On the Social Desirability of Patents for Sequential Innovations in a Vertically Differentiated Market[*]

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Abstract

We consider a market for vertically differentiated goods where firms enter over time, after having developed innovations characterised by different quality levels. We show that the adoption of a patent system limiting the quality range avalable to later entrants is socially undesirable, in that it builds a barrier to entry to the advantage of the first innovator.

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1 Introduction

An apparently well established result in the theory of vertically differentiated oligopoly states that earlier entrants supply goods of higher quality than later entrants, in that the high-quality products earn higher profits than low-quality alternatives (see, *inter alia*, Gabszewicz and Thisse, 1979, 1980; Shaked and Sutton, 1982, 1983; Donnenfeld and Weber, 1995). A general proof of this result for every convex fixed-cost function of quality improvement is provided by Lehmann-Grube (1997).[1]

In this literature, quality is thought of as being the result of firms' R&D activity, although this aspect is not explicitly tackled. This is done instead in a few recent papers where firms' efforts in developing quality is described in details (Dutta *et al.*, 1995; Rosenkranz, 1995; 1997; van Dijk, 1996). In particular, van Dijk (1996) investigates the design of patent height in a duopoly model where quality and time are measured along the same dimension.[2] This accounts for the idea that quality improvements take time. Adopting a model where the market is only partially served under monopoly and fully served under duopoly, van Dijk considers a patent system establishing that later innovators can enter the market if and only if they supply sufficient quality improvements as compared to incumbents (i.e., earlier innovators). van Dijk proves that the patent system does not affect the evolution of market structure as compared to the situation without patents. Moreover, he shows that later entrants may gain by producing goods whose quality is higher than the first innovator's. This result is due to the fact that it is more profitable to be the improver rather than the original innovator, in line with well known findings by previous research in the field of vertical differentiation.

We investigate a model of vertical differentiation where (i) partial market coverage emerges at equilibrium independently of the number of firms; (ii) time and quality are separate dimensions; and (iii) costs are convex in quality. Hypotheses (ii) and (iii) amount to assuming that the development of superior quality levels is not necessarily correlated with the time span over

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[1] Aoki and Prusa (1997) adopt a specific case of the cost function analysed by Lehmann-Grube, to investigate the consequences on profits, consumer surplus and social welfare of the timing of investment in product quality in a vertically differentiated duopoly where the market stage is played in the price space. See also Lambertini (1999).

[2] The optimal design of patents in terms of their breadth and length, and imitations alongside the patent, are analysed in several contributions (Gilbert and Shapiro, 1990; Klemperer, 1990; Scotchmer and Green, 1990; Gallini, 1992; Denicolò, 1996).

which R&D activity is carried out, and we allow for the choice of quality to be dictated by strategic incentives alone. That is, we take the view that a later entrant may supply lower quality product than the incumbent's, if it is profitable to do so. The time horizon is infinite, and two firms may enter the market at different dates, which are exogenously fixed. Under specific assumptions, we prove that it is convenient for the later entrant to produce a lower quality than the first innovator's, for the following reason. If the later innovator enters with a quality lower than the first innovator, both firms' profits are positive. Otherwise, when the second innovator enters with a higher quality, her profits are nil due to the fact that the incumbent finds it optimal to produce a limit quality, larger than the optimal monopoly quality, such that entry is blockaded and the first innovator remains a monopolist. In contrast with van Dijk's findings, this result allows us to conclude that a patent system requiring any given improvement on the incumbent's quality is socially undesirable, in that it produces an everlasting monopoly. The same holds for a patent system covering a subset of the product range below the incumbent's quality, if the extent of the patent's height is larger than a critical threshold.

Several real-world examples can be found to match with our theoretical setting. For instance, consider the evolution of the market for CD players (CDPs). Early innovators, Philips and Sony, first introduced top-notch CDPs in 1982. The diffusion of low-priced CDPs started in the late '80s, with the same brands as well as several others. The same holds for the adoption of kevlar in the manufacturing of loudspeaker cones, originally introduced by Bowers & Wilkins in expensive loudspeaker systems, and subsequently adopted by B&W and many other manufacturers in more popular products.

One could recast our model in terms of a more general perspective concerning the optimal behaviour of firms simultaneously activating R&D investments for innovations based on the same technology but eventually leading to marketable products characterised by some degree of substitutability, as in Cardon and Sasaki (1998). This is related, but not equivalent to, the problem of sequential patent races starting at different points in time, where later innovations must not infringe upon existing patents on earlier innovations (see Scotchmer, 1991, 1996; Green and Scotchmer, 1995; O'Donoghue, 1998; O'Donoghue, Scotchmer and Thisse, 1998).

The remainder of the paper is structured as follows. The basic model of vertical differentiation is laid out in section 2. Section 3 describes the two alternative entry games, where (i) the first entrant produces a high-quality

good while the second entrant produces a low-quality good; and (ii) the opposite situation where the second entrant sells a good of higher quality than the incumbent's. Section 4 contains the evaluation of the effects of patents on market structure, assessing their social desirability. Finally, section 5 provides concluding remarks.

2 The model

Consider the following full-information setting. Let a market for vertically differentiated products exist over $t \in [0, \infty)$. Two single-product firms, labelled as 1 and 2, produce goods of different qualities, q_1 and $q_2 \in [0, \infty)$, through the same technology, $C_i(q_k) = cq_k^2$, with $k = H, L$ and $i = 1, 2$. Development costs $C_i(q_k)$ are evaluated at the time t_i when firm i enters the market.[3] As usual, these costs can be interpreted as fixed costs due to the R&D effort needed to produce a certain quality, while variable production costs are negligible. Hence, without further loss of generality, we assume them to be nil. We characterize the technology represented by the above cost function as follows:

Assumption 1 *Suppose firm i develops a good of quality $\hat{q}$, at cost $C_i(\hat{q}) = c\hat{q}^2$. Then she can supply the market with any quality $q \in (0, \hat{q})$ without any additional costs. Once into the market, the firm cannot invest any more resources in R&D.*

The above amounts to assuming that any change in the quality level implies adjustment costs if and only if the change takes the form of a quality increase.[4] Conversely, once firm i has borne the cost of developing a given quality, she may decide to decrease the quality of her product costlessly. For later reference, we define $[0, \hat{q}]$ as the *quality production set* of the firm.

As to the design of the patent system, we can outline three alternative situations, analogous to some of those discussed in van Dijk (1996, pp. 159-61):

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[3] van Dijk (1996, p. 154) assumes that quality and time coincide, with $C_i(q) = t_i = q_i$.

[4] One could assume that adjustment costs be sufficiently large to make it unprofitable for firms to modify quality after the initial development phase. However, under full information, any such behaviour would clearly be irrational. Therefore, adjustment costs can be expected not to be borne by either firm at equilibrium.

Patent System 1 *The patent's length is infinite. If the first innovator develops quality $\widehat{q}$, then the height of the patent system covers the interval $[0, \widehat{q}]$, so that the second innovator must produce $\widetilde{q} > \widehat{q}$ in order not to infringe the patent over the first innovation.*

Patent System 2 *The patent's length is infinite. If the first innovator develops quality $\widehat{q}$, then the patent covers the interval $[0, \widehat{q} + \varepsilon]$, with $\varepsilon > 0$. Then, the second innovator must produce $\widetilde{q} > \widehat{q} + \varepsilon$ in order not to infringe the patent over the first innovation.*

Patent System 3 *The patent's length is infinite. If the first innovator develops quality $\widehat{q}$, then the patent covers the interval $[\widehat{q} - \varepsilon, \widehat{q}]$, with $\varepsilon > 0$. Then, the second innovator must produce $\widetilde{q} < \widehat{q} - \varepsilon$ or $\widetilde{q} > \widehat{q}$ in order not to infringe the patent over the first innovation.*

Observe that patent system 2 imposes a *minimum quality improvement* ε, arbitrarily large, on the part of the second entrant. That is, the protection ensured by the patent to the incumbent is strictly larger than the incumbent's quality production set, $[0, \widehat{q}]$. Patent system 3 covers a proper subset of the incumbent's quality production set. A fourth patent system would be one that covers only $\widehat{q}$. This is immediately ruled out as it is obvious that the incentive toward product differentiation suffices to guarantee that later entrants would never produce $\widehat{q}$.

Products are offered on a market where consumers have unit demands, and buy if and only if the net surplus derived from consumption $v_\theta(q_k, p_i(q_k)) = \theta q_k - p_i(q_k) \geq 0$, where $p_i(q_k)$ is the unit price charged by firm i on a good of quality q_k, purchased by a generic consumer whose marginal willingness to pay is $\theta \in [0, \overline{\theta}]$. We assume that θ is uniformly distributed with density one over such interval, so that the total mass of consumer is $\overline{\theta}$. Assuming that the lower bound to θ is zero implies that, throughout the following analysis, we will consider partial market coverage only.

Firms' entry dates are exogenous. Firm 1 enters at date $t_1 \geq 0$; in the remainder, we shall label the first entrant as the *leader*. Firm 2 enters at date $t_2 \in [t_1, \infty)$, and we shall refer to her as the *follower*. However, we do not assume that quality level and entry time are positively related. A firm might enter the market early with a high quality or late with a low quality, depending upon strategic incentives. Over the time span $[t_1, t_2)$, the leader remains a monopolist. At any $t \in [t_1, t_2)$, firm 1 maximizes w.r.t. price the

instantaneous revenues:

$$R_M = p_M x_M = p_M \left(\overline{\theta} - \frac{p_M}{q_M} \right) \tag{1}$$

where p_M is monopoly price and $x_M = \overline{\theta} - p_M/q_M$ is market demand. We can prove the following:

Lemma 1 *The leader's revenues during the monopoly phase are everywhere increasing in quality.*

Proof. The first order condition (FOC) of the monopolist w.r.t. p_M is:

$$\frac{\partial R_M}{\partial p_M} = \frac{\overline{\theta} q_M - 2 p_M}{q_M} = 0 \tag{2}$$

yielding $p_M = \overline{\theta} q_M / 2$ as the optimal monopoly price, so that (1) simplifies to:

$$R_M = \frac{\overline{\theta}^2 q_M}{4} \tag{3}$$

which is everywhere increasing in q_M. ■

Concerning firms' behavior after the follower's entry, we are going to examine two alternative perspectives:

A. The follower enters at t_2 with a product whose quality is lower than the leader's. We label this case as *high-quality leadership*.

B. The follower enters at t_2 with a product whose quality is higher than the leader's. We label this case as *low-quality leadership*.

In both cases, over $t \in [t_2, \infty)$, firms compete in prices. The solution concept is the subgame perfect equilibrium by backward induction.[5] We borrow from Aoki and Prusa (1997) and Lehmann-Grube (1997) the assumption that downstream Bertrand competition is simultaneous. Market demands for the high- and low-quality good are, respectively:

$$x_H = \overline{\theta} - \frac{p_H - p_L}{q_H - q_L} \tag{4}$$

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[5] In particular, we solve both stages in the duopoly regime through a simultaneous Nash equilibrium. This simplifies calculations withouth significantly affecting the main results.

and

$$x_L = \frac{p_H - p_L}{q_H - q_L} - \frac{p_L}{q_L} \tag{5}$$

Duopoly revenue functions are $R_H = p_H x_H$ and $R_L = p_L x_L$. Solving for the price equilibrium, the revenue function of firm 1 can be rewritten in terms of qualities only, as follows:[6]

$$R_H = \frac{4\bar{\theta}^2 q_H^2 (q_H - q_L)}{(4q_H - q_L)^2} \quad \text{if} \quad q_1 = q_H \tag{6}$$

$$R_L = \frac{\bar{\theta}^2 q_H q_L (q_H - q_L)}{(4q_H - q_L)^2} \quad \text{if} \quad q_1 = q_L \tag{7}$$

Per-period revenues accruing to the follower are defined residually. On the basis of expressions (6-7), previous literature, dealing with single-period models, established that the first entrant would choose to supply the high-quality good.

3   The entry games

Here, we briefly expose the outcomes of the two entry games, in order to outline firms' profit incentives which are going to shape the subgame perfect equilibrium of the model.

3.1   High-quality leadership

We first consider case A, where under monopoly $(t \in [t_1, t_2))$

the first entrant (firm 1) supplies quality q_M, while in the duopoly regime $(t \in [t_2, \infty))$ the first entrant produces $q_H \geq q_M$ and the second entrant (firm 2) produces $q_L \leq q_H$. Hence, the development cost born by firm 1 is that associated with q_H, i.e., $C_1(q_H)$, whose present value is $cq_H^2 \cdot e^{-rt_1}$. The discounted flow of profits accruing to firm 1 is:

$$\Pi_{1H} = \int_{t_1}^{t_2} R_M \cdot e^{-rt} dt + \int_{t_2}^{\infty} R_H \cdot e^{-rt} dt - cq_H^2 \cdot e^{-rt_1} \tag{8}$$

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[6]The proof is omitted here, as it is provided by several authors (Gabszewicz and Thisse, 1979; Choi and Shin, 1992; Motta, 1993; Aoki and Prusa, 1997; Lehmann-Grube, 1997).

while the discounted profits to firm 2 are:

$$\Pi_{2L} = \int_{t_2}^{\infty} R_L \cdot e^{-rt} dt - cq_L^2 \cdot e^{-rt_2} \tag{9}$$

Instantaneous revenues R_M, R_H and R_L are given by (3), (6) and (7), respectively. Firms non-cooperatively maximize discounted intertemporal profits w.r.t. qualities. The outcome of this game is summarized by the following:

Proposition 1 *When the leader produces a higher quality than the rival, both firms enjoy positive profits, with $\Pi_{1H} > \Pi_{2L} > 0$. Hence, over $t \in [t_2, \infty)$ the market is a duopoly.*

Proof. See appendix A.1.

3.2 Low-quality leadership

Consider now the alternative case where, under duopoly, the leader (firm 1) produces $q_L \leq q_M$, while the follower (firm 2) produces $q_H \geq q_M \geq q_L$. The discounted flow of profits accruing to firm 1 is:[7]

$$\Pi_{1L} = \int_{t_1}^{t_2} R_M \cdot e^{-rt} dt + \int_{t_2}^{\infty} R_L \cdot e^{-rt} dt - cq_M^2 \cdot e^{-rt_1} \tag{10}$$

while the discounted profits to firm 2 are:

$$\Pi_{2H} = \int_{t_2}^{\infty} R_H \cdot e^{-rt} dt - cq_H^2 \cdot e^{-rt_2} \tag{11}$$

The outcome of this game is summarized by:

Proposition 2 *When the leader produces a lower quality than the rival, $\Pi_{1L} > 0$ and $\Pi_{2H} = 0$. As a result, the incumbent remains a monopolist forever.*

Proof. See appendix A.2.

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[7]It might happen that $q_M < q_L$ (see appendix A2). In general, the cost function of the leader would write $C_1 (\max \{q_L, q_M\})$. This would not influence the main result we derive in this section. Moreover, as the *ad interim* monopoly revenues are everywhere increasing in q_M, it appears reasonable to consider the situation where the lead time be exploited as much as possible by the first entrant. This coincides with the view taken by van Dijk (1996, p. 156).

4   Evaluation of patent systems

An intermediate step towards the design of the patent system consists in assessing firms' preferences as to their respective locations along the quality spectrum, based upon their pure profit incentives, i.e., independently of exogenous conditions imposed by a patent system.

Consider the perspective facing the first firm to innovate. The leader's decision as to whether to supply a low or a high-quality good depends on the comparative evaluation of the following profit flows:

$$\Pi_{1H}(x,\nu,\zeta) = \left(8x^3\left(4x-7\right) + \left(\nu-1\right)\left(4x-1\right)^3\right)\frac{4x^2-3x+2}{2\left(4x-1\right)^6}\zeta \qquad (12)$$

$$\Pi_{1L}(\nu,\zeta) = \left[\frac{1}{64}\left(1-\nu\right)^2 + \frac{7}{2304}\right]\overline{\theta}^4\zeta \qquad (13)$$

If $\Pi_{1H}(x,\nu,\zeta) > \Pi_{1L}(\nu,\zeta)$, then it is profitable to supply the high-quality good, and conversely if $\Pi_{1H}(x,\nu,\zeta) < \Pi_{1L}(\nu,\zeta)$. Hence, we have to evaluate the following expression:

$$\Pi_{1L}(\nu,\zeta) - \Pi_{1H}(x,\nu,\zeta) = \left\{\left[\frac{\left(1-\nu\right)^2}{64} + \frac{7}{2304}\right]\overline{\theta}^4 + \right. \qquad (14)$$

$$\left. - \left(8x^3\left(4x-7\right) + \left(\nu-1\right)\left(4x-1\right)^3\right)\frac{\left(4x^2-3x+2\right)}{2\left(4x-1\right)^6}\right\}\zeta$$

It is immediate that (14) is positive if $\overline{\theta}$ is sufficiently large, i.e., if the market is sufficiently rich. However, we are also interested in evaluating (14) in terms of the time span separating entries by the two firms, that is, the length of the period $t_2 - t_1$ wherein firm 1 remains a monopolist. To this aim, it is convenient to use the expression in (22) and the definition $\nu = \mu/\eta$, so as to rewrite both $\Pi_{1L}(\nu,\zeta)$ and $\Pi_{1H}(x,\nu,\zeta)$ in terms of μ and η only, and to set $\overline{\theta} = 1$. Under all practical respects, this re-parameterization permits to evaluate the profit perspectives of firm 1 in terms of t_1 and t_2, in that $\mu = e^{-rt_1}$ and $\eta = e^{-rt_2}$. The problem reduces to evaluating $\Pi_{1L}(\mu,\eta) - \Pi_{1H}(\mu,\eta)$ in the admissible parameter range $\mu > \eta$. The implicit plot of $\Pi_{1L}(\mu,\eta) - \Pi_{1H}(\mu,\eta) = 0$ in the space $\{\mu,\eta\}$ is in figure 1.

Figure 1 : $\Pi_{1L}(\mu,\eta)$ -$\Pi_{1H}(\mu,\eta)$
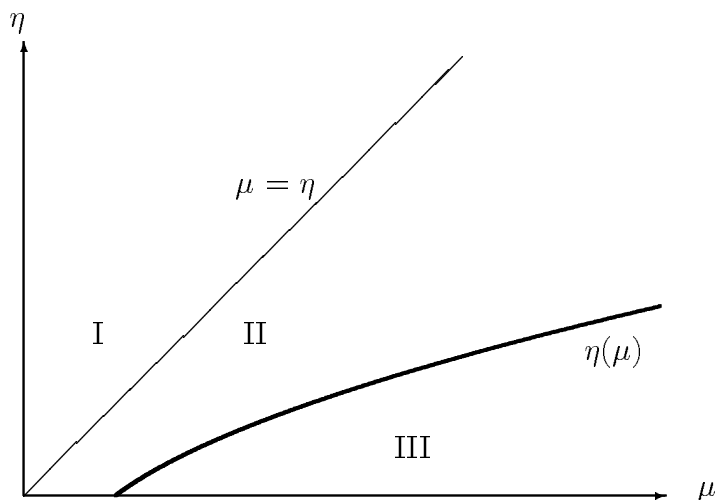
The thick curve denotes a solution $\eta(\mu)$ to $\Pi_{1L}(\mu,\eta) - \Pi_{1H}(\mu,\eta) = 0$. Together with $\eta = \mu$, $\eta(\mu)$ allows us to partition the parameter space into three regions.

- Region I is not admissible, in that it must be $\mu = e^{-rt_1} > \eta = e^{-rt_2}$.

- In region II, $\Pi_{1L}(\mu,\eta) < \Pi_{1H}(\mu,\eta)$ in that t_2 is relatively close to t_1. The time span over which the leader remains a monopolist is too short to justify entering with a low quality good.

- In region III, $\Pi_{1L}(\mu,\eta) > \Pi_{1H}(\mu,\eta)$. The entry date of the follower is far enough in the future to make it more profitable entering with a low-quality good.

The above taxonomy highlights that there exists a trade-off between enjoying monopoly power as long as possible and enjoying an advantage over the rival when the latter enters the market and duopoly competition starts. We can summarize the foregoing discussion in the following:

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Proposition 3 *The leader prefers to enter with a low-quality good if the transitory monopoly power outweighs the disadvantage of being the low-quality firm in the duopoly regime. Otherwise, the leader prefers to enter with a high-quality good.*

Observe that, if $\mu \in (0, \eta(\mu))$, the first entrant produces a (limit) low quality given by (41), that keeps the follower out of the market. That is, obliging the follower to employ a larger amount of resources in the development of a higher quality turns into a sort of predatory behavior preserving the monopoly power of firm 1.

In order to simplify the analysis, let us assume that a monopolist who started production at t_1 and ended it at t_2 would always produce an intermediate quality with respect to the low and the high quality leader. It is easy to check that a monopolist would produce the quality level $q_M = \frac{1}{8}\frac{\bar{\theta}^2}{rc}\left(e^{-rt_1} - e^{-rt_2}\right) = \frac{1}{8\gamma}\left(\nu - 1\right)$. Our assumption amounts to saying that $q_{1L} \leq \frac{1}{8\gamma}\left(\nu - 1\right) \leq q_{1H}$.

Assumption 2 $q_{1L} \leq \frac{1}{8\gamma}\left(\nu - 1\right) \leq q_{1H}$.

The assumption seems perfectly legitimate and ensures that, if there is a subgame perfect equilibrium with a low quality leadership, then that equilibrium implies that (i) the leader in the monopoly phase produces with quality $q_M = \frac{1}{8\gamma}\left(\nu - 1\right)$; and (ii) if the follower entered with a higher quality, it would respond with the quality $q_{1L} = \frac{7}{48\gamma\mu}$. However, we can prove that this is not an equilibrium because the follower finds it profitable to enter with a low quality when the leader tries to preempt the market, correctly anticipating that offering a superior quality would not pay off. Therefore, the only possible equilibrium is that with high quality leadership. This is proved in the following Proposition.

Proposition 4 *In the absence of patent protection, a unique duopoly equilibrium exists over $t \in [t_2, \infty)$, with the leader supplying the high-quality good and the follower supplying the low-quality good.*

Proof. See appendix A.3.

Since entry dates are exogenous, firms cannot influence the outcome of the game through the speed and/or intensity of their respective R&D activities.

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Indeed, the above Proposition reveals that the intertemporal model closely replicates the conclusions reached by previous literature dealing with static one-shot models,[8] which points out the incentive for earlier entrants to supply high-quality goods. However, the static approach usually adopted so far cannot highlight the incentive for later innovators to enter the market in the lower part of the quality spectrum. Judging on the basis of the existing literature, one is lead to believe that later entrants produce low-quality goods because they have missed the chance of doing any better. On the contrary, our model depicts a situation where profit incentives drive newcomers towards inferior varieties. Moreover, the static approach is by definition unable to account for the possibility of trading off the market power attached to a dominant position in the duopoly regime against transitory monopoly power, which represents the prize for the earlier entrant.[9]

We are now able to evaluate the desirability of a patent regime. Consider patent system 1. If it is adopted, the follower cannot enter the market profitably without infringing the leader's patent, and the leader can stay a monopolist forever by developing and patenting the limit quality (41), which can be kept asleep indefinitely. It is then trivial to prove that it is socially preferable to allow for a duopoly equilibrium with the follower producing the low-quality product. The same considerations hold, a *fortiori*, for patent system 2. In the case of patent system 3, if the height of the patent is small enough (that is, $\varepsilon \in [0, \max\{q_M, q_{1H}\} - q_{2L}]$), then the patent is actually inoperative and the follower may enter the market from below. Otherwise, if $\varepsilon > \max\{q_M, q_{1H}\} - q_{2L}$, entry is blockaded. This yields the following:

Proposition 5 *Under convex costs of quality improvement and partial market coverage, a patent system limiting the height of later innovations in any direction is socially undesirable.*

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[8] Relevant references include Gabszewicz and Thisse (1979, 1980), Shaked and Sutton (1982, 1983), Motta (1993), Aoki and Prusa (1997) and Lehmann-Grube (1997).

[9] Modelling an R&D race under uncertainty, Dutta *et al.* (1995) confirm the established wisdom, in that they find that the *ad interim* monopoly rent cannot counterbalance the advantage associated with high quality. However, their result relies upon a specific reduced form where production costs are completely absent.

5 Concluding remarks

We have investigated a model of sequential entry in a market for vertically differentiated goods. The foregoing discussion points out that, when quality and time are separate dimensions, firms' strategic incentives are such that the follower will enter the market from below and gain positive profits. This is also socially preferable to the alternative situation where the first innovator produces a limit quality and remains a monopolist. As a consequence, it appears socially undesirable to adopt a patent system so as to protect the first innovator.

Appendix

A.1 Proof of Proposition 1

Expressions (8) and (9) rewrite as follows:

$$\frac{\Pi_{1H}}{\overline{\theta}^2 e^{-rt_2}} = 4\frac{q_H^2 \left(q_H - q_L\right)}{\left(4q_H - q_L\right)^2} + \frac{\left(e^{-rt_1} - e^{-rt_2}\right) q_M}{4e^{-rt_2}} - \frac{rcq_H^2}{e^{-rt_2}\overline{\theta}^2} = \quad (15)$$

$$= 4\frac{q_H^2 \left(q_H - q_L\right)}{\left(4q_H - q_L\right)^2} + \frac{\left(e^{-t_1 r} - e^{-t_2 r}\right) q_M}{4e^{-t_2 r}} - \gamma q_H^2$$

$$\frac{\Pi_{2L}}{\overline{\theta}^2 e^{-rt_2}} = \frac{q_H q_L \left(q_H - q_L\right)}{\left(4q_H - q_L\right)^2} - \frac{e^{-rt_1} rc}{\overline{\theta}^2 e^{-rt_2}} q_L^2 = \frac{q_H q_L \left(q_H - q_L\right)}{\left(4q_H - q_L\right)^2} - \mu\gamma q_L^2 \quad (16)$$

where $\dfrac{rc}{e^{-rt_2}\overline{\theta}^2} = \gamma$ and $\mu = e^{-rt_1}$. First order conditions (FOCs) w.r.t. qualities are:[10]

$$\frac{\partial \Pi_{1H}}{\partial q_H} = 2\overline{\theta}^2 q_H \left[\frac{8q_H^2 - 6q_H q_L + 4q_L^2 - \gamma\left(4q_H - q_L\right)^3}{e^{t_2 r}\left(4q_H - q_L\right)^3}\right] = 0 \quad (17)$$

$$\frac{\partial \Pi_{2L}}{\partial q_L} = \frac{\overline{\theta}^2 \left[4q_H^3 - 7q_H^2 q_L - 2\mu\gamma q_L\left(4q_H - q_L\right)^3\right]}{e^{t_2 r}\left(4q_H - q_L\right)^3} = 0 \quad (18)$$

In order to simplify calculations, set $q_L = q$ and $q_H = xq$. This makes it possible to rewrite FOCs (17) and (18) in terms of q/γ, to obtain:

$$-\frac{q^2}{\gamma^2}\left(-8x^2 + 6x - 4 + 64qx^3 - 48qx^2 + 12xq - q\right) = 0 \quad (19)$$

From (17) and

$$-\frac{q^3}{\gamma^3}\left(-4x^3 + 7x^2 + 128\mu qx^3 - 96\mu qx^2 + 24\mu qx - 2\mu q\right) = 0 \quad (20)$$

From (18). Excluding the trivial case $q = 0$, the system (19-20) has the following solution:

$$q(x) = \frac{2\left(4x^2 - 3x + 2\right)}{\left(4x - 1\right)^3} \quad (21)$$

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[10]Second order conditions are met here as well as in the next section. They are omitted for the sake of brevity.

$$x = \frac{\alpha^2 + \alpha(16\mu + 7) + 256\mu^2 + 80\mu + 49}{12\alpha} \tag{22}$$

where

$$\alpha = \Big[8(512\mu^2 + 240\mu + 321)\mu + 343+ \tag{23}$$

$$+24\sqrt{3\left(5888\mu^3 + 3552\mu^2 + 2967\mu + 686\right)\mu} \Big]^{1/3}$$

The second first order condition could also be solved for μ, obtaining:

$$\mu = \frac{1}{2}x^2 \frac{4x - 7}{q\left(4x - 1\right)^3}$$

and using (21)it would yield:

$$\mu\left(x\right) = \frac{1}{4}x^2 \frac{4x - 7}{4x^2 - 3x + 2} \tag{24}$$

Since $0 \leq \mu \leq 1$, the admissible interval for x is $[1.75, 5.2512]$.

Rearranging (6) and (7), we obtain:

$$R_{1H}\left(\frac{xq}{\gamma}, \frac{q}{\gamma}, \gamma\right) = \frac{x^2 \left[4\left(x - 1\right) - \left(4x - 1\right)^2 q\right] q}{\left(4x - 1\right)^2 \gamma} \tag{25}$$

$$R_{2L}\left(\frac{xq}{\gamma}, \frac{q}{\gamma}, \gamma\right) = \frac{x\left(x - 1\right) - \left(4x - 1\right)^2 \mu q}{\left(4x - 1\right)^2 \gamma}q \tag{26}$$

In particular, R_{1H} can be rewritten as

$$R_{1H}\left(x, \gamma\right) = \frac{4x^3(4x^2 - 3x + 2)(4x - 7)}{(4x - 1)^2\gamma} \tag{27}$$

Moreover, we know from Lemma 1 that the monopoly revenues accruing to the leader are everywhere increasing in the quality level, so that (3) rewrites as follows:

$$R_M(x, \gamma) = \frac{(4x^2 - 3x + 2)(e^{-rt_1} - e^{-rt_2})}{2\gamma\left(4x - 1\right)^3 e^{-rt_2}} \tag{28}$$

Hence, the overall intertemporal profits of firm 1 are:

$$\Pi_{1H}\left(x, \gamma\right) = \left[\frac{(4x^2 - 3x + 2)(e^{-rt_1} - e^{-rt_2})}{2\gamma\left(4x - 1\right)^3 e^{-rt_2}}+ \right. \tag{29}$$

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$$+\frac{4x^3(4x^2-3x+2)(4x-7)}{(4x-1)^2\gamma}\Bigg]\frac{e^{-rt_2}}{\overline{\theta}^2 r}$$

which, setting

$$e^{-rt_2}=\eta\ ;\ \frac{\mu}{\eta}=\frac{e^{-rt_1}}{e^{-rt_2}}=\nu\ ;\ \frac{\eta}{\gamma\overline{\theta}^2 r}=\frac{1}{c}\left(\frac{e^{-rt_2}}{r}\right)^2=\zeta \qquad (30)$$

can be further simplified as follows:

$$\Pi_{1H}(x,\nu,\zeta)=\left(8x^3\left(4x-7\right)+\left(\nu-1\right)\left(4x-1\right)^3\right)\frac{4x^2-3x+2}{2\left(4x-1\right)^6}\zeta \qquad (31)$$

The discounted flow of profits accruing to the follower is:

$$\Pi_{2L}\left(x,\zeta\right)=\frac{x\left(4x^2-3x+2\right)^2}{\gamma(4x-1)^6}\cdot\frac{e^{-rt_2}}{\overline{\theta}^2 r}=\frac{x\left(4x^2-3x+2\right)^2}{(4x-1)^6}\cdot\zeta \qquad (32)$$

Observe that $\Pi_{1H}(x,\nu,\zeta)>\Pi_{2L}\left(x,\zeta\right)>0$ always. ∎

A.2 Proof of Proposition 2

Consider first the leader's maximum problem. Expression (10) writes as follows:

$$\frac{\Pi_{1L}}{\overline{\theta}^2 e^{-rt_2}}=\frac{q_H q_L\left(q_H-q_L\right)}{\left(4q_H-q_L\right)^2}+\frac{\left(e^{-rt_1}-e^{-rt_2}\right)q_M}{4e^{-rt_2}}-\frac{e^{-rt_1}rcq_M^2}{\overline{\theta}^2 e^{-rt_2}}= \qquad (33)$$

$$=\frac{q_H q_L\left(q_H-q_L\right)}{\left(4q_H-q_L\right)^2}-\gamma q_M^2+\frac{\left(e^{-rt_1}-e^{-rt_2}\right)q_M}{4e^{-rt_2}}$$

where $q_M=\overline{\theta}^2\left(e^{-rt_1}-e^{-rt_2}\right)/(8rc)$ and net monopoly profits over $t\in[t_1,t_2]$ amount to:

$$\pi_{1M}=\frac{1}{4}\overline{\theta}^2 q_L\int_{t_1}^{t_2}e^{-rt}dt-cq_L^2=\frac{\overline{\theta}^2\left(e^{-t_2 r}-e^{-t_1 r}\right)^2}{64cr^2} \qquad (34)$$

Firm 2's profit flow (11) simplifies to:

$$\frac{\Pi_{2H}}{\overline{\theta}^2 e^{-rt_2}}=4\frac{q_H^2\left(q_H-q_L\right)}{\left(4q_H-q_L\right)^2}-\frac{rcq_H^2}{\overline{\theta}^2 e^{-rt_2}}=4\frac{q_H^2\left(q_H-q_L\right)}{\left(4q_H-q_L\right)^2}-\mu\gamma q_H^2 \qquad (35)$$

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The FOC for the leader w.r.t. q_L is:

$$\frac{\partial \Pi_{1L}}{\partial q_L} = \frac{\overline{\theta}^2 q_H^2 (4q_H - 7q_L)}{e^{rt_2} (4q_H - q_L)^3} = 0 \tag{36}$$

Therefore, the optimal strategy for firm 1 is given by the following best reply:[11]

$$q_L = \frac{4}{7} q_H \tag{37}$$

The behavior of firm 2 is summarized by:

$$\frac{\partial \Pi_{2H}}{\partial q_H} = \frac{2q_H \left(8q_H^2 - 6q_H q_L + 4q_L^2 - (4q_H - q_L)^3 \mu\gamma\right)}{(4q_H - q_L)^3} = 0 \tag{38}$$

The equilibrium at the first stage can now be characterized. Plugging (37) into (38) and rearranging, we get:

$$\text{sign} \left. \frac{\partial \Pi_{2H}}{\partial q_H} \right|_{q_L = \frac{4}{7}q_H} = \text{sign} \left\{ \frac{288 q_H^2}{49} - \frac{13\,824 \mu\gamma q_H^3}{343} \right\} \tag{39}$$

The r.h.s. expression in (39) has three critical points w.r.t. q_H, the unique relevant solution being

$$q_H = \frac{7}{48\mu\gamma} \tag{40}$$

which yields the following equilibrium quality for firm 1:[12]

$$q_L = \frac{4}{7} \frac{7}{48\gamma_2} = \frac{1}{12\mu\gamma} \tag{41}$$

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[11] Observe that (37) is the same best reply function identified by Choi and Shin (1992) in a one-shot game where quality is costless. This coincidence is due to our assumption that decreasing quality requires no additional costs.

[12] It can be verified that $q_M > q_L$ if

$$t_2 > \frac{1}{r} \ln \left[\frac{e^{rt_1}(3 + 2e^{rt_1})}{3} \right] \ .$$

If $t_1 = 0$, the above condition simplifies to:

$$t_2 > \frac{\ln(5/3)}{r} \approx \frac{0.5108}{r} \ .$$

However, notice that in the opposite case, where $q_M < q_L$, the leader could choose to develop the limit quality (41) and keep it 'asleep' until t_2.

As a result, the discounted profits accruing to the leader over the horizon $[t_1, \infty)$ simplify as follows:

$$\Pi_{1L}(\nu, \zeta) = \left[\frac{1}{64} (1 - \nu)^2 + \frac{7}{2304} \right] \overline{\theta}^4 \zeta \tag{42}$$

while those of firm 2, supplying the high-quality good, are $\Pi_{2H} = 0$. ∎

A.3 Proof of Proposition 4

In order to prove the proposition is sufficient to prove that there cannot be a sub-game perfect equilibrium with a low quality leader. Suppose the opposite. Then the leader would choose the quality level $q_{1L} = \frac{7}{48\gamma\mu}$, which induces zero profits for the follower. However, if the follower tries to undercut the leader's quality, it gains the profit function

$$R_{2L}\left(\frac{7}{48\gamma\mu}, \frac{q_L}{\gamma\mu} \right) = \frac{1}{16} \frac{q_L}{\gamma\mu} \frac{49 - 1120 q_L + 2688 q_L^2 - 2304 q_L^3}{(-7 + 12 q_L)^2} \tag{43}$$

which is positive for very low q_L. Hence we obtained a contradiction. ∎

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