# THE CHOICE TO MIGRATE: WHERE TO WORK AND WHERE TO LIVE

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#### Introduction

In this paper we consider an overlapping generation model in which people, born in two different countries, decide where to spend their working period of life, and where to retire after it. The two countries differ in technology and quality of life. The "northern" country, N, is more productive, while in the "southern" country, S, people enjoy a better quality of life. Locational preferences are influenced not only by "objective" criteria –like climate, pollution, congestion—, but also by "cultural" factors. Thus we assume that people prefer to live in their country of origin. footnote

It will be shown that, under plausible assumptions, locational decisions cause migration from *S* to *N*, but the importance of this phenomenon depends on how locational preferences are made endogenous.

In Section 2 we describe the production side of our model, and in Section 3 its consumption side. We then examine the influence of locational preferences that are taken as exogenous, in Section 4, and made endogenous, at least for their subjective component, in Section 5.

## The production sector

Indigenous population is assumed to be constant in both countries, and equal to  $2P_N$  and  $2P_S$  respectively, so that  $P_N$  and  $P_S$  are the working populations in the two countries. footnote Technology in the two countries is represented by Cobb-Douglas production functions. Country N has a factor neutral technological advantage on country S. More precisely:

$$Y_S = K_S^{\beta} L_S^{1-\beta}$$
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$$Y_N = (1+\sigma)K_N^{\beta}L_N^{1-\beta}$$
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where the parameter  $\sigma > 0$  measures the technological advantage enjoyed by N. Note that, because of migration,  $L_i$  does not necessarily coincide with  $P_i$  (i = N, S). We assume that capital is formed by costless reshaping of the consumption good, so that we do not need a separate price for it.

Each individual inelastically supplies one unit of labor in the first period of life, thereby earning income; part of it is consumed, and the rest is invested in order to allow consumption of capital and interest in the second period of life, when the individual is retired.

Firms operate in a perfectly competitive setting, so that factors of production are paid according their marginal productivity; we therefore have

$$\omega_S = (1 - \beta)k_S^{\beta}$$
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$$\omega_N = (1 + \sigma)(1 - \beta)k_N^{\beta}$$
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$$r_S = \beta k_S^{\beta - 1}$$
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$$r_N = (1+\sigma)\beta k_N^{\beta-1}$$
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where  $\omega_i$  is real wage,  $r_i$  is the rental rate of capital, and  $k_i = \frac{K_i}{L_i}$  is per-worker capital in country i (i = N, S).

We assume perfect international capital mobility, so that in equilibrium interest rates are equalized as between the two countries. The following condition must therefore hold in equilibrium:

$$\frac{k_N}{k_S} = \frac{\omega_N}{\omega_S} = (1+\sigma)^{1/(1-\beta)} > 1$$

Thus, in absence of labour movement, country *N* has higher per-worker capital and higher real wages than country *S*.

## The consumption sector

Individuals born in country *i* maximise an intertemporal utility function, the value of which depends on the levels of consumption in the two periods of life, and on the countries in which individuals spend their two periods of life. We assume the following Cobb-Douglas form for such a function:

$$U_{i}(C_{j,t}, C_{j,t+1}, j(t), j(t+1)) = (Z_{i,j(t)}C_{j,t})^{\alpha} (Z_{i,j(t+1)}C_{j,t+1})^{1-\alpha}$$
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Note that consumption levels in both periods are linked to the country j where individuals work during the first period of life; in fact only that period is relevant to determine life-long income. As for  $Z_{i,j(t)}$ , it is a parameter describing how individuals born in country i enjoy living in country j at time t. Note also that this unusual intertemporal utility function can be derived, by an increasing monotonic transformation, from the more usual function:

$$U_i(C_{j,t}, C_{j,t+1}, j(t), j(t+1)) = U_t(C_t) + \frac{1}{1+\rho} U_{t+1}(C_{t+1})$$

by letting

$$U_t(C_t) = \log(Z_{i,j(t)}C_{j,t})$$

$$U_{t+1}(C_{t+1}) = \log(Z_{i,j(t+1)}C_{j,t+1})$$

taking antilogs and defining

$$\alpha \equiv \frac{1+\rho}{2+\rho}$$

Thus there is a one-to-one positive function between the intertemporal discount rate  $\rho$  and the parameter  $\alpha$ , which corresponds to the share of total income devoted to consumption in period t. Note that when  $\rho=0$ , then  $\alpha=1/2$ , so that when individuals do not discount the future, consumption and saving in the working period of their life are equal. Note also that, as the intertemporal discount rate goes to infinity, individuals tend to consume all their income while young.

Maximization of (8) is subject to the budget constraint

$$p_t C_{i,t} + p_{t+1} C_{i,t+1} = p_t \omega_{i,t}$$

where j = N, S and j(t) may be different from j(t + 1), and where  $p_t$  is the product price at time t. This leads to the following consumption functions:

$$C_{i,t} = \alpha \omega_{i,t}$$
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$$C_{i,t+1} = (1 - \alpha)(1 + r)\omega_{i,t}$$
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where in equilibrium

$$1 + r = \frac{p_t}{p_{t+1}}$$

Consumption levels are not the only decision that individuals must make in order to maximize (8), as they must also decide where to work and where to spend the retirement period of their life. In other words, individuals must choose the countries j(t) and j(t+1). This locational pair [j(t), j(t+1)] must satisfy the condition

$$U_i(C_{j,t}, C_{j,t+1}, j(t), j(t+1)) \geq U_i(C_{j',t}, C_{j',t+1}, j(t)', j(t+1)')$$
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for all  $[j(t)', j(t+1)'] \neq [j(t), j(t+1)]$ . Note that this inequality does not imply that **both**  $j(t)' \neq j(t)$  and  $j(t+1)' \neq j(t+1)$ , but rather that in the pair [j(t)', j(t+1)'] either  $j(t)' \neq j(t)$  or  $j(t+1)' \neq j(t+1)$ .

This problem cannot be solved by differentiation, because the choice set is not continuous; however some reasonable assumptions about  $Z_{i,j(t)}$  and  $Z_{i,j(t+1)}$  provide a solution.

# **Exogenous locational preferences**

Let us first use equations (3), (4), (8), (9), and (10) to rewrite the optimality condition as

$$Z_{i,j(t)}^{\alpha} Z_{i,j(t+1)}^{1-\alpha} k_{j(t)}^{\beta} (1+\sigma_{j(t)}) \ge Z_{i,i(t)'}^{\alpha} Z_{i,j(t+1)'}^{1-\alpha} k_{j(t)'}^{\beta} (1+\sigma_{j(t)'})$$
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where  $\sigma_{j(t)} = 0$  if j(t) = S and  $\sigma_{j(t)} = \sigma$  if j(t) = N.

As anticipated above,  $Z_{i,j}$  is an index that captures both the "objective" quality of life in country j, and the "subjective" locational preference that individuals born in country i have for country j relative to country i. We can therefore think of  $Z_{i,j}$  as composed of two factors,  $Q_j$  and  $A_{i,j}$ , where  $Q_j$  represents the "objective" quality of life in country j, while  $A_{i,j}$  measures how individuals born in i like living in country j. Thus:

$$Z_{i,j} \equiv Q_j A_{i,j}$$
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Assuming that S offers a better quality of life, the following condition holds:

$$Q_S > Q_N$$
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Moreover we assume that, others things equal, individuals prefer living in their country of origin, footnote so that:

$$A_{ii} > A_{ii}$$
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if  $j \neq i$ . Finally we assume that individuals born in country N have a very strong preference for their own country, so that the following condition holds:

$$Q_N A_{NN} > Q_S A_{NS}$$
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Using conditions (14), (15), and (16), and considering that in equilibrium  $k_N > k_S$ , we can solve the individuals' locational problem. Using definition (13) we can rewrite (12) as

$$(Q_{j(t)}A_{i,j(t)})^{\alpha}(Q_{j(t+1)}A_{i,j(t+1)})^{1-\alpha}k_{j(t)}^{\beta}(1+\sigma_{j(t)})\geq$$

$$(Q_{j(t)'}A_{i,j(t)'})^{\alpha}(Q_{j(t+1)'}A_{i,j(t+1)'})^{1-\alpha}k_{j(t)'}^{\beta}(1+\sigma_{j(t)'})$$

Using equations (7) and (16) to analyse the alternatives facing individuals born in country N, it is easy to see that

$$U_N(C_{N,t},C_{N,t+1},N,N) = (Q_N A_{NN})^{\alpha} (Q_N A_{NN})^{1-\alpha} k_N^{\beta} (1+\sigma) >$$

$$(Q_N A_{NN})^{\alpha} (Q_S A_{NS})^{1-\alpha} k_N^{\beta} (1+\sigma) = U_N (C_{N,t}, C_{N,t+1}, N, S)$$

$$U_N(C_{N,t}, C_{N,t+1}, N, N) = (Q_N A_{NN})^{\alpha} (Q_N A_{NN})^{1-\alpha} k_N^{\beta} (1+\sigma) >$$

$$(Q_S A_{NS})^{\alpha} (Q_N A_{NN})^{1-\alpha} k_S^{\beta} = U_N(C_{N,t}, C_{N,t+1}, S, N)$$

$$U_N(C_{N,t}, C_{N,t+1}, N, N) = (Q_N A_{NN})^{\alpha} (Q_N A_{NN})^{1-\alpha} k_N^{\beta} (1+\sigma) >$$

Thus, individuals born in N choose to work and grow old in their country of origin.

Consider now the alternatives facing individuals born in country S. First note that, whatever the country chosen for the working period, they decide to grow old in their country of origin. In fact, using equations (7), (14), and (15), it is easy to see that:

 $(O_S A_{NS})^{\alpha} (O_S A_{NS})^{1-\alpha} k_S^{\beta} = U_N (C_{N,t}, C_{N,t+1}, S, S)$ 

$$U_{S}(C_{N,t}, C_{N,t+1}, N, S) = (Q_{N}A_{SN})^{\alpha}(Q_{S}A_{SS})^{1-\alpha}k_{N}^{\beta}(1+\sigma) >$$

$$(Q_{S}A_{SN})^{\alpha}(Q_{N}A_{SN})^{1-\alpha}k_{N}^{\beta}(1+\sigma) = U_{S}(C_{N,t}, C_{N,t+1}, N, N)$$

$$U_{S}(C_{S,t}, C_{S,t+1}, S, S) = (Q_{S}A_{SS})^{\alpha}(Q_{S}A_{SS})^{1-\alpha}k_{S}^{\beta} >$$

$$(Q_{S}A_{SS})^{\alpha}(Q_{N}A_{SN})^{1-\alpha}k_{S}^{\beta} = U_{S}(C_{S,t}, C_{S,t+1}, S, N)$$

This result depends on the fact that, in the second period of life, individuals make a choice only in relation to their locational preferences (their income has been determined in the first period), and since country S offers a better quality of life, and is strictly preferred to country N by those born in S, then individuals born in country S decide to grow old there, regardless of where they have chosen to spend the working period of their life. On the contrary, individuals born in N, in order to determine their locational choice must consider the additional condition (16); in fact, even though their preferences are "biased" in favour of their country of origin, they recognise that country S offers a better quality of life; thus preference for their country of origin must be strong enough.

It remains now to be determined whether individuals born in *S* choose to work there or in *N*. Individuals born in *S* will migrate and work in *N* if the following condition holds:

$$U_{S}(C_{N,t}, C_{N,t+1}, N, S) = (Q_{N}A_{SN})^{\alpha}(Q_{S}A_{SS})^{1-\alpha}k_{N}^{\beta}(1+\sigma) >$$

$$(O_{S}A_{SS})^{\alpha}(O_{S}A_{SS})^{1-\alpha}k_{S}^{\beta} = U_{S}(C_{S,t}, C_{S,t+1}, S, S)$$

Using the equilibrium condition (7) and simplifying, we can equivalently write:

$$(O_N A_{SN})^{\alpha} (1+\sigma)^{1/(1-\beta)} > (O_S A_{SS})^{\alpha}$$
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or

$$Z_{SN}^{\alpha}(1+\sigma)^{1/(1-\beta)} > Z_{SS}^{\alpha}$$

i.e.

$$Z_{SN}/Z_{SS} > (1+\sigma)^{1/[\alpha(1-\beta)]}$$

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Thus, with exogenous locational preferences we get the result that, if individuals born in S do not appreciate life in country N too badly, or if the wage differential is large enough, all the working population of country S will migrate to N. Of course an inverse conclusion holds if inequality (19) is reversed. In this case there will not be migration. It is immediate to verify that only in a particular case (19) can be satisfied with equality, thereby implying indifference between the two alternative locations for workers born in country S.

#### **Endogenous locational preferences**

More interesting conclusions hold if the ratio  $Z_{SN}/Z_{SS}$  is made endogenous, at least in its "objective" components,  $Q_S$ ,  $Q_N$ . Note that the endogeneity of  $Z_{SN}/Z_{SS}$  is empirically very reasonable; in fact, this ratio, although possibly linked to exogenous elements, such as the climate of the two countries, depends also on pollution, congestion, and locational preferences, all of which may be function of the distribution of labour. Thus migration to country N makes it more crowded, and possibly more polluted, thereby worsening its quality of life. Moreover, if many workers from country S migrate to N, indigenous population in N may become more hostile to individuals born in S. On the other hand, country N may become more pleasant for S nationals, as more workers born in S may live close to other migrants of their same culture and nationality.

We can determine the equilibrium distribution of workers from (19), holding it as an equality. However, as the result depends on the functional form chosen to endogenise the ratio on the left-hand side of (19), we limit our analysis to show how equilibrium is affected when the relevant parameters change under some reasonable functional forms.

#### Quality of life dependent on pollution

In this first case we assume that the "subjective" factors in the individuals' preferences for their country of origin  $(A_{ij})$  are given, while the relative quality of life in the two countries  $(Q_i)$  is a negative function of their respective levels of production. More specifically, we assume that greater production causes greater levels of pollution, and this deteriorates the quality of life. Formally we have:

$$A_{SN}/A_{SS} \equiv a < 1$$

$$Q_S/Q_N = q(Y_S/(Y_S + Y_N)) > 1$$

with q' < 0. Thus the quality of life is higher in country S, but detoriorates as its share of total production increases.

Using (1), (2), (3), and (7), and defining

$$\gamma \equiv L_N/(P_N + P_S)$$

we get

$$Y_S/(Y_S + Y_N) = [1 + (1 + \sigma)^{1/(1-\beta)}\gamma/(1 - \gamma)]^{-1}$$

and we can determine the sign of the relevant partial derivatives of q(.):

$$q_{\gamma}' = \frac{\partial q(.)}{\partial \gamma} > 0$$
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$$q'_{\sigma} \equiv \frac{\partial q(.)}{\partial \sigma} > 0$$
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$$q'_{\beta} \equiv \frac{\partial q(.)}{\partial \beta} > 0$$
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Thus an increase in  $\gamma$ ,  $\sigma$ , or  $\beta$ , by raising the share of total production of country N, improves the quality of life in country S relative to that in N.

The equilibrium condition (19) can now be written as

$$q(\gamma, \sigma, \beta)^{\alpha} = a^{\alpha} (1 + \sigma)^{1/(1 - \beta)}$$

and lend itself to exercises in comparative statics.

First note that, by differentiating (23) with respect to  $\gamma$  and  $\alpha$ , we get:

$$d\gamma/d\alpha = \frac{\left[a^{\alpha}\log(a)(1+\sigma)^{1/(1-\beta)} - q(.)^{\alpha}\log(q(.))\right]}{\alpha q(.)^{\alpha-1}q'_{\gamma}} < 0$$

because the denominator is positive, while the numerator is unambiguously negative (remember that a < 1, so that log(a) < 0). If the share of income devoted to consumption during the first period increases, the relevance of the quality of life in the place where that period is spent increases, and the incentive to migrate and work in N decreases.

Second, by differentiating (23) with respect to  $\gamma$  and a, we can see that when the preference for one's own country decreases (so that a becomes higher) a greater part of the working population of country S will migrate. In fact:

$$d\gamma/da = \frac{\alpha a^{\alpha-1} (1+\sigma)^{1/(1-\beta)}}{\alpha q(.)^{\alpha-1} q'_{\gamma}} > 0$$

Third, differentiating (23) with respect to  $\gamma$  and  $\sigma$ , we get an ambiguous result. This should not come as a surprise, because a higher technological advantage in favour of N raises the wage differential in favour of that country, but also worsens its quality of life with respect to that in S, because of higher pollution levels:

$$d\gamma/d\sigma = \frac{(1-\beta)^{-1}a^{\alpha}(1+\sigma)^{\beta/(1-\beta)} - \alpha q(.)^{\alpha-1}q'_{\sigma}}{\alpha q(.)^{\alpha-1}q'_{\sigma}}$$

Note that the denominator is positive, as are both terms in the numerator. Thus a higher wage differential in favour of country N raises migration only if the negative pollution effect (measured by  $q'_{\sigma}$ ) is not too high.

Forth, extending similar arguments to the differentiation of (23) with respect to  $\gamma$  and  $\beta$ , the result is again ambiguous; in fact  $\beta$  is positively linked to the wage differential between the two countries, but also negatively related to the share of country S in total production:

$$d\gamma/d\beta = \frac{a^{\alpha}(1+\sigma)^{1/(1-\beta)}log(1+\sigma)(1-\beta)^{-2} - \alpha q(.)^{\alpha-1}q'_{\beta}}{\alpha q(.)^{\alpha-1}q'_{\gamma}}$$

This derivative is also positive only if the "pollution effect" of an increase in  $\beta$  (as measured by  $q'_{\beta}$ ) is not too high.

#### Quality of life dependent on congestion

In this case we assume that the relative quality of life is affected by people crowding in a country, rather than by the country levels of production. We know from our previous analysis that individuals decide to grow old in the same country where they are born, so that we can assume q(.) to be a function of the international distribution of workers only. Thus we assume:

$$A_{SN}/A_{SS} \equiv a < 1$$

$$Q_S/Q_N = q(\gamma) > 1$$

with q' > 0.

Now, instead of (23), we differentiate the simpler equilibrium condition:

$$q(\gamma)^{\alpha} = a^{\alpha} (1+\sigma)^{1/(1-\beta)}$$

and we get the following results:

$$d\gamma/d\alpha = \frac{a^{\alpha} \log(a)(1+\sigma)^{1/(1-\beta)} - q(\gamma)^{\alpha} \log(q(\gamma))}{\alpha q(\gamma)^{\alpha-1} q' \gamma} < 0$$

$$d\gamma/da = \frac{\alpha a^{\alpha-1} (1+\sigma)^{1/(1-\beta)}}{\alpha q(\gamma)^{\alpha-1} q'_{\gamma}} > 0$$

$$d\gamma/d\sigma = \frac{(1-\beta)^{-1} a^{\alpha} (1+\sigma)^{\beta/(1-\beta)}}{\alpha q(\gamma)^{\alpha-1} q'_{\gamma}} > 0$$

$$d\gamma/d\beta = \frac{a^{\alpha} (1+\sigma)^{1/(1-\beta)} (1-\beta)^{-2} \log(1+\sigma)}{[\alpha q(.)^{\alpha-1} q'_{\gamma}]} > 0$$

and the ambiguity of the effects of  $\sigma$  and  $\beta$  on  $\gamma$  is eliminated.

# **Bibliography**

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