Firms' Network Formation Through the Transmission of Heterogeneous Knowledge

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Abstract

This paper studies the evolution of a network of leading firms that are engaged in an active search to improve their technological capability through interaction with knowledge-heterogeneous firms. Through the simulation of a linear model of technological spillovers we show the emergence of paradigm setters and the impact of search routines on the system's average performance

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1 Introduction

Recent literature has highlighted the crucial importance that networks have in spreading innovation-relevant information among interacting firms and consequently foster technological efficiency spillovers. It is indeed widely recognised that firm interaction is the process that accounts for much of the learning and useful knowledge acquisition that enable firms to innovate and that eventually renders some of them technological leaders. In economies at the cutting edge of their frontier the competitive drive compels leading firms to engage in these processes lest their advantage be lost to competitors and imitators. It is also well established that because of bounded rationality, the gleaning of relevant information occurs within the confines of neighbourhoods within which networking becomes both viable and result-bearing. This fact owes to firms' limited ability to explore a given system's cognitive complexity and comprehend all the agents that inhabit it. Those firms that at any particular point in time can be reached, understood and finally exploited in terms of their spillover potential are normally only a small share of the entire firms' space. Nevertheless, an active process of searching and learning is the tool that eventually leads them to set up viable information linkages and gain technological capability. It has been persuasively argued that, thanks to this process, networks evolve: they change membership and mode of functioning. It is, therefore, their dynamics that determine a specific architecture the characteristics of which are conducive to a firm's innovative performance and, in the aggregate, to that of the economy as a whole. This quest for information is largely an adaptive, gradual process in which internal, in-house resources generating innovation-worthy knowledge are woven together with those obtained through technological spillovers proceeding from other firms.

In this context, the economy appears as a large interactive system (Kirman 1997a, 1997b). The importance of networks and their properties in the process of interaction has been highlighted in recent literature investigating technological and knowledge diffusion, see for example, Cowan and Jonard (2004, 2005), Silverberg and Verspagen (2002), Arenas et Alii, (2001, 2002). Seminal work in network connectivity and dynamics has been done by Albert and Barabasi, (2002) for an exhaustive review, Watts and Strogatz (1998), for the emergence of small worlds and Jackson (2005) and Jackson and Rogers (2005) for the economic implications. These findings have been deemed as quite relevant in the literature of innovation diffusion. This is because a 'Small World' architecture, as represented in a graph, compounds the benefits of a localised transmission of spillovers with those obtainable from information broadcast by relatively distant nodes (agents), enhancing the average innovative performance of an economic system. In this framework, these nodes have the role of augmenting the pool of cognitive capabilities without the excessive dispersion that would occur if relational edges were wholly random. While these are undoubtedly relevant results that shed light on network properties, it is nevertheless necessary to take into account that evolution and architecture changes come about because of the specific searching behaviour and information gathering strategies that firms implement in striving towards greater innovative prowess. Normally, in a rationally bounded world, these procedures take the form of routines, Cohen et Alii (1995), Egidi and Ricottilli (1997), that are aimed at the discovery and eventual adoption of more performing neighbours and to the dismissal of less effective ones. New edges in the firms' graph are set up thanks to this adaptive but endogenous process. It can be shown that in a framework in which interaction takes place unfettered, if routines are adopted merging localised and moderately frequent across-the-board search of new neighbours, paradigm setters are likely to emerge (Andergassen, Nardini and Ricottilli; ANR, 2004). The latter are defined as firms whose technological features set a paradigm for most other firms with a positive probability as they are elected as spillover providers. Thus, while small worlds are normally well distributed and scale free networks, the world of paradigm setters is a technologically hierarchical one.

In this paper, we distinguish two different but definitely complementary and overlapping ways through which searching and learning occur. The first exploits the spillover potential that lies in a firm's network and thanks to which gathering innovation-useful information is actually possible. The second rests with the autonomous capacity that a firm possesses in order to carry out in-house innovative research. While these two searching processes not only coexist but are also reciprocally sustaining, we find it expedient to separate them by integrating a knowledge diffusion mechanism that propagates technological capabilities with an independent stochastic process capturing innovation arrivals due to internal R.&D. A network's evolution depends on how firms assess their performance in terms of innovation-enabling spillovers. In a bounded rationality framework, firms normally explore a limited part of the firms' space and require a protocol to target their information gathering efforts. The paper addresses this issue by designing a routinised behaviour according to which firms periodically reshape the neighbourhood that they observe to glean information by reassessing other firms' contributions to their own capability. The way the specific neighbour-choosing routine is accordingly organised determines in a significant way firms' average innovative capability. This feature is modelled by changing the span of network observation from a very broad setting, the whole economy, to a very narrow one, namely the most proximate neighbourhood membership. As a result of the structure of the model presented in the next section, there are two distinct but to some extent overlapping neighbourhoods which are relevant for firms' interaction. The first is the neighbourhood whose members are observed by each firm and from which capability contributions are obtained. We term this neighbourhood *inward.* The second is the one made up by a firm's observers, i.e. by firms observing and learning from it: it evolves as an active search for new inward members is carried out. We call this neighbourhood *outward.* This process of information interaction leads to the emergence of some firms that are observed by most of the remaining ones. It is they that provide some or much of the overall technological capability and that we term *paradigm setters.* We also assume that the in-house acquired capability is subject to structural shifts by means of periodic random shocks. A main feature of the model that is to follow is knowledge heterogeneity. Spillovers occur to the extent that a firm's technology can effectively be observed and learnt. Seen it from the point of view of those firms that actually provide information, a spillover depends on the share of their innovative capability that a common knowledge base permits to pass on to other firms. Technologies, however, are grounded on knowledge that is normally firm-specific rendering most economies cognitively heterogeneous, a fact that sets hurdles to the flow of information. We accordingly emphasise that, in consequence, knowledge and capability diffusion occur as an interaction of firms that possess differing broadcasting or understanding abilities.

To keep the model mathematically tractable, we formalise the features stated above by means of a linear system in which technological capabilities are made to depend on a matrix of interaction with evolving outward neighbours as well as on a vector of in-house generated knowledge. The matrix records areas of firms' different cognitive understanding , or areas of differing degrees of knowledge base translating into differing diffusion capacity. Within such areas firms feature a homogeneous diffusion coefficient. For simplicity's sake, we limit the number of these cognitive areas to just two. The model is then simulated to determine the emergent properties of neighbourhood formation and stability together with average capability. We aim to identify (i) under what conditions the emergence of technological paradigm setters occurs, (ii) the pattern of neighbourhood formation and (iii) the average relative efficiency in terms of technological capability of the economy as a whole. Our findings suggest that the type of searching routine chosen is crucial to determine the emergence of paradigm setters. Furthermore, while average innovative performance is strongly affected by the selected routine it also shown that its performance worthiness crucially depends on how knowledge heterogeneous the economy happens to be.

The plan of the paper is as follows: section two illustrates the linear model that is implemented to run simulations and the procedures utilised; section three discusses results obtained and section four draws conclusions and sets an agenda for further research.

2 Firms' Technological Capabilities and Spillover Potential

We view a firm's technological capability as the outcome of an evolutionary process owing to learning, searching and gathering of information ultimately leading to innovation. We further regard this process to be largely but not entirely explained by the interaction taking place within the system thanks to information flows that proceed from sources that are cognitively heterogeneous in relation to the searching and learning firm. In the main, this heterogeneity is a consequence of knowledge specificity and diversity and it is, therefore, both a hurdle and a challenge setting bounds to the understanding and broadcasting of relevant information. In this section, we direct our analysis to investigate firms that are assumed to be technological leaders and whose major interest lies with innovation. We, accordingly, postulate that they possess 'in house' innovative capabilities resulting from past investment and that we distinguish, in a somewhat artificial manner but useful for modelling, from those that are entirely due to spillovers. It is important to stress that the latter do not accrue effortlessly but result from a steady attempt to observe other firms of which little is known *a-priori* and whose technological characteristics and thus whose worthiness to yield useful information must be discovered by the searching activity referred to above. Technological capabilities can be viewed and measured in a way akin to the more general category of a firm's knowledge base, cognitive potential or set of skills, know-how and competencies: they can actually be modelled as either a vector arranging different indicators or more simply as a scalar compounding the whole. We choose the latter approach and propose as reference a general model that later will be simulated in a more simplified form.

Let $V_i(t)$ be the scalar that at time t designates firm i's innovative capability or, to use a term borrowed from biology, its innovative fitness. Then, $V(t)$ is the vector $V(t) = [V_i(t)]$, $i = 1, 2....J$ arraying the fitness of all firms in the economy. By $C_i(t)$ we further designate the in-house capability cumulated until time t. This magnitude is intended as an index measuring a firm's capacity to innovate thanks to cumulated knowledge achieved by means of investment specifically aimed at this purpose. Indeed, investment is necessary not only to augment it but also merely to maintain it. Considerable efforts are therefore necessary to remain on the forefront of technological prowess, efforts which need not always prove successful; they may fail entailing a fall in capability rather than an improvement. $C_i(t)$ is measured on a $0 - 1$ scale, $C_i(t) \in (0, 1)$, and it is accordingly assumed to be stochastically subject to change. $C(t)$ is the corresponding vector.

As it has been mentioned, a significant part of total technological capability is explained by interaction and therefore by the ease with which each firm is observable by other firms when broadcasting information. How much and how well a firm is capable to pass on information depend on the cognitive distance that a firm's searching must ascertain. In the end, the intensity of interaction depends crucially on cognitive proximity. In a straightforward sense, we assume that the higher is the latter, the stronger is interaction and the greater is the associated spillover. Accordingly, the ability to broadcast relevant technological information can, in general, be postulated to be measured by a basic index specific to each pair ij of firms in the economy, although simulation in the following sections will consider only broad areas of proximity to simplify, without loss of generality, the dynamics of network formation and of average performance. Let a_{ij} indicate such an index in terms of the part of each firm j's total innovative fitness that can cognitively be passed on to firm i should the latter be in a position to observe the former. The entire web of interfirm technological spillover can then be designated by a square, JxJ, matrix A, its main diagonal being made up by $a_{ii} = 0$ since no firm broadcasts information to itself. Therefore, A simply indicates the structure of cognitive proximity and thus of the technological information broadcasting capability of this economy.

Firms possess bounded rationality. This is a stylized fact that carries the important implication that the actual number of firms each can observe is a small subset of the whole. The neighbourhhood from which firms glean useful information, however, is subject to change since firms carry out a search for better alternatives. To single out neighbours better suited to pass on information when chance allows them to do so, firms resort to a routine, to a search protocol that leads them to identify new neighbourhood members, The breadth and range of this routine in terms of the sample of new firms from which to randomly choose from is a control parameter of ensuing simulations. It is, accordingly, assumed that each firm i searches among its potential information suppliers j with broadcasting capacity $a_i = (a_{ij})$, $j = 1, 2...J$, those that at each point in time it is able to choose and that it can actually observe. This choice can be formalised by introducing the adjacency matrix $B(t) = [b_{ij}(t)]$ where each $b_{ij}(t) = 1$ or 0 according to whether neighbour j has or hasn't been identified as a useful contributor. This procedure defines matrix $M(t) = (a_{ij}b_{ij}(t))$. Thus, the innovative capability that is determined by interaction can be formalised by the system $M(t)V(t)$ where actually observed firms are restricted to a limited number of neighbours. The general equation for firm i's innovative capability is[1]

$$V_i(t) = \sum_{j=1}^{J} a_{ij} b_{ij}(t) V_j(t) + C_i(t) \tag{1}$$

and the system for all firms:

$$V(t) = [I - M(t)]^{-1} C(t) \tag{2}$$

where $[I - M(t)]^{-1}$ plays the role of an endogenous matrix multiplier of in-house capabilities: different neighbourhood configurations lead to different multipliers as $M(t)$ changes thanks to active searching. In order to evaluate the impact of cognitive heterogeneity, we assume that the economy is partitioned in clusters of roughly homogeneous cognitive areas and in order to simplify the exposition we postulate that matrix $A = [a_{ij}]$ features only two different coefficients $a_2 < a_1 < 1$. This procedure defines two areas, one of cognitive similarity or of homogeneous proximity in which belonging firms broadcast and retrieve information according to parameter $a_1$and an area of homogeneous cognitive distance made up by all other firms from which information flows according to a_2.

2.1 Neighbourhood Structure

The structure through which we describe firms' innovative capability can be represented by a directed graph of J nodes each of which is connected with other nodes in two different but overlapping ways. The first is the number of connections that each firm establishes when observing other firms to determine its own innovative capability. The number of $k_{i,in} << J$ connections defines

[1] Absorbing the impact of spillovers is clearly a process that requires an adjustment in time. We simplify this problem by assuming that the time required to complete adjustment is negligible in relation to the system evolution.

for firm i the dimension of its *inward neighbourhood.* This number is substantially smaller than J since searching is costly and observation bounded . This neighbourhood can formally be defined as

$$\Gamma_i(t) = \{j : j = 1, 2...J \wedge b_{ij}(t) = 1\}$$

This is the set of firms from which at any time t firm i is able to glean innovative capability through observation and learning.

The second kind of neighbourhood, which we term *outward,* is made up for each firm j by firms that actually observe it. It results as a consequence of their networking activity. Let it be defined by:

$$\Psi_j(t) = \{i : i = 1, 2...J \wedge b_{ij}(t) = 1\}$$

Its size determines the impact of an observed firm's technological capability as it propagates throughout the economy contributing to overall performance. For this purpose, we classify the population of firms according to quantiles of their outward neighbourhood size and then define an impact factor by ranking them.

Definition 1 *Global technological paradigm setters emerge when the probability of each impact factor rank defined over the entire economy is positive. Local paradigm setters emerge when the probability of each impact factor defined over the subset of cognitively homogeneous firms is positive.*

Therefore, we consider as global paradigm setters firms included in the last quantile, that is those being or that have been observed by almost all firms in the whole economy. On the contrary, if we consider just the cognitively homogeneous clusters of firms, we may observe the emergence of local paradigm setters likewise defined as firms being observed by almost all other firms within the cluster.

2.2 Evolution

Given this neighbourhood structure, evolution owes to two basic determinants: search routines and exogenous changes of individual firms' in-house innovative capabilities. Searching, while bounded by the neighbourhood in which the firm happens to be nested, may take place according to a variety of algorithms. We have chosen one that responds to the criteria of bounded rationality and *satisficing.* We propose two versions that respectively capture a strong and a weak form of bounded rationality. In both, we first conjecture that the cardinality of Γ_i is $|\Gamma_i| = k_{i,in} \ll J$ and generate the choice of neighbours and the evolution of this neighbourhood according to the following *routine*: each firm i assesses the fitness contribution of its existing neighbours and picks out the least contributing one:

$$\gamma_i(t-1) = \arg\min_{j\in\Gamma_i(t)} [a_{ij}b_{ij}\,(t-1)\,V_j\,(t-1)]$$

We then consider two alternative procedures, a local and a global one. In the case of weak bounded rationality, the local procedure, we take each firm's neighbours' neighbours as the actual set of reachable, information-wise, technological sources. Thus, the identified firm is substituted with a new one by randomly drawing from this set. In the case of strong bounded rationality, the global procedure, we instead allow the firm to randomly draw from the remaining $J - k_{i,in} - 1$ members of the entire economy. In either case, to generate a new $\Gamma_i(t)$ it is necessary that this simple condition be satisfied:

$$V_i(t) > V_i(t-1)$$

If not, firms reinstate the neighbour they have chosen for substitution. This is because other firms's in-house capability and transmission coefficient are not observable. This procedure redefines at each time step $M(t)$ and the system then generates a new set of solutions.

Next to the dynamics generated by neighbourhood adjustment we introduce in the system the autonomous and independent dynamics involving the in-house capability $C(t)$. This vector is subject to change by a random draw of some $i \in (1, 2...J)$ and by randomly redefining the i^{th} component by a new random value $C_i(t)$ uniformly chosen between 0 and 1. These occurrences are arrivals that take place according to a predetermined mean waiting time μ.

The two crucial variables that are tuned in following simulations are (i) the neighbour searching routine designated by τ and (ii) the relative cognitive distance $\delta = \dfrac{a_2}{a_1}$, a measure of the economy's knowledge heterogeneity. Variable τ measures how local the search for a new neighbour is. $\dfrac{1}{\tau}$ is then the probability of engaging in global search and $1 - \dfrac{1}{\tau}$ that of engaging in local search. Thus, when $\tau = 1$ the search routine is always global and as τ increases searching becomes increasingly local, when $\tau \to \infty$ it is accordingly always local.

3 Simulation results

In this section we run simulations with a population $J = 64$ of firms, setting the number of inward neighbours $k_{in} = 3$. For simplicity's sake, we carry out the experiment by assuming the economy to be divided in four symmetric blocks of 16 firms that have the same knowledge base and that interact with each other by swapping spillovers by means of parameter a_1. Symmetrically, each of these four blocks is surrounded by three, equally numbered, blocks of distant knowledge firms with which interaction occurs through parameter a_2. Simulations with only two symmetric blocks have also been carried out with no appreciable qualitative difference with this more general case. More refined differentiation is clearly possible but this simple framework suffices to check for the impact of knowledge heterogeneity. To insure solutions for system (2) we constrain a_1 to be $a_1 \leq \dfrac{1}{k_{in}}$.

The results shown below are obtained by subjecting the economy to idiosyncratic shocks according to a mean waiting time that we conventionally fix at $\mu = 16$, i.e. on average every sixteen simulation periods a randomly drawn firm is shocked to determine a change in its $C_i(t)$. As shown in ANR 2004, varying μ upsets the adjustment process, slower when shorter but faster (less subject to oscillations) when longer, without major qualitative difference in performance patterns and in the emergence of paradigm setters. We keep it, therefore, constant at the specified value.

3.1 Outward Neighbourhoods and Paradigm Setters' Emergence

We wish to deal first with the pattern of interaction emerging from searching behaviour. For this purpose it is interesting to observe what connections are established between the heterogeneous parts of these economies. Figure 1 plots an interconnectivity index as a function of $\delta \in \{.6, .8, 1\}$. Since there is no qualitatively significant difference in interconnectivity for different search routines, we simply chart this figure for a $\tau = 2$ routine. This index is simply calculated as the ratio of the number of outward linkages across the a_1, a_2 divide over their number within the homogeneous a_1 area. As it is to be expected, connectivity between the two cognitive areas increases as δ rises. The more accessible and understandable the whole economy is, the greater is the number of linkages that are established between different areas. This finding implies that when δ is low, the economy effectively splits up into separate parts and firms remain bounded in their own cognitively homogeneous block.

The following figures show diagrams in which the x-axis represents quantiles of outward neighbours (i.e. the number of firms by which each firm is observed) and the y-axis represents the average percentage number of firms belonging to each quantile within the considered time span. The population of firms is made up by $J = 64$ individuals split in 16 quantiles: thus, the first quantile in each diagram includes firms having from 0 to 3 neighbours, the last one from 60 to 63 neighbours.

Figures 2-4 show quantile distributions in decreasing order of δ, that is from a completely homogeneous economy ($\delta = 1$) to a fairly heterogeneous one ($\delta = .6$). Finally, each curve in each diagram corresponds to a specific τ, that is to a particular search routine. The continuous line refers to $\tau = 7$, the dashed line to $\tau = 4$ and the dotted line to $\tau = 1$.

Data points to a rather robust pattern. Figure 2 shows an economy that is cognitively homogeneous ($\delta = 1$). When this is the case, paradigm setters definitely emerge for search routines above $\tau > 2$ and only barely for $\tau = 2$. These results had already been obtained and extensively commented upon in ANR 2004 where it was shown that only the very broad, across-the-board search ($\tau = 1$) does not give rise to paradigm setters. What engenders this result is the nature of the search protocol. When searching targets the whole economy, there exists a nearly equal probability of finding either high or mediocre performers that are just barely better than the neighbour that each firm wishes to substitute

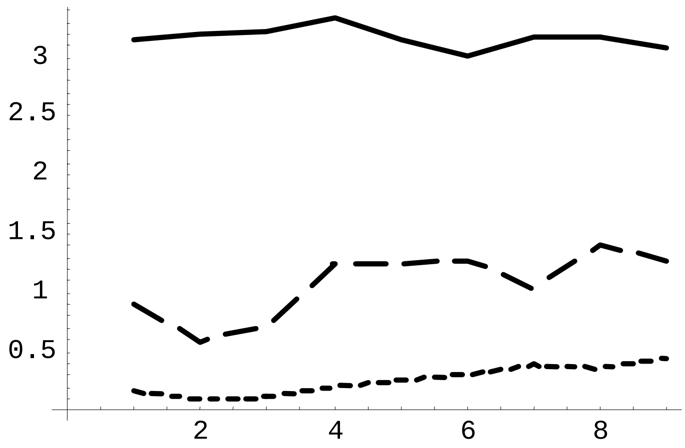

Figure 1: Interconnectivity for $\delta = 1$ (continuous line), $\delta = 0.8$ (dashed line) and $\delta = 0.6$ (dotted line).

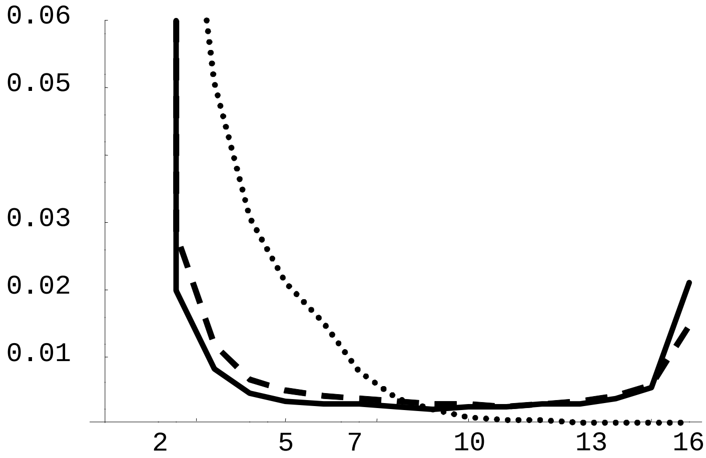

Figure 2: Quantile distribution for $\delta = 1$ and $\tau = 1$ (dotted line), $\tau = 4$ (dashed line) and $\tau = 7$ (continuous line).

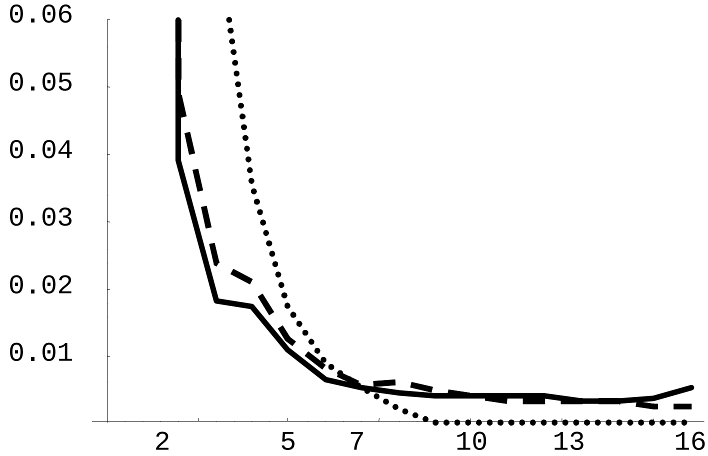

Figure 3: Quantile distribution for $\delta = 0.8$ and $\tau = 1$ (dotted line), $\tau = 4$ (dashed line) and $\tau = 7$ (continuous line).

when it gets the chance of doing so. This causes a wide dispersion of firms that are observed with no particular one emerging as a general technological leader. On the other hand, when firms pursue a highly local search, checking only their neighbours' neighbourhoods for a fruitful substitution they are quite likely to discover the same high performers that other firms are currently including or are about to include in their own set of neighbous. Once this happens, firms remain locked within a neighbourhood that almost everybody else shares. This pattern emerges more strongly the more search becomes local. In Figure 2 it is seen that the probability of paradigm setters emerging is higher the higher is τ, i.e. the more local search becomes. In this figure it also appears that frequency of firms that are or have been paradigm setters is quite large in the simulation period considered, larger than the intermediate classes implying that as soon as a high capability contributor is found a sizable band wagon effect is set off only to be frustrated by random negative shocks: the previous leaders being then replaced by others whose performance is found to be improving. Figures 3 shows the quantile distribution for $\delta = .8$ .The frequency of global paradigm setters is found to sharply decline as δ decreases, as the economy tends to be more heterogeneous, and local ones begin to appear. No global ones are found for $\tau = 1$ and 2. Figures 4 shows the quantile distribution of a highly heterogeneous economy, $\delta = .6$. In this case, global paradigm setters do not emerge at all whilst local ones appear for any of the routines taken in consideration ($\tau = 4, ..., 7$) with the notable exception of $\tau = 1$.

The increase in the interconnectivity index evidenced in Figure 1, when δ

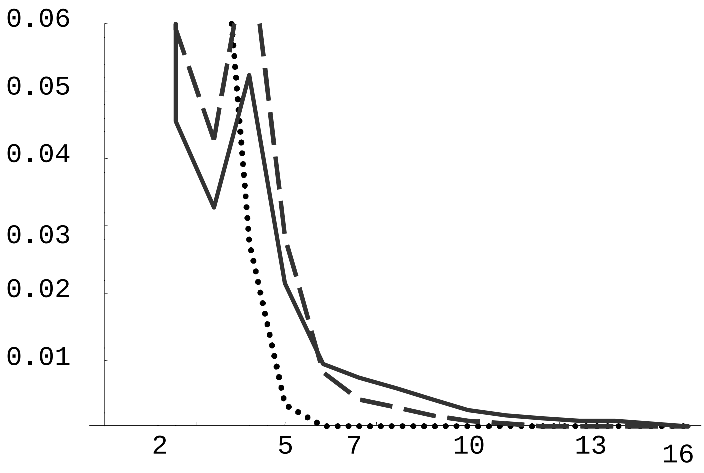

Figure 4: Quantile distribution for $\delta = 0.6$ and $\tau = 1$ (dotted line), $\tau = 4$ (dashed line) and $\tau = 7$ (continuous line).

rises from .6 to .8, is mainly due to local paradigm setters connecting to groups of firms belonging to one or two cognitively different parts of the economy. The case where firms succeed to acquire neighbours belonging to all other cognitive sectors is less frequent; thus, the probability of emergence of a global paradigm setter, though positive, is very small.

3.2 The performance pattern

The logic of this model is such that for any given (δ, τ) the average performance of the economy would gradually improve because of firms' adjusting behaviour if it were not for shocks that randomly hit with an arrival rate that is conventionally set to be $\mu = 16$. These random events clearly upset the state of the economy, either positively or negatively, giving new scope for searching better, more contributing neighbours. Performance is measured by $V(t)$, the vector determined by system of equations(2). Comparisons of these measures across economies differing on account of knowledge heterogeneity are not significant. Heterogeneity in the context of this paper takes the form of stronger or weaker information broadcasting capabilities or, viewed from the receiving firm's standpoint, susceptibility to absorb spillovers. In this sense, heterogeneity sets hurdles to the flow of informational externalities and thus to the building up of technological capabilities. A purely quantitative comparison would, then, have to discount the degree of knowledge heterogeneity. Furthermore, it is quite likely that a highly heterogeneous economy, because of high specialisation exhibit higher spillovers within the homogeneous area and possibly less without.

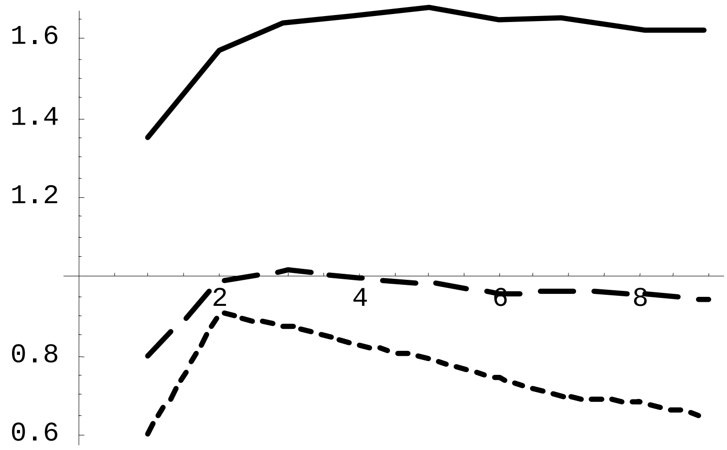

Figure 5: Performance pattern for $\delta = 1$ (continuous line), $\delta = 0.8$ (dashed line) and $\delta = 0.6$ (dotted line).

In this section, instead, we illustrate the qualitative differences affecting searching routines and their performance profile. Results show that independently of knowledge heterogeneity measured by δ, global search across the whole economy ($\tau = 1$) is not a well performing option, the same holding for very localised search routines. These results confirm the findings of ANR 2004 which dealt only with the homogeneous case ($\delta = 1$). That this should be the case for the very low and very high δ is to be expected. Firms in these two polar cases act and search in a homogeneous framework, by default in the first and by the actual situation in the second. What happens is that when the economy is very heterogeneous, firms cannot improve their capabilities by looking into the knowledge distant part of the economy; thus, they remain bounded in their own homogeneous environment. Figure 5 illustrates in detail the performance pattern. In the homogeneous case, when $\delta = 1$, performance clearly improves as the search routine rises from $\tau = 1$ to $\tau \sim 5$ where a peak appears to occur. Past this point, it slides down, with some fluctuations, as τ becomes more local. This pattern is reproduced with significant differences in economies exhibiting knowledge heterogeneity. In a $\delta = .8$ economy, a case of moderate heterogeneity, very broad, economy wide searching is very inefficient but the best performance peak is reached at a lower τ than in the previous case: $\tau \sim 3$, as shown in Figure 5. Best performance, therefore, requires a routine with more frequent attempts to draw a new neighbour by sampling the whole economy. This means that more localised searching carries a greater danger of a lock-in into a not-so-performing neighbourhood. It is also interesting to note that, as a consequence, while the slide down towards worse performing routines past the peak is similar to the homogeneous case the drop is steeper. Likewise, in a $\delta = .6$ economy, a case of high heterogeneity, performance increases as routines become more local but the peak occurs at an even lower τ, at approximately $\tau = 2$ and, furthermore, it worsens more for higher $\tau's$ as indicated by Figure 5. Thus, the more local searching becomes past the best performance peak the greater is the probability of a mediocre lock-in. Simulation results lead us to conclude that the greater is the knowledge heterogeneity, the less efficient is a very localised searching routine.

These results must be interpreted in the context of the connectivity properties that have been shown in foregoing paragraphs. Consider the $\delta = .6$ case: high heterogeneity conjures up an economy that ends up in a landscape of cognitive near islands. In this case, global paradigm setters do not emerge whatever the search routine. Instead but with the exception of $\tau = 1$, *local* paradigm setters appear within the boundaries of each cognitive environment as soon as searching becomes more local. It follows that the still frequent sampling outside one's own neighbourhood that is necessary to reach a high performance (the peak occurring at $\tau = 2$) is actually an attempt almost entirely confined within the cognitive area firms belong to. The apparent paradox according to which the best search routine is one that quite often looks across the whole of an economy that is fragmented in almost cognitively independent blocks is easily explained. It owes to the fact that since each cognitive island is relatively small, confinement in a local, neighbours'neighbourhood, search carries a relatively high probability of locking into a poor performing environment. Hence, the necessity of often broadening the searching pattern. What is at stake, in these cases, is avoiding the double constraint of a rather small pool of cognitively reachable firms and that of a narrow neighbourhood of potential performance contributors. This result is confirmed by the evidence provided by the $\delta = .8$ case. In this case, the constraint of cognitively reachable spillover contributors is partly eased. Thus, it pays firms to restrict their search routine to one that is somewhat more local (the peak is at $\tau \sim 3$). The most local of the best searching routines is, indeed, observable when the economy is cognitively homogeneous, i. e. when $\delta = 1$.

4 Conclusions

The foregoing analysis highlights the importance of searching and networking in fostering the development of technological capabilities to innovate in a context of bounded rationality and when firms' knowledge base is heterogeneous. Firms obtain information and learn when crucially placed in a cognitive and information providing neighbourhood. Technological spillovers flow and give other firms the opportunity to learn only if networks come into being to give shape to searching and make learning possible. This paper depicts this process as an effort by firms, which do carry out their own in-house innovation capability building, to seek out high performers able to contribute to the latter. Routines differ according to the breadth of this search.

The paper main findings can be summarised in the following points.

(i) Global paradigm setters emerge when the cognitive heterogeneity of the economy is not very high. They begin to emerge only for intermediate values of the measure of heterogeneity.

(ii) For high levels of cognitive heterogeneity, the economy becomes partitioned into separate parts; in each homogeneous one, local paradigm setters emerge.

(iii) Highest technological capabilities are achieved neither with a general searching routine that spans the whole economy nor with very local ones in which only neighbours' neighbourhoods are sampled. Thus, tuning short-sightedness and farsightedness improves the system's innovative efficiency. Past a given combination of the two, the system slides towards increasing mediocrity but paradigm setters emerge as a permanent and systematic feature of the economy.

(iv) How local searching should be to attain the best performance peak depends crucially on how heterogenous the knowledge base is. Seeking good contributors to technological capability is subject to the double constraint of cognitive attainability and neighbourhood narrowness. Our findings show that to ease this constraint the more heterogeneous is the firms' knowledge base, the more wide ranging should the required search routine be: firms ought to look across the whole economy more often to replace a poorly contributing neighbour. Thus, tuning far-sightedness with short-sightedness is very much a knowledge dependent task.

Further research is required to investigate the trade off between knowledge heterogeneity and average innovative performance when paradigm setters' emergence occurs. A highly heterogeneous economy, knowledge wise, is a highly specialised one in which broadly diffusing information flows and spillovers are likely to be specificity constrained. Such an economy may yet be more innovative.

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