# SENSITIVITY ANALYSIS AS A MANAGERIAL DECISION MAKING TOOL

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**SUMMARY**

Decision making is an integral part of operations management. It may be useful to a decision maker to have some indication of how sensitive an alternative choice might be to the changes in one or more of those values. Unfortunately, it is not possible to explore all the possible combinations of all the variables in a typical problem. In spite of this, there are some elements that a decision maker can use to assess the sensitivity of assumption probabilities. One of the tools useful for the analysis in some decision making problems is sensitivity analysis. It provides a range of feasibility over which the choice of alternative remains the same.

Successful decision making consists of several steps, the first and most important being carefully defining the problem. Given that linear problems can be extensive and complex, they are solved by using sophisticated computer methods. This paper will present software solutions available for personal computers (Lindo, POM). For a manager taking the decision, however, a solution model is only part of the answer. Sensitivity analysis offers a better understanding of the problem, different effects of limitations and "what if" questions. The insights obtained are frequently much more valuable that a specific numerical answer. One of the advantages of linear programming lies in the fact that it provides rich information on sensitivity analysis as a direct part of the solution.

Key words: feasibility range, linear programming, Lindo, POM, optimum solution, optimum range, sensitivity analysis.

## 1. ON SENSITIVITY ANALYSIS

Sensitivity analysis is a process of varying input parameters of a model within allowed area and observing the resulting changes in the model solution. It explores how changes in the model output can be qualitatively and quantitatively attributed to different change sources.

The purpose of sensitivity analysis is to indicate the sensitivity of simulation to uncertainties in the values of input data in the model.

Sensitivity analysis attempts to establish how the model depends on the attributed values, on its structure and on the assumptions it is based on. It is an important method for testing the quality of a given model, and it is also used for checking the reliability of the analysis.

Sensitivity analysis was originally devised to deal only with uncertainties in input variables and model parameters. Over time the analysis was expanded, insecurity was included also in the model structure, its assumptions and specifications. Generally speaking, sensitivity analysis is used to increase the reliability of a given model and its predictions by providing understanding of how model variables react to the input changes, whether they are data used for adjustments in the model structure, or factors, i.e. independent model variables.

Sensitivity analysis can be used to determine the following:
> Similarity of the model with the observed process,
> Quality of the model definition,
> Factors contributing the most to output variables,
> Areas in the input factor space for which there is maximum variation,
> Optimum area in the space of factors, which are used in subsequent exploration of adjustments,
> Interaction between factors.[1]

Using sensitivity analysis is popular in financial applications, risk analysis, neural networks, and in all the areas where models are developed. Sensitivity analysis can also be used in model-based policy assessment studies.

Sensitivity analysis can be useful in different cases: when identifying key assumptions, or when comparing alternative model structures, when initiating future data collections, establishing important criteria, optimizing resource allocation, simplifying a model, etc.

## 2. LINEAR PROGRAMMING

Linear programming methods are the most important instrument in operations research, and one of the disciplines in mathematical optimizing.

Each linear programming method consists of the following:
a) LINEAR OBJECTIVE FUNCTION – The objective of optimizing comes to the foreground here. It can be maximizing profits, but also minimizing costs. The objective is chosen depending on the problem structure. The objective function is a set that should meet the requirements of the decision maker.

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[1] Sensitivity analysis – Wikipedia, The free encyclopedia, http://en.wikipedia.org/wiki/sensitivity_analysis (20.01.2007.)

b) LINEAR RESTRICTIONS – They come from the environment of the decision maker and impose certain restrictions / conditions in terms of achieving the objective. The real limitation is the product of multiplication of time required to make one unit with the quantity. This product of multiplication must not be bigger than available capacity. The restrictions are always specified in the form of equations or inequations.

c) CONDITIONS FOR VARIABLE NON-NEGATIVITY – for example, negative quantities of products cannot be produced.

THE GENERAL MATHEMATICAL FORMULA FOR LINEAR PROGRAMMIN IS AS FOLLOWS: [2]

$$\max Z = c_1x_1 + c_2x_2 + \ldots + c_nx_n$$

$$a_1\,x_1 + a_2\,x_2 + \ldots + a_{1j}x_j + \ldots + a_{1n}x_n \le b_1$$

$$a_1\,x_1 + a_2\,x_2 + \ldots + a_{2j}x_j + \ldots + a_{2n}x_n \le b_2$$

$$\wedge \qquad \wedge \qquad \wedge \qquad \wedge \qquad \wedge$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{jn}\,x_j + \ldots + a_m\,x_n \le b_m$$

$$x_1 \ge 0, x_2 \ge 0, x_n \ge 0$$

where:

m – is the number of restrictions,

n – is the volume of particular activities,

$x_j$ – is the unknown activity j (structural variable),

$a_{ij}$ – are technical coefficients which indicate how many units of a resource (restriction) are being spent for producing a unit $x_j$,

$b_i$ – is the size of the restricting factor,

$c_j$ – is the profit per unit j.

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[2] Barković, D.: Operacijska istraživanja, Ekonomski fakultet u Osijeku, Osijek, 2002, p. 58.

### 2.1. Computer software LINDO and POM for Windows

In order to explain sensitivity analysis, we shall use here the following problem in linear programming:

$$\max Z = 300x_1 + 500x_2$$
$$x_1 + 2x_2 \leq 170$$
$$x_1 + x_2 \leq 150$$
$$3x_2 \leq 180$$

$$x_{1,2} \geq 0$$

To achieve the optimum solution we have used the widely available software for linear programming LINDO and POM for Windows.

The name LINDO is an acronym for Linear INteractive Discrete Optimization. The term "discrete" means skipping from one to another basic solution instead of circling within the permissible area seeking the optimum solution (if there is one).[3]

This software has an emphasis on operations research. It is a specialized programme which covers the topics such as linear/non-linear programming and combinatorial optimizations. It is not really intended for solving managerial problems; rather, it is more suitable as a learning tool for the methods of finding solutions in operations research, as well as for learning how to formulate an optimization problem in mathematical terms. This software could not be easily incorporated into the curriculum of the Operations Management course.

In contrast, software intended for studying operations management covers the same, if not a larger scope of material, and is generally well suited for solving the problems from one or several course books dealing with operations management.

The software with an emphasis on operations management is called POM for Windows. The name POM is the acronym for Production and Operations Management.

This programme offers elementary solutions for a wide range of operating problems. It contains several modules connected to operations research. The additional modules are assignation, linear programming, simulation, transport, etc.

Within the POM software, the user first has to select the desired module, or the window for entering the data will not open. After that, parameters are selected that will be used in the formulation, and finally, the data are entered.

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[3] Barković, D.: Operacijska istraživanja u investicijskom odlučivanju, Ekonomski fakultet u Osijeku, Osijek, 2004., str. 23.

### 3. AN EXAMPLE OF SENSITIVITY ANALYSIS

Both software programmes presented here use a simplex table. The majority of information required for sensitivity analysis is contained in the final simplex table. This and the data from the initial problem is sufficient to perform an analysis.

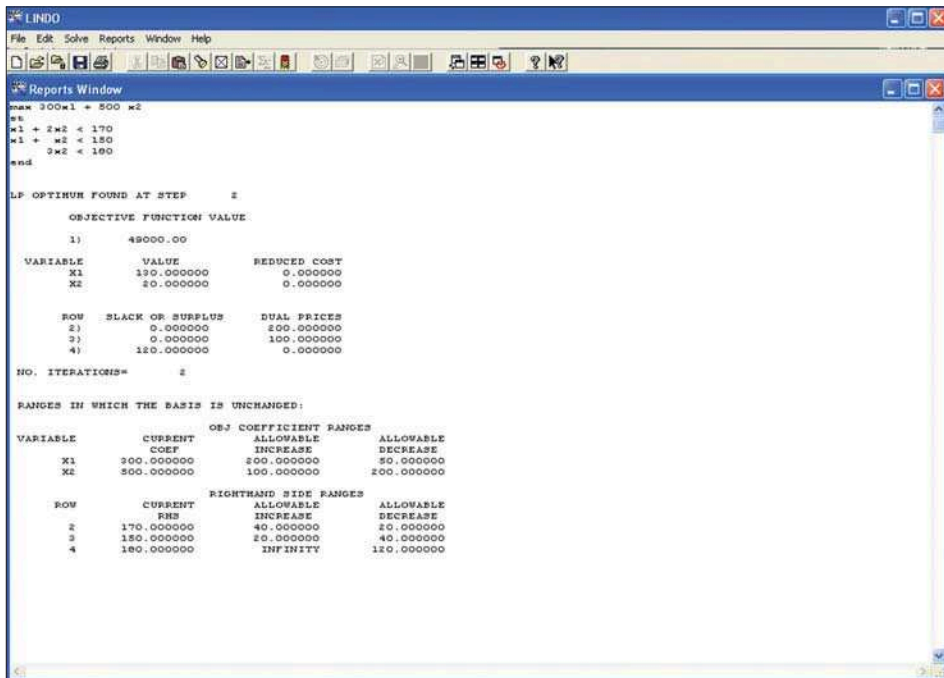Figures 1 and 2 indicate the solutions provided by those two software programmes.



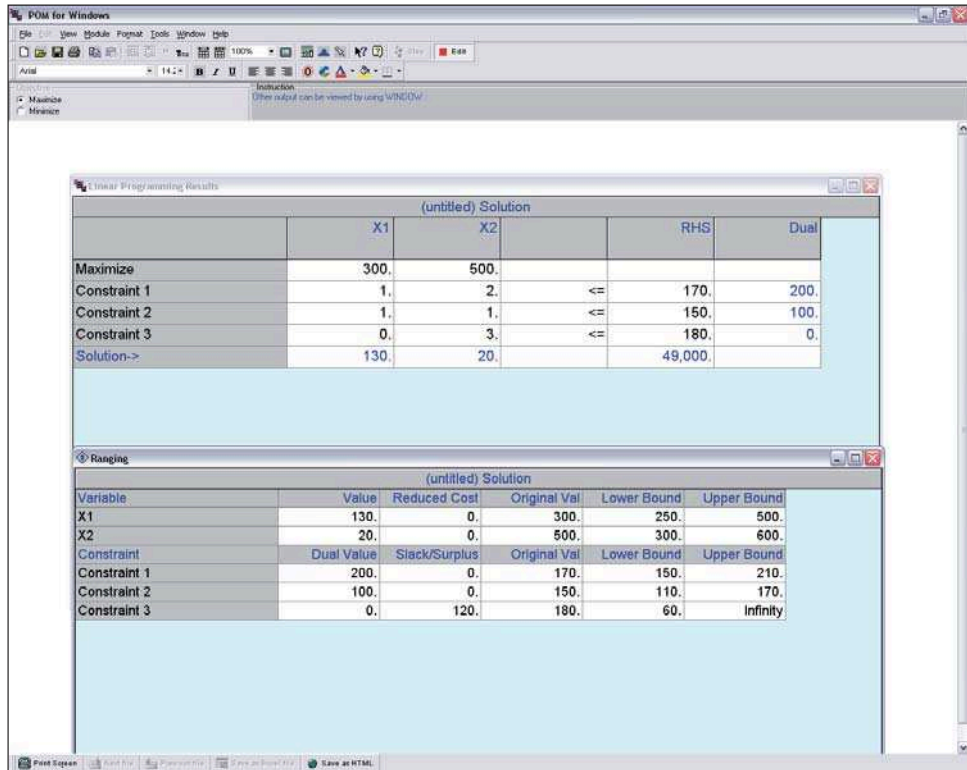**Figure 1.** SOLUTION WITH A COPIED PROBLEM LP FROM THE INITIAL WINDOW (LINDO PROGRAMME)

**Figure 2.** TWO WAYS OF SHOWING THE SOLUTION (POM for Windows)

After running the programme, the computer printout of sensitivity analysis for this particular problem shows us that the optimum solutions are:

$$x_1 = 130, \text{ and } x_2 = 20$$

at which the maximum profits of $49,000.00 are achieved.

OPTIMUM SOLUTION is that combination of products which can result in the highest profit, i.e. the lowest cost (depending on the optimization objective).

As stated above, sensitivity analysis is a means for assessing the scope and effectiveness of potential changes on parameters of LP model.

a) COEFFICIENT CHANGES IN THE LINEAR OBJECTIVE FUNCTION

OPTIMUM RANGE is the range of values over which optimum values of decision

making variables will not be changed. (upper part of the table Ranging)

We differentiate LOWER BOUND and UPPER BOUND which indicates that the optimum solution will not change if the profit coefficient is moving between the lower and upper limit.

$$x_1 \quad 250 - 500$$
$$x_2 \quad 300 - 600$$

The optimum solution remains, $x_1 = 130$, and $x_2 = 20$, however, the profit is no longer $49,000.00. If the profit per unit $x_1$ is increased from 300 to 350, the total profit is then:

$$350 \times 130 + 500 \times 20 = \$55,500.00$$

since we receive  $50.00 of additional profit per product unit $x_1$.

Let us assume that we have to report to our supervisor on the quantities of particular products our company should produce in order to maximize profits. Our superviser could pose the following problem for us: our engineers have found the way of increasing the profit for the product $x_2$ from $500.00 to $550.00, and we should think about this and propose a new product mix. We do not really need time for considering this and can answer immediately. The optimum product mix remains $x_1 = 130$, and $x_2 = 20$ because the profit of $550.00 is within optimality limits (which is observable from the LOWER – UPPER BOUND). The profit will be changed as we are now earning additional 50 dollars per product $x_2$.

The profit will increase by $50 \times 20 = \$1,000.00$.

We cannot use sensitivity analysis to evaluate the changes in several coefficients simultaneously.

b) CHANGES IN RESTRICTION VALUES

FEASIBILITY RANGE is the range of values for RHS (right-hand side) restrictions over which dual prices remain the same.

Dual values or shadow prices are defined as the amount at which the values of the objective function will be changed with the change of one unit in the value of restriction. Dual values are marginal values for restrictions in our problem. A marginal value is the amount we are prepared to pay for an additional resource unit. This is not the original sales price for a resource but the price we need to pay in order to get more of a resource. (lower part of the table Ranging)

Dual values indicate that we will not be willing to engage an additional unit of restriction 3, since we are left with 120 units already when producing the optimum product mix.

$$0 \times 130 + 3 \times 20 \leq 180$$
$$60 \leq 180$$

These are UNTIED RESTRICTIONS which have either slack or surplus[4]. An additional unit of such a restriction would result only in the increase of slack or surplus, and not in the increase of profits (or decrease of costs).

Unlike untied restrictions, TIED RESTRICTIONS force us to go beyond the optimum point. They have a certain amount for dual values (different from 0), and slack/surplus is 0. An additional unit of this restriction can enable us to increase the total profit (or decrease the total cost) by finding a new optimum solution.

Dual values 200 and 100 indicate that we will be willing to pay $200.00 for each additional unit of restriction 1, and $100.00 for each additional unit of restriction 2. We should not be prepared to pay more than $200.00 for the restriction 1, because if we get one unit of restriction 1 more, the profit will rise by only $200.00. Marginal values help in determining resource prices and making decisions on the purchase of new resources.

The purchase of additional units will change the optimum solution.

Dual values are not viable for unlimited supply of restrictions. From LOWER – UPPER BOUND we can see that the dual value for restriction 1 is valid in the range from 150 to 210.

We cannot use the feasibility range to evaluate the changes on more than one restriction simultaneously.

Let us assume that our boss says that he can acquire 30 additional units of restriction 1 for $75.00 per unit. We would immediately advise him to buy all the 30 units since the quantity to date, i.e. $170 + 30 = 200$, which is within lower and upper limit. Each additional unit within feasibility range adds $200.00 of profit. The total increase in profits will be

$$30 \times (\$200.00 - \$75.00) = \$3,750.00$$

On the other hand, if our boss were to tell us he can acquire 20 additional units of restriction 3 for only $0.50, we would advise him not to buy any additional units of restriction 3 as $180 + 20 = 200$, which is within feasibility range, and in that case the

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[4] SLACK is an ADDITIONAL VARIABLE, it is an unused resource. SURPLUS is the production surplus.

dual value is 0. We already have 120 units more than we need (slack), and if we buy additional 20 units, our surplus will rise to 140 units.

The restriction is 180, and we have:

| | | |
|---|---|---|
| 60 | => | SLACK |
| 220 | => | SURPLUS |

## 4. CONCLUSION

Decision makers are often concerned with more things than just a solution to a problem. He or she could benefit from the results of sensitivity analysis in several ways. While preparing the problem for processing by means of linear programming, a decision maker could use subjective assumptions for particular parameters (e.g. restriction coefficients, objective function coefficients and values of right-hand side restrictions). It is understandable that a decision maker wishes to know how sensitive the optimum solution is to the changes in values of one or more parameters. If the solution is relatively insensitive to reasonable changes, the decision maker can have more confidence in the application of his solution. In contrast, if the solution is sensitive to changes, he or she will want to determine much more precise assumptions on the basis of parameters in question.

The problem related to sensitivity analysis includes specific changes in parameters. For example, there can be a change in price which can alter the objective function coefficient. Furthermore, a change in the production process can require a change in the coefficient of one of the restrictions. Sensitivity analysis can provide an adequate insight into the problem for the decision maker. The third possibility is the question whether the purchase of high quantities of a resource would be beneficial. This question often arises when large quantities of resources are not used in the optimum solution; by adding more of other resources it is possible to achieve a disproportionate increase in the objective function value.

There are obviously numerous reasons why a decision maker may want to use sensitivity analysis.

## 5. REFERENCES

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