# On Contextually Embedded Choices

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**Abstract**: Starting from Amartya Sen's works on rational behaviour, in this essay a contextually-embedded choice theory is presented. Using concepts and tools from poset mathematics, we show how to inject in rational choice theory cultural and social effects. Specifically, we define some *choice super-structures* seen as choice sets' transformations imposed by accepted external consistency of choice requirements. As we will argue, these applications can be of some help in explaining preference changes within different contexts fo choice. Hence, using the same analytical framework, some well-known results about maximizing and optimizing behaviour may be confirmed as well as some insights on intransitive choices phase out.

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# 1 Introduction

*Rational Choice Theory* (RCT, henceforth) is based on preference-driven, internally consistent, optimizing behaviour. Individuals choose optimal elements from their choice sets accordingly with their exogeneous preferences which may take into account several psychological factors (like regret, risk aversion etc...) or behavioral norms (like empathy or altristic motives). Individual preferences are assumed invariant with respect to everything not included in preference's description and internally consistent. Internal consistency of choice is usually defined by two properties: *contraction consistency* (i.e. choosing an object from a given set of choice implies to choose the same object from any proper subset of choice) and *expansion consistency* (i.e. choosing an object from a given intersection of choice sets involves to choose the same object from the union of these sets). Whereas choice sets are finite, there properties are necessary and sufficient conditions for the binariness of the choice function and this allows to rationalise behaviour through choice using a revealed preference argument (Sen (1994a)).

In last decades, many refinements of RCT have been undertaken by scholars as well as many alternative approaches have been suggested by heterodox economists and social scientists. Among the former, a particularly crucial issue has been how to explain complex decision makings in which many order relations or evaluation criteria are used by individuals. In this perspective, Ehrgott (1998) proposes a multi-criteria decision making theory mainly based on the evaluation of an alternative through a multi-attribute value function that maps alternatives on the Reals with respect to $n$ criteria. More recently, Kalai et al. (2002) provide a multiple rationales theory in which any decision maker is endowed by $k$ order relations through which behaviour may rationalized. In fact, these rationalization methods are *context free*, in the sense that they do not make use of information other than choice behaviour. Thus directly or indirectly, they are linked with the hicksian preference theory explaining behaviour throught order relations/criteria totally determined and controlled by individual will, attitudes and desires[2].

Exactly removing this hicksian assumption, other scholars have suggested theories in which peers pressures or causal conditions of choice (i.e. *the degree of freedom of choice*, *individual identities*, *social positions*) may strongly influence individual desires, will and behaviour. For instance, Harsany's (1977) and Broome's (1994) *extended preference theory* mainly focused on personal, causal and environmental conditions of choice. As it will clearer below (see Section2), they starting idea is to extend preference relation's domain from sets of alternatives to sets of vectors formed by an object and a causal condition of choice. Even if, as stressed by Puppe (1995) and Gravel (1998), some problem

[2] Other recent papers on RCT in an hicksian approach are: Podinovski and Podinovski (1998) for a decision analysis under partial information; Nehring (2000) for a theory of rational choice under ignorance; Ray and Zhou (2001) and Quesada (2002) for an analysis of rationality in strategic games.

of *basedness* between this extended preference relation and its related binary relation over alternatives do emerge, this approach has been a useful starting point in investigations about how contexts of choice affect human behaviour.

Not surprisingly, difficulties in building extended preference orderings have left rooms to other theories of socially complex choice behaviour. Among other, *collective preference theory* recently proposed by Sudgen (2000) and discussed by Gilbert (2001) deals explicitly with *non-individualistic* choices. Individuals are here supposed to belong to teams or groups with collective objectives, group preferences and joint commitments. Nevertheless, these collective preferences are not reducible to a set of correlative individual preferences and hence they do not rationalize behaviour other than formal groups'one (like football teams or armies).

However, contexts of choice may influence choice behaviour in a more subtle way through informal and cultural norms, traditions or individualistic moral obbligations. Exactly formthis viewpoint, Sen (1994) introduces a dinstinction between *culmination outcomes* (i.e results of choice) and *comprehensive outcomes* valued not only with respect to chosen alternatives, but also with regards to the act of choice and its coherence with something external to the choice itself (like agency of choice, menù of choice, urgency of choice or chooser's social position). His main result is that with menù (or chooser) dependent preferences (i.e. different preference orderings with respect to different choice menù or different choosers) binariness of choice is not verified and traditional revealed preference arguments cannot be applied. Thus, menù (and chooser) dependency of preference orderings have shown that human decision making must be viewed as a *contextually embedded* process and external consistency with some normative values or factors has to be seek in order to explain individuals'choice behaviour. In this perspective have to be read, in order to appreciate authors' findings, Munro and Sudgen's (2003) *reference-dependent preference theory* and Katzner's (2002) *culturally-determined choice theory.* As it will be more deeply discussed in next section, both theories consider how chooser's starting conditions influence choice behaviour using respectively preference structures and preferences over extended alternatives.

In what follows, we provide a *contextually embedded choice theory* built using analytical tools of *poset mathematics* [3]. Surprisingly so far, fews contributions in which this theory is applied to choice problems exist. Some papers on the structure of non-representable preferences (see Beardon et al. (2002) or Candeal et al. (1998)) or on path dependency of choice functions (Johnson and Dean (2001)) are remakable exceptions. Nevertheless, as best as we know, no contributions have made use of these tools to model culturally or socially embedded rational choices. The theory here presented has exactly this feature investigating on preferences' changes within different contexts of choice. The essay is organized as follows. Section 2 reviews some recent articles which deal

[3] For an introduction to these mathematical tools see Davey and Prestley (2000).

with contextually embedded rational choices and external consistency of choice. Hence, Section 3 presents preliminary definitions and concepts from poset theory used in the rest of the paper. In Sections 4, 5 and 6 our main results are presented and discussed, while Section 7 traditionally concludes.

## 2 Embedded Choices: literature review

As stressed above, several contributions have raised questions to mainstream RCT from different perspectives and using different theoretical tools. For sure, this hasn't make easier the debate. However, what after all has been accepted among sholars is that individual choice behaviour cannot be deeply understood if cultural, social and positional influences are ignored. Among other, Amartya K. Sen has argued

> against the *neoclassical* approach to choice and behaviour *showing* the inescapable need to go beyond the internal features of a choice function to understand its cogency and consistency (Sen, 1994a, p.497, italics added).

His methodological appraisal may be fully appreciated only if philosophical linkages with his concept of *positional objectivity* (Sen (1997), Sen (2002)) are fully recognized. Roughly speaking, beliefs and alternatives' evaluations are objective not if they are context free (a sort of view from nowhere) but whereas they are consistent with agents'position within groups, communities or societies. Hence, modeling context dependency of preferences may be meaningful since:

> in many circumstances [...] there is no way- internal to the choice function- of determinating whether a particular behaviour pattern is or is not consistent. The necessity of bringing in something outside choice is the issue. (Sen, 1994a, p. 498)

As Sen recognizes, quoting Davidson (1980), what has to be brought in are: "*desires, wanting, urges, promptings and a great variety of moral views, aesthetic principles, social conventions, public and private values*"; in short, anything that determining contexts of choice and choosers' identity and/or social position. An example may be used to show how richer information about the context of choice may explain apparently irrational behaviour.

**Example 1:** *Mangos, Apples and Kindness (Sen (1997))*

Suppose that agents $i$ and $k$ face the following choice set $S = \{m^1; a^1; a^2\}$ where $m$ and $a$ indicate respectively mangos and apples contained in a fruit basket. Person $i$ prefers mangos to apples but when he/she has to choose by first his revealed preference is for $\{a^1\}$. Some hours later, the two friends face a similar situation. Now, the choice set is given by $T = \{m^1; m^2; a^1; a^2\}$ and agent $i$ choosing by first does select $\{m^1\}$. This choice violates a contraction consistency property (also called property $\alpha$)[4] and the weak axiom of revealed preferences. Hence, it cannot be explained through internal features of the choice function. Nevertheless, agent $i$'s behaviour can be rationalized using menù-dependent preferences[5] such that

$$a^1 \succsim_i^S m^1 \wedge \; m^1 \succsim_i^T a^1 \text{ for } \forall S \subseteq T$$

Menù-dependency of $\succsim_i$can be caused by many factors as the desire to not be greedy, the importance to leave friends all choice opportunities (i.e. freedom of choice) and go on. ■

Thus, Sen's researches on rational choice have been focused on what menù-dependency entails for a well-known result in RCT: the binariness of a choice function and its generated revealed preference relation. Defining a menù-independent preference relation as a binary relation over an universal set of alternatives $X$ such that for $\forall S \subseteq X$, $\succsim^S = \succsim^{X-S}$(i.e. $\succsim^S$ is exactly the restriction of $\succsim^X$over $S$), it may be prooved that menù-independence of $\succsim$ involves menù-independence of the generated choice function, a necessary and sufficient condition for binariness of choice[6]. Moreover, menù-dependent preferences entail choices in full contrast with Property $\alpha$'s predictments. In these cases, RCT's reasoning is not able to rationalize behaviour anymore, opening rooms for contextually embedded choices or choice behaviour strongly influenced by causal conditions of choice[7].

Exactly dinstinctions between objects and causes of preference have been deeply investigated in Broome (1994). As this author points out, an object of preference tipically is an attainment that may be judged differently by different agents with divergent causes of preference. In Broome's own words:

> [causes of preferences] determine the structure of people's preferences to be different from another's. My preferences are causally influenced by the life I lead. (Broome, 1994, p.10)

Thus, people's preference structures have to be described using *extended alternatives*, that are combinations of attainments together with particular personal characteristics. Formally, an extended alternative is a couple $(x, k)$ where

[4] Property $\alpha$ may be formally written as:

$$[x \in C(T) \wedge x \in S \subseteq T] \Rightarrow x \in C(S)$$

where $C$ is the choice function.

[5] In what follows strict preference relation and indifference relations are traditionally defined. Furthemore, whenever not differently specified preference relations are assumed to be transitive, reflexive and anti-symmetric.

[6] Formal proofs can be found in Sen (1971), Herzberg (1973), Sen (1997).

[7] Sen also discusses chooser-dependent preferences even if his main results are obtained for the case of menù-dependency. Worth noting, Sen's view has been strongly influenced by Stig Kanger's work on background-dependent preferences. On this topic see Sen (1994b).

$x \in X$ is the attainment vector and $k \in K$ a parameter which shortly describes chooser's characteristics. Thus, an extended preference relation $\succsim^e$ is a binary relation over couples $(x, k) \in X \times K$. Declining $k$ as a subset $A$ of the power set of $X$ (i.e. $k \equiv A \subseteq 2^X$), we may use an extended preference relation for dealing directly with Sen's menù-dependency. The following example can give us the basic idea of such an intersection between above approaches.

**Example 2:** *Mango, Apple and Extended Alternatives (Baharad and Nitzam (2000))*

Suppose that the two friends above deal with the following choice problem. Let $S$ be the choice set with $S = \{m^1; a^1; a^2\}$ and suppose that

$$m^1 \succ_i a^1 \sim_i a^2 \text{ with } C(S; \succsim_i) = \{a^1\} \text{ for } \forall i$$

Then, assume that a new choice menù is proposed to both agents. As above, the new choice set is $T = \{m^1; m^2; a^1; a^2\} \supset S$ with $C(T; \succsim_i) = \{m^2\}$. In this case, menù-dependency of preferences can be model using an extended preference relation such that:

$$(m^2; T) \sim^e (m^1; T) \succ_i^e (a^1; T) \sim^e (a^1; S) \succ_i^e (m^1; S)$$

Under such an extended preferences specification of the problem, positive appreciation of freedom of choice can cause a substantial changes in agent $i$'s preference rankings, hence rationalizing apparently irrational choices. ■

Even if extended preference theory suffers of a deep constitutive problem linked with, as discussed in Baharad and Nitzam (2000), the simultaneous satisfaction of a positive (or negative) respect for freedom fo choice (as in example 2) and some form of logical compatibility between $\succsim_i$ and its extended version (the so called basedness condition), the recognized significance of the distinction between objects and causes of preferences has generated interesting advancements in the idea that something else than possible attainments may explain choice behaviour. Causes of preferences may be internally determined (as personal attitudes, emphatic virtues or individual values) or externally shaped by social, cultural or moral norms. In any case, taking into account these elements does allow to properly explain behaviour usually not rationalized by RCT.

An interesting illustration of the last issue can be undertaken considering Katzner's (2000) recent contribution. Katzner suggests a culturally embedded choice theory starting by the above definition of extended alternatives. Individual preferences, traditionally characterized, are referred to an extended choice set $S_0(X \times K)$ where $X$ is the universal set of alternatives and $K$ the set of culturally determined causal conditions of choice. He firstly shows that if a choice function $C$ defined on $S$ satisfies property $\alpha$ then always exist an associated preference relation which rationalizes $C$ (see Theorem 2, p.244). Hence, he demonstrates that for any non-rationalizable choice function defined on $S(X)$

there exist a finite set $K$ together with an associated rationalizable choice function defined on $S_0(X \times K)$ (see Theorem 3, p.248). Roughly speaking, well-specified cultural influences on choice may rationalize what should be viewed as irrational using RCT's insights.

Behind existance results, Katzner's approach is operatively based on what he calls *a two step procedure*:

> first [an agent] selects $S_0 \subset S$ then he/she selects as his/her choice from $S$ the most preferred element of $S_0$. The selection of $S_0$ may be interpreted as an expression of the primary motive in determining choice action in that it eliminates from consideration those options that are less attractive, and therefore unacceptable, for cultural reasons. (Katzner, 2000, p252)

Through this,

> by selecting an appropriate *context,* all choice behaviour is explainable in terms of rationality as defined by property $\alpha$. (Katzner, 2000, p250, italics added)

Working with choice set contractions Katzner can razionalize apparently irrational choices through different culturally-driven selections of $S_0$ from $S$. A slightly modified version of The Mangos and Apples example may help us in understanding how the above procedure works.

**Example 3:** *Mangos, Apples and Cultural Norms (Katzner (2000))*

Suppose that choice sets are given by $S = \{m^1; a^1; a^2\}$ and $T = \{m^1; m^2; a^1; a^2\}$. Since, as above, $C(S; \succsim_i) = \{a^1\}$ and $C(T; \succsim_i) = \{m^2\}$ property $\alpha$ is violated and $C$ is a non-rationalizale choice function for RCT. Now consider a culturally determined vector of individual characteristics given by $k = \{k_1; k_2; k_3; k_4\} \in K$ and extended alternatives $(x; k) \in X \times K$. Original preferences over alternatives are supposed to be:

$$m^2 \succsim_i m^1 \succsim_i a^1 \succsim_i a^2$$

Consistenly, preferences over extended alternatives are given by

$$(m^2; k_j) \succsim_i (m^1; k_j) \succsim_i (a^1; k_j) \succsim_i (a^2; k_j) \text{ for } j = 1, 2, 3, 4$$

with

$$\begin{aligned}
(m^2; k_2) &\succsim_i (m^1; k_1) \\
(a^1; k_3) &\succsim_i (m^1; k_1) \\
(a^2; k_4) &\succsim_i (m^1; k_1) \\
(a^1; k_3) &\succsim_i (m^2; k_2) \\
(a^2; k_4) &\succsim_i (m^2; k_2) \\
(a^2; k_4) &\succsim_i (a^1; k_3)
\end{aligned}$$

Hence, using transitivity it may be shown that the choice function $C^*$ rationalized by this ordering is:

$$C^*\{(\otimes;k_j)\} = (\otimes;k_j) \text{ with } \otimes = \{m^2;m^1;a^2;a^1\} \text{ and } j=1,2,3,4$$
$$C^*\{(m^1;k_2);(a^2;k_4)\} = (a^2;k_4)$$
$$C^*\{(a^1;k_3);(a^2;k_4)\} = (a^2;k_4)$$
$$C^*\{(m^1;k_2);(a^1;k_3)\} = (a^1;k_3)$$
$$C^*\{(m^2;k_1);(a^2;k_4)\} = (a^2;k_4)$$
$$C^*\{(m^2;k_1);(m^1;k_2)\} = (m^1;k_2)$$
$$C^*\{(m^2;k_1);(a^2;k_4)\} = (a^2;k_4)$$
$$C^*\{(a^1;k_3);(m^2;k_1);(m^1;k_2)\} = (a^1;k_3)$$
$$C^*\{(m^1;k_1);(a^2;k_4);(a^1;k_3)\} = (a^2;k_4)$$
$$C^*\{(m^1;k_2);(a^2;k_4);(m^2;k_1)\} = (a^2;k_4)$$
$$C^*\{(m^1;k_2);(a^2;k_4);(a^1;k_3)\} = (a^2;k_4)$$
$$C^*\{(m^2;k_j);(m^1;k_j);(a^2;k_j);(a^1;k_j)\} = (m^2;k_j) \text{ for } j=1,2,3,4$$

As Katzner shows, the last function satifies property $\alpha$ and choice consistency is valued on the basis of some cultural norms external to the choice itself. Finally, $C^*$ can also be explained in terms of choice set contractions with $T_0 = T - \{\delta\}$ where $\delta$ is one of the best elements in $T$. ■

Noteworthy, Katzner's proposal goes in the direction of a contextually embedded choice theory which internalizes some external consistency requirments with respect to socially, culturally or positionally determined chooser's personal characteristics or causal conditions of choice (i.e. moral obligations, normative principles etc). Such a complex paradigm ought to be able to deal with menù-dependent, chooser-dependent and positional preferences.

However, this is not the unique path followed by researches in last decades. Arguments of different nature have been presented by several authors in order to show how a neoclassical (or substantive) view of human rationality is inadequate to explain real world choice behaviour not completely driven by self-interested motivations. Let us review these last contributions in order to better focus our proposal introduced in the following sections.

H.Simon (2000) provides an excellent review of recent philosophical and methodological appraisals to RCT. He writes in his conclusions:

> as human beings are adaptive organisms and social organisms that can preserve body of learning *or being influenced by socially, culturally or positionally determined norms, values or habits*, studying their behaviour will not return us to permanently invariant laws, for human learning and social influence will continue to change people's way of making rational decisions (Simon (2000), p.252, italics added).

As well-known, Simon has suggested a procedural view of rational behaviour specifying how rational choices vary under different choice procedures and different notions of what a satisfying choice is[8]. In what follows, we propose an alternative approach. Accepting Simon's view that rational human behaviour will not return us to permanently invariant laws, individuals are assumed to be rational maximizers facing some contextually determined constraints or influences which change their choice sets structures. Therefore, embedded rationality is not modeled assuming a given choice structures and some satisfying procedures designed by the context of choice, but, conversely, assuming certain social and cultural influences that may make rational a substantively non-optimalalternative[9] . Hence, our proposal is a theory of contextually-embedded rationality as Katzner's (2000) one.

Nevertheless, differently by Katzner, we will not use choice set contractions but choice sets' trasformations to model context-dependency of choice. Contexts of choice are thought not as frames in the sense of Kahneman and Tversky (1979)[10], but, following Elliot and Hayward (1998), as a complex compact within which choice behaviour is considered, selected, interpreted and evaluated. Echoing Etioni (1988), we recognize that "*most general frames are social and cultural norms and institutions*" generally actived by context cues which determine what kind of frame of choice actually holds. Ignoring what context cue activates a relevant frame, our proposal allows to take into account *context effects* on choice behaviour. Finally, contextually-embedded rationality here refers to maximizing behaviour towards higher well being achievements within limits and obligations imposed by some choice *super-structures* [11]. These structures operate as formal and/or informal contraints and they are co-determined by culturally specific factors, social and group obligations, norms and ethical values, personal identities and attitudes. Moreover, their are *super*-structures since, in many cases, they are not under direct individual control. A classic dilemma[12], called Sophie's Choice, may be used to illustrate how a choice super-structures works.

[8] See Simon (1955), (1972), (1982). Furthermore, an elegant step forward in modeling procedural rationality has been done by Rubinstein (1998). His choice structure is given by a triple: a choice set, a decisional procedure and a similarity relation among alternatives (see p.28). He models choices between lotteries and he shows that some usual criticisms to RCT (like *framing effects*) may be easily reconciled using a procedural approach. Similarly, Mullainathan (2002) provide a memory-based model of bounded rationaly.

[9] We will deal with the dinstinction between maximization and optimization in Section 5.

[10] In their seminal contribution, Kahneman and Tversky (1979) discuss how frames characterized by some *reflection effects* (like being relatively more (less) risk-adverse after some past episodes of gains (losses)) affects choices. For a survey on frames of choice based on reflection effects see Levin et al. (1996). A recent application of framing effects in explaining non-transitive choices can be found in Humphrey (2001).

[11] We derive our concept of choice super-structure by Folbre's (1994) idea of structures of contraints. In her own words, "*individuals are embedded in a complex structure of individual and collective identities and competing interpretations of these that sometimes they do not even know whose interests they are acting on*" (Folbre, 1994, p.16). These elements characterize agents' structures of constraints within which motivations for action and criteria of behaviour are embedded.

[12] The Sophie's Dilemma is discussed in Bailey (1998).

**Example 4:** *Sophie's Choice*

Sophie is a young mother of two twins. A sadistic officer in a concentration camp proposes her the following alternatives: "*choose before night one of your babies to be killed, or I shall kill them both*". A neoclassical chooser would probably suggest to flip a coin in order to maximize expected utility. Nevertheless, at the sunset, Sophie decides for killing both babies and herself. Indeed, her decision can be viewed as irrational. Nevertheless, it should be also a contextually-embedded rational choice where peer-group safety is an accepted value (or a moral contraint). In this case Sophie's behaviour could be rationalized as perfectly consistent with her will to signal that "*none of us will accept to gamble with our children*". ■

As we will see below, choice super-structures are modeled through socially or culturally determined choice sets' transformations which do not necessarly maintain orders among alternatives (named in short *allomorphic applications*) as well as through the imposition of *equivalence relations* between alternatives previously perceived as not-equivalent[13]. In order to appreciate how these are defined, we have to introduce some concepts of poset mathematics. To this task is devoted the next section.

# 3 Definitions and Notations

Throughout this paragraph, we introduce definitions and notations[14]. Take a non-empty, finite set of alternatives $X$ and a binary relation $\succsim$ on $X$. Suppose that $\succsim$ is reflexive, transitive and anti-symmetric[15]. Let $(X;\succsim)$ be a finite, non-empty poset. Whereas each $(Z;\succsim) \subseteq (X;\succsim)$ is a closed set, then any $Z$ has a supremum and an infimum element. Traditionally, we can write these using the following notation:

$$\begin{aligned} x \wedge y &\equiv \sup(x,y) \\ x \vee y &\equiv \inf(x,y) \end{aligned} \quad (1)$$

for some $x, y \in Z$. In what follows, symbols $\vee$ and $\wedge$ are traditionally named *join* and *meet*. There can be also the case that $(Z;\succsim)$ has a top and a bottom element respectively denoted by $\top$ and $\bot$. Ordinary representability of posets

[13]Sen (1986) discusses invariance requirements of choice. He uses a notion of *isoinformation set* to express similarity with respect relevant information. If two alternatives $x$ and $y$ belong to the same isoinformation set then they must be treated in the same way. Something very similar to an equivalence relation. He also suggests isomorphic transformations of alternatives in order to deal with moral identity of alternatives of the kind: if $x = y$ then $f(x) = f(y)$ for any $f \in F$ where $F$ is a class of morally equivalent transformations.

[14]For an introduction to ordered set theory see Davey and Priestley (1990) and Rival (1982). For its application to preference orderings see Bridges and Metha (1995).

[15]For $x \precsim y$ with $x, y \in X$ we mean that $\neg(x \succ y)$.

through *Hesse diagrams* is also assumed. For illustrative porposes, the following diagrams depicts ordered choice sets for the mangos and apples example:

**[Insert here Figure 1]**

Then, let us assume that $X$ has finite lenght $l$ with

$$l := \max d\,(m; x) + 1 \tag{2}$$

where $m$ is a maximal element of $X$. In this case, we can decompose $X$ in a l-tuple $(L_1; ...; L_l)$ of *levels*. Each level is a set of alternatives defined as follows:

$$L_k := \{Y \subset X |\ \forall x, y \in Y\ rk\,(x) = rk\,(y)\} \tag{3}$$

where we assume that always exists a *rank function* $rk : X \to I_+$ such that $rk\,(x) > rk\,(y)$ if $d\,(m; x) > d\,(m; y)$ and $rk(x) = rk(y) + 1$ if $y$ covers $x$.

Hence, let $P$ and $Z$ be two posets. A map $\varphi : X \to Z$ is said to be *order-preserving* if:

$$x \leq y \text{ in } X \quad \text{then} \quad \varphi\,(x) \leq \varphi\,(y) \text{ in } Z \tag{4}$$

or, alternatively, when

$$rk\,(\eta\,(x)) = rk\,(x) \text{ for } \forall x \in X \tag{5}$$

A one-to-one and onto order preserving map is generally named *isomorphism.* Hence, an *allomorphic application* is a non-order-preserving map $\eta \in \Upsilon : X \to Z$ such that:

$$rk\,(\eta\,(x)) = \eta\,(rk\,(x)) \neq rk\,(x) \tag{6}$$

for at least one $x \in X$. The finite, non-empty set $Z$ is simply named induced poset. Then, an *equivalence relation* $\theta \in \Omega$ on a set $Z \subseteq X$ is a reflexive, symmetric and transitive binary relation which gives raise to a partition of $Z$ into disjoint *blocks.* A typical block is of the form:

$$[y]_\theta := \{x \in Z \mid x \equiv y\,(\mathrm{mod}\,\theta)\} \tag{7}$$

When for all $x, y, z, v \in Z$

$$\begin{aligned} x &\equiv y \,(\mathrm{mod}\, \theta) \\ z &\equiv v \,(\mathrm{mod}\, \theta) \end{aligned} \tag{8}$$

then

$$\begin{aligned} x \vee z &\equiv y \vee v \,(\mathrm{mod}\, \theta) \\ x \wedge z &\equiv y \wedge v \,(\mathrm{mod}\, \theta) \end{aligned} \tag{9}$$

and $\theta$ is said to be join (resp.meet)-compatible. Finally, we can define a *choice super-structure* as a contextually induced transformation of a poset through an equivalence relation and/or a non-order-preserving application [16]. Formally, a *choice super structure* can be defined as:

$$(\mathrm{mod}\, \theta; \eta) \in \Omega \cup \theta_0 \times \Upsilon \cup \eta_0 \tag{10}$$

with $\theta_0 :=^\neg \{\theta | \theta \in \Omega\}$and $\eta_0 :=^\neg \{\eta | \eta \in \Upsilon\}$ . Consistently, the induced poset is equal to:

$$Z_{(\mathrm{mod}\, \theta; \eta)} := \left\{ [x]_{(\mathrm{mod}\, \theta; \eta)} \text{ for } \forall x \in Z \right\} \tag{11}$$

with $Z_{(\theta_0; \eta_0)} \equiv Z$ and $Z_{(\mathrm{mod}\, \theta; \eta)} \neq \emptyset$ for $\forall\, (\mathrm{mod}\, \theta; \eta) \in \Omega \times \Upsilon$.

As it will clearer in next sections, the basic idea is that these transformations may be used in modeling contextually embedded choices since they allow us to describe how choice super-structures effectively work.Finally, let us define the narrowest family of choice super-structures.

Let us call a *pure choice super-structure* a couple $(\mathrm{mod}\, \theta; \eta)$ with $\theta \in \Omega$ and $\eta \in \Upsilon$ that is a categorial transformation in which both an equivalent relation and an allo-morphic application work. in opposition, if $\theta \equiv \theta_0$ or $\eta \equiv \eta_0$ we have *impure* choice super-structures. These may be divided in two classes: *permutative choice superstructures* (i.e. $(\theta_0; \eta) \in \theta_0 \times \Upsilon$) and *equivalence choice super-structures* i.e. $((\theta; \eta_0) \in \Omega \times \eta_0)$. In the first class of transformations, a non-order-preserving map modifies alternatives' ranks, in the second one an equivalence relation makes contextually equivalent (i.e. on a par) some previously distinct alternatives. Finally, whereas $\theta \equiv \theta_0$ and $\eta \equiv \eta_0$ no choice superstructures affect choice behaviour.

[16] In recent pure mathematics, a *category* is defined as a combination of an ordered set and some structure-preserving morphisms. For an introduction to this topic see Blyth (1982).

# 4 Choice Super-Structures and External Consistency of Choice

We have now almost all theoretical tools to deal with external consistency of choice. Few additional definitions complete our set-up. Take a non-empty, finite poset of alternatives $(X)$ and denotes with $\widehat{Z} := \{Z_i\}_{i\in X}$ all non-empty families of non-empty subsets of $X$. Assume that there exists a choice function defined as a map:

$$C := X \to \bigcup_{i\in X} Z_i \text{ such that } (\forall i \in X)\, C(i) \in Z_i \tag{12}$$

with $Z_x := \{y \in X | y > x\}$ for $\forall x \in X$. Hence, by the Zorn's Lemma[17], any non-empty family of subsets of $X$ has at least one maximal element. Maximal elements are defined as follows:

**Definition 1:** *Let $X$ be a finite, non-empty poset and let $Z \subseteq X$. Then $a \in Z$ is a maximal element of $Z$ whereas if $a \precsim x \in Z$ then $a \sim x$. Moreover, if $a \succsim x$ for $\forall x \in Z$ then $a \in Z$ is the (maximum) optimal element of $Z$.*

Thus, consistently with Sen (1971), (1997) and Suzumura (1976), (1983), we can define maximal set and optimal set choice functions as follows:

**Definition 2:** *Let $Z \subseteq X$ be finite, non-empty posets with at least one maximal element. A maximal set choice function is a map*

$$C_M(Z; \succsim) : Z \to M(Z; \succsim)$$

*with* $M(Z; \succsim) := \{x | x \in Z \text{ and } \nexists y \in Z : y \succ x\}$.

**Definition 3:** *Let $Z \subseteq X$ finite, non-empty posets with at least one optimal element. An optimal set choice function is a map*

$$C_B(Z; \succsim) : Z \to B(Z; \succsim)$$

*with* $B(Z; \succsim) := \{x | x \in Z \text{ and } \forall y \in Z : x \succsim y\}$

Trivially, whereas $\succsim$ is assumed complete, $Z$ has a top element $(\top)$ and $B(Z; \succsim) \equiv M(Z; \succsim) \equiv (\top)$ as shown in Sen's (1997) Theorem 5.3. Hence, a set value choice function is *non-rationalizable* if it does not satifies a slightly modified version of property $\alpha$. More precisely,

[17] See Hamilton (1982).

**Definition 4:** *Given $Z \subseteq X$ and $P \subseteq Z \subseteq X$ finite, non-empty posets, the set-value choice function $C$ is said to be non-rationalizable by $\succsim$ if*

$$\begin{aligned} C(Z; \succsim) &\equiv S \neq \emptyset \text{ and} \\ C(P; \succsim) &\equiv S^{'} \neq \emptyset \text{ with} \\ S \bigcap S^{'} &= \emptyset \text{ and } S \bigcap P \neq \emptyset \end{aligned} \tag{13}$$

Example 5 does illustrate our last definition:

**Example 5:** *A Non-Rationalizable Maximal Choice Function*

Let $Z$ be a finite, non-empty poset and let $P$ a non-empty subset of $Z$. Both sets are characterized by the following Hesse diagrams in which order relations are traditionally denoted :

**[Insert here Figure 2]**

Looking at Definition 4, $C_M$ is non-rationalizable if $C_M(Z; \succsim) := \{x, z, k\}$ and $C_M(P; \succsim) := \{y; w\}$. As the reader may easily check, such a situation phases out in mangos and apples-like choice problems. ■

As we have discussed above, menù restrictions, cultural norms or positional obligations may explain apparently irrational choice behaviour. These contextually determined influences are modeled using choice super-structures defined as a categorial transformations $(\text{mod}\,\theta; \eta)$ of ordered choice sets. For illustrative pourposes, let us consider, once more, Sophie's tragic choice.

**Example 6:** *Sophie's Choice and Choice Super-Structures*

Remind Sophie's choice. Her choice set is given by:

$$Z = \{x; y; z\}$$

with $x$ : save one baby throwing a coin; $y$ : loose both babies; $z$ :kill both babies and herself. Anyone agrees that outside a concentration camp a natural order among alternatives should be $x \succ y \succ z$ and that $C_B(Z; \succsim) \equiv C_M(Z; \succsim) := \{x\}$. Nevertheless, inside a camp things are tragically different. Group-oriented norms of the kind "*...this is a monstrous game ! We must signal that we do not accept these kind of gamblings*" may cause that $C_B\left(Z_{(\text{mod}\,\theta^*; \eta^*)}; \succsim\right) \equiv C_M\left(Z_{(\text{mod}\,\theta^*; \eta^*)}; \succsim\right) := \{z\}$ with

$$\begin{aligned} \theta^* \quad &: \quad = \{y \equiv x\,(\text{mod}\,\theta)\} \\ \eta^* \quad &: \quad = \begin{cases} r(\eta(x)) = r(z) \\ r(\eta(z)) = r(x) \end{cases} \end{aligned}$$

Hence, an oppurtunely defined transformation of Sophie's choice set may explain as heroically rational an apparently irrational choice. Her group loyalty is an ethical principle able to rationalize what is un-acceptable for a neoclassical chooser: self-sacrifice without personal future rewards. ■

Following Example 6's intuition, we are able to show the following:

**Proposition 1:** *Let* $P \subseteq Z$ *be non-empty, finite posets,* $C_M$ *a maximal set choice function and* $(\bmod\,\theta;\eta) \in \Omega \times \Upsilon$ *a pure choice super-structure. Then, it always exists a*

$$\begin{aligned} (\bmod\,\theta^*;\eta^*) \text{ such that if } C_M(Z;\succsim) &\equiv S \neq \emptyset \text{ and } C_M(P;\succsim) \equiv S' \neq \emptyset \\ \text{with } S \cap S' &\equiv \emptyset \text{ and } S \cap P \neq \emptyset \\ \text{then } C_M\left(Z_{(\bmod\,\theta;\eta)};\succeq\right) &\equiv \widetilde{S} \text{ with } \widetilde{S} \subseteq P \text{ and} \\ C_M(P;\succsim) &\equiv S' \text{with } S' \cap \widetilde{S} \neq \emptyset \end{aligned}$$

**Proof.** Suppose that $C_M$ is not rationalized by $\succsim$ . By definition, it is true that $r(s) = 0$ for any $s \in S$ and that $r(z) > 0$ for any $z \in Z - S$. Pick any $m \in \Lambda = (Z - S) \cap P = P - (S \cap P)$ with $r(m) = k > 0$. and consider a pure choice superstructures $(\bmod\,\theta^*;\eta^*) \in \Omega \times \Upsilon$.

We have two cases. Let us deal with them separately.

**CASE 1:** $s \neq m\,(\bmod\,\theta^*)$ for any $s \in S$ and $s = z\,(\bmod\,\theta^*)$ for some $z \in Z - S$

In this case, take an allo-morphism $\eta^*$ such that

$$r(\eta^*(z)) = r(\eta^*(s)) = r(m) \text{ and } r(\eta^*(m)) = r(s) \tag{14}$$

Using (14) it is possible to get the following induced poset:

$$Z_{(\bmod\,\theta^*;\eta^*)} := \left\{[x]_{(\bmod\,\theta^*;\eta^*)} \text{ for } \forall x \in Z\right\}$$

with

$$C_M\left(Z_{(\bmod\,\theta^*;\eta^*)};\succsim\right) = \widetilde{S} \neq \emptyset \text{ and } \widetilde{S} \subseteq P$$

Since $m \in \Lambda$ then

$$C_M(P;\succsim) = S' \supseteq \Lambda \text{ and } C_M(P;\succsim) \cap C_M\left(Z_{(\bmod\,\theta^*;\eta^*)};\succsim\right) \supseteq \Lambda \neq \emptyset$$

hence $C_M$ is rationalized by an order relation and an opportunely defined categorial transformation.

**CASE 2:** $s = m\,(\bmod\,\theta^*)$ for any $s \in S$ and $s \neq z\,(\bmod\,\theta^*)$ for some $z \in Z - S$

In this case, take simply $\eta^*$ such that

$$r(\eta^*(m)) = r(s) \text{ and } r(\eta^*(s)) = r(s) \tag{15}$$

Thus, we have that

$$C_M\left(Z_{(\bmod\,\theta^*;\eta^*)};\succsim\right) = \widetilde{S} \neq \emptyset \text{ and } \widetilde{S} \subseteq P$$

and as above $C_M$ can be rationalized using $\succsim$ and $(\operatorname{mod}\theta^*;\eta^*)$ .This concludes the proof. ■

Proposition 1's insights are prefectly consistent with Katzner's conclusion that all choice behaviour are explainable in terms of condition $\alpha$ once culturally determined factors are appropriately taken into account. Intuitively, this finding shows that all non-rationalizable maximal set choice functions can be rationally explained with respect to some external consistency requirements. External references of choice are here depicted as choice super-structures which has to be correctly specified[18]. These super-structures can be self-imposed or not and their effect is to modify how the choice conditions are perceived and understood. Decoding them, it would be more likely to understand menu-dependent or background-conditions-dependent choices[19]. For illustrative purposes, before moving to the next section, we provide, as an application of Proposition 1, a quick rationalization of the mangos and apples choice problem.

**Example 7:** *Menu-Dependent Preferences and Choice Super-Structures*

Consider the following choice set

$$Z = \left\{m^1; m^2; a^1; a^2\right\}$$

with the order structure represented in Figure 1. As discussed above, $C_M(Z;\succsim) := \{m_1; m_2\}$. Then, take a subset $P = \left\{m^1; a^1; a^2\right\}$ with $C_M(Z;\succsim) \cap P := \left\{m^1\right\}$ and $\Lambda := \left\{a^1; a^2\right\}$. As known, in order to contraddict property $\alpha$ we must have that $C_M(P;\succsim) \neq \left\{m^1\right\}$ like in the orginal case (see Example 1). Now, let us imagine the following pure choice super-structure. An equivalence relation of the kind "*given fruits'aspect and shape any mango (and any apple) is equivalent to the other*" is accepted. Formally,

$$m^1 = m^2 (\operatorname{mod}\theta^*)$$

Furthermore, a kindness-oriented allo-morphism suggests to value other agent's freedom of choosing what he or she likes more. This modifies choice set structure in the following way:

$$\begin{aligned} r\left(\eta^*\left(a^1\right)\right) &= r\left(\eta^*\left(a^2\right)\right) = r\left(m_1\right) \\ r\left(\eta^*\left(m^1\right)\right) &= r\left(a^1\right) = r\left(a^2\right) \end{aligned}$$

[18] In fact, there exixt other conceptions of what external references of choice are. For instance, Sen uses the adjective *external* referred to anything that drives choices (preference or utility maximization, well-being maximization, other values and so on) in opposition to internal features of the process of choice (like contraction or expantion consistency). Differently, Gaertner and Xu (1996), (1999) show that different choice functions can be rationalized taking into account different external references here interpreted as axioms that a choice function has to satisfy (precisely properties of path dependency or balanced choice).

[19] Note that what Sen (1997) calls a *permissibility function*, in symbol $K$ such that $K(S) \subseteq S$ where $S$ is the option set, might be seen as a special choice super-structure that defines what is and is not permissible and what is maximal given actual permissibility conditions.

Hence, using the induced ordered subset $Z_{(\bmod\,\theta^*;\eta^*)}$ it is possible to show that

$$C_M\left(Z_{(\bmod\,\theta^*;\eta^*)};\succsim\right) := C_M\left(P;\succsim\right) := \left\{a^1;a^2\right\}$$

Then, the maximal set value choice function is not rationalizable through an order relation but choice behaviour can be explained using a well-shaped transformation of the choice set. ■

# 5 Maximization and Optimization

The reader has surely noticed that so far we have only dealt with maximal set choice functions. Let us now consider which relation occurs between maximization and optimization in a contextually embedded choice theory. In his 1997 paper, "*Maximization and the Act of Choice*", Sen defines relations between maximizing and optimizing choice behaviour. His main findings may be summarized as follows:

**(i)** main contrasts between maximization and optimization arises from imcompleteness of preference rankings. Where an order relation on a non-empty, finite choice set is complete, reflexive and transitive maximal and optimal sets coincide.

**(ii)** for partially incomplete preference rankings, any optimal set of alternatives is a proper subset of the maximal set defined on the same choice set

**(iii)** any maximization problem can be viewd as an optimization one with respect to an opportunely defined *as if* order relation. Nevertheless, the converse is not always true.

On issues (i) and (ii), Quizilbash (2002) rightly notices that Sen's analysis is mainly referred to *commensurate alternatives* in Broome's (2000) sense[20]. Nevertheless, following Griffin's (1986) notion of "*roughly equal in value*" options, alternatives may be *on a par*, that is being comparable without being B-commensurable. Hence, Quizilbash defines a "*roughly equal in value*" binary relation $\succsim^A$as an order relation such that for $\forall x, y \in X$

$$x \succsim^A y \iff x \text{ and } y \text{ are comparable and } \neg\left(y \succ x\right) \tag{16}$$

reformulating Sen's results in terms of $\succsim^A$ . His findings show that any optimal set of alternatives with respect to a finite, non-empty choice set and a

[20] B-commensurability of an order relation involves that for $\forall x, y \in X \Rightarrow x \succsim y$ and $y \succsim x$.

"roughly equal in value" order relation can be exactly replicated by a maximal set defined on B-commensurate options and a traditionally defined preference relation. Thus, any maximizing behaviour can be matched to an optimization exercise taking into account $\succsim^A$ and allowing for some degree of incommesurability among alternatives. Hence, roughness has to be read as something ineradicably in alternatives themselves which makes us unable to commisurate differences among choice options even with more available information or more penetrating examination[21]. Roughly equal in value alternatives are options perceived as equivalent and non-commensurable because of their intrinsic nature (for instance human development dimensions) or the context in which choice has to be undertaken (as in Sophie Choice-like examples).

Hence, using Quizilbash's insights, equivalence choice super-structure may be of some help in investigating effects that contextually determined roughness among options have on the relation between maximal and optimal choice sets (point (iii) above).

Let $Z \subseteq X$ be finite, non-empty ordered choice sets and $\theta$ be an equivalence relation defined consistently with some external (to the choice itself) normative principles, values or norms. Furthermore, let $C_B\left(Z_{(\theta;\eta_0)};\succsim\right)$ denote the optimal set choice function whereas an equivalence choice super-structure is imposed on $Z$. Using $\theta$'s properties it is possible to prove the following:

**Proposition 2:** *For any finite, non-empty ordered choice set $Z \subseteq X$ exists an equivalence choice super-structure $(\theta^*;\eta_0) \in \Omega \times \eta_0$ and an induced poset $Z_{(\theta;\eta_0)} \subseteq X$ such that*

$$C_M(Z;\succsim) \equiv C_B\left(Z_{(\theta^*;\eta_0)};\succsim\right)$$

**Proof.**

Suppose that $C_M(Z;\succsim) := \Phi := \{a_i\}_{i=1,\ldots,M} \subseteq Z$ is the maximal set with respect to $Z$. Then, by definition, $\nexists c \in Z$ such that $c \succ a$ for $\forall a_i \in \Phi$. Hence, for $\forall c \in Z$ it must be that either $c \sim a_i \in \Phi$, but then $c \in \Phi$ or that $c \prec a_i \in \Phi$ and hence $c \in Z - \Phi$.

If this is the case, for $\forall c \in Z - \Phi$ and $\forall a_i \in \Phi$ it is true that

$$a_i \wedge c = a_i \text{ and } a_i \vee c = c$$

Take an equivalence choice super-structure $(\theta^*;\eta_0) \in \Omega \times \eta_0$ such that for $\forall a_i \in \Phi$

$$a_1 = \ldots = a_M \,(\operatorname{mod} \theta^*)$$

Then, properties of an equivalence relation state that

$$a_i = a_i \vee a_{-i} \,(\operatorname{mod} \theta^*) \text{ and } a_i = a_i \wedge a_{-i} \,(\operatorname{mod} \theta^*)$$

[21] In Sen's (1997) words, we can speak of *assertive incompleteness*.

Thus, for any $c \in Z - \Phi$, it is true that for $i = 1, ..., M$

$$\begin{aligned} [a_i \wedge a_{-i} (\operatorname{mod} \theta^*)] \wedge a_i &= [a_i \wedge a_{-i} (\operatorname{mod} \theta^*)] = a_i \\ [a_i \wedge a_{-i} (\operatorname{mod} \theta^*)] \vee c &= c \\ [a_i \vee a_{-i} (\operatorname{mod} \theta^*)] \wedge a_i &= [a_i \vee a_{-i} (\operatorname{mod} \theta^*)] = a_i \\ [a_i \vee a_{-i} (\operatorname{mod} \theta^*)] \vee c &= c \end{aligned}$$

and then it must be that for any $a_i \in \Phi$

$$a_i \succsim c (\operatorname{mod} \theta^*)$$

Hence, using an order isomorphism $f_{\theta^*} : Z \rightarrow Z_{(\theta^*;\eta_0)}$ associated with $\theta^*$, we can define the following induced poset:

$$Z_{(\theta^*;\eta_0)} := \{f_{\theta^*}(x) \text{ for any } x \in Z\}$$

Then

$$C_B \left(Z_{(\theta^*;\eta_0)}; \succsim\right) := \Phi = C_M (Z; \succsim)$$

This concludes the proof. ■

Intuitively, $(\theta; \eta_0)$ may be seen as contextually imposed assertive incompleteness which modify choice set's structure. Noteworthy, it may also viewed as a binary relation though which roughly equality can be directly defined. As in Quizilbash's (2002), choice super-structures ensure that any maximizing behaviour can be mimicked by an opportunely defined optimizing behaviour with respect to some "on a par" alternatives.

The following example illustrates our last result.

**Example 8:** *Equivalence Choice Super-Structures*

Suppose that $Z := \{x, y, z, v, w\}$ with order relations as in Figure 2 (a)'s diagram and that $C_M (Z; \succsim) := \{x, y, z\}$. Now, let us consider an equivalence choice super-structure $(\theta^*; \eta_0) \in \Omega \times \eta_0$ with a related order isomorphism $f_{\theta^*}$ such that

$$\begin{aligned} x &= y = z (\operatorname{mod} \theta^*) \text{ and } v = w (\operatorname{mod} \theta^*) \\ f_{\theta^*}(x) &= f_{\theta^*}(y) = f_{\theta^*}(z) \succsim f_{\theta^*}(v) = f_{\theta^*}(w) \end{aligned}$$

Thus, taking $Z_{(\theta^*;\eta_0)} := \{f_{\theta^*}(x) \text{ for any } x \in Z\}$, it must be that

$$C_B \left(Z_{(\theta^*;\eta_0)}; \succsim\right) = C_M (Z; \succsim) := \{x, y, z\} \blacksquare$$

Finally, using Proposition 1 and 2 it is immediate to show that:

**Proposition 3:** *For any finite, non-empty ordered choice set induced by an equivalence relation $\theta^* \in \Omega - \theta_0$, exists a pure choice super-structure which rationalizes not rationalizable optimal set choice functions.*

**Proof.** Suppose a non-rationalizable optimal set choice function defined on $Z_{(\theta^*;\eta_0)}$. By Theorem 2, using an inverse isomorphism $f_{\theta^*}^{-1} : Z_{(\theta^*;\eta_0)} \to Z$ and removing equivalences, we may determine an poset $Z \subseteq X$ such that

$$C_B\left(Z_{(\theta^*;\eta_0)}; \succsim\right) = C_M\left(Z; \succsim\right)$$

with $C_M\left(Z; \succsim\right)$ non-rationalizable by $\succsim$. But then, by Theorem 1, exists a pure choice super-structure $(\operatorname{mod}\theta^*; \eta^*) \in \Omega \times \Upsilon$ which may rationalize $C_M\left(Z; \succsim\right)$ and $C_B\left(Z_{(\theta^*;\eta_0)}; \succsim\right)$. ■

# 6 Intransitive Choices and Permutative Choice Super-Structures

Let us conclude mentioning what role permutative choice super-structures have inside our theory. As defined above, a permutative choice super-structure is a choice set transformation which simply modifies alternatives' ranks at a given time. As other choice super-structures, it may be due to internalized moral principles, cultural norms or social influences. Differently by other transformations, it is not accompained by an equivalence relation which makes some options B-incommensurable.

Nevertheless, changing options' relative rank in agent's choice set, some choice intransitivities might phase out expecially whereas transitivity is applied to preferences at a single point of time[22]. Instrantivity has been ruled out by RCT since it does not allow preferences' representability and give raise to the well-known *money pump argument*, a *reductio ad absurbum* which shows that an intransitive chooser can be made to give up all his money. As argued in Anand (1993), intransitive agents are seen by "*tellers of the money pump story*" as Panglossian fools with contraddicting preference structures of the kind:

$$a \succsim b,\ b \succsim c \text{ and } c \succsim a$$

Intrasitivity involves that agents are willing to trade $c$ with $b$ paying some money, $b$ and some wealth for $a$ and again they are willing to give up $a$ for $c$ paying again. The circle goes on and intransitive agents are pumped dry of money. Surely, inside RCT money-pump-like-arguments have disruptive effects. As Anand recognises, these are mainly consequences of thinking transitivity as something referred to preferences at a given time. Since the above paradox is a multi-period example, endogeneous preference changes (Etioni (1985)) or framing effects (Kahneman D. and Tversky A. (1984)) cannot be ruled out at

[22] The same observation can be easily extended to pure choice super-structures. Hence, our argument has to be applied to those transformations too.

least whereas a dynamic solution of the paradox is searched. Moreover, he continues, even a simultaneous explaination of the money pump story cannot have disastrous consequences as expected. In fact, introducing the notion of *counterfactual connective*[23] (i.e. an assertive proposition of the kind "*if it were the case that...then the following would happen*") and defining three dinstinct choice sets $Z_1 := \{a, b\}$, $Z_2 := \{b, c\}$, $Z_3 := \{a, c\}$ , intransitive choice may be rationalized by unexpected changes in choice counterfactuals.

Once more, a slightly modified version of the mangos and apples example can be useful in order to appreciate Anand's argument.

**Example 9:** *Mangos, Apples and The Intransitivity of Choice (Anand (1993))*

During a dinner in a friend's house, some fruits are offered to our chooser. If he/she is offered a mango and a small apple, he/she would choose to have a mango, and if his/her host proposes a mango versus a big apple, the choice would be for the latter. Nevertheless, once two apples of different dimensions are offered, our chooser will opt for the small one, violating transtivity. This apparently irrational behaviour can be rationalized using a particular counterfactual connective of the kind "*if it were the case that two fruits of different dimensions are offerred, then, if etiquette matters, it would be better to leave the biggest one to our host*". As Anand underlines, it is difficult to judge such a behaviour as irrational. ■

Similarities between our approach and Anand's idea of counterfactual connectives are evident. Moral principles, cultural norms or other contextually determined external constraints may, *via* counterfactuals, radically transform chooser's preference ranking, generating intransitive, but not irrational, behavior[24]. In our theory's words, the same conclusion can be reached using *permutative choice super-structures* which may be seen as functional versions of counterfactual connectives for ordered choice sets. To see this, let us consider the three choice sets defined above. Whereas preferences are like the money pump paradox's ones, it is immediate to check that $C_B(Z_1; \succsim) := \{a\}$ and $C_B(Z_2; \succsim) := \{b\}$ and $C_B(Z_3; \succsim) := \{c\}$. In the last case, intransitivity of choice can be explained defining a permutative choice super-structure $(\theta_0; \eta^*) \in \theta_0 \times \Upsilon$ such that

$$\begin{aligned} \eta^*(r(a)) &= r(c) \\ \eta^*(r(c)) &= r(a) \end{aligned}$$

which may be semantically declined invoking etiquette.

[23] For an analysis of counterfactuals see Lewis (1986).

[24] Anand (1987) discusses under which conditions a transitive description of an intransitive behaviour can be given. His suggestion is to introduce more complex choice primitives than simple and static order relations. Exactly counterfactuals may works well for these porposes.

# 7 Final Remarks

In this essay, we have proposed a possible direction for extending RCT using some concepts of poset mathematics. Specifically, following Sen's work on rationality, we have dealt with the issue of how the context of choice may affect choice behaviour through culturally, socially or positionally imposed norms, principles or values. As we have seen, using equivalence relations, isomorphisms and allo-morphic applications, it is possible to explain chooser-dependent preferences, menù-dependent preferences as well as it is relatively straightforward relating optimizing and miximizing behaviour. Hence, some starting elements for a *contextually-embedded choice theory* (which takes into account self imposed constraints, permissible behaviour given some cultural and social norms, values, chooser's identity, preferences' assertive incompleteness and intransite choices) do emerge. Surely, these tools have to be further developed (even using different analytical frameworks) in order to reach a full account of what behaviour, within influencing groups, communities or informal networks, may be valued as a rational. Our proposal is a first attempt to internalise in the economic discourse contextually-provided criteria of rational behaviour. This seems us consistent with what recently reminded by Lukes (2000):

> rational behaviour not only must possess *internal* criteria of truth and logic, but also it has to be related with *contextually-provided criteria*, specifying which beliefs and behaviour may acceptably go together and accoding to which beliefs may count as true or false, meaningful or non-sensical, appropriate or inappropriate in the circumstances, soundly or unsoundly reached, properly or improperly held and, in general, based on good or bad actions (Lukes (1970), p.263).

Contextually-provided criteria of rationality may easily explain second/third best choice behaviour, as shown by Katzner (2000), but they also could play a role in the understanding of dynamically inconsistent behaviours through description of how frames of choice are evolved. Case-based reasoning and decision can be easily approached as well, specifying which context cues are in action and what frame of choice they determine.

Obviously, some vagueness in this approach is not avoidable. The reason of this is twofold. On the one hand, different descriptions or perceptions of the same context of choice can co-exist as well as choosers' identities and attitudes may be reflected in different choice super-structures. On the other hand, the same social, cultural or positional norm or factor may be differently internalized by individuals as well as individuals' behaviour can be influenced by some cultural, social or positional values instead of others. Using Sen's expressions, our theory puts identity before reason and not reason before identity as RCT does (Sen (1999)). Moreover, his vagueness introduces rooms for a non-mechanical approach to rationality (Sen (1985)): choice super-structures as well as social relations or cultural influences must be decoded and understood case by case, individual by individual. Suited-for-all criteria of rationality (as property $\alpha$ or the axioms of revealed preferences) are automatically ruled out.

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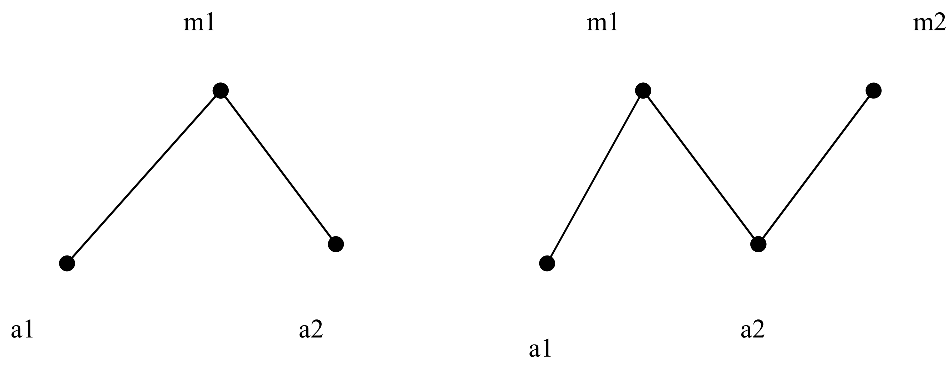


**FIGURE 1**

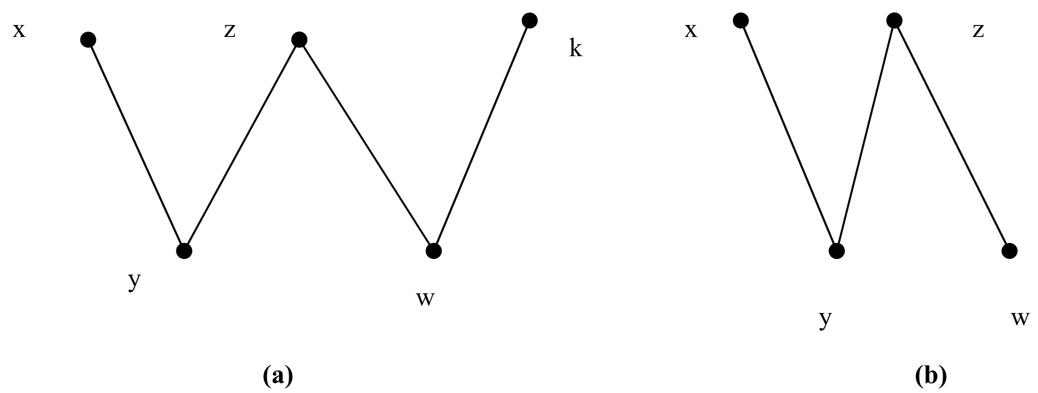


**FIGURE 2**