Investment in transport and communication technology in a Cournot duopoly with trade*

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Abstract

We investigate the role of R&D investment in transport and communication (TCRD) in a Cournot duopoly with trade. Our analysis suggests that firms may invest in TCRD at reasonable levels of effciency of TCRD even when they do not maximize their aggregate payoffs, i.e., when the game is a prisoner's dilemma. As we consider countries of different size, it appears that for low levels of efficiency of TCRD only small countries invest. Welfare effects suggest that investing in TCRD is superior for the country that undertakes it. The terms of trade improve as the foreign market gets larger and, in some cases, if the foreign firm invests in TCRD.

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1 Introduction

Transport and communication (TC) costs are at the heart of international trade either in perfectly competitive or in non perfectly competitive markets. TC costs put a wedge on transactions across borders because of distance and differences of various kind (language, administrative procedures, technical requirements etc.) among countries.

Despite their crucial role in international trade transport and communication costs have mostly had a marginal role in theoretical models.

So far most literature on TC costs has emphasized the role of public infrastructure in reducing them both on the domestic market and in cross border trade (Barro, 1990). Agglomeration of economic activity and the surgeance of cluster of firms is partly associated with the public investment in infrastructure and the economies of scale that made initially convenient to establish in one particular region (Krugman, 1991a,b). Trade then appears as the exchange of goods and services between agglomeration points, when *latu sensu* distance is taken into account. The large literature on geography and trade covers most of the aspects of the effect of better TC facilities on trade and geographic concentration.

The transport cost issue has been dealt also in a different framework, mainly due to Hotelling (1929), or, to be more precise and just, due to Launhardt (1885, 1993), as a recent contribution of Dos Santos Ferreira and Thisse (1996) has made it clear. In all these models, the distance between sellers becomes synonymous of differentiation. In the original Launhardt (1993) model, adopting a particular transport technology can be interpreted as a choice of quality or, in other words, a vertical differentiation commitment. Last, but not least, there is no cost associated with the quality and/or transport technology chosen by a firm.

The question of TC costs can be addressed also by an analysis confined to a domestic scenario (Lambertini, Mantovani, Rossini, 2001) with a set of results that are partially relevant also for trade. A parallel distinction, yet between public infrastructures facilitating domestic trade *vis à vis* international trade, may be found in Martin and Rogers (1995).

So far, there has been no attempt to investigate the role of strategic investment to reduce the burden of TC costs on firms.[1] This should actually

[1] In the existing literature, iceberg transportation costs are taken as given (see Helpman and Krugman, 1985, 1989; Grossman, 1992, for exhaustive surveys), while R&D investment for process innovation has been investigated (Spencer and Brander, 1983, *inter alia*).

be on the agenda because most of the new technologies related to the Internet are going to reduce private TC costs according to the investment effort of each firm and make cross border trade less expensive only for those firms that are better equipped.

Our main purpose is to go through private investment in TC in a strategic environment. To this aim, we analyse the largely unexplored field of investment undertaken by firms to reduce the distance between themselves and customers when products are homogeneous, transport and communication are costly and oligopoly markets are there. We then depart from the literature on the effect of public investment in infrastructure, and concentrate on the strategic issues involved in the technology of TC for the part that is under the control of the firm.

To this purpose, we investigate the issue of investment in R&D devoted to the improvement of the TC technology (TCRD) in a Cournot duopoly setting where each firm sells in two markets, the domestic one and the foreign one.

Whenever a firm ships an item to her customers, the efficiency of her action is influenced by both the state of public infrastructures and by the technology she adopts for TC. We take as exogenously given the state of public infrastructure, and we dwell on the strategic decisions of firms to decrease the burden of TC costs, that translates into the ability to shift the level of transportation costs, in an international trade scenario.

Firms may reduce the weight of TC costs by investing in TCRD to make their shipment to foreign markets less expensive and faster, in other words to minimize the amount of value that is lost under way due to TC costs, deterioration, damage and so on.

We investigate the outcomes of interactions of firms as the efficiency of TCRD changes and we evaluate the social desirability of it. We then go through some calibrated example to see the effect of asymmetric market sizes on TCRD and equilibrium outcomes, finding some confirmation of stylized facts suggesting that firms based in large countries tend to invest relatively less in TCRD than those based in smaller countries. We then go through the analysis of the terms of trade effects of TCRD obtaining some further insight into the welfare effects of trade in the presence of modifiable TC costs.

In the next section we provide the general setting of the model. In section 3 we go through the solution of the two stage game. In section 4 we provide a calibrated example to analyse the effect of the size of the market on TCRD. In section 5 we provide social welfare assessments of the different market solutions. In section 6 we close with the terms of trade effects of TCRD.

Section 7 proposes some concluding remarks.

2 The setting

We consider two firms H and F selling in two markets, their domestic market and the foreign market, i.e., the rival's domestic market. The two markets are separated by natural barriers that require firms to bear transport and communication costs whenever they wish to sell in the foreign market.

Firms play a two stage game. In the first stage they face a binary strategy choice and decide whether to invest in TCRD or not. In the second stage they compete in the market. The solution of the game requires a subgame perfect equilibrium by backward induction.

We assume quadratic consumer surplus functions and, consequently, linear demand functions. We allow for different reservation prices across markets, i.e. different market sizes:

$$p_H = a - hh - tf \tag{1}$$

$$p_F = b - ff - th, \tag{2}$$

where $p_{F,H}$ are the good prices in the two markets, a and b are the respective reservation prices, hh is the quantity sold by firm H in its own domestic market while h is the quantity sold in the foreign market, ff is the quantity sold by firm F in its domestic market, while f refers to what she sells abroad. TC costs are modeled by the iceberg way, introduced for the first time by Samuelson (1954). For any quantity produced at home only a fraction $t \in]0, 1]$ can reach the foreign market, since the remaining fraction $1 - t$ is used up in transport and communication. Prohibitive TC costs or very bad public infrastructure push t towards 0. In case of no TC costs t goes to one. In our analysis initial t is exogenously given to the firms by the state of public facilities. For the sake of simplicity we confine to a binary choice set whereby a firm investing in TCRD gets $t = 1$, while a firm that is not investing in TCRD is stuck to the exogenously given $t \in]0, 1[$. Moreover, we assume that each firm faces a constant marginal production cost c. Products are homogeneous.

We then distinguish 3 cases. i) both firms invest in TCRD, ii) no firm invests in TCRD iii) one firm invests while the other does not.

2.1 Both firms invest (kk)

The demand functions (1 and 2) for country H and country F, become, respectively:

$$_{KK}p_H = a -_{KK} hh -_{KK} f \tag{3}$$

and

$$_{KK}p_F = b -_{KK} ff -_{KK} h. \tag{4}$$

Profits functions are:

$$_{KK}\pi_H = (_{KK}p_H - c)_{KK}hh + (_{KK}p_F - c)_{KK}h - k \tag{5}$$

and

$$_{KK}\pi_F = (_{KK}p_F - c)_{KK}ff + (_{KK}p_H - c)_{KK}f - k, \tag{6}$$

where k is the common amount spent for TCRD by both firms. Each firm maximises its profits with respect to the two controls, i.e. the quantity sold on the domestic market and the quantity sold abroad. We then have 4 first order conditions (FOC):

$$\frac{\partial_{KK}\pi_H}{\partial_{KK}hh} = 0 \tag{7}$$

$$\frac{\partial_{KK}\pi_F}{\partial_{KK}f} = 0 \tag{8}$$

$$\frac{\partial_{KK}\pi_F}{\partial_{KK}ff} = 0 \tag{9}$$

$$\frac{\partial_{KK}\pi_H}{\partial_{KK}h} = 0 \tag{10}$$

Since there are constant marginal costs and perfect symmetry between the two firms the two FOCs (7 and 8) can be solved independently of the other two remaining FOCs and vice versa[2]. From the above FOCs we get equilibrium quantities:

$$_{KK}hh^* = \frac{a-c}{3}; \quad _{KK}h^* = \frac{b-c}{3}; \quad _{KK}f^* = \frac{a-c}{3}; \quad _{KK}ff^* = \frac{b-c}{3}. \tag{11}$$

[2] See Brander (1981) p.4.

with the usual feasibility condition that $a \geq c$, $b \geq c$.
Equilibrium profits are:

$$_{kk}\pi^*_H =_{kk} \pi^*_F = \frac{a^2 + b^2 - 2ac - 2bc + 2c^2}{9} - k. \tag{12}$$

2.2 No firm invests (00)

Demand functions are:

$$_{00}p_H = a -_{00} hh - t_{00}f$$

$$_{00}p_F = b -_{00} ff - t_{00}h.$$

Profits are respectively:

$$_{00}\pi_H = (_{00}p_H - c)_{00}hh + (_{00}p_F t - c)_{00}h \tag{13}$$

$$_{00}\pi_F = (_{00}p_F - c)_{00}ff + (_{00}p_H t - c)_{00}f. \tag{14}$$

From FOCs we obtain equilibrium quantities:

$$_{00}hh^* = \frac{c + at - 2ct}{3t}$$

$$_{00}ff^* = \frac{c + bt - 2ct}{3t}$$

$$_{00}f^* = \frac{-2c + at + ct}{3t^2}$$

$$_{00}h^* = \frac{-2c + bt + ct}{3t^2}.$$

Nonnegativity constraints on optimal quantities ($_{00}f^*$and $_{00}h^*$) and non-negative marginal revenues on the foreign markets require $t \geq c$ and:

$$a \geq 1 \geq t \geq \frac{2c}{a + c} =_H t_F \text{ and } b \geq 1 \geq t \geq \frac{2c}{b + c} =_F t_F. \tag{15}$$

These conditions say that t has to be high for the domestic firm to consider exporting when the foreign market is relatively small. In other words small foreign markets will be served only if TC barriers are not too high. Notice that this result does not depend on any economies of scale since they are assumed away in production.

We then turn to equilibrium profits:

$$_{00}\pi_H^* = \frac{5c^2 + 2act - 4bct - 8c^2t + a^2t^2 + b^2t^2 + 2bct^2 - 4act^2 + 5c^2t^2}{9t^2}, \quad (16)$$

$$_{00}\pi_F^* = \frac{5c^2 + 2bct - 4act - 8c^2t + a^2t^2 + b^2t^2 + 2act^2 - 4bct^2 + 5c^2t^2}{9t^2}. \quad (17)$$

2.3 Only one firm invests (0k,k0)

If only firm H invests in TCRD, the demand functions are

$$_{K0}p_H = a -_{K0} hh - t_{K0}f \quad (18)$$

$$_{K0}p_F = b -_{K0} ff -_{K0} h \quad (19)$$

Profits are respectively

$$_{K0}\pi_H = (_{K0}p_H - c)_{K0}hh + (_{K0}p_F - c)_{K0}h - k \quad (20)$$

$$_{K0}\pi_F = (_{K0}p_F - c)_{K0}ff + (_{K0}p_Ht - c)_{K0}f. \quad (21)$$

Solving best reply functions we can obtain optimal quantities:

$$_{K0}hh^* = \frac{at - 2ct + c}{3t}$$

$$_{K0}f^* = \frac{-2c + at + ct}{3t^2}$$

$$_{K0}h^* =_{K0} ff^* = \frac{b - c}{3}.$$

For$_{K0}$ $f^* \geq 0$, it must be $t \geq 2c/(a + c)$, while for $_{K0}hh^* \geq 0$, it must be $a \geq 2c$.

Equilibrium profits are:

$$_{K0}\pi_H^* = \frac{(b - c)^2}{9} + \frac{(-2ct + at + c)^2}{9t^2} - k \quad (22)$$

$$ {}_{K0}\pi_F^* = \frac{(b-c)^2}{9} + \frac{(at-2c+ct)^2}{9t^2} \qquad (23) $$

The case of firm F investing in TCRD while the rival H does not, leads to the reversed above profits[3].

3 The reduced form game

To find the solution of the two stage game we consider the reduced (normal) form of the game, i.e., the matrix of payoffs obtained by adopting either the strategy of investing in TCRD or not investing.

		firm F	
		0	k
firm H	0	${}_{00}\pi_H^*, {}_{00}\ \pi_F^*$	${}_{0k}\pi_H^*, {}_{0k}\ \pi_F^*$
	k	${}_{k0}\pi_H^*, {}_{k0}\ \pi_F^*$	${}_{kk}\pi_H^*, {}_{kk}\ \pi_F^*$

Matrix 1

From the reduced form of the game we can derive the following:

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[3]Quantities produced are:

$$ {}_{0K}f^* = {}_{0K}\ hh^* = \frac{a-c}{3} $$

$$ {}_{0K}ff^* = \frac{bt-2ct+c}{3t} $$

$$ {}_{0K}h^* = \frac{-2c+bt+ct}{3t^2}. $$

The feasibility condition is again given by $t \geq 2c/(b+c)$.

Equilibrium profits are:

$$ {}_{0k}\pi_H^* = \frac{(a-c)^2}{9} + \frac{(bt-2c+ct)^2}{9t^2} $$

and

$$ {}_{0k}\pi_F^* = \frac{(a-c)^2}{9} + \frac{(-2ct+bt+c)^2}{9t^2} - k. $$

Theorem 1 *In an international Cournot duopoly where countries are separated by natural barriers giving rise to TC costs, firms adopt their strategies according to the efficiency of TCRD.*

Suppose $a = b$. We then have 3 different symmetric solutions.

A) At high levels of efficiency (i.e., for $k \leq k_1$), there exists a unique equilibrium in dominant strategies in which both firms invest in TCRD and maximize their aggregate payoff.

B) For lower levels of efficiency of TCRD (i.e., for $k_1 \leq k \leq k_2$), the game is a Prisoner's dilemma with an equilibrium in dominant strategies with both firms investing without being able to maximize their total payoff.

C) For even lower levels of efficiency of TCRD, there still exists a unique equilibrium in dominant strategies where no firm undertakes TCRD and the aggregate payoff is maximised.

Proof. We prove the above theorem confining the proof to the case in which the two markets are of the same size, i.e., $a = b$.

Compare equilibrium profits along the principal diagonal. We have that ${}_{kk}\pi^*_{F,H} \geq {}_{00}\pi^*_{F,H}$ if

$$k \leq k_1 = \frac{c(t-1)(5c - 2at - 3ct)}{9t^2}.$$

Then compare ${}_{00}\pi^*_{F,H}$ with ${}_{0k}\pi^*_H$. It appears that ${}_{00}\pi^*_{F,H} \leq {}_{0k}\pi^*_H$ if

$$k \leq k_2 = \frac{4c(1-t)(at-c)}{9t^2}.$$

Moreover, ${}_{0k}\pi^*_F \leq {}_{0k}\pi^*_H$ if

$$k \leq k_3 = \frac{-c(2at^2 - ct^2 + c - 2at)}{3t^2}.$$

The sequence the critical $k's$ is

$$k_1 \leq k_2 \leq k_3.$$

Finally, in the acceptable region of parameters it is always true that: ${}_{0k}\pi^*_H \geq {}_{kk}\pi^*_{H,F}$ and ${}_{00}\pi^*_{F,H} \geq {}_{0k}\pi^*_F$.

Therefore, for $k \leq k_1$ the sequence of payoffs is

$${}_{0k}\pi^*_H \geq {}_{kk}\pi^*_{H,F} \geq_{00} \pi^*_{F,H} \geq {}_{0k}\pi^*_F.$$

While for $k \in [k_1, k_2]$, the sequence becomes:

$${}_{0k}\pi^*_H \geq \; {}_{00}\pi^*_{H,F} \geq_{kk} \pi^*_{F,H} \geq \; {}_{0k}\pi^*_F.$$

Finally, for $k \geq k_2$, we get:

$${}_{00}\pi^*_{F,H} \geq \; {}_{0k}\pi^*_H \geq_{0k} \pi^*_F \geq \; {}_{kk}\pi^*_{F,H}.$$

■

4 The effects of market size on TCRD: an example

We provide a calibrated example to assess the effect of the size of the market on investment in TCRD. To this purpose we assume that the home market is ten times as large as the foreign one. We evaluate again the strategic choices of the firms in the same environment as above. The new payoffs may be easily derived from previous sections. The matrix of the game in reduced form is no longer symmetric with respect to the principal diagonal. By comparing the payoffs, as the level of required investment TCRD changes, we can derive the following:

Remark 1 *If TCRD is not very efficient, only the firm belonging to the small country invests in TCRD, while the firm from the larger country shuns TCRD. Only when TCRD becomes highly profitable the firm from the large country follows the one from the small country investing in TCRD.*

Proof. Assume that $a = 10b$. Calculate the payoffs of the two firms and compare them. Beginning with[4] ${}_{KK}\pi_{F,H}$, we face the only case in which both firms have the same payoff, since they both invest in TCRD and therefore the two markets loose any separation. Compare it with ${}_{00}\pi_H$. It appears that it is always true that ${}_{00}\pi_H >_{KK} \pi_{H,F}$. While ${}_{KK}\pi_{H,F} \geq_{00} \pi_F$ if

$$k \leq z_2 = \frac{c(t-1)(5c - 38bt - 3ct)}{9t^2}.$$

[4]We do not use * for equilibrium profits in the example.

Then ${}_{KK}\pi_{H,F} \geq_{0K} \pi_H$ and ${}_{K0}\pi_H \geq_{00} \pi_H$ if

$$k \leq z_1 = \frac{4c(t-1)(c-bt)}{9t^2}.$$

Proceeding further we have that: ${}_{K0}\pi_H \geq_{00} \pi_F$ if

$$k \leq z_5 = \frac{2c(t-1)(2c-29bt)}{9t^2}.$$

While it can be easily verified that ${}_{0K}\pi_H >_{0K} \pi_F$ is always true. While ${}_{K0}\pi_H \geq_{K0} \pi_F$ if

$$k \leq z_6 = \frac{c(t-1)(c-20bt+ct)}{3t^2}.$$

Then ${}_{0K}\pi_F \geq_{00} \pi_F$ and ${}_{KK}\pi_{H,F} \geq_{K0} \pi_F$ if

$$k \leq z_4 = \frac{4c(t-1)(c-10bt)}{9t^2}.$$

Finally ${}_{K0}\pi_H >_{KK} \pi_{H,F}$ always holds and ${}_{0K}\pi_F \geq_{K0} \pi_F$ if

$$k \leq z_3 = \frac{c(t-1)(c-14bt+ct)}{3t^2}.$$

Since $z_1 \leq z_2 \leq z_3 \leq z_4 \leq z_5 \leq z_6$, we have that:

a) for $k \leq z_1$ the sequence of payoffs is ${}_{K0}\pi_H \geq_{00} \pi_H \geq_{KK} \pi_{F,H} \geq_{0K} \pi_H \geq_{0K} \pi_F \geq_{K0} \pi_F \geq_{00} \pi_F$ and there exists a unique equilibrium in dominant strategies in which both firms invest in TCRD;

b) for $k \in [z_1, z_2]$ the sequence of payoffs becomes: ${}_{K0}\pi_H \geq_{00} \pi_H \geq_{0K} \pi_H \geq_{KK} \pi_{F,H} \geq_{0K} \pi_F \geq_{K0} \pi_F \geq_{00} \pi_F$ and there is again a unique equilibrium in dominant strategies in which firm H (the one living in the larger market) does not invest in TCRD, while the rival firm F, based in the smaller market invests in TCRD. This sort of equilibrium holds regardless of slight changes in the sequence of payoffs for all $k \in [z_1, z_4]$;

c) for $k \geq z_4$ neither firm invests in TCRD as a result of a unique equilibrium in dominant strategies. ■

The above remark, based on a calibrated proof, suggests that large countries tend to invest relatively less in TCRD than smaller countries. Holland, for road and maritime transportation, and Greece, for maritime transportation may provide instances of such behaviour.

5 Welfare comparisons

We are now going to find out the welfare implications of investment in TCRD. To this purpose we consider the social welfare of each country as represented by the sum of the domestic consumer surplus plus the profits the domestic firm obtains on both the foreign and the domestic market. The consumer surplus is quadratic. Confining the analysis to country H, it can take two different forms according to whether the foreign firm invests in TCRD or not:

$$C_{SHK} = a({}_{0K}hh +_{0K} f) - \frac{({}_{9K}hh +_{0K} f)^2}{2}$$

if the foreign firm invests, or

$$C_{SH0} = a({}_{K0}hh + t_{K0}f) - \frac{({}_{K0}hh + t_{K0}f)^2}{2}$$

if she doesn't.

Profits vary according to the interactions between TCRD investment of the domestic firm vis à vis the foreign firm.

We then have 4 cases, as from section 2, and the relative 4 welfare functions:

$$_{00}W_H = \frac{(9c^2 + 2act - 8bct - 18c^2t + 10a^2t^2 + 2b^2t^2 - 10act^2 + 4bct^2 + 9c^2t^2)}{18t^2},$$

when neither firm invests in TCRD.

If both firms invest, we have:

$$_{KK}W_H = \frac{(5a^2 + b^2 - 4ac - 2bc)}{9} - k,$$

while, if only the domestic firm invests, we get:

$$_{K0}W_H = \frac{(c^2 + 2act - 10c^2t + 10a^2t^2 + 2b^2t^2 - 10act^2 - bct^2 + 9c^2t^2)}{18t^2} - k.$$

Yet, if only the foreign firm invests it is:

$$_{0K}W_H = \frac{(-2c^2 - bct + 5c^2t + 5a^2t^2 + b^2t^2 - 4act^2 - bc^2 + 3c^2t^2)}{9t^2}.$$

From the above welfare functions we can obtain some comparative statics information concerning market size effects. Consider the effect of the increase in the dimension of the domestic market. It appears that

$$\frac{\partial_{KK}W_H}{\partial a} \propto \frac{\partial_{K0}W_H}{\partial a} \propto \frac{\partial_{0K}W_H}{\partial a} \propto \frac{\partial_{00}W_H}{\partial a} \geq 0$$

in the feasible set of parameters. The increase in the size of the foreign market has a similar nonnegative effect on social welfare of country H, regardless of the strategy adopted by firms. If we go through the relative effects on the consumer surplus and on the profits of the domestic firm, we find that the consumer surplus does not, obviously, change as the size of the foreign market varies and increases as the domestic market gets larger. Profits react to an increase in the size of any of the two markets in a nonnegative fashion for all feasible sets of parameters.

More interesting is to find out the regime that the social planner would prefer in a second best comparison of different market outcomes. Again we consider country H and we assume, for the sake of simplicity, that the dimension of the two markets is the same, i.e.: $a = b$. From feasibility conditions it appears that we can consider markets whose dimension is weakly larger than $t \geq c$. Then we obtain the following:

Theorem 2 *We consider 3 different market sizes.*

i) Small *market size*: $a = b \leq \frac{17}{5}c$. *It appears that for low levels of* t *the most desired market solution are* ${}_{K0}W_H$ *for low levels of* k, *and then* ${}_{00}W_H$ *for high* k. *As we move towards higher* t, ${}_{KK}W_H$ *becomes larger at low* k *and* ${}_{00}W_H$ *at high* k. *At even higher* t *and* k, ${}_{0K}W_H$ *is preferred.*

ii) Medium *market size:* $a = b \in \left[\frac{17}{5}c, \frac{9}{2}c\right]$.

At high levels of transport costs and/or bad public infrastructure, i.e., for $t \in \left[t_F, t_2 = \frac{7c}{b+12c}\right]$, *we have that* ${}_{K0}W_H$ *is the largest and it is surpassed only when* k *gets larger, respectively by* ${}_{00}W_H$ *for all* $t \in \left[t_F, t_1 = \frac{13c}{4b+15c}\right]$ *when* $k \geq k_{3S}$, *and by* ${}_{0K}W_H$ *for all* $t \in \left[t_1, t_2 = \frac{7c}{b+12c}\right]$. *As* t *increases, the socially preferred outcome become* ${}_{KK}W_H$ *for low investment levels and* ${}_{0K}W_H$ *for high levels of* k.

iii) Large *market size:* $a = b \in \left[\frac{9}{2}c, \infty\right)$*,we have that* ${}_{K0}W_H$ *is again the* *largest welfare level for low values of k. As k gets larger, ${}_{0K}W_H$ and ${}_{00}W_H$ prevail for lower and higher levels of t, respectively.*

We provide the detailed proof in the Appendix.

A brief comment suggests that, almost regardless of market size, the social planner always prefers the domestic firm to invest in TCRD, provided that it is efficient and that initial endowment of public infrastructure is poor, i.e. for low t. When TCRD becomes highly costly, in some cases, the social planner would like the foreign firm to invest to increase its presence in the domestic market so as to keep prices at a reasonable level.

6 Terms of trade effects

We now wish to see how the terms of trade (TOT) react to some key variables, such the dimension of markets, the initial level of endowment of TC infrastructure, i.e. TC costs, and the level of sheer production costs. TOT are defined as the ratio between the price of exports over the price of imports of a country. In our case the price of exports is equal to the foreign price p_F while the price of imports is equal to the domestic price p_H. We then calculate the TOT in the 4 regimes termed $0k, k0, kk, 00$ and we get:

$$
{}_{0K}TOT = \frac{c + bt + ct}{at + 2ct}
$$

$$
{}_{00}TOT = \frac{c + bt + ct}{c + at + ct}
$$

$$
{}_{K0}TOT = \frac{b + 2c}{a + c + c/t}
$$

$$
{}_{KK}TOT = \frac{b + 2c}{a + 2c}.
$$

It can be easily seen that the TOT always improve as the dimension of the foreign market increases. Moreover, if the two markets are of the same dimension the TOT equal unity in the symmetric cases and become larger than unity if the foreign market gets larger. Sticking to markets of equal dimension, it appears that ${}_{K0}TOT \leq 1$, while ${}_{0K}TOT \geq 1$. Some comparative statics may be added to see whether:

i) better public infrastructure (higher t)

ii) better production technologies (lower c)

iii) changes in market dimensions

affect the TOT in the asymmetric cases.

Let's start with t :

$$\frac{\partial\, _{0K}TOT}{\partial\, t} = -\frac{c}{(a+2c)t^2}$$

$$\frac{\partial\, _{K0}TOT}{\partial\, t} = \frac{c(b+2c)}{(at+ct+c)^2}.$$

The first expression is negative while the second is positive, which implies that a general improvement in public infrastructure benefits more the country that invests in TCRD.

Let's analyse the effects of c :

$$\frac{\partial\, _{0K}TOT}{\partial\, c} = \frac{a+at-2bt}{(a+2c)^2 t} \geq 0$$

$$\frac{\partial\, _{K0}TOT}{\partial\, c} = \frac{t(-b-bt+2at)}{(c+at+ct)^2} \leq 0.$$

Here it appears that better technologies benefit the TOT of the country that invests in TCRD.

Finally look at the effects of market dimension:

$$\frac{\partial\, _{K0}TOT}{\partial\, a} = -\frac{(b+2c)t^2}{(c+at+ct)^2}$$

$$\frac{\partial\, _{K0}TOT}{\partial\, b} = \frac{1}{c+a+c/t}$$

$$\frac{\partial\, _{0K}TOT}{\partial\, a} = -\frac{(c+bt+ct)}{(a+2c)^2 t}$$

$$\frac{\partial\, _{0K}TOT}{\partial\, b} = \frac{1}{2c+a}.$$

The own market effect is always negative for the TOT in both cases, while the foreign market effect is positive. This is due to fact that a larger internal market implies a higher internal price that coincides with the price of imports since products are homogeneous.

7 Conclusions

In a Cournot duopoly setting in which firms sell on both the domestic and the foreign market, export takes place towards small markets only if well linked, or in other words if the initial state of public infrastructure for TC purposes is not too bad in both large and small countries.

With countries of equal size, both firms undertake TCRD at high levels of efficiency of their investment. If the profitability of TCRD decreases, both of them still invest and yet fail to maximise their aggregate payoff, in that the game is a Prisoner's Dilemma. For even less efficient TCRD, they both stop investing.

It seems that small countries invest in TCRD even when large countries stop doing it since they find it not profitable enough. This is consistent with the stylized fact that can be mainly observed in Holland and Greece, highly specialised in TC industries.

When markets are small and public infrastructure not much efficient social welfare is maximised if the domestic firm invests in TCRD.

Related TOT analysis shows that they improve as the size of the foreign market increases and as the foreign firm invests in TCRD, that, then, provides a positive externality to domestic consumers. A general improvement in TC public infrastructure is going to benefit more the TOT of the country that has the firm already investing in TCRD.

8 Appendix

Here we provide the proof of theorem 2.

Proof. We omit the deponent H since we always refer to welfare levels of country H.

We provide a proof divided between a common initial section and a set made up of 3 parts according to the 3 regions mentioned in the proposition.

INITIAL COMPARISONS

First we consider the overall levels of welfare and we compare them. Then $_{KK}W \geq {}_{00}W$ if

$$k \leq k_1 = \frac{(-9c^2 + 6bct - 6bct^2 + 18c^2t - 9c^2t^2)}{18t^2}.$$

While $_{KK}W \geq {}_{0K}W$ if

$$k \leq k_2 = \frac{(2c^2 + bct - 5c^2t - bct^2 + 3c^2t^2)}{9t^2}.$$

And $_{K0}W \geq {}_{0K}W$ if

$$k \leq k_3 = \frac{(5c^2 + 4bct - 20c^2t - 4bct^2 + 5c^2t^2)}{18t^2}.$$

And $_{K0}W \geq {}_{00}W$ if

$$k \leq k_4 = \frac{(-4c^2 + 4bct + 4c^2t - 4bct^2)}{9t^2}.$$

Finally $_{KK}W \geq {}_{K0}W$ if

$$\frac{c(t-1)(c + 2bt - 9ct)}{18t^2} \geq 0.$$

In this last case, if $b \geq 4.5c$, we have that $_{KK}W \leq {}_{K0}W$. While if $b \leq 4.5c$ it must be $t \geq t_0 = \frac{c}{9c-2b}$ for $_{KK}W \geq {}_{K0}W$. Finally we have that $_{00}W \geq {}_{0K}W$ if

$$\frac{c(t-1)(-13c + 4bt + 15ct)}{18t^2} \geq 0,$$

that requires $t \leq t_1 = \frac{13c}{15c+4b}$.

Compare now the $k's$. It appears that $k_1 \geq k_2$ if $t \geq t_1$, while $k_1 \geq k_3$ if $t \geq t_2 = 7c/(b + 12c)$, and $k_2 \geq k_3$ if $t \geq t_0$ and $b \leq 4.5c$. The opposite, that is $k_2 \leq k_3$ happens if $b \geq 4.5c$. Moreover $k_4 \geq k_2$ always holds. $k_4 \geq k_2$ if $t \geq t_1$. And, finally $k_4 \geq k_1$ if $b \geq 4.5c$. If $b \leq 4.5c$ we have $k_4 \leq k_1$ provided that $t \leq t_0$.

Now we consider the feasible set of market sizes and we divide it into the 3 mentioned regions.

REGION 1: $a = b \leq 17/5c = 3.4c$.

REGION 2: $17/5c \leq b \leq 4.5c$,

REGION 3: $b \geq 4.5c$.

Consider first *REGION 1.*

In this region the sequence of the transport parameter is $t_0 \leq t_2 \leq t_1$.

For $t \leq t_0$ the sequence of k is $k_1 \leq k_2 \leq k_4 \leq k_3$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{00} W \geq_{0K} W$ for $k \leq k_1$; ${}_{K0}W \geq {}_{00}W \geq_{KK} W \geq_{0K} W$ for $k \in [k_1, k_2]$; ${}_{K0}W \geq {}_{00}W \geq_{0K} W \geq_{KK} W$ for $k \in [k_2, k_4]$; ${}_{00}W \geq {}_{K0}W \geq_{0K} W \geq_{KK} W$ for $k \in [k_4, k_3]$; ${}_{00}W \geq {}_{0K}W \geq_{K0} W \geq_{KK} W$ for $k \geq k_3$.

For $t \in [t_0, t_2]$ the sequence of k is $k_4 \leq k_1 \leq k_3 \leq k_2$ and welfare ranking is ${}_{KK}W \geq {}_{K0}W \geq_{00} W \geq_{0K} W$ for $k \leq k_4$;${}_{KK} W \geq {}_{00}W \geq_{K0} W \geq_{0K} W$ for $k \in [k_4, k_1]$;${}_{00} W \geq {}_{KK}W \geq_{K0} W \geq_{0K} W$ for $k \in [k_1, k_3]$;${}_{00} W \geq {}_{KK}W \geq_{0K} W \geq_{K0} W$ for $k \in [k_3, k_2]$;${}_{00} W \geq {}_{0K}W \geq_{KK} W \geq_{K0} W$ for $k \geq k_2$.

For $t \in [t_2, t_1]$ the sequence of k is $k_2 \leq k_3 \leq k_1 \leq k_4$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{0K} W \geq_{00} W$ for $k \leq k_2$;${}_{K0} W \geq {}_{0K}W \geq_{KK} W \geq_{00} W$ for $k \in [k_2, k_3]$;${}_{0K} W \geq {}_{K0}W \geq_{KK} W \geq_{00} W$ for $k \in [k_3, k_1]$;${}_{0K} W \geq {}_{K0}W \geq_{00} W \geq_{KK} W$ for $k \in [k_1, k_4]$;${}_{0K} W \geq {}_{00}W \geq_{K0} W \geq_{KK} W$ for $k \geq k_4$.

For $t \geq t_1$ the sequence of k is $k_2 \leq k_1 \leq k_3 \leq k_4$ and welfare ranking is ${}_{KK}W \geq {}_{K0}W \geq_{00} W \geq_{0K} W$ for $k \leq k_4$;${}_{KK} W \geq {}_{00}W \geq_{K0} W \geq_{0K} W$ for $k \in [k_4, k_3]$;${}_{KK} W \geq {}_{00}W \geq_{0K} W \geq_{K0} W$ for $k \in [k_3, k_1]$;${}_{00} W \geq {}_{KK}W \geq_{0K} W \geq_{K0} W$ for $k \in [k_1, k_2]$;${}_{00} W \geq {}_{0K}W \geq_{KK} W \geq_{K0} W$ for $k \geq k_2$.

Then consider *REGION 2.*

In this case the sequence of t is $t_1 \leq t_2 \leq t_0$.

For $t \leq t_1$ the sequence of k is $k_1 \leq k_2 \leq k_4 \leq k_3$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{00} W \geq_{0K} W$ for $k \leq k_1$; ${}_{K0}W \geq {}_{00}W \geq_{KK} W \geq_{0K} W$ for $k \in [k_1, k_2]$; ${}_{K0}W \geq {}_{00}W \geq_{0K} W \geq_{KK} W$ for $k \in [k_2, k_3]$; ${}_{00}W \geq {}_{0K}W \geq_{K0} W \geq_{KK} W$ for $k \geq k_3$.

For $t \in [t_1, t_2]$ the sequence of k is $k_2 \leq k_1 \leq k_3 \leq k_4$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{0K} W \geq_{00} W$ for $k \leq k_2$;${}_{K0} W \geq {}_{0K}W \geq_{KK} W \geq_{00} W$ for $k \in [k_2, k_1]$;${}_{K0} W \geq {}_{0K}W \geq_{00} W \geq_{KK} W$ for $k \in [k_1, k_3]$;${}_{0K} W \geq {}_{K0}W \geq_{00} W \geq_{KK} W$ for $k \in [k_3, k_4]$;${}_{0K} W \geq {}_{00}W \geq_{K0} W \geq_{KK} W$ for $k \geq k_4$.

For $t \in [t_2, t_0]$ the sequence of k is $k_2 \leq k_3 \leq k_1 \leq k_4$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{0K} W \geq_{00} W$ for $k \leq k_2$;${}_{K0} W \geq {}_{0K}W \geq_{KK} W \geq_{00} W$ for $k \in [k_2, k_3]$;${}_{0K} W \geq {}_{K0}W \geq_{KK} W \geq_{00} W$ for $k \in [k_3, k_1]$;${}_{0K} W \geq {}_{K0}W \geq_{00} W \geq_{KK} W$ for $k \in [k_1, k_4]$;${}_{0K} W \geq {}_{00}W \geq_{K0} W \geq_{KK} W$ for $k \geq k_4$.

For $t \geq t_0$ the sequence of k is $k_1 \leq k_2 \leq k_4 \leq k_1$ and welfare ranking is ${}_{KK}W \geq {}_{K0}W \geq_{0K} W \geq_{00} W$ for $k \leq k_3$;${}_{KK} W \geq {}_{0K}W \geq_{K0} W \geq_{00} W$ for $k \in [k_3, k_2]$;${}_{0K} W \geq {}_{KK}W \geq_{K0} W \geq_{00} W$ for $k \in [k_2, k_4]$;${}_{0K} W \geq {}_{KK}W \geq_{00} W \geq_{K0} W$ for $k \in [k_4, k_1]$;${}_{0K} W \geq {}_{00}W \geq_{KK} W \geq_{K0} W$ for $k \geq k_1$.

Now we are left with *REGION 3*.

Here the sequence is just represented by $t_1 \leq t_2$.

For $t \leq t_1$ the sequence of k is $k_1 \leq k_2 \leq k_4 \leq k_3$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{00} W \geq_{0K} W$ for $k \leq k_1$; ${}_{K0}W \geq {}_{00}W \geq_{KK} W \geq_{0K} W$ for $k \in [k_1, k_2]$; ${}_{K0}W \geq {}_{00}W \geq_{0K} W \geq_{KK} W$ for $k \in [k_2, k_4]$; ${}_{00}W \geq {}_{K0}W \geq_{0K} W \geq_{KK} W$ for $k \in [k_4, k_3]$;${}_{00} W \geq_{0K} W \geq_{K0} W \geq_{KK} W$ for $k \geq k_3$.

For $t \in [t_1, t_2]$, the sequence of k is $k_2 \leq k_1 \leq k_3 \leq k_4$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{0K} W \geq_{00} W$ for $k \leq k_2$; ${}_{K0}W \geq {}_{0K}W \geq_{KK} W \geq_{00} W$ for $k \in [k_2, k_1]$; ${}_{K0}W \geq {}_{0K}W \geq_{00} W \geq_{KK} W$ for $k \in [k_1, k_3]$; ${}_{0K}W \geq {}_{K0}W \geq_{00} W \geq_{KK} W$ for $k \in [k_3, k_4]$;${}_{0K} W \geq_{00} W \geq_{K0} W \geq_{KK} W$ for $k \geq k_4$.

For $t \geq t_2$ the sequence of k is $k_2 \leq k_3 \leq k_1 \leq k_4$ and welfare ranking is ${}_{K0}W \geq {}_{KK}W \geq_{0K} W \geq_{00} W$ for $k \leq k_2$; ${}_{K0}W \geq {}_{0K}W \geq_{KK} W \geq_{00} W$ for $k \in [k_2, k_3]$; ${}_{0K}W \geq {}_{K0}W \geq_{KK} W \geq_{00} W$ for $k \in [k_3, k_1]$; ${}_{0K}W \geq {}_{K0}W \geq_{00} W \geq_{KK} W$ for $k \in [k_1, k_4]$;${}_{0K} W \geq_{00} W \geq_{K0} W \geq_{KK} W$ for $k \geq k_4$. ■

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