frac{1}{\sigma_0^4}Z_{2n}'u_n \\ \sim, & \sim, & q_1(u_n) + q_2(u_n), & q_3(u_n) \\ \sim, & \sim, & \sim & q_4(u_n) \end{pmatrix}$$

where $q_1(u_n) = \mathtt{tr}(\Omega_{n\gamma}^{*2}) - \frac{1}{\sigma_0^2}u_n'\Omega_{n\gamma}^{*2}u_n$, $q_2(u_n) = \frac{1}{2\sigma_0^2}u_n'B_n^{-1}\Omega_{n\gamma\gamma}B_n'^{-1}u_n - \frac{1}{2}\mathtt{tr}(\Omega_n^{-1}\Omega_{n\gamma\gamma})$, $q_3(u_n) = \frac{1}{\sigma_0^2}\mathtt{tr}(\Omega_{n\gamma}^*) - \frac{1}{\sigma_0^4}u_n'\Omega_{n\gamma}^*u_n$, and $q_4(u_n) = \frac{n}{\sigma_0^4} - \frac{1}{\sigma_0^6}u_n'u_n$. Thus, the elements of $H_n(\xi_0) + I_n(\xi_0)$ are either linear or quadratic forms of u_n, which can easily be shown to be $o_p(n)$ by applying the Chebyshev's inequality. The result of Lemma 6 follows.

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Table 1. Mean and RMSE for the QMLEs, DGP1 with Normal Errors
n = 50, 100, and 200, for upper, middle and lower panels, respectively

		MC Estimate of Mean						MC Estimate of RMSE					
ρ	γ	β_0	β_1	β_2	σ	ρ	γ	β_0	β_1	β_2	σ	ρ	γ
.0	.0	5.000	2.000	2.000	.934	.001	-.041	.178	.048	.050	.121	.029	.199
	.2	5.003	1.999	1.999	.944	-.001	.150	.206	.051	.050	.115	.029	.199
	.5	5.015	1.998	2.000	.943	-.001	.433	.300	.052	.050	.118	.029	.181
	.8	5.006	2.000	2.001	.958	-.001	.740	.755	.056	.052	.113	.030	.132
.2	.0	5.002	2.000	2.001	.936	.200	-.046	.155	.049	.048	.120	.030	.198
	.2	4.998	2.000	1.999	.940	.200	.142	.171	.051	.048	.118	.028	.203
	.5	4.999	1.998	2.001	.944	.200	.433	.254	.054	.052	.115	.028	.185
	.8	5.002	2.002	2.002	.961	.199	.741	.639	.058	.053	.115	.027	.131
.5	.0	5.002	1.999	2.001	.936	.500	-.047	.133	.048	.047	.118	.032	.202
	.2	5.004	1.999	2.000	.941	.500	.144	.143	.048	.049	.118	.028	.203
	.5	5.006	1.999	2.000	.948	.499	.428	.199	.053	.053	.115	.026	.183
	.8	4.991	2.001	2.001	.958	.501	.734	.529	.062	.056	.112	.026	.136
.8	.0	5.005	2.001	2.000	.938	.800	-.047	.114	.045	.046	.118	.034	.197
	.2	4.998	2.000	2.000	.943	.800	.132	.121	.045	.048	.117	.030	.202
	.5	5.002	2.003	2.001	.950	.800	.439	.174	.052	.051	.115	.026	.176
	.8	4.992	2.001	2.000	.961	.799	.735	.416	.059	.056	.110	.024	.134
.0	.0	5.005	1.999	2.001	.972	.000	-.013	.185	.035	.036	.075	.025	.138
	.2	4.999	2.000	2.002	.972	.000	.175	.203	.035	.036	.076	.026	.137
	.5	5.004	2.000	1.999	.974	-.000	.476	.252	.036	.036	.076	.026	.116
	.8	5.010	1.999	1.998	.979	-.002	.773	.519	.037	.038	.078	.028	.075
.2	.0	5.001	2.001	1.999	.970	.200	-.022	.151	.034	.035	.075	.025	.137
	.2	5.003	1.999	2.000	.969	.200	.176	.158	.035	.037	.077	.024	.133
	.5	5.014	2.000	1.999	.974	.199	.473	.210	.037	.039	.078	.025	.114
	.8	5.003	2.001	2.000	.984	.200	.771	.428	.038	.040	.078	.025	.076
.5	.0	5.004	1.999	2.000	.971	.500	-.021	.121	.034	.035	.075	.025	.136
	.2	5.003	2.000	2.000	.967	.500	.175	.125	.036	.036	.078	.024	.137
	.5	5.002	1.999	2.000	.973	.500	.469	.159	.038	.040	.077	.022	.120
	.8	5.006	2.000	2.000	.982	.499	.770	.347	.040	.042	.076	.023	.076
.8	.0	5.004	1.999	2.000	.968	.799	-.023	.101	.034	.034	.078	.026	.137
	.2	5.005	1.999	2.000	.970	.799	.171	.106	.037	.036	.075	.024	.131
	.5	5.001	2.000	2.000	.972	.800	.473	.137	.037	.038	.079	.022	.115
	.8	4.995	2.002	2.000	.982	.800	.768	.297	.041	.044	.077	.021	.080
.0	.0	5.001	1.999	2.000	.984	-.000	-.009	.110	.025	.028	.053	.017	.095
	.2	5.007	2.000	2.000	.986	-.001	.189	.121	.025	.028	.053	.018	.091
	.5	4.994	2.000	2.000	.985	.001	.487	.166	.026	.029	.054	.018	.080
	.8	5.009	2.000	2.001	.988	.000	.786	.359	.027	.029	.054	.018	.049
.2	.0	5.005	2.000	1.999	.985	.200	-.006	.095	.025	.028	.053	.018	.099
	.2	5.005	2.001	2.000	.987	.199	.187	.099	.024	.028	.052	.018	.095
	.5	4.999	2.001	2.001	.987	.200	.488	.139	.027	.029	.054	.017	.076
	.8	5.005	2.001	2.000	.989	.200	.786	.296	.028	.030	.052	.017	.048
.5	.0	5.000	1.999	2.001	.984	.500	-.010	.073	.023	.028	.052	.018	.095
	.2	4.998	2.000	2.000	.984	.500	.184	.080	.025	.028	.053	.017	.095
	.5	5.001	2.001	2.000	.987	.500	.482	.106	.027	.031	.052	.016	.081
	.8	5.000	2.000	1.998	.990	.500	.785	.246	.029	.032	.054	.015	.049
.8	.0	5.001	2.000	1.999	.984	.800	-.011	.064	.023	.027	.054	.019	.096
	.2	5.003	2.000	1.999	.985	.799	.183	.067	.024	.028	.053	.017	.096
	.5	5.007	2.000	2.000	.989	.799	.480	.091	.027	.032	.052	.015	.080
	.8	5.002	2.000	2.000	.992	.800	.785	.200	.031	.034	.054	.014	.049

Table 2. Mean and RMSE for the QMLEs, DGP1 with Normal Mixture Errors
n = 50, 100, and 200, for upper, middle and lower panels, respectively

		MC Estimate of Mean						MC Estimate of RMSE					
ρ	γ	β_0	β_1	β_2	σ	ρ	γ	β_0	β_1	β_2	σ	ρ	γ
.0	.0	5.003	1.998	1.999	.931	.001	-.048	.195	.050	.060	.146	.031	.200
	.2	4.993	2.000	2.001	.941	.002	.147	.230	.053	.060	.141	.033	.204
	.5	5.011	1.997	1.998	.941	-.001	.429	.318	.054	.066	.143	.033	.187
	.8	5.035	2.000	2.001	.952	-.000	.738	.822	.058	.070	.141	.037	.137
.2	.0	5.005	2.003	2.000	.936	.201	-.047	.168	.048	.057	.141	.032	.192
	.2	5.000	2.001	1.998	.937	.200	.145	.185	.050	.060	.143	.031	.195
	.5	5.011	1.998	1.999	.946	.199	.434	.259	.056	.066	.142	.032	.184
	.8	5.035	1.996	1.999	.956	.198	.737	.664	.059	.072	.143	.035	.134
.5	.0	4.999	1.999	2.002	.930	.501	-.056	.139	.047	.053	.145	.033	.201
	.2	5.004	2.001	2.001	.935	.499	.125	.147	.050	.057	.146	.031	.202
	.5	4.997	2.000	2.002	.947	.501	.421	.209	.054	.064	.141	.030	.188
	.8	4.983	1.998	2.000	.959	.499	.734	.567	.061	.077	.140	.030	.135
.8	.0	5.004	2.000	2.001	.934	.800	-.059	.122	.045	.051	.146	.035	.198
	.2	4.999	2.001	2.002	.938	.801	.133	.131	.047	.054	.145	.032	.199
	.5	4.997	2.000	2.000	.948	.800	.421	.172	.053	.063	.142	.029	.187
	.8	5.004	1.996	2.001	.956	.801	.731	.406	.061	.076	.140	.027	.137
.0	.0	5.007	2.000	2.001	.969	-.000	-.021	.192	.036	.033	.099	.026	.134
	.2	4.999	2.001	1.999	.967	.000	.175	.216	.036	.034	.095	.027	.132
	.5	5.004	2.000	1.999	.971	-.001	.472	.276	.037	.034	.097	.029	.117
	.8	5.005	2.000	2.001	.982	.001	.770	.516	.038	.038	.099	.031	.078
.2	.0	5.008	2.000	2.001	.969	.199	-.022	.165	.035	.033	.098	.027	.135
	.2	5.002	2.001	2.000	.968	.200	.173	.178	.036	.033	.096	.027	.132
	.5	5.002	2.000	2.001	.969	.200	.474	.222	.039	.037	.102	.028	.117
	.8	4.994	2.000	2.001	.980	.200	.774	.448	.039	.037	.100	.029	.076
.5	.0	5.002	2.000	2.000	.968	.500	-.025	.132	.035	.032	.097	.028	.134
	.2	4.998	2.001	2.000	.971	.501	.166	.140	.037	.034	.097	.027	.135
	.5	4.999	2.000	1.998	.976	.500	.460	.184	.038	.036	.094	.027	.121
	.8	5.002	2.000	2.002	.984	.499	.769	.357	.041	.039	.100	.027	.078
.8	.0	5.003	1.999	1.999	.968	.800	-.021	.115	.035	.031	.097	.030	.135
	.2	4.995	1.999	1.999	.968	.801	.166	.117	.036	.033	.098	.028	.136
	.5	4.994	1.999	1.999	.972	.801	.466	.145	.040	.037	.101	.026	.121
	.8	4.992	1.999	2.000	.979	.800	.768	.296	.042	.040	.099	.025	.081
.0	.0	5.001	1.999	2.001	.984	.000	-.008	.113	.023	.022	.070	.016	.095
	.2	4.997	2.001	2.001	.984	.000	.189	.126	.024	.022	.068	.016	.093
	.5	4.993	2.001	2.001	.987	.001	.485	.170	.024	.023	.069	.017	.076
	.8	4.992	2.001	2.001	.990	.000	.784	.367	.026	.024	.070	.019	.052
.2	.0	5.001	2.000	2.000	.984	.200	-.011	.093	.023	.022	.068	.016	.095
	.2	5.000	1.999	2.001	.984	.200	.187	.103	.024	.022	.067	.016	.091
	.5	4.999	2.001	1.999	.986	.200	.486	.141	.024	.022	.069	.016	.079
	.8	4.996	2.000	2.000	.989	.200	.787	.304	.026	.024	.070	.017	.051
.5	.0	5.000	1.999	2.000	.986	.500	-.010	.077	.024	.021	.068	.017	.095
	.2	5.000	2.000	2.000	.982	.500	.183	.081	.023	.021	.072	.016	.094
	.5	4.999	2.000	2.000	.987	.500	.487	.108	.025	.023	.069	.015	.077
	.8	5.001	2.000	2.000	.991	.500	.787	.238	.026	.025	.070	.016	.049
.8	.0	5.002	2.000	2.000	.984	.800	-.009	.065	.022	.021	.068	.018	.096
	.2	4.998	2.000	2.000	.984	.801	.187	.072	.024	.022	.067	.017	.091
	.5	5.003	2.001	2.000	.986	.800	.484	.092	.025	.024	.069	.015	.079
	.8	5.003	2.000	2.001	.991	.800	.784	.202	.028	.026	.068	.015	.049

Table 3. Mean and RMSE for the QMLEs, DGP1 with Normal-Gamma Mixture Errors
n = 50, 100, and 200, for upper, middle and lower panels, respectively

		MC Estimate of Mean						MC Estimate of RMSE					
ρ	γ	β_0	β_1	β_2	σ	ρ	γ	β_0	β_1	β_2	σ	ρ	γ
.0	.0	5.007	2.001	2.000	.938	-.000	-.037	.222	.056	.064	.146	.035	.193
	.2	5.007	2.000	1.999	.935	-.000	.158	.235	.057	.064	.142	.034	.193
	.5	5.020	1.998	1.997	.943	-.002	.444	.323	.059	.065	.147	.035	.175
	.8	5.042	2.000	2.000	.945	-.000	.747	.821	.060	.070	.142	.036	.125
.2	.0	5.008	1.999	1.998	.935	.199	-.032	.176	.056	.064	.143	.032	.199
	.2	5.002	1.999	2.001	.935	.200	.151	.198	.059	.066	.149	.032	.201
	.5	4.992	2.000	2.000	.945	.200	.437	.268	.060	.067	.145	.032	.178
	.8	5.008	1.999	2.002	.955	.200	.745	.723	.063	.072	.144	.033	.127
.5	.0	5.005	2.000	2.000	.938	.499	-.051	.136	.053	.062	.147	.032	.202
	.2	5.002	2.000	2.000	.941	.500	.150	.149	.056	.065	.144	.030	.193
	.5	5.001	1.999	1.999	.945	.500	.436	.213	.060	.070	.142	.027	.174
	.8	5.003	1.998	1.999	.956	.498	.739	.550	.065	.073	.140	.027	.131
.8	.0	5.001	1.999	1.999	.938	.799	-.034	.115	.052	.059	.147	.031	.189
	.2	5.010	2.001	1.996	.941	.799	.148	.121	.053	.065	.142	.028	.198
	.5	5.012	2.000	2.002	.947	.800	.440	.171	.061	.071	.141	.025	.182
	.8	5.001	2.000	2.002	.959	.801	.736	.407	.067	.076	.140	.023	.134
.0	.0	5.007	2.000	2.000	.965	.000	-.014	.140	.037	.042	.099	.023	.138
	.2	4.998	2.001	2.000	.968	.001	.179	.149	.036	.043	.101	.022	.135
	.5	5.008	2.000	2.000	.972	-.001	.475	.220	.039	.044	.099	.023	.114
	.8	5.014	2.000	2.000	.976	-.000	.774	.508	.042	.045	.101	.025	.077
.2	.0	5.005	2.001	2.002	.964	.199	-.017	.116	.036	.043	.099	.023	.134
	.2	5.000	2.000	1.999	.971	.200	.176	.131	.037	.043	.102	.023	.136
	.5	5.001	2.000	2.001	.974	.200	.468	.175	.038	.044	.096	.022	.120
	.8	5.002	2.000	2.000	.979	.199	.773	.415	.041	.046	.100	.022	.076
.5	.0	4.994	1.999	2.000	.965	.501	-.017	.098	.034	.042	.099	.024	.134
	.2	4.999	2.001	1.999	.971	.500	.173	.108	.036	.043	.099	.023	.134
	.5	5.010	2.001	2.000	.975	.499	.464	.147	.040	.046	.099	.021	.121
	.8	5.014	2.000	2.001	.982	.500	.771	.365	.045	.049	.101	.020	.077
.8	.0	5.000	2.001	2.000	.967	.801	-.021	.086	.034	.041	.097	.026	.137
	.2	4.997	2.001	1.999	.968	.801	.170	.092	.036	.044	.098	.023	.133
	.5	5.002	2.002	2.000	.971	.800	.466	.121	.040	.048	.101	.020	.118
	.8	5.004	2.000	2.001	.978	.801	.771	.281	.047	.052	.101	.018	.075
.0	.0	5.003	2.001	2.001	.982	-.000	-.006	.118	.027	.021	.069	.017	.096
	.2	5.004	1.999	2.000	.982	.000	.189	.128	.026	.022	.070	.017	.095
	.5	5.005	2.000	2.000	.989	.000	.486	.173	.026	.023	.071	.017	.078
	.8	4.996	2.000	2.000	.989	.000	.788	.361	.026	.023	.072	.018	.048
.2	.0	4.998	1.999	1.999	.984	.201	-.011	.097	.026	.022	.069	.017	.095
	.2	4.998	2.000	2.000	.984	.200	.188	.105	.027	.022	.071	.017	.094
	.5	4.999	1.999	2.001	.986	.200	.486	.141	.027	.023	.070	.016	.077
	.8	5.000	2.000	2.000	.989	.201	.788	.299	.028	.023	.074	.016	.048
.5	.0	5.003	2.000	2.000	.982	.500	-.010	.080	.026	.021	.070	.018	.094
	.2	5.001	2.000	2.000	.983	.499	.186	.086	.027	.022	.070	.017	.093
	.5	5.003	2.000	2.000	.984	.500	.487	.110	.028	.023	.069	.016	.078
	.8	5.006	1.999	1.999	.989	.499	.786	.248	.029	.025	.072	.015	.050
.8	.0	5.003	2.000	1.999	.985	.800	-.014	.067	.024	.021	.071	.018	.095
	.2	4.998	2.000	2.000	.984	.801	.185	.071	.027	.022	.068	.017	.095
	.5	5.003	2.000	2.000	.986	.799	.483	.093	.029	.024	.070	.015	.077
	.8	4.997	1.999	2.000	.989	.800	.784	.202	.031	.025	.071	.014	.049

Table 4. Empirical Sizes (%) for the Four Tests, DGP1 with Normal Errors

		$n = 50$				$n = 100$				$n = 200$			
ρ_0	γ_0	T_{1n}	T_{2n}	T_{3n}	T_{4n}	T_{1n}	T_{2n}	T_{3n}	T_{4n}	T_{1n}	T_{2n}	T_{3n}	T_{4n}
-.8	-.8	8.55	7.75	8.15	9.15	6.60	6.75	6.75	7.85	6.10	5.45	4.90	5.30
	-.5	9.55	8.30	10.15	11.65	6.95	6.45	6.90	7.35	6.10	6.30	6.30	6.75
	-.2	9.80	6.80	11.20	10.60	6.85	7.05	6.60	7.65	6.80	5.95	5.50	5.70
	.0	9.80	8.55	10.60	13.05	6.90	7.50	6.70	8.05	5.90	5.30	7.10	7.10
	.2	9.30	9.40	9.85	12.70	7.20	6.30	6.85	7.60	6.50	5.95	6.50	6.60
	.5	12.05	9.75	12.55	14.10	8.10	7.40	6.95	8.00	5.85	6.10	6.40	6.50
	.8	12.70	8.20	9.75	11.20	7.75	6.25	7.30	7.45	6.35	5.95	6.00	6.90
-.5	-.8	9.80	8.55	8.45	10.20	7.05	5.35	7.40	6.85	6.90	5.90	5.30	5.90
	-.5	10.80	8.15	10.60	10.70	6.75	6.05	6.40	7.55	6.10	6.10	6.15	6.05
	-.2	10.25	8.10	11.40	12.30	6.65	6.05	8.00	7.30	5.55	5.80	6.10	6.90
	.0	9.65	7.60	9.75	11.05	8.40	7.40	6.60	7.75	6.00	6.30	6.65	7.30
	.2	10.50	7.85	10.75	11.00	6.95	5.80	6.90	7.60	5.80	5.40	5.75	5.45
	.5	10.80	8.20	8.85	10.40	7.20	5.95	6.90	6.55	6.00	5.20	6.20	6.55
	.8	15.00	7.15	12.20	13.10	8.80	6.30	7.70	7.60	7.70	6.15	5.60	6.15
-.2	-.8	9.95	7.40	7.55	8.70	7.05	5.85	6.65	7.10	6.75	5.50	6.75	6.35
	-.5	10.80	8.25	9.70	11.35	7.10	5.60	6.55	6.70	5.30	5.05	5.75	5.90
	-.2	10.35	8.40	10.65	11.40	6.40	6.20	6.80	7.15	6.20	5.15	6.70	6.70
	.0	9.65	7.40	11.40	11.85	5.95	5.55	6.55	6.35	6.95	5.50	5.60	5.50
	.2	9.35	7.65	9.30	10.55	7.15	6.75	7.25	7.15	5.50	5.15	4.80	5.15
	.5	10.90	7.05	9.15	10.35	7.45	6.80	6.50	6.85	6.25	5.70	4.80	5.35
	.8	13.45	8.05	10.15	11.00	9.50	6.20	7.00	8.05	7.65	5.45	6.45	6.05
.0	-.8	8.65	7.95	7.30	8.75	7.20	5.95	6.50	6.95	6.30	5.55	5.65	5.85
	-.5	10.15	8.50	11.50	12.00	6.40	6.60	6.60	7.65	5.80	5.50	5.80	5.70
	-.2	9.25	7.80	11.90	12.00	6.90	6.85	7.15	8.25	5.30	5.25	6.65	6.30
	.0	9.60	7.70	10.25	10.70	6.80	5.40	7.70	7.20	6.40	5.25	6.05	6.45
	.2	10.45	7.40	9.95	10.15	7.30	5.80	7.60	7.95	5.65	5.90	5.60	5.05
	.5	11.40	7.55	8.30	9.60	6.45	5.20	6.00	6.10	6.90	5.95	6.50	6.65
	.8	14.10	6.65	9.50	9.90	9.85	6.80	6.45	6.90	7.50	5.10	5.90	5.35
.2	-.8	9.00	6.85	8.40	9.40	6.60	5.55	6.45	6.70	6.80	5.05	6.20	5.80
	-.5	10.45	8.25	10.00	11.30	6.30	5.80	7.90	8.75	6.10	5.50	6.10	6.20
	-.2	9.30	6.70	11.30	11.10	6.65	4.65	7.55	7.70	6.65	5.85	6.75	6.30
	.0	10.60	7.10	10.45	10.60	6.80	6.20	7.20	7.15	6.30	6.25	7.05	7.40
	.2	9.70	6.95	10.10	10.25	6.25	6.65	5.90	7.15	5.05	6.15	6.60	6.95
	.5	11.85	7.40	9.30	9.25	7.70	6.55	6.35	7.10	6.45	5.60	4.75	6.10
	.8	14.05	6.95	9.95	9.75	9.00	5.20	6.05	6.50	7.40	6.05	5.45	5.45
.5	-.8	8.55	7.50	7.65	9.00	8.00	6.85	6.80	8.30	6.00	4.90	5.70	5.80
	-.5	10.75	7.90	10.45	10.95	7.40	6.15	7.25	8.40	6.15	6.65	5.90	6.70
	-.2	9.25	7.30	9.30	9.90	6.95	6.05	6.75	7.05	5.75	4.95	5.70	6.40
	.0	10.25	7.65	11.10	11.75	6.90	5.50	6.40	7.20	6.20	4.70	5.30	5.25
	.2	11.20	6.80	10.85	10.65	7.20	6.85	7.80	8.35	6.25	6.15	6.70	6.40
	.5	12.30	7.05	9.70	9.25	6.65	5.90	6.95	7.90	6.00	5.80	6.95	6.80
	.8	17.00	6.95	9.60	9.10	10.60	5.65	5.70	6.00	7.30	4.75	5.55	5.30
.8	-.8	9.20	8.20	9.05	10.30	7.70	7.30	6.15	7.50	6.80	5.95	5.95	6.65
	-.5	10.25	7.85	10.25	11.20	8.10	7.15	7.25	8.00	6.50	6.05	6.05	6.35
	-.2	10.85	8.15	9.80	11.80	8.00	5.75	7.90	8.05	5.55	5.85	6.75	6.70
	.0	10.55	7.55	10.50	10.60	7.30	6.35	7.70	7.70	7.60	5.75	6.25	6.10
	.2	10.30	6.80	9.30	9.65	7.85	6.15	6.10	7.10	5.80	5.60	6.60	7.05
	.5	12.80	6.00	8.65	8.85	6.15	6.55	6.35	7.35	7.05	5.25	5.70	5.50
	.8	15.40	6.80	8.55	8.80	9.35	5.85	6.50	6.70	7.70	5.55	4.55	5.60

Table 5. Empirical Sizes (%), DGP1 with Normal Mixture Errors

		$n=50$				$n=100$				$n=200$			
ρ_0	γ_0	T_{1n}	T_{2n}	T_{3n}	T_{4n}	T_{1n}	T_{2n}	T_{3n}	T_{4n}	T_{1n}	T_{2n}	T_{3n}	T_{4n}
-.8	-.8	9.60	7.80	7.65	9.10	6.35	6.20	6.35	7.00	5.25	5.35	5.60	5.70
	-.5	10.45	6.60	10.40	10.85	7.75	6.90	7.30	8.55	6.15	4.30	5.60	5.95
	-.2	10.50	8.25	9.55	10.95	7.80	7.25	6.55	7.05	6.90	5.65	5.75	6.50
	.0	9.95	8.15	9.75	11.10	7.55	7.15	6.75	7.80	6.50	6.60	5.55	6.75
	.2	10.20	10.05	9.85	12.20	6.20	7.45	6.75	8.80	5.65	5.45	5.60	6.20
	.5	11.25	9.55	10.80	13.10	8.05	7.45	6.95	7.80	6.45	7.15	5.20	7.05
	.8	10.15	8.20	9.70	11.20	8.00	6.50	7.30	9.20	6.55	5.30	6.30	6.45
-.5	-.8	8.85	7.60	8.05	9.60	6.45	5.75	6.15	6.75	5.35	5.40	5.50	5.35
	-.5	8.20	7.30	9.10	9.70	7.00	6.15	7.10	7.05	5.85	4.40	5.50	5.85
	-.2	9.90	7.85	8.75	10.25	8.45	6.75	7.55	8.80	5.95	5.55	5.05	6.10
	.0	8.40	7.55	9.90	11.10	7.80	5.60	7.75	7.20	5.65	5.10	5.65	6.45
	.2	10.85	7.90	8.95	10.10	7.15	5.95	6.15	6.45	5.85	5.00	5.65	5.90
	.5	10.75	8.55	8.60	9.75	7.95	7.00	5.60	6.65	4.95	5.80	6.00	6.45
	.8	13.85	7.00	12.45	13.10	7.85	6.85	7.45	8.75	8.15	4.70	5.60	5.45
-.2	-.8	8.15	5.85	7.65	8.25	7.20	6.15	5.60	6.75	6.15	5.55	5.05	5.15
	-.5	8.70	6.80	8.95	9.90	7.20	5.70	7.30	7.35	6.95	5.80	5.55	6.20
	-.2	9.65	7.95	9.95	11.10	7.85	6.40	6.80	6.85	5.75	5.60	5.30	5.45
	.0	9.30	8.30	10.45	11.15	6.95	6.05	7.55	7.45	6.95	5.65	6.10	5.70
	.2	10.85	7.65	10.65	12.20	7.65	7.15	6.65	7.95	6.25	5.75	5.60	5.85
	.5	10.45	9.45	10.00	11.70	7.00	6.80	5.90	6.95	6.50	5.60	5.70	6.40
	.8	13.90	7.70	10.80	11.90	8.20	5.85	6.65	7.70	7.65	5.35	6.15	6.00
.0	-.8	9.00	6.35	7.35	8.95	6.10	5.50	5.95	6.25	6.10	6.10	5.55	6.05
	-.5	9.65	7.25	9.45	10.35	7.00	5.25	6.35	6.40	6.05	5.70	6.05	6.65
	-.2	9.75	7.75	9.60	10.85	7.40	6.80	7.45	8.80	5.45	5.35	5.60	5.60
	.0	9.95	7.35	10.30	11.10	6.45	4.80	6.85	6.65	5.80	5.65	5.55	5.85
	.2	10.35	9.15	10.60	12.30	8.35	6.15	6.35	6.60	6.95	5.40	5.65	5.85
	.5	11.45	7.40	9.35	10.45	7.30	6.85	7.35	7.70	5.90	5.20	5.55	5.50
	.8	15.45	8.20	9.65	11.60	8.95	6.30	6.95	7.75	7.80	6.20	6.65	6.45
.2	-.8	8.65	7.40	7.75	9.10	7.20	6.75	5.50	7.10	5.80	5.25	5.90	5.75
	-.5	9.90	7.95	8.75	10.50	6.45	6.40	8.10	8.30	5.30	5.45	6.30	6.20
	-.2	10.05	8.65	10.30	11.65	6.85	5.45	7.05	7.55	6.05	5.25	7.15	6.85
	.0	8.75	7.90	9.25	10.10	7.05	6.25	7.00	7.65	6.45	5.30	6.15	6.10
	.2	10.20	6.60	9.55	9.80	6.55	5.70	6.75	7.05	6.20	6.25	5.55	5.85
	.5	11.20	8.35	9.60	10.85	8.70	7.00	7.30	8.35	6.00	5.40	5.30	5.85
	.8	15.55	7.85	8.95	9.95	8.55	5.85	6.20	6.75	7.15	5.25	6.60	6.20
.5	-.8	9.65	7.20	7.55	8.75	7.55	6.55	5.45	6.50	6.40	5.95	5.10	5.30
	-.5	9.75	7.45	9.50	10.55	6.65	6.30	7.75	8.25	5.95	5.50	6.10	6.60
	-.2	10.80	6.75	10.05	9.70	7.55	5.70	7.40	7.70	6.75	5.50	6.65	6.35
	.0	9.75	7.10	10.20	11.25	5.95	5.80	6.75	7.00	5.60	5.75	6.05	5.85
	.2	10.80	7.40	9.80	9.60	7.40	6.05	7.35	7.60	5.85	4.80	5.90	6.10
	.5	12.75	6.65	9.60	10.30	7.40	5.75	6.80	7.15	6.65	5.25	5.40	5.10
	.8	16.35	7.25	8.60	8.25	9.50	5.80	7.05	6.80	6.95	5.90	6.15	5.95
.8	-.8	8.70	7.50	7.85	9.60	7.05	4.95	6.80	5.85	4.90	4.40	5.65	5.45
	-.5	10.10	7.95	8.60	9.70	7.55	5.45	7.75	7.70	6.40	4.65	6.60	5.75
	-.2	10.20	6.90	9.75	10.10	8.40	5.90	7.20	7.55	5.50	5.05	6.95	6.40
	.0	10.80	7.10	9.30	10.55	7.40	7.40	6.60	7.50	6.50	5.50	6.00	6.25
	.2	10.15	7.35	9.50	10.30	7.60	6.20	7.45	7.85	6.95	6.00	5.40	6.15
	.5	11.05	7.50	9.60	9.85	8.85	6.80	7.60	8.05	6.85	5.05	5.75	6.35
	.8	16.80	6.80	9.60	8.30	9.75	5.55	7.85	6.60	7.70	5.55	5.55	5.70

Table 6. Empirical Sizes (%), DGP1 with Normal-Gamma Mixture Errors

		$n=50$				$n=100$				$n=200$			
ρ_0	γ_0	T_{1n}	T_{2n}	T_{3n}	T_{4n}	T_{1n}	T_{2n}	T_{3n}	T_{4n}	T_{1n}	T_{2n}	T_{3n}	T_{4n}
-.8	-.8	10.95	8.50	7.35	10.15	6.65	5.35	6.95	6.95	6.15	5.30	5.70	5.60
	-.5	11.55	9.20	9.15	10.55	7.40	6.45	6.85	7.70	6.50	5.70	6.25	6.35
	-.2	11.15	10.30	10.15	12.85	7.10	6.85	7.95	7.45	6.05	6.65	6.65	6.85
	.0	11.30	10.50	10.55	13.15	7.25	7.75	6.10	8.20	6.20	6.45	6.05	7.25
	.2	10.25	10.00	10.10	12.60	7.10	6.60	7.00	7.30	6.30	6.80	5.45	7.05
	.5	10.80	9.60	10.50	13.45	7.35	6.45	7.10	8.15	6.35	5.75	6.30	6.10
	.8	11.35	7.70	10.55	11.75	7.45	6.15	7.40	7.90	7.30	6.35	6.15	7.05
-.5	-.8	10.55	7.05	6.90	8.55	7.40	7.65	6.35	7.95	6.15	4.80	5.65	4.60
	-.5	11.00	7.80	9.70	11.20	6.45	5.70	7.70	7.25	6.50	5.95	6.15	5.70
	-.2	10.00	8.10	9.75	10.75	6.85	6.55	7.15	7.05	7.05	6.25	5.55	6.25
	.0	8.60	7.10	8.35	9.50	6.65	6.20	7.90	7.85	5.35	5.60	5.70	5.90
	.2	10.10	8.80	8.60	10.45	7.30	6.65	7.25	7.95	5.90	6.60	4.95	6.60
	.5	9.45	8.15	8.60	10.35	6.00	5.05	7.30	6.85	4.95	5.85	5.55	5.55
	.8	15.45	7.30	10.00	11.10	8.60	6.55	6.35	7.75	6.20	5.05	5.35	5.80
-.2	-.8	9.25	7.25	7.90	9.10	5.90	5.75	7.85	7.25	7.10	6.00	5.05	6.40
	-.5	9.05	6.80	8.90	9.30	7.10	7.55	6.25	7.55	6.15	6.15	6.35	6.55
	-.2	10.25	7.40	9.75	10.90	7.70	6.35	7.10	7.45	6.20	4.70	5.70	6.00
	.0	9.25	6.75	8.85	9.15	7.80	5.70	8.70	8.40	6.50	5.40	6.35	6.65
	.2	9.60	7.35	9.80	10.15	7.25	6.45	6.55	7.75	5.50	5.05	6.55	6.15
	.5	10.15	7.00	9.75	9.60	6.70	6.50	7.00	7.55	5.80	5.75	5.30	6.35
	.8	14.50	6.55	10.50	10.30	9.20	7.30	7.20	7.45	6.85	5.95	6.65	6.40
.0	-.8	7.85	5.85	6.95	7.80	7.50	6.35	6.35	7.10	5.75	4.95	5.70	5.75
	-.5	9.80	7.80	8.80	9.60	7.60	6.50	6.75	7.00	6.15	4.95	6.30	6.90
	-.2	8.35	6.45	9.65	10.25	7.60	5.20	6.45	6.10	6.60	4.95	6.20	6.80
	.0	11.35	8.85	9.30	10.05	7.50	6.35	8.15	7.65	6.05	6.05	6.70	5.95
	.2	9.15	6.90	9.25	9.90	6.80	5.65	7.05	7.25	5.65	5.40	6.15	6.60
	.5	9.70	8.10	8.20	9.95	8.15	6.20	6.75	6.55	6.80	6.30	5.55	6.70
	.8	15.70	6.85	8.85	10.05	8.75	6.05	6.35	7.55	6.35	6.30	5.40	5.50
.2	-.8	9.40	8.05	7.85	9.20	7.35	5.35	6.75	7.00	5.40	5.40	5.80	6.00
	-.5	8.95	8.85	8.95	11.05	7.30	5.75	6.95	7.20	6.75	5.40	5.40	5.50
	-.2	10.55	8.35	10.35	11.65	6.65	5.65	7.85	7.40	5.50	6.00	5.75	6.50
	.0	11.45	7.25	9.40	10.60	7.25	6.50	6.55	7.20	5.70	5.90	5.80	5.40
	.2	11.10	6.55	10.70	9.80	7.65	6.30	7.00	7.50	6.90	5.75	5.75	6.25
	.5	10.75	7.05	8.10	9.15	7.30	5.65	7.25	6.85	5.85	5.90	5.05	6.25
	.8	13.70	7.00	9.10	9.85	8.85	5.35	6.65	5.65	6.80	5.10	5.55	5.40
.5	-.8	9.35	7.65	8.00	9.35	6.65	6.55	6.15	6.75	5.30	5.85	5.60	5.45
	-.5	9.00	8.25	8.35	10.20	7.20	6.85	7.70	7.95	6.05	5.65	5.30	6.05
	-.2	9.10	7.95	9.00	10.10	6.90	5.45	6.70	6.65	5.75	5.55	5.45	5.70
	.0	9.50	7.95	10.90	11.60	7.60	6.30	7.05	7.85	5.65	5.90	5.75	6.50
	.2	10.25	6.95	8.95	9.15	6.25	6.30	6.70	7.75	5.80	5.25	6.30	5.80
	.5	11.25	6.75	8.00	8.05	7.90	5.50	7.00	7.35	6.55	5.85	6.00	5.60
	.8	15.40	6.65	9.25	9.45	9.50	6.55	5.95	7.20	7.15	5.70	5.80	5.90
.8	-.8	9.10	7.35	8.25	9.70	5.95	5.00	6.75	7.20	5.25	4.55	6.25	5.90
	-.5	9.75	7.45	9.50	10.20	6.90	7.55	6.25	7.65	6.05	5.85	6.05	6.30
	-.2	9.25	6.20	9.45	10.10	6.80	7.20	7.25	8.10	6.75	5.55	6.10	5.70
	.0	9.60	7.30	8.75	9.10	6.50	5.65	7.35	7.35	5.90	5.35	6.50	5.95
	.2	9.60	7.40	10.05	9.90	7.70	6.30	6.30	7.25	5.70	5.00	7.20	6.80
	.5	10.65	7.65	10.50	10.30	8.55	5.50	6.75	7.25	5.80	5.75	4.90	5.70
	.8	14.50	6.75	8.45	8.45	9.20	5.50	5.85	5.75	6.40	5.90	5.40	5.90

Table 7. Empirical Sizes (%) of the Four Tests, DGP2 with Normal Errors

		$n=50$				$n=100$				$n=200$			
ρ	γ	T_{1n}	T_{2n}	T_{3n}	t_{4n}	T_{1n}	T_{2n}	T_{3n}	t_{4n}	T_{1n}	T_{2n}	T_{3n}	t_{4n}
-.8	-.8	9.80	9.35	7.85	7.30	7.25	6.70	7.05	6.85	5.95	7.00	5.50	5.50
	-.5	9.80	8.35	8.50	8.40	6.30	6.65	6.65	6.55	5.60	4.90	5.50	5.60
	-.2	10.10	7.60	9.60	9.40	8.10	5.90	7.50	7.35	5.95	5.25	5.55	5.60
	.0	9.40	8.15	9.10	9.00	7.70	5.80	7.95	7.65	5.20	5.65	6.40	6.40
	.2	10.25	7.25	7.90	7.70	7.50	6.10	7.50	7.55	5.90	4.90	6.55	6.55
	.5	10.55	7.80	8.50	8.60	7.85	5.95	7.10	7.35	6.40	5.50	6.10	6.10
	.8	14.00	7.85	12.35	12.45	8.10	5.80	7.55	7.40	5.40	4.90	6.10	6.10
-.5	-.8	9.80	8.80	7.95	6.65	7.05	8.05	6.15	6.00	6.00	6.15	6.15	5.85
	-.5	9.70	8.35	8.75	8.35	6.10	5.80	8.10	7.35	5.40	6.15	6.20	6.50
	-.2	9.90	7.50	8.95	9.30	7.40	7.25	6.65	6.55	6.70	5.40	6.50	6.65
	.0	10.20	6.05	9.20	9.35	7.05	6.20	6.20	6.65	5.60	5.00	6.05	6.00
	.2	10.95	7.15	9.15	9.60	6.45	5.50	7.10	7.20	7.10	5.80	5.90	5.75
	.5	11.20	6.85	8.45	8.25	7.05	5.85	6.35	5.90	6.15	6.15	6.35	6.35
	.8	13.25	5.45	11.40	11.00	8.55	6.35	6.75	6.40	7.25	4.80	5.75	6.05
-.2	-.8	8.95	7.55	7.95	6.30	7.10	6.45	5.70	6.10	5.60	5.60	5.35	5.35
	-.5	8.35	7.35	7.80	7.90	6.90	6.40	7.05	6.65	5.10	4.35	6.05	5.80
	-.2	9.40	6.65	9.65	9.05	6.30	5.25	7.50	8.05	5.25	5.80	6.25	7.05
	.0	9.70	7.40	9.05	8.95	6.55	6.15	6.40	6.65	5.20	5.95	5.70	5.90
	.2	10.55	7.65	9.30	9.35	7.80	6.75	7.80	7.45	6.90	5.90	5.65	5.80
	.5	10.85	7.20	8.75	8.50	8.25	5.05	6.65	6.35	6.20	5.55	5.55	5.35
	.8	14.60	7.45	9.80	9.65	9.25	6.05	5.90	6.55	6.65	5.20	6.00	5.60
.0	-.8	9.25	7.65	8.25	6.90	6.85	5.60	5.50	5.15	5.85	4.55	5.50	5.60
	-.5	9.80	7.85	9.45	8.75	6.85	6.15	7.75	8.00	6.25	5.85	5.50	5.30
	-.2	9.45	7.40	9.40	9.40	6.90	6.40	7.40	7.35	6.35	6.10	6.80	6.55
	.0	9.35	6.90	9.20	8.90	6.45	5.60	7.65	7.30	5.65	5.65	5.55	5.95
	.2	10.35	8.05	10.15	10.85	7.65	6.65	6.80	7.15	5.75	5.90	5.80	6.20
	.5	10.35	7.95	9.70	9.55	6.95	6.10	6.85	7.30	5.85	6.10	6.15	6.70
	.8	13.25	7.10	10.25	9.95	8.40	5.85	7.10	7.05	7.35	4.80	6.00	6.50
.2	-.8	8.30	7.70	7.70	6.55	6.00	5.50	5.60	6.35	6.20	5.30	5.45	5.05
	-.5	8.65	6.90	8.70	9.05	6.55	6.05	6.85	6.60	4.90	6.05	5.20	5.10
	-.2	11.35	8.80	9.20	8.60	7.70	6.00	6.80	6.55	6.40	6.25	5.40	5.30
	.0	10.10	7.95	10.05	9.55	7.80	6.80	7.45	7.50	6.35	5.55	6.85	6.25
	.2	9.30	7.35	9.95	9.40	6.60	7.20	6.85	7.05	6.20	6.10	6.15	6.35
	.5	10.15	8.30	9.85	10.30	5.75	5.95	6.85	6.70	6.00	5.50	5.50	5.40
	.8	11.70	8.90	7.90	9.25	9.30	5.65	6.25	7.50	7.05	5.65	4.65	5.00
.5	-.8	9.55	6.75	7.20	6.30	7.25	6.05	6.85	6.95	6.90	5.55	5.95	5.90
	-.5	9.10	6.40	9.35	9.15	7.00	7.50	6.85	7.00	5.15	5.50	5.20	5.20
	-.2	10.90	8.90	11.40	11.55	7.05	6.05	7.80	8.20	5.75	5.65	6.50	6.60
	.0	10.45	7.70	10.10	9.75	7.55	6.50	7.55	7.45	6.45	5.00	6.15	6.05
	.2	10.60	8.00	10.65	10.80	7.40	6.35	7.15	7.55	5.85	6.10	6.30	6.40
	.5	9.65	7.80	9.65	9.85	7.75	6.00	7.50	7.30	6.50	6.40	6.00	5.75
	.8	11.30	10.15	10.00	10.10	8.00	7.80	7.95	8.10	6.00	5.85	5.95	6.10
.8	-.8	10.25	7.60	7.45	7.20	7.35	5.60	6.70	6.90	6.05	5.05	5.40	5.15
	-.5	9.80	7.20	9.15	9.25	7.35	5.95	7.05	7.25	6.25	5.55	6.55	6.60
	-.2	10.95	7.30	11.20	11.20	7.70	6.25	8.85	8.90	6.70	5.15	6.65	6.65
	.0	12.45	8.05	12.10	12.05	8.90	6.80	8.20	8.40	6.00	5.25	6.55	6.55
	.2	11.00	7.90	10.90	10.55	9.20	7.50	8.90	8.75	6.50	5.95	7.05	6.90
	.5	12.70	8.90	11.90	12.10	8.35	7.10	8.05	8.35	6.75	6.05	6.00	5.85
	.8	12.85	10.55	10.90	10.70	8.30	8.20	8.10	8.35	6.80	6.85	5.75	6.40

Table 8. Empirical Sizes (%) of the Four Tests, DGP2 with Normal Mixture Errors

		$n=50$				$n=100$				$n=200$			
ρ	γ	T_{1n}	T_{2n}	T_{3n}	t_{4n}	T_{1n}	T_{2n}	T_{3n}	t_{4n}	T_{1n}	T_{2n}	T_{3n}	t_{4n}
-.8	-.8	6.95	9.75	8.25	7.25	6.15	7.85	6.25	6.15	5.15	6.70	5.70	5.05
	-.5	9.55	8.30	8.90	8.80	6.70	7.90	7.45	7.50	6.00	6.05	5.80	5.65
	-.2	8.40	6.85	10.25	10.20	8.05	6.55	7.80	7.65	5.80	5.85	5.30	5.30
	.0	10.15	8.45	8.50	8.40	7.55	6.60	6.90	6.90	5.25	5.25	5.35	5.35
	.2	9.15	6.75	8.80	8.65	6.35	6.75	5.85	6.05	5.45	5.35	5.70	5.50
	.5	9.80	6.00	9.10	9.20	6.45	6.00	6.05	5.95	5.85	5.20	5.85	5.80
	.8	13.30	6.60	13.00	12.85	8.40	5.45	7.85	7.85	6.50	4.55	6.00	6.05
-.5	-.8	9.05	9.95	8.35	6.30	6.75	8.45	6.65	5.85	5.55	5.95	5.70	4.95
	-.5	7.90	7.65	8.55	8.05	6.15	7.00	6.60	6.95	5.85	5.00	6.05	6.20
	-.2	9.65	8.10	7.45	7.95	7.00	6.25	7.15	7.15	6.20	5.40	6.35	6.50
	.0	9.65	7.50	8.40	8.65	6.95	6.45	7.05	6.95	6.05	5.70	6.85	7.15
	.2	10.40	8.20	9.10	8.90	7.50	5.90	6.35	6.05	7.30	5.15	5.45	5.45
	.5	9.50	5.75	9.00	8.70	9.30	6.55	6.35	6.50	7.00	5.65	5.20	4.95
	.8	14.60	6.95	11.70	11.00	9.35	4.95	6.60	6.55	7.00	5.40	6.40	6.25
-.2	-.8	10.00	8.40	8.50	8.00	8.15	6.40	5.80	5.65	6.45	6.85	5.80	5.05
	-.5	9.30	8.05	9.15	9.50	6.85	6.00	7.10	6.55	5.65	4.95	5.55	5.60
	-.2	8.95	8.05	10.15	9.40	7.55	6.20	7.00	7.20	6.20	5.75	5.70	5.85
	.0	9.50	6.70	8.75	8.70	7.35	5.95	7.05	7.10	6.10	5.05	6.65	6.25
	.2	10.25	7.20	9.20	9.85	6.75	5.15	6.55	6.55	5.15	4.45	6.60	6.40
	.5	10.65	6.60	7.65	7.80	9.00	6.35	6.50	6.10	6.75	5.45	6.15	6.35
	.8	14.85	8.90	8.95	8.35	9.55	6.00	7.10	7.35	7.20	6.05	5.95	5.95
.0	-.8	9.45	8.30	7.50	6.90	6.70	5.55	6.35	5.95	5.85	5.50	5.30	5.30
	-.5	9.85	7.50	10.05	9.75	7.70	7.50	7.30	7.10	5.40	4.65	5.85	5.45
	-.2	9.20	7.65	9.00	9.75	5.75	6.00	6.95	7.35	6.00	5.80	6.20	6.50
	.0	10.35	8.10	9.75	10.20	8.05	5.70	7.60	7.80	5.15	5.65	6.50	6.40
	.2	8.60	8.10	8.55	8.25	7.15	6.55	7.40	7.85	6.65	4.70	5.95	5.80
	.5	9.25	7.50	7.70	8.35	7.75	6.45	6.05	6.80	5.15	5.15	5.55	5.30
	.8	13.95	6.90	7.85	8.30	9.30	5.90	6.50	7.00	7.00	6.15	5.35	5.80
.2	-.8	9.60	7.00	7.10	7.90	7.00	5.55	6.40	5.95	6.05	6.00	4.65	4.75
	-.5	9.50	8.55	8.00	7.50	7.65	7.15	6.75	6.45	6.50	6.65	6.55	7.00
	-.2	11.55	9.30	10.45	10.00	8.00	6.40	8.20	7.70	6.50	6.05	6.70	6.40
	.0	10.25	8.65	10.70	10.60	7.50	6.30	6.35	6.30	5.90	5.80	6.40	6.15
	.2	11.65	6.75	9.20	9.40	6.60	6.90	6.80	7.30	6.75	5.25	6.05	5.35
	.5	9.65	7.40	8.35	8.35	7.70	6.10	7.25	7.50	6.85	5.75	6.45	6.30
	.8	11.85	7.25	8.60	9.65	8.00	6.20	6.45	6.35	6.45	5.85	5.70	5.30
.5	-.8	9.40	7.15	7.25	6.70	7.00	5.35	6.20	5.95	5.35	4.05	5.85	5.60
	-.5	9.60	8.25	8.80	8.55	7.60	6.30	7.45	7.55	6.10	5.70	5.75	5.85
	-.2	10.50	7.70	9.95	9.50	8.95	6.50	7.05	7.20	6.50	5.75	6.40	6.35
	.0	11.70	9.35	10.95	10.80	7.15	7.60	7.15	7.05	7.10	5.85	6.30	6.30
	.2	11.50	8.50	10.40	10.65	8.65	6.70	7.05	7.20	7.25	5.80	6.85	6.60
	.5	11.95	9.80	10.55	10.25	7.95	7.35	7.80	7.15	6.30	5.25	6.15	6.05
	.8	10.95	10.15	9.90	9.75	9.15	7.55	8.40	8.25	7.00	6.30	5.55	6.00
.8	-.8	9.75	8.25	8.70	8.05	7.25	6.10	6.15	6.05	5.60	5.15	5.85	5.75
	-.5	11.05	7.55	10.25	10.20	8.15	7.15	7.70	7.65	6.15	5.10	5.75	5.75
	-.2	11.05	8.10	10.80	10.80	7.65	6.70	7.40	7.50	6.10	6.10	6.40	6.40
	.0	13.00	8.25	11.55	11.45	8.35	6.60	9.05	8.85	6.70	5.40	5.95	6.20
	.2	13.75	9.05	12.75	12.85	10.15	8.00	8.05	7.75	6.60	5.50	6.10	5.90
	.5	14.90	11.40	11.95	12.45	10.55	7.90	9.35	9.40	6.20	5.10	6.85	7.00
	.8	17.10	16.45	13.95	14.20	11.05	10.80	9.55	9.40	7.35	7.35	7.40	7.40

Table 9. Empirical Sizes (%) of the Four Tests, DGP2 with Normal-Gamma Mixture

		$n=50$				$n=100$				$n=200$			
ρ	γ	T_{1n}	T_{2n}	T_{3n}	t_{4n}	T_{1n}	T_{2n}	T_{3n}	t_{4n}	T_{1n}	T_{2n}	T_{3n}	t_{4n}
-.8	-.8	8.55	11.00	8.30	7.40	7.45	7.00	6.60	6.40	6.25	6.60	5.60	5.40
	-.5	10.55	9.00	9.10	9.45	6.45	7.40	7.70	7.65	6.45	6.15	6.40	6.65
	-.2	9.20	7.70	8.55	8.35	7.70	6.40	7.00	6.95	6.75	5.90	6.20	6.25
	.0	10.55	6.90	9.00	8.90	7.35	6.00	7.30	7.00	5.90	6.45	5.90	5.90
	.2	10.85	7.85	10.10	10.20	6.65	5.25	5.95	5.95	5.15	5.75	5.15	5.15
	.5	10.55	6.85	9.40	9.30	8.20	6.45	7.30	7.35	5.85	5.65	5.85	5.85
	.8	15.90	7.60	12.70	12.55	9.85	6.70	6.45	6.50	7.05	4.85	5.30	5.45
-.5	-.8	9.00	9.55	9.85	8.25	6.65	6.15	6.15	6.10	5.10	6.35	6.05	6.05
	-.5	9.40	8.60	8.90	8.35	6.30	6.20	6.85	6.70	5.75	5.25	6.40	6.15
	-.2	8.10	6.85	8.30	7.90	7.10	6.85	7.65	7.70	6.05	5.50	6.15	6.40
	.0	9.90	6.80	9.40	9.50	7.50	6.35	7.55	7.50	5.50	5.25	5.75	5.60
	.2	10.45	6.35	10.10	9.90	8.00	5.95	6.15	5.95	6.55	5.55	6.50	6.15
	.5	10.60	7.00	7.75	7.95	7.80	6.95	7.45	7.65	5.85	5.55	5.95	5.95
	.8	12.50	6.50	10.60	10.60	9.30	6.05	7.05	6.45	8.20	6.80	5.25	5.45
-.2	-.8	8.05	7.70	6.45	6.75	7.45	6.65	5.80	5.10	6.20	5.45	5.50	5.20
	-.5	9.30	6.60	8.95	9.05	7.10	6.25	7.30	7.15	6.95	5.45	4.70	4.60
	-.2	10.90	8.10	8.90	9.40	6.65	6.70	6.85	5.85	6.30	5.55	6.40	6.05
	.0	9.95	8.05	8.80	8.50	7.20	6.10	6.75	6.60	6.25	5.85	6.10	6.00
	.2	9.10	6.90	9.50	9.55	7.45	6.05	7.40	7.80	6.60	5.25	5.85	6.05
	.5	10.75	7.95	8.65	8.70	9.15	5.65	6.95	6.65	5.70	5.75	6.20	6.60
	.8	13.90	7.00	10.80	10.35	8.65	5.30	6.50	6.30	7.90	5.95	6.05	6.00
.0	-.8	8.85	6.60	8.65	6.85	6.60	6.45	5.95	5.65	5.90	6.05	4.80	5.05
	-.5	9.20	7.45	9.00	9.00	7.80	6.05	6.15	5.95	6.00	5.65	6.25	5.85
	-.2	10.35	7.45	10.35	10.80	6.35	6.80	8.40	8.30	5.85	5.45	6.15	6.30
	.0	11.00	8.00	10.60	10.85	8.85	6.20	8.15	8.35	6.30	6.30	6.25	5.95
	.2	11.05	7.80	9.10	8.80	6.75	5.75	6.95	7.00	7.35	5.55	6.50	6.05
	.5	11.40	9.50	9.75	9.85	7.80	5.65	8.10	7.45	6.15	6.75	4.50	4.45
	.8	15.10	8.15	8.45	9.15	9.65	5.80	5.55	5.75	6.60	5.25	5.00	5.40
.2	-.8	9.10	6.80	7.00	7.25	8.20	6.25	6.60	5.75	6.25	6.15	5.80	5.70
	-.5	9.35	8.00	9.80	9.15	7.40	7.00	7.30	7.40	5.95	5.40	6.20	6.30
	-.2	10.95	8.90	9.10	9.10	8.55	5.95	8.30	8.10	6.30	6.35	5.00	5.05
	.0	10.60	8.70	10.65	10.70	7.50	6.50	6.90	6.60	5.90	6.00	5.80	5.90
	.2	10.55	8.60	9.90	9.35	7.85	6.75	7.20	7.35	5.25	5.50	6.30	6.20
	.5	10.40	9.35	11.05	10.95	6.95	7.00	6.75	6.90	6.15	6.20	6.10	6.15
	.8	13.00	9.55	9.40	9.65	8.25	6.00	6.20	6.95	7.15	5.45	6.25	5.65
.5	-.8	9.65	8.25	8.60	8.35	5.70	5.45	5.55	5.25	4.95	5.65	4.65	4.80
	-.5	10.05	7.25	10.50	10.90	6.40	6.30	7.80	7.50	5.35	4.80	5.75	5.75
	-.2	12.55	8.95	10.65	10.50	6.60	5.40	7.25	7.20	6.25	5.75	5.95	5.95
	.0	11.75	8.90	9.60	10.10	7.95	5.30	7.85	7.70	5.60	5.15	6.05	5.75
	.2	11.95	8.70	11.20	11.00	8.35	5.85	7.50	7.25	7.35	6.30	6.50	6.65
	.5	14.65	9.55	12.25	12.40	7.60	7.50	7.25	7.25	7.00	5.55	6.55	6.60
	.8	14.10	10.35	11.65	10.70	8.65	7.70	7.45	7.80	6.45	6.85	5.90	6.30
.8	-.8	10.00	7.95	8.80	8.75	6.95	6.05	6.95	6.85	6.90	6.10	6.15	6.25
	-.5	9.10	7.05	10.90	10.60	7.50	6.05	7.05	7.15	6.35	4.95	5.30	5.60
	-.2	11.30	9.05	11.70	11.60	7.55	6.25	7.30	7.00	6.40	5.90	6.30	6.65
	.0	15.10	10.90	12.30	12.40	7.35	6.20	8.25	8.00	6.85	6.25	7.75	7.70
	.2	15.25	9.10	14.25	14.20	7.90	6.50	9.40	9.10	6.60	5.65	6.30	6.30
	.5	17.85	10.55	15.65	15.80	9.45	8.05	7.05	7.95	7.15	6.30	7.45	7.30
	.8	19.45	13.95	16.60	16.60	8.35	8.20	7.95	8.25	7.30	6.95	6.50	6.20