WP-2008-002

On Measuring Group Differential: Some Further Results

Hippu Salk Kristle Nathan and Srijit Mishra

Indira Gandhi Institute of Development Research, Mumbai
March 2008
http://www.igidr.ac.in/pdf/publication/WP-2008-002.pdf

On Measuring Group Differential:

Some Further Results[1]

Hippu Salk Kristle Nathan and Srijit Mishra

Indira Gandhi Institute of Development Research (IGIDR)
General Arun Kumar Vaidya Marg
Goregaon (E), Mumbai- 400065, INDIA
Emails: hnathan@igidr.ac.in, and srijit@igidr.ac.in

Abstract

We impose a value judgment that a decrease in failure should be accompanied by a decrease in gap (difference or ratio) between sub-groups. In other words, the same gap at lower levels of failure is to be considered worse off. This, in line with transfer sensitivity axiom of poverty indices, is formalized by Mishra and Subramanian (2006) through two level-sensitive axioms in group differential measures. In addition, Mishra (2007) imposes an axiom of normalization. At a basic level it means that the group differential measure lies between zero and unity. However, at a fundamental level it should also mean that zero indicates no differential between the two sub-groups whereas unity indicates maximum differential between the two sub-groups. A group differential measure discussed in the above-mentioned two papers satisfied the level-sensitivity axioms but failed the normalization axiom at a fundamental level. Further, the comparison between two situations under this measure also happened to be dependent on the choice of some parameters. Both these problems are done away with in the measure proposed in this paper. Empirical illustration with infant mortality rate data for selected Indian states has also been given.

Key words: Indicator of failure, Level sensitivity (difference-based and ratio-based), Normalization

JEL Codes: D63, O15

[1] This has been prepared for an edited volume on Human Development. The second author's discussions with student in the class on Contemporary Issues in Human Development and Policy (January-May 2008) at the Indira Gandhi Institute of Development Research (IGIDR), Mumbai were helpful.

On Measuring Group Differential:

Some Further Results

Hippu Salk Kristle Nathan and Srijit Mishra

1. Introduction

Group differential is an important class of measures to know the gap with regard to failure (or attainment) indicators between two groups.[2] Conventionally, this has been expressed by simple difference or simple ratio. To be the basis for comparison, these measures should have certain properties In line with the transfer-sensitivity property of poverty indices (Kakwani 1993 and Sen 1976), Mishra and Subramanian (2006) have introduced two axioms on level sensitivity, difference-based level sensitivity (DBLS) and ratio-based level sensitivity (RBLS). These axioms indicate that for a failure (attainment) indicator a given hiatus between two groups should acquire a greater salience the lower (higher) the level at which the hiatus arises. It subscribes to a value judgment that a decrease in failure should be accompanied by a decrease in gap (difference or ratio between sub-groups). In other words, the same gap at lower levels of failure is to be considered worse off. They discuss three existing and a fourth new measure of group differential, which were later refined by Mishra (2007), who also added the axiom of normalization. At a basic level, it means that the group differential measure lies between zero and unity. However, at a fundamental level it should also mean that zero indicates no differential between the two sub-groups whereas unity indicates maximum differential between the two sub-groups. The suggested new measure in the above-mentioned two papers gave a positive non-zero value when there were no differences between sub-groups – a failure of the normalization axiom at a fundamental level. Further, the comparison between two situations under this measure also happened to be dependent on the choice of some parameters. This paper suggests a measure that tries to address these.

[2] There are genuine failure indicators like Infant Mortality Rate (IMR), Maternal Mortality Rate (MMR), and Death Rate. There are genuine attainment indicators like literacy rate and income. An attainment indicator can be converted as failure by taking its inverse, like when literacy rate is replaced with illiteracy rate, or in case of income, a maximum may be posited and the actual observations subtracted from this to obtain an indicator of failure. However, axioms of level sensitivity should be different for attainment indicators. This forms a larger exercise which is being currently carried out by the authors.

Empirical illustration has been provided with the same set of infant mortality rate data, as has been used by Mishra (2007).

2. Axiomatic characterization of group differential

Consider a socio economic failure indicator, $I_{js}\varepsilon[0,1]$; 0=no failure and 1=complete failure for j^{th} group ($j=a,b$), under situation s ($s=A,B$). Without loss of generality, given a situation s let group b be considered to be at lower failure level than a, $I_{as}>I_{bs}$ and given a group j situation A is at least as good as B so that $I_{jA}\leq I_{jB}$. Following are a number of intuitive properties that a measure of group differential, d or $d(I_{as},I_{bs})$ should satisfy.

Normalization (Axiom N): At a basic level, the measure of group differential should lie between zero and unity, $d\varepsilon[0,1]$. At a fundamental level the measure should have a minimum and a maximum such that 0=no group-differential and 1=highest group-differential.

Strong Monotonicity (Axiom M): The measure of group differential should be such that it is higher (lower) if one of the groups remaining constant at a particular level of failure; the other changes so that the absolute gap increases (decreases). Mathematically, $d(I_{aA},I_{bA})>d(I_{aB},I_{bB})$ when $I_{aA}=I_{aB}$ and $I_{bA}<I_{bB}$. Weak monotonicity means, $d(I_{aA},I_{bA})\geq d(I_{aB},I_{bB})$ when $I_{aA}=I_{aB}$ and $I_{bA}<I_{bB}$. Two corollaries of strong monotonicity are axioms of minimality and maximality.

Minimality (Axiom M_{min}): The measure of group differential should be higher than its minimum value if there is some group differential. Mathematically, $d>0$ if $(I_{as}-I_{bs})>0$.

Maximality (Axiom M_{max}): The measure of group differential should be lower than its maximum value if the group-differential is less than the highest. Mathematically, $d<1$ if $(I_{as}-I_{bs})<1$.

Difference based level sensitivity (DBLS) (Axiom D): The measure of group differential should be such that it is more pronounced if the difference level persists at a lower level of failure. Mathematically, if $I_{aA}-I_{bA}\geq I_{aB}-I_{bB}=h$; $h>0$, then the DBLS axiom requires that $d(I_{aA},I_{bA})>d(I_{aB},I_{bB})$.

Ratio based level sensitivity (RBLS) (Axiom R): The measure of group differential should be such that it is more pronounced if the ratio level persists at a lower level of failure. Mathematically, if $I_{aA}/I_{bA}\geq I_{aB}/I_{bB}=k$; $k>0$, then the RBLS axiom requires that $d(I_{aA},I_{bA})>d(I_{aB},I_{bB})$. RBLS is a stricter condition than DBLS, if $I_{aA}/I_{bA}\geq I_{aB}/I_{bB}$ then $I_{aA}-I_{bA}<I_{aB}-I_{bB}$.

3. Measures of group differential

Some of the differential measures with regard to a failure indicator discussed in literature are:

$$d_1=I_a-I_b \tag{1}$$

$$d_2=I_a^{\delta}-I_b^{\delta};\ 0<\delta<1 \tag{2}$$

$$d_3=1-I_b/I_a \tag{3}$$

$$d_4=1-I_b^{\alpha+\beta}/I_a^{\alpha};\ \alpha>0,\ \beta\geq 0 \tag{4}$$

Mishra (2007) indicates that d_1 satisfies DBLS in a weak sense, d_2 satisfies DBLS strongly, d_3 satisfies RBLS in weak sense, and d_4 satisfies RBLS strongly. Keeping in mind that RBLS is a stricter condition for failure indicators, d_1 and d_2 would not be considered as serious contenders of a differential measure. The measure of d_3 satisfies RBLS weakly and also fails the strong monotonicity axiom at $I_b=0$. We have d_4 that satisfies RBLS strongly but fails normalization at a fundamental level when there is no differential between sub-groups, $0<I_a=I_b<1$. Further, in d_4 the comparison between two situations is dependent on the subjective choice of α and β parameters and as in the previous case it also fails the strong monotonicity test. It is a measure that takes us out of the non-frying pan to the fire. The problems are addressed by a proposed alternative,

$$d_5=(1-I_b/I_a)*(1-I_b) \tag{5}$$

The measure of d_5 satisfies the level sensitivities and normalization axioms but still fails the strong monotonicity, particularly the maximality version when $I_b=0$. In fact, it is quite intuitive to show that at $I_b=0$ the RBLS and maximality axiom cannot be satisfied together. RBLS indicates that for the same ratio (in this case zero) as level decreases (in this case I_a because $I_b=0$) then d should increase. Whereas maximality indicates that if value for one sub-group is constant (in this case $I_b=0$) then a decrease in the value of the other sub-group, I_a, should lead to a decrease in d. Table 1 indicates the applicability of axioms to various differential measures.

Table 1: Applicability of axioms to various differential measures

Measure	Axioms					
	N	M	M_{min}	M_{max}	D	R
$D_1 = I_a - I_b$	Yes	Yes	Yes	Yes	Yes (weakly)	No
$D_2 = I_a^{\delta} - I_b^{\delta}$	Yes	Yes	Yes	Yes	Yes	No
$D_3 = 1 - I_b / I_a$	Yes	Yes (weakly)#	Yes	Yes (weakly)#	Yes	Yes (weakly)
$D_4 = 1 - I_b^{\alpha+\beta} / I_a^{\alpha}$	Yes (Basic)$	Yes (weakly)#	Yes	Yes (weakly)#	Yes	Yes
$D_5 = (1 - I_b/I_a) * (1 - I_b)$	Yes	Yes (weakly)#	Yes	Yes (weakly)#	Yes	Yes

Note: # fails at $I_b=0$; $ satisfies normalization at a basic level, $d_4 \varepsilon [0,1]$, but fails it at a fundamental level because at no differential, $0<I_a=I_b<1$, $d_4>0$. Further note that comparison between two situations in d_4 would be dependent on α and β parameters.

3. Empirical illustration

We use infant mortality rate (IMR) data of selected Indian states, the same as in Misrha (2007), for empirical illustration. Case 1 is equal difference case, $I_{aA}-I_{bA}=I_{aB}-I_{bB}$, where d_1 satisfies DBLS weakly, whereas other measures (d_2, d_3, d_4, and d_5) satisfy it strongly. Case 2 is equal ratio case, $I_{bA}/I_{aA}=I_{bB}/I_{aB}$, where d_1 and d_2 do not satisfy RBLS, whereas d_3 satisfies it weakly and d_4 and d_5 satisfy it strongly. Case 3 illustrates no group differentiation, $I_{aA}=I_{bA}$ & $I_{aB}=I_{bB}$, where d_4 gives f a non-zero positive value indicating a failure of the normalization axiom at a fundamental level whereas other measures (d_1, d_2, d_3, and d_5) satisfy it.

Table 2: Comparing various group differential measures using Infant Mortality Rate data from selected Indian states

Cases	Situations	I_a	I_b	d_1	d_2	d_3	d_4	d_5
Case 1	Karnataka, 2003	0.052	0.051	0.0010	0.0022	0.0192	0.0221	0.0183
$I_{aA}-I_{bA}=I_{aB}-I_{bB}$	Orissa, 2003	0.083	0.082	0.0010	0.0017	0..0120	0.0145	0.0111
Case 2	Assam, 2003	0.070	0.035	0.0350	0.0775	0.5000	0.5017	0.4825
$I_{bA}/I_{aA}=I_{bB}/I_{aB}$	Assam, 1990	0.078	0.039	0.0390	0.0818	0.5000	0.5016	0.4805
Case 3	Kerala, Rural 2003	0.012	0.012	0	0	0	0.0044	0
$I_{aA}=I_{bA}$ & $I_{aB}=I_{bB}$	West Bengal, Rural 2003	0.048	0.048	0	0	0	0.0030	0

Notes: I_a and I_b denote infant mortality converted to the 0-1 range for sub-groups *a* and *b* respectively; d_1, d_2, d_3, d_4 and d_5 denote the five differential measures discussed in the text; d_2 has been computed for $\delta=0.5$; d_4 has been computed for $\alpha=1$ and $\beta=0.001$. In all cases, situations *A* and *B* are indicated in the first and second rows respectively. Sub-groups *a* and *b* refer to female and male respectively in cases 1 and 3, and rural and urban respectively in case 2.

Sources: *Sample Registration System Statistical Report 2003*, Report No. 2 of 2005, Registrar General, India, New Delhi. *Vital Statistics of India 1990 Based on the Civil Registration System*, Office of the Registrar General, India, Ministry of Home Affairs, New Delhi.

4. Concluding remarks

The paper discusses about measures of group differentials for failure indicators. It identifies the limitations of the measures in the literature and proposes an alternative which satisfies the axioms of level sensitivity and normalization. It also does away with the subjectivity associated in the choice of parameters in some existing measures. An empirical illustration using data for infant mortality rate from selected Indian states shows the advantages of the proposed alternative. For future work, providing a differential measure for attainment indicators would be useful. These measures can be used to evaluate success in some of the Millennium Development Goals.

References

Kakwani, N (1993) Performance in living standards: an international comparison, *Journal of Development Economics*, 41 (2): 307-336.

Mishra, Srijit (2007) On measuring group-differentials displayed by socio-economic indicators: an extension, *Applied Economics Letters*, 99999 (1): 1-4, http://www.informaworld.com/10.1080/13504850600972238, (accessed 28 September 2007). IGIDR working paper version is available at http://www.igidr.ac.in/pdf/publication/WP-2006-005.pdf.

Mishra, U. S. and S. Subramanian (2006) On measuring group-differentials displayed by socio-economic indicators, *Applied Economics Letters*, 13 (8): 519-521.

Sen, Amartya (1976) Poverty: an ordinal approach to measurement, *Econometrica*, 44 (2): 219-231.