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Optimal Devaluations

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Abstract

According to the conventional wisdom, when an economy enters a recession and nominal prices adjust slowly, the monetary authority should devalue the domestic currency to make the recession less severe. The reason is that a devaluation of the currency lowers the relative price of non-tradable goods, and this reduces the necessary adjustment in output relative to the case in which the exchange rate remains constant. This paper uses a simple small open economy model with sticky prices to characterize optimal fiscal and monetary policy in response to productivity and terms of trade shocks. Contrary to the conventional wisdom, in this framework optimal exchange rate policy cannot be characterized just by the cyclical properties of output. The source of the shock matters: while recessions induced by a drop in the price of exportable goods call for a devaluation of the currency, those induced by a drop in productivity in the non-tradable sector require a revaluation.

This paper—a product of the Growth and the Macroeconomics Team, Development Research Group—is part of a larger effort in the department to understand the role of fiscal and monetary policy in developing countries. Policy Research Working Papers are also posted on the Web at http://econ.worldbank.org. The author may be contacted at chevia@worldbank.org.

Optimal Devaluations

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INTRODUCTION

In most economies that can be described as small and open, the Central Bank targets the value of the nominal exchange rate by intervening in the foreign currency market. Sometimes, the nominal exchange is fixed at a given value for a period of time; most often, the interventions respond to economic conditions in a discretionary way.

According to the conventional wisdom, when the economy enters a recession it is convenient to devalue the domestic currency due to the existence of price stickiness. It is argued that a devaluation lowers the relative price of non-tradable goods–that would not fall otherwise because nominal prices are sticky–and, therefore, allows for a smaller adjustment in quantities, making the recession less severe[1]. This conventional wisdom has been supported by the implications of static reduced form models in which the way firms that cannot adjust prices behave, given that constraint, is not formally modeled. The policy implication derived from these models is attractive in that it can be characterized in a simple way: the nominal exchange rate must be counter-cyclical.

In this paper we characterize the optimal exchange rate policy in a fully specified dynamic general equilibrium model of a small open economy in which the price frictions are explicitly modeled and firms have rational expectations. The model identifies the distortion that price stickiness introduces and the way this distortion interacts with other ones, like income taxes and monopoly power. We can formally analyze how these different distortions should be traded off to maximize welfare, as it is traditionally done in the second-best literature. The purpose of the paper is to explore if a standard general equilibrium dynamic Ramsey model of a small-open economy supports the conventional wisdom.

We study a labor only representative agent economy with monopolistic competition–so firms have power to set prices–a cash-in-advance constraint, a final non-tradable good, and

[1] This was the logic behind several devaluations, like the ones in Brazil (1999) Argentina (2001) and, to some extent, Spain, UK and other western European countries in the early 90´s.

two tradable intermediate inputs, one that can be exported and the other that must be imported–so we can analyze the optimal policy response following both real exchange rate and terms of trade shocks. Our model extends the closed economy model in Correia, Nicolini and Teles (2008) to allow for international trade in goods and international capital mobility.

We show that the conventional wisdom (that is, that a devaluation must follow a recession) does not hold even in the simple model we analyze: it depends on the source of the shock generating the fluctuation and on the modeling details. For example, if the recession is due to a decline in the international price of exportable goods, the nominal exchange rate should be devalued. On the other hand, if the recession is due to a negative productivity shock in the non-tradable sector, the nominal exchange rate should be revalued. In addition, modeling details such as the distinction between cash and credit goods or which is the sector where the price friction is present also matter. For example, as we impose a cash-in-advance constraint solely on consumers, it is never optimal to move the nominal exchange rate after government spending shock, but an active exchange rate policy is called for after those same shocks if the government also faces a cash-in-advance constraint.

We derive general principles for the conduct of optimal monetary policy which hold in other contexts[2]: optimal policy ought to fully stabilize the "right" price index and reproduce the flexible prices allocation. However, the way that principle translate into actual policy instruments depends on both the source of the shock and on details of the economy such as where exactly the rigidity is, which are the traded goods and which are the cash goods.

The policy instruments that we consider are labor income taxes, dividend taxes, a tax on the return to foreign assets, and monetary policy. We also allow the government to issue state contingent bonds. With these instruments, the government has a complete set of instruments, as defined in Chari and Kehoe (1999). As it is standard in Ramsey analysis, we abstract from time inconsistency and assume full commitment. Thus, whichever role the

[2] See for instance Obstfeld and Rogoff (2000) and Woodford (2003).

exchange rate can have in fostering good (or bad) reputation will be absent in this analysis.

Following the seminal work of Obstfeld and Rogoff (1995, 1996) there has been a growing interest in studying optimal policy in open economies with frictions in the setting of prices. A branch of the literature, like Obstfeld and Rogoff (2000), Engel (2001) and many others[3] focused on the two-countries case. This literature emphasizes the relationship between the strategic interactions in two-countries models and optimal exchange rate policy, and in most cases it focused on the flexible versus fixed exchange rate regimes debate. In addition, it ignores - with the exception of Adao, Correia and Teles (2005) - the interaction between exchange rate and monetary policy with distortionary fiscal policy, an issue that we formally address.

The case of the small open economy we focus has been the subject of several papers. Gali and Monacelli (2005) and de Paoli (2004) study a similar problem but do not consider the fiscal instruments as we do. Their approach only allows solving for linear approximations very useful to compare alternative rules, but less so to characterize properties of the optimal policy as we do. Cunha (2001) analyzes a model similar to ours but without a complete set of instruments and he can only solve some examples. Closer to ours is the work of Faia and Monacelli (2004a) and (2004b), who use a framework similar to ours, but without jointly considering fiscal and monetary instruments.

The main contribution of this paper is the focus: rather than studying the welfare properties of families of policy rules in general, we want to answer the following question: should a government of a small open economy devalue during a recession? The key difference with the literature is the joint analysis of monetary and fiscal policy with distortionary taxation. It turns out that with a complete set of instruments, the analysis of the solution is rather simple since all known results on the Ramsey literature can be applied. Therefore, we can

[3] An incomplete list also includes Corsetti and Pesenti (2001) and (2005), Devereux and Engel (2003), Benigno and Benigno (2003), Duarte and Obstfeld (2004), Ferrero (2005) and Adao, Correia and Teles (2005).

provide a very sharp answer to the question we focus on.

The paper is organized as follows. Section 2 spells out the model. Section 3 solves for the optimal allocation and characterizes the optimal monetary and exchange rate policy as a function of the shocks to the economy. Section 4 discusses extensions and concludes.

1. THE MODEL

We model a dynamic small open economy with uncertainty. Time is discrete and the state of the economy at period $t = 0, 1, ...$ is denoted by s_t, which belongs to a finite set of events, with s_0 given. Let $s^t = (s_0, s_1, ..., s_t)$ be the history of events up to period t, and let $\pi(s^t)$ be the probability of s^t conditional on s_0.

Households: There is a representative household who derives utility from a composite good and leisure according to

$$\mathbb{U} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t U\left(c\left(s^t\right), \ell\left(s^t\right)\right) \pi\left(s^t\right) \tag{1}$$

where $0 < \beta < 1$, $U(\cdot)$ is increasing in each argument and concave, $\ell(s^t)$ denotes leisure, and $c(s^t)$ is a composite final goods defined as

$$c\left(s^t\right) = \left[\int_0^1 c\left(i, s^t\right)^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}}.$$

The variable $c(i, s^t)$ denotes consumption of a final good of variety $i \in [0, 1]$, and $\theta > 1$ is the elasticity of substitution across varieties.[4] We assume that all varieties are non-traded, so they must be produced domestically.

Markets are complete. We let $B(s^t, s_{t+1})$ and $B^*(s^t, s_{t+1})$ be one-period bonds denominated in domestic and foreign currency respectively. These bonds are issued at s^t and pay

[4]Existence of an equilibrium requires $\theta > 1$.

one unit of the corresponding currency at $t+1$ on the event s_{t+1} and zero otherwise. The price in domestic and foreign currency of these claims are $Q\left(s^{t+1}|s^{t}\right)$ and $Q^{*}\left(s^{t+1}|s^{t}\right)$ respectively. For $k>0$, let $Q\left(s^{t+k}|s^{t}\right)=Q\left(s^{t+1}|s^{t}\right)Q\left(s^{t+2}|s^{t+1}\right)...Q\left(s^{t+k}|s^{t+k-1}\right)$ be the price of one unit of currency at s^{t+k} in units of currency as s^{t}. A similar definition holds for $Q^{*}\left(s^{t+k}|s^{t}\right)$. As long as a no-arbitrage condition between domestic and foreign bonds is satisfied (see condition (18) below), we can assume, without loss of generality, that households do not hold foreign bonds.[5] Therefore, for all periods, households face the budget constraint

$$\begin{aligned} M^{h}\left(s^{t}\right)+\sum_{s_{t+1}}B\left(s^{t+1}\right)Q\left(s^{t+1}|s^{t}\right) \leq\; & M^{h}\left(s^{t-1}\right)-\int_{0}^{1}P_{t-1}\left(i,s^{t-1}\right)c\left(i,s^{t-1}\right)di \\ & +W\left(s^{t-1}\right)\left(1-\ell\left(s^{t-1}\right)\right)\left(1-\tau^{n}\left(s^{t-1}\right)\right)+B\left(s^{t}\right), \end{aligned} \tag{2}$$

where $W\left(s^{t}\right)\left(1-\tau^{n}\left(s^{t-1}\right)\right)$ is the net of taxes nominal wage rate, $M^{h}\left(s^{t}\right)$ is the household's demand for currency, and $P_{t}\left(i,s^{t}\right)$ is the price of the final good of variety i. We assume that initial nominal wealth is zero: $M_{-1}+B_{0}=0$.[6]

We model a money demand by imposing a cash-in-advance constraint on a subset of goods. The timing protocol follows Lucas (1984) and we assume that goods in the set $\Gamma\subseteq[0,1]$ are cash goods, so we impose the following cash-in-advance constraint

$$\int_{\Gamma}P\left(i,s^{t}\right)c\left(i,s^{t}\right)di\leq M^{h}\left(s^{t}\right). \tag{3}$$

By making Γ a proper subset of the unit interval, we allow for cash and credit goods in the model.

[5] Note that markets are complete with the domestic securities, so the foreign currency denominated securities are redundant. It is convenient to introduce these bonds for two reasons, first, to let all intertemporal trade with the rest of the world be carried out by issuing foreign securities, and second, to derive the interest parity conditions.

[6] We assume this in order to avoid the well known problem of hyperinflation incentives in period 0 (Lucas and Stokey 1983).

If we use the compact notation x to denote the contingent sequence $\{x(s^t)\}$ for any x, the household's problem is to maximize (1), by choice of $\left\{c(i), \ell, M^h, B\right\}$, subject to (2), (3), and an arbitrarily large negative lower bound on the real holding of assets. If the cash-in-advance constraint is binding, the optimality conditions can be expressed as

$$\frac{U_\ell(s^t)}{U_c(s^t)} = \frac{W(s^t)}{P_t(s^t)}\left(1-\tau^n\left(s^t\right)\right) \tag{4}$$

$$Q\left(s^{t+1}|s^t\right) = \frac{\beta U_c(s^{t+1})}{U_c(s^t)}\frac{P_t(s^t)}{P_{t+1}(s^{t+1})}\frac{R(s^{t+1})}{R(s^t)}\pi\left(s^{t+1}|s^t\right) \tag{5}$$

$$c\left(i,s^t\right) = c\left(s^t\right)\left(\frac{P_t(s^t)}{R(s^t)P_t(i,s^t)}\right)^\theta \quad \text{if } i\in\Gamma \tag{6}$$

$$c\left(i,s^t\right) = c\left(s^t\right)\left(\frac{P_t(s^t)}{P_t(i,s^t)}\right)^\theta \quad \text{if } i\in\Gamma^c \tag{7}$$

where $\pi(s^{t+1}|s^t)$ is the conditional probability of s^{t+1} given s^t, and $P_t(s^t)$ is a price index for households defined as

$$P_t\left(s^t\right) \equiv \left[R\left(s^t\right)^{1-\theta}\int_\Gamma P_t\left(i,s^t\right)^{1-\theta}di + \int_{\Gamma^c} P_t\left(i,s^t\right)^{1-\theta}di\right]^{\frac{1}{1-\theta}}.$$

Here $U_c(s^t)$ and $U_\ell(s^t)$ denote the marginal utility of consumption and leisure at s^t respectively, $R(s^t) = \left[\sum_{s_{t+1}} Q(s^{t+1}|s^t)\right]^{-1}$ is the nominal (gross) interest rate between periods t and $t+1$, and $P_t(s^t)$ is the price of a unit of consumption aggregate $c(s^t)$.[7] The price index incorporates the fact that the effective price of a cash good, $P_t(i,s^t)R(s^t)$, includes the financial cost of holding currency. Equation (4) is the standard intratemporal condition between leisure and the consumption aggregate; (5) is the intertemporal optimality condition; and (6) and (7) are the conditional demands of cash and credit goods, respectively.

[7] It is standard to show that if $R(s^t) > 1$ the cash-in-advance constraint binds, while if $R(s^t) = 1$ it does not. In this latter case we will focus on the equilibria where (3) holds at equality.

Further, (6) and (7) can be expressed as

$$\left(\frac{c\left(i,s^{t}\right)}{c\left(j,s^{t}\right)}\right)^{1/\theta}=\frac{P_{t}\left(j,s^{t}\right)}{R\left(s^{t}\right)P_{t}\left(i,s^{t}\right)}\quad\text{if } i\in\Gamma \text{ and } j\in\Gamma^{c}. \tag{8}$$

This condition equates the marginal rate of substitution between cash and credit goods to their relative price.

Government: The government has to finance an exogenous sequence of expenditures, which is also a composite of non-tradeable goods identical to that of the households,

$$g\left(s_{t}\right)=\left[\int_{0}^{1}g\left(i,s^{t}\right)^{\frac{\theta-1}{\theta}}di\right]^{\frac{\theta}{\theta-1}},$$

where $g\left(i,s^{t}\right)$ is the government purchase of the final good of variety i. Taking as given the prices $P_{t}\left(i,s^{t}\right)$, the government minimizes the cost of acquiring the aggregate $g\left(s_{t}\right)$. Cost minimization implies the conditional demands

$$g\left(i,s^{t}\right)=g\left(s_{t}\right)\left(\frac{P_{t}^{g}\left(s^{t}\right)}{P_{t}\left(i,s^{t}\right)}\right)^{\theta}. \tag{9}$$

where $P_{t}^{g}\left(s^{t}\right)=\left[\int_{0}^{1}P_{t}\left(i,s^{t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$ is the cost minimizing price index for the government. Households and the government face different price indexes because the former are subject to a cash-in-advance constraint.

Monetary policy consists of rules for the stock of money $M\left(s^{t}\right)$, and the nominal exchange rate between domestic and foreign currency $E\left(s^{t}\right)$. Fiscal policy consists of linear tax rates on labor $\tau^{n}\left(s^{t}\right)$; taxes on dividends $\tau^{d}\left(s^{t}\right)$; linear taxes on foreign securities $\tau^{*}\left(s^{t}\right)$; one-period bonds issued to domestic residents denominated in domestic currency $B\left(s^{t},s_{t+1}\right)$; and one-period state contingent bonds issued to foreigners denominated in foreign currency $B^{*}\left(s^{t},s_{t+1}\right)$. A government policy is defined as $\omega\equiv\left\{M,E,\tau^{n},\tau^{d},\tau^{*},B,B^{*}\right\}$, where we use the compact notation described above.

The introduction of taxes on foreign securities is useful for two reasons: first, it roughly captures the idea of taxing international capital flows, and second, they are used to implement the Ramsey allocation. In the last section we discuss the consequences of eliminating these taxes, and how they can be substituted by consumption taxes. In addition, since taxation of dividends is equivalent to lump-sum taxes, any optimal policy will set dividend taxes to its highest feasible value $\tau^d(s^t) = 1$ for all s^t.[8]

Tradeable Sector: The non-tradeable varieties are produced using two tradeable inputs. One of them can be produced domestically and the other must be imported.[9] Output of the home - exportable - input is given by the linear production function

$$X\left(s^t\right) = A\left(s_t\right) n\left(s^t\right)$$

where $A(s_t)$ is a technology shock and $n(s^t)$ is labor. Competitive pricing requires

$$P_x\left(s^t\right) = W\left(s^t\right)/A\left(s_t\right). \tag{10}$$

where $P_x(s^t)$ is the domestic currency price of the home input.

As this is a small open economy, we assume that international trade is carried out in foreign currency. Absence of arbitrage opportunities implies the purchasing power parity conditions

$$\begin{aligned} P_x\left(s^t\right) &= E\left(s^t\right) P_x^*\left(s_t\right) \qquad (11) \\ P_m\left(s^t\right) &= E\left(s^t\right) P_m^*\left(s_t\right). \end{aligned}$$

[8] The results also hold if $\tau^d(s^t) = 0$ or if it is constrained to be a positive number lower than one. The problem is much simpler with our assumption, otherwise the governemnt will try to manipulate other instruments, like monetary policy or exchange rates in order to tax profits.

[9] This allows consideration for terms of trade shocks.

where $P_m(s^t)$ is the domestic currency price of the foreign input and the terms of trade are defined as $d^*(s_t) = P_x^*(s_t)/P_m^*(s_t)$ for all t.

Production of Varieties: The technology to produce the final good $i \in [0,1]$ is given by the Cobb-Douglas production function

$$y\left(i, s^t\right) = Z\left(s_t\right) x\left(i, s^t\right)^{\eta} m\left(i, s^t\right)^{1-\eta}$$

where $x(i, s^t)$ is the home input, $m(i, s^t)$ the foreign input, and $Z(s_t)$ is an aggregate productivity shock common across varieties.[10]

The Cobb-Douglas technology implies that marginal cost is given by

$$MC\left(s^t\right) = \frac{P_x\left(s^t\right)^{\eta} P_m\left(s^t\right)^{1-\eta}}{Z\left(s_t\right)\eta^{\eta}\left(1-\eta\right)^{1-\eta}}.$$

Using (11) and the definition of $d^*(s_t)$, the marginal cost function can be written, in equilibrium, as

$$MC\left(s^t\right) = \frac{P_m\left(s^t\right)}{\eta^{\eta}\left(1-\eta\right)^{1-\eta}}\frac{d^*\left(s_t\right)^{\eta}}{Z\left(s_t\right)}$$

Cost minimization implies that all final goods firms choose the same inputs ratio

$$\frac{m\left(i, s^t\right)}{x\left(i, s^t\right)} = \left(\frac{1-\eta}{\eta}\right) d^*\left(s_t\right) \text{ for all } i. \tag{12}$$

Then, equilibrium production can be expressed as

$$y\left(i, s^t\right) = m\left(i, s^t\right)\frac{Z\left(s_t\right)}{d^*\left(s_t\right)^{\eta}}\left(\frac{\eta}{1-\eta}\right)^{\eta}. \tag{13}$$

[10]Our results generalize to any constant returns to scale technology $y(i, s^t) = F(x(i, s^t), m(i, s^t))$ but at the cost of additional notation, see Hevia and Nicolini (2004).

We assume that each variety is produced by a monopolist.[11] We also assume that there is price stickiness. In particular, firms $i \in [0, \alpha]$ are constrained to set prices at period t conditional on the information available up to period $t-1$. Equivalently, we can think of those firms, called *sticky firms*, as setting prices one period in advance.[12] The other firms, called *flexible firms*, are allowed to set prices at t conditional on s^t.

The demand faced by firm i is $y^d(i, s^t) = c(i, s^t) + g(i, s^t)$, and differs according to the cash-credit characteristic of the good. Equations (6), (7) and (9) imply

$$y^d\left(i, s^t\right) = P_t\left(i, s^t\right)^{-\theta} \widetilde{Y}\left(i, s^t\right),$$

where

$$\widetilde{Y}\left(i, s^t\right) = \begin{cases} c\left(s^t\right)\left[P_t\left(s^t\right)/R\left(s^t\right)\right]^{\theta} + g\left(s_t\right) P_t^g\left(s^t\right)^{\theta} & \text{if } i \in \Gamma \\ c\left(s^t\right) P_t\left(s^t\right)^{\theta} + g\left(s_t\right) P_t^g\left(s^t\right)^{\theta} & \text{if } i \in \Gamma^c \end{cases}.$$

Note that both the marginal cost and the elasticity of demand that each monopolist face is the same across varieties. Thus, from the point of view of preferences and technologies, the model exhibits total symmetry. However, the frictions in transactions and in the setting of prices do introduce heterogeneity across firms. Indeed, there are four types of final goods firms: flexible firms with and without cash-in-advance constraint, and sticky firms with and without cash-in-advance constraints.

Flexible firms face the static optimization problem of maximizing nominal dividends,[13]

$$\max_{P_t(i,s^t)} \left[P_t\left(i, s^t\right) - MC\left(s^t\right)\right] y^d\left(i, s^t\right).$$

[11] This assumption is essential to have firms that can set prices in advance.

[12] The results also hold for other forms of price stickyness, as Calvo staggered pricing for instance. For details, see Correia, Nicolini and Teles (2002).

[13] Strictly speaking, full taxation of dividends imply that the pricing and production decisions of the firms are indeterminate. It is convenient, instead, to think of each firm maximizing after-tax dividends for $\tau_j^d < 1$, and then considering the limit economy as $\lim_{j\to\infty} \tau_j^d = 1$ for all s^t.

Independently of whether the good is cash or credit, the optimal pricing rule determines the price as a constant mark-up over the marginal cost,

$$P_t\left(i,s^t\right)=\frac{\theta}{\theta-1}\;\frac{P_m\left(s^t\right)}{\eta^{\eta}\left(1-\eta\right)^{1-\eta}}\frac{d^*\left(s_t\right)^{\eta}}{Z\left(s_t\right)}\quad \text{for } i\in(\alpha,1]. \tag{14}$$

Sticky firms set prices at period t conditional on information available up to period $t-1$. This is equivalent to choosing prices at $t-1$ to maximize the value of the next period's dividends,[14]

$$\max_{P_t\left(i,s^{t-1}\right)}\sum_{s^t|s^{t-1}}Q\left(s^t|s^{t-1}\right)\left[P_t\left(i,s^{t-1}\right)-MC\left(s^t\right)\right]y^d\left(i,s^t\right)$$

The optimal pricing rule is

$$P_t\left(i,s^{t-1}\right)=\frac{\theta}{\theta-1}\sum\nolimits_{s^t|s^{t-1}}\psi\left(i,s^t\right)\frac{P_m\left(s^t\right)}{\eta^{\eta}\left(1-\eta\right)^{1-\eta}}\frac{d^*\left(s_t\right)^{\eta}}{Z\left(s_t\right)}\quad \text{for } i\in[0,\alpha]\,, \tag{15}$$

where

$$\psi\left(i,s^t\right)=\frac{Q\left(s^t|s^{t-1}\right)\;\widetilde{Y}\left(i,s^t\right)}{\sum_{s^t|s^{t-1}}Q\left(s^t|s^{t-1}\right)\;\widetilde{Y}\left(i,s^t\right)}.$$

The optimal price is set as a mark-up over a weighted average of the marginal cost across states. Unless the government follows the Friedman rule (i.e. $R\left(s^t\right)=1$), sticky firms will choose different prices depending on whether the good they produce is cash or credit.

In addition, we assume throughout that the initial price at period 0 of *all* sticky firms are identical and given by P_0^s.

Foreign Sector: The trade balance measured in units of the home input is defined as

$$TB\left(s^t\right)=X\left(s^t\right)-\int_0^1 x\left(i,s^t\right)di-\frac{P_m\left(s^t\right)}{P_x\left(s^t\right)}\int_0^1 m\left(i,s^t\right)di, \tag{16}$$

[14] We assume that shocks are sufficiently small so that sticky firms always find it optimal to remain active.

where $X\left(s^t\right)-\int_0^1 x\left(i,s^t\right)di$ is net exports of the home input and $\frac{P_m\left(s^t\right)}{P_x\left(s^t\right)}\int_0^1 m\left(i,s^t\right)di$ denotes the imports of the foreign input measured in units of the home input.

The evolution of the country's foreign debt is given by

$$\sum_{s^{t+1}|s^t} B^*\left(s^{t+1}\right)Q^*\left(s^{t+1}|s^t\right)+P_x^*\left(s_{t-1}\right)TB\left(s^{t-1}\right)=B^*\left(s^t\right)$$

where $B^*\left(s^t\right)$ denotes the stock of foreign debt of the economy as a whole and $TB\left(s^{-1}\right)=0$. Solving the previous equation starting from period 0 forward and ruling out Ponzi schemes, we obtain the economy foreign sector feasibility constraint,

$$\sum_{t=0}^{\infty}\sum_{s^t} P_x^*\left(s_t\right)\frac{Q^*\left(s^t|s^0\right)}{R^*\left(s^t\right)}TB\left(s^t\right)=B_0^*$$

where B_0^* is initial debt and $R^*\left(s^t\right)=\left[Q^*\left(s^{t+1}|s^t\right)\right]^{-1}$ is the foreign interest rate.

Foreign investors are risk neutral and discount the future at the same rate as domestic residents, hence the price Q^* satisfies

$$Q^*\left(s^{t+1}|s^t\right)=\beta\pi\left(s^{t+1}|s^t\right)\frac{P_x^*\left(s_t\right)}{P_x^*\left(s_{t+1}\right)}\frac{R^*\left(s^{t+1}\right)}{R^*\left(s^t\right)}$$

Thus, the foreign sector constraint becomes

$$\sum_{t=0}^{\infty}\sum_{s^t}\beta^t TB\left(s^t\right)\pi\left(s^t\right)=\frac{B_0^*\ R^*\left(s_0\right)}{P^*\left(s_0\right)} \tag{17}$$

Finally, by arbitrage, the price of domestic and foreign bonds must be related through the covered interest parity condition

$$Q\left(s^{t+1}|s^t\right)=Q^*\left(s^{t+1}|s^t\right)\left(1+\tau^*\left(s^{t+1}\right)\right)\frac{E\left(s^t\right)}{E\left(s^{t+1}\right)}. \tag{18}$$

Market Clearing: Feasibility in the final goods, labor and currency market require

$$c\left(i, s^t\right) + g\left(i, s^t\right) = y\left(i, s^t\right), \text{ for } i \in [0,1], \tag{19}$$

$$n\left(s^t\right) = 1 - \ell\left(s^t\right) \tag{20}$$

$$M^h\left(s^t\right) = M\left(s^t\right). \tag{21}$$

Definition: An allocation a and a price system $\mathcal{P}$ are contingent sequences $a = \{c\,(i),\, \ell,$ $x\,(i),\, m\,(i),\, y\,(i)\}$ and $\mathcal{P} = \{Q,\, R,\, P_x,\, P_m,\, P\,(i),\, W\}$ for $i \in [0,1]$.

Definition: Given a government policy ω, an allocation a and a price system $\mathcal{P}$ are an equilibrium if (i) households solve their utility maximization problem; (ii) the price of the home input satisfies (10); (iii) final goods producers act optimally: (12) hold for all $i \in [0,1]$ and they follow the pricing rules (14) if $i \in (\alpha, 1]$ and (15) if $i \in [0, \alpha]$; (iv) the market clearing conditions (19), (20) and (21) are satisfied; (v) the economy-wide feasibility constraint (17) holds; (vi) the no-arbitrage conditions (11) and (18) hold; and $vii)$ net nominal interest rates are non-negative, $R\,(s^t) \geq 1$ for all s^t. (By Walras' Law, the government budget constraint is also satisfied.)

2. THE RAMSEY PROBLEM

As it is standard in the literature, we assume the government can commit to a particular policy chosen at period 0. We define an allocation rule $a\,(\omega)$ as the mapping from the set of government policies into equilibrium allocations. Specifically, $a\,(\omega)$ is the set of allocation that satisfy conditions $i)$ to $vii)$ in definition 2 given ω, and we call these allocations implementable. The Ramsey problem is to choose the implementable allocation that maximizes the household's utility (1). Let the solution, called the Ramsey allocation, be denoted by a^R.

The standard approach (Lucas and Stokey (1983), Chari and Kehoe (1999)) is to find necessary and sufficient conditions that an allocation has to satisfy to be implementable. Let the set of allocations that satisfy the necessary and sufficient conditions, called the *implementable set,* be denoted by $J(\alpha, \Gamma)$.[15] Then, a^R is defined as

$$a^R \in \arg \max_{a \in J(\alpha,\Gamma)} \mathbb{U}(a).$$

As in this model the sufficient conditions cannot be characterized in terms of the allocations alone, we follow a different approach to characterize the Ramsey allocation. We first describe a set of *necessary* conditions any equilibrium allocation must satisfy.

Proposition 1: *Given a government policy ω, if an equilibrium exists, the allocation $a(\omega)$ satisfies the following conditions:*

i) feasibility:

$$c\left(i, s^t\right) + g\left(i, s^t\right) = m\left(i, s^t\right) \frac{Z(s_t)}{d^*(s_t)^\eta} \left(\frac{\eta}{1-\eta}\right)^\eta; \tag{22}$$

ii) current account sustainability:

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t TB\left(s^t\right) \pi\left(s^t\right) = \frac{B_0^* \; R^*(s_0)}{P^*(s_0)}, \tag{23}$$

where

$$TB\left(s^t\right) = A(s_t)\left(1 - \ell\left(s^t\right)\right) - \frac{\int_0^1 m\left(i, s^t\right) di}{(1-\eta)\, d^*(s_t)};$$

and

iii) household's optimization

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \left[U_c\left(s^t\right) c\left(s^t\right) - U_\ell\left(s^t\right)\left(1 - \ell\left(s^t\right)\right)\right] \pi\left(s^t\right) = 0. \tag{24}$$

[15] Given a policy, the equilibrium allocation in general depends on the parameters α and Γ.

Proof: Condition $i)$ follows from (13) and (19). To obtain condition $ii)$, use the definition of $X(s^t)$ and (20) into (16), and note that (11) and (12) imply

$$\int_0^1 x\left(i,s^t\right)di + \frac{P_m\left(s^t\right)}{P_x\left(s^t\right)}\int_0^1 m\left(i,s^t\right)di = \frac{\int_0^1 m\left(i,s^t\right)di}{(1-\eta)\,d^*\left(s_t\right)}.$$

Finally, from (2) construct the household's present value budget constraint

$$\sum_{t=0}^{\infty}\sum_{s^t}\frac{Q\left(s^t|s_0\right)}{R\left(s^t\right)}\left\{\int_0^1 P_t\left(i,s^t\right)c\left(i,s^t\right) + M\left(s_t\right)\left(R\left(s_t\right)-1\right) - W\left(s^t\right)\left(1-\tau^n\left(s^t\right)\right)\left(1-\ell\left(s^t\right)\right)\right\} = 0$$

where we assume $B_0 + M_{-1} = 0$. Condition (24) follows, as it is standard, by using (3), (4) and (5) into the previous equation.□

Conditions (22) and (23) are feasibility in the final goods market and foreign-sector feasibility, and (24) summarizes the household's behavior.

It is important to stress that (22), (23) and (24) are not sufficient conditions of an equilibrium allocation. In addition to those conditions, an equilibrium allocation must satisfy a symmetry constraint across final goods firms of each type. There are four types of final goods firms, the four possible combinations of sticky-flexible, and cash-credit goods. In equilibrium, firms of each type choose the same prices and inputs. These constraints imposed by the price setting and financial restrictions make the equilibrium set $J(\alpha,\Gamma)$ a complicated object to work with. Thus, we find it convenient to focus on a larger set $\widetilde{J}$ defined as follows,

Definition: Let $\widetilde{J}$ be the set of allocations satisfying the conditions in proposition 1.

As the conditions in proposition 1 are necessary, it is clear that $\widetilde{J}$ includes the equilibrium set $J(\alpha,\Gamma)$ for any pair (α,Γ).[16] Our strategy consists on finding the best allocation in the set $\widetilde{J}$, and then showing that the argmax belongs to the set $J(\alpha,\Gamma)$ for any pair (α,Γ). In other words, since the equilibrium set $J(\alpha,\Gamma)$ is a complicated object, we find the allocation

[16]It is easy to see that the set $\widetilde{J}$ is independent of both α and Γ.

that maximizes the household's utility in the set $\widetilde{J}$. Since $\widetilde{J}$ includes $J(\alpha, \Gamma)$, we are solving a relaxed Ramsey problem which maximizer allocation may not be an equilibrium. We then show, however, that for all (α, Γ) there is a government policy ω^R and a price system $\mathcal{P}^R$ such that the relaxed Ramsey allocation satisfies the additional equilibrium conditions. Consequently, the relaxed Ramsey allocation can be implemented as an equilibrium and therefore, it is the Ramsey allocation. Interestingly, since the larger set $\widetilde{J}$ is independent of the degree of price stickiness α, and of the set of cash-goods Γ, it follows that the Ramsey allocation a^R is also independent of α and Γ.[17]

The *relaxed* Ramsey allocation $\widetilde{a}$ is defined as

$$\widetilde{a} = \arg\max_{a \in \widetilde{J}} \mathbb{U}(a).$$

Since $J(\alpha, \Gamma) \subseteq \widetilde{J}$, it follows that $\mathbb{U}(\widetilde{a}) \geq \mathbb{U}\left(a^R\right)$. Proposition 2 below states that, in fact, $\widetilde{a} = a^R$, so $\mathbb{U}(\widetilde{a}) = \mathbb{U}\left(a^R\right)$.

Relaxed Ramsey Problem: The relaxed Ramsey problem is to maximize (1), by choice of $\{c(i), \ell, m(i)\}$, subject to the necessary conditions (22), (23), and (24). We solve this problem by posing the Lagrangian

$$\begin{aligned} L = & \sum_t \sum_{s^t} \beta^t \pi\left(s^t\right) \left\{ U\left(c\left(s^t\right), \ell\left(s^t\right)\right) + \lambda\left[U_c\left(s^t\right) c\left(s^t\right) - U_\ell\left(s^t\right)\left(1 - \ell\left(s^t\right)\right)\right] \right. \\ & - \int_0^1 \xi\left(i, s^t\right)\left[c\left(i, s^t\right) + g\left(i, s^t\right) - m\left(i, s^t\right) \frac{Z(s_t)}{d^*(s_t)^\eta}\left(\frac{\eta}{1-\eta}\right)^\eta\right] di \\ & \left. + \zeta \left[A(s_t)\left(1 - \ell\left(s^t\right)\right) - \frac{\int_0^1 m\left(i, s^t\right) di}{(1-\eta) d^*(s_t)}\right] \right\}.^{18} \end{aligned}$$

where $\beta^t \pi\left(s^t\right) \xi\left(i, s^t\right)$, ζ and λ are the Lagrange multipliers on (22), (23) and (24), respectively.

[17] For a further discussion of this result in a closed economy model, see Correa, Nicolini and Teles (2002).

Lemma 1: *The relaxed Ramsey allocation satisfies* $c(i,s^t) = c(s^t)$ *for all* i *and* s^t. *In addition, if* $\widetilde{a}$ *is implementable, then* $g(i,s^t) = g(s_t)$, $x(i,s^t) = x(s^t)$ *and* $m(i,s^t) = m(s^t)$ *for all* i *and* s^t. *Furthermore, prices satisfy* $P_t(i,s^t) = P_t(s^t)$ *for all* i *and* s^t, *and the Friedman rule is optimal, that is* $R(s^t) = 1$ *for all* s^t.

Proof: Taking the necessary first order conditions of the Lagrangian with respect to $m(i,s^t)$ and $c(i,s^t)$, we obtain

$$\xi\left(i,s^t\right) = \frac{\zeta}{\eta^{\eta}\left(1-\eta\right)^{1-\eta}} \frac{1}{Z\left(s_t\right) d^*\left(s_t\right)^{1-\eta}}$$

$$\left\{U_c\left(s^t\right)\left(1+\lambda\right) + \lambda\left[U_{cc}\left(s^t\right) c\left(s^t\right) - U_{\ell c}\left(s^t\right)\left(1-\ell\left(s^t\right)\right)\right]\right\}\left(\frac{c\left(i,s^t\right)}{c\left(s^t\right)}\right)^{-1/\theta} = \xi\left(i,s^t\right)$$

The last two equations and the definition of $c(s^t)$ imply $c(i,s^t) = c(s^t)$ for all i and s^t. It follows from the definition of $P_t(s^t)$ and conditions (6) and (7) that $P_t(s^t) = P_t(i,s^t)$ for all i and s^t. Then (8) and the previous results imply $R(s^t) = 1$ for all s^t. Further, $g(i,s^t) = g(s_t)$ for all i and s^t follows from (9), and equations (22) and (12) imply $m(i,s^t) = m(s^t)$ and $x(i,s^t) = x(s^t)$ for all i and s^t.$\square$

Corollary: *If* $\alpha > 0$, *all prices at period* t *depend on* s^{t-1}. *That is, the price of the final goods are perfectly forecastable one period in advance.*

The intuition is as follows: the set $\widetilde{J}$ does not restrict the behavior of the final goods firms. Since any pair of final goods $c(i,s^t)$ and $c(j,s^t)$ for $i \neq j$ are imperfect substitutes in the homogeneous consumption aggregate $c(s^t)$, it is optimal to set a symmetric allocation in which $c(i,s^t) = c(s^t)$ for all i. If implementable, a symmetric allocation requires an equal price across varieties. The prices that matter are those *faced by the households and the government,* and incorporate the financial cost of holding currency in order to buy cash-goods. Therefore, this requires an identical nominal price $P(i,s^t)$ across varieties and following the Friedman rule $R(s^t) = 1$. In addition, if $\alpha > 0$, a fraction of firms set prices at t, conditional on the information at $t-1$, hence equal nominal prices requires the prices of *all* varieties to depend on information available up to period $t-1$.

The next proposition shows that indeed, the relaxed Ramsey allocation $\widetilde{a}$ is implementable as an equilibrium for any degree of price stickiness α.

Proposition 2: *For any pair* (α, Γ) *there is a government policy* ω^R *and a price system* $\mathcal{P}^R$ *that implement the relaxed Ramsey allocation* $\widetilde{a}$. *Therefore,* $\widetilde{a} = a^R$. **Proof:** in the appendix.

It is interesting to note that the relaxed Ramsey problem solves a first best for a 'distorted' utility function defined as

$$\widetilde{U}\left(c\left(s^t\right), \ell\left(s^t\right); \lambda\right) \equiv U\left(c\left(s^t\right), \ell\left(s^t\right)\right) + \lambda\left[U_c\left(s^t\right) c\left(s^t\right) - U_\ell\left(s^t\right)\left(1 - \ell\left(s^t\right)\right)\right].$$

If $\widetilde{U}\left(c\left(s^t\right), \ell\left(s^t\right); \lambda\right)$ is well behaved (i.e. increasing and concave),[19] the qualitative response of the Ramsey allocation to the different shocks is identical to the response in the first best, where the planner maximizes (1) subject to the feasibility constraints.

Using that the allocation is symmetric and the definition of $\widetilde{U}(c, \ell; \lambda)$, the necessary first order conditions of the Ramsey problem can be expressed as

$$\widetilde{U}_c\left(c\left(s^t\right), \ell\left(s^t\right); \lambda\right) = \frac{\zeta}{\eta^\eta (1-\eta)^{1-\eta}} \frac{1}{Z(s_t)\, d^*(s_t)^{1-\eta}} \tag{25}$$

$$\widetilde{U}_\ell\left(c\left(s^t\right), \ell\left(s^t\right); \lambda\right) = \zeta\ A(s_t) \tag{26}$$

[19] This may not be the case, since $U_c(s^t)\, c(s^t)$ may not be concave in consumption.

Feasibility in the final goods market becomes

$$c\left(s^{t}\right)+g\left(s_{t}\right)=m\left(s^{t}\right)\frac{Z\left(s_{t}\right)}{d^{*}\left(s_{t}\right)^{\eta}}\left(\frac{\eta}{1-\eta}\right)^{\eta}. \tag{27}$$

Equations (25), (26) and (27) together with constraints (23) and (24) characterize the relaxed Ramsey allocation, where the trade balance becomes

$$TB\left(s^{t}\right)=A\left(s_{t}\right)\ \left(1-\ell\left(s^{t}\right)\right)-\frac{m\left(s^{t}\right)}{\left(1-\eta\right)d^{*}\left(s_{t}\right)}. \tag{28}$$

Specifically, equations (25) and (26) can be used to solve for consumption and leisure as a function of the multipliers λ and ζ. Then we can use (12) and (27) to solve for $m\left(s^{t}\right)$ and $x\left(s^{t}\right)$ as a function of λ and ζ. The present value constraints (23) and (24) can be used to find the values for ζ and λ.

Equations (25), (26), (27) and (28) imply that the Ramsey allocation only depends on the realization of the stochastic processes dated at period t, as government purchases $g\left(s_{t}\right)$, productivity shocks $A\left(s_{t}\right)$ and $Z\left(s_{t}\right)$, and the terms of trade shock $d^{*}\left(s_{t}\right)$, and not on the history of realizations s^{t}. In addition, government expenditure shocks do not affect $c\left(s^{t}\right)$ nor $\ell\left(s^{t}\right)$ since both can be solved as a function of the shocks $A\left(s_{t}\right)$, $Z\left(s_{t}\right)$ and $d^{*}\left(s_{t}\right)$, and of the two multipliers λ and ζ, which do not depend on the particular realization of $g\left(s_{t}\right)$. For example, if government expenditures increase, equation (27) shows that in order to keep private consumption constant, the usage of the tradeable inputs $m\left(s^{t}\right)$ and $x\left(s^{t}\right)$ increase. Then, (28) shows that the trade balance decreases. In other words, the availability of international credit allows the planner to insure all government expenditure shocks through borrowing and lending.

In what follows we assume that utility is separable between consumption and leisure and that is well behaved, so $\widetilde{U}(c,\ell) = H(c;\lambda) - V(1-\ell;\lambda)$ is increasing and concave.[20] We will analyze these cases first, and we discuss other cases in the last section.

Response to Shocks: Here we study how the different shocks affect the optimal allocation. Suppose that we want to analyze a shock to the final good technology $Z(s_t)$. Strictly speaking, given s^{t-1}, the analysis compares two states s_t and s_t' such that $Z(s_t) > Z(s_t')$. However, given the stationarity of the Ramsey allocation in our set-up, this is equivalent to analyze the experiment $Z(s_t) > Z(s_{t-1})$. Thus, both interpretations follow from our analysis. In addition, since government expenditure shocks were analyzed above, here we focus on productivity shocks $Z(s_t)$ and $A(s_t)$, and terms of trade shocks $d^*(s_t)$.

1) Consider a negative shock to the final goods technology $Z(s_t)$. Leisure remains constant, consumption decreases. The change in the trade balance and the change in usage of inputs $m(s_t)$ and $x(s_t)$ are indeterminate.

Proof: Conditions (25) and (26) imply that $c(s_t)$ decreases and $\ell(s_t)$ remains constant. From (27) it follows that $x(s_t)$ could either increase or decrease. Hence from (12) and (28), $m(s_t)$ and $TB(s_t)$ could increase or decrease.□

The constancy of labor supply is a consequence of the separable utility function and the fact that the country has access to international capital markets. This can be seen in equation (26), where it is evident that the allocation of labor depends solely on the shocks to the home input sector and the multipliers, which capture the wealth of the country in present value terms. The usage of inputs $x(s_t)$ and $m(s_t)$ could increase or decrease depending on the curvature (i.e. the willingness to substitute intertemporally) of the utility function.

2) Consider a negative shock to the terms of trade $d^*(s_t)$. Leisure remains constant and consumption decreases. The usage of the foreign input $m(s_t)$ decreases. The change in $x(s^t)$ and $TB(s_t)$ are indeterminate.

[20] In this case, charaterizing the reaction of the optimal allocation after a shock is equivalent to a first best and the intuitions are straighforward.

Proof: (25) and (26) imply that $c(s_t)$ decreases and $\ell(s_t)$ does not change. From (27) it follows that $m(s_t)$ decreases. Then (12) imply that $x(s_t)$ could increase or decrease. Finally, from (28), $TB(s_t)$ could increase or decrease.□

Intuitively, a negative shock to the terms of trade has, in addition to a negative income effect, a substitution effect. The country is poorer since the price of the exportable input is lower. The negative income effect implies a reduction in consumption. Labor remains constant for the same reason as in *1)* above. The income effect implies a lower usage of the inputs $x(s_t)$ and $m(s_t)$ to produce less final goods. But the reduction in the relative price of $x(s_t)$ relative to $m(s_t)$ implies a substitution toward the home input. Both effects imply an reduction in $m(s_t)$, but the net effect in $x(s_t)$ and the trade balance, are indeterminate.

3) Consider a negative shock to the home input technology $A(s_t)$. $c(s_t)$, $m(s_t)$ and $x(s_t)$ do not change. Leisure increases and the trade balance decreases.

Proof: Equations (12), (25) and (27) imply that neither $c(s_t)$, $m(s_t)$, nor $x(s_t)$ change. Condition (26) implies that $\ell(s_t)$ increases, and hence, from (28), $TB(s^t)$ also decreases.□

The shock to the home-input technology determines when it is good a time to export and when it is not. Periods with low values of $A(s_t)$ are bad periods to export, hence leisure increases and the trade balance decreases.

The table below summarizes these results

	$c(s_t)$	$\ell(s_t)$	$x(s_t)$	$m(s_t)$	$TB(s_t)$
$\downarrow Z(s_t)$	$\downarrow$	$=$	$=$	$\downarrow$	$\downarrow$
$\downarrow d^*(s_t)$	$\downarrow$	$=$	$=$	?	?
$\downarrow A(s_t)$	$=$	$\uparrow$	$\uparrow$	$=$	$\downarrow$

Table I. Response of the allocation to negative shocks

Note that for any shock, either consumption or labor remains constant. This is because of the joint effect of two assumptions: separability of labor and consumption in preferences and the fact that labor does not enter the production function of the varieties. How changing any of these assumptions affect the results will be discussed in the last section.

3. DECENTRALIZATION

This section studies the policy implications regarding monetary and exchange rate policy in a partially sticky economy (i.e. $0 < \alpha < 1$) with separable utility. The complete characterization of the optimal policy for any $\alpha \in [0,1]$ and any utility function is in the appendix.

As mentioned above, the Friedman rule is optimal since it eliminates the distortion between cash and credit goods, then $R(s^t) = 1$ for all s^t. Given the Ramsey allocation a^R and the initial price P_0^s of sticky firms, we consider the following equilibrium conditions

$$\frac{H'(c(s_t);\lambda)}{P_t(s^{t-1})} = \beta \sum_{s^{t+1}|s^t} \frac{H'(c(s_{t+1});\lambda)}{P_{t+1}(s^t)} \pi\left(s^{t+1}|s^t\right), \tag{29}$$

which is the Fisher equation, obtained by summing (5) for all $s^{t+1}|s^t$ and setting $R(s^t) = 1$ for all s^t,

$$M\left(s^t\right) = P_t\left(s^{t-1}\right) c(s_t)\, \mu_\Gamma \tag{30}$$

is the cash-in-advance constraint, where $\mu_\Gamma = \int_\Gamma di$ is the 'size' of the set Γ,

$$P_m\left(s^t\right) = \frac{\theta-1}{\theta}\eta^\eta (1-\eta)^{1-\eta} \frac{Z(s_t)}{d^*(s_t)^\eta} P_t\left(s^{t-1}\right) \tag{31}$$

is the pricing rule of flexible firms once we impose that it must be equal to the price of the sticky firms, and

$$\frac{V'(1-\ell(s_t),\lambda)}{H'(c(s_t),\lambda)} = \left(\frac{\theta-1}{\theta}\right)\eta^\eta (1-\eta)^{1-\eta} Z(s_t)\, A(s_t)\, d^*(s_t)^{1-\eta} \left(1-\tau^n\left(s^t\right)\right), \tag{32}$$

the household's equilibrium condition (4) after using (10), (11) and (31).

The equilibrium policies and prices are obtained as follows. Given the initial price $P_0(s^{-1}) = P_0^s$, (29) determines recursively the price of the final good. The cash-in-advance constraint (30) determines the money supply at period t that implements the price $P_t(s^{t-1})$. Condition (31) determines the foreign-input price $P_m(s^t)$ and (32) pins down the labor tax rate $\tau^n(s^t)$. The exchange rate and home-input price follow from (11), and (10) determines the nominal wage rate $W(s^t)$. The prices $Q(s^{t+1}|s^t)$ follow from (5), and the taxes on foreign securities $\tau^*(s^{t+1})$ are pinned down from (18). (The decentralization of asset holdings is described in the appendix.)

Policy Response to Shocks: In all cases, the behavior of labor taxes is indeterminate, except for a government purchase shock, where they stay constant. Moreover, note that none of the instruments or prices change with government expenditure shocks. This is a consequence of the previously mentioned result that expenditure shocks are fully insured through international capital markets. We study the optimal response of policy after the other three shocks.

1) Negative shock to the final goods technology $Z(s_t)$. Consumption decreases, and labor remains constant. Since $P_t(s^{t-1})$ is given, it follows from (30) that a lower consumption is implemented through a reduction in $M(s^t)$. In addition, the input prices, the nominal wage rate and the exchange rate all decrease. The decline in $P_m(s^t)$, $P_x(s^t)$ and $E(s^t)$ follow from (31) and (11), and the decrease in $W(s^t)$ follows from (10).

2) Negative shock to the terms of trade $d^*(s_t)$. Consumption decreases, labor remains constant and $m(s_t)$ decreases. The decentralization of a terms of trade shock differs depending on whether the decline in $d^*(s_t)$ is driven by a lower $P_x^*(s_t)$ or a higher $P_m^*(s_t)$. In any case, from (30), the lower consumption level is implemented through a decline in $M(s^t)$.

Condition (31) implies that $P_m(s^t)$ increases but $d^*(s_t) P_m(s^t)$ does not change.[21] Since $P_x(s^t) = d^*(s_t) P_m(s^t)$, the home-input price does not change, and therefore, (10) implies that $W(s^t)$ also remains constant. If the negative shock to $d^*(s_t)$ is driven by a decline in $P_x^*(s_t)$, (11) implies that $E(s^t)$ increases. If the decline in $d^*(s_t)$ is driven solely by an increase in $P_m^*(s_t)$, from (11) and (31), the nominal exchange rate does not change. (Due to this result, the next table considers changes in $P_x^*(s_t)$ and $P_m^*(s_t)$ separately.)

3) Negative shock to the home-input technology $A(s_t)$. Consumption remains constant and labor decreases. (30) implies that $M(s_t)$ does not change. (31) implies that $P_m(s^t)$, and from the (11) conditions, $P_x(s^t)$ and $E(s^t)$ do not change. Finally, condition (10) implies that $W(s^t)$ declines.

The following table summarizes the previous results

	$E(s^t)$	$M(s^t)$	$W(s^t)$	$P_m(s^t)$	$P_x(s^t)$
$\downarrow Z(s_t)$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$	$\downarrow$
$\downarrow P_x^*(s_t)$	$\uparrow$	$\downarrow$	$=$	$\uparrow$	$=$
$\uparrow P_m^*(s_t)$	$=$	$\downarrow$	$=$	$\uparrow$	$=$
$\downarrow A(s_t)$	$=$	$=$	$\downarrow$	$=$	$=$

Table II. Response of the optimal policies and prices to negative shocks

Table II summarizes the main message of the paper: it is not possible to characterize optimal policy by the cyclical properties of the economy even in this very simple model. The table considers several negative exogenous shocks hitting the economy. Some shocks call for a depreciation of the exchange rate, others call for an appreciation of the exchange rate, and others call for a constant exchange rate.

[21] The last result is special to the Cobb-Douglas specification.

4. EXTENSIONS AND CONCLUSIONS

In this section we discuss extensions to the model we just analyzed and show that they reinforce the results obtained. We assumed that labor enters only in the production of the tradeable input and that consumption and labor were separable in preferences. These two assumptions imply that either leisure or consumption would remain constant after any of the shocks analyzed and made the discussion of the decentralization straightforward and simple. The model can be extended by relaxing any of these assumptions. For instance, if we introduce labor as an additional input in the production of the continuum of final goods, a negative shock to the home-input technology $A(s_t)$ can be shown to imply a devaluation of the exchange rate and an increase in the money supply.[22] These results contrasts with those in Table II, where the nominal exchange rate and the money supply do not change. We draw two conclusions from the discussion. First, that modelling details are important for the qualitative predictions of the model, and second, that the main message that it is not possible to characterize optimal policy in terms of simple rules based on the cyclical properties of the economy is a robust result.

Another key assumption in the model is the existence of taxes on foreign securities $\tau^*(s^t)$. This instrument gives the planner the ability to manipulate the intertemporal prices faced by the domestic residents without violating the interest parity condition (18). The same allocation can be implemented if, instead of having the taxes $\tau^*(s^t)$, the government has access to a uniform consumption tax over varieties $i \in [0,1]$. If, however, the government does not have access to consumption taxes, the constraint (18) binds. The allocation will be different, and welfare will be lower. Further, even though the consumption aggregate $c(s^t)$ is homogeneous of degree one in the individual varieties, a condition known to imply that the Friedman rule is optimal with a cash-in-advance constraint on a subset of varieties (Chari,

[22] This version of the model is analyzed in Hevia and Nicolini (2004).

Christiano and Kehoe, 1991), it will not be so in this case, since the government does not have a complete set of instruments. Intuitively, there is a trade-off between distorting the relative consumption of cash and credit varieties, and the constraint imposed by (18), which ties down the intertemporal prices faced by households. By manipulating the nominal interest rate the government affects the intertemporal prices, but distorts the relative consumption of varieties.[23] The same happens if the tax rates cannot be made state contingent or if the government cannot issue state contingent debt.

We imposed frictions in the setting of final good prices. An alternative, widely used in the literature, is to assume frictions in the setting of wages. In addition, we imposed a one period rigidity, while most of the literature assumes staggered price setting. If we relax both assumptions, the Ramsey allocation is still independent of both the set of cash goods and the set of firms with sticky wages, and the same as in our model. However, the decentralization would be very different, since monetary and exchange rate policy would aim at stabilizing nominal wages, rather than prices. So, the optimal response of the nominal exchange rate—to implement the same Ramsey allocation—would be different in general.

Finally, if the conditions imposed on the utility function in Section 3 are not satisfied, some of the conclusions regarding the optimal response of the allocation may be different. This is a standard feature of optimal taxation Ramsey problems and means that some of the results described in Table I may be different from what we obtained. However, given a particular movement of the optimal allocation, the way to decentralize it is the same as before.

This discussion reinforces the message of the paper: optimal policy is about dealing with distortions in response to shocks. The optimal exchange rate policy critically depends on which is the set of instruments available, on how -given those instruments- the optimal allocation responds to the shocks and on the mapping from policies to allocations.

[23] These results are discussed in Hevia and Nicolini (2004).

There is a general policy principle that can be derived from our analysis, which is consistent with previous work: price stability is optimal, since this price stability is the one that avoids the distortion between flexible and sticky varieties. But to make this principle operational, one needs to know, first, where the price stickiness is, and second, which is the transmission mechanism from the exchange rate to those sticky prices. The qualitative relationship between movements in aggregate output and optimal movements on the exchange rate critically depends on those modeling details.

Appendix.

Proof of Proposition 2: We find a government policy ω and a price system $\mathcal{P}$ such that the relaxed Ramsey allocation satisfies conditions $i)$ through $vii)$ in definition 2. Throughout, fix the relaxed Ramsey allocation $\widetilde{a}$. Recall that $\widetilde{a}$ satisfies $c(i, s^t) = c(s^t)$ and $m(i, s^t) = x(s^t)$. As shown above, this requires $P(i, s^t) = P(s^t)$ for all i, and $R(s^t) = 1$ for all s^t. Summing (5) over all $s^{t+1}|s^t$ and using $\sum_{s^{t+1}|s^t} Q(s^{t+1}|s^t) = 1$, it follows that

$$1 = \frac{P_t(s^t)}{U_c(s^t)} \sum_{s^{t+1}|s^t} \beta \frac{U_c(s^{t+1})}{P_{t+1}(s^{t+1})} \pi\left(s^{t+1}|s^t\right) \tag{A1}$$

Using (21) into (3) we obtain the equilibrium condition,

$$M\left(s^t\right) = P_t\left(s^t\right) c\left(s^t\right) \mu_\Gamma, \tag{A2}$$

where $\mu_\Gamma = \int_\Gamma di$ is the measure of the set Γ. Solving the households budget constraint starting from period t forward and using $R(s^t) = 1$ we find

$$B\left(s^t\right) = \sum_{j=t}^{\infty} \sum_{s^j|s^t} \frac{Q(s^j|s^t)}{R(s^j)} \left[P_j\left(s^j\right) c\left(s^j\right) - W\left(s^j\right)\left(1 - \ell\left(s^j\right)\right)\left(1 - \tau^n\left(s^j\right)\right)\right] \tag{A3}$$

and solving the economy-wide budget constraint starting from period t forward,

$$B^*\left(s^t\right) = \sum_{j=t}^{\infty} \sum_{s^j|s^t} P_x^*\left(s^j\right)\left[A(s_j)\left(1 - \ell\left(s^j\right)\right) - \frac{m(s^j)}{(1-\eta)\, d^*(s_t)}\right] Q^*\left(s^j|s^t\right) \tag{A4}$$

Using (5), (11) and (14) into (18) we find that the tax rate on foreign bonds is uniquely determined by

$$1 + \tau^*\left(s^{t+1}\right) = \frac{U_c(s^{t+1})}{U_c(s^t)} \frac{R^*(s^t)}{R^*(s^{t+1})} \left(\frac{d^*(s_{t+1})}{d^*(s_t)}\right)^{1-\eta} \frac{Z(s_{t+1})}{Z(s_t)}. \tag{A6}$$

There are three cases to consider: $0 < \alpha < 1$, $\alpha = 0$, and $\alpha = 1$. These are, respectively, a partially sticky economy, a flexible prices economy, and a fully sticky prices economy. All three cases have $R(s_t) = 1$, $\tau^d(s^t) = 1$, $\tau^*(s^{t+1})$ determined from (A6) and holdings of foreign bonds B^* pinned down from (A4).

Case I. A partially sticky economy, i.e. $0 < \alpha < 1$. In this case, the policy that decentralizes $\widetilde{a}$ is uniquely determined. Since there are sticky firms, the price level at period t depends on s^{t-1}. Hence, $P_t(s^{t-1}) = P_t(i, s^{t-1})$ for all i. At period 0, $P_0(s^{-1}) = P_0^s$ is given. Then, (A1) can be expressed as

$$P_{t+1}\left(s^t\right) = \frac{P_t(s^{t-1})}{U_c(s^t)} \sum_{s^{t+1}|s^t} \beta U_c\left(s^{t+1}\right) \pi\left(s^{t+1}|s^t\right) \tag{A5}$$

Given $\widetilde{a}$ and $P_0(s^{-1}) = P_0^s$, (A5) determines recursively the price level $P_t(s^{t-1})$ that justifies the Friedman rule $R(s^t) = 1$. Given the sequence of prices, (A2) determines the required money supply, and the equilibrium prices $Q(s^{t+1}|s^t)$ follow from (5).

The pricing equation (14) together with the condition $P_t(i, s^t) = P_t(s^{t-1})$ can be used to pin down the foreign-input price $P_m(s^t)$. Given $P_m(s^t)$, (11) determines the nominal exchange rate $E(s^t)$ and the home input price $P_x(s^t)$. The wage rate $W(s^t)$ follows from (10), and (4) pins down the labor tax rate $\tau^n(s^t)$. Finally, the stock of domestic debt $B(s^t)$ is obtained from (A3).

By construction, flexible and sticky firms find it optimal to set a common price $P_t(s^{t-1})$. Since all the conditions of an equilibrium are satisfied, the relaxed Ramsey allocation is implementable with a unique policy ω^R. Thus, $\widetilde{a}$ is also the Ramsey allocation.

Case II. Flexible economy, i.e. $\alpha = 0$. In this case, as in Lucas and Stokey (1983), monetary policy $\{E, M\}$ and domestic bonds policy $\{B\}$ are indeterminate. Given the allocation $\widetilde{a}$, the real wage $W(s^t)/P_t(s^t)$ is determined from (10) and (14). Then, (4) pins down the labor tax rate τ^n. To see the indeterminacy of monetary policy, note that by symmetry, all final goods firms set a common price $P_t(s^t)$ given by (14). This price, in general, depends on the current shock s_t. Specifically, given the allocation, *any* sequence of prices satisfying (A1) can be an equilibrium path for the price level. In particular, the equilibrium price sequence $P_t(s^{t-1})$ obtained in case I can be implemented as an equilibrium in a flexible prices economy. Then, (14) and (11) determines the nominal exchange rate $E(s^t)$, and given the prices and allocation, (A3) pins down the bonds holdings B. The rest of the equilibrium prices are obtained as in case I.

Case III. Fully sticky prices economy, i.e. $\alpha = 1$. This case also features an indeterminacy, but of a very different nature. Here, all policy instruments except labor taxes and nominal exchange rates are uniquely determined. First, given P_0^s and the allocation $\widetilde{a}$, the sequence of prices $P_t(s^{t-1})$ is obtained from (A5). The money supply $M(s^t)$ and the equilibrium prices $Q(s^{t+1}|s^t)$ are given by (A2) and (5). To see the indeterminacy, note, first, that in period 0 any combination of τ_0^n and W_0 satisfying (4) can be implemented as an equilibrium. Second, using $R(s^t) = 1$, (4), (5), (10) and (11) into (15) for all $t \geq 1$, and noting that $P_t(s^{t-1})$ is independent of s_t, we obtain

$$1 = \frac{\theta}{\theta - 1} \sum\nolimits_{s^t|s^{t-1}} \psi(s^t) \; \frac{U_\ell(s^t)/U_c(s^t)}{(1 - \tau^n(s^t))} \; \frac{d^*(s_t)^{\eta-1}}{Z(s_t)\, A(s_t)\, \eta^\eta (1-\eta)^{1-\eta}} \tag{A6}$$

where

$$\psi(s^t) = \frac{U_c(s^t)\;\; [c(s_t) + g(s_t)]\;\; \pi(s^t|s^{t-1})}{\sum_{s^t|s^{t-1}} U_c(s^t)\;\; [c(s_t) + g(s_t)]\;\; \pi(s^t|s^{t-1})}.$$

Any labor tax rate $\tau^n(s^t)$ satisfying (A6) can be decentralized. In particular, any pair of labor tax policies $\tau_1^n(s^t)$ and $\tau_2^n(s^t)$ satisfying (A6) decentralizes the Ramsey allocation. According to (5), however, they induce different nominal wages $W_1(s^t)$ and $W_2(s^t)$, which, by virtue of (10) and (11) induce different nominal prices $\{E_1(s^t), P_{x1}(s^t), P_{m1}(s^t)\}$ and $\{E_2(s^t), P_{x2}(s^t), P_{m2}(s^t)\}$.

In all three cases I, II and III, there are prices and policy instruments ω^R such that relaxed Ramsey allocation $\widetilde{a}$ is an equilibrium allocation. Therefore, $\widetilde{a}$ is indeed the Ramsey allocation a^R.$\square$

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