Complementarity between product and process innovation in a monopoly setting.[1]

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January 2005

[1]This paper is part of my PhD Thesis at CORE, Université Catholique de Louvain. I would like to thank Rabah Amir, Claude d'Aspremont, Luca Lambertini, Giordano Mion, Jacques-François Thisse, Vincent Vannetelbosch and the seminar audience at Université Catholique de Louvain for useful comments and suggestions. Financial support from CORE is gratefully acknowledged. The usual disclaimer applies.

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Abstract

We study complementarity between product and process innovation in a monopoly setting. First we consider the possibility for the firm to alternatively invest only along one of the two directions and compare the incentives of process *vs* product innovation. Then we allow the firm to invest simultaneously in both activities, showing that both investment levels as well as the profit are higher than in case of individual investment. Product and process innovations are then complementary and the firm always prefers a simultaneous adoption.

Keywords: Complementarity, product innovation, process innovation.

JEL classification: L12, O31.

1 Introduction

The theory of comparative statics, initially investigated by Samuelson, has recently received renewed attention thanks to the improvement in the theory of supermodular games, introduced by Topkis (1978) and further studied by Milgrom and Roberts (1990a, 1990b, 1995), Vives (1990) and Amir (1996), *inter alia*. The issue of complementarity has become a leading field of research both in games with strategic complementarities among the strategies of the players and in games with players that face multidimensional strategy space.

The study of innovative activity has represented one of the first field where complementarity has led to very interesting results. In particular, the analysis of product innovation and process innovation has been recently enriched by many contributions. While in a recent past the focus has been on the choice *between* these two types of activities, recent works tend to stress the possibility that some complementarities may appear when the two innovative activities are taken together.

Athey and Schmutzler (1995) analyze several features of a firm's long-run decisions about organizational structure that affect its short-run innovative activity. In particular, they consider an investment in research capabilities that improves future opportunities for new product and process innovation. Product innovation is formalized as an upward shift in the demand curve, while process innovation lowers the marginal cost of production. The authors focus on the conditions under which product and process innovations, as defined in this way, are complementarity and the returns accruing from one type of innovation are at highest when the firm also adopts the other type. This clearly induces the firm to implement both types of innovation at the same time.

Lin and Saggi (2002) investigate the relationship between process and product innovation both under Bertrand competition and under Cournot competition in a duopoly model with differentiated goods. Following the conventional view on the timing of adoption, they build a three stage game where firms first choose their investment in product innovation, then the investment in process innovation and finally they compete either in prices or in quantities.[1] In their model product innovation affects the degree of product differentiation by reducing product substitutability. The effect on the output by those two kinds of R&D leads to a two-way complementarity in which the investment in one makes the other more attractive. As a consequence, firms invest more in product innovation when they can undertake also process innovation. Moreover, the equilibrium level of process innovation is an increasing function of the degree of product differentiation.

Other papers deal with the topic of product and/or product innovation without taking into explicit account the issue of complementarity. Lambertini and Orsini (2000) analyze the incentive to introduce process and product innovation in a vertically differentiated monopoly. Rosenkranz (2003) considers a two-stage Cournot duopoly model with horizontal differentiation, where in the first stage firms simultaneously choose the product characteristic and the unit cost. Eswaran and Gallini (1996) examine the role of patent policy in redirecting the mix of product and process innovation towards a more efficient technological change. Bonanno and Haworth (1998) provide an interesting duopoly model which explains the R&D choice according to the type of competitive regime where firms operate (Cournot vs. Bertrand), even if firms are not allowed to conduct both types of R&D.

In this paper we analyze a monopoly case where the firm faces the possibility of investing

[1] Abernathy and Utterback (1975 and 1982) and Klepper (1996) propose a "technological life-cycle" model in which firms initially direct most of their R&D resources to product innovation and then move to process innovation. The main argument is that the returns to product innovation are at highest at the very beginning because they depend on the acquisition of new consumers, while the returns to process are very attractive in a second time becuase they are proportional to the level of output produced by the firm. Adner and Levinthal (2001) recently moved some critics to what they called a "supply-side" view of technological change in favor of a "demand side" approach where technology changes are driven by the interaction between technology development and consumers' heterogeneous demands.

both in product and in process innovation. Product innovation shifts the demand curve by increasing consumers' willingness to pay, while process innovation takes the conventional form of a reduction in the unit cost of production.

We initially consider the alternative for the firm to invest only along one direction and compare the profitability of process *vis à vis* product innovation.[2] Two factors will determine which innovative activity has to be carried out: (i) the relative efficiency of product *vs* process innovation; (ii) the potential market expansion *vs* the potential cost reduction. The first factor reflects the conventional approach based on comparing the cost of implementing different innovative activities, while the second factor points out that initial conditions may play a fundamental role in such a decision.

The second part of the paper deals with the possibility for the firm to invest in both activities at the same time. The aim is to show that demand enhancing product innovation and cost reducing process innovation are complementary. The firm obtains then a higher profit when it invests simultaneously along both directions, because the gains brought about by the market enlargement expand the returns arising from the reduction of the unit cost of production. Moreover, we verify that complementarity holds even in circumstances where the firm is impeded from market constraints to invest at the optimal level.

The paper proceeds as follows. In Section 2 we present the basic model and consider the cases where the firm invests only along one direction, either process or product innovation. Section 3 provides a comparison between the two types of innovative activities. In Section 4 we move to the case where the firm invests in both activities at the same time, focusing on the presence of complementarity and its effects on equilibrium investment levels and overall profit. Section 6 concludes the paper.

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[2] Halmenschlager (2004) considers a modification of the standard two-stage model wherein two high-cost firms conduct cost-reducing R&D, in a setting with spillovers, and then Cournot compete against a low-cost firm that engages in no R&D.

2 The model

Consider a monopoly case where a firm produces a homogeneous good at the marginal cost c. The inverse demand function is linear and given by

$$p = a - bq \tag{1}$$

where a represents the initial marginal willingness to pay and b the slope (in absolute value) of the demand function. We assume that the firm produces without incurring in any fixed cost. The profit function is then given by:

$$\pi = (p - c)\, q \tag{2}$$

From profit maximization we get the equilibrium quantity $q^* = \dfrac{a-c}{2b}$ by which it can be easily computed the equilibrium profit:

$$\pi^* = \frac{(a-c)^2}{4b}. \tag{3}$$

Let introduce the alternative for the monopolist to activate R&D along two different directions, either process innovation or product innovation.

2.1 Process innovation

The firm is allowed to invest in a type of R&D that lowers the marginal cost of production, i.e. it does process innovation R&D. Following the common assumption of diminishing returns for R&D expenditures (see d'Aspremont and Jacquemin, 1988; Qiu, 1997), the cost is given by $c(x) = \gamma \dfrac{x^2}{2}$, where x indicates the investment level in process R&D while γ inversely measures the efficiency of such R&D technology: the higher its value, the more expensive (i.e. the less efficient) is process R&D, and viceversa. The profit function becomes:

$$\pi_x = \frac{(a-c+x)^2}{4b} - \gamma \frac{x^2}{2} \tag{4}$$

The monopolist maximizes w.r.t. x; the profit function is concave when $\gamma > \gamma_1 = \dfrac{1}{2b}$ and by using first-order conditions we get the equilibrium investment level in process R&D:

$$x^* = \frac{a-c}{2b\gamma - 1} \tag{5}$$

Moreover, we take into account a feasibility condition that guarantees the presence of non-negative costs of production. This requires $x^* \leq c$, where c represents an upper bound in the possibility of invest to abate production costs, and it holds when:

$$\gamma \geq \gamma_2 = \frac{a}{2bc}. \tag{6}$$

It is easy to verify that $\gamma_2 > \gamma_1$. In $\gamma_1 < \gamma < \gamma_2$, the profit function is concave in x but the optimal value x^* is out of reach, given that it would entail a negative cost of production. The monopolist sets then $x = c$ and obtains the profit:

$$\pi_c = \frac{a^2 - 2bc^2\gamma}{4b} \tag{7}$$

which is always positive and greater than (3) in the interval under consideration.

When $\gamma \geq \gamma_2$ the monopolist is 'free' to select the optimal investment level (5) and obtains:

$$\pi_x^* = \frac{(a-c)^2}{2(2b\gamma - 1)} \tag{8}$$

Lastly, in $0 < \gamma < \gamma_1$ the profit function is convex in x and one has to compare the profits accruing at the extrema of the variable choice, i.e. π_c (where $x = c$) *vis à vis* π^* (where $x = 0$). It is straightforward to verity that $\pi_c > \pi^*$ and the firm invests in process innovation up to the admissable limit, due to the very high efficiency of such activity (very low values of γ). We can summarize the above discussion as follows:

Remark 1 *When $0 < \gamma \leq \gamma_2$, process R&D is very efficient and the firm sets $x = c$ obtaining the profit π_c, while for lower levels of process R&D efficiency, i.e. when $\gamma > \gamma_2$, the firm chooses $x = x^* < c$ and gets the profit π_x^*.*

2.2 Product innovation

The firm invests in product innovation to raise consumers' reservation price. Similarly to the previous case, the cost is given by $c(y) = \delta\frac{y^2}{2}$, where y indicates the investment level and δ inversely measures the efficiency of product innovation R&D. We assume that implementing product innovation shifts the market demand outward. The profit function becomes:

$$\pi_y = \frac{(a - c + y)^2}{4b} - \delta\frac{y^2}{2}, \tag{9}$$

which is concave w.r.t. y when $\delta > \delta_1 = \frac{1}{2b}$. In this case, the first-order condition gives the equilibrium investment level in product R&D:

$$y^* = \frac{a - c}{2b\delta - 1} \tag{10}$$

As for feasibility requirements, we assume that the demand expansion due to the increase in the reservation price cannot exceed a maximum level parameterized by A. As a consequence, we impose an upper bound on the optimal investment level in product innovation given by $y^* \leq A - a$, which holds if:

$$\delta \geq \delta_2 = \frac{A - c}{2b(A - a)}. \tag{11}$$

Notice that $\delta_2 > \delta_1$; as before, we have to consider two subintervals. In $\delta_1 < \delta < \delta_2$ the equilibrium level y^*exceeds the maximum demand enlargement; the monopolists is obliged to fix $y = A - a$, obtaining

$$\pi_A = \frac{(A - c)^2 - 2b\,(A - a)^2\,\delta}{4b} \tag{12}$$

which is always positive and greater than (3) in the region that we consider.

In $\delta \geq \delta_2$ the monopolist chooses the optimal investment level (10) and gets:

$$\pi_y^* = \frac{(a - c)^2}{2(2b\delta - 1)} \tag{13}$$

Finally, in $0 < \delta < \delta_1$, the profit function is convex in y and by comparing π_A (where $y = A - a$) *vis à vis* π^* (where $y = 0$) we verify that $\pi_A > \pi^*$: the firm invests in product up to the admissable market expansion. Then:

Remark 2 *When* $0 < \delta \leq \delta_2$*, product R&D is very efficient and the firm sets* $y = A - a$ *obtaining* π_A*, while for* $\delta > \delta_2$*, i.e. for lower levels of product R&D efficiency, the firm chooses* $y = y^* < A - a$ *and gets the profit* π^*_y*.*

Caveat: market expansion and utility function In our model product innovation shifts the demand function by raising consumers' willingness to pay. The net effect of such R&D investment is doubtlessly equivalent to an investment in advertising that shifts demands curves. In general, any demand enhancing investment activity gives rise to a complex issue if it also shifts the individual utility function from which the demand curve is derived, as initially pointed out by Dixit and Norman (1978).[3] The usual criticism regarding the welfare comparisons also applies to the profit comparisons carried out in our model.

We can therefore use a *relative* measure that captures the ability of the monopolist to extract profit from total surplus, usually defined as the sum of consumers' and producers' surplus. In the benchmark case, where the monopolist does not carry out any investment activity, demand is stuck at the initial level (1) and profit is given by (3). Total surplus can be easily computed and amounts to:

$$TS^* = \frac{(a-c)^2}{2b}. \tag{14}$$

It follows immediately that:

$$EE^* = \frac{\pi^*}{TS^*} = \frac{1}{2} \tag{15}$$

[3] See also Martin (2002), Ch. 9, for a comprehensive discussion.

where EE stands for *extraction efficiency,* the fraction of total surplus that the monopolist is able to capture.

Take now into account the case where the monopolist is allowed to invest in product innovation. Demand function becomes $p_y = a - bq + y$ and total surplus is given by:

$$TS_y = \frac{(a + y - c)^2}{2b}. \tag{16}$$

Backward induction solution gives rise to first-stage profit function (9) and the firm's extraction efficiency is the given by:

$$EE_y = \frac{\pi_y}{TS_y} = \frac{1}{2} - \frac{\delta b y^2}{(a + y - c)^2}. \tag{17}$$

As it can be easily noticed:

$$EE^* > EE_y \tag{18}$$

meaning that the fraction of total surplus that the monopolist is able to extract diminishes in the product innovation case. In absolute terms, however, cash flows are higher in case of demand expansion generated by product R&D than in case of no investment, and the monopolist decides to invest. The same reasoning applies for the process innovation case.

3 Process *vs* product innovation

In the previous part we have considered the choice of process R&D separately from the choice of product R&D. Remind that the monopolist sets $x = x^* < c$ for $\gamma > \gamma_2$ and $x = c$ for $0 < \gamma \leq \gamma_2$, while it chooses $y = y^* < A - a$ for $\delta > \delta_2$ and $y = A - a$ for $0 < \delta \leq \delta_2$. In Figure 1 we represent all the possible regions of interest in the comparison between the two types of innovation.

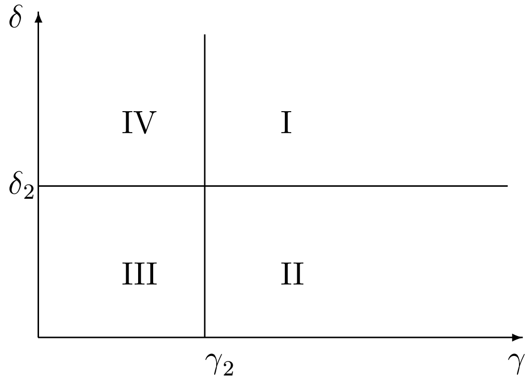

Figure 1: Process vs product R&D

We have then to perform the following comparisons:

- in region I ($\gamma > \gamma_2$ and $\delta > \delta_2$): π_x^* vs π_y^*;
- in region II ($\gamma > \gamma_2$ and $\delta < \delta_2$): π_x^* vs π_A;
- in region III ($\gamma < \gamma_2$ and $\delta < \delta_2$): π_c vs π_A;
- in region IV ($\gamma < \gamma_2$ and $\delta > \delta_2$): π_y^* with π_c.

First of all we compare γ_2 and δ_2 and find that:

$$\gamma_2 > (<)\delta_2 \;\; iff \;\; A - a > (<)c. \tag{19}$$

We define the potential increase in consumers' reservation price as *large* (*small)* when $A - a > (<)c$, and product (process) innovation is likely to be more profitable for firms. The following analysis will shed light on the effect of product and process innovation in the decision between the two kinds of activities.

Let us begin from Region I. It is easy to prove that:

$$\pi_x^* < (>)\pi_y^* \;\; \text{when } \delta < (>)\gamma.$$

Furthermore, when $\delta < (>) \gamma$ then $y^* > (<)\ x^*$. In line with the theoretical assumption of our model, when $\delta < \gamma$, the firm prefers to invest in product innovation because it can be implemented at a lower cost. The opposite holds when $\delta > \gamma$ and the firm chooses process innovation instead.

In the other regions the result crucially depends not only on the relative efficiency of process *vs* product innovation, but also on the potential shift in the demand function as compared to the potential reduction in the marginal cost of production. We find the following results:

- When $A - a > c$:
  - in Region II $\pi_A > \pi_x^*$;
  - in Region III $\pi_A > \pi_c$ for $0 < \delta < \dfrac{(A-c)^2 + 2bc^2\gamma - a^2}{2(A-a)^2 b} = \delta_A (< \delta_2)$;
  - in Region IV $\pi_y^* > \pi_c$ for $0 < \delta < \dfrac{2bc^2\gamma - a^2}{2bc^2(1+2b\gamma) - 4abc} = \delta_c (> \delta_2)$.
- When $A - a < c$:
  - in Region II $\pi_x^* > \pi_A$ for $0 < \gamma < \dfrac{(A-c)^2 - 2(A-a)^2 b\delta}{2b(A-a)\left[A + a - 2c - 2(A-a)b\delta\right]} = \gamma_A$ $(> \gamma_2)$;
  - in Region III $\pi_c > \pi_A$ for $0 < \gamma < \dfrac{a^2 - (A-c)^2 + 2(A-a)^2 b\gamma}{2bc^2} = \gamma_c (< \gamma_2)$;
  - in Region IV $\pi_c > \pi_y^*$.

Moreover, it is important to notice that the size of the four regions, being delimited by values taken by γ_2 and δ_2, depends in turn on the relative shift in the demand function, as we know from (19). In Figure 2 we represent the decision choice between process and product innovation in the $\{\gamma,\ \delta\}$ space in the two cases introduced before. In the left panel the potential demand expansion is supposed to be large, while in the right panel it is taken as small.[4]

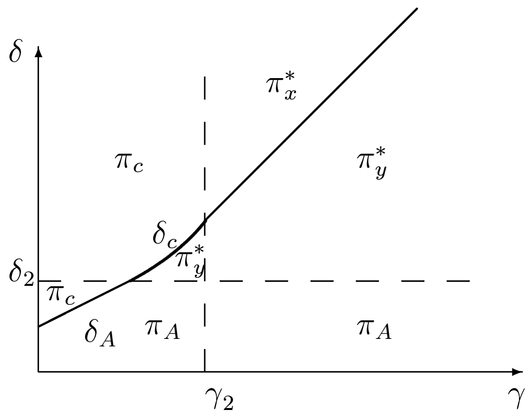

Figure 2a: $A - a > c$

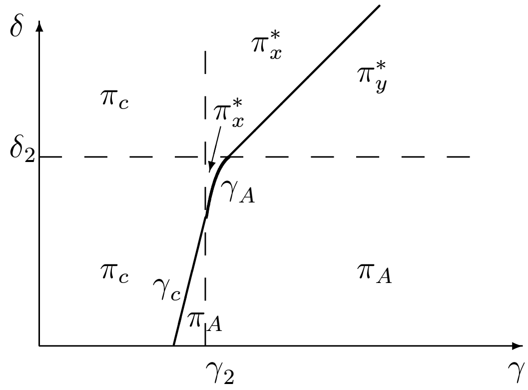

Figure 2b: $A - a < c$

The above representation clarifies the importance of initial conditions in deciding whether to invest in product or in process innovation. When the potential demand enhancement is large, the firm prefers product innovation. In particular, when both activities are very efficient, i.e. when both γ and δ are sufficiently low, the firm chooses product innovation because it arrives at the maximum expansion of the demand by spending a limited amount of money. Not only product innovation is very efficient, but also the perspective of market enlargement is very appealing. The opposite holds when the potential demand expansion is small: the firm prefers process innovation because it finds unprofitable to devote resources to product R&D in a situation where the perspectives of raising consumer's reservation price are limited.

The following proposition summarizes the above discussion:

Proposition 1 *When the potential demand enlargement is large, i.e. when $A - a > c$, the firm prefers to invest in product innovation, while the opposite holds when $A - a < c$.*

[4] We draw both pictures by taking $b = 1$, $c = 1$, $a = 1.5$. In the left panel we further impose that $A = 3$, hence $A - a > c$, while in the right panel $A = 2$ and $A - a < c$.

Hence, the choice between product and process innovation is dictated not only by the relative efficiency of the two activities but also by the potential demand expansion as compared to potential cost reduction. This result clearly indicates that *initial conditions play a crucial role* in determining the investment decision between product and process innovation. In other words, it suggests a closer attention to the nature of the product itself. In case of technologically "mature" goods, the possibility of furtherly shifting market demand is somehow limited and the firm concentrates on process innovation. On the contrary, new-to-the-market goods are usually very attractive for consumers and product innovation turns out to be more profitable.

4 Process *and* product innovation

In this section we analyze the possibility for the firm to invest in both kinds of R&D. The aim is to show that process and product R&D, as they are defined in our model, are complementary.

The profit function is given by:

$$\pi_{x,y} = \frac{(a-c+x+y)^2}{4b} - \gamma\frac{x^2}{2} - \delta\frac{y^2}{2} \tag{20}$$

Separate concavity with respect to each investment level still requires:

$$\frac{\partial\pi_{x,y}^2}{\partial x^2} < 0 \;\text{ for } \gamma > \gamma_1 \;\text{ and }\; \frac{\partial\pi_{x,y}^2}{\partial y^2} < 0 \;\text{ for } \delta > \delta_1, \tag{21}$$

while joint concavity with respect to both investment decisions holds if:

$$\gamma > \gamma_3 = \frac{\delta}{2b\delta - 1}\ , \tag{22}$$

where $\gamma_3 > \gamma_1$.

First, notice that:

$$\frac{\partial^2 \pi_{x,y}}{\partial x \partial y} = \frac{\partial^2 \pi_{x,y}}{\partial y \partial x} = \frac{1}{2b} > 0, \tag{23}$$

hence the two activities are complementary. Following Topkis (1978), in fact, a function is supermodular when cross-partial derivatives between each pair of variables are positive. This formalizes the notion of complementary investment opportunities. Any increase in one of the two variables raises the profitability of implementing the other. The economic interpretation is that the marginal returns from a demand enhancement investment activity increase with a reduction of the marginal cost due to process innovation. Alternatively, since supermodularity is a two-way complementarity relationship, an equivalent interpretation is that the benefit that the firm derives from a reduction of the marginal cost of production increases with the overall demand, i.e. with the acquisition of new consumers.

Assume for the moment that (22) holds; equilibrium investment levels are:

$$x^*_{x,y} = \frac{\delta (a-c)}{2b\gamma\delta - \gamma - \delta} > 0, \tag{24}$$

$$y^*_{x,y} = \frac{\gamma (a-c)}{2b\gamma\delta - \gamma - \delta} > 0. \tag{25}$$

Obviously, the firm devotes more resources to product (process) when $\gamma > \delta$ ($\gamma < \delta$) and its R&D portfolio depends on the relative efficiency of the two activities. It is easy to show that:

$$x^*_{x,y} > x^* \text{ and } y^*_{x,y} > y^*.$$

Due to the complementarity effect, both the investment level in process innovation and the one in product innovation are higher than in the case of investment along one direction.

As for the equilibrium profit, by plugging (24) and (25) in (20) and rearranging, we get:

$$\pi^*_{x,y} = \frac{(a-c)^2 \gamma\delta}{2(2b\gamma\delta - \gamma - \delta)} > 0 \tag{26}$$

which is higher than (8) and (13).

Another consideration deserves attention. As it stands, the overall effect of process innovation is exactly the same as the one of product innovation, given that both shift the demand curve outward. It can be argued that the separation of the two innovative activities taken into account in our model is somewhat unnecessary. However, this simply implies the presence of *cost complementarity* between the two options. When there are diminishing returns for R&D expenditures, two separated units working on demand expansion, one directly through an increase of consumers' willingness to pay and the other indirectly through a cost reduction, exploit complementarity and yield a higher profit for the firm.

Finally, one may argue that the results derived in the previous section are strongly affected by the adoption of a linear demand function, given that the solution basically depends on a change in $(a - c)$. Even if similar results can be shown to hold if one considers more complex demand and /or cost functions, we prefer to present an easy case where complementarity always holds because this allows to propose a model that combines technological and demand factors and to stress on the role played by initial conditions. Moreover, due to its simple structure, we are able to completely characterize the equilibrium solutions also in presence of market constraints, as it will be discussed in the following section.

4.1 Market constraints

In this section we consider the feasibility constraints that the firm faces when invests simultaneously in product and process innovation. Under (22) equilibrium investment levels are (24) and (25). Nonetheless, we need to verify the feasibility conditions, $x^*_{x,y} \leq c$ and $y^*_{x,y} \leq A - a$, yielding:

Lemma 1 *(i)* $x^*_{x,y} \leq c$ *when* $\gamma \geq \gamma_4 = \dfrac{a\,\delta}{c(2b\delta - 1)}$, *with* $\gamma_4 > \gamma_3$, *(ii)* $y^*_{x,y} \leq A - a$ *when* $\gamma \geq \gamma_5 = \dfrac{(A-a)\delta}{2b\delta(A-a) - (A-c)}$, *with* $\gamma_5 > \gamma_4$ *in* $\delta_2 \leq \delta \leq \delta_3 = \dfrac{A}{2b(A-a)}$.

In Figure 3 we represent in the $\{\delta, \gamma\}$ plan the additional feasibility requirements:

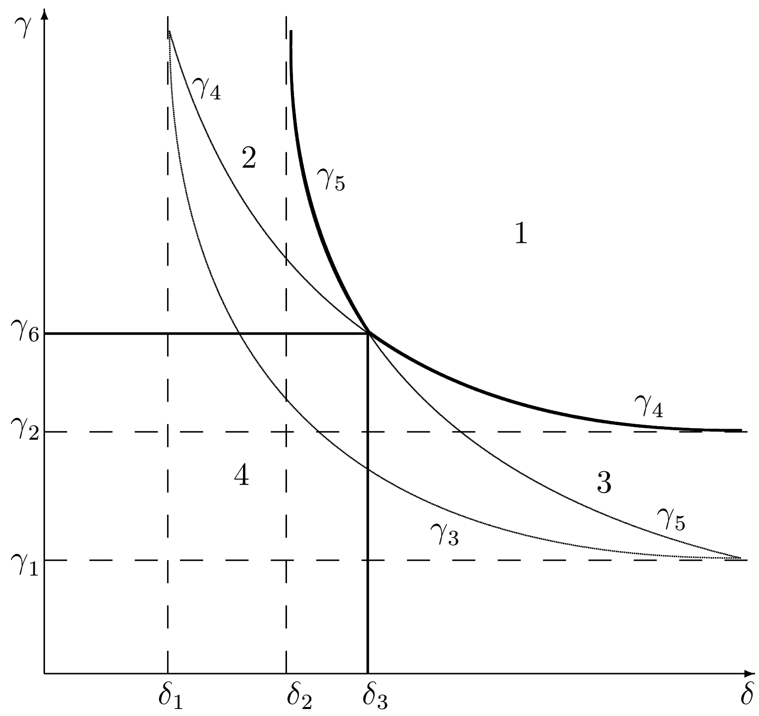

Figure 3: Feasibility conditions

In Region 1, where $\gamma > \max\{\gamma_4, \gamma_5\}$, the firm is 'free' to choose (24) and (25). As we pointed out, the equilibrium investment level in both activities as well as the profit are higher than in case of investment only along one direction.

In Region 2, where $\delta < \delta_3$ and $\gamma_4 < \gamma < \gamma_5$, the firm hits the constraint on product innovation. As a consequence, it sets the highest admissable level, $y = A - a$, and recomputes the optimal investment in process innovation, which is equal to:

$$\widehat{x}_a = \frac{A - c}{2b\gamma - 1}. \tag{27}$$

Concavity of the profit function now only requires $\gamma > \gamma_1$. It is immediate to prove that $x^* < \widehat{x}_a < x^*_{x,y}$. The equilibrium level of process innovation is still higher than in the individual case but it is smaller than in the unconstrained case, due to a weaker complementarity effect that derives from a lower investment in product innovation than in the optimal value. Moreover, notice that $\widehat{x}_a < c$ for $\gamma > \gamma 6 = \dfrac{A}{2bc}(> \gamma 1)$ and Region 2 expands then to include $\delta < \delta_3 \ \cup \ \gamma_6 \ < \gamma < \gamma_5$. The equilibrium profit becomes:

$$\widehat{\pi}_a = \frac{(A-c)^2 \gamma - (A-a)^2 \delta (2b\gamma - 1)}{2(2b\gamma - 1)} > 0 \tag{28}$$

and it can be easily proved that it is lower than the profit obtained in Region 1. One might wonder whether the monopolist still prefers to invest in both activities when at least one of them is constrained by the market conditions. By considering all the admissable alternatives, i.e. π_x^*, π_y^* (for $\delta > \delta_2$) and π_a (for $0 \leq \delta \leq \delta_2$) it is possible to show that $\widehat{\pi}_a$ corresponds to the highest admissable profit. Hence, in this parameter's region, there still exists a sufficient complementarity to make the monopolist willing to invest in both directions.

In Region 3, where $\delta > \delta_3$ and $\gamma_5 < \gamma < \gamma_4$, we face the opposite situation with respect to the previous case, but the results are analogous. The monopolist is obliged to fix $x = c$ and recomputes the optimal level of product innovation, given by:

$$\widehat{y}_c = \frac{a}{2b\delta - 1} \tag{29}$$

and $y^* < \widehat{y}_c < y_{x,y}^*$. Concavity holds if $\delta > \delta_1$, while $\widehat{y}_c < A - a$ for $\delta > \delta_3$. The area that we consider expands to $\delta > \delta_3 \cup \ \gamma < \gamma_4$. The equilibrium profit is then:

$$\widehat{\pi}_c = \frac{a^2 \delta - c^2 \gamma (2b\delta - 1)}{2(2b\delta - 1)} > 0. \tag{30}$$

The monopolist is obliged to set a lower value of process innovation then the optimal one and this reduces both the equilibrium level of product innovation and the equilibrium profit. By considering all the admissable alternatives, i.e. π_x^*, π_y^* and π_c it is possible to show that $\widehat{\pi}_c$ corresponds to the highest admissable profit. Complementarity is again strong enough for the monopolist to invest in both activities.

Finally, in Region 4, that after the previous considerations amounts to $\delta < \delta_3 \ \cup \ \gamma < \gamma_6$, the firm either faces constraints on both directions or neither joint concavity nor single concavity are preserved. In $\gamma > \gamma_3$, for example, the two activities are still complementary and the profit function is joint concave but the optimal investment levels cannot be reached. The firm could set $x = c$ and $y = A - a$ and get:

$$\widehat{\pi}_{ac} = \frac{A^2 - 2bc^2\gamma - 2b(A-a)^2\delta}{4b} \tag{31}$$

which is always positive in the parameter region that we consider. Moreover, it is easy to demonstrate that such a profit is the highest the firm can obtain. The same argument holds also when $\gamma < \gamma_3$ and either the profit function is not jointly concave or it is concave only w.r.t. one investment activity, and it even holds when it is convex w.r.t both process and product innovation investment levels.

We can summarize the main results as follows:

- 1) $x = x^*_{x,y}$, $y = y^*_{x,y}$ and $\pi = \pi^*_{x,y}$;
- 2) $x = \widehat{x}_a$, $y = A - a$ and $\pi = \widehat{\pi}_a$;
- 3) $x = c$, $y = \widehat{y}_c$ and $\pi = \widehat{\pi}_c$;
- 4) $x = c$, $y = A - a$ and $\pi = \widehat{\pi}_{ac}$.

From the above discussion, it emerges that:

Proposition 2 *Complementarity between process innovation and product innovation is preserved under feasibility constraints on equilibrium investment levels.*

As a result, the firm *always* prefers to activate both types of R&D activity, even in parameters' regions where feasibility constraints limit the amount that can be invested. However, this happens when either one or both innovative activities are very efficient and the firm decides to invest a lower-than-optimal level and enjoy complementarity gains.

Finally, it is worth noting that the combination between technological features, summarized by the relative efficiency of process and product innovation, and initial conditions on market demand and cost structure determine the amount of resources devoted to each innovative activity.

5 Conclusions

We have investigated a monopoly case where a firm can invest in product innovation which expands market demand and in process innovation which reduces the marginal cost of production. We have initially considered the two activities as separate decisions and have drawn interesting conclusions regarding the choice of the firm on whether to invest in process innovation or in product innovation. In particular, we have focused on the role played by the initial condition of the market, i.e. on potential demand improvement *vs* effective cost abatement.

The second part of the paper has dealt with the simultaneous implementation of process and product innovation. We have shown the existence of complementarity between a demand enhancing activity and a cost reducing one. As a consequence, the firm always prefers to invest in both activities at the same time to maximise its profit. Furthermore, we have demonstrated that such complementarity is preserved even in case of constraints on equilibrium investment levels. In other words, even in circumstances where market conditions do not allow for a considerable cost reduction and/or demand expansion, the firm gets a higher profit when invests in both activities rather than when it implements only one of them. Initial conditions act in combination with efficiency considerations to determine firm's optimal R&D portfolio.

Finally, we have highlighted the existence of a cost complementarity that reinforces the result that a joint adoption of the two activities is more profitable than the adoption of a single activity.

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