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Appendix

Assume ten varieties are being supplied by the profit maximizing monopolist. From expression (10), it can be checked that $\bar{\theta}$ must be greater than 19/10 for the lowest quality, q_L^m, to be positive. Compare then the total profits accruing to the monopolist in the two alternative settings, i.e., full and partial market coverage, as defined by expressions (27) and (28), respectively. In the case under analysis , the difference between the two amounts to:

$$\pi_{pc}^m - \pi_{fc}^m = \frac{55\bar{\theta}^3}{1323t} - \frac{100\bar{\theta}^3 - 200\bar{\theta}^2 + 133}{400t}, \qquad (a1$$

which simplifies to

$$\pi_{pc}^m - \pi_{fc}^m = \frac{(10\bar{\theta} - 21)^2 (220\bar{\theta} - 399)}{529200t}. \qquad (a2$$

It is immediate to verify that the above difference is non-negative for all $\bar{\theta} \in]19/10, \infty[$, and it is nil in correspondence of $\bar{\theta} = 21/10$. Besides, straightforward calculations are needed to show that the critical value of $\bar{\theta}$ for which the monopolist is indifferent between partial and full market coverage (which we have seen to correspond to 3 for n=1 and 21/10 for n=10) is decreasing in the number of varieties, so that as the latter becomes arbitrarily large, the profit seeking monopolist strictly prefers not to serve the entire market.

given the number of products. Consequently, one can conclude that if the monopolist can choose the type of distortion, she prefers to restrict output rather than providing customers with suboptimal qualities.

$$\frac{\bar{\theta}^3}{27t} \geq \frac{(\bar{\theta}-1)^2}{4t} \qquad (29$$

for all $\bar{\theta} \geq 1$. In particular, condition (25) holds as a strict inequality for all values of $\bar{\theta}$ except $\bar{\theta} = 3$, for which it holds as an equality. Analogous calculations are needed to obtain the same results when more than one product is supplied. The calculations concerning the case in which n=10 are provided in the Appendix. Accordingly, the following relevant corollary holds:

COROLLARY 1. For a given number of varieties, if the profit seeking monopolist is given the option between restricting output and biasing qualities, she prefers to exert her monopoly power by excluding the poorer individuals from consumption.

3. Conclusions

The behaviour of a multiproduct monopolist in a market for vertically differentiated goods has been described. I have shown that if the multiproduct monopolist is assumed to serve the entire market with goods of different qualities, she undersupplies all qualities as compared to the social optimum, as long as the number of varieties is finite. When the latter becomes infinite, or equivalently when the quality range being supplied becomes continuous, the monopolist supplies the socially optimal quality exclusively to the consumer with the highest valuation for quality, while increasing the distortion in the varieties offered to poorer consumers. Thus the results already highlighted by Mussa and Rosen (1978) and other authors emerges here as the asymptotic result of a discrete model. Furthermore, when the alternative assumption of partial market coverage is adopted, I have shown that the profit seeking monopolist supplies the same qualities as the social planner, though she produces half the output associated with social planning. Finally, the comparison between the monopolist’s profit in the two settings yields the result that under partial market coverage she is at least as well off as under full market coverage, Hence,

$$\lim_{n \to \infty} X^{m} = \frac{1}{2}; \quad \lim_{n \to \infty} X^{sp} = 1, \tag{26}$$

so that as the number of varieties tends to infinity the private monopolist serves the richer half of the market, while the social planner serves all consumers[4]. *Q.E.D.*

Compare now the profits accruing to the monopolist under the alternative assumptions of partial and complete market coverage. Under full market coverage the overall profit amounts to

$$\pi^{m}_{fc} = \sum_{i} \pi^{m}_{i}, \quad \pi^{m}_{i} = \frac{\bar{\theta} n^{2}(\bar{\theta} - 2) + 2n(2i - 1) - 2i(i - 1) - 1}{4n^{3}t}, \tag{27}$$

while under partial market coverage it corresponds to

$$\pi^{m}_{pc} = \sum_{i} \pi^{m}_{i}, \quad \pi^{m}_{i} = \frac{i\bar{\theta}^{3}(2n - i + 1)}{2t(2n + 1)^{3}}, \tag{28}$$

where subscripts *fc* and *pc* stand for full and partial market coverage, respectively. It can be shown that $\pi^{m}_{pc} \geq \pi^{m}_{fc}$ for all admissible values of parameter θ. An illustrative example is now provided. Consider the case of a single variety, and compare the profits associated with the two alternative assumptions:

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4. This results was already highlighted by Mussa and Rosen (1978, p.313) for the case where low income consumers cannot buy any variety by definition, i.e., $\underline{\theta} = 0$.

$$\frac{\partial p}{\partial q} = \frac{\overline{\theta}}{2} + qt. \qquad (20$$

The average valuation is obtained as follows:

$$\frac{1}{x}\int_{p/q}^{\overline{\theta}} \frac{\partial p}{\partial q} d\theta = \frac{\overline{\theta}}{2} + qt. \qquad (21$$

The two magnitudes coincide because the demand function is linear (see Spence, 1975, pp.421-2). The same considerations obviously apply to the multiproduct setting.

Focus now on the case where more than one quality is produced, and define $X^m = \sum_i x_i^m$, and $X^{sp} = \sum_i x_i^{sp}$, i=1, 2...n. In the case of two varieties, I obtain:

$$q_H^m = q_H^{sp} = \frac{2\overline{\theta}}{5t}; \quad q_L^m = q_L^{sp} = \frac{\overline{\theta}}{5t}; \qquad (22$$

$$X^m = \frac{2\overline{\theta}}{5} = \frac{X^{sp}}{2}, \quad x_i^m = \frac{\overline{\theta}}{5} = \frac{x_i^{sp}}{2}, \quad i = H, L, \qquad (23$$

and so on as the number of varieties increases. Extending the analysis to n varieties, the following results obtain:

$$q_i^m = q_i^{sp} = \frac{i\overline{\theta}}{t(2n+1)}, \quad i = 1, 2...n; \qquad (24$$

$$X^m = \frac{n\overline{\theta}}{2n+1} = \frac{X^{sp}}{2}; \quad x_i^m = \frac{X^m}{n} = \frac{x_i^{sp}}{2}, \quad i = 1, 2...n. \qquad (25$$

$$x = \bar{\theta} - \frac{p}{q}, \tag{16}$$

so that the monopolist's profit function is:

$$\pi^m = (p - tq^2)x. \tag{17}$$

Observe that, potentially, the monopolist could choose not to exclude any individual from consumption by setting the price-quality ratio below $\underline{\theta}$, so as to serve the entire market. If this does not obtains at equilibrium, it implicitly means that the monopolist prefers quantity restriction to quality distortion. Optimal quality and price can be obtained by solving the first order conditions (it can be easily shown that second order conditions are also satisfied):

$$\frac{\partial \pi^m}{\partial q} = tp + \frac{p^2}{q^2} - 2\bar{\theta}qt = 0; \tag{18}$$

$$\frac{\partial \pi^m}{\partial p} = \bar{\theta} - \frac{2p}{q} + qt = 0; \tag{19}$$

yielding $p^m = 2\bar{\theta}^2/(9t)$ and $q^m = \bar{\theta}/(3t)$. The equilibrium quantity is $x^m = \bar{\theta}/(3t)$ and profit amounts to $\pi^m = \bar{\theta}^3/(27t)$. It is easily shown that a social planner maximizing welfare would set the same quality as the profit maximizing monopolist, producing though $q^{sp} = 2\bar{\theta}/3$.[3] This is due to the fact that the average valuation of quality icrements coincides with the marginal one (Spence, 1975, p.419). The latter is given by the derivative of price w.r.t. quality:

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3. In this setting the social welfare function is defined as in (3) above, but for the integration limits, which are now *p/q* and $\bar{\theta}$.

$$\lim_{n \to \infty} \Delta q_H = 0; \quad \lim_{n \to \infty} \Delta q_L = \frac{1}{2t}. \tag{15}$$

This states that as the number of varieties tends to infinity, the monopolist ends up supplying the socially optimal highest quality, while she undersupplies the lowest quality to an extent equal to the size of the whole spectrum of consumers' preferences.[2] As emphasized by Besanko *et al*. (1987, p.749), "the essence of the monopolist's quality distortion is that the quality levels provided to some groups of consumers are distorted so as to protect the higher profitability of sales to other groups". *Q.E.D.*

2.2. Partial market coverage

Here, condition (1) is allowed to be violated for a non empty set of consumers. The behaviour of the private monopolist and the social planner in such circumstances is summarized by

PROPOSITION 2. For a given number of varieties, the monopolist supplies the same qualities as the social planner, while producing half the output of the social planner, both overall and for each variety. When the number of varieties tends to infinity, the social planner serves all the market, while the monopolist serves only the upper half.

PROOF. Again, I proceed by induction. First, consider a monopolist who is only partially serving the market, selling a single variety. The demand for her product is:

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2. Analogously, it could also be shown that, as n increases, the difference between the monopolist's quality and the corresponding social planner's one, $q_i^{sp} - q_i^m, i \in]L, H[$, shrinks as one moves from the bottom to the top of the quality range.

$$q_H^m = \frac{n\overline{\theta} - 1}{2nt}; \quad q_L^m = \frac{n\overline{\theta} - 2n + 1}{2nt}; \qquad (10$$

$$q_H^{sp} = \frac{2n\overline{\theta} - 1}{4nt}; \quad q_L^{sp} = \frac{2n\overline{\theta} - 2n + 1}{4nt}. \qquad (11$$

Thus, when n varieties are available, the differentiation degrees under the two market regimes are, respectively:

$$q_H^m - q_L^m = \Delta q^m = \frac{n-1}{nt}; \quad q_H^{sp} - q_L^{sp} = \Delta q^{sp} = \frac{n-1}{2nt}, \qquad (12$$

with $\Delta q^m / \Delta q^{sp} = 2$ and

$$\lim_{n \to \infty} \Delta q^m = \frac{1}{t}, \quad \lim_{n \to \infty} \Delta q^{sp} = \frac{1}{2t}. \qquad (13$$

The result in (13) implies that as the number of varieties tends to infinity, the social planner provides each individual with his own most preferred quality, so that under social planning the degree of differentiation coincides in the limit with the range of consumers' preferences in terms of quality.

As for the extent to which the profit maximizing monopolist undesupplies quality, observe that

$$q_H^{sp} - q_H^m = \Delta q_H = \frac{1}{4nt} = \left(\frac{1}{2n}\right)\left(\frac{1}{2t}\right); \quad q_L^{sp} - q_L^m = \Delta q_L = \frac{2n-1}{4nt} = \left(\frac{2n-1}{2n}\right)\left(\frac{1}{2t}\right), \qquad (14$$

with

Let's take into account the problem of the private monopolist. Provided that p_L is such that the poorest consumer is indifferent between purchasing or not, the price of the high quality good can be obtained from the first order condition (FOCs) derived from (8). Thus, equilibrium qualities are univocally determined by the FOCs of (8) w.r.t. q_H and q_L, with $q_H^m = (2\overline{\theta} - 1)/4t$ and $q_L^m = (2\overline{\theta} - 3)/4t$. Solving then the social planner's problem yields $q_H^{sp} = (4\overline{\theta} - 1)/8t$ and $q_L^{sp} = (4\overline{\theta} - 3)/8t$. It immediately appears that the monopolist undersupplies both qualities, with $q_H^{sp} - q_H^m = 1/8t$ and $q_L^{sp} - q_L^m = 3/8t$, i.e., the "distance" between varieties in the high quality segment of the market amounts to one quarter of the range of consumers' preferred qualities, while it amounts to three quarters in the low quality segment. Furthermore, the degree of differentiation adopted by the profit-seeking monopolist is twice as wide as that adopted by the social planner, due to the the monopolist's attempt to extract as much consumer surplus as possible by enhancing differentiation beyond the socially preferable level. This, coupled with the fact that the monopolist undersupplies quality since by assumption she cannot restrict output, yields the result observed here.

The same problem can be easily reformulated in the case of three varieties, where the private monopolist supplies $q_L^m = (3\overline{\theta} - 5)/6t$, $q_M^m = (\overline{\theta} - 1)/2t$ and $q_H^m = (3\overline{\theta} - 1)/6t$, while the social planner produces $q_L^{sp} = (6\overline{\theta} - 5)/12t$, $q_M^{sp} = (2\overline{\theta} - 1)/4t$ and $q_H^{sp} = (6\overline{\theta} - 1)/12t$. Subscript M indicates the *intermediate* quality. As compared to the previous case, enlarging the number of varieties leads to (i) an increase in the degree of differentiation under both market regimes; (ii) an increase (decrease) in the differentiation degree between the low (high) qualities supplied in the two market regimes, with the monopolist always undersupplying all qualities respect to the social optimum.

Thus, I am now able to extend the analysis to the setting where n varieties are offered, or, in other terms, where the distance between contiguous varieties tends to zero so that quality becomes a continuous variable, with q_H and q_L indicating now the highest and lowest qualities being supplied, respectively. In such a case, The derivative w.r.t. price is always positive. Consequently, the monopolist will set the maximum price consistent with the assumption of full market coverage, i.e., the price at which the poorest consumer gives up all his surplus to purchase the good, $p = \underline{\theta} q$. Accordingly, the profit function can be rewritten as follows:

$$\pi^m = \underline{\theta} q - tq^2, \qquad (6$$

which is concave and single-peaked, with the maximum at $q^m = \overline{\theta}/2t$, that is the quality preferred by the poorest consumer. This allows to conclude that when a single variety is available, the monopolist undersupplies quality as compared to the social optimum, and the "distance" $q^{sp} - q^m = 1/4t$ amounts to one half of the interval of consumers' preferred qualities.

Assume now that two varieties are being supplied, $q_H > q_L > 0$, produced at costs $C_i = tq_i^2 x_i$, $i=H, L$. Market demands are given by:

$$x_H = \overline{\theta} - \frac{(p_H - p_L)}{q_H - q_L}; \quad x_L = \frac{(p_H - p_L)}{q_H - q_L} - (\overline{\theta} - 1). \qquad (7$$

The objective function of the profit-seeking monopolist and the social planner are, respectively:

$$\pi^m = (p_H - tq_H^2)x_H + (p_L - tq_L^2)x_L; \qquad (8$$

and

$$sw = \int_{\underline{\theta}}^{h} (\theta q_L - tq_L^2) d\theta + \int_{h}^{\overline{\theta}} (\theta q_H - tq_H^2) d\theta. \qquad (9$$

maximization of social welfare, defined as the sum of profit and consumer surplus:

$$\max_{q} \quad sw = \int_{\underline{\theta}}^{\overline{\theta}} (\theta q - tq^2) d\theta. \tag{3}$$

It can be quickly verified that the price level is irrelevant as for the problem described in (3), since it can only redistribute surplus from the consumers to the producer or *viceversa*, without modifying the overall level of welfare.[1] Thus, from the first order condition of (3) w.r.t q, I obtain

$$q^{sp} = \frac{\overline{\theta} - 1}{4t}, \tag{4}$$

i.e., the social planner locates her product exactly in the midpoint of the spectrum of consumers' preferred qualities, as it also happens in spatial models *à la* Hotelling (see Bonanno, 1987, and Lambertini, 1995).

Consider now the problem faced by a profit-maximizing monopolist, whose objective function is:

$$\pi^m = p - tq^2. \tag{5}$$

1. Obviously, the same cannot be expected to hold under partial market coverage, since in such a case a price change brings about a change in quantity as well.

$$U = \theta q_i - p_i \geq 0, \quad i = 1, 2 ... n, \qquad (1$$

where q_i is the quality of variety i and p_i is the price charged by the monopolist for that quality. Production involves variable costs only:

$$C = t q_i^2 x_i, \quad i = 1, 2 ... n, \qquad (2$$

where x_i is the quantity produced and t is a positive parameter. Hence, if marginal cost pricing is considered, it is possible to define the range of consumers' preferred qualities as the interval $[\underline{\theta}/2t, \overline{\theta}/2t]$ (see Cremer and Thisse, 1994).

2.1. Full market coverage

Under the full market coverage assumption, condition (1) holds as an equality for the poorest consumer, while it holds as a strict inequality for any other individual. The main features of the behaviour of the social planner and the private monopolist as far as the provision of product quality is concerned are summarized in the following

PROPOSITION 1. As long as the number of varieties is finite, the profit seeking monopolist strictly undersupplies all qualities as compared to the social optimum. As the number of varieties tends to infinity, the highest quality she offers tends to coincide with the socially optimal one, while the difference between her lowest quality and the social planner's lowest quality is increasing in the number of varieties and in the limit it is equal in size to the whole range of consumers' preferred qualities.

PROOF. In order show what is stated in the above Proposition, I shall proceed by induction. Hence, assume first that a single quality is supplied. The objective of the social planner is the

Considering a continuum of consumers characterized by different valuation for quality, under the assumption of full market coverage I show that, as long as the the quality spectrum of the private monopolist is discrete, i.e., the number of varieties being provided is finite, the monopolist undersupplies all qualities. Furthermore, the extent to which she distorts quality is inversely related to the marginal willingness to pay, both for a given number of varieties and as their number increases, so that one can rather paradoxically conclude that rich consumers would favourably consider product proliferation, while the opposite holds for poor consumers.

Then, it is shown that if the monopolist only partially serves the market, she offers the same qualities that would be supplied by a social planner aiming at the maximization of social welfare. In such a situation, though, the monopolist produces only half the output of the social planner, both overall and for each variety. Finally, it appears that the monopolist is at least as well off by restricting output than she is by serving all the market, so that she should be expected to exert her monopoly power so as to exclude some individuals from consumption rather than providing them with a range of suboptimal qualities.

2. The setting

Consider a monopolistic market for a vertically differentiated good where the firm supplying the good may be thought of as being alternatively run by a social planner taking care of social surplus or by a profit seeking monopolist. I shall compare the behaviour of these two agents assuming they provide the same number of varieties, *n*. Consumers are uniformly distributed with unit density over the interval $[\underline{\theta}, \overline{\theta}]$, $\underline{\theta} > 0$, $\overline{\theta} = \underline{\theta} + 1$, so that the total number of individuals is normalised to 1. Parameter θ represents consumer's marginal willingness to pay for quality, and it may be interpreted as the reciprocal of the marginal utility of nominal income, or money (Tirole, 1988, pp.96-7). Each consumer buys one unit of the variety of the product maximizing the net surplus he obtains, provided that the latter is non-negative:

1. Introduction

The issue of evaluating the behaviour of a profit seeking monopolist under vertical product differentiation, as compared to the social optimum, has been the focus of several influential papers. Spence (1975, 1976) and Sheshinski (1976) have established that, although the monopolist tends to restrict output for a given quality, she introduces a bias in the provision of quality *for a given output level*, since in selecting quality the private monopolist takes into account the willingness to pay of the *marginal* consumer, while a social planner would take into account that of the *average* consumer. Thus, the monopolist ends up undersupplying quality for a given output level if the average consumer's valuation for quality is higher than the marginal consumer's, and *viceversa*.

The monopolist may offer several qualities of the same good in order to extract more consumer surplus. This is the subject of the contributions due to Mussa and Rosen (1978), Itoh (1983), Maskin and Riley (1984), and Besanko *et al*. (1987). A conclusion common to all these authors is that the monopolist resorts to an enlargement of the quality spectrum, as compared to the social optimum, as a screening device which enables her to discriminate among consumers. In particular, it has been shown that with either (i) two types of consumers (rich-poor) and two qualities (high-low) or (ii) with a continuum of consumers and qualities, the richest consumers, i.e., those with the highest valuation for quality, are provided with the socially optimal quality, while the poorest ones buy a suboptimal quality (for an illustrative argument, see Tirole, 1988, p.150). The above contributions, though, leave largely unanswered the following questions: If there exixts a continuum of consumers characterized by different incomes and thus different marginal willingness to pay for quality, what kind of distortion shall the multiproduct monopolist prefer to introduce in the market? Will she exploit exclusively the quality distortion or alternatively a restriction in output, or rather a mix of both?

In order to provide an answer, I shall adopt here a model which respects the general assumptions made in Mussa and Rosen (1978) as for technology and consumer tastes.

THE MULTIPRODUCT MONOPOLIST
UNDER VERTICAL DIFFERENTIATION:
AN INDUCTIVE APPROACH

Luca Lambertini

Keywords: monopoly, quality distortion, output restriction
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Abstract

The behaviour of a multiproduct profit seeking monopolist is evaluated *vis à vis* that of a social planner, in a model where there is a continuum of consumers characterized by different marginal willingness to pay for quality. When the market is completely covered, the monopolist undersupplies all qualities as long as their number is finite. When quality becomes continuous, the richest consumer is provided with the socially optimal quality. Under the alternative assumption of partial market coverage, the monopolist supplies the same qualities as the social planner, restricting though total output. Finally, it turns out that, for a given number of varieties, under partial market coverage the monopolist can make at least as good as under full market coverage, so that she prefers to distort quantity rather than quality.

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