A Class of Best-Response Potential Games*

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Abstract

We identify a class of noncooperative games in continuous strategies which are best-response potential games. We identify the conditions for the existence of a best-response potential function and characterise its construction, describing then the key properties of the equilibrium. The theoretical analysis is accompanied by applications to oligopoly and monetary policy games.

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1 Introduction

A recent stream of research, stemming from Hofbauer and Sigmund (1988) and Monderer and Shapley (1996), has investigated the concept of exact potential game, whereby the information about Nash equilibria is nested into a single real-valued function (the potential function) over the strategy space. The specific feature of a potential function defined for a given game is that the gradient of the corresponding potential function coincides with the vector of first derivatives of the individual payoff functions of the original game itself.[1]

However, the relevant information about the Nash equilibrium can be collected by imposing a weaker requirement, leading to the class of best-response potential games, as defined by Voorneveld (2000), which contains the class of exact potential games as a proper subset. The requirement used by Voorneveld is that the strategy profile maximizing the best-response potential function coincides with that identifying the Nash equilibrium.

In this paper we investigate a class of games in which the i-th player has a payoff function consisting in the product of her strategy times the generic power of a function of all players' strategies, the latter function being invariant across the population of players. First, we identify a sufficient condition for the existence of the best-response potential function for this game and illustrate the explicit construction procedure of the best-response potential function. Then, we also identify a sufficient condition for (i) the Nash equilibrium of the original game to be unique, and (ii) the correspond-

[1]The main directions along which the research on potential games has been carried out in recent years are, *inter alia*, (i) the relationship between strategic complementarity/substitutability and potential games (Brânzei *et al.*, 2003; Dubey *et al.*, 2006); (ii) the construction of a potential function for perfect information games (Kukushkin, 2002); (iii) the existence of correlated equilibria in potential games (Neyman, 1997; Ui, 2008); and coalition formation in potential games (Ui, 2000; Slikker, 2001).

ing best-response potential function to have a linear-quadratic form. Additionally, in the two-player case, we establish that if the Nash equilibrium is asymptotically stable (respectively, unstable), then the best-response potential function is concave (resp., convex), and conversely. This is followed by examples related to oligopoly theory and international monetary policy.

2 The best-response potential game

We consider a one-shot noncooperative full information game defined as $\Gamma = \left\langle N, (S_i)_{i\in N}, (\pi_i(s))_{i\in N} \right\rangle$, where

- $N = \{1, 2, 3, ...n\}$ is the set of players;
- $S_i \subset [0, +\infty)$ is the compact strategy space for player i;
- $\pi_i(s)$ is the profit function attributing a payoff to i in correspondence of any given admissible outcome $s = (s_1, \ldots, s_n)$, $s \in S = S_1 \times \ldots \times S_n$, of the following kind:

  $$\pi_i(s) = (f(s))^k s_i, \tag{1}$$

  where $f(s) \in C^2(S),\ f(s) \geq 0,\ k \in \mathbb{R}_+$. The game takes place under simultaneous play, and we confine our attention to the case where $\pi_i(s)$ is such that at least one Nash equilibrium in pure strategies does exist. Our analysis in the remainder of this section treats the case where the system of first order conditions has more than one critical point.

As a preliminary step, we recall the definition of potential function borrowed from physics:

Definition 1 *A vector field $F = (F_1(s), \ldots, F_n(s))$ is called **conservative** if there exists a differentiable function $P(s)$ such that:*

$$\frac{\partial P(s)}{\partial s_i} = F_i(s), \quad i = 1, \ldots, n.$$

$P(s)$ is called a ***potential function*** *for F.*

If a game admits a potential function P, then it is an exact potential game in the sense of Monderer and Shapley (1996). However, there are games that do not admit a potential function but meet a weaker requirement leading to the construction of a best-response potential function in the sense of Voorneveld (2000). The definition of a best-response potential game is the following:

Definition 2 (Voorneveld, 2000) *$\Gamma = \left\langle N, (S_i)_{i\in N}, (\pi_i(s))_{i\in N} \right\rangle$ is a best-response potential game if there exists a function $\widehat{P}(s) : S \to \mathbb{R}$ such that*

$$\arg\max_{s_i\in S_i} \pi_i(s) = \arg\max_{s_i\in S_i} \widehat{P}(s)\, \forall i \in N,\, \forall s_{-i} \in S_{-i}.$$

The function $\widehat{P}(s)$ is called a ***best-response potential function*** *for the game Γ.*

Our interest in the chance to build a best-reponse potential function $\widehat{P}(s)$ for a strategic game stems from the fact that if $\widehat{P}(s)$ attains its maximum over S in correspondence of $s^* = (s_1^*, \ldots, s_n^*)$, then the game has a Nash equilibrium in s^* (see Voorneveld, 2000, Proposition 2.2, p. 290).

In order to construct the best-response potential function for the game under consideration, we need the following:

Definition 3 *We call* ***fictitious profit functions*** *the functions $\widehat{\pi}_i : S \longrightarrow \mathbb{R}$ obtained by the following integration:*

$$\widehat{\pi}_i(s) = \int \left(f(\cdot) + ks_i \frac{\partial f(\cdot)}{\partial s_i} \right) ds_i + \xi_i(s),$$

where $\partial \xi_i(s)/\partial s_i = 0$, for all $i = 1, \ldots, n$.

Relying upon Definitions 1-3, we may prove:

Proposition 4 *s^* is an interior maximum for the fictitious profit functions $\widehat{\pi}_i$ if and only if it is a maximum for the profit functions π_i too.*

Proof. s^* is an interior maximum for the fictitious profit functions if and only if the following conditions hold:

$$f(s^*) + ks_i^* \frac{\partial f(s^*)}{\partial s_i} = 0, \tag{2}$$

$$\frac{\partial^2 \widehat{\pi}_i(s^*)}{\partial s_i^2} = \frac{\partial f(s^*)}{\partial s_i} + k\frac{\partial f(s^*)}{\partial s_i} + ks_i^* \frac{\partial^2 f(s^*)}{\partial s_i^2} < 0. \tag{3}$$

It follows that if we evaluate the second partial derivative of the profit functions in s^*, we have:

$$\frac{\partial^2 \pi_i(s^*)}{\partial s_i^2} = (f(s^*))^{k-1} \frac{\partial^2 \widehat{\pi}_i(s^*)}{\partial s_i^2} < 0,$$

because $f(\cdot)$ is nonnegative by definition. This condition, together with the first order condition, ensures that s^* is an interior maximum for π_i too.

Vice versa, suppose that s^* maximizes all profit functions π_i. Then, the first order conditions on π_i imply (2), and by substitution in the second partial derivative we obtain (3). ■

W.r.t. the game Γ defined above, we can prove:

Proposition 5 *If the vector field*

$$\left(f(s) + ks_1 \frac{\partial f(s)}{\partial s_1}, \ldots, f(s) + ks_n \frac{\partial f(s)}{\partial s_n} \right) \tag{4}$$

is conservative in S, then Γ is a best-response potential game.

Proof. We start by writing down the first order conditions for the i-th player:

$$\begin{aligned} \frac{\partial \pi_i(s)}{\partial s_i} &= (f(s))^k + k(f(s))^{k-1} s_i \frac{\partial f(s)}{\partial s_i} \\ &= (f(s))^{k-1} \left(f(s) + ks_i \frac{\partial f(s)}{\partial s_i} \right) = 0. \end{aligned} \tag{5}$$

If there exists a strategy $s^* = (s_1^*, \ldots, s_n^*) \in S$ such that $(f(s^*))^{k-1} = 0$, then $\pi_i(s^*)$ vanishes too and that is a zero profit strategy, so we can restrict our analysis to the second factor. Analogously, if $(f(s))^{k-1}$ has no zeros, we have to focus on the second factor as well. We define:

$$\frac{\partial \widehat{\pi}_i(s)}{\partial s_i} = f(s) + k s_i \frac{\partial f(s)}{\partial s_i} = 0$$

as the fictitious first order condition. If the vector field $\left(\frac{\partial \widehat{\pi}_1}{\partial s_1}, \ldots, \frac{\partial \widehat{\pi}_n}{\partial s_n}\right)$ is conservative in S, then there exists a function $\widehat{P}(s)$ such that

$$\frac{\partial \widehat{P}}{\partial s_i} = \frac{\partial \widehat{\pi}_i}{\partial s_i}, \qquad i = 1, \ldots, n.$$

Therefore, on the basis of Definitions 2-3 and Proposition 4, Γ is a best-response potential game. ■

2.1 Constructing the best-response potential function

Here we illustrate the construction procedure yielding $\widehat{P}(s)$. The first step to this aim is the following definition:

Definition 6 *We call the* ***total fictitious profit function*** *the sum of all fictitious profit functions:*

$$\widehat{\Pi}(s) = \sum_{j=1}^{n} \widehat{\pi}_j(s). \tag{6}$$

In the remainder of the section we will omit the arguments for simplicity. The partial derivative of the total fictitious profit function can be decomposed as follows:

$$\frac{\partial \widehat{\Pi}}{\partial s_i} = \frac{\partial \widehat{\pi}_i}{\partial s_i} + \phi_i, \tag{7}$$

where

$$\phi_i = \frac{\partial}{\partial s_i}\left(\sum_{j \neq i} \widehat{\pi}_j\right). \tag{8}$$

The crucial additional step is to build up the potential function, that we label as $\widehat{\Phi}$, of the conservative vector field $(\phi_1, \ldots, \phi_n)$ measuring the difference between the total fictitious profit function $\widehat{\Pi}$ and the best-response potential function $\widehat{P}$ of Γ :

Proposition 7 *If the vector field* $(\phi_1, \ldots, \phi_n)$ *is conservative with a potential* $\widehat{\Phi}$*, the function*

$$\widehat{P} := \widehat{\Pi} - \widehat{\Phi} \tag{9}$$

is the best-response potential function of the game Γ.

Proof. If $\widehat{\Phi}$ is the potential function for the conservative vector field $(\phi_1, \ldots, \phi_n)$, then

$$\phi_i = \frac{\partial \widehat{\Phi}}{\partial s_i},$$

then it immediately follows that

$$\frac{\partial \widehat{P}}{\partial s_i} = \frac{\partial \widehat{\Pi}}{\partial s_i} - \frac{\partial \widehat{\Phi}}{\partial s_i} = \frac{\partial \widehat{\pi}_i}{\partial s_i} + \phi_i - \frac{\partial \widehat{\Phi}}{\partial s_i} = \frac{\partial \widehat{\pi}_i}{\partial s_i},$$

for all $i = 1, \ldots, n$. ■

2.2 Equilibrium properties

Propositions 5 and 7 immediately entail the following relevant Corollary:

Corollary 8 *For any* $k \in \mathbb{R}_+$*, if* f *is linear in* s_i *for all* $i = 1, 2, ...n$*, then the fictitious first order conditions are linear as well. Consequently, if a Nash equilibrium exists in pure strategies, it is necessarily unique and follows from the maximization of a best-response potential function* $\widehat{P}$ *which is linear-quadratic in* s_i *for all* $i = 1, 2, ...n$.

That is, if f is linear, then, irrespective of k, a simple linear-quadratic function sums up all of the relevant information associated with the optimal behaviour of players taking part in Γ.[2] The next Proposition establishes a relation between interior maxima of the best-response potential function $\widehat{P}$ and the asymptotic stability of Nash equilibria of the game Γ :

Proposition 9 *Suppose the game admits a best-response potential function, and take $N = \{1, 2\}$. If the Nash equilibrium is asymptotically stable (unstable), then the best-response potential function is concave (convex), and conversely.*

Proof. In a two-player best-response potential game endowed with a best-response potential $\widehat{P}(s_1, s_2)$, the Hessian matrix $H\left(\widehat{P}(s^*)\right)$ evaluated in the Nash equilibrium $s^* = (s_1^*, s_2^*)$ takes the following form:

$$H\left(\widehat{P}(s^*)\right) = \begin{pmatrix} \frac{\partial f(s^*)}{\partial s_1} + k\frac{\partial f(s^*)}{\partial s_1} + ks_1^*\frac{\partial^2 f(s^*)}{\partial s_1^2} & \frac{\partial f(s^*)}{\partial s_2} + ks_1^*\frac{\partial^2 f(s^*)}{\partial s_1 \partial s_2} \\ \frac{\partial f(s^*)}{\partial s_1} + ks_2^*\frac{\partial^2 f(s^*)}{\partial s_1 \partial s_2} & \frac{\partial f(s^*)}{\partial s_2} + k\frac{\partial f(s^*)}{\partial s_2} + ks_2^*\frac{\partial^2 f(s^*)}{\partial s_2^2} \end{pmatrix}.$$

The conditions for $s^* = (s_1^*, s_2^*)$ to be a maximum are:

$$\frac{\partial f(s^*)}{\partial s_i} + k\frac{\partial f(s^*)}{\partial s_i} + ks_i^*\frac{\partial^2 f(s^*)}{\partial s_i^2} < 0, \; i = 1, 2 \tag{10}$$

$$\det H\left(\widehat{P}(s^*)\right) > 0. \tag{11}$$

On the other hand, the conditions for the equilibrium point to be an asymptotically stable Nash equilibrium are:[3]

$$\left(\frac{\partial^2 \pi_1 (s^*)}{\partial s_1^2}\right)\left(\frac{\partial^2 \pi_2 (s^*)}{\partial s_2^2}\right) - \left(\frac{\partial^2 \pi_1 (s^*)}{\partial s_1 \partial s_2}\right)\left(\frac{\partial^2 \pi_2 (s^*)}{\partial s_2 \partial s_1}\right) > 0. \tag{12}$$

Since this game's profit functions are $\pi_i(s) = (f(s))^k s_i$, $i = 1, 2$, condition (12) becomes $\Omega > \Xi$, where:

$$\Omega \equiv \left(\frac{\partial f(s^*)}{\partial s_1} + k\frac{\partial f(s^*)}{\partial s_1} + ks_1^*\frac{\partial^2 f(s^*)}{\partial s_1^2}\right)\left(\frac{\partial f(s^*)}{\partial s_2} + k\frac{\partial f(s^*)}{\partial s_2} + ks_2^*\frac{\partial^2 f(s^*)}{\partial s_2^2}\right)$$

[2] Of course, an analogous form obtains in the special case where f is linear and $k = 1$, in which, however, there's no need to resort to a potential function in the first place.

[3] See Fudenberg and Tirole (1991, pp. 24-25), *inter alia*.

and

$$\Xi \equiv \left(\frac{\partial f(s^*)}{\partial s_1} + ks_2^* \frac{\partial^2 f(s^*)}{\partial s_1 \partial s_2}\right)\left(\frac{\partial f(s^*)}{\partial s_2} + ks_1^* \frac{\partial^2 f(s^*)}{\partial s_2 \partial s_1}\right),$$

and it is easy to see that (11) and (12) coincide. An analogous proof holds in the case of convexity. ■

This result, in addition to being useful in the remainder where we will provide examples, represents also a desirable instrument to identify stable Nash equilibria in those cases where the system of first order conditions (5) yields more than one critical point, by looking at the Hessian matrix of the best-response potential function only.

3 Examples

We start with an example related to oligopolistic Cournot competition (to this regard, see also Slade, 1994, and Monderer and Shapley, 1996). The next proposition investigates the simple case in which $f(\cdot)$ is a demand function, linear in the quantities of n firms, each one selling a differentiated variety of the same product. Hence, the demand function for variety i writes as

$$p_i(s) = \left(a - bs_i - \gamma \sum_{j \neq i} s_j\right)^k, \; a, b > 0 \tag{13}$$

where (i) the exogenous parameter $\gamma \in [-b, b]$ measures product complementarity/substitutability (as in Singh and Vives, 1984, *inter alia*), and (ii) $k > 0$, so that (13) is convex for all $k \in (0, 1)$, linear if $k = 1$ and concave for all $k > 1$. Production costs are normalised to zero.

Proposition 10 *Every game* Γ *where the profit functions are:*

$$\pi_i(s) = \left(a - bs_i - \gamma \sum_{j \neq i} s_j\right)^k s_i, \tag{14}$$

with $k \in \mathbb{R}_+$, admits the unique best-response potential function (up to an additive constant):

$$\widehat{P}(s) = -\frac{(k+1)b}{2}\sum_{j=1}^{n} s_j^2 + \sum_{j=1}^{n}\left(a - b\sum_{l\neq j} s_l\right) s_j + \gamma \sum_{l\neq i} s_l s_i. \qquad (15)$$

Proof. The first order conditions of the maximization problem are the following:

$$\frac{\partial \pi_i}{\partial s_i} = -kb\left(a - bs_i - \gamma\sum_{j\neq i} s_j\right)^{k-1} s_i + \left(a - bs_i - \gamma\sum_{j\neq i} s_j\right)^{k} =$$

$$= \left(a - bs_i - \gamma\sum_{j\neq i} s_j\right)^{k-1} \cdot \left(-kbs_i + a - bs_i - \gamma\sum_{j\neq i} s_j\right) = 0.$$

Given that $a - bs_i - \gamma\sum_{j\neq i} s_j > 0$, the fictitious first order condition is:

$$\frac{\partial \widehat{\pi}_i}{\partial s_i} = -kbs_i + a - bs_i - \gamma\sum_{j\neq i} s_j.$$

Integration with respect to the i-th strategy yields:

$$\widehat{\pi}_i(s) = \int \frac{\partial \widehat{\pi}_i}{\partial s_i} ds_i = -\frac{(k+1)b}{2} s_i^2 + \left(a - \gamma\sum_{j\neq i} s_j\right) s_i + z.$$

Consequently, by summation we obtain the total fictitious profit function:

$$\widehat{\Pi}(s) = \sum_{i=1}^{n}\left[-\frac{(k+1)b}{2} s_i^2 + \left(a - \gamma\sum_{j\neq i} s_j\right) s_i\right] + Z,$$

where z, Z are the constants of integration. It is easy to check that:

$$\frac{\partial \widehat{\Pi}}{\partial s_i} = \frac{\partial \widehat{\pi}_i}{\partial s_i} + \phi_i,$$

where

$$\phi_i = \sum_{j\neq i} \frac{\partial \widehat{\pi}_j}{\partial s_i} = -\gamma\sum_{j\neq i} s_j.$$

Finally, by calling

$$\widehat{P} = \widehat{\Pi} + \gamma \sum_{j \neq i} s_i s_j,$$

we have that:

$$\frac{\partial \widehat{P}}{\partial s_i} = -kbs_i + a - bs_i - \gamma \sum_{j \neq i} s_j = \frac{\partial \widehat{\pi}_i}{\partial s_i},$$

whereby $\widehat{P}$ qualifies as a best-response potential function for the game Γ. ■

The previous proposition can be applied to the Cournot oligopoly game as in Anderson and Engers (1992) by taking the following values: $a = 1,\ b = 1,\ \ \gamma = 1,\ \ k = 1/\alpha,\ \alpha > 0:$

Proposition 11 *For all $\alpha > 0$, the Cournot game with non linear demand is a best-response potential game with a linear-quadratic best-response potential function:*

$$\widehat{P}(s) = \sum_{i=1}^{n} \frac{s_i}{2} \left[2\alpha \left(1 - \sum_{i \neq j} s_j \right) - s_i (1 + \alpha) \right] + \sum_{i \neq j} \alpha s_i s_j. \tag{16}$$

Proof. See Dragone and Lambertini (2008). ■

To conclude this example, note that here $f(s)$ is linear and therefore the related best-response potential function has indeed a linear-quadratic form.

The second example deals with a well known model belonging to international monetary economics, that can be traced back to Hamada (1976, 1979).[4] This is a simple Keynesian two-country model where countries are connected by international trade. The price level is p worldwide, and it is exogenously given. The exchange rate is also fixed. The nominal money supply in country i is $M_i = R_i + D_i$, where R_i is the amount of international reserves held by i's central bank and D_i measures total liabilities of its banking system. Per-country as well as worldwide international reserves

[4]See also Canzoneri and Gray (1983, 1985).

$W = R_i + R_j$ are assumed to be fixed in the short run. The equilibrium in the money market of country i corresponds to the condition $M_i/p = L_i(Y_i, r_i)$, where $L_i(Y_i, r_i) = \eta Y_i - \nu r_i$ is the demand for money, with $\eta, \nu > 0$.

The utility function of country i's policy maker is

$$U_i(D_i, Y_i, Y_j) = (Y_i + \sigma Y_j)^k \left(\overline{D} - D_i\right), \tag{17}$$

where $\sigma \in [0, k+1]$ measures the role played by country j's national income Y_j in relation to the objective of the policy maker of country i, manoeuvring liabilities $D_i \in \left[0, \overline{D}\right]$ so as to maximise (17) in a non-cooperative simultaneous game. Parameter $\overline{D}$ is the ceiling beyond which, by assumption, the price level ceases to be stable at p; i.e., any $D_i > \overline{D}$ would cause inflation at the world level.

When the money market of country i is in equilibrium, the following relation holds:

$$Y_i = \frac{R_i + D_i + p\nu r}{\eta p} \tag{18}$$

showing, in line with Hamada (1976, 1979), that $\partial Y_i/\partial D_i = 1/(\eta p) > 0$. The utility function of country i's policy maker becomes:

$$U_i(D_i, D_j) = \left(\frac{R_i + D_i + p\nu r}{\eta p} + \sigma \frac{R_j + D_j + p\nu r}{\eta p}\right)^k \left(\overline{D} - D_i\right), \tag{19}$$

with the first order condition:

$$\frac{\partial U_i(D_i, D_j)}{\partial D_i} = \left\{\frac{1}{\eta p}\left[k\overline{D} - D_i(1+k) - R_i - \sigma R_j + \right.\right. \tag{20}$$

$$\left.\left. -\nu r p(1+\sigma) - \sigma D_j\right]\left(\frac{R_i + D_i + p\nu r}{\eta p} + \sigma \frac{R_j + D_j + p\nu r}{\eta p}\right)^{k-1}\right\} = 0$$

so that the relevant fictitious first order condition is

$$\frac{1}{\eta p}\left[k\overline{D} - D_i(1+k) - R_i - \sigma R_j - \nu r p(1+\sigma) - \sigma D_j\right] = 0. \tag{21}$$

Let $\tilde{D}_i = \overline{D} - D_i$, $\tilde{D}_j = \overline{D} - D_j$, $a = \left[R_i + \sigma R_j + (\overline{D}_j + p\nu r)(1+\sigma)\right] / (\eta p)$, $b = 1/(\eta p)$ and $\gamma = \sigma/(\eta p)$, so that (19) can be written as

$$\begin{aligned} U_i\left(\tilde{D}_i, \tilde{D}_j\right) &= \left(\frac{R_i + \overline{D} - \tilde{D}_i + p\nu r + \sigma\left(R_j + \overline{D} - \tilde{D}_j + p\nu r\right)}{\eta p}\right)^k \tilde{D}_i \\ &= \left(a - b\tilde{D}_i - \gamma\tilde{D}_j\right)^k \tilde{D}_i. \end{aligned}$$

Then Propositions 5 and 7 can be applied to derive the best-response potential function of the game:

$$\begin{aligned} \widehat{P}(\tilde{D}_i, \tilde{D}_j) &= -\frac{(k+1)}{2\eta p}\sum_{j=1}^{2}\tilde{D}_j^2 + \\ &\sum_{j=1}^{2}\left(\frac{R_i + \sigma R_j + (\overline{D} + p\nu r)(1+\sigma) - \tilde{D}_i}{\eta p}\right)\tilde{D}_j + \frac{\sigma}{\eta p}\tilde{D}_i\tilde{D}_j. \end{aligned} \tag{22}$$

This function is a semidefinite negative quadratic form if and only if $\sigma \leq k+1$. That is, the corresponding Hessian matrix is semidefinite negative in the range $\sigma \leq 1+k$, wherein the Nash equilibrium is stable. Hence, we may state:

Proposition 12 *The monetary policy coordination game based upon objective functions (17) is a best-response potential game.*

As in the previous example, also here $f(s)$ is linear and therefore Corollary 8 applies.

4 Concluding remarks

We have investigated a class of noncooperative full information games, outlining necessary and sufficient conditions whereby the corresponding vector field is conservative, and therefore these games are indeed best-response potential ones. In addition to the theoretical analysis, we have also provided examples based on Cournot oligopoly and a game of international coordination of monetary policies.

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