Market Expansion and Elasticity Improvement as Complementary Marketing Activities

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Abstract

Consider a marketing division of a monopoly firm that faces two marketing options: Market enlargement and elasticity improvement. These options are conceived in terms of the target of the firm's advertising campaigns: Potential new consumers versus existing consumers.

Using a CES demand function in a simple model, we demonstrate that the two activities are complementary, so that for some cost configurations, the firm will find it profitable to implement the two options together when either option alone would result in a loss. This calls for the marketing division to be integrated, rather than decentralized.

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1 Introduction

The concept of economic complementarity, initially introduced by Edgeworth and further developed as part of early modern economic theory by many pioneering scholars such as Samuelson (1974), has recently received renewed attention in the management science/operations research literature through the work of Topkis (1978, 1998), which gave it its modern elegant and unified mathematical structure. More recently, the topic resurfaced again in economics, emerging as a leading theme of economic research providing the ideal methodology for the study of games with strategic complementarities and their use in industrial economics (Vives, 1990; Milgrom and Roberts, 1990 and Amir, 1996) and for comparative statics analysis (Milgrom and Shannon, 1994).

Another major sphere where the modern concept of economic complementarity has proved fertile and inspiring is in the study of interconnected subsytems linked by complementarity relationships. This strand comprises scholarly work in social sciences, broadly construed, and includes the theory of the firm (Milgrom and Roberts, 1995), the emergence of modern manufacturing (Milgrom and Roberts, 1990), and the wholistic approach to socio-economic systems along many inter-related dimensions, including economic, social, financial, and historical factors (Hall and Soskice, 2001). The main idea that ties together the otherwise disparate studies that form the latter strand is that, in the presence of complementarities linking the various subsystems of a global system, a separate study of any one subsystem alone, ceteris paribus, is likely to be substantially off the mark in its key conclusions, simply due to its ignorance of the underlying complementarity relationships[1].

[1] For instance, one cannot simply compare the financial systems of the United States and Germany *in isolation* from their overall socio-economic context, and conclude that one of them is superior to the other. Instead, each of the systems is embedded in a much larger whole through a web of complementarity relationships that have direct impact on its performance. As a result, each of the

The present study is an attempt to apply this fundamental but simple idea to one of the main functions of the firm: Marketing/product development[2]. Our starting point is the well-known model of Dixit and Norman (1978), who evaluated the social desirability of advertizing. In their model, a firm facing an isoelastic demand curve can engage in advertizing activity as follows. It can make one type of advertizing expenditure that has two somewhat inter-related effects: *Market expansion* or an upward shift in the demand curve and *elasticity improvement*[3]. The former effect reflects any attempt to reach out to new potential consumers while the latter aims at increasing existing consumers' willingness to pay for the product at hand. Both effects may be achieved in the form of *informative* advertizing, which provides information about the existence or potential quality of a product, or in the form of *persuasive* advertizing, which aims at convincing the consumer, in one way or another, that what he really wants is the advertized brand. As to elasticity improvement, some types of advertizing could reduce product substitutability for a particular brand, and thus lower its elasticity of demand. This is true in particular for persuasive advertizing, which in some cases creates a kind of *perceived* product differentiation that does not correspond to a real difference in the product's performance or characteristics[4].

We modify the Dixit and Norman model insofar as we consider the firm's overall

two systems is probably better than the other in terms of fitting well within its own whole.

[2]In a similar vein, there are some studies dealing with the complementarity between process and product innovation for a firm, including Athey and Schmutzler (1995), Adner and Levinthal (2001) and Lin and Saggi (2002).

[3]For a discussion of the theories of advertizing and the evidence of its use by firms, see Mansfield (1996); Pepall, Richards and Norman (1999, Ch 10); Friedman, 1983; and Martin, (2001, Ch. 6).

[4]In the marketing literature, there is an extensive empirical strand aiming at categorizing ads in the various media according to the number of information "cues" contained in them. The overall conlcusion is that ads contain little actual information that would usefully guide consumers' purchase decisions. See e.g. Resnik and Stern, (1997) and Abernathy and Franke (1996).

marketing activity as consisting of two *a priori* independent activities, each controlling one of the aforementioned effects separately. This division rests thus fundamentally on whether a particular advertizing campaign is aimed at new potential consumers or at existing consumers. Our objective is to establish in a very simple framework the complementarity of the two activities, from the point of view of the marketing division of the firm, thereby justifying the centralization of marketing activities within the firm. To this end, we assume that the monopolist has at its disposal three different marketing scenarios: *(i)* market expansion alone, (*ii*) demand elasticity improvement alone, *(iii)* both goals simultaneously. For the sake of simplicity, the two activities are described in the most elementary manner possible that allows us to make our point: By expanding a fixed outlay, the firm will be able to achieve a fixed shift in the characteristic of interest, for each of the two options.

For the modified Dixit and Norman model, we establish the simple fact that market enlargement is complementary to elasticity improvement, when both options are modelled in the elementary binary manner described above. As a consequence, there is a region of intermediate fixed cost levels for the two separate options such that neither of them alone is profitable while the simultaneous implementation of both is profitable. Furthermore, we will show that for such intermediate fixed cost levels, the simultaneous investment in both options generates a profit that is higher than the sum of the profit gains derived from the two separate activities. It follows that the marketing division of such a firm should be not be tempted to decentralize its operations in its attempts to reach out to new consumers and to existing consumers. Such decentralization might well miss out on jointly profitable operations in cases when each operation alone is unprofitable.

2 The model

We consider a profit maximizing monopolist that faces a constant marginal production cost c. As for the demand side, we adopt a modified version of Dixit and Norman (1978) using the isoelastic demand function:

$$Q(p) = a\,p^{-b} \tag{1}$$

where a represents a measure of market size and $-b$ is the constant elasticity of demand. We assume that $b > 1$, so that the profit-maximizing solution is interior (see below.)

The choice of such a demand function is not innocuous. First of all, the multiplicative form is deemed more realistic than the linear additive form in the managerial economics/marketing literature. Furthermore, such a demand can be easily linearized by taking logarithms, so as to allow for its parameters to be easily estimated via standard regression.[5] For the present paper, as will be seen, it allows for a potential separation between the two marketing effects that we wish to analyze.

The corresponding profit function is given by:

$$\pi(p) = (p - c)\,Q(p) \tag{2}$$

From the first-order conditions, it is easily seen that the optimal price is then a constant mark-up over marginal cost, another attractive of this demand function:

$$P^* = \frac{c}{1 - 1/b}. \tag{3}$$

The level of mark-up thus depends in a simple way on the elasticity of demand. By plugging (3) into (2) and rearranging, we get an expression of the firm's optimal

[5] See Mansfield (1996, Ch. 3) for an illustrative discussion of the usefulness of the isoelastic utility function in managerial economics.

profit:

$$\pi^* = ab^{-b}(\frac{c}{b-1})^{1-b} \tag{4}$$

The elasticity of demand, defined in general as $\in = \frac{p}{q}\ \frac{dq}{dp}$, is here equal to $-b$, a negative constant independent of the output level. It is well-known that a monopolist always prices at a point where $|\in| > 1$, whence our assumption that $b > 1$. It is useful to note that an increase in demand elasticity (corresponding to lowering the value of b) is profit-improving, i.e.:

$$\frac{\partial \pi^*}{\partial b} = -ab^{-b}(\frac{c}{b-1})^{1-b}\left[\log b + \log\left(\frac{c}{b-1}\right)\right] < 0.$$

It follows that the monopolist has an incentive to increase demand elasticity (i.e. move $\in$ closer to zero), which is what we term elasticity improvement throughout the present note.

Without taking costs into account, it is easy to see that[6]:

Lemma 1 *Market expansion and elasticity improvement are complementary activities.*

[6]The formal concept of complementarity invoked here is due to Edgeworth, and corresponds to the mathematical notion of increasing differences of supermodularity, which for a function of two scalar variables is defined as follows. For all $x' > x$ and $y' > y$,

$$F(x', y') + F(x, y) \geq F(x, y') + F(x', y)$$

For a twice continuously differentiable function, this is equivalent to $\partial^2 F(x, y)/\partial x \partial y \geq 0$ for all x, y. This makes it apparent that the economic interpretation of the complementarity between two variables is that having more of either one increases the marginal returns to having more of the other.

The mathematical analysis of complementarity, and its far-reaching implications are covered in full detail in Topkis (1998).

Proof. As elasticity improvement corresponds to a lower b, we need only verify that the cross-partial derivative $\dfrac{\partial^2 \pi^*}{\partial a \partial b}$ is < 0 (see Footnote 5). Indeed, given that $b > 1$, we have

$$\frac{\partial^2 \pi^*}{\partial a \partial b} = -b^{-b}(\frac{c}{b-1})^{1-b}\left[\log b + \log\left(\frac{c}{b-1}\right)\right] < 0.$$

3 The Three Marketing Options

Suppose that the firm's marketing possibilities can be described as follows: Expend resources on pursuing one of the following three courses of action.

(i) *Option 1*: Market expansion alone.

(ii) *Option 2*: Elasticity improvement alone.

(ii) *Option* 3: Both market expansion and elasticity improvement.

In the simplest theoretical framework, these three options can be described as follows. For option 1, a fixed investment of K_a enlarges the market by a fixed amount Δ_a. For option 2, a fixed investment in elasticity improvement of K_b increases demand elasticity by Δ_b, provided $b - \Delta_b > 1$. For option 3, the associated cost is simply the sum of the separate fixed costs, or $K_a + K_b$. (We are thus ignoring the possibility that the cost of realizing the two types of marketing campaigns may be less than the sum of the costs of the two separate campaigns, i.e. that there is a cost complementarity, or economies of scope, between them. We will come back to this point in the discussion at the end of the paper.)

The profit gain in case of market expansion alone is given by (cf. (4)):

$$\pi_a = (a + \Delta_a)\, b^{-b}(\frac{c}{b-1})^{1-b} - K_a \tag{5}$$

Market expansion alone is profitable if $\pi_a > \pi^*$, which reduces to the condition:

$$K_a < \Delta_a \, \Phi \triangleq \widehat{K_a} \tag{6}$$

where $\Phi = b^{-b}(\frac{c}{b-1})^{1-b}$.

As for elasticity improvement alone, the profit gain is given by:

$$\pi_b = a(b-\Delta_b)^{-(b-\Delta_b)}(\frac{c}{b-\Delta_b-1})^{1-(b-\Delta_b)} - K_b \tag{7}$$

Hence, option 2 is profitable (i.e. $\pi_b > \pi^*$) if

$$K_b < a(\,\Psi - \Phi) \triangleq \widehat{K_b} \tag{8}$$

where $\Psi = (b-\Delta_b)^{-(b-\Delta_b)}(\frac{c}{b-\Delta_b-1})^{1-(b-\Delta_b)}$.

As for option 3, if the monopolist undertakes both options at the same time, the profit will be equal to:

$$\pi_{a,b} = (a+\Delta_a)(b-\Delta_b)^{-(b-\Delta_b)}(\frac{c}{b-\Delta_b-1})^{1-(b-\Delta_b)} - K_a - K_b \tag{9}$$

Hence, option 3 is worthwhile for the monopolist if

$$K_a + K_b < (a-\Delta_a)\Psi - a\Phi \triangleq \widehat{K_{ab}} \tag{10}$$

A key property of this model is as follows (this is easy to verify computationally):

$$\widehat{K_{ab}} > \widehat{K_a} + \widehat{K_b} \tag{11}$$

There are three different parameter configurations of interest for the analysis at hand. The first is the low costs case, where both (6) and (8) hold. The monopolist would then implement either option alone, as well as both options simultaneously.

The second is the high costs case, where $K_a + K_b > (a-\Delta_a)\Psi - a\Phi$. In this case, the monopolist would neither implement both options simultaneously, nor either option alone.

The third case is the intermediate costs situation, where (6) and (8) do not hold, but (10) holds.[7] In this case, of greatest interest to us, the monopolist would not implement either option alone but would implement both options together.

As a numerical illustration, consider the following parameter values: $a = 5$, $b = 2$, $c = 3$, $\Delta_a = 2$, $\Delta_b = 0.5$. If $K_a = 0.2$ and $K_b = 0.7$, hence $K_a > \widehat{K_a} = 0.167$ and $K_b > \widehat{K_b} = 0.694$ but $K_a + K_b < \widehat{K_{ab}} = 1.139$. The threshold level of aggregate investment after which the firm does not invest at all is then higher in case of both options implemented together or, in other words, the interval where the firm is willing to invest is larger.[8]

It can easily be verified that, in both the first and the third case, the profit gain from implementing both options together is higher than the sum of the profit gains from implementing the two options separately.

All in all then, the firm would always implement both options together whenever it is worthwhile to implement each of them separately, but the converse fails to hold in the intermediate-costs case. This result is very robust, and in no way a reflection of the binary nature of the investment options, as adopted in this paper. A version of this result can easily be obtained when the two options are allowed to be continuous decisions with a variable cost structure. Since the essential message is conveyed in this simple framework rather succinctly, we opted for the resulting simplicity of the exposition.

[7] In view of (11), it is clear that this case is possible, and in specific examples, it can be seen that the parameter configuration implied by the three conditions here is robust and far from degenerate.

[8] Following the numerical specification, we have in fact that $\pi^* = 0.417$, $\pi_a = 0.383$, $\pi_b = 0.411$, $\pi_{a,b} = 0.65$.

4 Complementarity and Investment

Since the firm faces binary decisions, for each investment opportunity, let 0 and 1 represent the decisions "not invest" and "invest" respectively. Denote by $i \in \{0,1\}$ and $j \in \{0,1\}$ the decisions to invest in market enlargement and elasticity rigidity enhancing, respectively. Then the overall profit function of the firm may be written as

$$\Pi(0,0) = ab^{-b}(\frac{c}{b-1})^{1-b}$$

$$\Pi(1,0) = (a+\Delta_a)\, b^{-b}(\frac{c}{b-1})^{1-b} - K_a$$

$$\Pi(0,1) = a(b-\Delta_b)^{-(b-\Delta_b)}(\frac{c}{b-\Delta_b-1})^{1-(b-\Delta_b)} - K_b$$

$$\Pi(1,1) = (a+\Delta_a)(b-\Delta_b)^{-(b-\Delta_b)}(\frac{c}{b-\Delta_b-1})^{1-(b-\Delta_b)} - K_a - K_b$$

It is easily verified that:

Proposition 1 Π *is supermodular on* $\{0,1\} \times \{0,1\}$, *i.e.:*

$$\Pi(1,1) + \Pi(0,0) \geq \Pi(1,0) + \Pi(0,1).$$

Subtracting $2\Pi(0,0)$ from both sides, this can be rewritten as

$$\Pi(1,1) - \Pi(0,0) \geq [\Pi(1,0) - \Pi(0,0)] + [\Pi(0,1) - \Pi(0,0)],$$

which says that the profit gain due to the simultaneous implementation of both options exceeds the sum of the profit gains of the two separate options. This formalizes the notion of complementary investment opportunities.

The marginal returns from market expansion increase with increases in the elasticity of demand. This finding is very intuitive in this context as it simply follows from the fact that the firm will make higher profits if the newly attracted consumers have a higher willingness to pay, which a higher elasticity presumes. Conversely, since supermodularity is a two-way complementarity relationship, an alternative and equivalent interpretation is that the benefit that the firm derives from an increased elasticity increases with the overall demand, i.e. with the enticement of new consumers.

This conclusion has a simple implication for the organization of the firm's marketing division: A centralized decision-making structure is necessary, in that coordination of the two options considered in this note is warranted in view of the simple complementarity argument developed here. In the intermediate range of the marketing cost parameters, two decentralized units each controlling one of the two options would independently conclude that implementation would result in a loss for their unit, while joint appraisal of the two options would call for joint implementation.

In reality, it can be argued that the separation of the two marketing options considered here is somewhat artificial and unrealistic, in that many types of advertizing campaigns will always reach both types of consumers (though they may well be designed to reach one type of consumers more effectively than the other.) Observe that this criticism reinforces our conclusion in that it implies that there is also a *cost complementarity* between the two options that we have not taken into account. Indeed, if implementing them jointly is less costly than implementing them separately, then there is a new incentive for joint implementation that we have not considered. This incentive would reinforce the demand side complementarity analyzed in this paper.

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