R&D Incentives and Market Structure: A Dynamic Analysis[1]

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Abstract

We investigate dynamic R&D for process innovation in an oligopoly where firms invest in cost-reducing activities. We focus on the relationship between R&D intensity and market structure, proving that the industry R&D investment monotonically increases in the number of firms. This Arrowian result contradicts the established wisdom acquired from static games on the same topic.

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1 Introduction

We propose a dynamic analysis of the relationship between market power and R&D efforts, in order to reassess a well-known issue in the theory of industrial organization, that can be traced back to the debate between Schumpeter (1942) and Arrow (1962). The so-called Schumpeterian hypothesis maintains that there exists an inverse relationship between the intensity of competition and the pace of technical progress. That is, according to Schumpeter, monopoly is the market structure that should ensure the fastest and largest technical progress. This relies upon the idea that monopoly ensures the highest profit level and therefore the larger internal sources for funding R&D activities. Exactly the opposite view is expressed by Arrow, since he focuses upon the replacement effect, according to which a monopolist should be induced to rest on his laurels, while a firm operating in a competitive environment should strive for new technologies or new products, in order to throw her rivals out of business.[1]

In order to assess this issue, we take a differential game perspective, proposing a dynamic version of a model first introduced in a static framework by d'Aspremont and Jacquemin (1988). We consider an oligopoly where n firms sell a homogeneous product and compete in quantities. Moreover, they also invest at each point in time in R&D for process innovation, i.e., reducing the marginal cost of production of the final good. The R&D activity is characterized by positive externalities, entailing that each firm receives a positive spillover from the investments carried out by all other firms in the industry.

Our model has the desirable property of being *state-redundant* or *perfect*, so that the open-loop solution is a Markovian equilibrium. We proceed

[1] For an exhaustive overview of the related literature, see Reinganum (1989) and Martin (2001).

in two steps. First, we characterize the individually optimal path of R&D investment for a given level of marginal production cost. Second, we obtain the steady state levels of investment and marginal cost. With respect to both the optimal path and the steady-state level of R&D investment, the following conclusions hold. The individual effort is always decreasing in the number of firms while the opposite holds for the aggregate R&D investments. This result has an Arrowian flavour, since as the degree of competition becomes tougher, the aggregate investment becomes larger. This is in sharp contrast with the conclusions drawn from the static version of the same model (Hinloopen, 2000) where a non-monotone relationship exists between aggregate R&D investment and market structure. Under this perspective, our model highlights the value added of a properly dynamic analysis over the static approach based upon a multistage game.

The remainder of the paper is structured as follows. Section 2 illustrates the basic setup. The solution of the open-loop game is investigated in section 3, while the industry R&D performance is assessed in section 4. Section 5 contains concluding remarks.

2 The setup

We consider an oligopoly with n firms selling a homogeneous goods over continuous time, $t \in [0, \infty)$. In every instant, the market demand function writes as follows:

$$p(t) = A - q_i(t) - Q_{-i}(t)\,. \tag{1}$$

where $Q_{-i}(t)$ is the output supplied by all firms other than i. Each firm supplies the market through a technology characterized by a constant marginal cost, c_i. Accordingly, her instantaneous cost function for the production of the final good is $C_i(c_i, q_i, t) = c_i(t)q_i(t)$. The marginal cost borne by firm i

evolves over time according to the following kinematic equation:

$$\frac{dc_i(t)}{dt} \equiv \dot{c}_i = c_i(t)\left[-k_i(t) - \beta K_{-i}(t) + \delta\right] , \tag{2}$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t, while $K_{-i}(t)$ is the aggregate R&D effort of all other firms and parameter $\beta \in [0,1]$ measures the positive technological spillover that firm i receives from the R&D activity of the rivals. Parameter $\delta \in [0,1]$ is a constant depreciation rate measuring the instantaneous decrease in productive efficiency due to the ageing of technology. Equation (2) is indeed a dynamic version of the linear R&D technology employed by d'Aspremont and Jacquemin (1988).

The instantaneous cost of running R&D activity is:

$$\Gamma(k_i, t) = b\left[k_i(t)\right]^2 , \tag{3}$$

where b is a positive parameter. Throughout the game, firms discount future profits a the common and constant discount rate $\rho > 0$.

Firms adopt a strictly noncooperative behaviour in choosing both the output levels and the R&D efforts, each firm operating her own R&D division.[2] The objective of firm i consists in maximizing discounted profits:

$$\Pi_i = \int_0^\infty \left\{\left[A - q_i(t) - Q_{-i}(t) - c_i(t)\right] q_i(t) - b\left[k_i(t)\right]^2\right\} e^{-\rho t} dt \tag{4}$$

subject to the set of dynamic constraints (2). The corresponding Hamiltonian function is:

$$\begin{aligned}\mathcal{H}_i(\mathbf{q}, \mathbf{k}, \mathbf{c}, t) = e^{-\rho t}\{&\left[A - q_i(t) - Q_{-i}(t) - c_i(t)\right] q_i(t) - b\left[k_i(t)\right]^2 + \\ &-\lambda_{ii}(t) c_i(t)\left[k_i(t) + \beta K_{-i}(t) - \delta\right] + \\ &-\sum_{j \neq i} \lambda_{ij}(t) c_j(t)\left[k_j(t) + \beta\left(k_i(t) + \sum_{l \neq i,j} k_l(t)\right) - \delta\right]\Bigg\}\end{aligned} \tag{5}$$

[2] For a discussion of R&D cooperation in the same model, see Cellini and Lambertini (2003).

where $\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c_j(t)$, and $\mathbf{q}, \mathbf{k}, \mathbf{c}$ are the vectors of control and state variables.

3 The open-loop solution

Here we characterize the Nash equilibrium under the open-loop information structure. As a first step, we prove the following result:

Lemma 1 *The open-loop Nash equilibrium of the game is subgame (or Markov) perfect.*

Proof. We are going to show that the present setup is a *perfect game* in the sense of Leitmann and Schmitendorf (1978) and Feichtinger (1983). In summary, a differential game is *perfect* whenever the closed-loop equilibrium collapses into the open-loop one, the latter being thus strongly time consistent, i.e., subgame perfect.[3] Consider the closed-loop information structure. The relevant first order conditions (FOCs) are:

$$\frac{\partial \mathcal{H}_i\,(.,t)}{\partial q_i(t)} = A - 2q_i(t) - Q_{-i}(t) - c_i(t) = 0\ ; \tag{6}$$

$$\frac{\partial \mathcal{H}_i\,(.,t)}{\partial k_i(t)} = -2bk_i(t) - \lambda_{ii}(t)c_i(t) - \beta\sum_{j\neq i}\lambda_{ij}(t)c_j(t) = 0\ . \tag{7}$$

As a first step, observe that (6) only contains firm i's state variable, so that in choosing the optimal output at any time during the game firm i may disregard the current efficiency of the rival. That is, there is no feedback effect in the

[3]The label 'perfect game' is due to Fershtman (1987), where one can find a general technique to identify any such games. Another class of games where open-loop equilibria are subgame perfect is investigated by Reinganum (1982). For further details, see Mehlmann (1988, ch. 4) and Dockner *et al.* (2000, ch. 7).

output choice. Conversely, at first sight there seem to be a feedback between the R&D decisions, as (7) indeed contains all state variables, at least for any positive spillover effect.[4] The core of the proof consists in showing that no feedback effect are actually present, even for positive spillover levels.

Taking the above considerations into account, the adjoint or co-state equations are:

$$-\frac{\partial \mathcal{H}_i(.,t)}{\partial c_i(t)} - \sum_{j\neq i} \frac{\partial \mathcal{H}_i(.,t)}{\partial k_j(t)} \cdot \frac{\partial k_j^*(.,t)}{\partial c_i(t)} = \frac{\partial \lambda_{ii}(t)}{\partial t} - \rho\lambda_{ii}(t) \Leftrightarrow \tag{8}$$

$$\frac{\partial \lambda_{ii}(t)}{\partial t} = q_i(t) + \lambda_{ii}(t)\left[k_i(t) + \beta K_{-i}(t) + \rho - \delta\right] + \tag{9}$$

$$-\frac{\beta}{2b}\sum_{j\neq i}\lambda_{ji}(t)\left[\beta\lambda_{ii}(t)c_i(t) + \lambda_{ij}(t)c_j(t) + \beta\sum_{l\neq i,j}\lambda_{il}(t)c_l(t)\right]$$

$$-\frac{\partial \mathcal{H}_i(.,t)}{\partial c_j(t)} - \frac{\partial \mathcal{H}_i(.,t)}{\partial k_i(t)} \cdot \frac{\partial k_i^*(.,t)}{\partial c_j(t)} - \sum_{l\neq i,j}\frac{\partial \mathcal{H}_i(.,t)}{\partial k_l(t)} \cdot \frac{\partial k_l^*(.,t)}{\partial c_j(t)} = \frac{\partial \lambda_{ij}(t)}{\partial t} - \rho\lambda_{ij}(t) \Leftrightarrow \tag{10}$$

$$\frac{\partial \lambda_{ij}(t)}{\partial t} = \lambda_{ij}(t)\left[k_j(t) + \beta k_i(t) + \beta\sum_{l\neq i,j}k_l(t) + \rho - \delta + \right. \tag{11}$$

$$\left. -\frac{\beta}{2b}\left(2bk_i(t) + \lambda_{ii}(t)c_i(t) + \beta\sum_{j\neq i}\lambda_{ij}(t)c_j(t)\right)\right] +$$

$$-\frac{\beta}{2b}\sum_{l\neq i,j}\lambda_{lj}(t)\left[\beta\lambda_{ii}(t)c_i(t) + \lambda_{il}(t)c_l(t) + \beta\sum_{j\neq i,l}\lambda_{ij}(t)c_j(t)\right]$$

where each term

$$\frac{\partial \mathcal{H}_i(.,t)}{\partial k_j(t)} \cdot \frac{\partial k_j^*(.,t)}{\partial c_i(t)} \tag{12}$$

captures the feedback effect from j to i, and partial derivatives $\partial k_j^*(.,t)/\partial c_i(t)$ are calculated using the optimal values of investments as from FOC (7):

$$k_j^*(.,t) = -\frac{\lambda_{jj}(t)c_j(t) + \beta\lambda_{ji}(t)c_i(t)}{2b}. \tag{13}$$

[4]Intuitively, if $\beta = 0$, then the investment plans are completely independent and therefore it is apparent that no feedback effect operates.

These conditions must be evaluated along with the initial conditions $\{c_i(0)\} = \{c_{0,i}\}$ and the transversality conditions

$$\lim_{t\to\infty} \lambda_{ij}(t) \cdot c_j(t) = 0 \ , \ i, j = 1, 2. \tag{14}$$

Note that, on the basis of *ex ante* symmetry across firms, $\lambda_{lj}(t) = \lambda_{ij}(t)$ for all l. Accordingly, from (11), we have $\partial\lambda_{ij}(t)/\partial t = 0$ in $\lambda_{ij}(t) = 0$. Then, using this piece of information, we may rewrite the expression for the optimal investment of firm i as follows:

$$k_i^*(.,t) = -\frac{\lambda_{ii}(t)c_i(t)}{2b} \ , \tag{15}$$

which entails that $\partial k_i^*(.,t)/\partial c_j(t) = 0$ for all $j \neq i$, i.e., feedback (cross-)effects are nil along the equilibrium path. Accordingly, the open-loop equilibrium is a degenerate closed-loop one, and it is strongly time consistent, or equivalently, subgame perfect. It is also worth observing that this procedure shows that FOCs are indeed unaffected by initial conditions as well. The property whereby the FOCs on controls are independent of states and initial conditions after replacing the optimal values of the co-state variables is known as *state-redundancy*, and the game itself as *state-redundant* or *perfect*. ■

On the basis of Lemma 1, we can proceed with the characterization of the open-loop solution. The FOCs on controls as well as the transversality conditions are the same as above, while the co-state equations simplify as follows:

$$-\frac{\partial\mathcal{H}_i(.,t)}{\partial c_i(t)} = \frac{\partial\lambda_{ii}(t)}{\partial t} - \rho\lambda_{ii}(t) \Leftrightarrow \tag{16}$$

$$\frac{\partial\lambda_{ii}(t)}{\partial t} = q_i(t) + \lambda_{ii}(t)\left[k_i(t) + \beta K_{-i}(t) + \rho - \delta\right]$$

$$-\frac{\partial\mathcal{H}_i(.,t)}{\partial c_j(t)} = \frac{\partial\lambda_{ij}(t)}{\partial t} - \rho\lambda_{ij}(t) \Leftrightarrow \tag{17}$$

$$\frac{\partial\lambda_{ij}(t)}{\partial t} = \lambda_{ij}(t)\left[k_j(t) + \beta K_{-j}(t) + \rho - \delta\right]$$

From FOCs (6-7) we have, respectively:

$$q_i^*(t) = \frac{A - Q_{-i}(t) - c_i(t)}{2}\,, \tag{18}$$

$$k_i\,(t) = -\frac{\lambda_{ii}(t)c_i(t)}{2b}\,, \tag{19}$$

since $\lambda_{ij}(t) = 0$ for all $j \neq i$, at any $t \in [0, \infty)$. While (18) has the usual appearance of a standard Cournot best reply function, the optimal R&D effort in (19) depends upon i's co-state variable. Such expression can be differentiated w.r.t. time to get the dynamic equation of $k_i(t)$:

$$\frac{dk_i(t)}{dt} \equiv \dot{k}_i = -\frac{1}{2b}\left[c_i(t)\dot{\lambda}_{ii}(t) + \lambda_{ii}\,(t)\,\dot{c}_i(t)\right] \tag{20}$$

with $\dot{\lambda}_{ii}(t)$ obtaining from (9). Then, (20) can be further simplified by using

$$\lambda_{ii}\,(t) = -\frac{2bk_i\,(t)}{c_i\,(t)} \tag{21}$$

which obtains from (7). This yields:

$$\dot{k}_i = -\frac{1}{2b}\left[c_i\,(t)\,q_i\,(t) - 2bk_i\,(t)\right]\,. \tag{22}$$

The next step consists in imposing the symmetry conditions $c_j(t) = c_i(t)$, $k_j(t) = k_i(t)$ and $q_j(t) = q_i(t)$ for all j, and solve the system of the best reply functions (18), yielding the Cournot-Nash output level of each firm:

$$q^N(t) = \frac{A - c(t)}{n+1} \tag{23}$$

which can be plugged into (22). Accordingly, we may simplify the dynamics of the R&D effort of any single firm as follows:

$$\dot{k} = -\frac{1}{2b}\left[\frac{c\,(t)\,[A - c\,(t)]}{n+1} - 2b\rho k\,(t)\right]\,. \tag{24}$$

Imposing the stationarity condition $\dot{k} = 0$, we obtain:

$$k^N(t) = \frac{c\,(t)\,[A - c\,(t)]}{2b\rho\,(n+1)} \geq 0 \text{ for all } c\,(t) \in [0,\, A\,]\,, \tag{25}$$

where the superscript N stands for *Nash equilibrium*.

The steady state level of marginal cost $c(t)$ can be found by solving:

$$\dot{c} = -c(t)\left[k^N(t)\left(1+\beta\left(n-1\right)\right) - \delta c(t)\right] = 0 \qquad (26)$$

which yields $c = 0$ and

$$c = \frac{A\left(1+\beta\left(n-1\right)\right) \pm \sqrt{\left(1+\beta\left(n-1\right)\right)\left[A^2\left(1+\beta\left(n-1\right)\right) - 8b\delta\rho\left(n+1\right)\right]}}{2\left(1+\beta\left(n-1\right)\right)} \qquad (27)$$

All solutions in (27) are real if and only if $\delta\rho \leq A^2\left(1+\beta\left(n-1\right)\right)/\left[8b\left(n+1\right)\right]$. If so, they also satisfy the requirement $c \in [0, A]$. By checking the stability conditions, we may prove the following:

Proposition 2 *Provided that* $\delta\rho \leq A^2\left(1+\beta\left(n-1\right)\right)/\left[8b\left(n+1\right)\right]$, *the steady state point*

$$c^{ss} = \frac{A\left(1+\beta\left(n-1\right)\right) - \sqrt{\left(1+\beta\left(n-1\right)\right)\left[A^2\left(1+\beta\left(n-1\right)\right) - 8b\delta\rho\left(n+1\right)\right]}}{2\left(1+\beta\left(n-1\right)\right)}$$

$$k^{ss} = \frac{\delta}{1+\beta\left(n-1\right)}$$

is the unique saddle point equilibrium.

Proof. Omitted for brevity. ■

4 Comparative statics

Now we focus on the interplay between market structure (as measured by the number of firms) and the industry incentive to invest in process R&D. To this aim, we examine effect of a change in n on individual and aggregate R&D efforts, both along the equilibrium path (expression (25)) and in steady state.

This discussion revisits the debate between Schumpeter (1942) and Arrow (1962). Their respective views can be summarized as follows. According to the Schumpeterian hypothesis, R&D investments and technical progress are positively related to the flow of profits and therefore we should expect to observe higher R&D efforts and a faster innovation process under monopoly than any other market form. Conversely, Arrow claims that the incentive to generate technical progress is negatively affected by market power, being then maximized under perfect competition. The Arrowian position relies upon the idea that innovation is more attractive for a competitive firm than for a monopolist who, by definition, can not improve his market power.

In order to assess this issue in the present model, we proceed as follows. The aggregate R&D investments along the equilibrium path and in steady state are, respectively:

$$K^N(t) = \frac{c(t)\left[A - c(t)\right]n}{2b\rho(n+1)}\,;\; K^{ss} = \frac{\delta n}{1+\beta(n-1)} \tag{28}$$

It is immediate to verify that:

$$\begin{aligned} \frac{\partial K^N(t)}{\partial n} &= \frac{2b\rho c(t)\left[A - c(t)\right]}{4\left[b\rho(n+1)\right]^2} > 0 \\ \frac{\partial K^{ss}}{\partial n} &= \frac{\delta(1-\beta)}{\left[1+\beta(n-1)\right]^2} \geq 0 \end{aligned} \tag{29}$$

which entails that the behaviour of the industry is clearly Arrowian. If instead we examine the individual investment, we obtain $\partial k^N(t)/\partial n, \partial k^{ss}/\partial n < 0$ everywhere. This entails that any increase in the number of firms brings about a decrease in individual R&D effort. This is caused by two facts: on the one hand, tougher market competition reduces profits and therefore that funds available to any given firm for conducting R&D activity; on the other, a larger population of firms means a larger amount of positive externality that any firm receives from the rivals. On the aggregate, a scale effect prevails, so that the overall expenditure of the industry is monotonically increasing in n.[5]

Hinloopen (2000) has solved the oligopoly equilibrium with n firms in the static case, finding that both aggregate and individual R&D efforts are non-monotone (first increasing and then decreasing) w.r.t. n. Under this respect, the static approach proves to fall short of appropriately accounting for the inherently dynamic nature of research and development which is not captured by multistage game modelling.

5 Concluding remarks

We have analyzed dynamic R&D investments for cost-reducing innovation in a Cournot oligopoly in order to evaluate the influence of market structure on R&D incentives.

The setup employed in the present paper is a dynamic version of the static game examined in d'Aspremont and Jacquemin (1988). Two features are worth stressing. First, the game is state-redundant, so that the open-loop solution is Markovian. Second, an Arrowian conclusion is established, as the aggregate R&D effort is everywhere increasing in the number of firms. The drastic difference between our results and the ambiguous conclusions drawn from the static model relies upon smoothing the investment efforts over a long time horizon, a perspective which is ruled out by definition in a static setting.

[5] For a similar result concerning the incentives towards R&D for product innovation, see Cellini and Lambertini (2002, 2004).

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