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Implementation of Social Choice Functions via Demanding Equilibria

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Abstract

We consider agents who do not have any information about others' preferences. In this situation they attempt to behave such as to maximize their chances to obtain their most preferred alternative. This defines a solution concept for games symmetrical to Barberà and Dutta's protective equilibrium, the demanding equilibrium. Necessary and sufficient conditions for self implementation in demanding equilibria (s.i.d.e.) of social choice functions are provided.

1 Introduction

In implementation theory one takes as given a social choice function or correspondence which gives for any situation those social alternatives that are desirable according to some criteria. Given that it may be impossible for a social planner, to directly use all data about a given situation he needs to determine the outcomes, be it for lack of information, or because such information is not verifiable, the objective then is to design a mechanism to implement the social choice rule under consideration. Different approaches are possible with regard to the informational assumptions among the agents, to the solution concept employed in the implementation, and further restrictions put on the mechanism. With respect to the latter point, often direct mechanisms are considered, for which agents' message spaces in the mechanism coincide with the space of those characteristics they have private knowledge about. Furthermore, often some notion of simplicity of the mechanism is invoked. Since any social choice function can itself be interpreted as a direct mechanism, one could consider this the most natural one to be used for implementation, where the problem becomes to check whether one gets positive

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results using some appropriate equilibrium concept. This is termed self-implementation by Barberà and Dutta (cf. [4]): Agents are asked to report their characteristics (so the mechanism is direct), and then the mechanism carries out what the social choice rule prescribes for the reported data.

Another reason to take such an approach lies in the fact that in some areas it is well known that the ideal social choice rule does not exist. In voting theory, for example, the results of Arrow, Gibbard and Satterthwaite (cf. [1, 10, 24]) tell us, that there are no voting rules satisfying a list of desirable (and seemingly innocent) requirements. Therefore, the voting rules considered in the literature in themselves already are an attempt to do as well as possible in the given limits. In this sense, voting rules already have the character of a mechanism rather than just being a normative prescription of the socially desirable outcomes. Hence, it may not seem very convincing to come up with yet another mechanism to implement these rules.

In this line of thinking, Barberà and Dutta [2, 3, 4] present results on social choice rules which are self implementable in an equilibrium concept they call *protective equilibrium*, which is based on an extreme type of risk-aversion of agents faced with decisions under complete ignorance. This type of behavior has been characterized by Barberà and Jackson [6] and has been used elsewhere in matching models (Barberà and Dutta [5]) as well as in game theory (Fiestra-Janeiro, Borm and van Mergen [9]).

One motivation for this paper lies in the fact that in the realm of voting rules, while the antiplurality rule turns out to be self implementable in protective equilibrium, such is not the case for the plurality rule.[1] Given that the latter is much more widely used, we felt it would be of interest to consider an alternative behavioral assumption which would support the plurality rule as being self implementable. This is *demanding behavior*, the characterization of which can be obtained by re placing a convexity (risk-aversion) axiom in the characterization of protective behavior by concavity (risk-loving) as has been demonstrated by Naeve [17]. This intuition is further supported by the fact that each characterization of the plurality rule has its counterpart for the antiplurality rule, and vice versa. This generic result has been highlighted by Saari [19, 21, 22], who remarked that *reversal symmetry* governs the mirror behavior between these two voting rules.

This paper should be viewed as part of a more general research project. The Gibbard and Satterthwaite ([10, 24]) impossibility result for choice functions is based upon the concept of Nash equilibrium. Nevertheless, in certain contexts one may argue that the behavior of the agents is governed by a different logic. This is precisely the line of inquiry followed by Barberà and Dutta who studied, with the concept of protective equilibrium, voting situations where the agents have no information at all about other agents' preferences and only use "protective" strategies of a lexical maximin type. As a consequence of their results, the use of the antiplurality rule as a democratic self implementable mechanism could be recommended in environments where their behavioral assumptions are fulfilled. Several other positive results in the literature make a similar

[1] For a formal definition of both rules as special cases of scoring rules see 6.1 below.

connection between reasonable voting rules and game theoretic solution concepts which capture certain types of rationality for the agents. Without being exhaustive, we could mention Moulin's results, for the positional rules and the concept of sophisticated voting [15], Dutta and Sen's for the Condorcet social choice functions and backward induction [8] or, more recently, Sanver and Sertel [27], who characterized the outcomes one gets by considering the strong Nash equilibria of mechanisms the outcome functions of which are voting rules. Thus, a possible interpretation of our results could be a justification of the use of the plurality rule in decision contexts where all the agents are risk-lovers and have no information about the other agents' behavior.

As most of the results are symmetrical to Barberà and Dutta's, the organization of the paper is similar to their *Implementability via Protective Equilibria* [3]. After having introduced the basic setup in Section 2, Section 3 presents the concept of *self implementation in demanding equilibria* and the first theorem about truthful revelation. Next, four necessary and sufficient conditions for implementation of social choice correspondences are proposed in Section 4. As one may guess, these axioms are mirror conditions of the ones used by Barberà and Dutta for the characterization of choice functions which are directly implementable via protective equilibria (d.i.p.e.).[2] Section 5 presents eight choice functions that serve to prove the independence of the four axioms. In the following Section 6, we give several examples of voting rules that are s.i.d.e.. The connection of Barberà and Dutta's or our approach, respectively, to Moulin's results on implementation under prudent behavior (cf. [15]) is clarified in Section 7. Finally we conclude with indicating possible lines for further research.

2 Notation

Let $A = \{a_1, \ldots, a_m\}$ be the finite set of alternatives. Let $I = \{1, \ldots, n\}$ be the finite set of individuals. $\mathcal{P}$ denotes the set of linear orderings on A, called preferences. $\mathcal{P}^n$ is the set of preference profiles, a typical element of which is $\pi = (P_1, \ldots, P_n)$.

For $P \in \mathcal{P}$ and $r \in \{1, \ldots, m\}$, we denote the rth ranking worst alternative in P by $b_r(P)$ and the kth ranking best alternative in P by $t_k(P)$, i. e.,

$$\begin{aligned} b_r(P) &= \left\{a \in A \,\middle|\, \left|\{a' \in A \mid a\,P\,a'\}\right| = r-1\right\} \\ \text{and} \quad t_k(P) &= \left\{a \in A \,\middle|\, \left|\{a' \in A \mid a'\,P\,a\}\right| = k-1\right\}. \end{aligned}$$

Note that $b_r(P) = t_{m-r+1}(P)$ and $t_k(P) = b_{m-k+1}(P)$. Also we define the *l-bottom* $B(l, P) = \{b_r(P) \mid r \leq l\}$, and the *l-top* $T(l, P) = \{t_k(P) \mid k \leq l\}$.

Given a preference profile $\pi = (P_1, \ldots, P_n)$ an agent $i \in I$, and a preference $P_i \in \mathcal{P}$, we write P_{-i} for the preferences in π of all agents other than i, and π/P_i' for the preference profile obtained by replacing P_i in π by P_i', leaving the other preferences unchanged. So we have $P_{-i} = (P_1, \ldots, P_{i-1}, P_{i+1}, \ldots, P_n)$, $\pi = (P_i, P_{-i})$, and $\pi/P_i' = (P_i', P_{-i})$.

[2] Actually, they use only three axioms, one of which is the conjunction of two more basic requirements. Instead of reproducing this structure we followed the advice of an anonymous referee, and differentiate four conditions thereby getting a clearer view on the differences between their result and ours.

A *social choice function (SCF)* is a mapping $f : \mathcal{P}^n \to A$. Given any SCF f, $P_i \in \mathcal{P}$, and $x \in A$ we define $g_f(x, P_i) = \{P_{-i} \in \mathcal{P}^{n-1} \mid f(P_{-i}, P_i) = x\}$.

Definition 1 *Let $i \in I$, $P_i, P_i' \in \mathcal{P}$, and $Y \subseteq A$. The preferences P_i and P_i' are* Y-equivalent for i under f *iff for all $a \in Y$ we have $g_f(a, P_i) = g_f(a, P_i')$.*

$P_i, P_i' \in \mathcal{P}$ are called equivalent under f, denoted $P_i \sim_f P_i'$ if they are A-equivalent under f.

3 Demanding Equilibrium

Agents facing a situation in which they lack information on others' preferences could employ very different strategic behavior. The protective equilibrium of [3] describes the case that agents are extremely prudent in their behavior. Here we deal with another extreme case. Agents use a lexicographic maxmax behavior. They aim to maximize the chance of their most preferred alternative to be the solution. Formally, this idea is captured in the following definitions.

Definition 2 *Let f be a given SCF. For $i \in I$ with preference P_i, a strategy $\hat{P}_i$* dominates $\tilde{P}_i$ relative to f, *denoted $\hat{P}_i d_f(P_i) \tilde{P}_i$, if there exists $k \in \{1, \ldots, m\}$ such that*

$$\begin{array}{rrcll} & g_f\big(t_k(P_i), \hat{P}_i\big) & \supsetneq & g_f\big(t_k(P_i), \tilde{P}_i\big) & \\ \textit{and} & g_f\big(t_r(P_i), \hat{P}_i\big) & = & g_f\big(t_r(P_i), \tilde{P}_i\big)\,, & \forall\, r < k. \end{array}$$

The set of undominated strategies is :

$$D_f(P_i) = \left\{\bar{P}_i \in \mathcal{P} \mid \nexists\, P_i^* \in \mathcal{P} \text{ s.t. } P_i^* d_f(P_i) \bar{P}_i\right\}\,.$$

First, it is useful to note that the dominance relation is transitive.

Proposition 3 *For all $i \in I$, for all $P_i \in \mathcal{P}$, and for all SCFs f, the dominance relation $d_f(P_i)$ is transitive. Hence, $D_f(P_i) \neq \emptyset\ \forall P_i \in \mathcal{P}$.*

The proof follows directly from the definition of the dominance relation.

Definition 4 *Let $\pi \in \mathcal{P}^n$. A strategy profile $\bar{\pi} \in \mathcal{P}^n$ is called a* demanding equilibrium with respect to the SCF f *iff $\bar{P}_i \in D_f(P_i)$ for all $i \in I$.*

Definition 5 *A SCF f is* self implementable in demanding equilibrium *(*s.i.d.e.*) iff for all pairs of preference profiles π and $\bar{\pi}$, $f(\pi) = f(\bar{\pi})$ whenever $\bar{\pi}$ is a demanding equilibrium with respect to f under π.*

Our main goal in this section is to prove the analogue of Theorem 1 of Barberà and Dutta [3] for our concept of demanding equilibrium. To prepare for this we first state a series of lemmata concerning properties of the dominance relation $d_f(P_i)$ and the set $D_f(P_i)$ for SCFs that are s.i.d.e..
The first is the analogue to (a) in the proof of Theorem 1 in [3].

Lemma 6 *Let f be s.i.d.e. Let $\hat{P}_i, \tilde{P}_i \in \mathcal{P}$ with $\hat{P}_i \not\sim_f \tilde{P}_i$. Then $D_f(\hat{P}_i) \cap D_f(\tilde{P}_i) = \emptyset$.*

Proof of Lemma 6: Suppose there were $\bar{P}_i \in D_f(\hat{P}_i) \cap D_f(\tilde{P}_i)$. Since $\hat{P}_i \not\sim_f \tilde{P}_i$, there exists $P_{-i} \in \mathcal{P}^{n-1}$ such that $a \neq f(\hat{P}_i, P_{-i})$ but $a = f(\tilde{P}_i, P_{-i})$, or $a = f(\hat{P}_i, P_{-i})$ but $a \neq f(\tilde{P}_i, P_{-i})$. Assume the first case holds. Let $\bar{P}_{-i} \in \mathcal{P}^{n-1}$ be such that $\bar{P}_k \in D_f(P_k)$, for all $k \neq i$. Since $\bar{P}_i \in D_f(\hat{P}_i)$ and f is s.i.d.e., we have $f(\hat{P}_i, P_{-i}) = f(\bar{P}_i, \bar{P}_{-i})$; also $f(\tilde{P}_i, P_{-i}) = f(\bar{P}_i, \bar{P}_{-i})$ because $\bar{P}_i \in D_f(\tilde{P}_i)$ and f is s.i.d.e. This results in the contradiction $a \neq f(\bar{P}_i, \bar{P}_{-i})$ and $a = f(\bar{P}_i, \bar{P}_{-i})$.

The second case leads to a contradiction in exactly the same way. $\diamond$

The next lemma is is the analogue of (b) in Barberà and Dutta's proof.

Lemma 7 *Let f be s.i.d.e. For all $P_i \in \mathcal{P}$ we have $P_i \in D_f(P_i)$.*

Proof of Lemma 7: Suppose there were P_i^0 such that $P_i^0 \notin D_f(P_i^0)$. Then there is $P_i^1 \in D_f(P_i^0)$ such that $P_i^1 d_f(P_i^0) P_i^0$ (here transitivity of the dominance relation enters). This means that there exists $P_{-i}^0 \in \mathcal{P}^{n-1}$, and an alternative $a \in A$, such that

$$a \neq f(P_i^0, P_{-i}^0)\,, \tag{1}$$

$$a = f(P_i^1, P_{-i}^0)\,, \tag{2}$$

$$g_f(a, P_i^1) \supset g_f(a, P_i^0)\,. \tag{3}$$

Since P_i^0 and P_i^1 are not equivalent, Claim 6 yields $P_i^1 \notin D_f(P_i^1)$. Therefore we can iterate the above argument to construct a sequence $P_i^0, P_i^1, P_i^2, \ldots$ of elements in $\mathcal{P}$ such that, for all $t \in \mathbb{N}$, $P_i^t \notin D_f(P_i^t)$ and $P_i^t \in D_f(P_i^{t-1})$.

Since $\mathcal{P}$ is finite, there must be some integers $T, S \in \mathbb{N}$, such that P_i^T and P_i^{T+S} are equivalent (actually even such that they are equal). $P_i^T \in D_f(P_i^{T-1})$ and $P_i^{T+S} \in D_f(P_i^{T+S-1})$, $P_i^T \sim_f P_i^{T+S}$, and Claim 8 yield $P_i^T \in D_f(P_i^{T+S-1})$ (and $P_i^{T+S} \in D_f(P_i^{T-1})$). Thus $D_f(P_i^{T-1}) \cap D_f(P_i^{T+S-1}) \neq \emptyset$ and hence by Claim 6 also $P_i^{T-1} \sim_f P_i^{T+S-1}$. This argument can be repeated to arrive at the conclusion that, in particular, $P_i^0 \sim_f P_i^S$.

Now consider a sequence $\{P_{-i}^0, P_{-i}^1, \ldots, P_{-i}^S\}$ of elements in $\mathcal{P}^{n-1}$ such that, for all $t \in \{1, \ldots, S\}$, and for all $j \neq i$, $P_j^t \in D_f(P_j^{t-1})$. Such a sequence exists, since $D_f(P_k) \neq \emptyset$, for all k and all $P_k \in \mathcal{P}$.

Equation (1), $\pi^1 \in D_f(\pi^0)$, and the fact that f is s.i.d.e. imply

$$a \neq f(P_i^1, P_{-i}^1) = f(P_i^0, P_{-i}^0)\,. \tag{4}$$

This and equation (3) yield

$$a \neq f(P_i^0, P_{-i}^1)\,. \tag{5}$$

Again we can iterate this type of argument. So Equation (5), $(P_i^1, P_{-i}^2) \in D_f\big((P_i^0, P_{-i}^1)\big)$ and the fact that f is s.i.d.e. imply

$$a \neq f(P_i^1, P_{-i}^2) = f(P_i^0, P_{-i}^1)\,. \tag{6}$$

This and equation (3) yield

$$a \neq f(P_i^0, P_{-i}^2)\,, \tag{7}$$

and so on. Finally,

$$a \neq f(P_i^0, P_{-i}^{S-1})\,. \tag{8}$$

Starting from equation (2), and repeatedly using that for all $t \in \{0,\dots,S\}$ we have $(P_i^{t+1}, P_{-i}^t) \in D_f\big((P_i^t, P_{-i}^{t-1})\big)$, and the fact that f is s.i.d.e., we get

$$a = f(P_i^S, P_{-i}^{S-1})\,. \tag{9}$$

But equations (8) and (9) contradict $P_i^0 \sim_f P_i^S$. $\diamond$

We continue with two observations which are not made explicit in the original proof by [3] but are used there implicitly.

Lemma 8 *Let f be s.i.d.e. Let $\hat{P}_i, \tilde{P}_i \in \mathcal{P}$ with $\hat{P}_i \sim_f \tilde{P}_i$. Then, for all $P_i \in \mathcal{P}$, we have $\hat{P}_i \in D_f(P_i) \Leftrightarrow \tilde{P}_i \in D_f(P_i)$.*

This lemma states that for any $P_i \in \mathcal{P}$ the set $D_f(P_i)$ is the union of equivalence classes of preferences. The proof follows directly from the definitions.

Lemma 9 *Let f be s.i.d.e. For any $P_i \in \mathcal{P}$, $\hat{P}_i \in D_f(P_i)$ and $\tilde{P}_i \in D_f(P_i)$ implies $\hat{P}_i \sim_f \tilde{P}_i$.*

This means that for any $P_i \in \mathcal{P}$ any two elements in $D_f(P_i)$ are equivalent. So Claims 8 and 9 together say that each $D_f(P_i)$ is exactly one equivalence class. (Recall that $D_f(P_i) \neq \emptyset$, for all $P_i \in \mathcal{P}$.)

Proof of Claim 9: Let $\tilde{P}_i \in D_f(\hat{P}_i)$ for some $\hat{P}_i \in \mathcal{P}$. We will show that $\tilde{P}_i \sim_f \hat{P}_i$. Take any $P_{-i} \in \mathcal{P}^{n-1}$. Since $P_j \in D_f(P_j)$, for all $j \neq i$ by Lemma 7, and f is s.i.d.e. we have $f(\hat{P}_i, P_{-i}) = f(\tilde{P}_i, P_{-i})$. Hence we have $f(\hat{P}_i, P_{-i}) = f(\tilde{P}_i, P_{-i})$, for all $P_{-i} \in \mathcal{P}^{n-1}$, which means $\hat{P}_i \sim_f \tilde{P}_i$. $\diamond$

Now we are ready for this section's main result which tells us, that if a social choice function is s.i.d.e., to tell the truth is at least as good as any other strategy when the criterion for individual i is given by the dominance relation $d_f(P_i)$. So whenever we can implement in demanding equilibrium we can assume that agents report their true preference.

Theorem 10 *A social choice function f is self implementable via demanding equilibrium iff for all $i \in I$, and all $P_i \in \mathcal{P}$,*

$$D_f(P_i) = \{P_i^* \mid P_i^* \sim_f P_i\}.$$

Proof If the condition on $D_f(P_i)$ is satisfied, f is obviously s.i.d.e.

On the other hand, let f be s.i.d.e. Then we know from Lemma 7 that $P_i \in D_f(P_i)$ and by Lemma 9 $\tilde{P}_i \sim_f P_i$ for any $\tilde{P}_i \in D_f(P_i)$.

This closes the proof of Theorem 10. $\diamond\diamond$

We close this section with a another lemma about properties of the dominance relation that will be used later on in the proof of Theorem 19.

Lemma 11 *Let f be a SCF that is s.i.d.e.. Let $P_i, P_i' \in \mathcal{P}$ such that $P_i \not\sim_f P_i'$. Then $P_i d_f(P_i) P_i'$ (and also $P_i' d_f(P_i') P_i$, of course).*

Proof of Lemma 11: If $P_i \not\sim_f P_i'$ and f is s.i.d.e., it follows from follows from Theorem 10 that $P_i' \notin D_f(P_i)$. So there must be a preference $P_i^1 \in \mathcal{P}$ such that $P_i^1 d_f(P_i) P_i'$. Now there are two possibilities: Either, $P_i^1 \sim_f P_i$, in which case we would have $P_i d_f(P_i) P_i'$, or $P_i^1 \notin D_f(P_i)$. In the latter case we find $P_i^2 \in \mathcal{P}$ with $P_i^2 d_f(P_i) P_i^1$. Continuing with the same argument we must arrive at some l for which $P_i^l \sim_f P_i$ because $\mathcal{P}$ is finite. So $P_i \sim_f P_i^l d_f(P_i) P^{l-1} d_f(P_i) \ldots d_f(P_i) P_i'$ and hence by the definition of $\sim_f$ and the transitivity of the dominance relation we have $P_i d_f(P_i) P_i'$. $\diamond$

4 Characterization Result

A necessary and sufficient condition for the implementation of a social choice function in Nash equilibrium is Muller and Satterthwaite's [16] strong positive association. This condition reads.

Definition 12 *A SCF f satisfies* strong positive association (SPA) *if for all $i \in I$, for all $\pi \in \mathcal{P}^n$, and for all $P_i' \in \mathcal{P}$ the following implication holds.*

$$[a = f(\pi) \text{ and } (aP_ib \Rightarrow aP_i'b)\ \forall\, b \in A] \Rightarrow a = f(\pi/P_i')\,.$$

Barberà and Dutta present the following three conditions which together are equivalent to SPA.

Definition 13 *A SCF f satisfies* monotonicity (MON) *if for all $i \in I$, for all $\pi \in \mathcal{P}^n$, and for all $P_i' \in \mathcal{P}$ the following implication holds.*

$$\left.\begin{array}{rl} a = f(\pi) & , \\ P_i \text{ and } P_i' \text{ agree on } A\backslash\{a\} & , \\ (aP_ib \Rightarrow aP_i'b)\ \forall\, b \in A & \end{array}\right\} \Rightarrow a = f(\pi/P_i')\,.$$

Definition 14 *A SCF f satisfies* top-invariance (TI) *if for all $i \in I$, for all $\pi \in \mathcal{P}^n$, and for all $P_i' \in \mathcal{P}$ the following implication holds.*

$$\left.\begin{array}{rl} b_r(P_i) = f(\pi) & , \\ B(r, P_i) = B(r, P_i') & , \\ P_i \text{ and } P_i' \text{ agree on } B(r, P_i) & \end{array}\right\} \Rightarrow b_r(P_i) = f(\pi/P_i')\,.$$

Definition 15 *A SCF f satisfies* bottom-invariance (BI) *if for all $i \in I$, for all $\pi \in \mathcal{P}^n$, and for all $P_i' \in \mathcal{P}$ the following implication holds.*

$$\left.\begin{array}{rl} t_k(P_i) = f(\pi) & , \\ T(k, P_i) = T(k, P_i') & , \\ P_i \text{ and } P_i' \text{ agree on } T(k, P_i) & \end{array}\right\} \Rightarrow t_k(P_i) = f(\pi/P_i')\,.$$

Proposition 16 *For all social choice functions, SPA is equivalent to the conjunction of MON, TI and BI.*

The proof is immediate from the definitions.

To charakterize social choice functions that are d.i.p.e. Barberà and Dutta [3] keep monotonicity and top-invariance, the conjunction of which they term *upper strong positive association (USPA)*, and replace bottom-invariance by two conditions which are weaker, namely *lower conditional independence (LCI)* and *bottom equivalence (BE)*.

We will instead stick to monotonicity and bottom-invariance, which we will continue to consider as two seperate conditions, and replace top-invariance by two weaker conditions, which are as follows.

Definition 17 *A SCF f satisfies* upper conditional independence (UCI) *if for all $i \in I$, and for all $P_i, P_i' \in \mathcal{P}$, the following implication holds.*

$$\left.\begin{array}{rr} t_{k+1}(P_i) = f(\pi) & , \\ T(k,P_i) = T(k,P_i') = T & , \\ P_i \text{ and } P_i' \text{ are } T\text{-equivalent and agree on } A \backslash T & \end{array}\right\} \Rightarrow t_{k+1}(P_i) = f(\pi/P_i')\,.$$

This condition states that some reshuffling is also possible in $T(k, P_i)$ without changing the status of $t_{k+1}(P_i)$. However, the admissible P_i's are severely constrained: they should be equivalent to P_i for every alternative in $T(k, P_i)$, have the same top, and agree with P_i on $A \backslash T(k, P_i)$.

Definition 18 *A SCF f satisfies* top equivalence (TE) *if for all $i \in I$, and for all $P_i, P_i' \in \mathcal{P}$, the following implication holds.*

$$\left.\begin{array}{r} P_i \text{ and } P_i' \text{ are } T(k,P_i)\text{- but not } T(k+1,P_i)\text{-equivalent} \\ P_i \text{ and } P_i' \text{ agree on } A \backslash T(k,P_i) \end{array}\right\} \Rightarrow T(k,P_i) = T(k,P_i').$$

A consequence of TE is that P_i and P_i' have exactly the same bottom $(A \backslash T(k, P_i))$. They might differ on the ranking of the $T(k, P_i)$, but this changes won't alter whether or not any of the alternatives in $T(k, P_i)$ are picked by the social choice function, irrespective of the preferences of other agents. It will only have some influence on whether $t_{k+1}(P_i)$ is the socially chosen alternative or not.

Theorem 19 *A SCF f is self implementable in demanding equilibrium iff it satisfies MON, BI, UCI, and TE.*

Proof We will first show that all four conditions are necessary for self implementability in demanding equilibrium.

Claim 20 *If a SCF f is s.i.d.e., it satisfies MON.*

Proof of Claim 20: The proof is by contradiction. Suppose f fails to satisfy MON but is s.i.d.e.. Then there exist $i \in I$, $\pi \in \mathcal{P}^n$, and $P_i' \in \mathcal{P}$ such that

$$
\begin{aligned}
& a = f(\pi)\,, && (10)\\
& P_i \text{ and } P_i' \text{ agree on } A\backslash\{a\}\,, && (11)\\
& (a\, P_i\, b \;\Leftarrow\; aP_i'b)\ \forall b \in A && (12)\\
\text{and}\quad & a \neq a' = f(\pi/P_i'). && (13)
\end{aligned}
$$

So obviously $P_i \not\sim_f P_i'$ from equations (10) and (13) and thus by Lemma 11 $P_i d_f(P_i) P_i'$ and $P_i' d_f(P_i') P_i$.

As we have $P_i d_f(P_i) P_i'$ there must exist $l \in \{1, \ldots, m\}$ such that

$$
\begin{aligned}
& g_f\big(t_l(P_i), P_i\big) \supsetneq g_f\big(t_l(P_i), P_i'\big)\\
\text{and}\quad & g_f\big(t_r(P_i), P_i\big) = g_f\big(t_r(P_i), P_i'\big)\,, \qquad \forall\, r < l.
\end{aligned}
$$

Let $k \in \{1, \ldots, m\}$ be such that $t_k(P_i) = a$. Then we know from equations (11) and (12), that there is $\bar{k} < k$ such that $t_{\bar{k}}(P_i') = a$.

For $s < \bar{k}$ we have $t_s(P_i) = t_s(P_i')$, so it cannot be the case that $l < \bar{k}$ because otherwise we would get a contradiction to $P_i' d_f(P_i') P_i$. But we know from equations (10) and (13) that $g_f\big(t_{\bar{k}}(P_i'), P_i\big) \not\subseteq g_f\big(t_{\bar{k}}(P_i'), P_i'\big)$ which again contradicts $P_i' d_f(P_i') P_i$. $\diamond$

Claim 21 *If a SCF f is s.i.d.e., it satisfies BI.*

Proof of Claim 21: Again, the proof is by contradiction. Suppose f fails to satisfy BI but is s.i.d.e.. Then there exist $i \in I$, $\pi \in \mathcal{P}^n$, $P_i' \in \mathcal{P}$ and $k \in \{1, \ldots, m\}$ such that

$$
\begin{aligned}
& t_k(P_i) = f(\pi)\,, && (14)\\
& T(k, P_i) = T(k, P_i')\,, && (15)\\
& P_i \text{ and } P_i' \text{ agree on } T(k, P_i)\,, && (16)\\
\text{and}\quad & t_k(P_i) \neq a' = f(\pi/P_i'). && (17)
\end{aligned}
$$

So we know that $P_i \not\sim_f P_i'$ and hence by Lemma 11 $P_i d_f(P_i) P_i'$ and $P_i' d_f(P_i') P_i$. The first says that there must exist $l \in \{1, \ldots, m\}$ such that

$$
\begin{aligned}
& g_f\big(t_l(P_i), P_i\big) \supsetneq g_f\big(t_l(P_i), P_i'\big)\\
\text{and}\quad & g_f\big(t_r(P_i), P_i\big) = g_f\big(t_r(P_i), P_i'\big)\,, \qquad \forall\, r < l.
\end{aligned}
$$

If l were less than k, there would be a contradiction to $P_i' d_f(P_i') P_i$ since P_i and P_i' share the same k-top and agree on that set. But since $t_k(P_i) = f(\pi) \neq f(\pi/P_i')$ we know $g_f\big(t_k(P_i), P_i)\big) \not\subseteq g_f\big(t_k(P_i), P_i'\big)$ which (with $t_k(P_i) = t_k(P_i')$) is equivalent to $g_f\big(t_k(P_i'), P_i')\big) \not\supseteq g_f\big(t_k(P_i'), P_i\big)$ leading to a contradiction of $P_i' d_f(P_i') P_i$. $\diamond$

Claim 22 *If a SCF f is s.i.d.e., it satisfies UCI.*

Proof of Claim 22: Suppose that f violates UCI. Then there exist $\pi \in \mathcal{P}^n$, $i \in I$ and $P_i' \in \mathcal{P}$ such that $t_{k+1}(P_i) = f(\pi)$, $T(k, P_i) = T(k, P_i')$, P_i and P_i' are $T(k, P_i)$-equivalent and agree on $A \backslash T(k, P_i)$, but $t_{k+1}(P_i) \neq f(\pi/P_i')$. As P_i' and P_i are $T(k, P_i)$-equivalent, and $t_{k+1}(P_i) = f(\pi)$ while $t_{k+1}(P_i) \neq f(\pi/P_i')$, it can't be that $P_i' d_f(P_i')P_i$. Thus, $P_i \in D_f(P_i')$, which contradicts the fact that $D_f(P_i')$ is the class of strategies equivalent to P_i'. $\diamond$

Claim 23 *If a SCF f is s.i.d.e., it satisfies TE.*

Proof of Claim 23: If TE does not hold, there exists $i \in I$, P_i, $P_i' \in \mathcal{P}$ such that

$$P_i \text{ and } P_i' \text{ are } T(k, P_i)\text{-equivalent} \tag{18}$$

$$P_i \text{ and } P_i' \text{ are not } T(k+1, P_i)\text{-equivalent} \tag{19}$$

$$P_i \text{ and } P_i' \text{ agree on } A \backslash T(k, P_i) \tag{20}$$

$$\text{and} \quad T(k, P_i) \neq T(k, P_i'). \tag{21}$$

Let $x = t_{k+1}(P_i)$. (20) and (21) imply that $x \in T(k, P_i')$. Since P_i and P_i' are not x-equivalent there are two possibilities :
Case 1. There exists $P_{-i}^\star \in \mathcal{P}^{n-1}$ such that $x = f(P_i, P_{-i}^\star)$ and $x \neq f(P_i', P_{-i}^{star})$. Let $x = t_l(P_i')$. P_i and P_i' are $T(l-1, P_i)$-equivalent by (18). Then, $P_i \in D_f(P_i')$, which contradicts the fact that f is s.i.d.e. as P_i and P_i' are not equivalent.
Case 2. There exists $P_{-i}^{\star\star} \in \mathcal{P}^{n-1}$ such that $x = f(P_i', P_{-i}^{\star\star})$ and $x \neq f(P_i, P_{-i}^{\star\star})$. As P_i and P_i' are $T(k, P_i)$-equivalent, it cannot be that $P_i d_f(P_i)P_i'$ and $P_i' \in D_f(P_i)$, which is a contradiction.

Thus, in either case f is not s.i.d.e. and TE is necessary for s.i.d.e. This concludes the necessity part. $\diamond$

Let us now consider a SCF f which satisfies MON, BI, UCI and TE. We shall prove that f is s.i.d.e. More precisely, for any two non-equivalent strategies P_i and P_i', we shall prove that $P_i d_f(P_i)P_i'$. Thus, $D_f(P_i)$ the set of non dominated strategies is the set of strategies that are equivalent to P_i, and for any $\pi' \in D_f(\pi)$, $f(\pi) = f(\pi')$.

Suppose P_i and P_i' are not equivalent. Let $t_k(P_i)$ be such that P_i and P_i' are $T(k-1, P_i)$-equivalent, but are not $\{t_k(P_i)\}$-equivalent. Thus, we have for some $P_{-i} \in \mathcal{P}^{n-1}$

$$t_k(P_i) = f(P_i, P_{-i}) \quad \text{and} \quad t_k(P_i) \neq f(P_i', P_{-i}) \text{ or} \tag{22}$$

$$t_k(P_i) \neq f(P_i, P_{-i}) \quad \text{and} \quad t_k(P_i) = f(P_i', P_{-i}) \tag{23}$$

or both for different profiles. As the second case is in contradiction with the fact that f is s.i.d.e., we shall demonstrate that it cannot happen by showing that the assumption that (23) holds leads to a contradiction.

Now, construct $P_i^\star$ such that :

$$T(k-1, P_i) = T(k-1, P_i^\star)\,, \tag{24}$$

$$P_i \text{ and } P_i^\star \text{ agree on } T(k-1, P_i)\,, \tag{25}$$

$$\text{and} \quad P_i' \text{ and } P_i^\star \text{ agree on } A \backslash T(k-1, P_i)\,. \tag{26}$$

Since f satisfies BI, any reshuffling of the alternatives below $x \in T(k-1, P_i)$ keeps the status of x unchanged. Thus, by (24) and (25), P_i and $P_i^\star$ are $T(k-1, P_i)$-equivalent. Thus, P_i' and $P_i^\star$ are also $T(k-1, P_i)$-equivalent and $T(k-1, P_i^\star)$-equivalent. Since P_i' and $P_i^\star$ are $T(k-1, P_i^\star)$-equivalent and agree on $A \backslash T(k-1, P_i')$, by TE, either we get (a) $P_i^\star$ and P_i' are not $\{t_k(P_i^\star)\}$-equivalent and $T(k-1, P_i') = T(k-1, P_i^\star) = T(k-1, P_i)$ or (b) P_i' and $P_i^\star$ are $\{t_k(P_i')\}$-equivalent.

Case (a). $P_i^\star$ and P_i' share the same top and are perfectly identical on $A \backslash T(k-1, P_i^\star)$. Consider now the profiles in $g_f(t_k(P_i^\star), P_i^\star)$. Since $T(k-1, P_i^\star) = T(k-1, P_i')$, $P_i^\star$ and P_i' are $T(k-1, P_i^\star)$-equivalent and agree on $A \backslash T(k-1, P_i^\star)$, by UCI, $g_f(t_k(P_i^\star), P_i^\star) \subseteq g_f(t_k(P_i^\star), P_i')$. As $t_k(P_i^\star) = t_k(P_i')$, by using the same argument for P_i', we get that $g_f(t_k(P_i^\star), P_i^\star) = g_f(t_k(P_i^\star), P_i')$. This contradicts the fact that $P_i^\star$ and P_i' are not $\{t_k(P_i^\star)\}$-equivalent and case (b) holds.

Case (b). $P_i^\star$ and P_i' are $\{t_k(P_i^\star)\}$-equivalent. Using the same argument as in case (a), we can prove that $P_i^\star$ and P_i' are $\{t_{k+1}(P_i^\star)\}$-equivalent, $\{t_{k+2}(P_i^\star)\}$-equivalent, etc. Thus $P_i^\star$ and P_i' are equivalent. By construction, we have $t_k(P_i) = t_l(P_i^\star)$, with $l > k$. Consider now the initial profile for which $t_k(P_i) \neq f(P_i, P_{-i})$ and $t_k(P_i) = f(P_i', P_{-i})$. Thus $t_k(P_i) = f(P_i^\star, P_{-i})$, and using MON, $t_k(P_i) \in \phi(P_i, P_{-i})$. This is in contradiction with (23), and only (22) is compatible with MON, BI, UCI and TE. Thus, $P_i d_f(P_i) P_i'$.

$\diamond\diamond$

5 Independence of the Axioms

We present a list of examples to show that the axioms MON, BI, TE and UCI are logically independent. We first show that none of the four axioms is redundant, i. e. implied by the other three.

Example 1 (A rule satisfying MON, BI and TE but not UCI) *Let $I = \{1, \ldots, n\}$, with $n \geq 2$, and $A = \{a_1, \ldots, a_m\}$, with $m \geq 4$. Let $Q \in \mathcal{P}$ be the ordering $a_3 a_4 \ldots a_m a_2 a_1$. Define f_1 by the following rule.*

$$f_1(\pi) = \begin{cases} a_1 & \textit{if } a_1 P_1 a_2 \textit{ or } P_1 = Q \\ a_2 & \textit{otherwise.} \end{cases}$$

Proof Since the outcome is determined by individual 1's preferences alone, we only need to check the properties for changes in 1's preference.

f_1 satisfies MON. If $f_1(\pi) = a_1$ we have $P_1 = Q$ or $a_1 P_1 a_2$. So for any preference P_1' in which a_1 has moved up, we have $a_1 P_1' a_2$ and hence $f(\pi/P_1') = a_1$. If $f_1(\pi) = a_2$ we must have $a_2 P_1 a_1$ and $P_1 \neq Q$. This will still hold when a_2 moves up in 1's preference, so a_2 remains chosen.

f_1 satisfies BI. In the case $P_1 = Q$, the chosen alternative is a_1 and this is at the bottom, so no reshuffling is possible. In all other cases, what matters is the relative position of a_1 and a_2 in 1's preference which will not be changed by reshuffling below the chosen alternative.

f_1 satisfies TE. Consider P_1 and P_1' wich are $T(k,P_1)$- but not $T(k+1,P_1)$-equivalent for some $k \in \{1,\ldots,m\}$ and agree on $A \backslash T(k,P_1)$. All preferences in $\mathcal{P}$ are $\{a_3,a_4,\ldots,a_m\}$-equivalent since none of these alternatives is ever chosen. If a_1 or a_2 are in $T(k,P_1)$, P_1 and P_1' would be equivalent. So $a_1,a_2 \in A \backslash T(k,P_1)$. Indeed one of a_1 and a_2 has to be $t_{k+1}(P_1)$ and since both P_1 and P_1' agree on $A \backslash T(k,P_1)$ the other one must be ranked below in both. P_1 cannot be Q because then there would be no P_1' with the required properties. So in both cases the alternative which is $t_{k+1}(P_1)$ is chosen.

f_1 violates UCI. Consider $P_1 = a_4a_3a_2a_1$ and $P_1' = Q = a_3a_4a_2a_1$. Then $f(\pi) = a_2 = t_3(P_1)$, $T(2,P_1) = \{a_3,a_4\} = T(2,P_1')$, P_1 and P_1' are $\{a_3,a_4\}$-equivalent and agree on $A \backslash \{a_3,a_4\} = \{a_1,a_2\}$ but $f(\pi/P_1') = a_1$.

$\diamond$

Example 2 (A rule satisfying MON, BI and UCI but not TE) *Let $m \geq 3$ and $n \geq 2$. Define f_2 as follows.*

$$f_2(\pi) = \begin{cases} a_2 & \textit{if } T(1,P_i) = a_2 \ \forall\, i \in I\,, \\ a_1 & \textit{otherwise}\,. \end{cases}$$

Proof f_1 satisfies MON. This is obvious, since a_2 is chosen if and only if it is everybody's top choice. In this case it cannot be moved further up, while in all other cases a_1 is chosen and a_2 cannot become the top choice by moving a_1 up.

f_1 satisfies BI. Again this is trivial. Reshuffling bottoms will not change whether or not a_2 is at the top.

f_1 satisfies UCI. Consider P_i and P_i' such that $f_2(\pi) = t_{k+1}(P_i)$, $T(k,P_i) = T(k,P_i') = T$, P_i and P_i' are T-equivalent and agree on $A \backslash T$. $t_{k+1}(P_i)$ must be a_1. If a_2 is not in T, obviously $f_2(\pi/P_i') = a_1$. But if $a_2 \in T$, T-equivalence of P_i and P_i' tells us that either it is the top choice according to both preferences or it is not the top for both. In the latter case, clearly $f_2(\pi/P_i') = a_1$ because of P_i'; but in the former we know from $f_2(\pi) = a_1$ that also $f_2(P_{-i},P_i') = a_1$, this time because a_2 cannot be everybody else's top choice.

f_1 violates TE. Consider $P_i = a_3a_2\ldots$ and $P_i' = a_2a_3\ldots$. Both are $T(1,P_i)$-equivalent (since a_3 is never selected) but not $T(2,P_i)$-equivalent. They agree on $A \setminus T(1,P_i)$ but $T(1,P_i) \neq T(1,P_i')$. $\diamond$

Example 3 (A rule satisfying BI, TE and UCI but not MON) *Let $m \geq 3$ and $n \geq 2$. Let $S^1(a_j) = \left|\{i \in I \mid a_j \in T(1,P_i)\}\right|$ Then,*

$$a_j = f_3(\pi) \Leftrightarrow \begin{cases} S^1(a_j) < S^1(a_k)\,\forall a_k \in A \setminus \{a_j\} \textit{ or} \\ S^1(a_j) \leq S^1(a_k)\,\forall a_k \in A \setminus \{a_j\} \textit{ and } S^1(a_j) = S^1(a_k) \Rightarrow j < k. \end{cases}$$

So effectively this rule selects the alternative that does worst in terms of plurality scores, using the smallest index as a tie-breaking rule.

Proof f_1 satisfies BI. This is obvious because reshuffling below any alternative will not change the top alternative and hence the scores S^1 remain unchanged.

f_1 satisfies TE. This property is trivially satisfied since the condition one needs to ckeck is never satisfied. The reason is that for two preferences P_i and P'_i to be $T(k, P_i)$-equivalent, they need to have the same top, in which case they are equivalent.

f_1 satisfies UCI. If P_i and P'_i are T-equivalent, they have the same top and hence $f_3(P_{-i}, P_i) = f_3(P_{-i}, P'_i)$ irrespective of P_{-i}.

f_1 violates MON. Consider a situation where a_1 is chosen because it is tied for the least score with some other alternative. Then there must be some individual not having a_1 at the top. By moving it up to the top in this individuals preference, then, a_1 will no longer be chosen by f_3. $\diamond$

Example 4 (A rule satisfying MON, TE and UCI but not BI) *Let $m \geq 3$ and $n \geq 2$. Let $S^2(a_j) = \left|\{i \in I \mid a_j \in T(2, P_i)\}\right|$ Then,*

$$a_j = f_4(\pi) \Leftrightarrow \begin{cases} S^2(a_j) > S^2(a_k)\, \forall a_k \in A \setminus \{a_j\} \text{ or} \\ S^2(a_j) \geq S^2(a_k)\, \forall a_k \in A \setminus \{a_j\} \text{ and } S^2(a_j) = S^2(a_k) \Rightarrow j < k. \end{cases}$$

Here an alternative scores if it belongs to the top two alternatives of an individual. The highest score wins and ties are broken using the smallest index.

Proof f_1 satisfies MON. This is obvious, since by moving any alternative up in anybody's ranking its score cannot decrease.

f_1 satisfies TE. This property is trivially satisfied since the condition on preferences that needs to be checked can never be satisfied. If preferences P_i and P'_i are $T(k, P_i)$-equivalent it follows that $T(2, P_i) = T(2, P_i)$. Otherwise there would be two alternatives one getting one point with P_i and zero with P'_i while this is reversed for the other. This fact could be used to make one of the two alternatives a winner under preference P_i or P'_i while something from $T(k, P_i)$ wins under the other preference. Hence $T(k, P_i)$-equivalence implies equivalence.

f_1 satisfies UCI. As we have seen P_i and P'_i being $T(k, P_i)$-equivalent implies $P_i \sim_{f_4} P'_i$. Therefore what is chosen with P_i will also be chosen with P'_i.

f_1 violates BI. Consider $P_1 = a_2a_3a_1$ and $P'_1 = P_2 = a_2a_1a_3$. Then we have $f_4(P_1, P_2) = a_2$ but $f_4(P'_1, P_2) = a_1$. $\diamond$

Next we demonstrate that each of the four axioms can be satisfied in the absence of the remaining three, i. e., violation of any triple of axioms does not imply the fourth one to be violated as well.

Example 5 (A rule satisfying TE but neither MON, BI nor UCI) *Let $m \geq 5$ and $n \geq 3$. Consider an ordering $Q = a_1a_2 \ldots a_m$. Define f_5 by the following rule. If $T(2, P_1) = T(2, Q)$ and both preferences agree on $T(2, Q)$, then apply f_4 to the preference P_2 restricted to $A \backslash T(2, P_1)$.[3] In all other cases, apply f_4 to the preference P_3 restricted to $A \backslash T(2, P_1)$.*

[3]Strictly speaking, we only define all social choice functions for at least two individuals. This is, because to define equivalence, for example, we need others' preferences. For those rules, like f_4, that only depend on one individuals preferences, however, there is no problem to also extend them to the case of $n = 1$.

PROOF f_1 satisfies TE. Since f_4 satisfies TE (see Example 4) TE is clearly satisfied unless we consider changes in 1's preferences, i. e. pairs of preferences P_1 and P_1'. There are several cases to be checked. The interesting thing to look at is the set of the two top ranked alternatives, where a_1 and a_2 play a special role, because they are in the top of Q. So we need to distinguish whether they are both included in the top two, just one of them, or none. For P_1 the ranking within the top two matters (because it is equivalence with respect to $T(k, P_1)$ that we need to ckeck), while for P_1' the ranking among the top two makes a difference if and only if $T(2, P_1') = \{a_1, a_2\}$. All possibilities are given in the following table, where the numbers refer to the list of different reasons why TE is satisfied in each case.

P_1 \ P_1'	a_1a_2	a_2a_1	a_1a_3 or a_3a_1	a_2a_3 or a_3a_2	a_3a_4 or a_4a_3
a_1a_2	1.(a)	2.	3.	4.	4.
a_2a_1	2.	1.(b)	4.	3.	4.
a_1a_3	3.	3.	1.(b)	4.	4.
a_3a_1	4.	4.	1.(b)	3.	3.
a_2a_2	3.	3.	4.	1.(b)	4.
a_3a_2	4.	4.	3.	1.(b)	3.
a_3a_4	4.	4.	3.	3.	1.(b)
a_4a_3	4.	4.	4.	4.	1.(b)

TE is satisfied for the following reasons according to which case applies.

1. (a) $f_5(P_1, P_{-1}) = f_5(P_1', P_{-1})$ because both are determined by P_2 using f_4. Therefore P_1 and P_1' are equivalent, so TE cannot be checked.

   (b) $f_5(P_1, P_{-1}) = f_5(P_1', P_{-1})$ because both are determined by P_3 using f_4. Therefore P_1 and P_1' are equivalent, so TE cannot be checked.

2. P_1 and P_1' are $T(2, P_1)$- but not $T(3, P_1)$- equivalent and $T(2, P_1) = T(2, P_1')$, hence TE is satisfied.

3. P_1 and P_1' are $T(1, P_1)$- but not $T(2, P_1)$- equivalent but they do not agree on $A \setminus T(1, P_1)$, so TE cannot be checked.

4. P_1 and P_1' are not $T(1, P_1)$-equivalent, so TE cannot be checked.

f_1 violates MON. Consider preferences $P_1 = P_2 = P_3 = a_1a_2a_3a_4a_5$ and $P_1' = a_1a_3a_2a_4a_5$. Then $f_5(P_1, P_2, P_3) = a_3$ because this is chosen according to f_4 applied to P_2 restricted on $A \setminus T(2, P_1) = \{a_3, a_4, a_5\}$. a_3 moves up from P_1 to P_1' the rest remaining unchanged. But now $f_5(P_1', P_2, P_3) = a_2$ since now f_4 is applied using P_3 restricted on the set $A \setminus T(2, P_1') = \{a_2, a_4, a_5\}$.

f_1 violates BI. Consider $P_1 = Q$, $P_2 = a_1a_2a_4a_5a_3$, $P_2' = a_1a_2a_4a_3a_5$ and P_3 arbitrary. Then $f_5(P_1, P_2, P_3) = a_4$ (use f_4 with P_2 on $\{a_3, a_4, a_5\}$). We get P_2' from P_2 by reshuffling below a_4, but $f_5(P_1, P_2', P_3) = a_3$ (because now a_3 has moved up and wins because of the tie-breaking rule of f_4).

f_1 violates UCI. Consider preferences $P_1 = a_1a_2a_3a_4a_5$, $P_1' = a_2a_1a_3a_4a_5$, $P_2 = a_1a_2a_3a_4a_5$ and $P_3 = a_1a_2a_4a_5a_3$. Then $f_5(P_1, P_2, P_3) = a_3 = t_3(P_1)$. $T(2, P_1) = T(2, P_1') = T$, P_1 and P_1' are T- equivalent and agree on $A \setminus T$, but $f_5(P_1', P_2, P_3) = a_4$. $\diamond$

Example 6 (A rule satisfying MON but neither BI, TE nor UCI) *Let $m \geq 5$ and $n \geq 2$. Define f_6 as always choosing a_3 unless $a_1P_1a_2$ and $a_4P_2a_5$, in which case a_4 is chosen.*

Proof f_1 satisfies MON. Moving a_3 up (or down, for that matter) never changes any of the relevant conditions, so if a_3 is chosen it will still be chosen after any monotonic change of preferences. If a_4 is chosen we have $a_1P_1a_2$ and $a_4P_2a_5$ and this will not be changed by moving up a_4 leaving the ranking of other alternatives unchanged.

f_1 violates BI. Consider $P_1 = a_1a_2\ldots$, $P_2 = a_3a_5a_4\ldots$, and $P_2' = a_3a_4a_5\ldots$ Then $f_6(P_1, P_2) = a_3$. We get P_2' by reshuffling 2's preference below a_3 but $f_6(P_1, P_2') = a_4$.

f_1 violates TE. Consider $P_1 = a_1, a_2\ldots$, $P_2 = a_5a_4a_3\ldots$, and $P_2' = a_4a_5a_3\ldots$ P_2 and P_2' are $T(1, P_2)$- but not $T(2, P_2)$-equivalent and agree on $A \setminus T(1, P_2)$. However, $T(1, P_2) \neq T(1, P_2')$.

f_1 violates UCI. Consider $P_1 = a_2a_1a_3\ldots$, $P_1' = a_1a_2a_3\ldots$, and $P_2 = a_4a_5\ldots$ $f_6(P_1, P_2) = a_3 = t_3(P_1)$, $T(2, P_1) = T(2, P_1') = T$, P_1 and P_1' are T-equivalent and agree on $A \setminus T$ but $f_6(P_1', P_2) = a_4$. $\diamond$

Example 7 (A rule satisfying BI but neither MON, TE nor UCI) *Let $m \geq 4$ and $n \geq 2$. Define f_7 as always choosing a_3 unless $P_1 = a_1a_2a_3\ldots$, $P_1 = a_1a_3\ldots$, or $P_1 = a_3\ldots$, in which case a_4 is chosen.*

Proof Since f_7 depends on 1's preferences, only, we just need to consider possible changes in the first individual's preferences.

f_1 satisfies BI. This is quite clear. Either P_1 has one of the three forms that lead to a_4 being chosen which will not be changed by reshuffling below a_4, or this is not the case, a_3 is chosen, and reshuffling below a_3 never leads to the first case.

f_1 violates MON. Consider $P_1 = a_4a_3\ldots$, $P_1' = a_3a_4\ldots$ a1nd arbitrary other preferences. Obviously $f_7(\pi) = a_3$, a_3 has moved up from P_1 to P_1', the ranking of all other alternatives remaining unchanged, but $f_7(\pi/P_1') = a_4$.

f_1 violates TE. Consider $P_1 = a_1a_2a_3\ldots$ and $P_1' = a_2a_3\ldots$ P_1 and P_1' are $T(2, P_1)$- but not $T(3, P_1)$-equivalent and agree on $A \setminus T(2, P_1)$ but $T(2, P_1) \neq T(2, P_1')$.

f_1 violates UCI. Consider $P_1 = a_1a_2a_3\ldots$ and $P_1' = a_2a_1a_3\ldots$ We have $T(2, P_1) = T(2, P_1') = T$, P_1 and P_1' are T-equivalent and agree on $A \setminus T$ but $f_7(\pi) = a_4 \neq a_3 = f_7(\pi/P_1')$. $\diamond$

Example 8 (A rule satisfying UCI but neither MON, BI nor TE) *Let $m \geq 4$ and $n \geq 2$.*

$$f_8(\pi) = \begin{cases} a_2 & \text{if } a_2 \in B(2, P_i)\, \forall i \in I \\ a_1 & \text{otherwise.} \end{cases}$$

Proof f_1 satisfies UCI. The only two possible choices are a_1 and a_2. If $f_8(\pi) = a_1$ there must be some i with $a_2 \notin B(2, P_i)$. Any changes in preferences other than i's will not change anything. Unless a_1 is ranked last, any change from P_i which keeps the same set above a_1 and does not change the ranking below cannot result in a_2 being in the 2-bottom. But if a_1 is ranked last, P_i and P_i' have to be a_2-equivalent and hence a_2 must stay higher in the ranking than the worst two. If a_2 is chosen, it is in $B(2, P_i)$ for all i. This will not be changed by any change in preferences keeping the same alternatives being ranked above a_2.

f_1 violates MON. Consider $P_1 = P_2 = a_1a_2a_3$ and $P_1' = a_2a_1a_3$. Then $f_8(P_1, P_2) = a_2$, a_2 has moved up, the other alternatives' ranking stays the same, but $f_8(P_1', P_2) = a_1$.

f_1 violates BI. Consider $P_1 = a_1a_2a_3a_4$ and $P_1' = P_2 = a_1a_3a_2a_4$. Then $f_8(P_1, P_2) = a_1$ and we get P_1' by reshuffling below a_1 but $f_8(P_1', P_2) = a_2$.

f_1 violates TE. Consider $P_1 = a_3a_4a_2a_1$ and $P_1' = a_3a_2a_4a_1$. P_1 and P_1' are $T(2, P_1)$- but not $T(3, P_1)$-equivalent and they agree on $A \setminus T(2, P_1)$ but $T(2, P_1) \neq T(2, P_1')$. $\diamond$

6 Some Side Social Choice Functions

In the previous sections, we described necessary and sufficient conditions for self implementation via demanding equilibria. We here check whether some famous voting rules are s.i.d.e. or not, and describe many s.i.d.e. SCF's. This section adds to the results obtained by Barberà and Dutta [4], who describe which rules are d.i.p.e.

6.1 Scoring rules

A scoring rule is characterized by a scoring vector $w = (w_1, \ldots, w_m) \in \mathbb{R}^m$. The rank of alternative a in preference P_i, denoted by $r(P_i, a)$, is defined by:

$$r(P_i, a) = k \Leftrightarrow t_k(P_i) = a$$

The score of a for the profile π and scoring vector w is:

$$S_w(\pi, a) = \sum_{i \in I} w_{r(P_i,a)}.$$

For any w, we can define the scoring rule f_w as selecting the alternative with the highest score, ties being broken according to the indices.

$$f_w(\pi) = a_j \quad \Leftrightarrow \quad \begin{cases} [S_w(\pi, a_j) > S_w(\pi, a_k)\ \forall a_k \in A \setminus \{a_j\}\ ] \text{ or} \\ [S_w(\pi, a_j) \geq S_w(\pi, a_k)\ \forall a_k \in A \setminus \{a_j\} \\ \text{and } S_w(\pi, a_j) = S_w(\pi, a_k)\ \Rightarrow\ j < k.] \end{cases}$$

Table 1: Profile π_1

P_1 :	a_1	P_1	a_3	P_1	a_4	P_1	$\dots$	a_s	P_1	a_2	P_1	a_{s+1}	P_1	$\dots$	P_1	a_m
P_2 :	a_m	P_2	a_1	P_2	a_3	P_2	$\dots$	a_{s-1}	P_2	a_s	P_2	a_2	P_2	$\dots$	P_1	a_{m-1}
P_3 :	a_{m-1}	P_3	a_m	P_3	a_1	P_3	$\dots$	a_{s-2}	P_3	a_{s-1}	P_3	a_s	P_3	$\dots$	P_3	a_{m-2}
$\vdots$	$\vdots$															
P_m :	a_3	P_4	a_4	P_4	a_5	P_4	$\dots$	a_2	P_4	a_{s+1}	p_4	a_{s+2}	P_4	$\dots$	P_4	a_1

Of course, the natural way to use scoring rules is to assume that $w_r \geq w_{r+1}$ for $r = 1, \dots, m-1$ and $w_1 > w_m$. Without loss of generality, we can also assume that $w_1 = 1$ and $w_m = 0$. Nevertheless, Smith [25] and Young [28] give characterizations of the scoring rules without these assumptions.

The three most famous scoring rules are the *plurality rule*, which selects as a social choice the alternative with the greatest number of first place votes ($w = (1, 0, \dots, 0)$), the *antiplurality rule* where each voter awards one point to any alternative except to the last ranked in her preference ordering ($w = (1, \dots, 1, 0)$), and the *Borda count*, which assigns $(m-1)$ to a candidate each time she appears first in one's preference ordering, $(m-2)$ points each time she appears second, and so on down to zero point each time she appears last ($w = (m-1, m-2, \dots, 1, 0)$ or $w = (1, \frac{m-2}{m-1}, \dots, \frac{1}{m-1}, 0)$ in an equivalent way). The constant scoring rule assigns the same number of points to any rank.

Theorem 24 *Let $m \geq 3$. The only non constant scoring rule which is s.i.d.e. for any population size is the plurality rule.*

Claim 25 *A scoring rule satisfies BI if and only if $w_r = b \; \forall r = 2, \dots, m$.*

Proof of Claim 25: If $w_1 = b$, we get the constant SCF that always selects a_1, and it trivialy satisfies BI. Whenever $w = (a, b, \dots b)$, any change in the preferences below the winner does not affect any score.

Now, let us assume that $w_r > w_s$ for $r > 1$ and $s > 1$. Consider the profile π_1 with m voters, displayed on Table 1.

Each alternative fills each position once and only once, and a_2 is ranked sth when a_1 is ranked first. Thus, all the alternatives get the same score and $f_w(\pi_1) = a_1$ according to the tie breaking rule. Consider now the preference P'_1, in which a_2 changes its position with $t_r(P_1)$ in P_1, everything else being unchanged. Thus,

$$S_w(\pi_1/P'_1, a_2) = S_w(\pi_1, a_2) + w_r - w_s > S_w(\pi_1, a_2)$$

Clearly a_2 obtains the highest score alone, and $f_w(\pi_1/P'_1) = a_2$, which contradicts BI. $\diamond$

Proof of Theorem 24: A scoring rule satisfies BI iff $w = (a, b, \dots b)$. It is non constant whenever $a \neq b$. If $b > a$, the scoring rule cannot be monotonic; indeed, it is

Table 2: Profile π_2

$$\begin{array}{lllllll} P_1 & : & a_1 & P_1 & a_2 & P_1 & a_3 \\ P_2 & : & a_3 & P_2 & a_1 & P_2 & a_2 \\ P_3 & : & a_2 & P_3 & a_3 & P_3 & a_1 \end{array}$$

equivalent to f_3 and selects the alternative with the smallest plurality score. So, only the case $a > b$ is left. This rule is equivalent to the plurality rule, $w = (1, 0, \ldots, 0)$. The strategies equivalent to P_i are the preferences P_i' such as $T(1, P_i) = T(1, P_i')$. So, MON, BI, UCI and TE are satisfied and the plurality rule is the only non constant s.i.d.e. scoring rule. $\diamond$

6.2 Condorcet Social Choice Functions

The Condorcet criterion is one of the most famous normative condition in social choice literature. It asserts that a candidate should be elected each time she gathers a majority of votes against any opponent in pairwise comparisons. We propose here a slightly weakened version of this requirement.

Definition 26 *Let A be a set of alternatives, I the set of voters and $\pi \in \mathcal{P}^n$. Then, the alternative a dominates the alternative b for the profile π, denoted by $aM(\pi)b$ if:*

$$\#\{i \in I \mid aP_ib\} > \#\{i \in I \mid bP_ia\}$$

We define the set of weak Condorcet winner, $CW(\pi)$, as the set of undominated alternatives:

$$CW(\pi) = \{a \in A \mid bM(\pi)a \ \text{ for no } b \in A \setminus \{a\}.\}$$

Definition 27 *f is a Condorcet Social Choice Function (CSCF) if:*

$$\forall \pi \in \mathcal{P}^n, \ CW(\pi) \neq \emptyset \Rightarrow f(\pi) \subset CW(\pi).$$

Theorem 28 *If $m \geq 3$, any CSCF f violates BI, except for the case $m = 3, n = 4$.*

Proof of Theorem 28: Consider first the case $m = 3$, $n = 3$, and the profile π_2 displayed on Table 2. We get $a_1M(\pi_2)a_2$, $a_2M(\pi_2)a_3$, $a_3M(\pi_2)a_1$, and $CW(\pi_2) = \emptyset$. Assume that $f(\pi_2) = a_1$. By BI, a change in the preferences below a_1 should not affect its status. Consider $P_1' = a_1 \, P_1' \, a_3 \, P_1' \, a_2$. Then, $f(\pi_2/P_1') = a_3 = CW(\pi_2/P_i')$, which contradicts BI. The same reasoning holds if we assume $f(\pi_2) = a_2$ or $f(\pi_2) = a_3$. We can generalize the reasoning to $m > 3$ by adding the alternatives a_4, a_5, etc... below a_3, a_1 and a_2 in the profile π_2. We can also generalize to $n > 4$, building a cycle similar to the one proposed in π_1. For $n = 4$, $m \geq 4$, we can check that BI is not satisfied from the profile π_3:

For $n = 4$, $m = 3$, $CW(\pi) \neq \emptyset$. Thus, any alternative in $CW(\pi)$ stays in this set when we affect the preferences below her. $\diamond$

Table 3: Profile π_3

$$
\begin{array}{lllllllll}
P_1 : & a_1 & P_1 & a_2 & P_1 & a_3 & P_1 & a_4 \\
P_2 : & a_4 & P_2 & a_1 & P_2 & a_2 & P_2 & a_3 \\
P_2 : & a_3 & P_3 & a_4 & P_3 & a_1 & P_3 & a_2 \\
P_3 : & a_2 & P_4 & a_3 & P_4 & a_4 & P_4 & a_1
\end{array}
$$

6.3 Other s.i.d.e. Voting Rules

We identified two s.i.d.e. voting rules: the constant SCF and the plurality rule. Nevertheless, we can design more s.i.d.e. rules. First, any rule based upon the plurality scores will be s.i.d.e., as long as it is monotonic. This condition rules out f_3 and any process that eliminates the alternatives progressively on the basis of the plurality scores (Smith [25] proves that such rules, called scoring run-offs, are not monotonic), but keeps all the voting procedures that use the plurality scores with thresholds. For example, we can decide to apply the plurality rule, unless a_1 already gets 20% of the total vote, in which case she is directly elected. For more on voting rules with thresholds, see Saari [20]. We can also attribute different weights to the voters when they cast their plurality vote, the extreme case being dictatorship. Using a tie breaking rule on the set of Pareto alternatives or on the set of alternatives which are ranked first by at least one voter would also lead to a s.i.d.e. SCF.

7 Protective Behavior versus Prudence, Demanding Behavior versus Risk Loving

The result we obtained for the plurality rule can be compared to the ones Barberà and Dutta [4] get for the antiplurality rule: on one hand, the plurality rule is the only s.i.d.e. scoring rule, and on the other hand, the antiplurality rule is the only d.i.p.e. scoring rule. In other words, when voters are extremely prudent, asking them to reveal their last ranked alternative is a good and simple way to avoid manipulation and when voters have an exaggerated preference for their top choice, asking them to report it will also avoid strategic behavior.

This typology has to be compared with some results of Moulin [15], who proposes a different way to model risk aversion, the prudent behavior. In the process of selecting her optimal strategies, a prudent voter will consider the number of profiles which lead to the selection of an outcome, i.e. the cardinality of the sets $g_f(a, P_i)$, rather than searching for inclusion relationships among these sets. Formally, for $i \in I$ with preference P_i, a strategy $\hat{P}_i$ *is prudent iff there does not exist* $\tilde{P}_i$ *such that for some* $k \in \{1, \ldots, m\}$ *the following holds*:

$$
\begin{array}{lrcl}
 & \#g_f\big(b_k(P_i), \hat{P}_i\big) & > & \#g_f\big(b_k(P_i), \tilde{P}_i\big) \\
\text{and} & \#g_f\big(b_r(P_i), \hat{P}_i\big) & = & \#g_f\big(b_r(P_i), \tilde{P}_i\big)\,, \qquad \forall\, r < k.
\end{array}
$$

As noted by Barberà and Dutta, the prudent behavior assumes implicitly that all the preferences profiles $\pi_{-i} \in \mathcal{P}^{n-1}$ are equally likely. On the contrary, the demanding or the protective behavior are applicable even if agents have no subjective probability distribution about others' strategies.

Using the same assumption as Moulin on the likelihood of profiles, we can define in a similar way a *"risk loving"* or *"admiriting "* behavior, by only considering the cardinalities of the sets $g_f(a, P_i)$. For $i \in I$ with preference P_i, a strategy $\hat{P}_i$ *is risky iff there does not exist* $\tilde{P}_i$ *such that for some* $k \in \{1, \ldots, m\}$ *the following holds*:

$$\begin{aligned} &\#g_f\big(t_k(P_i), \hat{P}_i\big) &<& \ \#g_f\big(t_k(P_i), \tilde{P}_i\big) \\ \text{and} \quad &\#g_f\big(t_r(P_i), \hat{P}_i\big) &=& \ \#g_f\big(t_r(P_i), \tilde{P}_i\big), \qquad \forall r < k. \end{aligned}$$

For a SCF f and a preference P_i, the set of risky strategies is denoted by $R_f(P_i)$, and the set of prudent strategies is denoted by $P_f(P_i)$.

Definition 29 *Let* $\pi \in \mathcal{P}^n$. *A strategy profile* $\bar{\pi} \in \mathcal{P}^n$ *is called a prudent equilibrium with respect to the SCF* f *iff* $\bar{P}_i \in P_f(P_i)$, *for all* $i \in I$.

Definition 30 *A SCF* f *is self implementable with a prudent behavior iff for all pairs of preference profiles* π *and* $\bar{\pi}$, $f(\pi) = f(\bar{\pi})$ *whenever* $\bar{\pi}$ *is a prudent equilibrium with respect to* f *under* π.

Definition 31 *Let* $\pi \in \mathcal{P}^n$. *A strategy profile* $\bar{\pi} \in \mathcal{P}^n$ *is called a risky equilibrium with respect to the SCF* f, *iff* $\bar{P}_i \in R_f(P_i)$, *for all* $i \in I$.

Definition 32 *A SCF* f *is self implementable with a risky behavior iff for all pairs of preference profiles* π *and* $\bar{\pi}$, $f(\pi) = f(\bar{\pi})$ *whenever* $\bar{\pi}$ *is a risky equilibrium with respect to* f *under* π.

While the protective behavior (resp. demanding behavior) clearly isolates the antiplurality rule (resp. plurality rule) among the scoring rules, Moulin [15] states that both plurality rule and Borda count are implementable with a prudent behavior, but leaves the proof to the reader. In fact, the next proposition extends his comments.

Proposition 33 *Both plurality and antiplurality rules are self implementable with a prudent behavior and with a risky behavior as soon as* $n \geq 2$.

Proof of Proposition 33: We handle the plurality case, the antiplurality case being symmetric. First, it is obvious that the plurality rule is implementable with a risky behavior, as the inclusion relationships between sets $g_f(a, P_i)$ lead to dominance in terms of cardinality.

We propose a detailed proof of the fact that the plurality is also prudent in the case $m = 3$; one can extend the arguments to $m > 4$, tough the number of cases to analyze increases rapidly. Let A $= \{a, b, c\}$ and use the alphabetical order to break ties. We partition $\mathcal{P}^{n-1}$ in 7 subsets:

$$
\begin{aligned}
J_1 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, a) > S^1(\pi_{-i}, b) \textit{ and } \right| S^1(\pi_{-i}, a) > S^1(\pi_{-i}, c)\} \\
J_2 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, b) > S^1(\pi_{-i}, a) \textit{ and } \right| S^1(\pi_{-i}, b) > S^1(\pi_{-i}, c)\} \\
J_3 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, c) > S^1(\pi_{-i}, a) \textit{ and } \right| S^1(\pi_{-i}, c) > S^1(\pi_{-i}, b)\} \\
J_4 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, a) = S^1(\pi_{-i}, b) > S^1(\pi_{-i}, c)\} \right. \\
J_5 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, a) = S^1(\pi_{-i}, c) > S^1(\pi_{-i}, b)\} \right. \\
J_6 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, b) = S^1(\pi_{-i}, c) > S^1(\pi_{-i}, a)\} \right. \\
J_7 &= \{\pi_{-i} \in \mathcal{P}^{n-1} \left| S^1(\pi_{-i}, a) = S^1(\pi_{-i}, b) = S^1(\pi_{-i}, c)\} \right.
\end{aligned}
$$

The scoring rules are anonymous and neutral when we don't use tie breaking rules[4]: all the individual have the same power, and all the alternatives are equally treated. Thus, $\#J_1 = \#J_2 = \#J_3$ and $\#J_4 = \#J_5 = \#J_6$. J_1, J_2 and J_3 are non empty as soon as $n \geq 2$, J_4, J_5 and J_6 for $n = 3$ and $n \geq 5$, and J_7 exists only if $n = 3k + 1$, $k \in \mathbb{N}$. There are only three possible strategies for the plurality rule: $P_a = aPbPc$ *or* $aPcPb$, $P_b = bPaPc$ *or* $bPaPc$ and $P_c = cPaPb$ *or* $cPbPa$.

Assume that $P_i = aPbPc$. We want to identify first the strategies that minimize the number of profiles such that c is selected. Clearly, P_c is not a prudent strategy as using it instead of P_a will lead to the selection of c instead of a for all the profiles in J_5 and J_7 and for some profiles in J_3 and J_6. Using P_b instead of P_a is useful in J_3 each time $S^1(\pi_{-i}, c) = S^1(\pi_{-i}, b) + 1 > S^1(\pi_{-i}, a)$. By neutrality and anonimity, there is an equal number of profiles in J_3 where $S^1(\pi_{-i}, c) = S^1(\pi_{-i}, a) + 1 > S^1(\pi_{-i}, b)$, in which cases $f(\pi_{-i}/P_a) = a$ and $f(\pi_{-i}/P_b) = c$. Thus, P_a and P_b are equivalent in order to avoid the selection of c. Secondly, we want to minimize the number of cases where b is selected; P_a and P_b are the only left strategies. Clearly, P_a does better than P_b in J_2. Voting for a is a prudent strategy. Similar conclusions can be raised for the other preference types; this concludes the proof. $\diamond$

So, why is the plurality prudent, but not d.i.p.e. ? At some point in the proof, P_b is the only strategy that can avoid the selection of c. So we cannot neither state that $g_f(c, P_b) \subset g_f(c, P_a)$, nor $g_f(c, P_b) \subset g_f(c, P_a)$! A protective voter cannot choose between P_b and P_a, while a prudent voter can use a cardinility argument to eliminate P_b from her set of admissible strategy. This subtle difference explains the different results we get for the implementation of the plurality rule and the antiplurality rule.

8 Concluding Remarks

As we have already remarked in the introduction, the concept of self implementation asks for a given social choice function which behavioral assumptions on the agents are

[4]For precise statements of these conditions and these results for social choice correspondences and social welfare functions, see Smith [25] and Young [28].

compatible with the SCF in the sense that for any profile of individual characteristics everybody behaving strategically in accordance with that assumption leads to the outcome prescribed by the social choice function for that profile. Of course, generally there will be more than one such behavioral assumption. The plurality rule, for example, is self implementable if we assume demanding behavior (Theorem 24) and also if we assume prudent or risky behavior (Proposition 33). This is in the same flavor as the concept of double implementation (cf. Maskin [12]) and should be seen as good news: even if we do not know exactly how individuals behave we have a whole list of behavioral assumptions for which we are confident that the SCF under consideration will work. And even if the plurality is not implementable for some behavioral assumption, it is of interest to find the domain of profiles under which it is implementable. For example, Lepelley and Mbih [11] examined its vulnerability to coalitional manipulations, and Dhillon and Lockwood [7] proposes condition under which the plurality game is dominance solvable.

One may say that these good news even sound better if among the tenable behavior assumptions are two that seem to cover two extreme positions, as prudent versus risky behavior do. Nevertheless, in the last section, we stressed a different perspective: Knowing which type of behavior leads to self implementability of a SCF enhances our understanding of the SCF by telling us in which environments it may or may not be suitable. In this sense it becomes more important to consider behavioral assumptions discriminating between different SCFs. It is in this respect that the protective versus demanding behavior fares better than prudent versus risky because the former pair drives a clear cut edge between the antiplurality rule and the plurality rule, respectively, each of them in fact signaling out exactly one from the set of all scoring rules. ¿From an axiomatic point of view, it would be also interesting in further reasearch to derive original characterization of the plurality and anti-plurality rules respectively based upon BI and TI.

Even though we hope we gave a complete picture of the relationships among the different concepts of risky and prudent behavior we can model for the implementation of SCFs, we did not tackled this issues for the case of social choice correspondence, i.e. when the choice set can be multivalued. A preliminary paper [13] proves that it is easy to adapt the definitions of section 2 and 3 and the proofs of section 4 and 5 to the case of correspondences. Unfortunately, the simple transposition of the Barberà and Dutta technique to the correspondence case, although possible, leads to consequence that are not convincing in term of interpretation: the behavior of the agents implicitly defines an ordering of the subsets of alternative which does not fit with a risk loving behavior (see Merlin and Naeve [13] for details). Given these observation, the logical nest step would be to explecitly introduce ranking on subsets induced by the agent's rankings on alternative in a way that is compatible with the demanding behavior we want to model. We feel that the natural candidate would be the lexicographic maximax extension characterized by Pattanaik an Peleg [18].

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