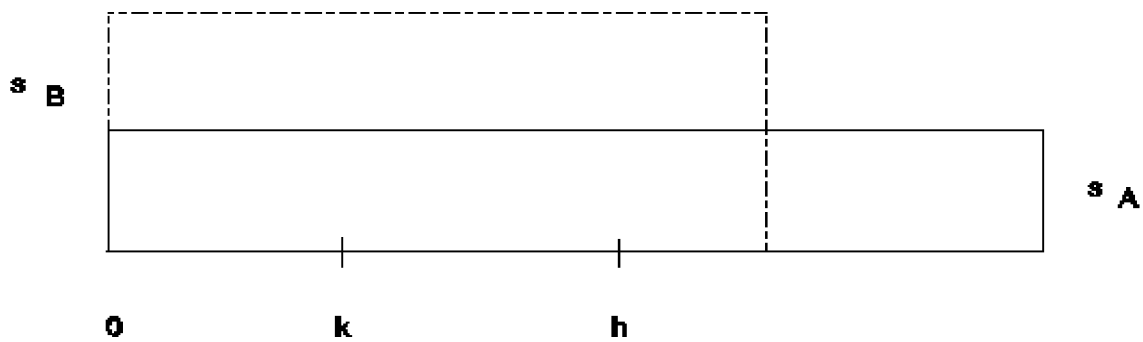**Figure 1.** *One-way trade*

Figure 2. *Two-way trade*

While the upper bound of the interval in (a10), i.e., r_2, is always greater than two for all positive values of n, the lower bound, r_1, lies in the interval $[(3 - \sqrt{13})/2, (9 - \sqrt{73})/2]$, for $n \in [1/2, 2]$. As a consequence, provided $n > r$ and $r \in \,]0, 1]$ if $n > 1$, as far as the analysis carried out in the paper is concerned, the above condition must be considered as satisfied for $r \in \,]r_1, 1]$.

Appendix C. Marginal willingness to pay of the indifferent consumers under one and two-way trade

Provided firms do not modify their respective qualities after the opening of trade, the locations of the consumers who are indifferent (i) between the two varieties, and (ii) between buying the low-quality good or not buying at all, are not invariant with respect to the kind of trade observed, since prices are different under one and two-way trade. The two values of the marginal willingness to pay identifying these consumers are h and k. Under one-way trade, they correspond to:

$$h^{1w} = \frac{\overline{\theta}_A(2n - 2r + 2n^2r - r^2 - nr^2)}{4n - 4r + 4n^2r - nr^2}; \quad k^{1w} = \frac{\overline{\theta}_A r(n + 2)(n - r)}{4n - 4r + 4n^2r - nr^2}. \tag{a8}$$

Under two way trade, they are

$$h^{2w} = \frac{\overline{\theta}_A r(n + 1)(2n - r)}{(4n - r)(nr + 1)}; \quad k^{2w} = \frac{\overline{\theta}_A r(n + 1)(n - r)}{(4n - r)(nr + 1)}. \tag{a9}$$

It immediately appears that $h^{2w} > k^{2w}$ for all admissible value of parameters. Furthermore, it is quickly verified that

$$\overline{\theta}_B > h^{2w} \quad iff \quad r \in \,]r_1, r_2[, \tag{a10}$$

where

$$r_1 = \frac{4n + 1 - \sqrt{16n^2 + 9}}{2}, \quad r_2 = \frac{4n + 1 + \sqrt{16n^2 + 9}}{2}. \tag{a11}$$

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Mussa and Rosen (1978) and Itoh (1983).

Appendix B. Social welfare under free trade

The levels of social welfare in the two countries under one and two-way trade are the following:

i) One-way trade

$$SW_A^{1w} = \overline{\theta}_A^4 nr^2(32n^3 - 48n^2r + 64n^4r - 16n^2r^2 - 56n^3r^2 + 32n^5r^2 + 16r^3 + 32nr^3 + 8n^2r^3$$
$$-4n^4r^3 - 16r^4 - 20nr^4 - n^3r^4 + 4r^5)/(64t(4n - 4r + 4n^2r - nr^2)^2); \qquad (a4$$

$$SW_B^{1w} = \overline{\theta}_A^4 r^4(32n^2 + 16n^3 + 12n^4 - 64nr - 32n^2r + 40n^3r + 8n^5r + 32r^2 + 16nr^2 - 60n^2r^2 + 8n^3r^2$$
$$+40n^4r^2 + 8nr^3 + 40n^4r^2 + 8nr^3 - 8n^2r^3 - 24n^3r^3 + 3n^2r^4)/(64t(4n - 4r + 4n^2r - nr^2)^2); \quad (a5$$

ii) Two-way trade

$$SW_A^{2w} = \overline{\theta}_A^4 n^2r^2(48n^2 + 8nr + 64n^3r - 29r^2 + 16nr^2 + 48n^2r^2 + 32n^3r^2 + 32n^4r^2 - 16r^3$$
$$-70nr^3 - 40n^2r^3 - 4n^3r^3 + 12r^4 + 8nr^4 - n^2r^4)/(64t(4n - r)^2(nr + 1)^2); \qquad (a6$$

$$SW_B^{2w} = \overline{\theta}_A^4 r^3(16n^3 - 32n^4 + 16n^5 + 12n^2r + 40n^3r + 44n^4r - 64n^5r - 8n^2r^2 - 4n^3r^2 + 96n^4r^2$$
$$+75n^5r^2 - r^3 - 32n^3r^3 - 56n^4r^3 - 2nr^4 + 12n^3r^4 - n^2r^5)/(64t(4n - r)^2(nr + 1)^2). \qquad (a7$$

Appendix

Appendix A. The behaviour of the social planner

A social planner sets both price (or quantity) and quality in order to maximize social welfare, defined as the sum of producer and consumer surpluses:

$$SW = \pi + CS = px - tq^2 + s \int_{g}^{\bar{\theta}} (\theta q - p)d\theta, \qquad (a1$$

where $g = p/q$. Differentiating (a1) w.r.t. p and q, one gets:

$$p^{sp} = 0, \quad q^{sp} = \frac{\bar{\theta}^2 s}{4t}. \qquad (a2$$

Substituting and simplifying,

$$SW^{sp} = \frac{\bar{\theta}^4 s^2}{16t}, \quad \pi^{sp} = -\frac{\bar{\theta}^4 s^2}{16t}, \quad CS^{sp} = \frac{\bar{\theta}^4 s^2}{8t}, \qquad (a3$$

while the equilibrium quantity amounts to $x^{sp} = \bar{\theta}s$. These results imply that the social planner supplies a quality that is twice as high as that of the profit-seeking monopolist, and sets price equal to marginal cost, serving the whole population of consumers, instead of the richer half, as the profit-maximizing monopolist would do. The divergence between a profit-maximizing and a welfare-maximizing monopolist can also emerge when production involves variable instead of fixed costs. The monopolist's inefficiency under this respect has received wide attention in the existing literature. The main references are Spence (1975) and Sheshinski (1976),

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Krishna, K., 1990, Protection and the Product Line: Monopoly and Product Quality, *International Economic Review* 9, 469-479.

Krugman, P., 1979, Increasing Returns, Monopolistic Competition and International Trade, *Journal of International Economics* 9, 469-479.

Lambertini, L. and G. Rossini, 1994, Trade and Welfare under Vertical Differentiation, SPES working paper, University of Bologna and University of Nottingham.

Lancaster, K., 1979, *Variety, Equity and Efficiency* (Columbia University Press, New York).

firm in the richer market while keeping her monopolistic position at home.

The present analysis can be extended and amended under several aspects. First, it has been carried out under the hypothesis that firms set the quality of their respective products under autarky, so that after trade liberalization they can only adjust prices. In a richer model, this could be considered as the *short run* or *impact effect* of trade liberalization, letting firms reoptimize with respect to quality thereafter.[8] Alternatively, if one prefers to maintain the assumption that quality must be set once and for all due to the existence of sunk costs, then it can be figured that firms choose quality under autarky, anticipating that at some date trade shall open, so that they set quality in order to maximize a discounted flow of profits over a time interval that stretches beyond the time at which liberalization occurs. This would properly embed the analysis in a dynamic perspective. Finally, the general setting presented here opens the way to the analysis of strategic trade policy by the governments of the countries involved. Some instances of the effects exerted by tariffs and quotas are already described by Krishna (1987, 1990) and Lambertini and Rossini (1994), where it is shown that the introduction of a tariff on imports by the rich country may benefit both firms and increase both countries' welfare. The issue of export rivalry on the world market between firms operating in countries characterized by different levels of economic development is tackled by Chang and Kim (1989) and Chang and Chen (1994). In these two papers, the firm operating in the developed country is appointed the Stackelberg leadership, and the follower relies on an imported key input which is needed for the production of a low-quality good that is exported to the world market. The authors establish that the government of the developing country should either introduce a tariff on the input imports or tax its final good exports.

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8. This is done by Motta, Thisse and Cabrales (1995) by introducing a convex adjustment cost which is completely absent under autarky. However, this is not fully satisfactory, since it seems to imply that firms face two different technologies under the two market regimes.

high-quality firm in country B, the low-quality firm's home market, adds to the loss due to the competitive regime associated with trade, so that firm B always prefers one-way trade.

The opposite preferences obviously characterize consumers living in both countries, since $\Delta CS_i^{21w} = CS_i^{2w} - CS_i^{1w}$ is always positive. As for social welfare, while $\Delta SW_A^{21w} = SW_A^{2w} - SW_A^{1w}$ is always positive, $\Delta SW_B^{21w} = SW_B^{2w} - SW_B^{1w}$ is positive for all values of r if the two countries have the sime size ($n=1$), while, if $n=2$, two-way trade is socially preferable to one-way trade only if country B is sufficiently rich, and precisely for $r>0.49218$. Otherwise, the loss suffered by the low-quality firm outweighs the gain in terms of consumer surplus. Accordingly, I can finally state

CLAIM 10. Two-way trade is preferable to one-way trade from the consumers' viewpoint. The same applies to social welfare in the rich country, while it holds for the poor country if the latter is not excessively poor or small as compared to the rich country.

9. Conclusions and suggestions for future research

I have investigated the issue of free trade in vertically differentiated goods between two countries characterized by different dimensions and income distributions, in a framework suitable to describe North-South trade.

Several results have been derived. First, according to the relative size as well as wealth of their respective domestic markets, firms may have conflicting interests as for (i) the opening of trade, be that one or two-way, as against the autarkic *status quo ante*; and (ii) one vs two-way trade. The conclusions that the model suggests in terms of consumer surplus and social welfare are rather clearcut. Under both perspectives, trade is generally preferred to autarky and two-way trade is preferred to one-way trade, since the former implies that duopolistic competition extends to both countries, while in the latter the low-quality firms competes against the high-quality

meaningful only when the two country are of about the same size ($n=1$) or the rich country is larger than the poor country ($n=2$).

As for prices' and quantities' behaviour in the two settings, I can state the following:

CLAIM 8. Both prices are lower under two-way trade than under one-way trade. The quantity sold by the high-quality firm is higher under two-way trade, while that sold by the rival is higher under one-way trade.

Again, define

$$\Delta p_i^{21w} = p_i^{2w} - p_i^{1w}; \quad \Delta x_i^{21w} = x_i^{2w} - x_i^{1w}; \quad i = A, B. \tag{44}$$

One quickly checks that $\Delta p_i^{21w} \leq 0$ for both firms, while $\Delta x_A^{21w} > 0$ and $\Delta x_B^{21w} < 0$ over the entire range of parameter r.

Furthermore, as far as the low-quality firm's performance is concerned, the following holds:

CLAIM 9. The low-quality firm is always better off under one-way trade than under two-way trade.

This obtains by checking that

$$\Delta \pi_B^{21w} = \pi_B^{2w} - \pi_B^{1w} < 0 \tag{45}$$

for all admissible values of r. Under two-way trade, the loss due to the competition by the

$$\Delta\pi_B^{2wm} > 0 \quad iff \quad r \in \,]0.39795, 0.471026[. \tag{43"}$$

Condition (43') implies that the high-quality firm may profit from two-way trade roughly over the upper half of the admissible range for r, while condition (43") says that the same happens to the low-quality firm if her own domestic consumers are sufficiently poor as compared to foreign consumers. The effect involving firm A can be given the following explanation: if the poor country is smaller but not significantly poorer than the rich country, when trade opens firm A, who sells a high-quality good, profits from a non-trivial increase in the demand for her product by high-income consumers living abroad, while exactly the opposite happens to firm B. As for the situation described by (43"), it may be thought to work like this: if the poor country is both appreciably smaller and significantly poorer than the rich country, when trade opens the low-quality firm's profit increases because she is now able to serve a large number of relatively richer consumers who cannot though afford to buy the high-quality good being sold by firm A. On the contrary, the latter looses from trade because the increase in the overall market size and demand is not sufficient to make up for the decrease in profits due to competition.

8. Two vs one-way trade

The comparison between two-way trade and one-way trade remains to be carried out. Obviously, it shall be limited to the restricted range of parameters where both kinds of trade are possible.

It has already been established in section 5 that the arising of one type of trade or the other depends upon the relative performance of the high-quality firm in these two settings. In particular, we know that if the rich country is considerably smaller than the poor country ($n=1/2$), two-way trade shall not occur. This implies that the comparison between one and two-way trade is

and consumer surplus, and thus also on social welfare, should be clearcut. Actually, this is not exactly the case, at least as far as firms' profits are concerned. These results are summarized in the following:

CLAIM 6. Two-way trade unambiguously decreases both firms' profits if the two countries have the same size. If the rich country is larger than the poor one, then trade increases the profit of the high-quality firm if the poor country is sufficiently rich, while it increases the profit of the low-quality firm under the opposite circumstances.

CLAIM 7. Two-way trade increases both consumer surplus and social welfare in both countries as compared to autarky.

I take into account firstly the case where $n=1$, i.e., where both countries have the same overall dimension, so that any market size-effect is ruled out. In such a case, it can be verified that

$$\Delta\pi_i^{2wm} = \pi_i^{2w} - \pi_i^m < 0; \quad \Delta CS_i^{2wm} = CS_i^{2w} - CS_i^m > 0; \quad \Delta SW_i^{2wm} = SW_i^{2w} - SW_i^m > 0, \quad i = A, B, \text{(42}$$

for all admissible values of r. The results displayed in (42) are fully in line with intuition. Things go a slightly different way if the rich country is larger than the poor country, e.g., if $n=2$. In such a case, although one reaches the same conclusions as above as for consumer surplus and social welfare, two-way trade happens to increase both firms' profits in two distinct parameter ranges:

$$\Delta\pi_A^{2wm} > 0 \quad iff \quad r \in]0.5877, 0.938364[; \tag{43'}$$

the whole admissible range of r in both countries. Analogously, it increases consumer surplus in the rich country. The same holds for consumer surplus in country B, except when $n=1/2$. In such a case, $\Delta CS_B^{1wm} > 0$ if $r \in]2 - \sqrt{3}, 0.341325[$. Notice that, since ΔWS_B^{1wm} is always positive, any loss suffered by consumers is always more than compensated by the increase in firm B's profit.

7. Two-way trade vs autarky

I shall now focus on the setting where both varieties are traded. The procedure and methods I shall adopt here are completely analogous to those explained in the previous section, so I can proceed rather quickly. The parameter ranges within which the comparison between two-way trade and autarky makes sense are: (i) if $n=1$, $r \in]0.328173, 1]$; (ii) if $n=2$, $r \in]0.39795, 1]$. Notice that if $n=1/2$, in the viable range of parameters only one-way trade may occur (see section 5). As before, I start by treating prices and quantities.

CLAIM 5. Under two-way trade, prices are always lower and quantities are always larger than under autarky.

Define:

$$\Delta p_i^{2wm} = p_i^{2w} - p_i^{m}; \quad \Delta x_i^{2wm} = x_i^{2w} - x_i^{m}; \quad i = A, B, \tag{41}$$

as the differences between two-way trade and autarky, as far as prices and quantities are concerned. It turns out that $\Delta p_i^{2wm} < 0$ and $\Delta x_i^{2wm} > 0$ for both countries over the whole admissible range of parameters. This leads one to think that the consequences of two-way trade on producer

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convex in r over the relevant range.

Focus now on profits. Trade exerts opposite effects on the two firms' performances, as stated in

CLAIM 3. The opening of trade decreases the profit of the high-quality firm while it increases that of the low-quality firm.

Again, define

$$\Delta\pi_A^{1wm} = \pi_A^{1w} - \pi_A^m, \quad \Delta\pi_B^{1wm} = \pi_B^{1w} - \pi_B^m. \tag{40}$$

It appears that $\Delta\pi_A^{1wm}$ is always negative. This result is intuitive and needs no further comments. I can only add that the decrease in firm A's profit after trade liberalization is increasing (in absolute value) and concave in r, i.e., it becomes larger at a decreasing rate as the maximum willingness to pay of the poor country gets closer to that of the rich country. This happens because, as r increases, the varieties offered by the two firms becomes more similar, enhancing thus price competition. As for $\Delta\pi_B^{1wm}$, it is always positive, increasing and convex in r.

The consequences of free trade on consumer surplus and total welfare in the two countries remain to be described.

CLAIM 4. Trade liberalization increases both consumer surplus and social welfare in the rich country. The same generally holds for the poor country as well, with the exception that when the latter is larger than the rich country, consumer surplus may be lower than in autarky.

The relevant magnitudes, ΔCS_i^{1wm} and ΔSW_i^{1wm}, $i=A,B$, are defined according to the same criteria adopted above. Trade, if only one-way, increases welfare as compared to autarky over

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alternatively.[7]

To begin with, consider prices and quantities. The consequences of one-way trade on these magnitudes is summarized by

CLAIM 2. While trade liberalization unambiguously lowers the price of the high-quality good, the price of the low-quality good may change either way. Both firms sell larger quantities after the opening of trade.

Define the following differences:

$$\Delta p_A^{1wm} = p_A^{1w} - p_A^m, \quad \Delta p_B^{1wm} = p_B^{1w} - p_B^m; \tag{38}$$

$$\Delta x_A^{1wm} = x_A^{1w} - x_A^m, \quad \Delta x_B^{1wm} = x_B^{1w} - x_B^m. \tag{39}$$

It is easy to verify that Δp_A^{1wm} is always negative in the relevant range, independently of the relative size of the two countries. As it could be expected from the outset, the competition implicit in the opening of trade lowers the price of the high-quality good. A slightly different story must be told about the price of the low-quality good. In fact, it turns out that if $n=1/2$, Δp_B^{1wm} is positive for all $r \in \ ]0, 0.267952[$. This means that the price of the low-quality good is bound to increase after trade liberalization if country B is significantly poorer than country A. In the other two cases ($n=1$ and $n=2$), Δp_B^{1wm} is positive over the entire parameter range. As for quantities, a quick exam suffices to conclude that both differences in (39) are positive, increasing and

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7. However, this need will emerge only under a few circumstances. Fortunately, in most cases no numerical simulations are required in order to establish the results I am going to expose in what follows.

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it can be established that (i) if $n=1/2$, $\Delta\pi_A^{21w} < 0$ $\forall r \in$ $]0, 1/2[$; (ii) if $n=1$, $\Delta\pi_A^{21w} > 0$ $\forall r \in$ $]0.328173, 0.913517[$; and (iii) if $n=2$, $\Delta\pi_A^{21w} > 0$ $\forall r \in$ $]0.39795, 1[$. Thus, if $n=1/2$ and consequently r lies in the interval $]0,1/2]$, i.e., the rich country is half the size of the poor country, but the richest consumers of the latter have a marginal willingness to pay that is at most half the corresponding marginal willingness to pay of the richest consumers in the rich country, then in principle two way trade is possible for $r \in$ $]0.341325, 1/2]$, but firm A is better off under one-way trade, so that she will decide not to set her price low enough to allow for the high-quality good to be purchased by consumers living in country B, since their number is not sufficient to compensate for the loss due to the decrease in the price charged by firm A and her consequent inability to appropriate a large share of consumer surplus in her domestic market.[6] *Mutatis mutandis*, for the cases where $n=1$ or $n=2$, it can be claimed one-way trade shall occur for $r \in$ $]0, 0.328173[$, or $r \in$ $]0, 0.39795[$, respectively. Notice that, since in such cases the parameter range where one-way trade is *possible* overlaps that where firm A prefers not to export to country B, the upper bounds of the intervals that I shall adopt in the following sections are set according to the optimal behaviour of the high-quality firm.

6. One-way trade vs autarky

In this section, I shall proceed to the comparative evaluation of the equilibrium values of individual and collective surpluses as well as the other relevant magnitudes under autarky and one-way trade. Obviously, the comparison will be carried out taking into account that r must lie within the relevant ranges established in the previous section, for n equal to 1/2, 1 or 2,

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6. This is clearly due to the fact that there is no market segmentation, i.e., firms are assumed to be unable to price discriminate between consumers in the two countries. For an analysis of such a setting, though exclusively carried out under two-way trade, see Motta, Thisse and Cabrales (1995).

CLAIM 1. If the rich country is too small as compared to the poor country, one-way trade occurs although two-way trade would be possible. Otherwise, if the dimension of the two countries is the same, or the rich country is larger than the poor country, two-way trade generally obtains, while one-way trade is observed only if the poor country is considerably poorer than the rich country.

Consider first one-way trade. As for the first point, notice that in such a case the following sequence of inequalities must hold:[5]

$$h > \overline{\theta}_B > k, \quad \overline{\theta}_B = r\overline{\theta}_A. \tag{36}$$

It can be easily established that $\overline{\theta}_B > k$ for all admissible values of r and n. As for the first inequality in (36), it can be established through numerical calculations that (i) if $n=1/2$, $h > \overline{\theta}_B$ $\forall r \in]0, 0.341325[$; (ii) if $n=1$, $h > \overline{\theta}_B$ $\forall r \in]0, 0.428007[$; and (iii) if $n=2$, $h > \overline{\theta}_B$ $\forall r \in]0, 0.466888[$. All the claims that can be found in the remainder of the paper are based on numerical simulations carried out by using the same three values adopted here for parameter n. This amounts to investigating three major cases, namely those in which country A is (i) half the size of country B; (ii) as large as country B; and (iii) twice as large as country B.

As for two-way trade, it turns out that the inequalities needed for this kind of trade to arise, i.e., $\overline{\theta}_B > h > k$, are satisfied for all admissible values of r (see Appendix C).

Consider now the preferences of firm A as for the kind of trade that may arise. By evaluating the sign of

$$\Delta\pi_A^{21w} = \pi_A^{2w} - \pi_A^{1w}, \tag{37}$$

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5. The equilibrium values of h and k for both one and two-way trade are in Appendix C.

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$$\pi_B^{2w} = \frac{\overline{\theta}_A^4 r^4 (8n^4 + 16n^3 - 8n^2 - 16n^2 r - 24n^3 r - r^2 + 8n^2 r^2 - nr^3)}{64t(nr+1)(4n-r)^2}. \tag{33}$$

Furthermore, consumer surplus in the two countries amounts to

$$CS_A^{2w} = \overline{\theta}_A^4 n^2 r^2 (16n^2 - 8nr - 16n^2 r + 16n^3 r + r^2 + 20nr^2 + 8n^2 r^2 - 8n^3 r^2 + 4n^4 r^2$$
$$-4r^3 - 9nr^3 + 6n^2 r^3 + 5n^3 r^3 + 3r^4 + 2nr^4)/(16t(nr+1)^2(4n-r)^2); \tag{34}$$

$$CS_B^{2w} = \overline{\theta}_A^4 n^2 r^3 (4n - 8n^2 + 4n^3 + 5r + 6nr + 9n^2 r - 16n^3 r + 2r^2 + 7nr^2 + 20n^2 r^2 + 16n^3 r^2$$
$$-2r^3 - 4nr^3 - 8n^2 r^3 + nr^4)/(16t(nr+1)^2(4n-r)^2). \tag{35}$$

Finally, the social welfare levels in the two countries, SW_A^{2w} and SW_B^{2w}, can be obtained by adding (34) to (32) and (35) to (33), respectively. Both magnitudes can be found in Appendix B.

5. One or two-way trade?

Before proceeding to the comparison of the results observed under one and two-way trade with what happens under autarky, it must be firstly established what set of relationships between the relevant parameters of the model, i.e., n and r, can lead to one kind of trade or the other; and it must be also taken into account that the choice between the two alternative trade regimes may well depend upon the performance of the high-quality firm, who can decide whether to export or not to the poor country by comparing the profits she can gain in the two settings. The factors determining the pattern of trade and the arising of a specific kind of trade are summarized in the following

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Superscript *2w* stands for *two-way trade*. Solving the system (26-27), one gets the Nash equilibrium prices:

$$p_A^{2w} = \frac{\overline{\theta}_A^3 nr^2(n+1)(n-r)}{4t(4n-r)(nr+1)};$$
(28

$$p_B^{2w} = \frac{\overline{\theta}_A^3 r^3(n+1)(r-n)}{8t(4n-r)(nr+1)}.$$
(29

As for equilibrium quantities for the two firms, they turn out to be the following:

$$x_A^{2w} = \frac{2\overline{\theta}_A nr(n+1)}{4n-r};$$
(30

$$x_B^{2w} = \frac{\overline{\theta}_A nr(n+1)}{4n-r},$$
(31

so that $x_B^{2w} = x_A^{2w}/2$, i.e., the high quality firm located in country *A* sells twice as much as the low-quality firm located in country *B*.

By substituting prices (28-29) into the objective functions and simplifying, the equilibrium profits under two-way trade obtain:

$$\pi_A^{2w} = \frac{\overline{\theta}_A^4 n^2 r^2(40nr - 16n^2 + 64n^2r + 16n^3r - 33r^2 - 64nr^2 - 24n^2r^2 - nr^3)}{64t(nr+1)(4n-r)^2};$$
(32

Consumer surplus in the two countries is given by:

$$CS_A^{1w} = \overline{\theta}_A^4 nr^2(4n^3 + 4n^2r + 8n^4r - 20nr^2 - 12n^2r^2 + 8n^3r^2 + 4n^5r^2 + 12r^3 + 24nr^3 - 12n^2r^3$$

$$-8n^3r^3 + 5n^4r^3 - 12r^4 - 7nr^4 + 8n^2r^4 + 3r^5)/(16t(4n - 4r + 4n^2r - nr^2)^2); \tag{24}$$

$$CS_B^{1w} = \frac{\overline{\theta}_A^4 r^4(n^2 - 2n + 2r - nr - 4n^2r + nr^2)^2}{16t(4n - 4r + 4n^2r - nr^2)^2}. \tag{25}$$

The equilibrium values of social welfare in the two countries, SW_A^{1w} and SW_B^{1w}, can be obtained by summing (22) to (24) and (23) to (25), respectively. The expressions for SW_A^{1w} and SW_B^{1w} are displayed in Appendix B.

4.2. Two-way trade

Consider now the setting in which both varieties are traded, i.e., not only the low-quality good produced in country B is exported to country A, but also the high-quality good produced in country A is made available for purchase by consumers living in country B. Demands are now defined as in expression (11) above. As in the case of one-way trade previously treated, after trade liberalization firms noncooperatively and simultaneously set prices. The FOCs w.r.t. prices are:

$$\frac{\partial \pi_A^{2w}}{\partial p_A} = \frac{\overline{\theta}_A^3 r^2(r + nr - n^2 - n) - 8tp_A + 16tp_B + 16tnrp_A - 8tnrp_B}{\overline{\theta}_A^2 r(r - n)} = 0; \tag{26}$$

$$\frac{\partial \pi_B^{2w}}{\partial p_B} = \frac{8t(nr + 1)(2np_B - rp_A)}{\overline{\theta}_A^2 r^2(r - n)} = 0. \tag{27}$$

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where the superscript *1w* stands for *one-way trade*. By solving the system (16-17), one gets the following equilibrium prices:

$$p_A^{1w} = \frac{r\overline{\theta}_A^2(n-r)(2n-2r+2n^2r+r^2)}{8t(4n-4r+4n^2r-nr^2)};$$
(18

$$p_B^{1w} = \frac{\overline{\theta}_A^3 r^3(n+2)(n-r)}{8t(4n-4r+4n^2r-nr^2)}.$$
(19

Thus, the equilibrium quantities for the two goods can be easily calculated:

$$x_A^{1w} = \frac{\overline{\theta}_A nr(2n-2r+2n^2r+r^2)}{4n-4r+4n^2r-nr^2};$$
(20

$$x_B^{1w} = \frac{\overline{\theta}_A r(2+n)(n-r+n^2r)}{4n-4r+4n^2r-nr^2}.$$
(21

I can now focus on the distribution of surplus between producers and consumers at equilibrium, in each country. Equilibrium profits are:

$$\pi_A^{1w} = -\overline{\theta}_A^4 nr^2(64n^2r - 16n^3 - 32n^4r - 80nr^2 - 32n^2r^2 + 88n^3r^2 - 16n^5r^2$$

$$+32r^3 + 64nr^3 - 56n^2r^3 - 32n^3r^3 + 24n^4r^3 - 32r^4 - 8nr^4 + 32n^2r^4 + n^3r^4$$

$$+8r^5)/(64t(4n-4r+4n^2r-nr^2)^2);$$
(22

$$\pi_B^{1w} = -\overline{\theta}_A^4 r^4(32nr - 16n^2 - 32n^3 - 8n^4 + 64n^2r + 16n^3r - 32n^4r - 8n^5r - 16r^2 - 32nr^2$$

$$-16n^2r^2 + 32n^3r^2 + 24n^4r^2 + 8nr^3 - 8n^3r^3 + n^2r^4)/(64t(4n-4r+4n^2r-nr^2)^2).$$
(23

country A is at least as large as country B, and $r \in]0, n]$ if instead the richer country is smaller than the poorer one.[4]

The two profit functions appear now as follows:

$$\pi_A^d = p_A x_A - tq_A^2; \quad \pi_B^d = p_B x_B - tq_B^2, \tag{15}$$

where the superscript d stands for *duopoly*, and market demands x_A and x_B are defined as in (10) if one-way trade occurs, or alternatively as in (11) if two-way trade is observed.

4.1. One-way trade

Assume now that trade liberalization leads to a one-way trade from the poor to the rich country, i.e., the low-quality good is exported from the poor country (B) to the rich country (A), while the high-quality good produced in country A is not traded. Market demands are thus given by the expressions in (10), and after the opening of trade, firms simultaneously compete in prices. The first order conditions (FOCs) for profit maximization are:

$$\frac{\partial \pi_A^{1w}}{\partial p_A} = \frac{n(\overline{\theta}_A^3 r^2 - \overline{\theta}_A^3 rn + 16tp_A - 8tp_B)}{\overline{\theta}_A^2 (r-n)} = 0; \tag{16}$$

$$\frac{\partial \pi_B^{1w}}{\partial p_B} = r\overline{\theta}_A - (1-rn)\frac{16tp_B}{r^2 \overline{\theta}_A^2} - (2p_A - p_B)\frac{8tn}{\overline{\theta}_A^2 (r-n)} = 0, \tag{17}$$

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4. Notice that these conditions are also sufficient to ensure that under autarky the profit of firm A is at least as large as the profit of firm B as described by expression (6), since $\pi_A^m \geq \pi_B^m$ if $\overline{\theta}_A^4 s_A^2 \geq \overline{\theta}_B^4 s_B^2$, which is true for all $n \geq r$.

when two-way trade obtains; h and k, identifying the marginal willingness to pay of the consumers indifferent between the two goods and between the low quality good and nothing at all, are respectively:

$$h = \frac{(p_A - p_B)}{q_A - q_B}; \quad k = \frac{p_B}{q_B}, \tag{12}$$

where both qualities are fixed at the levels chosen by each firm under autarky.

As for product quality, the conditions needed for the quality of the variety being produced in country A to be higher than that of the variety being produced in country B can be established in the following way. Without loss of generality, set $s_B=1$, and $\bar\theta_B = r\bar\theta_A$, with $r \in]0, 1]$. Accordingly, from (7) $s_A=nr$ obtains. This set of assumptions allows to reduce significantly the number of parameters involved in the model and ease calculations without prejudicing the validity of the results. Consequently, it can be stated that

$$q_A > q_B \quad iff \quad s_A \bar\theta_A^2 > r^2 \bar\theta_A^2, \tag{13}$$

i.e.,

$$q_A > q_B \quad iff \quad n > r. \tag{14}$$

In the remainder of the paper I shall assume that condition (14) holds. Provided that r cannot be greater than one, the above condition implies that I shall consider $r \in]0, 1]$ if $n \geq 1$, i.e., if

In the second, two-way trade obtains, with both qualities being purchased in both countries, giving rise to a proper intraindustry trade. This happens when the richest consumer in country B is located above the marginal willingness to pay of the consumer indifferent between buying either of the two varieties (h). This situation is described by Figure 2.

Under free trade, the firm located in the richer country (A) offers a good of higher quality as compared to the firm operating in country B (see below), so that their respective market demands can be indexed as A and B, and are now defined as follows:

$$x_A = (\overline{\theta}_A - h)s_A; \quad x_B = (h - \overline{\theta}_B)s_A + (\overline{\theta}_B - k)(s_A + s_B) \tag{10}$$

if one-way trade occurs, and

$$x_A = (\overline{\theta}_A - \overline{\theta}_B)s_A + (\overline{\theta}_B - h)(s_A + s_B); \quad x_B = (h - k)(s_a + s_B), \tag{11}$$

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$$s_A > s_B \left(\frac{\overline{\overline{\theta}}_B}{\overline{\overline{\theta}}_A} \right)^2. \tag{7}$$

Since it appears natural to think that the possibility of serving richer consumers provides an incentive to produce a good of higher quality as compared to a market where consumers are characterized by a lower marginal willingness to pay, in the next section I will specify the conditions under which the above inequality holds.

As for consumer surplus, it is defined as follows:

$$CS_i^m = s_i \int_{p/q}^{\overline{\theta}} (\theta q - p) d\theta, \tag{8}$$

while social welfare corresponds to the sum of consumer and producer surplus. Then, straightforward calculations show that consumer surplus and social welfare under autarky amount to

$$CS_i^m = \frac{\overline{\theta}_i^4 s_i^2}{64t}; \quad SW_i^m = \frac{\overline{\theta}_i^4 s_i^2}{32t}. \tag{9}$$

4. Free trade

When trade opens between the two countries, two alternative settings can emerge. In the first, one-way trade occurs, with the firm located in country B exporting to country A. This happens when the richest consumer in country B is located between the levels of marginal willingness to pay associated with the consumers who are indifferent between buying either of the two varieties (h) and between buying the low quality good or nothing (k), respectively. This situation is depicted in Figure 1.

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$$x_i = \left(\bar{\theta}_i - \frac{p_i}{q_i} \right) s_i, \quad i = A, B. \tag{4}$$

From the first order conditions for profit maximization w.r.t. quality and price, we have

$$q_i^m = \frac{\bar{\theta}_i^2 s_i}{8t}; \quad p_i^m = \frac{\bar{\theta}_i^3 s_i}{16t}, \tag{5}$$

yielding

$$x_i^m = \frac{\bar{\theta}_i s_i}{2}; \quad \pi_i^m = \frac{\bar{\theta}_i^4 s_i^2}{64t} \tag{6}$$

as the optimal quantity and maximum profit. It appears thus that the monopolist always serves the upper (or richer) half of the market.[3] Besides, all equilibrium magnitudes increase as $\bar{\theta}_i$ and s_i increase. This implies that the monopolist will find it advantageous to improve product quality as the marginal willingness to pay of the richest consumer increases. Analogously, she will increase quality as consumer density increases, provided that the burden of any increase in quality falls upon fixed costs only. These linkages between quality and marginal willingness to pay as well as consumer density entail that the higher quality good is not necessarily being produced in the richer country, unless it the following inequality is met:

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3. A social planner aiming at the maximization of social welfare would supply a higher quality as compared to the profit-seeking monopolist. Furthermore, the planner would price at marginal cost in order to serve all consumers. See Appendix A.

where q is the quality of the good and p the price at which it is sold.

On the supply side, one firm is active in each country under autarky, offering a good whose production requires a fixed cost which is convex in quality:

$$F = tq^2, \quad t > 0. \tag{2}$$

Variable costs are assumed away. This hypothesis may be given the following justification: quality can be thought of as the result of investments in R&D, whose size is increasing in the quality level of the good being supplied, while it is completely unrelated to the scale of production. It could easily be shown that the introduction of a constant unit variable cost would not modify significantly the results that I am going to derive in the following sections. Consequently, it can be normalised to zero without loss of generality.[2] Finally, I shall assume that fixed costs are sunk, implying that firms choose quality once and for all.

3. The autarky equilibrium

Under autarky each firm operates as a monopolist in her own market. Her obective function is

$$\pi_i^m = p_i x_i - tq_i^2, \quad i = A, B, \tag{3}$$

where x_i is market demand, defined as follows:

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2. Instead, the assumption of variable costs increasing in quality would radically change the picture. This setting is investigated in Lambertini and Rossini (1994).

to a main consideration, namely that while free trade is generally preferable to autarky from a social standpoint, the choice between one and two-way trade essentially depends on the preferences of the high-quality firm, and a proper form of intraindustry trade may benefit the high-quality firm if the income difference between the consumers in the two countries is not too wide, while is not necessarily preferred to simple one-way trade by the firm located in the poor country, that is specialized in the production of a low-quality good, unless her home market is very poor as compared to the foreign one. Neat conclusions can be reached as far as consumer surplus and social welfare are concerned. Under this respect, two-way trade is generally preferred to one-way trade by both countries.

The paper is structured in the following way. The basic model is introduced in section 2. Section 3 describes the autarkic regime. Then, sections 3 and 4 deal with the alternative settings of one-way trade and two-way trade, respectively. The issue of the choice between the two free trade regimes is tackled in section 5. Sections 6 through 8 contain a comparative evaluation of the results. Finally, section 9 provides concluding remarks and suggestions for future research.

2. The model

Label the two countries as *A* and *B*, respectively. In each country, consumers are characterized by a marginal willingness to pay for quality θ and are uniformly distributed with density s_i over the interval $[0, \overline{\theta}_i]$, $i=A,B$, with $\overline{\theta}_A \geq \overline{\theta}_B$. The latter assumption means that country *A* is at least as rich as country *B*, in that the marginal willingness to pay of the richer consumer living in country *A* is not lower than that of the richest consumer in country *B*. The global dimension of each market is given by $s_i \overline{\theta}_i$, and I assume that $s_A \overline{\theta}_A = n s_B \overline{\theta}_B$, with $n>0$, so that we shall say that country *A* is larger than country *B* if $n>1$. Each consumer buys at most one unit of the product if and only if the net surplus he gets from consumption is non negative:

$$U = \theta q - p \geq 0, \tag{1}$$

low quality goods.[1] A dramatic change in the pattern of production and trade may be observed if technical progress is faster in the South than in the North. More recently, Motta (1992) has analysed a duopolistic model of trade in vertically differentiated goods between two countries which differ in size, showing that under certain conditions the small country may loose from trade liberalization. Here I want to focus on the interplay between preferences and income on one side and vertical differentiation on the other side as determinants of intraindustry trade between two countries characterized by different income distributions as well as consumer densities, extending the analysis carried out in Lambertini and Rossini (1994). The model I adopt shares many features with the one in Motta, Thisse and Cabrales (1995), though they address a completely different question, namely, whether a country that, under autarky, produces a good whose quality is lower than that of the good produced in the other country, can catch up after trade liberalization. They show that, when trade opens, two possible equilibria may arise. In the first, the high-quality firm maintains its leadership, while in the second, leapfrogging is observed. The latter event is possible only if the initial quality gap is not excessively large. These results hold for both integrated and segmented markets.

I shall assume that the product variety offered by each firm is first determined under autarky. When trade opens, provided that quality cannot be changed due to a sunk cost, firms adjust their respective prices in order to compete in the international market. I adopt the hypothesis that markets are integrated, i.e., firms cannot price-discriminate by charging different prices in the two countries. This may be due to the possibility of arbitrage by consumers or to the existence of legal constraints. The two alternative cases of one-way and two-way trade are described. These settings are first assumed *a priori*; then, the conditions leading to the arising of one kind of trade or the other are assessed. The results obtained throughout the paper point

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1. This is also confirmed by the empirical literature available, according to which poor countries usually specialize in the production of low-quality goods. See, *inter alia*, Tharakan (1984); Tharakan, Kerstens and Glejser (1994).

1. Introduction

The role of product differentiation and different consumer preferences across countries have been first advocated as two major factors explaining intraindustry trade between developed countries by Linder (1961). He argued that the priciples governing trade in manufactured goods differ from those at the basis of trade in primary goods. While accepting the idea that trade in basic goods is determined by factor endowments, he put into question the notion that factor endowments are the main determinants of trade in manufactured items. Linder highlighted instead the role of demand, stressing that usually a large intraindustry trade occurs between developed countries which have comparable factor endowments or relatively easy access to endowments available in third countries, that according to the Heckscher-Ohlin theory we might not expect to observe. Then, a cause of trade other than factor endowment must be identified.

According to Linder, a manufactured good is produced by an entrepreneur in response to a perceived demand, which can be determined by the interaction of preferences and income. The role of preferences in explaining intraindustry trade has been largely investigated in several contributions resorting to the Chamberlinian approach (Krugman, 1979; Helpman, 1981; Markusen, 1981, to mention only a few). Lancaster (1979, 1980) has introduced this issue in the context of the address approach. The role of income as a determinant of trade flows has been emphasized by Hunter and Markusen (1987). In a different context, Shaked and Sutton (1984) have described the effect of free trade on the extent of vertical differentiation and the equilibrium number of firms able to gain positive profits after the liberalization of trade.

The issue of North-South trade in vertically differentiated products has been focused upon by Flam and Helpman (1987). They propose a model where two countries are endowed with production technologies characterized by different levels of efficiency. The authors show that the advanced country produces the top quality goods while the other supplies and exports the

INTRAINDUSTRY TRADE

UNDER VERTICAL PRODUCT DIFFERENTIATION

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Abstract

The paper focuses on trade between two countries where a vertically differentiated commodity is produced by a single firm in each country, operating initially in autarkic conditions. It is assumed that the two countries have overlapping income distributions, giving thus rise, under certain conditions, to two-way trade, i.e., a proper intraindustry trade. It emerges that while consumers always benefit from trade, especially if two-way trade arises, firms may have conflicting preferences on the choice between (i) autarky and trade, as well as (ii) one-way trade and two-way trade.