Time Consistency in Games of Timing[1]

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Abstract

This paper tackles the issue of choosing roles in duopoly games. First, it is shown that the two necessary (and su¢cient, if both satis…ed) conditions for sequential play to emerge at equilibrium are that both leader and follower are at least weakly better o¤ than under simultaneous play. Second, by means of a two-stage game of vertical di¤erentiation, it is shown that if …rms can commit to their respective timing decisions, there may exists a case where the leader is not necessarily better o¤ than in the simultaneous equilibrium. Finally, in the absence of any commitment devices, it is proved that the timing choice can be time inconsistent if it is taken before …rms proceed to play in both stages taking place in real time.

JEL Classi…cation: D43, L13

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1 Introduction

The way …rms can be expected to conduct oligopolistic competition has represented a relevant issue in the economists' research agenda for a long time. The earliest literature in this …eld treated a relevant feature such as the choice between simultaneous and sequential moves as exogenous (Stackelberg, 1934; Fellner, 1949). Later contributions considered as a sensible approach to investigate the preferences of …rms over the distribution of roles in price or quantity games (Gal-Or, 1985; Dowrick, 1986; Boyer and Moreaux, 1987a; 1987b). The preference for leadership (respectively, followership) in quantity (price) games can be established on the basis of the slope of …rms' reaction functions or, likewise, noting that products are strategic substitutes (complements).[1] Other authors have taken into account the possibility that cost asymmetry or uncertainty may lead to Stackelberg equilibria (Ono, 1982; Albæk, 1990).

The idea that preplay communication can allow agents to play a Stackelberg equilibrium, if there exists at least one dominating the Nash equilibrium (or equilibria) of the game can be traced back to d'Aspremont and Gérard-Varet (1980). Recent literature explicitly models the strategic choice of timing, which is often possible in reality. Robson (1990a) has proposed an extended duopoly model where price competition takes place in a single period, preceded by …rms' scattered price decisions, which cannot be altered. Only Stackelberg equilibria emerge from such a game. In an in‡uential paper, Hamilton and Slutsky (1990) investigated the endogenous choice of roles, i.e., the endogenous arising of Stackelberg or Cournot equilibria, in noncooperative two-person games (typically, duopoly games), by analysing an extended game where players (say, …rms) are required to set both the actual moves or actions and the time at which such actions are to be implemented. Their approach is close in spirit to Robson's, though they also consider Cournot competition and the mixed case where one …rm sets her price and the other …rm decides her output level. When …rms choose to act at di¤erent times, sequential equilibria obtain, while if they decide to move at the same time, simultaneous Nash equilibria are observed. The choice of the timing occurs in a preplay stage which does not take place in real time, so that there is no discounting associated with waiting and payo¤s are the same whether …rms choose to move as soon as possible or they delay as long as they can. The decision to play early or at a later time is not su¢cient per se to yield sequential play, since an analogous decision taken by the rival leads to simultaneous play.

Hamilton and Slutsky (1990, HS henceforth) assume that each of the games associated with simultaneous or sequential play has a unique equilibrium. The immediate consequence of this hypothesis is a lemma according to which each …rm strictly prefers her payo¤ as a leader to that accruing to her under simulta-

[1] The concept of strategic substitutability/complementarity is due to Bulow, Geanakoplos and Klemperer (1985).

neous moves. Building on such a lemma, HS show that a Stackelberg equilibrium with sequential play is selected as a subgame perfect equilibrium of the extended game with observable delay if and only if the outcome of sequential play Pareto-dominates the outcome associated with simultaneous play (HS, 1990, Theorems III and IV). Otherwise, if …rms are better o¤ playing simultaneously rather than accepting the follower's role, the subgame perfect equilibrium involves simultaneous play (HS, 1990, Theorem II). Summing up, in a game where …rms choose a single variable, their respective reaction functions are monotone in the rival's strategic variable and a unique and distinct Nash and Stackelberg equilibria exist in the interior of the action space, (a) if both reaction function have the same slope, then alternatively (i) neither intersects the Pareto-superior set, in which case the timing game has a unique equilibrium involving simultaneous play, or (ii) both reaction function intersect the Pareto-superior set, in which case both Stackelberg equilibria are equilibria of the timing game; (b) if reaction functions have opposite slopes, the timing game has a unique equilibrium where the …rm whose reaction function intersects the Pareto-superior set moves second (HS, 1990, Theorem V).[2] Recently, Amir (1995) has provided a counterexample to HS's Theorem V, showing that the monotonicity of best-reply functions is insu¢cient for HS's Theorem V to hold, and the characterization of the order of moves in the extended game requires the monotonicity of each player's (or …rm's) payo¤ function in the rival's actions.

The possible consequences of asymmetric information on the order of moves are accounted for by Mailath (1993). In a quantity game whit asymmetric information about demand, he shows that the informed …rm does not exploit her chance to move before the rival. Pal (1996) explicitly takes into account mixed strategies. He considers an extended quantity-setting game with two identical …rms and two production periods before the market-clearing instant. He shows that only three outcomes are possible: (i) both …rms produce in the second period, so that a simultaneous Cournot equilibrium obtains; (ii) …rms produce in di¤erent period, yielding a Stackelberg-like equilibrium (see also Robson, 1990b); (iii) Stackelberg warfare may arise when …rms produce in the …rst period, but both produce more than in the Cournot-Nash equilibrium.

The aim of this paper is threefold. First, I shall extend the analysis provided by HS by showing that their box of tools can be pro…tably used in a more general environment than the one they have described. Speci…cally, I am going to prove that sequential play obtains at the subgame perfect equilibrium of an extended game with observable delay if and only if both the leader and the follower are at least weakly better o¤ than under simultaneous play. Second, I will analyse a

[2] HS (1990, section IV) also consider an extended game with action commitment in the spirit of Dowrick (1986), where each …rm must commit to a particular action irrespectively of the rival trying to lead or follow. This yields multiple equilibria with both simultanous and sequential play.

two-stage game of vertical di¤erentiation where …rms choose the timing of moves, product quality and compete à la Bertrand on the market, allowing for the payo¤ sequence to be such that the leader's payo¤ is not necessarily preferred to the simultaneous play payo¤. Two di¤erent extended games can be conceived. In the …rst, …rms take their timing decisions between the quality and the price stage. Here, on the basis of strategic complementarity between prices, as well as the normal form of the game, it emerges that …rms decide to play sequentially. In the alternative extended game, the timing decisions are taken before playing both stages taking place in real time. Here, provided …rms can irreversibly commit to their respective timing decisions, unusual results may emerge in terms of preferences over the distribution of roles. The subgame perfect equilibrium of such an extended game drastically di¤ers from that observed under price competition when …rms cannot endogenously di¤erentiate their respective goods. Speci…cally, in the game I present, simultaneous play emerges when …rms bear variable production costs, due to the fact that the price leader's pro…t is lower than simultaneous play pro…t, so that both duopolists play at the latest opportunity in order to avoid being …rst. Otherwise, when costs take the form of R&D e¤orts, a sequential equilibrium emerges with the low-quality …rm taking the lead. This entails that most oligopoly models where market competition is preceded by a stage in which …rms proceed to take a commitment that a¤ects the ensuing price or quantity subgame (quality choice, location, delegation, or R&D) are likely to produce equilibrium outcomes where preferences over the distribution of roles are drastically di¤erent as compared to one-stage games where price or quantity is the only strategic variable.

Using the vertical di¤erentiation model as an example, the issue of time consistency of timing decisions is considered. This allows to reach several conclusions. The two di¤erent extended games with observable delay are characterized by different subgame perfect equilibria, determined by di¤erent sequences of moves. Hence, changing the location of the timing stage drastically changes the outcome of the extended game. It appears that, to be consistent (and thus also credible), timing announcements made before any move in real time need to be supported by a commitment technology forcing …rms to stick to such announcements once they reach the price stage. Otherwise, if such devices are not available, at the price stage any timing combination that does not yield sequential play is not credible. Therefore, to avoid time inconsistency the extension concerning timing decisions must be located between the …rst and the second stage of the basic game. It emerges that the choice of timing in multistage games can jeopardize HS's conclusions, in a way that closely mimics the point raised by Amir (1995). This has a last straightforward implication for multistage games. If players are required to set the timing of their respective moves at a particular stage, then locating the timing decision just upstream that stage will always avoid problems of time inconsistency.

The remainder of the paper is structured as follows. The generalization of the extended game approach is discussed in section 2. Section 3 is devoted to the description of the vertical differentiation setting. Sections 4 and 5 describe the extended games that can be envisaged under vertical differentiation. The issue of time consistency is then dealt with in section 6. Finally, section 7 contains concluding remarks.

2 The extended games with observable delay

Consider the extension of a two-stage game where firms can set a strategic variable (price or quantity) in the downstream stage and another variable (the R&D effort, product quality, location, etc.) in the upstream stage. Then, as in HS, the extension consists in choosing noncooperatively between moving first or second in the downstream market stage only, while moves are simultaneous in the upstream stage. I shall adopt here a symbology which largely replicates that in HS (1990, p. 32). Two different extended games are considered. In the first, the timing decisions pertaining to the moves in the second stage of the basic game are taken between the first and the second stage of the basic game. In the second extended game, the timing decisions are taken before any decision in real time takes place, that is, before deciding upon the variables pertaining to both stages forming the basic game.

2.1 The first extended game with observable delay

Define as $\overline{\Gamma} = (N; \Sigma; \overline{\Pi})$ the first extended game with observable delay, where the extension takes place between the first and the second stage of the basic game. The set of players (or firms) is $N = \{A; B\}$, and $\alpha(\xi_A; \xi_B)$ and $\beta(\xi_A; \xi_B)$ are the compact and convex intervals of $\mathbb{R}$ representing the actions available to agents A and B in the downstream stage, conditional upon the choices made in the upstream stage where they are required to set ξ_A and ξ_B, respectively. $\overline{\Pi}$ is the payoff function, such that individual payoffs are defined as $\pi_A(\overline{\xi}_A; \overline{\xi}_B) : \alpha(\overline{\xi}_A; \overline{\xi}_B) \times \beta(\overline{\xi}_A; \overline{\xi}_B) \to \mathbb{R}$ and $\pi_B(\overline{\xi}_A; \overline{\xi}_B) : \alpha(\overline{\xi}_A; \overline{\xi}_B) \times \beta(\overline{\xi}_A; \overline{\xi}_B) \to \mathbb{R}$. The bar indicates that ξ_A and ξ_B are a generic given pair which may or may not (and as a general rule they do not) coincide with the subgame perfect values of ξ_A and ξ_B, as determined by backward induction when one takes into account the timing chosen in the downstream stage. I assume that, for any given pair $(\overline{\xi}_A; \overline{\xi}_B)$; π_i is single-valued in the action chosen by player j. The set of times at which firms can choose to move is $T = \{F; S\}$, i.e., first or second. The set of strategies for player i is $\Sigma_i = \{F; S\} \times \Phi_i$, where Φ_i maps $T \times \beta(\overline{\xi}_A; \overline{\xi}_B)$ (or $\alpha(\overline{\xi}_A; \overline{\xi}_B)$) into $\alpha(\overline{\xi}_A; \overline{\xi}_B)$ (or $\beta(\overline{\xi}_A; \overline{\xi}_B)$).

If in the market subgame both firms choose to move at the same time (F-F or S-S), they obtain the payoffs associated with the simultaneous Nash equilibrium, $(a_n(\overline{\xi}_A; \overline{\xi}_B); b_n(\overline{\xi}_A; \overline{\xi}_B))$, otherwise they get the payoffs associated with the Stackelberg equilibrium, e.g., $(a_l(\overline{\xi}_A; \overline{\xi}_B); b_f(\overline{\xi}_A; \overline{\xi}_B))$ if A moves first and B

moves second, or vice versa. Define the set of pure-strategy equilibria at the timing stage as $\bar{\Omega} = \{(T_A(\bar{\xi}_A, \bar{\xi}_B), T_B(\bar{\xi}_A, \bar{\xi}_B))\}$:

2.2 The second extended game with observable delay

Define as $\Gamma^* = (N, \Sigma, \Pi^*)$ the second extended game with observable delay. Again, the set of players (or firms) is $N = \{A, B\}$, and $\alpha(\xi_A, \xi_B)$ and $\beta(\xi_A, \xi_B)$ are the compact and convex intervals of $\mathbb{R}$ representing the actions available to agents A and B in the downstream stage, conditional upon the choices made in the upstream stage where they are required to set ξ_A and ξ_B, respectively. Π^* is now the payoff function, such that individual payoffs are defined as $\pi_A(\xi_A^*, \xi_B^*) : \alpha(\xi_A^*, \xi_B^*) \times \beta(\xi_A^*, \xi_B^*) \rightarrow \mathbb{R}$ and $\pi_B(\xi_A^*, \xi_B^*) : \alpha(\xi_A^*, \xi_B^*) \times \beta(\xi_A^*, \xi_B^*) \rightarrow \mathbb{R}$. The star indicates that the choice of ξ_A and ξ_B (which firms accomplish through simultaneous moves) is part of the subgame perfect equilibrium path which is determined by backward induction when one takes into account the timing chosen in the downstream stage. The set of times at which firms can choose to move is $T = \{F, S\}$, i.e., first or second. The set of strategies for player i is $\Sigma_i = \{F, S\} \times \Phi_i^*$, where Φ_i^* maps $T \times \beta(\xi_A^*, \xi_B^*)$ (or $\alpha(\xi_A^*, \xi_B^*)$) into $\alpha(\xi_A^*, \xi_B^*)$ (or $\beta(\xi_A^*, \xi_B^*)$).

If in the market subgame both firms choose to move at the same time (F-F or S-S), they obtain the payoffs associated with the simultaneous Nash equilibrium, $(a_n(\xi_A^n, \xi_B^n), b_n(\xi_A^n, \xi_B^n))$, otherwise they get the payoffs associated with the Stackelberg equilibrium, e.g., $(a_l(\xi_A^l, \xi_B^f), b_f(\xi_A^l, \xi_B^f))$ if A moves first and B moves second, or vice versa. The superscripts n, l, and f associated with ξ_A and ξ_B indicate that the values of these variables are chosen optimally, according to the shape of downstream competition. Finally, define the set of pure-strategy equilibria at the timing stage as $\Omega^* = \{(T_A(\xi_A^*, \xi_B^*), T_B(\xi_A^*, \xi_B^*))\}$:

Both games can be described in normal form as in matrix 1, where $(\cdot, \cdot)$ stands either for $(\bar{\xi}_A, \bar{\xi}_B)$ or for the relevant (ξ_A^*, ξ_B^*).

		B	
		F	S
A	F	$a_n(\cdot,\cdot), b_n(\cdot,\cdot)$	$a_l(\cdot,\cdot), b_f(\cdot,\cdot)$
	S	$a_f(\cdot,\cdot), b_l(\cdot,\cdot)$	$a_n(\cdot,\cdot), b_n(\cdot,\cdot)$

Matrix 1

Notice that, in the absence of the upstream stage where firms must set ξ_A and ξ_B, this game coincide with that considered by HS, so that matrix 1 would collapse into their matrix (cf. HS, 1990, p. 33). In the remainder of the paper, I will assume what follows:

Assumption 1 Both $\bar{\Omega}$ and Ω^* are non-empty.

Assumption 1 rules out situations like the one that would arise if payoffs in matrix 1 were ranked as follows: $a_n(\cdot;\cdot) > a_l(\cdot;\cdot) > a_f(\cdot;\cdot)$; $b_l(\cdot;\cdot) > b_f(\cdot;\cdot) > b_n(\cdot;\cdot)$:

HS (1990, p. 31) assume that each of the basic games generated by a particular timing combination has a unique equilibrium, and that these differ from each other. Then, on this basis, HS (1990, Lemma I, p. 35) show that each player's (firm's) leadership payoff must exceed his payoff in simultaneous play because if he is the leader, he is obviously able to choose the best position along the follower's reaction function, so that the Nash equilibrium point is feasible for him. If he accepts to move first (and chooses a point which differs from the Nash equilibrium one), it must be true that he is at least as well off as in the simultaneous equilibrium. Per se, this argument appears intuitive and unquestionable. Though, intuition also suggests that analogous considerations must hold for the follower as well. Consider a firm that is contemplating the opportunity of moving second. Provided that by moving at the first occasion, she can at least obtain the Nash payoff, she will accept to move late only if she is better off as a follower than in any other situation. Notice that this is precisely what emerges from HS's Theorems II and III. Accordingly, I state the following:

Lemma 1 A necessary condition for sequential play in pure strategies to emerge at the subgame perfect equilibrium of the extended game with observable delay is that each player's leadership payoff be higher than his payoff under simultaneous play.

and

Lemma 2 A necessary condition for sequential play in pure strategies to emerge at the subgame perfect equilibrium of the extended game with observable delay is that each player's followership payoff be higher than his payoff under simultaneous play.

Considered jointly, lemma 1 and lemma 2 yields a necessary and sufficient condition for sequential play to obtain at the subgame perfect equilibrium of the extended game with observable delay. This is stated in

Proposition 1 The subgame perfect equilibrium of the extended game with observable delay involves sequential moves if and only if the basic game exhibits at least one Stackelberg equilibrium that Pareto-dominates the simultaneous Nash equilibrium.

In other words, the method for equilibrium selection proposed by HS holds with no specific requirement on the sequence of profits associated with the roles firms can play in the basic game. Note that this picture largely replicates the notion of Stackelberg-solvable game as defined by d'Aspremont and Gérard-Varet (1980, Theorem 1.1, p. 203).

As to the issue of time (in)consistency, I introduce the following definitions:

Definition 1 An extended game with observable delay is strictly time consistent if $\Omega^* \equiv \overline{\Omega}$.

Definition 2 An extended game with observable delay is weakly time consistent if $\Omega^* \subset \overline{\Omega}$.

Definition 3 An extended game with observable delay is weakly time inconsistent if $\Omega^* \cap \overline{\Omega} \neq \emptyset$.

Definition 4 An extended game with observable delay is time inconsistent if $\Omega^* \cap \overline{\Omega} = \emptyset$.

In words, an extended game is (i) strictly time consistent, if the location of the timing choice along the game tree is irrelevant as to the set of pure-strategy equilibria; (ii) weakly time consistent, if the set of equilibria of the game where the timing choice takes place before any other stage is played in real time is a proper subset of the set of equilibria observed if the timing choice is located just upstream the stage to which it refers; (iii) weakly time inconsistent, if the intersection between the two sets is non-empty and Ω^* is not a subset of $\overline{\Omega}$; (iv) time inconsistent, if the two sets of equilibria have no element in common. If players' timing decisions are unaffected by the location of the timing stage itself, the extended game is time consistent and timing announcements are indeed credible. Moreover, the two versions of the game are observationally equivalent and firms can disregard the issue of which kind of game they are actually playing. If, instead, it is in the interest of at least one player to renege ex post a previous declaration in at least one of the two extended games proposed here, then the two games are not observationally equivalent. This entails that announcements by at least one firm are not credible, and a further issue arises, namely, which of the two games will be endogenously selected by firms. In the latter setting, firms might be able to stick to previous announcements if and only if a commitment technology is available, as, e.g., a capacity choice (see Kreps and Scheinkman, 1983; Davidson and Deneckere, 1986, inter alia).

3 A differentiated duopoly model

Consider a duopolistic market where firms supply a vertically differentiated good, whose quality is denoted by q_i, $i = H, L$, with $q_H \geq q_L > 0$. They employ the same productive technology, which can alternatively involve variable costs of quality improvements,

$$C_i = q_i^2 x_i, \tag{1}$$

where x_i denotes the output of firm i; or fixed costs of quality improvements,

$$C_i = q_i^2; \tag{2}$$

which may be the case when the cost of increasing the quality level falls on R&D investments and is not related to the scale of production (see, inter alia, Gabszewicz and Thisse, 1979; Shaked and Sutton, 1982; 1983).

Consumers are uniformly distributed over the interval $[0; \bar{\mu}]$. Parameter μ represents each consumers' marginal willingness to pay for quality, and it can be thought of as the reciprocal of the marginal utility of nominal income or money (cf. Tirole, 1988, p. 96). As $\bar{\mu}$ increases, the size of the market increases. Consumers' density can be normalised to one, so that total population is also equal to one. The indirect utility function of the generic consumer is:

$$U = \mu q_i - p_i: \tag{3}$$

If the consumer buys, he buys just one unit of the product from the firm that offers the price-quality ratio ensuring the highest utility. Let h and k denote the marginal willingness to pay characterizing, respectively, the consumer who is indifferent between the high and the low-quality good, and that who is indifferent between buying the low-quality good or nothing at all:

$$h = \frac{p_H - p_L}{q_H - q_L}; \quad k = \frac{p_L}{q_L}: \tag{4}$$

Then, the market demands for the two varieties are, respectively,

$$x_H = \bar{\mu} - h \text{ if } h \in]k; \bar{\mu}[; \tag{5}$$

$$x_L = h - k \text{ if } k \in]0; h[: \tag{6}$$

By inverting the system (5-6), one obtains the demand functions pertaining to Cournot behavior:

$$p_H = \bar{\mu} q_H - q_H x_H - q_L x_L; \tag{7}$$

$$p_L = q_L(\bar{\mu} - x_H - x_L): \tag{8}$$

Competition takes place in two stages, the first played in the quality space, the second either in the price or in the quantity space. In the first extended game, the extension takes place between the quality stage and the market stage, so that the relevant payoff facing firms when they are required to set the timing of moves pertaining to the market stage are the profit functions defined for a generic pair of qualities $(\bar{q}_H; \bar{q}_L)$: In the second extended game the extension precedes both stages and the relevant payoffs are the equilibrium profits obtained in correspondence of the specific pair $(q_H^*; q_L^*)$ which is optimal given the sequence of moves in the market stage.

4 The first extended game with observable delay

In this section I consider the case where the extension concerning the choice of timing is inserted between the quality stage and the market stage, so that in choosing whether to move early or late firms face a matrix where profits are defined as a function of a generic pair of quality levels. The main aim of the following analysis is to establish that π_H (resp., π_L) is single-valued in L's (resp., H) price choice for any given quality pair, in the admissible range of $\bar{\theta}$.

4.1 Variable costs of quality improvement and Bertrand competition

Assume production costs are described by (1). Firms' objective functions are defined as follows:

$$\pi_H = (p_H - q_H^2)x_H; \; \pi_L = (p_L - q_L^2)x_L. \tag{9}$$

A preliminary observation concerning the viable quality range is that, given (1) and (3), any change in the quality level produced by either firm must respect the condition $\theta dq_i \geq 2q_i dq_i$. Since the upper bound of θ is $\bar{\theta}$, the latter inequality implies $q_i \in]0, \bar{\theta}/2]$ (cf. Delbono, Denicolò and Scarpa, 1996, p. 36). This information will be useful below. The game is solved by backward induction. Consider first the fully simultaneous game. The first order conditions (FOCs) at the price stage are:

$$\frac{\partial \pi_H}{\partial p_H} = \frac{p_L - 2p_H + \bar{\theta} q_H - \bar{\theta} q_L + q_H^2}{q_H - q_L} = 0; \tag{10}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H q_L - 2p_L q_H + q_H q_L^2}{q_L(q_H - q_L)} = 0. \tag{11}$$

The above FOCs implicitly define increasing reaction functions in the price space, i.e., as it is usually observed under price competition, there exists strategic complementarity (Bulow, Geanakoplos and Klemperer, 1985). Solving the system (10-11), I obtain the equilibrium prices:

$$p_H^n = \frac{q_H}{4q_H - q_L}[2\bar{\theta}(q_H - q_L) + 2q_H^2 + q_L^2]; \tag{12}$$

and

$$p_L^n = \frac{q_L}{4q_H - q_L}[\bar{\theta}(q_H - q_L) + q_H(q_H + 2q_L)]; \tag{13}$$

where superscript n stands for Nash equilibrium. This yields the following Bertrand-Nash equilibrium profits:

$$\pi_H^n = \frac{q_H^2(q_H - q_L)(2\bar{\theta} - 2q_H - q_L)^2}{(4q_H - q_L)^2}; \; \pi_L^n = \frac{q_H q_L(q_H - q_L)(\bar{\theta} + q_H - q_L)^2}{(4q_H - q_L)^2}. \tag{14}$$

The leader's problem in the price stage can be described as follows:

$$\max_{p_i} \pi_i \tag{15}$$

$$s.t.: \frac{\partial \pi_j}{\partial p_j} = 0; i \neq j; \tag{16}$$

for both firms, i.e., it consists in the maximization of the leader's profit under the constraint represented by the follower's reaction function, implicitly given by the derivative of her profit function w.r.t. her price.[3] The equilibrium prices that obtain in the two problems can be found in Appendix A. Equilibrium profits under sequential play are

$$\pi^l_H = \frac{q_H(q_H - q_L)(2\bar{\theta} - 2q_H - q_L)^2}{8(2q_H - q_L)^2}; \quad \pi^f_L = \frac{q_H q_L (q_H - q_L)(2\bar{\theta} + q_H - 3q_L)^2}{16(2q_H - q_L)^2}; \tag{17}$$

$$\pi^f_H = \frac{(q_H - q_L)(4\bar{\theta} q_H - \bar{\theta} q_L - 4q_H^2 - q_H q_L + q_L^2)^2}{16(2q_H - q_L)^2}; \quad \pi^f_L = \frac{q_L(q_H - q_L)(\bar{\theta} + q_H - q_L)^2}{8(2q_H - q_L)^2}: \tag{18}$$

It can be easily established that $\pi^f_H \geq \pi^l_H \geq \pi^n_H$ and $\pi^l_L \geq \pi^n_L$ for all $q_H \geq q_L > 0$: As to the comparison between the leadership profit and the followership profit for the low-quality firm, one obtains the following

$$\text{sign}\,(\pi^f_L - \pi^l_L) = \text{sign}\,(2\bar{\theta}^2 - 4\bar{\theta} q_L - 2q_H^2 + q_H q_L + 2^2_L): \tag{19}$$

The roots w.r.t. q_H of the polynomial in (19) are

$$q_{H1} = \frac{q_L - \sqrt{16\bar{\theta}^2 - 32\bar{\theta} q_L + 17q_L^2}}{4}; \quad q_{H2} = \frac{q_L + \sqrt{16\bar{\theta}^2 - 32\bar{\theta} q_L + 17q_L^2}}{4}; \tag{20}$$

where $q_{H1} \leq 0$ and $q_{H2} \in [0.64039\bar{\theta}; \bar{\theta}]$ for $q_L \in [0; \bar{\theta}/2]$: Hence, $\pi^f_L \geq \pi^l_L$ for all $q_H \in [q_L; \bar{\theta}/2]$: This entails that $\pi^f_L \geq \pi^l_L \geq \pi^n_L$ for all admissible quality levels. Consequently, the set of pure-strategy equilibria is $\beta = \{(F_H(\bar{q}_H; \bar{q}_L); S_L(\bar{q}_H; \bar{q}_L)); (F_H(\bar{q}_H; \bar{q}_L); S_L(\bar{q}_H; \bar{q}_L))\}$:

4.2 Variable costs of quality improvement and Cournot competition

Again, assume production costs are given by (1).[4] When firms set output levels simultaneously, the Cournot-Nash equilibrium at the market stage is the solution

[3] Several others equilibria could be investigated, if firms were assumed to be able to play sequentially also in the quality stage, or set quantities instead of prices in the market stage. For an analisys of such equilibria, see Lambertini (1996).

[4] The game where Cournot competition follows a product stage where quality improvements are obtained through a fixed cost is not described in that it yields the same results in terms of the choice of timing.

of the following FOCs:

$$\frac{\partial \pi_H}{\partial x_H} = \bar{\theta} q_H - q_H^2 - 2q_H x_H - q_L x_L = 0; \tag{21}$$

$$\frac{\partial \pi_L}{\partial x_L} = q_L(\bar{\theta} - x_H - x_L) - q_L x_L - q_L^2 = 0; \tag{22}$$

yielding

$$x_H^n = \frac{2\bar{\theta} q_H - 2q_H^2 - \bar{\theta} q_L + q_L^2}{4q_H - q_L}; \quad x_L^n = \frac{q_H(\bar{\theta} + q_H - 2q_L)}{4q_H - q_L}. \tag{23}$$

Plugging (23) into firms' objective functions, I obtain the Cournot-Nash equilibrium profits defined in terms of a generic quality pair:

$$\pi_H^n = \frac{q_H(2\bar{\theta} q_H - 2q_H^2 - \bar{\theta} q_L + q_L^2)^2}{(4q_H - q_L)^2}; \quad \pi_L^n = \frac{q_H^2 q_L(\bar{\theta} + q_H - 2q_L)^2}{(4q_H - q_L)^2}. \tag{24}$$

When sequential play is adopted, the leader's problem is as in (15-16), yielding

$$\pi_H^l = \frac{(2\bar{\theta} q_H - 2q_H^2 - \bar{\theta} q_L + q_L^2)^2}{2(2q_H - q_L)}; \quad \pi_L^f = \frac{q_L(2\bar{\theta} q_H + 2q_H^2 - \bar{\theta} q_L - 4q_H q_L + q_L^2)^2}{16(2q_H - q_L)^2}; \tag{25}$$

$$\pi_H^f = \frac{q_H(4\bar{\theta} q_H - 4q_H^2 - 3\bar{\theta} q_L + q_H q_L + 2q_L^2)^2}{16(2q_H - q_L)^2}; \quad \pi_L^l = \frac{q_H q_L(\bar{\theta} + q_H - 2q_L)^2}{8(2q_H - q_L)}. \tag{26}$$

The output levels corresponding to the two Stackelberg equilibria can be found in Appendix B. It can be quickly checked that $\pi_i^l > \pi_i^n > \pi_i^f$; $i = H, L$, for all $\bar{\theta} \geq q_H \geq q_L > 0$. Given that the viable range for q_i is $]0; \bar{\theta}/2]$, the above profit ranking holds everywhere. As a result, the set of pure-strategy equilibria is $\beta = \{(F_H(\bar{q}_H; \bar{q}_L); F_L(\bar{q}_H; \bar{q}_L))\}$. According to the definition of d'Aspremont and Gérard-Varet (1980, pp. 204-207), the quantity game is strictly competitive, meaning that since both firms aim at being the leader, the game cannot be played simultaneously even with preplay communication, or, as it is the case here, with a preplay stage where timing is noncooperatively decided upon.

4.3 Fixed costs of quality improvement and Bertrand competition

Production costs are given by (2), and can be thought of as R&D efforts. Market demands correspond to (5-6). Firms' profit functions can be written as:

$$\pi_i = p_i x_i - q_i^2; \; i = H; L. \tag{27}$$

Consider first the fully simultaneous game. Proceeding backwards, I calculate the FOCs pertaining to the price stage:

$$\frac{\partial \pi_H}{\partial p_H} = \bar{\theta} - \frac{p_H - p_L}{q_H - q_L} = 0; \tag{28}$$

$$\frac{\partial \pi_L}{\partial p_L} = \frac{p_H - p_L}{q_H - q_L} - 2\frac{p_L}{q_L} = 0: \tag{29}$$

Again, the FOCs reveal strategic complementarity between prices. Solving the system (28-29) yields the following equilibrium prices:

$$p_H^n = 2\bar{\theta} q_H \frac{(q_H - q_L)}{(4q_H - q_L)}; \; p_L^n = \bar{\theta} q_L \frac{(q_H - q_L)}{(4q_H - q_L)}: \tag{30}$$

The corresponding profits at the quality stage are:

$$\pi_H^n = \frac{4(\bar{\theta} q_H)^2 (q_H - q_L)}{(4q_H - q_L)^2} - q_H^2; \; \pi_L^n = \frac{\bar{\theta}^2 q_H q_L (q_H - q_L)}{(4q_H - q_L)^2} - q_H^2: \tag{31}$$

As in the previous subsections, in the cases where sequential play is adopted, the leader's problem is described by (15-16), yielding

$$\pi_H^l = \frac{\bar{\theta}^2 q_H (q_H - q_L)}{2(2q_H - q_L)} - q_H^2; \; \pi_L^f = \frac{\bar{\theta}^2 q_H q_L (q_H - q_L)}{4(2q_H - q_L)^2} - q_L^2; \tag{32}$$

$$\pi_H^f = \frac{\bar{\theta}^2 (4q_H - q_L)^2 (q_H - q_L)}{16(2q_H - q_L)^2} - q_H^2; \; \pi_L^l = \frac{\bar{\theta}^2 q_L (q_H - q_L)}{8(2q_H - q_L)} - q_L^2; \tag{33}$$

with $\pi_i^f \geq \pi_i^l \geq \pi_i^n$ for all admissible quality levels. Equilibrium prices can be found in Appendix C. Again, the set of pure-strategy equilibria is $\overline{\Omega} = \{(F_H(\bar{q}_H; \bar{q}_L); S_L(\bar{q}_H; \bar{q}_L)); (F_H(\bar{q}_H; \bar{q}_L); S_L(\bar{q}_H; \bar{q}_L))\}$:

As a consequence, regardless of the technology, in both games both Stackelberg outcomes dominate the simultaneous play outcome, so that the whole discussion above can be summarized in the following:

Proposition 2 *The subgame perfect equilibrium of the extended game where the choice of timing is taken between the quality and the price stage involves sequential play, independently of the technology adopted by firms. If instead the market stage is played in the quantity space, the extended game has a unique equilibrium involving simultaneous play.*

This obviously implies that each Bertrand game has also a correlated equilibrium and, finally, a mixed-strategy equilibrium where firms randomize over playing early or delay, hence with a strictly positive probability of moving simultaneously.

5 The second extended game with observable delay: irreversible commitment

I am now in a position to consider the game where the timing decision concerning the market stage takes place before the choice of qualities. Assume for the moment that firms can irreversibly commit to the timing decision. To begin with, I describe the setting where production technology involves variable costs.

5.1 Variable costs of quality improvement and Bertrand competition

From the FOCs (10-11) we can observe that, by choosing quality levels in the upstream stage of the game, the high-quality firm affects the intercept while the low-quality firm affects the slope of their respective reaction functions in the price space. This phenomenon is responsible for the outcomes I will illustrate below. The solution of the first stage of the game involves numerical calculations. Normalising $\bar{\theta}$ to one, it can be shown that $q_H^n = 0.40976$ and $q_L^n = 0.19936$.[5] The corresponding equilibrium profits amount to $\pi_H^n = 0.0164$ and $\pi_L^n = 0.0121$. Further numerical computations show that

$$q_i^n(\bar{\theta}) = \bar{\theta} q_i^n(1); \; \pi_i^n(\bar{\theta}) = \bar{\theta} \pi_i^n(1). \tag{34}$$

This holds independently of the timing of moves firms adopt in the market stage.

Obviously, equilibrium qualities are different in each of the games being considered, though the quality stage is always played simultaneously. The optimal qualities selected when the high-quality firm is appointed the leader's role in the ensuing price stage are:

$$q_H^l(\bar{\theta}) = \bar{\theta} q_H^l(1) = 0.41601\bar{\theta}; \quad q_L^f(\bar{\theta}) = \bar{\theta} q_L^f(1) = 0.21887\bar{\theta}; \tag{35}$$

and the corresponding equilibrium profits amount to:

$$\pi_H^l(\bar{\theta}) = \bar{\theta}^3 \pi_H^l(1) = 0.01506\bar{\theta}^3; \quad \pi_L^f(\bar{\theta}) = \bar{\theta}^3 \pi_L^f(1) = 0.01412\bar{\theta}^3. \tag{36}$$

It is immediate to verify that (i) both qualities increase as compared to the fully simultaneous game; and (ii) the follower's profit exceeds the simultaneous play profit, while the leader's does not.

Consider now the case where the low-quality firm acts as the price leader, which is perhaps hardly justifiable on both theoretical and empirical grounds, nevertheless needed to complete the picture. Equilibrium qualities and profits are:

$$q_H^f(\bar{\theta}) = \bar{\theta} q_H^f(1) = 0.39999\bar{\theta}; \quad q_L^l(\bar{\theta}) = \bar{\theta} q_L^l(1) = 0.19999\bar{\theta}; \tag{37}$$

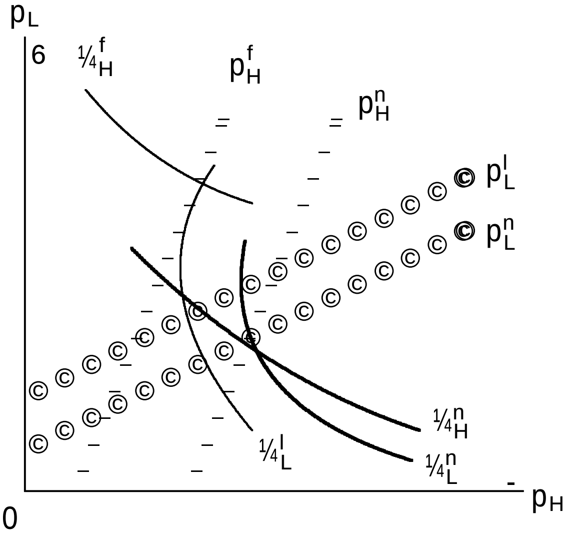
[5] Here, as well as in the remainder of the section, one should prove that neither firm has any incentive to leapfrog the rival. One such proof has been provided by Motta (1993) for the fully simultaneous setting, and is omitted here.

$$\pi^{f}_{H}(\overline{\theta}) = \overline{\theta}^{3}\pi^{f}_{H}(1) = 0{:}018\overline{\theta}^{3}; \quad \pi^{l}_{L}(\overline{\theta}) = \overline{\theta}^{3}\pi^{l}_{L}(1) = 0{:}01199\overline{\theta}^{3}: \tag{38}$$

Observe that, as compared to the fully simultaneous equilibrium, (i) the high quality decreases, while the low quality increases; and (ii) again, as in the previous case, the leader is worse o¤ than under simultaneous play.

An illustration is given in …gure 1, where the cases of simultaneous play and sequential play with the low-quality leading are described. In order not to hinder the explanatory power of the …gure, the remaining case where the high-quality …rm takes the lead is no illustrated. The reaction functions pertaining to simultaneous play are represented by thick lines, while those describing the setting where the low-quality …rm is leading are thin. The same applies to isopro…t curves. As quality levels change according to the speci…c timing chosen by …rms in the price stage, the position of reaction functions as well as the overall map of isopro…t curves change as well. Speci…cally, notice that the reaction functions of both …rms move upwards as each …rm (i) move at the same time as the rival; (ii) moves earlier than the rival; (iii) move later than the rival.

Figure 1



I am now in a position to consider the possibility that …rms choose the timing of moves pertaining to the price stage before setting qualities in the …rst stage. The outcome of such a game is summarized by the following:

Proposition 3 If …rms can set the timing of moves in the price stage before deciding their respective quality levels, and have a commitment device, both will choose to move late, so that simultaneous play emerges.

Proof. Since the size of $\bar{\theta}$ exerts only a scale effect on profits, I can confine to the case of $\bar{\theta} = 10$. Then, the game can be described by matrix 2.

		L: F	L: S
H	F	16.4; 12.1	15.06; 14.12
	S	18; 11.99	16.4; 12.1

Matrix 2

A quick inspection of matrix 2 suffices to verify that $\pi_i^f > \pi_i^N > \pi_i^l$, so that playing late (S) is a dominant strategy for both firms, and simultaneous play emerges at equilibrium, the latter being $\Sigma^* = \{S_H(q_H^*; q_L^*); S_L(q_H^*; q_L^*)\}$. ■

5.2 Variable costs of quality improvement and Cournot competition

Turn now to the case where the downstream stage takes the form of competition in output levels. On the basis of the discussion carried out in the previous section, this setting can be quickly dealt with. The optimal qualities chosen when firms compete simultaneously in the output stage are:

$$q_H^n(\bar{\theta}) = \bar{\theta} q_H^n(1) = 0.369648\bar{\theta}; \quad q_L^n(\bar{\theta}) = \bar{\theta} q_L^n(1) = 0.292788\bar{\theta}; \tag{39}$$

yielding

$$\pi_H^n(\bar{\theta}) = \bar{\theta}^3 \pi_H^n(1) = 0.0176282\bar{\theta}^3; \quad \pi_L^n(\bar{\theta}) = \bar{\theta}^3 \pi_L^n(1) = 0.017478\bar{\theta}^3. \tag{40}$$

When the high-quality firm takes the lead in the market stage, the relevant equilibrium magnitudes are:

$$q_H^l(\bar{\theta}) = \bar{\theta} q_H^l(1) = 0.35321\bar{\theta}; \quad q_L^f(\bar{\theta}) = \bar{\theta} q_L^f(1) = 0.228453\bar{\theta}; \tag{41}$$

$$\pi_H^l(\bar{\theta}) = \bar{\theta}^3 \pi_H^l(1) = 0.020598\bar{\theta}^3; \quad \pi_L^f(\bar{\theta}) = \bar{\theta}^3 \pi_L^f(1) = 0.0130477\bar{\theta}^3. \tag{42}$$

Finally, when the low-quality firm is appointed the leadership in the market stage, one obtains:

$$q_H^f(\bar{\theta}) = \bar{\theta} q_H^f(1) = 0.422087\bar{\theta}; \quad q_L^l(\bar{\theta}) = \bar{\theta} q_L^l(1) = 0.315747\bar{\theta}; \tag{43}$$

$$\pi_H^f(\bar{\theta}) = \bar{\theta}^3 \pi_H^f(1) = 0.0123216\bar{\theta}^3; \quad \pi_L^l(\bar{\theta}) = \bar{\theta}^3 \pi_L^l(1) = 0.0197048\bar{\theta}^3. \tag{44}$$

I am now in a position to state

Proposition 4 *If firms can set the timing of moves in the quantity stage before deciding their respective quality levels, and have a commitment device, both will choose to move early, so that simultaneous play emerges.*

Proof. Again, the size of $\bar{\theta}$ exerts only a scale effect on profits. Hence, I confine to the case of $\bar{\theta} = 10$. Then, the game can be described in reduced form by matrix 3.

		L	
		F	S
H	F	17.63; 17.48	20.6; 13.05
	S	12.32; 19.70	17.63; 17.48

Matrix 3

A quick inspection of matrix 3 reveals that $\pi_i^l > \pi_i^n > \pi_i^f$, so that playing early (F) is a dominant strategy for both firms. As a consequence, simultaneous play emerges at equilibrium, the latter being $\Omega^* = \{F_H(q_H^*, q_L^*), F_L(q_H^*, q_L^*)\}$. ■

Again, it is worth noting that, in the jargon of d'Aspremont and Gérard-Varet (1980, pp. 204-207), the quantity game is strictly competitive, i.e., it is not Stackelberg-solvable.

5.3 Fixed costs of quality improvement and Bertrand competition

Turn now to the case of a technology involving variable costs. From (28-29), it emerges that the choice of qualities affects the intercept of the reaction function of the high-quality firm in the price subgame, while it affects the slope of the low-quality firm's reaction function, in a way which reminds what we observed in the previous subsection. I can now look for the equilibrium qualities at the first stage. The FOCs of this problem are (cf. Motta, 1993, p. 116):

$$\frac{\partial \pi_H}{\partial q_H} = \frac{4\bar{\theta}^2 q_H (4q_H^2 - 3q_H q_L + 2q_L^2)}{(4q_H - q_L)^3} - 2q_H = 0; \quad (45)$$

$$\frac{\partial \pi_L}{\partial q_L} = \frac{\bar{\theta}^2 q_H^2 (4q_H - 7q_L)}{(4q_H - q_L)^3} - 2q_L = 0. \quad (46)$$

Manipulating appropriately (45-46), yields the following equilibrium quality levels:[6]

$$q_H^n(\bar{\theta}) = 0.12665\bar{\theta}^2; \quad q_L^n(\bar{\theta}) = 0.02412\bar{\theta}^2; \tag{47}$$

where $q_H^n(1) = 0.12665$ and $q_L^n(1) = 0.02412$ are the qualities selected when $\bar{\theta}$ is equal to one. The corresponding equilibrium profits are:

$$\pi_H^n = 0.01222\bar{\theta}^4; \quad \pi_L^n = 0.000764\bar{\theta}^4. \tag{48}$$

Again, this applies independently of the order of moves at the price stage. I turn now to the setting where each firm is alternatively appointed the leadership in the price stage. When the high-quality firm acts as the price leader, equilibrium qualities and profits are:

$$q_H^l(\bar{\theta}) = 0.12715\bar{\theta}^2; \quad q_L^f(\bar{\theta}) = 0.02949\bar{\theta}^2; \tag{49}$$

$$\pi_H^l = 0.01145\bar{\theta}^4; \quad \pi_L^f = 0.000942\bar{\theta}^4. \tag{50}$$

It can be quickly verified that (i) both qualities increase as compared to the fully simultaneous game; and (ii) the follower's profit exceeds the simultaneous play profit, while the leader's is lower than that associated with simultaneous play.

Finally, the case where the low-quality firm is the price leader remains to be described. The equilibrium levels of qualities and profits are:

$$q_H^f(\bar{\theta}) = 0.12613\bar{\theta}^2; \quad q_L^l(\bar{\theta}) = 0.02425\bar{\theta}^2; \tag{51}$$

$$\pi_H^f = 0.01234\bar{\theta}^4; \quad \pi_L^l = 0.000766\bar{\theta}^4. \tag{52}$$

Here, (i) the high quality decreases whereas the low quality increases as compared to the fully simultaneous game; and (ii) both the follower's and the leader's profits are higher than under simultaneous play.

Assume firms can decide the timing of their respective moves at the price stage before setting qualities in the first stage. The outcome of such an extended game is described by the following:

Proposition 5 *If firms can set the timing of moves in the price stage before deciding their respective quality levels, and have a commitment device, the high-quality firm will choose to move late whereas the low-quality firm will choose to move first, so that a unique equilibrium in pure strategies exists, involving sequential play.*

[6] From an inspection of (46), it can be noticed that, if costs were nil, the low quality would be $q_L^n = 4q_H^n/7$. See Choi and Shin (1992).

Proof. Again, provided that the size of $\bar{\mu}$ exerts only a scale e¤ect on pro…ts, I can con…ne to the case where $\bar{\mu} = 10$. Then, the game is described by matrix 4, which reveals that playing late (S) is a strictly dominant strategy for the high-quality …rm.

		L: F	L: S
H	F	122.2; 7.64	114.5; 9.42
	S	123.4; 7.66	122.2; 7.64

Matrix 4

As a consequence, it is optimal for the low-quality …rm to play early (F), and the unique equilibrium of this game, identi…ed by the combination of strategies (S-F) involves sequential play with the low-quality …rm in the price leader's position. Hence, in this game, $\Omega^* = \{S_H(q_H^*; q_L^*); F_L(q_H^*; q_L^*)\}$. ■

6 Time consistency

I am now in a position to discuss the issue of time consistency and the related role of commitment in the extended games with observable delay described above. In the …rst, …rms' timing choices depend solely on the slope of reaction functions in the market stage or, equivalently, they are taken on the basis of a matrix game where pro…ts are de…ned in terms of a generic quality pair. With both variable and …xed costs of quality improvement, price competition in the downstream stage leads …rms to declare that they will move sequentially. Once they actually reach the market place and set prices, none of them has any incentive to renege the announcement made in the extension taking place between stages. This setting exactly replicates the situation depicted by HS (1990, Theorem III). Analogous considerations hold when market interaction takes the form of a Cournot game. Facing downward sloping reaction functions, …rms …nd it optimal to move simultaneously at the earliest occasion (HS, 1990, Theorem II). When it comes to the second extended game with observable delay, where the extension is relocated upstream, before any decision in real time, the picture changes, and the credibility of announcements in some cases relies drastically on the existence of a commitment device. In the case of variable costs of quality improvements, the subgame perfect equilibrium involves simultaneous moves at the price stage, so that $\Omega^* \cap \bar{\Omega} = \emptyset$, and I can state

Remark 1 The extended game with observable delay where …rms bear variable costs of quality improvement and compete in prices is time inconsistent.

Under Cournot competition, it appears that locating the timing decision ahead of the two stages taking place in real time or between them is irrelevant, in that $-^{*} \equiv \bar{-}$: Hence,

Remark 2 The extended game with observable delay where firms bear variable costs of quality improvement and compete in quantities is strictly time consistent.

In the case of fixed costs of quality improvements the subgame perfect equilibrium is unique and entails a particular sequential play with the low-quality firm leading. In other words, in the former setting the upward relocation of the timing choice yields an equilibrium which is not in the set of equilibria arising from the first extended game, while in the latter setting the relocation of the timing choice shrinks the set of equilibria to a single component of the wider set of equilibria yielded by the first extended game, i.e., $-^{*} \subset \bar{-}$. As a result,

Remark 3 The extended game with observable delay where firms bear fixed costs of quality improvement and compete in prices is weakly time consistent.

This discussion finally leads to the following

Proposition 6 A sufficient condition for an extended game with observable delay to be strictly time consistent is that the timing choice immediately precede the stage which the choice itself refers to.

Hence, the picture emerging from the above analysis highlights that the choice of timing in a game where firms (or players) choose more than one variable potentially gives rise to a problem largely analogous to that spotted by Amir (1995). When a generic quality pair is considered, profit functions are single-valued, so that HS's Theorems II-V hold. However, strategic interaction in the first stage generates different quality pairs according to the specific sequence of moves adopted in the market stage, so that when Stackelberg and Nash payoffs are evaluated from the viewpoint offered by the root of the two-stage game, each player's profit (or payoff) may or may not be monotone in the rival's action. If it is not, then timing decisions are inconsistent, i.e., HS's Theorem V may fail to apply.

A last issue remains to be investigated, namely, what happens if firms can endogenously and noncooperatively decide whether to plug the extension pertaining the choice of timing at the root of the basic two-stage game, or to insert it between stages. This amounts to asking whether firms choose to play a time consistent game or not, or whether they prefer to set the timing at different points along the game tree. The case of Cournot competition is straightforward, in that any firm would always declare to move early irrespective of the location of the extension concerning herself as well as the rival. Hence, focus on the two Bertrand settings proposed above. In the case where production involves variable costs, the reduced form of the game in which …rms can endogenously establish the position of the extension is given by matrix 5.

		L	
		B	R
H	B	16.53; 13.05	15.06; 14.12
	R	18; 11.99	16.4; 12.1

Matrix 5

Strategies B and R stand for between and root, respectively. The payo¤s corresponding to $(B; B)$ are those yielded by the correlated equilibrium. In the asymmetric cases where …rm i set the timing of her price move at the root, while player j chooses between stages, player i declares she will move late, since at that stage qualities are already set and it becomes optimal to play a Stackelberg equilibrium. Observe that, since strategy R is strictly dominant for both …rms, the equilibrium is $(R; R)$, entailing that …rms would choose to set the extension at the root, i.e., they would play a time inconsistent game due to a prisoners' dilemma.

		L	
		B	R
H	B	118.95; 8.54	114.5; 9.42
	R	123.4; 7.66	123.4; 7.66

Matrix 6

Turn now to the …xed cost setting. The reduced form of the game is in matrix 6, where obviously $(R; B)$ and $(R; R)$ yield the same pro…ts. Again, R is a dominant strategy for both …rms, strictly for H and weakly for L, so that …rms choose to plug the extension at the root. As a result, they choose to play a weakly time consistent game. A …nal remark is in order. It appears from the analysis of matrices 5 and 6, as well as from the Cournot game which has not been explicitly investigated, that the taxonomy of the games in terms of time (in)consistency arising when both …rms set the timing at the same point along the game tree carries over to the more general setting where the location of each player's declaration on timing is fully endogenous.

7 Concluding remarks

In this paper I have analysed the nature of the equilibria that can be expected to arise in extended duopoly two-stage games where firms first set the timing of moves pertaining to the market stage of the game, and then proceed to play. This may be the case when firms set variables that are bound to heavily affect the ensuing market competition, such as the amount of R&D effort, product quality or location.

I have obtained three main results. First, I have shown that the criteria for equilibrium selection introduced by Hamilton and Slutsky (1990) hold even without the requirement that the leader's profit be at least as high as in the simultaneous equilibrium. Indeed, this must be true in order for sequential play to arise as a subgame perfect equilibrium of the extended game, but it must hold for the follower as well. This leads to the second result. I have established that sequential play will be observed if and only if both firms are at least weakly better off playing sequentially than playing simultaneously, i.e., if the game is Stackelberg-solvable (d'Aspremont and Gérard-Varet, 1980). Third, resorting to a model of endogenous differentiation followed by price competition, I have proved the existence of cases where the leader can indeed be worse off than under simultaneous play. Finally, I have discussed the issue of time consistency in timing games, showing that a sufficient condition for such a choice to be strictly time consistent is that the timing stage be located adjacent to the stage at which firms will indeed be required to implement their timing decisions. Otherwise, as in Amir (1995), each player's payoff (or profit) function may not be monotone in the other player's choice, and HS's conclusions may not hold. The complete endogenization of the choice of timing has highlighted that firms can be expected to locate the extension in such a way that the resulting game may not be strictly time consistent.

Hence, HS's analytical framework is applicable to one-stage games where the choice of timing concerning the relevant strategic variable is not affected by any other strategic consideration. In the light of the foregoing analysis, it appears that in multi-stage games the location of the timing decisions along the tree becomes crucial. The considerable range of models in which competition takes place in more than one stage suggests that the preferences over the distribution of roles and, consequently, the particular role distribution characterizing each specific model at equilibrium are issues to be carefully analysed in future research. Finally, the choice of timing could be extended to all the stages along which a game unravels (for applications to games with endogenous product differentiation, see Lambertini, 1996, 1997). A fully-fledged approach to this problem would certainly shed some new light on the explanatory value of the game-theoretic approach as to the behaviour of firms in the real world.

Appendix A: equilibrium prices under sequential play and variable production costs

i) The equilibrium prices when the high-quality firm is the price leader are $p^l_H = q_H[2\bar{\mu}(q_H - q_L) + 2q_H^2 - q_H q_L + q_L^2]/[2(2q_H - q_L)]$ and $p^f_H = q_L[2\bar{\mu}(q_H - q_L) + 2q_H^2 + 3q_H q_L - q_L^2]/[4(2q_H - q_L)]$.

ii) The equilibrium prices when the low-quality firm is the price leader are $p^f_H = q_H[\bar{\mu}(q_H - q_L) + q_H^2 - q_H q_L + q_L^2]/(2q_H - q_L)$ and $p^l_H = [\bar{\mu}q_L(q_H - q_L) + q_H^2 q_L + 2q_H q_L^2 - q_L^3]/[2(2q_H - q_L)]$.

Appendix B: equilibrium outputs under sequential play and variable production costs

i) The equilibrium outputs when the high-quality firm is the quantity leader are $x^l_H = (2\bar{\mu}q_H - 2q_H^2 - \bar{\mu}q_L + q_L^2)/[2(2q_H - q_L)]$ and $x^f_L = (2\bar{\mu}q_H - 2q_H^2 - \bar{\mu}q_L - 4q_H q_L + q_L^2)/[4(2q_H - q_L)]$:

ii) The equilibrium outputs when the low-quality firm is the quantity leader are $x^f_H = (4\bar{\mu}q_H - 4q_H^2 - 3\bar{\mu}q_L + q_H q_L + 2q_L^2)/[4(2q_H - q_L)]$ and $x^l_L = q_H(\bar{\mu} + q_H - 2q_L)/[2(2q_H - q_L)]$:

Appendix C: equilibrium prices under sequential play and fixed production costs

i) The equilibrium prices when the high-quality firm acts as the price leader are $p^l_H = \bar{\mu}q_H(q_H - q_L)/(2q_H - q_L)$ and $p^f_L = \bar{\mu}q_L(q_H - q_L)/[2(q_H - q_L)]$.

ii) The equilibrium prices when the low-quality firm acts as the price leader are $p^f_H = \bar{\mu}(4q_H - q_L)(q_H - q_L)/[4(2q_H - q_L)]$ and $p^l_L = \bar{\mu}q_L(q_H - q_L)/[2(2q_H - q_L)]$.

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