Rational destabilising speculation and the riding of bubbles

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Abstract

We present a model where it can be optimal for rational informed speculators/arbitragers to ride the bubble instead of using their information for stabilising purposes. This result stems from the interaction of speculators with behavioural traders. These latter in each period of time either discover the true fundamental value of the asset, or use a positive feedback strategy. We study the equilibrium strategy profiles of speculators in the case of short and long horizons and derive the resulting average expected excess deviation of the asset price. Further we consider the possibility of market manipulation and its consequences on the market efficiency.

Keywords: Rational Destabilising Speculation, Bubbles, Market Timing, Market Efficiency, Behavioural Finance.

J.E.L.: G12, G14, D84

1 Introduction

The efficient market hypothesis relies on the assumption of rationality of all agents and on the stabilising power of arbitrage: backwards induction or a transversality condition precludes bubbles[1] , and mispricings are corrected through arbitrage. Consequently, prices in each point in time reflect fully the information available about the fundamentals. Recent literature, relaxes the assumption that all agents are fully rational and focuses on the interaction between rational and irrational agents. This literature shows how limits to arbitrage arise and how these limits lead, for example, to excess deviations of asset price, overreaction to news and bubbles[2] .

Limits to arbitrage are not always sufficient to explain the behaviour of informed rational traders. Forces weakening the stabilising forces of arbitragers are highlighted, such as, for example, fundamental risk, (see, Wurgler and Zhuravskaya, 2002), noise trader risk, (see Shleifer and Vishny, 1997) and syncronisation risk (see Abreu and Brunnermeier, 2002). These models succeed in explaining the limited power of arbitragers in stabilising the asset market, but not why these informed traders invest in overpriced assets.

Brunnermeier and Nagel (2003) find that hedge funds, which are among the most sophisticated traders, during the time of the Technology Bubble on NASDAQ were heavily tilted towards technology stocks. Further, they find that hedge funds reduced their holdings before the price collapsed. Thus, hedge funds managers timed the market, i.e. they invested in the stock as long as prices continued to rise, while they sold the stock just before the price correction started. They conclude that these traders were riding the bubble for some time before attacking the same.

We build a model which rationalises this behaviour of informed arbitragers/speculators. We consider the case of an innovation to the asset occurring far away in the future, and arbitragers/speculators being informed about this innovation. In our model, rational speculators interact with behavioural traders. We assume that the latter, in each period of time, discover

[1] See for example Tirole (1982) and Santos and Woodford (1997) for a discussion.

[2] For a survey see Shiller (2002), Barberis and Thaler (2002) and Shleifer (2000).

with a given probability the true value of the asset which becomes in this way common knowledge. If they do not discover the true value of the asset, then they engage in positive feedback trading (as in De Long et al. , 1990), i.e. buy the asset if its price increased, sell it if its price decreased. Extrapolative expectations and trend chasing strategies are among the most prominent factors leading to positive feedback trading[3] . Thus, for example, if the asset price increased at level above its fundamental value, speculators have to trade off the opportunity of further destabilising the asset price, i.e. buying the asset, trying to anticipate positive feedback trading of behavioural traders in the next period, with the opportunity of stabilising the asset price, i.e. selling short the asset, trying to exploit the possibility that the asset price jumps back to its fundamental value in the next period of time.

The main purpose of the paper is to focus on the interaction between informed speculators and behavioural traders in a multiperiod framework, and to study the implications of short versus long horizons[4] and the implication of market manipulation power for the strategy profile of informed speculators and for the average expected excess deviation of the asset price from its fundamental value.

We consider an initial situation where the time horizon spanning the arrival of an innovation is much larger than the trade horizon of speculators. Speculators are informed about the arrival of these innovations, but have limited horizons indicating prohibitively large costs of long-term arbitrage/speculation. For the shake of simplicity we assume that these agents live only for two periods, and thus overlapping generations of speculators span the whole time horizon.

We study the optimal, conditional strategy profile of speculators and their consequences on the asset price. We show that if the probability that the true value of the asset becomes common knowledge in the next period is sufficiently large, then they engage in a stabilising strategy. On the other hand, the lower is this probability, the larger is the number of generations of speculators

[3] See Shleifer (2000), chapter 6, for a more detailed discussion on the use of positive feedback strategies.

[4] Froot et al. (1992) study the implications of short vs. long horizons of speculators for asset price dynamics in the context of information acquisition.

destabilising the asset market.

Thus, if the innovation to the asset occurs far away in the future, speculators can find it optimal to destabilise the asset price for some time and profit from the trading with uninformed behavioural agents. Informed speculators do not use the private information they have, but instead herd on the behaviour of past speculators. Speculators ride the bubble for some time before attacking the same. The stabilising force of arbitrage due to informed agents eventually works out, but with some delay.

We compare these result with the situation where speculators have long horizons. We show that the average expected excess deviation[5] of the asset price from its fundamental value is lower in the case of short horizons than in the case of long horizons. Thus, shortening the horizon of the speculators is beneficial for market efficiency. Thus, we recover a result of Shleifer and Vishny (1997): the smaller the time horizon of arbitragers/speculators, the lower is their impact on equilibrium prices. On the other hand, our conclusion in terms of asset market efficiency is the opposite: shortening speculators time horizon increases on average market efficiency.

Further, we study also the equilibrium in the case where speculators with short horizons can manipulate the information in the asset market. In particular, we assume that if behavioural traders observe subsequent generations of speculators buying the asset, then these traders increase even more their demand thinking that the asset is still undervalued. We show that, under certain conditions, the resulting equilibrium strategies of speculators turn out to depend also on the choice of subsequent speculators and consequently on the action chosen by the chain of speculators spanning the whole time horizon (see Dow and Gorton, 1994). Thus, speculators destabilise the asset market today, manipulating the information in the asset market, in order to trigger additional, future demand of behavioural traders. This can lead to a chain of speculative trading which destabilises completely the asset market. Further, we show that, under certain conditions,

[5] The average is taken over the probability that the true value of the asset becomes common knowledge in the next period.

the average expected excess deviation of the asset price from its fundamental value is an increasing function of the size of this additional demand of behavioural traders. Consequently, the larger is the manipulative power of speculators, i.e. the larger is the additional demand they can trigger, the less efficient is the asset market.

In Section 2 we briefly review the related literature. Section 3 introduces the model, in Section 3.1 we describe the equilibrium strategies in the case of short horizons, while in Section 4 we compare these results with the case where speculators have long horizons. In Section 5 we study the case of market manipulation and arbitrage chains. Section 6 concludes.

2 Related literature

The paper is most similar to De Long et al. (1990). The authors study the situation where speculators interact with behavioural traders using a positive feedback strategy. In their model is assumed that speculators have long horizons since they can hold their position without any temporal constraint. They show that speculators, instead of stabilising the asset price in the case of excess deviation, destabilise the asset price anticipating demand from positive feedback traders. Thus, informed speculators in their model have a destabilising function and not a stabilising function. In our model arbitragers/speculators can have either a destabilising or a stabilising function, depending on the probability of public disclosure of information and on their manipulative power.

Abreu and Brunnermeier (2003) highlight a different source leading informed, rational agents to ride a bubble until it reaches a critical level. In their model the stabilising forces of arbitragers are initially inhibited because of a syncronisation risk. Each informed agent has a negligible impact on the asset price, thus coordination is required among informed agents in order to move and correct the asset price. Further, in their model there is also a competition effect at work since arbitragers waiting too long miss the profit opportunity.

Other models generating speculative bubbles are Hart and Kreps (1978) and Scheinkman and Xiong (2003), where agents have heterogeneous beliefs, and agree to disagree. In Allen and Gorton

(1993) asymmetric information between investors and portfolio managers lead the latter to churn bubbles. In Hart and Kreps (1986) study a model where rational speculators with short horizons interact with consumers whose demand is random. The authors show that destabilising speculation can be a rational expectation equilibrium. Brunnermeier (2001) reviews numerous other papers on bubbles.

3 The model

Consider an economy consisting of T periods. There are two types of assets: a risky one and a riskless one. The latter yields zero net return ($r = 0$), while the former does not distribute any dividends, but its final value is not known to everyone in the economy. Further, we assume that the supply of the risky asset is unitary. For each $t < 0$, the value of the asset is V and its price is $P_t = V$. In $t = 0$ an innovation to the fundamentals occurs, becoming common knowledge with probability 1 only in $t = 4$, i.e. $P_4 = V_4 = V + \phi$, $\phi > 0$. Time is discrete.

There are two types of agents: informed speculators and behavioural traders. While the former have perfect information about the true value of the asset, the latter have only incomplete information.

Informed speculators have perfect information about the final value of the asset, but have limited horizons, capturing high costs of long-term arbitrage. In particular, we assume that they live for two periods only. Thus, there are overlapping generations of young and old speculators in each period of time. We abstract from strategic interaction between speculators assuming that each generation consists of a unit mass of speculators behaving in a competitive way (i.e. taking prices as given). When young, speculators have to decide either to engage in a stabilising strategy or in a destabilising one, while when old they have to close their position. For simplicity, we assume that these agents are risk neutral, but have limited funds D.[6] Thus, their position is

[6] Otherwise, assuming that arbitragers are risk-averse but have not limited funds lead to qualitatively similar results.

limited by $\pm D$ and their (aggregate) demand in period t is given by $\pm \frac{D}{P_t}$. Initial endowment of informed speculators is such that eventual losses can always be covered.

Behavioural traders have non-limited horizons. In particular, we assume that there is a continuum of influx and outflux of these agents from the market with different time horizons, and adding up to an aggregate demand. The single agents open and close continuously their position, making either profits or losses. Each behavioural trader receives a noisy signal ϕ' about the final, true value of the asset and update their demand accordingly. Assuming that there are many behavioural traders in the market and that the noise of the signal is uncorrelated with mean zero we have that $\phi' = \phi$. We introduce probability δ which indicates the probability that the information becomes common knowledge in the next period of time. Thus, with probability δ behavioural trader learn the true value of the asset which becomes common knowledge and consequently the price jumps towards its fundamentals. If behavioural traders have not learned the true value of the asset, then they engage in positive feedback trading. Thus, given that they did not discover the true value of the asset, their aggregate demand for the risky asset in period $t \geq 0$ is given by

$$Q(t) = \frac{V + \phi + D'(P_{t-1} - P_{t-2})}{P_t} \tag{1}$$

Thus, the larger is the price increase over a time period, the more are the behavioural traders buying the asset.

The optimal strategy of informed speculators depends on the strategies of past speculators, on the size of the feedback trading, on the probability δ and also on the action of the subsequent generation of speculators. Thus, prices and strategies are path dependent. First, we introduce some notation.

Definition 1 $P_t^{\{a\}_t}$*, where* $\{a\}_t = \{a_0, a_1, ..., a_i, ..., a_t\}$*, indicates the equilibrium price in period* t*, given the strategy* a_i *of speculators belonging to generation* i*, where* $a_i \in \{s, d\}$*, and* s *indicates a stabilising strategy and* d *indicates a destabilising strategy.*

Definition 2 $E\left(\pi^{a_t}_{\{a\}_{t-1}, a_{t+1}}\right)$ *is the expected profit of generation* t *speculators using strategy*

a_t, given the history of strategies of previous speculators $\{a\}_{t-1}$ and given that speculators of generation $t+1$ use strategy a_{t+1}; $E\left(\pi^{a_0}_{\{0\},a_1}\right)$ if $t = 0$.

Definition 3 *$\overline{\delta}_t$ is the critical value of δ where $a_n = d$, for each $0 \leq n \leq t-1$, and $a_{t+1} = d$ such that if $\delta < \overline{\delta}_t$ then $a_t = d$ is a best response. In other words, $E\left(\pi^d_{\{d\}_{t-1},d}\right) > E\left(\pi^s_{\{d\}_{t-1},d}\right)$ for each $\delta < \overline{\delta}_t$*

Definition 4 *$\underline{\delta}_t$ is the critical value of δ, where $a_t = d$, for each $0 \leq n \leq t$, and $a_{t+1} = s$ such that if $\delta < \underline{\delta}_t$ then $a_t = d$ is a best response. In other words, $E\left(\pi^d_{\{d\}_{t-1},s}\right) > E\left(\pi^s_{\{d\}_{t-1},s}\right)$ for each $\delta < \underline{\delta}_t$.*

3.1 Equilibrium

For the following we assume that $0 < D < D'\phi$ and that the innovation to the asset occurs at $T = 4$. We can state the first result:

Lemma 1

1) $\underline{\delta}_t \leq \overline{\delta}_t$ *for each* $0 \leq t \leq 2$

2) $\underline{\delta}_{t+1} < \underline{\delta}_t$ *for each* $0 \leq t \leq 1$

3) $\overline{\delta}_{t+1} < \overline{\delta}_t$ *for each* $0 \leq t \leq 1$

Proof. We have to calculate expected profits for each possible strategy, and find the critical value of δ. In the following we sketch the main results.

The condition such that $E\left(\pi^d_{0,d}\right) > E\left(\pi^s_{0,d}\right)$ and $E\left(\pi^d_{0,s}\right) > E\left(\pi^s_{0,s}\right)$ reduces to $\underline{\delta}_0 = \overline{\delta}_0 = 1$. Thus, as long as $\delta < 1$, speculators in 0 destabilise the asset price independently of the strategy used by the subsequent generation of speculators.

The optimal strategy of speculators in 1 depends on the strategy used by speculators in 2. Thus, if $a_2 = d$, then $a_1 = d$ is a best response as long as $E\left(\pi^d_{d,d}\right) > E\left(\pi^s_{d,d}\right)$, i.e.

$$\delta < \overline{\delta}_1 = \frac{(D'-1)\left[D'(\phi+D)-D\right]}{D'(\phi+D)+(D'-1)\left[D'(\phi+D)-D\right]}$$

On the other hand, given that $a_2 = s$, we have that $a_1 = d$ is a best response as long as $E\left(\pi^d_{d,s}\right) > E\left(\pi^s_{d,s}\right)$, i.e.

$$\delta < \underline{\delta}_1 = \frac{(D'-1)\left[D'(\phi+D)-D\right]-2D}{D'(\phi+D)+(D'-1)\left[D'(\phi+D)-D\right]-2D}$$

Straightforward algebra shows that $\underline{\delta}_1 < \overline{\delta}_1$, $\underline{\delta}_1 < \underline{\delta}_0 = 1$ and $\overline{\delta}_1 < \overline{\delta}_0 = 1$.

Now, consider the optimal strategy for speculators in 2. This latter depends on the strategy chosen by speculators in 3. Given that $a_3 = d$, $a_2 = d$ is a best response as long as $E\left(\pi^d_{dd,d}\right) > E\left(\pi^s_{dd,d}\right)$, i.e.

$$\delta < \overline{\delta}_2 = \frac{D'^2(D'-2)(\phi+D)-D(D'-1)}{D'^2(\phi+D)+D'^2(D'-2)(\phi+D)-D(D'-1)}$$

Further, given that $a_3 = s$, $a_2 = d$ is a best response as long as $E\left(\pi^d_{dd,s}\right) > E\left(\pi^s_{dd,s}\right)$, i.e.

$$\delta < \underline{\delta}_2 = \frac{D'^2(D'-2)(\phi+D)-D(D'+1)}{D'^2(\phi+D)+D'^2(D'-2)(\phi+D)-D(D'+1)}$$

It is easy to see that $\underline{\delta}_2 < \overline{\delta}_2$ and that $\underline{\delta}_2 < \underline{\delta}_1 < \underline{\delta}_0 = 1$ and $\overline{\delta}_2 < \overline{\delta}_1 < \overline{\delta}_0 = 1$. ■

The critical values of δ depend, among others, on the size of positive feedback trading, i.e. D'. For $D' < 1$ we observe that $\underline{\delta}_1$ as well as $\underline{\delta}_2$ are both negative. Further, as long as $D' < 2$, $\underline{\delta}_2 < 0$. In order to rule out the trivial case, we assume, for the following, that D' is sufficiently large, such that $\underline{\delta}_2 > 0$.

We are able to state the following Proposition which characterises the conditional strategy profile of speculators.

Proposition 1 *Conditional on the information not being revealed previous to period* 4*, the following results hold:*
a) if $0 < \delta < \underline{\delta}_2$*, then the equilibrium strategies are* $a_0 = a_1 = a_2 = d$ *and* $a_3 = s$
b) if $\underline{\delta}_2 \leq \delta < \underline{\delta}_1$*, then the equilibrium strategies are* $a_0 = a_1 = d$ *and* $a_2 = a_3 = s$
c) if $\underline{\delta}_1 \leq \delta < 1$*, then the equilibrium strategies are* $a_0 = d$ *and* $a_1 = a_2 = a_3 = s$

Proof. Since at $t = 4$ the value of the asset becomes common knowledge, $a_3 = s$ is optimal.

Part a) and b) of Proposition 1 follow straightforwardly from Lemma 2. In order to prove part c) of the proposition, we have to show that δ^*, defined as the critical level of δ such that

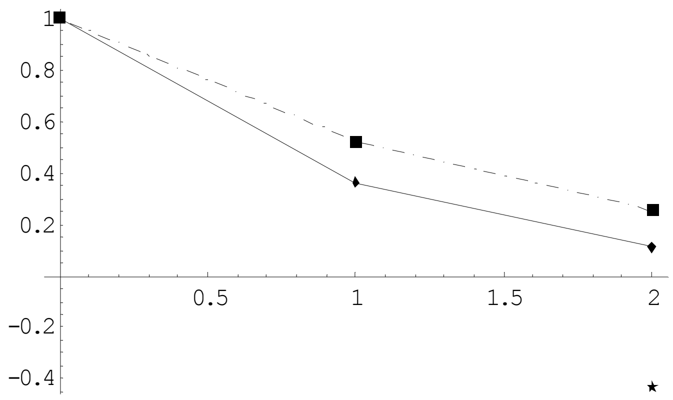

Figure 1: $\underline{\delta}_t$ (continuous line) and $\overline{\delta}_t$ (discontinuous line) as a function of t. $D' = 2.5$, $D = 2$, $\phi = 1$. The star indicates gδ^*.

$E\left(\pi^d_{ds,s}\right) < E\left(\pi^s_{ds,s}\right)$ for each $\delta > \delta^*$, is lower than $\underline{\delta}_1$, i.e. $\delta^* < \underline{\delta}_1$. Simple algebra shows that

$$\delta^* = \frac{D'^2\left(D'-2\right)\left(\phi+D\right) - D\left(D'-1\right)\left(2D'-1\right)}{D'^2\left(\phi+D\right) - 2DD' + D'^2\left(D'-2\right)\left(\phi+D\right) - D\left(D'-1\right)\left(2D'-1\right)}$$

It is easy to see that $\delta^* < \underline{\delta}_2$. From Lemma 1 we know that $\underline{\delta}_1 > \underline{\delta}_2$ and so the result is established.

■

Proposition 1 shows that if δ is sufficiently large, i.e. $\underline{\delta}_1 \leq \delta < 1$, then all but the first generation of speculators stabilise the asset price. On the other hand, the lower is δ, the larger is the number of speculators using a destabilising strategy. In particular, for $0 < \delta < \underline{\delta}_2$, i.e. very low values of δ, all but the last generation of speculators destabilise the asset price.

In Figure 1 we have that $\overline{\delta}_1 = 0.52381$, $\overline{\delta}_2 = 0.253731$, $\underline{\delta}_1 = 0.361702$, and $\underline{\delta}_2 = 0.112426$. Consequently, for each $\delta < 0.112426$ the optimal strategy is $a_0 = a_1 = a_2 = d$ and $a_3 = s$, i.e. complete destabilisation of the asset market. For $0.361702 > \delta \geq 0.112426$ the optimal strategy is $a_0 = a_1 = d$ and $a_2 = a_3 = s$, while for $1 > \delta \geq 0.361702$ the optimal strategy is $a_0 = d$ and $a_1 = a_2 = a_3 = s$.

4 Long horizons

In this section we focus on the problem of a representative speculator having long horizons. Also in this case we assume that speculators behave in a competitive way.

The representative speculator has again perfect information about the fundamental value of the asset, and he knows that with probability δ this information becomes common knowledge in the next period of time, while with probability $1-\delta$ positive feedback traders arrive in the market. Thus, in each period of time, speculators have to decide whether to continue to destabilise the asset market trying to gain from trading with behavioural traders in the next period, or to stabilise the asset market.

Speculators in 0 do not have any incentive to stabilise the asset price as long as $\delta < 1$. If speculators in period $t = 0$ buy the asset, then the problem of finding the optimal strategy profile reduces to the problem of finding the optimal period of attacking the bubble, i.e. stabilising the asset. The gain from stabilising at time t is given by

$$2\left(P^{\{d\}_{t-1},s} - V - \phi\right) - D$$

The expected gain of stabilising the asset market at time $t+1$ is given by

$$(1-\delta)\left[2\left(P^{\{d\}_{t},s} - V - \phi\right) - D\right] - \delta D$$

Stabilisation at time $t > 0$ is an optimal strategy as long as

$$\delta > \delta_t^L = \frac{P^{\{d\}_t,s} - P^{\{d\}_{t-1},s}}{P^{\{d\}_t,s} - V - \phi} \tag{2}$$

We can state the following Lemma.

Lemma 2

1) $\underline{\delta}_{t+1}^L < \underline{\delta}_t^L$ *for each* $0 \leq t \leq 1$

2) $\underline{\delta}_t < \underline{\delta}_t^L$ *for each* $1 \leq t \leq 3$, *while* $\underline{\delta}_0^L = \underline{\delta}_0 = 1$

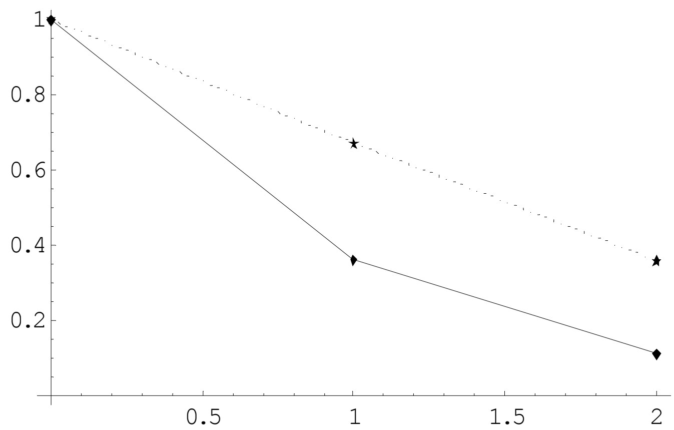

Figure 2: $\underline{\delta}_t$ (continuous line) and $\underline{\delta}_t^L$ (discontinuous line) as a function of t. $D' = 2.5$, $D = 2$, $\phi = 1$.

Proof. Here we just sketch the intermediate results. Using (2) we have that

$$\underline{\delta}_1^L = \frac{D'\left(D'-1\right)\left(D+\phi\right)}{D'^2\left(D+\phi\right)-D}$$

$$\underline{\delta}_2^L = \frac{D'^2\left(D'-2\right)\left(D+\phi\right)}{D'^2\left(D'-1\right)\left(D+\phi\right)-D}$$

■

We are able to state the following Proposition which characterises the conditional strategy profile of speculators having long horizons.

Proposition 2 *Conditional on the information not being revealed previous to period* 4, *the following results hold:*
a) if $0 < \delta < \underline{\delta}_2^L$, *then the equilibrium strategies are* $a_0 = a_1 = a_2 = d$ *and* $a_3 = s$
b) if $\underline{\delta}_2^L \leq \delta < \underline{\delta}_1^L$, *then the equilibrium strategies are* $a_0 = a_1 = d$ *and* $a_2 = a_3 = s$
c) if $\underline{\delta}_1^L \leq \delta < 1$, *then the equilibrium strategies are* $a_0 = d$ *and* $a_1 = a_2 = a_3 = s$

Proof. Proposition 2 is a consequence of Lemma 2. ■

For the following our statements are conditional on the true value of the asset not being revealed in periods previous to $T = 4$. Consider the situation depicted in Figure 2, where $\underline{\delta}_1 = 0.361702$, and $\underline{\delta}_2 = 0.112426$, while $\underline{\delta}_1^L = 0.671642$, and $\underline{\delta}_2^L = 0.358852$. Thus, for each $\delta < 0.112426$, speculators destabilise completely the asset market, independently of their time horizon. For $0.358852 > \delta \geq 0.112426$, we have that in the case of short horizons, the asset market is not completely destabilised (optimal strategy: $a_0 = a_1 = d$ and $a_2 = a_3 = s$), while in the case of long horizons, the asset market is completely destabilised ($a_0 = a_1 = a_2 = d$ and $a_3 = s$). Increasing further δ we observe that for $0.361702 > \delta \geq 0.358852$, independent of the time horizon speculators face, the optimal strategy is $a_0 = a_1 = d$ and $a_2 = a_3 = s$. For $0.671642 > \delta \geq 0.361702$ the asset market is less destabilised if speculators have short horizons (optimal strategy: $a_0 = d$ and $a_1 = a_2 = a_3 = s$) than in the case where they have long horizons (optimal strategy: $a_0 = a_1 = d$ and $a_2 = a_3 = s$). For $1 > \delta \geq 0.671642$ we observe that in cases the optimal strategy is $a_0 = d$ and $a_1 = a_2 = a_3 = s$.

We are able to prove the main proposition of this section.

Proposition 3 *The average expected excess deviation of the asset price, where the average has been taken over all possible values of δ, is lower if speculators have short horizons than in the case where speculators have long horizons.*

Proof. The difference between the average expected excess deviation of the asset price in the case of long horizons (E^L) and short horizons (E^S) is given by

$$\begin{gathered}
E^L - E^S = D + [D + D'(D+\phi)] \left[\int_0^{\underline{\delta}_1^L} (1-\delta) f(\delta)\, d\delta - \int_0^{\underline{\delta}_1} (1-\delta) f(\delta)\, d\delta \right] + \\
[D'(D+\phi) - D] \left[\int_{\underline{\delta}_1^L}^1 (1-\delta) f(\delta)\, d\delta - \int_{\underline{\delta}_1}^1 (1-\delta) f(\delta)\, d\delta \right] + \\
\left[D + D'^2 (D+\phi)\right] \left[\int_0^{\underline{\delta}_2^L} (1-\delta)^2 f(\delta)\, d\delta - \int_0^{\underline{\delta}_2} (1-\delta)^2 f(\delta)\, d\delta \right] + \\
\left[D'^2 (D+\phi) - D\right] \left[\int_{\underline{\delta}_2^L}^{\underline{\delta}_1^L} (1-\delta)^2 f(\delta)\, d\delta - \int_{\underline{\delta}_2}^{\underline{\delta}_1} (1-\delta)^2 f(\delta)\, d\delta \right] + \\
\left[D'^2 (D+\phi) - D - 2DD'\right] \left[\int_{\underline{\delta}_1^L}^1 (1-\delta)^2 f(\delta)\, d\delta - \int_{\underline{\delta}_1}^1 (1-\delta)^2 f(\delta)\, d\delta \right] + \\
+ \left[D'^2 (D'-1)(D+\phi) - D\right] \left[\int_0^{\underline{\delta}_2^L} (1-\delta)^3 f(\delta)\, d\delta - \int_0^{\underline{\delta}_2} (1-\delta)^3 f(\delta)\, d\delta \right] + \\
\left[D'^2 (D'-1)(D+\phi) - D - 2DD'\right] \left[\int_{\underline{\delta}_2^L}^{\underline{\delta}_1^L} (1-\delta)^3 f(\delta)\, d\delta - \int_{\underline{\delta}_2}^{\underline{\delta}_1} (1-\delta)^3 f(\delta)\, d\delta \right] + \\
\left[D'^2 (D'-1)(D+\phi) - D - 2DD'^2\right] \left[\int_{\underline{\delta}_1^L}^1 (1-\delta)^3 f(\delta)\, d\delta - \int_{\underline{\delta}_1}^1 (1-\delta)^3 f(\delta)\, d\delta \right]
\end{gathered}$$

where $f(\delta)$ is a probability density function of δ. Now from Lemma 2 we obtain the result. ■

Proposition 3 states that, shortening speculators time horizons is beneficial in the sense that the average expected deviation of the asset price is lower, and consequently the asset market is, on average, more efficient.

5 Market manipulation and arbitrage chains

In this section we consider the case where speculators can manipulate the asset market. We assume that behavioural traders can observe the action of speculators and if they observe that speculators continue to buy the asset, then they increase demand in period 2. In particular, we assume that a sequence of strategies $\{d, d, d\}$ can trigger additional demand of $\frac{2k}{P}$ units from behavioural traders in period 2. The size of $k \geq 0$ measures the market manipulation power of speculators.

We consider the case where arbitragers have short horizons. $\underline{\delta}_2^M(k)$ is the critical value of δ in period 2 such that for each δ larger than this critical value, stabilisation for generation 2 is optimal. More formally, $E\left(\pi^d_{\{d\}_1,s}\right) > E\left(\pi^s_{\{d\}_1,s}\right)$ for each $\delta < \underline{\delta}_2(k)$.

The following lemma can be proved.

Lemma 3

1) $\underline{\delta}_2^M(0) = \underline{\delta}_2$

2) $\underline{\delta}_2^M(k)$ *is an increasing function of* k.

3) $\lim_{k\to\infty} \underline{\delta}_2^M(k) = \frac{D'-1}{D'}$

Proof. Simple algebra shows that

$$\underline{\delta}_2^M(k) = \frac{D'^2(D'-2)(D+\phi) - D(D'+1) + (D'-1)k}{D'^2(D'-2)(D+\phi) - D(D'+1) + D'^2(D+\phi) + D'k}$$

Taking the first derivative of $\underline{\delta}_2^M(k)$ with respect to k and rearranging terms, we obtain

$$\frac{\partial}{\partial k}\underline{\delta}_2^M(k) = \frac{D'^2(D+\phi) + D(D'+1)}{\left[D'^2(D'-2)(D+\phi) - D(D'+1) + D'^2(D+\phi) + D'k\right]^2} > 0$$

■

We are able to prove the following proposition defining the optimal, conditional strategy profile of speculators.

Proposition 4 *Conditional on the information not being revealed previous to period* 4*, for each* $k \geq k^* = \frac{D'(D+\phi)\left(D+D'^2\phi\right)}{D(1+D')}$*, where* $\underline{\delta}_2^M(k^*) = \underline{\delta}_1$*, the following results hold.*

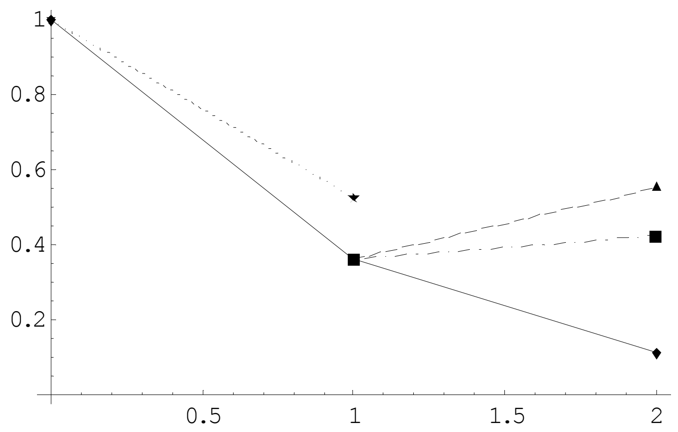

Figure 3: $\underline{\delta}_2^{M}(0)$ (continuous line), $\underline{\delta}_2^{M}(15)$ (- - - - line), $\underline{\delta}_2^{M}(75)$ (— — — line) and $\overline{\delta}_t$ (dots) as a function of t. $D' = 2.5$, $D = 2$, $\phi = 1$.

a) If $\underline{\delta}_2^{M}(k) < \overline{\delta}_1$ then for each $\delta < \underline{\delta}_2^{M}(k)$ the optimal strategy profile is $a_0 = d$, $a_1 = d$, $a_2 = d$, $a_3 = s$, while for each $1 > \delta \geq \underline{\delta}_2^{M}(k)$ the optimal strategy profile is $a_0 = d$, $a_1 = s$, $a_2 = s$, $a_3 = s$.
b) If $\underline{\delta}_2^{M}(k) > \overline{\delta}_1$ then for each $\delta < \overline{\delta}_1$ the optimal strategy profile is $a_0 = d$, $a_1 = d$, $a_2 = d$, $a_3 = s$, while for each $1 > \delta \geq \overline{\delta}_1$ the optimal strategy profile is $a_0 = d$, $a_1 = s$, $a_2 = s$, $a_3 = s$.

Proof. Propostion 4 is a consequence of Lemma 1 and Lemma 3. ■

From Proposition 4 we observe that if k is sufficiently large, then the destabilisation of the asset market occurs through arbitrage chains. In other words, speculators choice depend in a crucial way on the strategy used by the subsequent generation of speculators. Further, we observe that small changes in δ can lead to large changes in the strategy profile of speculators and consequently to a large change in the asset price dynamics, while large changes in δ lead to no change at all.

In Figure 3 we have that $\overline{\delta}_1 = 0.52381$, $\underline{\delta}_1 = 0.361702$, $\underline{\delta}_2 = \underline{\delta}_2^{M}(0) = 0.112426$, $\underline{\delta}_2^{M}(15) = 0.424307$, and $k^* = 8.83929$. Thus, for each given $k > 8.83929$, we have that, for each $\delta < \min\left\{\underline{\delta}_2^{M}(k), \overline{\delta}_1\right\}$, the asset market is completely destabilised, while for $1 > \delta \geq \min\left\{\underline{\delta}_2^{M}(k), \overline{\delta}_1\right\}$ just generation 0 speculators destabilise the asset market, while the subsequent generations stabilise it. $\min\left\{\underline{\delta}_2^{M}(k), \overline{\delta}_1\right\}$ is a threshold which separates the complete destabilisation regime from the quasi-complete stabilisation regime. In Figure 3, we observe that for $k = 15$, this threshold is $\underline{\delta}_2^{M}(15) = 0.424307$. For $k = 75$, we observe that $\underline{\delta}_2^{M}(75) = 0.550629 > \overline{\delta}_1 = 0.52381$ and

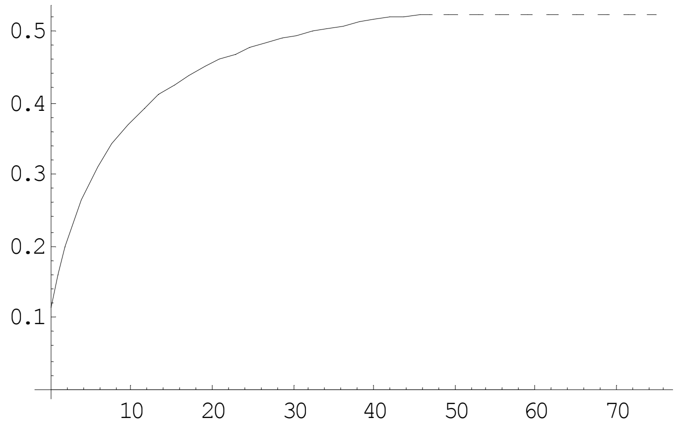

Figure 4: δ^c as a function of k; $\overline{k} = 45.625$. $D' = 2.5$, $D = 2$, $\phi = 1$.

consequently the threshold is $\overline{\delta}_1 = 0.52381$.

We can further characterise the complete destabilisation regime.

Lemma 4 *Call δ^c the critical value of δ such that complete destabilisation of the asset market occurs for each $\delta < \delta^c$. Then, for each $k \in [0, \overline{k})$,* where

$$\overline{k} = \frac{D'(D+\phi)\left(D + D'^2\phi + 2D'D\right)}{D(D'-1)}$$

$\delta^c = \underline{\delta}_2^M(k)$ is an increasing function of k, while for each $k \geq \overline{k}$, $\delta^c = \overline{\delta}_1$.

Proof. Notice that $\underline{\delta}_2^M\left(\overline{k}\right) = \overline{\delta}_1$, where $\overline{k} = \frac{D'(D+\phi)\left(D+D'^2\phi+2D'D\right)}{D(D'-1)}$. Thus, from the Lemma 1 we know that if $\delta > \overline{\delta}_1$, then, even though $a_2 = d$, the optimal strategy for generation 1 speculators is $a_1 = s$. This fact, together with the results stated in Proposition 4 proves the Lemma. ■

In Figure 4 we graph δ^c as a function of k using our previous numerical example.

We are able to prove the main proposition of this section.

Proposition 5 *The average expected excess deviation of the asset price, where the average has been taken over all possible values of δ, is an increasing function of k, for each $k \in [0, \overline{k})$.*

Proof. We define $\delta^a(k) = \underline{\delta}_1 - \underline{\delta}_2^M(k)$, and $E^S(k)$ as defined in the proof of Proposition 3 just instead of $\underline{\delta}_2$ we have now $\underline{\delta}_2^M(k)$. Consequently, $E^S(k)$ is an increasing function of k. The average expected deviation $E(k)$ is given by

$$
\begin{aligned}
E(k) = D + I(\delta^a(k) > 0)\, E^S(k) + \\
I(\delta^a(k) < 0) \Bigg\{ \int_0^{\min\left\{\overline{\delta}_1, \underline{\delta}_2^M(k)\right\}} [D + D'(D+\phi)](1-\delta) f(\delta)\, d\delta + \\
\int_{\min\left\{\overline{\delta}_1, \underline{\delta}_2^M(k)\right\}}^1 [D'(D+\phi) - D](1-\delta) f(\delta)\, d\delta + \\
\int_0^{\min\left\{\overline{\delta}_1, \underline{\delta}_2^M(k)\right\}} \left[D + D'^2(D+\phi)\right](1-\delta)^2 f(\delta)\, d\delta + \\
\int_{\underline{\delta}_1}^{\min\left\{\overline{\delta}_1, \underline{\delta}_2^M(k)\right\}} \left[D'^2(D+\phi) - D - 2DD'\right](1-\delta)^2 f(\delta)\, d\delta + \\
\int_0^{\min\left\{\overline{\delta}_1, \underline{\delta}_2^M(k)\right\}} \left[D'^2(D'-1)(D+\phi) - D\right](1-\delta)^3 f(\delta)\, d\delta + \\
\int_{\min\left\{\overline{\delta}_1, \underline{\delta}_2^M(k)\right\}}^1 \left[D'^2(D'-1)(D+\phi) - D - 2DD'^2\right](1-\delta)^3 f(\delta)\, d\delta + \Bigg\}
\end{aligned}
$$

where $I(\cdot)$ is an indicator function.

Using Lemma 4, we observe that $E(k)$ is an increasing function of k for each $k \in [0, \overline{k})$. ■

Proposition 5 states that, as long as $k < \overline{k}$, the larger the market manipulation power of behavioural traders, the larger is the average expected deviation of the asset price, and consequently the lower is, on average, the market efficiency.

6 Conclusion

We studied a simple multiperiod model where behavioural traders interact with rational informed speculators. We studied the incentives of speculators to destabilise the asset price instead of using their private information for stabilising purposes. We showed that rational speculators can find it optimal to destabilise the asset price, and to ride the bubble for some time. Private incentives in our model are a syncronisation device for rational speculators.

We studied the impact of short vs. long horizons on the incentives of speculators and showed that long horizons lead to increased incentives to destabilise the market. Further, if speculators have market manipulation power, the larger is this power, the larger are the incentives to destabilise the market. Thus, long horizons and a larger market power lead, on average, speculators to ride the bubble for a longer time and consequently the market efficiency is, on average, lower.

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