Dynamic R&D with Spillovers: Competition vs Cooperation[1]

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Abstract

We investigate dynamic R&D for process innovation in a duopoly where firms may either undertake independent ventures or form a cartel for cost-reducing R&D investments. By comparing the profit and welfare perfomances of the two settings in steady state, we show that private and social incentives towards R&D cooperation coincide for all admissible levels of the technological spillovers characterising innovative activity. This results stems from smoothing the investment reffort over the time horizon of the game.

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1 Introduction

The role of technological spillovers in shaping the incentives to conduct R&D for process innovation has attracted a wide attention in the existing literature on oligopoly theory. The most relevant contributions in this vein are those of d'Aspremont and Jacquemin (1988), Kamien *et al.* (1992), Suzumura (1992) and Amir (2000), to mention only a few. A general appraisal of the advantages associated with R&D cooperation, and the related policy measures, can be found in Katz and Ordover (1990).[1]

The theoretical debate on the private and social advantages generated by R&D cooperation was triggered by an analogous policy debate on the same issue, leading to the National Research Cooperation Act that passed in the US in 1984.[2] Then, following Katz (1986), a large body of literature has discussed the theoretical and empirical facets of welfare-improving technology policies based upon two forms of R&D cooperation, namely, R&D cartels and research joint ventures.[3] Here, we shall briefly summarise the approaches adopted in d'Aspremont and Jacquemin (1988) and Kamien *et al.* (1992).

d'Aspremont and Jacquemin (1988) consider a homogeneous Cournot duopoly, where each firm enjoys a spillover from the rival in terms of the final outcome of R&D activity, in the following sense. To firm i, investing k_i costs an amount bk_i^2, which captures the presence of decreasing returns to innovative activity, but the total effective R&D contributing to reduce

[1]The underlying relationship between innovation and market structure came to the fore even earlier, of course. To this regard, see Spence (1984) and Reinganum (1989), *inter alia*. The above mentioned contributions share with Brander and Spencer (1983) the concept of R&D as a cost-reducing activity, adding to the Brander-Spencer setup the possibility of information transmission or technological externalities.

[2]For the EU and Japan, see Goto and Wakasugi (1988) and the CE Commission (1990).

[3]A relatively scanty attention has been paid to the possibility that any form of R&D cooperation facilitates collusion, either in prices or in quantities. To this regard, see Martin (1995), Lambertini *et al.* (1998, 2002, 2003) and Cabral (2000).

firm i's marginal cost c_i is in fact $K_i = k_i + \beta k_j$, where β is the technological externality generated from the rival's investment k_j. Therefore, given a generic initial marginal cost $\overline{c}$, we have $c_i = \overline{c} - K_i$. In Kamien *et al.* (1992), instead, the spillover effect is measured in terms of Dollars or Euro, in the sense that they assume each firm to have a concave R&D technology $f(Y_i)$, where $Y_i = y_i + \beta y_j$ is the effective R&D effort, comprehensive of the external effect, and the reduction in firm i's marginal cost is given by $c_i = \overline{c} - f(Y_i)$. This technology is coupled with linear R&D costs equal to y_i for each firm. In other terms, what changes from the first to the second model is the way chosen to make the setup concave. In the former case, concavity is achieved through a convex R&D cost function, while in the latter case the same property rests upon a concave R&D technology. Using $f(Y_i) = \sqrt{y_i + \beta y_j}$, Amir (2000) shows that the two models are isomorphic up to the transformation $k_i = y_i/\sqrt{b}$. For this reason, one can focus upon d'Aspremont and Jacquemin (1988). They compare two different games: one where firms behave noncooperatively in choosing both R&D efforts and output levels, the other where firms form a cartel in the R&D stage, choosing thus R&D investments so as to maximise joint profits in that stage only, while they continue to adopt a Nash behaviour in the market stage. Comparing the two setups, d'Aspremont and Jacquemin (1988) find that (i) for high spillover levels $[\beta > 1/2]$, R&D investments - and also cost reduction, clearly - are higher under cooperative behaviour, and conversely for low spillovers; (ii) for high spillover levels $[\beta > 1/2]$, social welfare is higher under cooperative behaviour, and conversely. Unfortunately, they also find that cartel profits are higher than noncooperative profits when spillovers are *low* $[\beta < 1/2]$. This yields an undesirable conflict between private and social incentives towards R&D cooperation (or cartelisation).[4]

[4]The literature on this topic has also discussed the issue of equilibrium stability, as for low levels of R&D costs (i.e., low levels of parameter b) there exists no internal solution

The d'Aspremont-Jacquemin model, or some variation of it, has been used thereafter to investigate several related issues, e.g., the possibility of setting up research joint ventures in relation to absorptive capacity (Kamien and Zang, 2000), the efficiency comparison between Bertrand and Cournot behaviour with product differentiation (Qiu, 1997), the endogenisation of spillovers (Katsoulacos and Ulph, 1998; Poyago-Theotoky, 1999; Amir and Wooders, 1999, 2003) and the effects of increasing the number of firms in the market (Hinloopen, 2000).[5]

However, the above mentioned lack of overlapping between social and private incentives towards cooperation has remained unsolved. To tackle this problem, we adopt an explicitly dynamic approach to describe the R&D activity aimed at process innovation, modelled as a differential game whose basic components are as close as possible to the original ones contained in d'Aspremont and Jacquemin (1988). As in their paper, we confine our attention to the alternative cases where firm either behave fully noncooperatively or build up a cartel in R&D investments. We compare steady state profits and social welfare at the subgame perfect equilibria of the two cases, finding that irrespective of the spillover level, R&D cooperation is preferable to noncooperative behaviour from both a private and a social point of view. Intuitively, this result stems from investment smoothing, which is carried out by firms over the time horizon of the dynamic setting, while it is utterly impossible to achieve in a static two-stage game where firms are compelled to invest one-shot the full amount of resources required to achieve the equilibrium efficiency level of their productive technology.

The remainder of the paper is structured as follows. Section 2 illustrates

as second order conditions are not met. On this issue, see Henriques (1990), d'Aspremont and Jacquemin (1990), Qiu (1997) and Amir and Wooders (1998).

[5] A large amount of research has also been carried out on the empirical side. See Lambertini *et al.* (2004) and the references therein.

the dynamic setup. Independent ventures are investigated in section 3, while the performance of the R&D cartel is described in section 4. Section 5 contains some concluding remarks.

2 The setup

We consider a duopoly with homogeneous goods over continuous time, $t \in [0, \infty)$. In every instant, the market demand function writes as follows:

$$p(t) = A - q_1(t) - q_2(t) \,. \tag{1}$$

Each firm i supplies the market through a technology characterised by a constant marginal cost. Accordingly, her instantaneous cost function is $C_i(c_i, q_i, t) = c_i(t)q_i(t)$. the marginal cost borne by firm i evolves over time as described by the following kinematic equation:

$$\frac{dc_i(t)}{dt} \equiv \dot{c}_i = c_i(t)\left[-k_i(t) - \beta k_j(t) + \delta\right] \,, \tag{2}$$

where $k_i(t)$ is the R&D effort exerted by firm i at time t, while parameter $\beta \in [0, 1]$ measures the positive technological spillover that firm i receives from the R&D activity of firm j. Parameter $\delta \in [0, 1]$ is a constant depreciation rate measuring the instantaneous decrease in productive efficiency due to the ageing of technology. Equation (2) can be rewritten as follows:

$$\frac{\dot{c}_i}{c_i(t)} = -k_i(t) - \beta k_j(t) + \delta \,, \tag{3}$$

so as to highlight that the rate of change of firm i's marginal cost over time is linear in the instantaneous investment efforts. That is, (2) is indeed adynamic version of the linear R&D technology employed by d'Aspremont and Jacquemin in the static model.

The instantaneous cost of setting up a single R&D laboratory is:

$$\Gamma(k, t) = b\left[k(t)\right]^2 \,, \tag{4}$$

where $k(t)$ is the R&D effort carried out at time t within the laboratory, and b is a positive parameter. Now define the instantaneous R&D investment of firm i as $\Gamma_i(k_i, t)$. If firms undertake independent ventures (i.e., each firms sets up her own R&D division or laboratory), then:

$$\Gamma_i(k_i, t) = b\,[k_i(t)]^2\,. \tag{5}$$

In such a case, firms may behave either noncooperatively or collusively. Throughout the game, firms discount future profits a the common and constant discount rate $\rho > 0$.

3 Independent ventures

In this setting, firms adopt a strictly noncooperative behaviour in choosing both the output levels and the R&D efforts, each firm operating her own R&D division. The Hamiltonian of firm i is:

$$\mathcal{H}_i(\mathbf{q}, \mathbf{k}, \mathbf{c}, t) = e^{-\rho t}\{[A - q_1(t) - q_2(t) - c_i(t)]\,q_i(t) - b\,[k_i(t)]^2 + \tag{6}$$

$$-\lambda_{ii}(t)c_i(t)\,[k_i(t) + \beta k_j(t) - \delta] - \lambda_{ij}(t)c_j(t)\,[k_j(t) + \beta k_i(t) - \delta]\}$$

where $\lambda_{ij}(t) = \mu_{ij}(t)e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c_j(t)$, and $\mathbf{q}, \mathbf{k}, \mathbf{c}$ are the vectors of control and state variables.

As a first step, we prove the following result:

Lemma 1 *The open-loop Nash equilibrium of the game with independent ventures is subgame (or Markov) perfect.*

Proof. We are going to show that the present setup is a *perfect game* in the sense of Leitmann and Schmitendorf (1978) and Feichtinger (1983). In summary, a differential game is *perfect* whenever the closed-loop equilibrium collapses into the open-loop one, the latter being thus strongly time consistent, i.e., subgame perfect.[6] Consider the closed-loop information structure. The relevant first order conditions (FOCs) are:

$$\frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial q_i(t)} = A - 2q_i(t) - q_j(t) - c_i(t) = 0 \; ; \tag{7}$$

$$\frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial k_i(t)} = -2bk_i(t) - \lambda_{ii}(t)c_i(t) - \beta\lambda_{ij}(t)c_j(t) = 0 \; . \tag{8}$$

As a first step, observe that (7) only contains firm i's state variable, so that in choosing the optimal output at any time during the game firm i may disregard the current efficiency of the rival. That is, there is no feedback effect in the output choice. Conversely, at first sight there seem to be a feedback between the R&D decisions, as (8) indeed contains both state variables, at least for any positive spillover effect.[7] the core of the proof consists in showing that no feedback effect are actually present, even for positive spillover levels.

Taking the above considerations into account, the adjoint or co-state equations are:

$$-\frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial c_i(t)} - \frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial k_j(t)} \cdot \frac{\partial k_j^*\left(.,t\right)}{\partial c_i(t)} = \frac{\partial \lambda_{ii}(t)}{\partial t} - \rho\lambda_{ii}(t) \Leftrightarrow \tag{9}$$

$$\frac{\partial \lambda_{ii}(t)}{\partial t} = q_i\left(t\right) + \lambda_{ii}(t)\left[k_i(t) + \beta k_j(t) + \rho - \delta\right] - \frac{\beta}{2b}\lambda_{ji}(t)\left[\lambda_{ij}(t)c_j(t) + \beta\lambda_{ii}(t)c_i(t)\right]$$

$$-\frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial c_j(t)} - \frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial k_i(t)} \cdot \frac{\partial k_i^*\left(.,t\right)}{\partial c_j(t)} = \frac{\partial \lambda_{ij}(t)}{\partial t} - \rho\lambda_{ij}(t) \Leftrightarrow \tag{10}$$

$$\frac{\partial \lambda_{ij}(t)}{\partial t} = \lambda_{ij}(t)\left\{\left[k_j(t) + \beta k_i(t) + \rho - \delta\right] - \frac{\beta}{2b}\left[2bk_i\left(t\right) + \lambda_{ii}(t)c_i(t) + \beta\lambda_{ij}(t)c_j(t)\right]\right\}$$

[6]The label 'perfect game' is due to Fershtman (1987), where one can find a general technique to identify any such games. Another class of games where open-loop equilibria are subgame perfect is investigated by Reinganum (1982). For further details, see Mehlmann (1988, ch. 4) and Dockner *et al.* (2000, ch. 7).

[7]Intuitively, if $\beta = 0$, then the two investment plans are completely independent and therefore it is apparent that no feedback effect operates.

where

$$\frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial k_j(t)} \cdot \frac{\partial k_j^*\left(.,t\right)}{\partial c_i(t)} \tag{11}$$

capture the feedback effects, and partial derivatives $\partial k_j^*\left(.,t\right)/\partial c_i(t)$ are calculated using the optimal values of investments as from FOC (8):

$$k_j^*\left(.,t\right) = -\frac{\lambda_{jj}(t)c_j(t) + \beta\lambda_{ji}(t)c_i(t)}{2b}\,. \tag{12}$$

These conditions must be evaluated along with the initial conditions $\{c_i(0)\} = \{c_{0,i}\}$ and the transversality conditions

$$\lim_{t\to\infty} \lambda_{ij}(t) \cdot c_j(t) = 0\ ,\ i,j = 1,2. \tag{13}$$

From (10), we note that $\partial\lambda_{ij}(t)/\partial t = 0$ in $\lambda_{ij}(t) = 0$. Then, using this piece of information, we may rewrite the expression for the optimal investment of firm i as follows:

$$k_i^*\left(.,t\right) = -\frac{\lambda_{ii}(t)c_i(t)}{2b}\,, \tag{14}$$

which entails that $\partial k_i^*\left(.,t\right)/\partial c_j(t) = 0$. By the underlying symmetry of the model, this holds for both firms, i.e., feedback (cross-)effects are nil along the equilibrium path. Accordingly, the open-loop equilibrium is a degenerate closed-loop one, and it is strongly time consistent, or equivalently, subgame perfect. It is also worth observing that this procedure shows that FOCs are indeed unaffected by initial conditions as well. The property whereby the FOCs on controls are independent of states and initial conditions after replacing the optimal values of the co-state variables is known as *state-redundancy*, and the game itself as *state-redundant* or *perfect*. ■

On the basis of Lemma 1, we can proceed with the characterisation of the open-loop solution. The FOCs on control as well as the transversality conditions are the same as above, while the co-state equations simplify as follows:

$$-\frac{\partial \mathcal{H}_i\left(.,t\right)}{\partial c_i(t)} = \frac{\partial \lambda_{ii}(t)}{\partial t} - \rho\lambda_{ii}(t) \Leftrightarrow \tag{15}$$

$$\frac{\partial\lambda_{ii}(t)}{\partial t} = q_i(t) + \lambda_{ii}(t)\left[k_i(t) + \beta k_j(t) + \rho - \delta\right]$$

$$-\frac{\partial\mathcal{H}_i(.,t)}{\partial c_j(t)} = \frac{\partial\lambda_{ij}(t)}{\partial t} - \rho\lambda_{ij}(t) \Leftrightarrow \tag{16}$$

$$\frac{\partial\lambda_{ij}(t)}{\partial t} = \lambda_{ij}(t)\left[k_j(t) + \beta k_i(t) + \rho - \delta\right]$$

From FOCs (7-8) we have, respectively:

$$q_i^*(t) = \frac{A - q_j(t) - c_i(t)}{2}, \tag{17}$$

$$k_i(t) = -\frac{\left[\lambda_{ii}(t)c_i(t) + \beta\lambda_{ij}(t)c_j(t)\right]}{2b}. \tag{18}$$

While (17) has the usual appearance of a standard Cournot best reply function, the optimal R&D effort in (18) depends upon co-state variables. Such expression can be differentiated w.r.t. time to get the dynamic equation of $k_i(t)$:

$$\frac{dk_i(t)}{dt} \equiv \dot{k}_i = -\frac{c_i(t)\frac{\partial\lambda_{ii}(t)}{\partial t} + \lambda_{ii}(t)\frac{dc_i(t)}{dt} + \beta\left[c_j(t)\frac{\partial\lambda_{ij}(t)}{\partial t} + \lambda_{ij}(t)\frac{dc_j(t)}{dt}\right]}{2b} \tag{19}$$

with $\partial\lambda_{ii}(t)/\partial t$ and $\partial\lambda_{ij}(t)/\partial t$ obtaining from (9-10). Then, (19) can be further simplified by using

$$\lambda_{ii}(t) = -\frac{2bk_i(t) + \beta\lambda_{ij}(t)c_j(t)}{c_i(t)} \tag{20}$$

which obtains from (8). As to the second co-state variable, its dynamic equation (10) must be treated autonomously[8] and, by imposing stationarity,

[8]That is, whenever the FOCs of firm i cannot determine the optimal value of the co-state variable attached to the rival's state dynamics, the co-state equation pertaining to the state variable of firm j is to be treated as an additional state equation, on which one must impose stationarity in equilibrium. For more details on this issue, see Başar and Olsder (1982, 1995[2]), Mehlmann (1988) and Dockner *et al.* (2000).

i.e., $\partial\lambda_{ij}(t)/\partial t = 0$, we obtain $\lambda_{ij}(t) = 0$. This yields:

$$\dot{k}_i = -\frac{c_i(t)}{2b}\left[q_i(t) - \frac{2bk_i(t)}{c_i(t)}\right]. \tag{21}$$

The next step consists in solving the system of best reply functions (17), yielding the Cournot-Nash output level of firm i as a function of state variables:

$$q_i^{CN}(t) = \frac{A - 2c_i(t) + c_j(t)}{3} \tag{22}$$

which can be plugged into (21). After imposing the symmetry condition $c_j(t) = c_i(t) = c(t)$, we may characterise the dynamics of the R&D effort of firm i is terms of her own state and control variables only:

$$\dot{k}_i = \rho k_i(t) - \frac{c(t)[A - c(t)]}{6b}. \tag{23}$$

Imposing the stationarity condition $\dot{k}_i = 0$ we obtain:

$$k^{IV}(t) = \frac{c(t)[A - c(t)]}{6b\rho} \geq 0 \text{ for all } c(t) \in [0, A], \tag{24}$$

where the superscript IV stands for *independent ventures*. Before proceeding to the characterisation of the steady state equilibrium, it is worth noting that, in general, the level of $c(t)$ will depend upon the technological spillover β, so that we can write:

$$\frac{\partial k^{IV}}{\partial\beta} = \frac{[A - 2c(t)] \cdot \partial c(t)/\partial\beta}{6b\rho} \tag{25}$$

which, in principle, may take either sign, depending upon the relative size of A and $c(t)$ as well as the sign of $\partial c(t)/\partial\beta$.

The steady state level of marginal cost $c(t)$ can be found by solving:

$$\dot{c} = -c(t)\left[k^{IV}(t)(1+\beta) - \delta c(t)\right] = 0 \tag{26}$$

which yields:

$$c = 0;\ c = \frac{A(1+\beta) \pm \sqrt{(1+\beta)\left[A^2(1+\beta) - 24b\delta\rho\right]}}{2(1+\beta)} \tag{27}$$

All solutions in (27) are real if and only if $\delta\rho \leq A^2 (1+\beta) / (24b)$. If so, they also satisfy the requirement $c \in [0,\ A]$. By checking the stability conditions, we may prove the following:

Proposition 2 *Provided that* $\delta\rho \leq A^2 (1+\beta) / (24b)$ *, the steady state point*

$$\begin{aligned} c^{IV} &= \frac{A(1+\beta) - \sqrt{(1+\beta)\left[A^2(1+\beta) - 24b\delta\rho\right]}}{2(1+\beta)} \\ k^{IV} &= \frac{\delta}{1+\beta} \end{aligned}$$

is the unique saddle point equilibrium of the game with independent ventures.

Proof. See Appendix 1. ■

Equilibrium output and profits are:

$$q^{IV} = \frac{A(1+\beta) + \sqrt{(1+\beta)\left[A^2(1+\beta) - 24b\delta\rho\right]}}{6(1+\beta)}\ ; \tag{28}$$

$$\pi^{IV} = \frac{A^2(1+\beta)^2 - 6b\delta\left[3\delta + 2\rho(1+\beta)\right] + A\sqrt{(1+\beta)^3\left[A^2(1+\beta) - 24b\delta\rho\right]}}{18(1+\beta)}\ , \tag{29}$$

From the steady state expressions of R&D investment, one can immediately derive the following intuitive property:

$$\frac{\partial k^{IV}}{\partial \beta} = -\frac{\delta}{(1+\beta)^2} < 0, \tag{30}$$

which implies that, as the size of technological spillover effects increases, the incentive to invest in process innovation shrinks as it becomes increasingly difficult to internalise the benefits from R&D activity. Observe that, in steady state, R&D investment is needed only to make up for the depreciation rate, with each firm receiving a positive externality from the other. Indeed, the total effective investment perceived from the viewpoint of firm i is $k^{IV}(1+\beta) = \delta$, which entails that the reduction of firm i's individual R&D

effort if fully made up for by the spillover effect. This, however, accounts for the specific functional form of k^{IV} in the steady state only, keeping in mind (25).

Moreover, in steady state the following also holds:

$$\frac{\partial c^{IV}}{\partial \beta} = -\frac{6b\delta\rho}{\sqrt{(1+\beta)^3 \left[A^2 (1+\beta) - 24b\delta\rho\right]}} < 0. \tag{31}$$

This is due to the fact that any increase in β entails a reduction in k^{IV}, as we know from (30). Indeed, using (25) and (30), we can write:

$$\frac{\left[A - 2c(t)\right] \cdot \partial c(t) / \partial \beta}{6b\rho} = -\frac{\delta}{(1+\beta)^2} \tag{32}$$

which must hold in equilibrium. From the above condition, we obtain:

$$\frac{\partial c(t)}{\partial \beta} = -\frac{6b\delta\rho}{(1+\beta)^2 \left[A - 2c(t)\right]}; \tag{33}$$

Then, noting that $A > 2c^{IV}$, it follows that $\partial c^{IV} / \partial \beta < 0$.

Consumer surplus and welfare in steady state are:

$$CS^{IV} \equiv \frac{\left(A - p^{IV}\right) \sum_{i=1}^{2} q_i^{IV}}{2} = \frac{\left[A\sqrt{1+\beta} + \sqrt{A^2 (1+\beta) - 24b\delta\rho}\right]^2}{18(1+\beta)} \tag{34}$$

$$SW^{IV} \equiv 2\pi^{IV} + CS^{IV} = \tag{35}$$

$$= \frac{2\left[A^2 (1+\beta)^2 - 3b\delta\left[3\delta + 4\rho(1+\beta)\right] + A\sqrt{(1+\beta)^3 \left[A^2 (1+\beta) - 24b\delta\rho\right]}\right]}{9(1+\beta)}.$$

4 R&D cartel

Here, we examine the case where firms noncooperatively choose output levels, while maximising joint profits w.r.t. the choice of their respective R&D efforts. As in the previous section, each firm operates her own laboratory.

This amounts to imposing *a priori* the symmetry conditions $c_i(t) = c_j(t) = c(t)$ and $k_i(t) = k_j(t) = k(t)$. The state equation now looks as follows:

$$\dot{c} = c(t) \left[-(1+\beta) k(t) + \delta \right] . \quad (36)$$

Therefore, the Hamiltonian of firm i can be written as follows:

$$\mathcal{H}_i(\mathbf{q}, k, c, t) = e^{-\rho t} \{ [A - q_1(t) - q_2(t) - c(t)] \, q_i(t) - b \, [k(t)]^2 + \quad (37)$$
$$+ \lambda(t) c(t) \left[-(1+\beta) k(t) + \delta \right] \}$$

where $\lambda(t) = \mu(t) e^{\rho t}$ is the co-state variable (evaluated at time t) associated with the state variable $c(t)$, and $\mathbf{q}$ is the vector of individual outputs. As in the previous case, it can be shown that the open-loop equilibrium is subgame perfect.[9] The open-loop first order conditions for the optimum are:

$$\frac{\partial \mathcal{H}_i(.,t)}{\partial q_i(t)} = A - 2q_i(t) - q_j(t) - c(t) = 0 \; ; \quad (38)$$

$$\frac{\partial \mathcal{H}_i(.,t)}{\partial k(t)} = -2bk(t) - \lambda(t) (1+\beta) c(t) = 0 \; ; \quad (39)$$

$$-\frac{\partial \mathcal{H}_i(.,t)}{\partial c(t)} = \frac{\partial \lambda(t)}{\partial t} - \rho \lambda \; , \quad (40)$$

along with the initial conditions $\{c(0)\} = \{c_0\}$, and the transversality conditions

$$\lim_{t \to \infty} \lambda(t) \cdot c(t) = 0 \; . \quad (41)$$

Solving (39), we obtain $\lambda(t) = -2bk(t)/ \left[(1+\beta) c(t) \right]$, entailing also:

$$\frac{dk(t)}{dt} \equiv \dot{k} = -\frac{(1+\beta)}{2b} \left[c(t) \frac{\partial \lambda(t)}{\partial t} + \lambda(t) \frac{dc(t)}{dt} \right] . \quad (42)$$

From (40) we obtain:

$$\frac{\partial \lambda(t)}{\partial t} = q_i(t) - \left[\delta - \rho - (1+\beta) k(t) \right] \lambda(t) \, . \quad (43)$$

[9] The details are omitted for brevity, as they closely replicate the same line as in the proof of Lemma 1.

This expression, together with the optimal values of the co-state variable and the Cournot-Nash output level $q^{CN} = [A - c(t)]/3$, can be plugged into (42), which simplifies as follows:

$$\dot{k} = \rho k(t) - \frac{c(t)[A - c(t)](1+\beta)}{6b}. \tag{44}$$

Therefore, $dk(t)/dt = 0$ in correspondence of:

$$k^{Cl}(t) = \frac{c(t)[A - c(t)](1+\beta)}{6b\rho}, \tag{45}$$

where superscript Cl stands for *cartel*. Before proceeding, we may compare $k^{Cl}(t)$ against $k^{IV}(t)$, as defined in (24), to ascertain that the following property:

$$\frac{c(t)[A - c(t)](1+\beta)}{6b\rho} > \frac{c(t)[A - c(t)]}{6b\rho} \Rightarrow k^{Cl}(t) > k^{IV}(t) \tag{46}$$

holds for all $\beta \in (0, 1]$ and $c(t) > 0$. This entails:

Lemma 3 *Setting up a cartel in the R&D stage leads firms to invest more than in the fully noncooperative case, for all positive levels of spillovers and marginal cost.*

Of course, the reason for this result is that R&D cooperation permits to better internalise the beneficial externality, therefore boosting firms' incentives to invest. The consequence of the above Lemma is that the private and social desirability of R&D cooperation drastically hinges upon its ability of reducing marginal cost significantly below the level resulting from Nash behaviour.

Plugging $k^{Cl}(t)$ into the state dynamics and imposing the stationarity condition, we have:

$$\dot{c} = -c(t)\left[\frac{c(t)[A - c(t)](1+\beta)^2}{6b\rho} - \delta\right] = 0 \tag{47}$$

yielding:

$$c = 0;\ c = \frac{A(1+\beta) \pm \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{2(1+\beta)}. \tag{48}$$

The above analysis allows us to state:

Proposition 4 *Provided that* $\delta\rho \leq A^2(1+\beta)^2/(24b)$, *the steady state point*

$$\begin{aligned} c^{Cl} &= \frac{A(1+\beta) - \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{2(1+\beta)} \\ k^{Cl} &= \frac{\delta}{1+\beta} \end{aligned}$$

is the unique saddle point equilibrium of the game where firm set up a cartel in the R&D stage.

Proof. See Appendix 2. ■

The steady state R&D effort is exactly the same as in the noncooperative case. This is obviously due to the fact that, in both cases, the investment needed to keep constant firm i's marginal cost is $k^m(1+\beta) = \delta$, $m = IV, Cl$. What changes, instead, is the steady state level of the marginal cost. To this regard, it can be easily verified that

$$c^{IV} - c^{Cl} \propto \frac{\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{1+\beta} - \frac{\sqrt{A^2(1+\beta) - 24b\delta\rho}}{\sqrt{1+\beta}} \tag{49}$$

which is strictly positive for all $\beta \in (0,1]$.

Individual output and profits under R&D cartelisation are:

$$q^{Cl} = \frac{A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{6(1+\beta)}; \tag{50}$$

$$\pi^{Cl} = \frac{A^2(1+\beta)^2 - 6b\delta(3\delta+2\rho) + A(1+\beta)\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{18(1+\beta)^2}. \tag{51}$$

Consumer surplus and welfare in steady state are:

$$CS^{Cl} \equiv \frac{\left(A - p^{Cl}\right)\sum_{i=1}^{2} q_i^{Cl}}{2} = \frac{\left[A\left(1+\beta\right)+\sqrt{A^2\left(1+\beta\right)^2 - 24b\delta\rho}\right]^2}{18\left(1+\beta\right)^2} \tag{52}$$

$$SW^{Cl} \equiv 2\pi^{Cl} + CS^{Cl} = \tag{53}$$

$$= \frac{2\left[A^2\left(1+\beta\right)^2 - 3b\delta\left(3\delta+4\rho\right) + A\left(1+\beta\right)\sqrt{A^2\left(1+\beta\right)^2 - 24b\delta\rho}\right]}{9\left(1+\beta\right)^2}.$$

Propositions 2 and 4 immediately entail the following Corollary:

Corollary 5 *For all $\beta \in (0,1]$ the parameter region wherein the R&D cartel problem admits an internal optimum is wider than the parameter region wherein noncooperative R&D activity yields an internal optimum.*

Proof. To show this, it suffices to verify that

$$A^2\left(1+\beta\right)^2 / \left(24b\right) > A^2\left(1+\beta\right) / \left(24b\right) \tag{54}$$

for all $\beta \in (0,1]$. ■

This of course stems from the fact that R&D cooperation is substantially equivalent to a reduction in ρ (or δ, or both). That is, when investing within a cartel, firms behave *as if* they were more patient than in the alternative case.

5 Private and social incentives to R&D cooperation

Now we are in a position to assess the incentive to activate a cartel in the R&D stage, both from the standpoints of each firm and from the regulator's, in the parameter region where both organizational arrangements are admissible, i.e., $\delta\rho \leq A^2(1+\beta)/(24b)$. This task involves, respectively, evaluating π^{Cl} against π^{IV} and SW^{Cl} against SW^{IV}. In both cases, we obtain:

$$\pi^{Cl} - \pi^{IV} \propto SW^{Cl} - SW^{IV} \propto \Theta \tag{55}$$

$$\Theta \equiv 12b\beta\delta\rho + A(1+\beta)\left[\sqrt{A^2(1+\beta)^2 - 24b\delta\rho} - \sqrt{1+\beta}\sqrt{A^2(1+\beta) - 24b\delta\rho}\right]$$

with the expression Θ being positive for all $\beta \in (0,1]$. Therefore, we have proved our main result:[10]

Proposition 6 *Consider the parameter range $\delta\rho \leq A^2(1+\beta)/(24b)$. For all positive spillover levels, the R&D cartel is preferable to independent ventures from private and social standpoints alike.*

As a final remark, we may observe that the beneficial effect of R&D cartelisation on social welfare comes from both sides of the market, since:

$$\pi^{Cl} > \pi^{IV} \text{ and } CS^{Cl} > CS^{IV} \tag{56}$$

for all $\beta \in (0,1]$. This can be explained on the following grounds. Given that $c^{IV} - c^{Cl} > 0$, one expects firms to expand output under cooperative R&D, as against case where they undertake independent ventures. By comparing (28) and (50), there indeed emerges that $q^{Cl} > q^{IV}$ always. Accordingly, consumer surplus is enhanced by R&D cooperation because industry output is larger and market price is lower than in the fully noncooperative setting. As for the performance of firms, the increase in profits generated by the cartel arrangement is not obvious *a priori*, because the increase in productive efficiency is surely beneficial but the opposite holds for the output expansion. In balance, it appears that the first effects outweighs the second.

[10]Also note that, for all $\beta \in (0,1]$, the parameter region where the cartel solution is admissible, i.e., $\delta\rho \leq A^2(1+\beta)^2/(24b)$, is wider than the analogous region defined for independent ventures to yield an admissible solution, i.e., $\delta\rho \leq A^2(1+\beta)/(24b)$.

The interpretation of these results (in particular, Proposition 6) is straightforward. With convex R&D costs, allowing firms to smooth their investment plans over some time span[11] is advantageous both because it permits firms to enhance profits and because it yields a higher welfare, as Jensen's inequality applied to the cost function $\Gamma_i(k_i, t)$ trivially implies.

6 Concluding remarks

We have analysed dynamic R&D investments for cost-reducing innovation in a Cournot duopoly where firms may either compete or cooperate in the R&D phase. The foregoing analysis has shown that a unique stable equilibrium exists in each setting. By comparing the steady state profit and welfare performances of the industry in the two cases, there emerges that private and social incentives towards R&D cooperation coincide for all admissible levels of the technological spillovers characterising innovative activity, in the sense that cartelisation dominates competition from both standpoints over the whole admissible parameter range.

The setup employed in the present paper is a dynamic version of the static game examined in d'Aspremont and Jacquemin (1988). The drastic difference between our results and theirs relies upon smoothing the investment efforts over a long time horizon, a perspective which is ruled out by definition in a static setting.

[11] The length of the horizon and the assumption that time be treated as a continuous variable are both, in fact, immaterial to the conclusions. It can be easily shown that an analogous differential game over $t \in [0, T]$ (no matter whether in continuous or discrete time) would produce qualitatively equivalent results.

7 Appendices

7.1 Appendix 1: Proof of Proposition 2

Assume $\delta\rho \leq A^2(1+\beta)/(24b)$. The stability properties of the system (23-26) can be assessed by evaluating the trace and determinant of the following Jacobian matrix:

$$J_{IV} \equiv \begin{bmatrix} \dfrac{\partial \dot{c}}{\partial c} = \delta - (1+\beta)k & \dfrac{\partial \dot{c}}{\partial k} = -(1+\beta)c \\ \dfrac{\partial \dot{k}}{\partial c} = -\dfrac{A-2c}{6b} & \dfrac{\partial \dot{k}}{\partial k} = r \end{bmatrix}$$

in correspondence of the steady state values of c and k. At $c = k = 0$, the trace is $T(J_{IV}) = \delta + \rho > 0$ and the determinant is $\Delta(J_{IV}) = \delta\rho > 0$. Therefore, such a point is unstable. In correspondence of

$$k = \frac{c[A-c]}{6b\rho};\ c = \frac{A(1+\beta) + \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{2(1+\beta)} \tag{a1}$$

we obtain $T(J_{IV}) = \rho > 0$ and

$$\Delta(J_{IV}) = \frac{A^2(1+\beta) - 24b\delta\rho + A\sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{12b} \tag{a2}$$

which is clearly positive in the admissible parameter range. Finally, in

$$k = \frac{c[A-c]}{6b\rho};\ c = \frac{A(1+\beta) - \sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{2(1+\beta)} \tag{a3}$$

we have $T(J_{IV}) = \rho > 0$ again, and

$$\Delta(J_{IV}) = \frac{A^2(1+\beta) - 24b\delta\rho - A\sqrt{(1+\beta)[A^2(1+\beta) - 24b\delta\rho]}}{12b}. \tag{a4}$$

In this case, $\Delta(J_{IV}) < 0$ in the admissible parameter region. Therefore, (a3) is the unique stable steady state point of the dynamic system; in particular, it is a saddle point. Simplifying the expression for optimal investment in steady state, we obtain $k^{IV} = \delta/(1+\beta)$.■

7.2 Appendix 2: Proof of Proposition 4

Assume $\delta\rho \leq A^2 (1+\beta)^2 / (24b)$. The stability properties of the system (36-44) can be assessed by evaluating the trace and determinant of the following Jacobian matrix:

$$J_{Cl} \equiv \begin{bmatrix} \dfrac{\partial \dot{c}}{\partial c} = \delta - (1+\beta) k & \dfrac{\partial \dot{c}}{\partial k} = -(1+\beta) c \\ \dfrac{\partial \dot{k}}{\partial c} = -\dfrac{(1+\beta)(A-2c)}{6b} & \dfrac{\partial \dot{k}}{\partial k} = r \end{bmatrix}$$

in correspondence of the steady state values of c and k. Observe that the only difference between J_{Cl} and J_{IV} is to be found in $\partial \dot{k}/\partial c$, since in the cooperative case this partial derivative fully embodies the spillover effect, which is absent in the previous case.

At $c = k = 0$, the trace is $T(J_{Cl}) = \delta + \rho > 0$ and the determinant is $\Delta(J_{Cl}) = \delta\rho > 0$. Therefore, such a point is unstable. In correspondence of

$$k = \frac{c[A-c]}{6b\rho}\,;\; c = \frac{A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{2(1+\beta)} \tag{a5}$$

we obtain $T(J_{Cl}) = \rho > 0$ and

$$\Delta(J_{Cl}) = \frac{\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}\left[A(1+\beta) + \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}\right]}{12b} \tag{a6}$$

which is clearly positive in the admissible parameter range. Finally, in

$$k = \frac{c[A-c]}{6b\rho}\,;\; c = \frac{A(1+\beta) - \sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{2(1+\beta)} \tag{a7}$$

we have $T(J_{Cl}) = \rho > 0$ again, and

$$\Delta(J_{Cl}) = \frac{A^2(1+\beta)^2 - 24b\delta\rho - A(1+\beta)\sqrt{A^2(1+\beta)^2 - 24b\delta\rho}}{12b}. \tag{a8}$$

In this case, $\Delta(J_{Cl}) < 0$ in the admissible parameter region. Therefore, (a7) is the unique stable steady state point of the dynamic system; in particular, it is a saddle point. Simplifying the expression for optimal investment in steady state, we obtain again $k^{Cl} = \delta / (1 + \beta) = k^{IV}$.■

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