E¢ciency of joint enterprises with internal bargaining

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Abstract : In this paper we take a close look at those strategic incentives arising in a situation where …rms share the costs and pro…ts in a multi-…rm project, and bargain for their respective (precommitted) split of cost- and pro…t-shares. We establish that, when each …rm's e¤ort contribution to the joint undertaking is mutually observable (which is often the case in closely collaborative operations) and hence can form basis of the contingent cost- and pro…t-sharing scheme, it is not the gross economic e¢ciency but the super-/sub-additivity of the nett returns from e¤ort that directly a¤ects the sustainability of a pro…le of …rms' e¤ort contributions. The (in)e¢ciency result we obtain in this paper is of di¤erent nature from so-called "free riding" or "team competition" problems: the set of sustainable outcomes with bargaining over precommetted cost- and pro…t-shares is generally neither a superset nor a subset of the sustainable set without bargaining.

Keywords : cost sharing, pro…t sharing, repayment, subgame perfection.

JEL classi…cation : L13, G31, O32.

1 Introduction

Cost sharing between multiple economic bodies, such as a research joint venture undertaken by multiple …rms operating in the same industry, often tends to be encouraged from policy- and welfare-points of view.[1] Apparently, the leading reason for such “o¢cial” encouragement is the assertion that cost sharing can enhance cost-e¢ciency, eliminating otherwise wasteful e¤ort duplication. A relevant example is the possibility of carrying out joint R&D activities in the development of new products and processes (see Katz, 1986; d’Aspremont and Jacquemin, 1988; Katz and Ordover, 1990; Kamien et al., 1992; Suzumura, 1992). On the other hand it is well noted that, generally, each participant in any joint project has an incentive to “free ride” the e¤ort exerted by other participants, which tends to entail an ine¢cient outcome with a suboptimally low level of e¤ort chosen strategically by each participant.

Perhaps one of the most natural and the most spontaneous “solution” to this incentive problem is to allow …rms to make their cost- and pro…t-shares in the joint project directly contingent upon their exerted e¤ort levels. In a joint project where participating …rms closely collaborate with one another, it is often not at all unrealistic to assume that they can accurately monitor each other’s e¤ort levels. In this paper we model such a situation by a simple two-stage game where each …rm decides its own e¤ort investment in the …rst stage, followed by the realisation of their respective nett pro…t shares in the ensuing stage. By precommitting their nett pro…t shares before deciding their respective e¤ort investment, …rms can reward or punish each other’s e¤ort decisions and thereby enforce a certain pro…le of e¤ort investment even if their decisions (in the …rst stage of the game) are simultaneously and noncooperatively made.

Our main …nding in this paper is that, even though the aforementioned contingent pro…t-sharing scheme serves as a means to circumvent the classical free riding problem, it can harbour a di¤erent kind of ine¢ciency. The gist of the di¤erence between preceding studies and our model is not the mere fact that …rms can precommit with their nett pro…t shares and monitor each other’s e¤ort, but that there can be room for bargaining among …rms when they predetermine their nett pro…t shares. Essentially, each …rm’s bargaining

[1]See the National Cooperative Research Act in the US; EC Commission (1990); and Goto and Wakasugi (1988), inter alia.

power is closely related to its “outside alternative” which, in a game-theoretic context, is largely parallel to the firm’s unilateral deviation incentives. This inevitably implies that an outcome which is not susceptible to strong unilateral deviation incentives by any of the participating firms, can be sustained as a (subgame perfect) equilibrium whether economically efficient or not.

To summarise, if each firm’s cost share is determined independently of their respective contributions, it tends to entail the classical free riding problem. Alternatively, if profit sharing can be made contingent upon each firms’ strategic contribution decisions, then the bargaining between participating firms tends to give rise to multiple equilibria, among which the effort levels are complementary between firms (i.e., the locus of equilibria is downward-sloping) if nett joint returns to effort is submodular between firms; or complementary between firms (i.e., the locus of equilibria is upward-sloping) if nett joint returns to effort is submodular.

The basic structure of our model, together with its general qualitative feature, is laid out in section 2. We present illustrative examples in section 3 to develop intuition on how precommitted bargaining can affect the sustainability of economically efficient outcomes. Section 4 concludes the paper. A glimpse of extension to a more quantitative decisions made by each of the participating firms in a joint project, is given in Appendix B.

2 Basic model

We start our analysis from a simple model of cost- and profit-sharing between two economic bodies, referred to as “firm 1” and “firm 2” henceforth. Although these two “firms” are to launch a jointly undertaken project, each of them is to decide, simultaneously and noncooperatively, whether or not to make an incremental marginal contribution to the joint project. The nett joint returns from their decisions can be summarised in the following table.

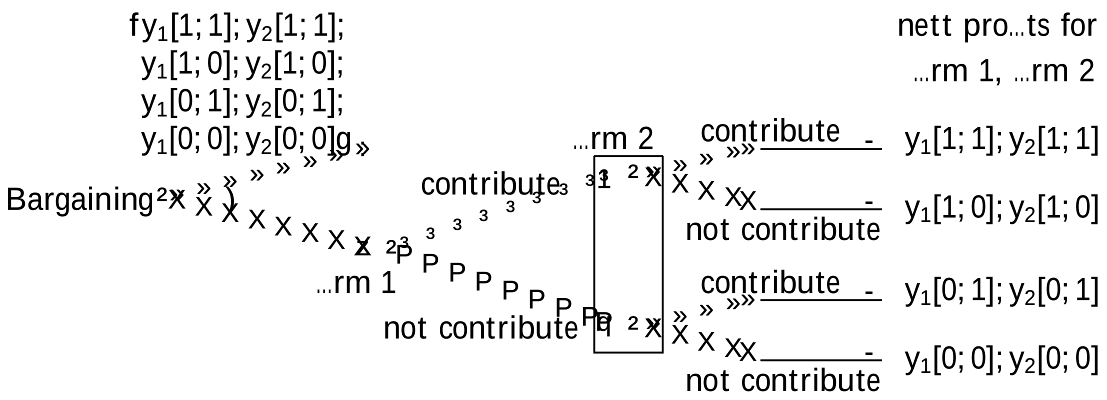
	firm 2	contribute	not contribute
firm 1	contribute	$Y[1;1]$	$Y[1;0]$
	not contribute	$Y[0;1]$	$Y[0;0]$

Throughout the paper we assume that contributions are mutually observable, so that firms can make their shares of nett profits $y_1[K]$ and $y_2[K]$ contingent directly upon the profile of their contributions subject, obviously, to

$$y_1[K] + y_2[K] = Y[K] \qquad [K] \in \{[1,1],[1,0],[0,1],[0,0]\}.$$

Hence, the procedural structure of this game can be summarised by the tree in figure 1.

Figure 1 : cost/profit sharing contingent upon contributions.



At the beginning, the two firms bargain[2] for the complete contingent set of nett profit shares. To retain as much generality as possible we avoid narrowly specifying the procedure of bargaining, other than imposing the following weak regularity requirement which seems plausible in any standard economic sense.

Definition : A bargaining function $b[\cdot,\cdot,\cdot] = (b_1[\cdot,\cdot,\cdot], b_2[\cdot,\cdot,\cdot])$ where

$$\begin{aligned} y_1[k_1,k_2] &= b_1\left[y_1[\neg k_1,k_2], y_2[k_1,\neg k_2], Y[k_1,k_2]\right] \\ y_2[k_1,k_2] &= b_2\left[y_1[\neg k_1,k_2], y_2[k_1,\neg k_2], Y[k_1,k_2]\right] \end{aligned} \qquad \{k_1,\neg k_1\} = \{k_2,\neg k_2\} = \{0,1\}$$

is said to be regular if

- b_1 increases in $y_1[\neg k_1,k_2]$, decreases in $y_2[k_1,\neg k_2]$, and increases in $Y[k_1,k_2]$;
- b_2 decreases in $y_1[\neg k_1,k_2]$, increases in $y_2[k_1,\neg k_2]$, and increases in $Y[k_1,k_2]$;
- $b_1 = y_1[\neg k_1,k_2]$ and $b_2 = y_2[k_1,\neg k_2]$ whenever $y_1[\neg k_1,k_2] + y_2[k_1,\neg k_2] = Y[k_1,k_2]$.

[2] It deserves heightened attention that the presence of bargaining should not be mistaken as if we were invoking any sort of cooperative decision making. As is well known, for instance, Nash bargaining can be viewed as a limiting solution for Rubinstein's bargaining, which is in fact a genuinely noncooperative game and is entirely devoid of any form of cooperative decision making (Rubinstein, 1982).

Note that our definition of regularity implies

$$\operatorname{sign}\left[y_1[k_1;k_2] - y_1[\neg k_1;k_2]\right] = \operatorname{sign}\left[y_2[k_1;k_2] - y_2[k_1;\neg k_2]\right].$$

This automatically implies the following general feature.

Proposition 1 : For any regular bargaining function $b[\cdot;\cdot;\cdot]$ and nett joint profit schedule $Y[\cdot;\cdot]$, whenever a solution $\left\{ \{y_i[k_1;k_2]\}_{k_1 \in \{0;1\}}^{k_2 \in \{0;1\}} \right\}_{i=1;2}$ exists, it sustains the equilibrium outcome such that :

- either both firms contribute or neither firm contributes if

$$Y[1;1] + Y[0;0] \geq Y[1;0] + Y[0;1]; \tag{2.1}$$

- only one of the two firms contributes if

$$Y[1;1] + Y[0;0] \leq Y[1;0] + Y[0;1]. \tag{2.2}$$

In words, {contribute, contribute} and {not contribute, not contribute} are equilibrium outcomes if the nett gains from the two firms' contributions are superadditive (as in inequality (2.1)).[3] Otherwise, if the strategic contribution decisions are to yield subadditive nett profits (indicated by inequality (2.2), then {contribute, not contribute} and {not contribute, contribute} are equilibrium outcomes.[4]

Economic implication : Proposition 1 implies that, in the prospect of bargaining between the participant firms inside the joint project, the equilibria resulting from each firm's "selfish" contribution decisions are determined solely on the grounds of super-/sub-additivity of the nett gains from contributions, not on the grounds of efficient outcomes

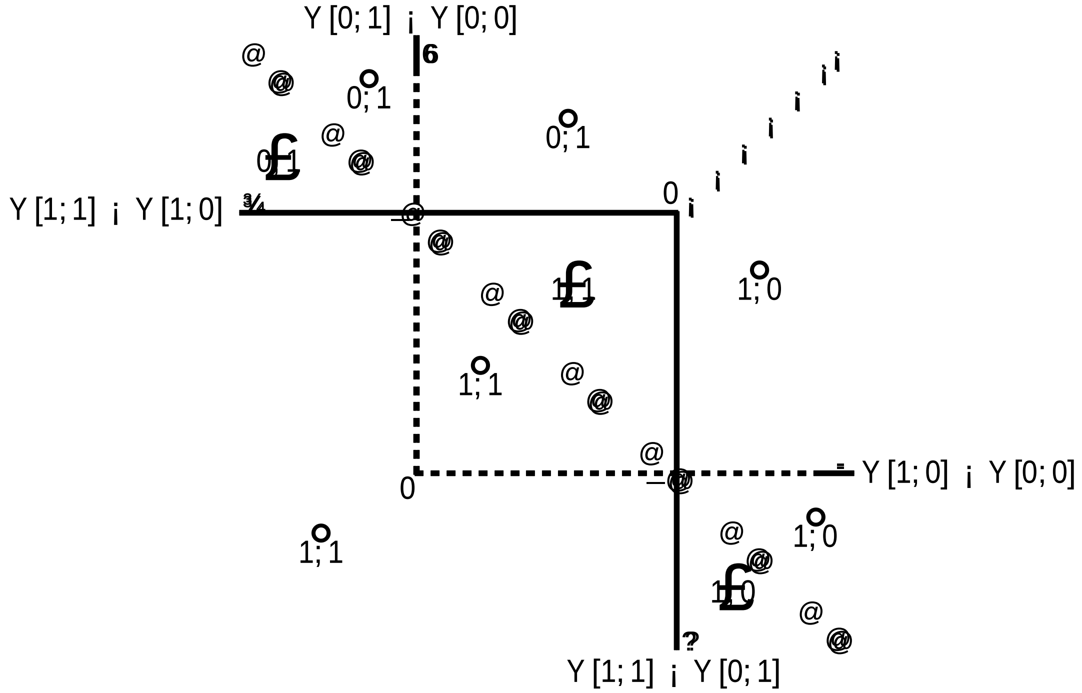
[3] Here, we refer to the discrete action version of superadditivity $Y[1;1] - Y[0;1] \geq Y[1;0] - Y[0;0]$ and submodularity $Y[1;1] - Y[0;1] \leq Y[1;0] - Y[0;0]$. In the former, the incentives for (or against) contribution are supermodular between the two firms (i.e., one firm's incentive to contribute is higher when the other firm does likewise than when the other firm does elsewise). In the latter, these incentives are submodular.

[4] Again, the analysis of R&D activity provides well known examples of both subadditivity and superadditivity, depending upon whether the choice of R&D efforts is followed by Cournot or Bertrand competition (see Brander and Spencer, 1983; Dixon, 1985; Bester and Petrakis, 1993).

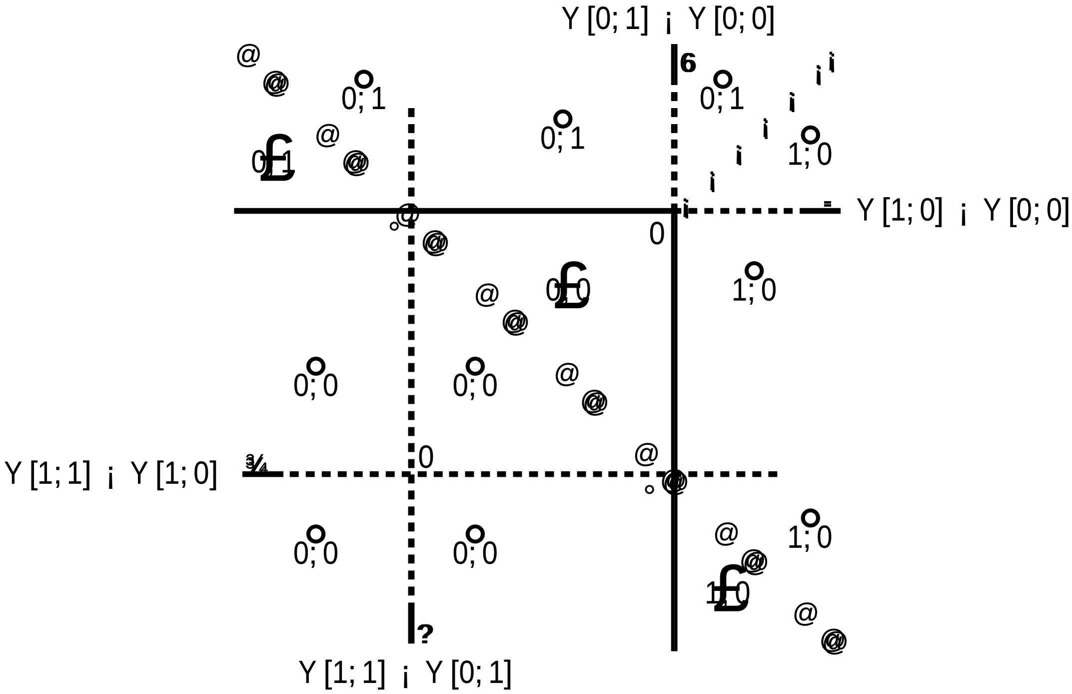
per se. This can be illustrated in …gure 2, where the most e¢cient contribution pro…les change across thickened borders. They are sustainable through Nash bargaining where circled, unsustainable where crossed out. For, to the upper-right of the dashed diagonal the equilibrium pro…les are $[1;0]$ and $[0;1]$, to its lower-left they are $[1;1]$ and $[0;0]$.

Figure 2 : e¢cient outcomes and their sustainability via Nash bargaining:

(2a) when $Y[1;1] - Y[0;0] = \beta \geq 0$;



(2b) when Y [1; 1] ¡ Y [0; 0] = ° · 0.



3 Illustrative examples

Probably the most popularly accepted bargaining solution concept is the Nash solution.[5] Let the ratio of bargaining power between the two …rms be $Ã_1 : Ã_2$. The two …rms' nett pro…t shares when both …rms contribute are determined as

$$\frac{y_1[1;1] \text{ ¡ } y_1[0;1]}{Ã_1} = \frac{y_2[1;1] \text{ ¡ } y_2[1;0]}{Ã_2}; \qquad y_1[1;1] + y_2[1;1] = Y[1;1]: \qquad (3:1)$$

In words, …rm 1's incentive (or disincentive) to "deviate" from contribution (C) to no contribution (N) should be matched against that for …rm 2, weighted by their respective

[5] For algebraic simplicity, we treat Nash bargaining as sharing of the nett excess surplus, i.e., the nett pro…t di¤erential in comparison to the "outside option" available to each …rm, be the excess surplus positive or negative. The qualitative nature of the game would stand largely una¤ected if we interpreted it as sharing of a positive excess surplus only, not adopting the same form of solution when the excess surplus is negative. The latter is technically more intricate but can be plausible depending upon what sort of economic circumstance is in question (see Appendix A).

bargaining power.[6] Likewise,

$$\frac{y_1[1;0] - y_1[0;0]}{\tilde{A}_1} = \frac{y_2[1;0] - y_2[1;1]}{\tilde{A}_2}; \qquad y_1[1;0] + y_2[1;0] = Y[1;0]; \tag{3:2}$$

$$\frac{y_1[0;1] - y_1[1;1]}{\tilde{A}_1} = \frac{y_2[0;1] - y_2[0;0]}{\tilde{A}_2}; \qquad y_1[0;1] + y_2[0;1] = Y[0;1]; \tag{3:3}$$

$$\frac{y_1[0;0] - y_1[1;0]}{\tilde{A}_1} = \frac{y_2[0;0] - y_2[0;1]}{\tilde{A}_2}; \qquad y_1[0;0] + y_2[0;0] = Y[0;0]: \tag{3:4}$$

The above system of eight equations with eight unknowns leaves one degree of freedom and gives us the general solution

$$\begin{aligned}
y_1[0;0] + y_2[0;0] &= Y[0;0]; \\
y_1[1;1] &= y_1[0;0] + \frac{Y[1;1] - Y[1;0] + Y[0;1] - Y[0;0]}{2}; \\
y_2[1;1] &= y_2[0;0] + \frac{Y[1;1] - Y[0;1] + Y[1;0] - Y[0;0]}{2}; \\
y_1[1;0] &= y_1[0;0] + \frac{\tilde{A}_1(Y[0;1] + Y[1;0] - Y[1;1] - Y[0;0])}{2(\tilde{A}_1 + \tilde{A}_2)}; \\
y_2[0;1] &= y_2[0;0] + \frac{\tilde{A}_2(Y[1;0] + Y[0;1] - Y[1;1] - Y[0;0])}{2(\tilde{A}_2 + \tilde{A}_1)}; \\
y_1[0;1] &= y_1[0;0] + \frac{\tilde{A}_2(Y[1;1] - Y[1;0]) + (2\tilde{A}_1 + \tilde{A}_2)(Y[0;1] - Y[0;0])}{2(\tilde{A}_1 + \tilde{A}_2)}; \\
y_2[1;0] &= y_2[0;0] + \frac{\tilde{A}_1(Y[1;1] - Y[0;1]) + (2\tilde{A}_2 + \tilde{A}_1)(Y[1;0] - Y[0;0])}{2(\tilde{A}_2 + \tilde{A}_1)}:
\end{aligned}$$

Example 1 : A priori equal bargaining power $\tilde{A}_1 = \tilde{A}_2$, the private cost of contribution is £8 million per …rm whilst the gross bene…t from contribution is £10 million if only one …rm contributes and £24 million if both contribute.

The nett joint return table becomes:

	…rm 2	contribute	not contribute
…rm 1	contribute	$Y[1;1] = 8$	$Y[1;0] = 2$
	not contribute	$Y[0;1] = 2$	$Y[0;0] = 0$

in £ million.

[6] Again, in our metaphor to Rubinstein's bargaining and its limiting solution, this corresponds to the case where the two …rms' degrees of patience, measured in terms of discount rates r_1 and r_2, are in the ratio $r_1 : r_2 = \frac{1}{\tilde{A}_1} : \frac{1}{\tilde{A}_2}$. As we see hereinafter, we introduce these bargaining power parameters for the sake of completeness, or di¤erently put, to demonstrate that our qualitative results do not hinge upon the distribution of bargaining power between the participating …rms.

The nett joint return from a sole …rm's contribution is £2 million whilst that from an additional …rm's contribution is £6 million, whereby nett returns from contribution is superadditive between the two …rms.

As aforementioned there are a continuum of bargaining solutions, but there is only one symmetric solution[7] (in £ million):

$$y_1[0;0] = y_2[0;0] = 0\,; \quad y_1[1;0] = y_2[0;1] = \text{¡}1\,;$$
$$y_1[0;1] = y_2[1;0] = 3\,; \quad y_1[1;1] = y_2[1;1] = 4:$$

This entails two pure strategy subgame perfect equilibria (simply "equilibria" hereinafter unless otherwise speci…ed), in one of which neither …rm contributes, in the other both …rms contribute and share the pro…t equally. This re‡ects the fact that nett returns from contribution is superadditive (see Proposition 1 in section 2). Obviously, the latter equilibrium entails the most e¢cient (…rst best) outcome.

Were there no bargaining, instead if the two …rms were to share the gross pro…t equally irrespective of their contributions, then the unique equilibrium outcome would be for neither …rm to contribute. This is due to the classical free riding problem leading to underincentives for each …rm to contribute. It is hereby concluded that the prospect of bargaining can help sustain the economically e¢cient outcome.

Example 2 : A priori equal bargaining power $\tilde{A}_1 = \tilde{A}_2$, the private cost of contribution is £2 million per …rm whilst the gross bene…t from contribution is £10 million if only one …rm contributes and £16 million if both contribute.

The nett joint return table becomes:

	…rm 2	contribute	not contribute
…rm 1	contribute	$Y[1;1] = 12$	$Y[1;0] = 8$
	not contribute	$Y[0;1] = 8$	$Y[0;0] = 0$

in £ million.

[7] We list the symmetric solution for nothing but concreteness. The set of equilibrium outcomes would, of course, be the same whether we selected the symmetric bargaining solution or an asymmetric solution.

The nett joint return from a sole …rm’s contribution is £8 million whilst that from an additional …rm’s contribution is £4 million, whereby nett returns from contribution is subadditive between the two …rms.

The unique symmetric solution (in £ million) is:

$$y_1[0;0] = y_2[0;0] = 0\,; \qquad y_1[1;0] = y_2[0;1] = 1\,;$$
$$y_1[0;1] = y_2[1;0] = 7\,; \qquad y_1[1;1] = y_2[1;1] = 6\,;$$

where the equilibria are for only one of the …rms to contribute and take £3 million out of the gross pro…t of £10 million whilst the other …rm takes the remainder, £7 million, without making contribution. This is the re‡exion of the subadditivity of nett returns from contribution.

Obviously, the nett return from contribution is always positive, hence economically the most e¢cient outcome would be for both …rms to contribute, which is nevertheless unsustainable through bargaining. Without bargaining, if the two …rms were to split the gross pro…t always evenly, then the e¢cient outcome would indeed be the unique equilibrium outcome. In this case, unlike in our previous example, bargaining hinders the sustainability of economically the most e¢cient outcome.

Example 3 : A priori equal bargaining power $\tilde{A}_1 = \tilde{A}_2$, the private cost of contribution is nil for …rm 1 and £10 million for …rm 2, whilst the gross bene…t from contribution is £4 million if only one …rm contributes and £12 million if both contribute.

The nett joint return table is:

	…rm 2	contribute	not contribute
…rm 1	contribute	$Y[1;1] = 2$	$Y[1;0] = 4$
	not contribute	$Y[0;1] = ¡6$	$Y[0;0] = 0$

in £ million.

As this game is a priori asymmetric between the two …rms, there is no “symmetric” solution. One of the solutions is

$$y_1[0;0] = y_2[0;0] = 0\,; \qquad y_1[1;0] = y_2[0;1] = ¡1\,;$$
$$y_1[0;1] = ¡5\,; \quad y_2[1;0] = 5\,; \qquad y_1[1;1] = ¡4\,; \quad y_2[1;1] = 6\,;$$

which accommodates two equilibria, in one of which neither …rm contributes, in the other both …rms contribute. Obviously in this case …rm 1's contribution is uniformly e¢cient whilst …rm 2's contribution is uniformly ine¢cient, hence the most e¢cient outcome is only for …rm 1 not for …rm 2 to contribute. This e¢cient outcome would be sustainable if gross pro…ts are shared evenly all the time, but is not sustainable with the bargaining for pro…t sharing schedules.

4 Conclusion

Standard game-theoretic literature has it that, in any jointly undertaken productive activity, if the pro…t sharing schedule cannot be made contingent upon the level of e¤ort exerted by each participating …rm and hence each …rm inevitably bears its own e¤ort costs, then there arises systematic underincentives for e¤orts, entailing lower aggregate e¤ort than economically e¢cient. This is the classical free riding problem.

What we have shown in this paper is that bargaining over a pro…t sharing schedule that is contingent upon observed e¤ort exerted by each participating …rm [I] can alleviate the free riding problem, yet [II] tends to entail a pro…le of e¤ort levels based solely upon the super-/sub-additivity of nett returns to e¤ort, irrespective of the nett joint productivity of e¤ort. The latter inevitably implies that [IIa] when nett returns to e¤ort are subadditive, a …rm's low e¤ort tends to be traded for another …rm's high e¤ort even when the joint nett returns to e¤ort is uniformly positive (in which case the e¢cient outcome of all …rms' high e¤ort becomes unsustainable) or when the joint nett returns is uniformly negative (where the e¢cient outcome would be all …rms' low e¤ort), and that [IIb] when nett returns to e¤ort are superadditive, a …rm's high e¤ort tends to link with another …rm's high e¤ort, whereby the system fails to select for an e¢cient …rm against an ine¢cient …rm.

Appendix A

The alternative, technically more intricate, scenario is that firms must follow Nash solution only if each firm's excess surplus share is positive. The system of equations (3.1) through (3.4) is now replaced with:

$$\frac{y_1[1;1]-y_1[0;1]}{\psi_1}=\frac{y_2[1;1]-y_2[1;0]}{\psi_2}\quad\text{if}\quad y_1[1;1]+y_2[1;1]=Y[1;1]\geq y_1[0;1]+y_2[1;0];$$
$$\frac{y_1[1;0]-y_1[0;0]}{\psi_1}=\frac{y_2[1;0]-y_2[1;1]}{\psi_2}\quad\text{if}\quad y_1[1;0]+y_2[1;0]=Y[1;0]\geq y_1[0;0]+y_2[1;1];$$
$$\frac{y_1[0;1]-y_1[1;1]}{\psi_1}=\frac{y_2[0;1]-y_2[0;0]}{\psi_2}\quad\text{if}\quad y_1[0;1]+y_2[0;1]=Y[0;1]\geq y_1[1;1]+y_2[0;0];$$
$$\frac{y_1[0;0]-y_1[1;0]}{\psi_1}=\frac{y_2[0;0]-y_2[0;1]}{\psi_2}\quad\text{if}\quad y_1[0;0]+y_2[0;0]=Y[0;0]\geq y_1[1;0]+y_2[0;1]:$$

It is intuitively clear that this leads to the same equilibrium result as in Proposition 1, although the exact set of sustainable solutions can now have one more degree of freedom than in section 2 (that is, two degrees of freedom in total).

Appendix B

Our basic analysis in section 2 can be straightforwardly extended to a descrete contributions space. Assume now that each of the two firms has a trinary, as opposed to the previously binary, choice of contributing either 0, 1, or 2 units to the jointly undertaken project. The nett joint returns table now becomes as follows.

	firm 2	2 units	1 unit	0 units
firm 1	2 units	$Y[2;2]$	$Y[2;1]$	$Y[2;0]$
	1 unit	$Y[1;2]$	$Y[1;1]$	$Y[1;0]$
	0 units	$Y[0;2]$	$Y[0;1]$	$Y[0;0]$

Accordingly, our concept of regularity needs to be redefined as follows.

Redefinition : A bargaining function $b[\cdot;\cdot;\cdot]=(b_1[\cdot;\cdot;\cdot];b_2[\cdot;\cdot;\cdot]):$

$$y_1[k_1;k_2]=b_1\left[\max y_1[:k_1;k_2];\max y_2[k_1;:k_2];Y[k_1;k_2]\right];$$
$$y_2[k_1;k_2]=b_2\left[\max y_1[:k_1;k_2];\max y_2[k_1;:k_2];Y[k_1;k_2]\right];$$

where

$$\max y_1[\,:k_1;k_2] \equiv \max\{y_1[k_1^{\varepsilon};k_2];\, y_1[k_1^{\delta};k_2]\} \qquad \{k_1;k_1^{\varepsilon};k_1^{\delta}\} = \{0;1;2\}\,;$$
$$\max y_2[k_1;\,:k_2] \equiv \max\{y_2[k_1;k_2^{\varepsilon}];\, y_1[k_1;k_2^{\delta}]\} \qquad \{k_2;k_2^{\varepsilon};k_2^{\delta}\} = \{0;1;2\}\,;$$

is said to be regular if

- b_1 increases in $\max y_1[\,:k_1;k_2]$, decreases in $\max y_2[k_1;\,:k_2]$, and increases in $Y[k_1;k_2]$;
- b_2 decreases in $\max y_1[\,:k_1;k_2]$, increases in $\max y_2[k_1;\,:k_2]$, and increases in $Y[k_1;k_2]$;
- $b_1 = \max y_1[\,:k_1;k_2]$ and $b_2 = \max y_2[k_1;\,:k_2]$ whenever $\max y_1[\,:k_1;k_2]+\max y_2[k_1;\,:k_2] = Y[k_1;k_2]$.

Our interest is not in an exhaustive equilibrium comparative statics result on this game, but instead in the following analogue of our foregoing Proposition 1 (see section 2).[8]

Proposition 2 :

- The two firms' contributing the same number of units, either 0, 1, or 2 units each, is an equilibrium outcome if

$$Y[2;2] + Y[1;1] \geq Y[2;1] + Y[1;2]\,; \tag{B.1}$$
$$Y[2;2] + Y[0;0] \geq Y[2;0] + Y[0;2]\,; \tag{B.2}$$
$$Y[1;1] + Y[0;0] \geq Y[1;0] + Y[0;1]\,. \tag{B.3}$$

- The two firms' contributing two units altogether, be it split 2-0 or 1-1, is an equilibrium outcome if

$$Y[2;0] + Y[1;1] \geq Y[2;1] + Y[0;1]\,; \tag{B.4}$$
$$Y[2;0] + Y[0;2] \geq Y[2;2] + Y[0;0]\,; \tag{B.5}$$
$$Y[1;1] + Y[0;2] \geq Y[1;0] + Y[1;2]\,. \tag{B.6}$$

[8] Note that Proposition 2, unlike Proposition 1, does not exhaust the entire feasible range of parameter values $Y[\cdot;\cdot]$. For instance, it is possible that inequality (B.1) may be satisfied whilst inequalities (B.2) and (B.3) may be violated. Our intention here is to focus on those two cases listed specifically on Proposition 2, as these two are the relevant cases hereinafter.

Namely, when the nett gains from the two firms' contributions are superadditive in the sense of positive affiliation (as in (B.1) through (B.3)) the two firms' equilibrium contributions are strategically perfectly complementary; otherwise when the nett gains are subadditive (as in (B.4) through (B.6)) the equilibrium contributions are strategically perfectly substitutable between the two firms.

The reason why these two "extreme" cases appeal to our interest is because they enable us to extrapolate our analysis further to a continuous contributions space. An analogue of Propositions 1 and 2 is as below.

Proposition 3 :

- The two firms' effort levels $k_1; k_2$ are complementary along the locus of equilibria if $\dfrac{\partial^2 Y[k_1; k_2]}{\partial k_1 \partial k_2} > 0$;
- The two firms' effort levels $k_1; k_2$ are substitutional along the locus of equilibria if $\dfrac{\partial^2 Y[k_1; k_2]}{\partial k_1 \partial k_2} < 0$.

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