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TAX INCIDENCE IN A MODEL WITH PERFECT FORESIGHT OF AGENTS AND RATIONING IN MARKETS

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This paper examines the incidence of several taxes in a macroeconomic model. Producers and consumers optimize with perfect foresight. Price inertia leads to rationing in the market for goods and for labour. In the long run the system tends towards Walrasian equilibrium. Meanwhile there may be Keynesian Unemployment, Classical Unemployment or Repressed Inflation, with possible switches of regimes. Balanced budget policies are analysed by working through numerical examples.

1. Introduction

Macroeconomic models are complex if a number of aspects are studied simultaneously. In general such models can only be solved numerically. The present paper aims at an analysis of tax incidence in a rather complicated model. Our approach builds on a seminal article by Blanchard and Sachs (1982), which integrates modern disequilibrium analysis with the theory of intertemporal choice under perfect foresight of economic agents.

Ignoring minor differences the BS model is extended in two directions. In the first place account is taken of labour scarcity, which allows for the regime of 'Repressed Inflation', following the terminology of Malinvaud (1977). This regime was eliminated for the benefit of simplicity in the BS model by assuming that firms can always get the amount of labour they want. In the experiments considered by Blanchard and Sachs, 'Repressed Inflation' does not matter. However, it can be shown easily that the latter regime may be of importance if the shocks analysed by Blanchard and Sachs are reversed. In fact, the behaviour of the model is strongly asymmetrical.

In the second place we consider labour supply as an endogenous variable. Households are supposed to determine the amount of leisure they want at any moment. As observed among others by Barro (1984), there seems to be no substantial structural change in the average number of hours worked in some industrialized countries. This can be explained by substitution and

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wealth effects cancelling each other. In an analysis of tax incidence, substitution effects come to the foreground if wealth effects are eliminated by compensating measures. It may therefore be worthwhile to assume that the supply of labour is endogenous.

As stressed by Atkinson and Stiglitz (1980) in analysing the impact of taxation, the basis for comparison is of critical importance, and the problem can be posed in several ways. Here we follow Abel and Blanchard (1983) and again Blanchard and Sachs (1982) by assuming balanced budget operations with lump-sum rebates or taxes as compensating instruments. It should be remarked at the outset that we are not concerned with the problem of optimal taxation over time as studied by Barro (1979) and Kremers (1984). It is obvious that in the case when the government wants to maximize the utility of its citizens, all taxes should be lump-sum. In practice there exists a number of other instruments such as a tax on profits, a sales tax, a wage tax or investment tax credits. We shall analyse the incidence of these taxes applying lump-sum payments as an analytical instrument only.

The paper is organized as follows. In section 2 the model used in the simulations is derived by analysing the intertemporal behaviour of rational producers and consumers. We mainly highlight the differences with the BS model. A more elaborate presentation can be found in Van de Klundert and Peters (1984). There are subsections on the behaviour of firms, the behaviour of households and market clearing. The incidence of the taxes mentioned above is discussed in section 3. The numerical results are obtained by the method of multiple shooting, as explained in Lipton, Poterba, Sachs and Summers (1982) and Mattheij (1982). The model specification and parameter values applied are presented in an appendix.

The long-run equilibrium in our model is Walrasian, as will be shown. This makes it possible to compare our long-run results with the outcomes of flex-price models, as for instance those of Abel and Blanchard (1983) and Barro (1984).

2. The model

2.1. The behaviour of firms

There are two factors of production, capital k and labour l . The technology of the representative firm is characterized by a neoclassical production function with constant returns of scale:

$$y = f(k, l). \tag{2.1.1}$$

Installation costs are introduced to derive a well-behaved investment function along the lines set out in Abel (1978) and Hayashi (1982). The method is

standard by now. To invest an amount of i goods, installation costs in the amount of $ih(i/k)$ are required ($h' > 0$). The opportunity costs of investment are therefore equal to

$$j = i(1 + h(i/k)). \quad (2.1.2)$$

Denote by τ_z, τ_y and τ_j a proportional tax rate on profits, a proportional tax rate on output and a proportional rate of an investment tax credit. The present value of the cash flow of firms can then be defined as

$$V_0 = \int_0^{\infty} \left\{ (1 - \tau_z) \left[(1 - \tau_y)y - \frac{W}{P}l \right] - (1 - \tau_j)j \right\} \rho_t dt, \quad (2.1.3)$$

where W stands for the wage rate and P for the price of output. The discount factor $\rho \equiv \exp(-\int_0^t r_s ds)$ gives the rate at which output at time t can be traded for output at time zero. Firms take the time paths of wages, prices and interest rates as given. In addition, there is the possibility that the amount of goods they can sell is restricted: $y \leq \bar{y}$, or that the amount of investment goods they can buy has an upper bound: $i \leq \bar{i}$ (and $j \leq \bar{j}$). If firms are not rationed in the output market, firms may be confronted with a constraint on the amount of labour they can hire: $l \leq \bar{l}$.¹ It should be noted that the values of the endogenous variables at which the constraints become binding are themselves endogenous to the model, though from the point of view of individual agents they are predetermined.

The decision problem of the representative firm is to choose a time path of investment and employment that maximizes V_0 subject to the inequalities mentioned above and the accumulation equation:

$$\dot{k} = i - \delta k, \quad (2.1.4)$$

with δ symbolizing the rate of exponential depreciation. As usual dotted variables indicate time derivatives.

Applying the standard solution technique of the maximum principle the following necessary conditions for an optimum are obtained:²

$$[(1 - \tau_y)(1 - \tau_z) - \lambda_y] f_l(k, l) = (1 - \tau_z) \frac{W}{P} + \lambda_l, \quad (2.1.5)$$

$$(1 - \tau_j) \left[1 + h\left(\frac{i}{k}\right) + \frac{i}{k} h'\left(\frac{i}{k}\right) \right] = q - \lambda_i, \quad (2.1.6)$$

¹As already observed by Malinvaud (1977), firms cannot be simultaneously rationed in the goods market and the labour market unless there is a discrepancy between output and sales and stockpiling is possible.

²The transversality condition is as usual: $\lim_{t \rightarrow \infty} \rho_t q_t k_t = 0$.

$$\dot{q} = (r + \delta)q - [(1 - \tau_y)(1 - \tau_z) - \lambda_y]f_k(k, l) - (1 - \tau_j) \left(\frac{i}{k}\right)^2 h\left(\frac{i}{k}\right), \quad (2.1.7)$$

together with the Kuhn–Tucker conditions:

$$\begin{aligned} \lambda_y(\bar{y} - y) &= 0, & \lambda_y &\geq 0, \\ \lambda_i(\bar{i} - i) &= 0, & \lambda_i &\geq 0, \\ \lambda_l(\bar{l} - l) &= 0, & \lambda_l &\geq 0. \end{aligned} \quad (2.1.8)$$

The symbol q stands for the costate variable associated with the accumulation equation. It can be interpreted as the shadow price of capital. As shown by Hayashi (1982), in the special case of no market constraints the shadow price is equal to Tobin's q . The symbols λ_y, λ_i and λ_l are familiar Lagrange multipliers associated with the inequality constraints.

Under *Keynesian Unemployment* ($\lambda_y > 0, \lambda_i = 0, \lambda_l = 0$) firms are rationed in the market for goods. The constraint on y is binding and output equals

$$y = \bar{y}. \quad (2.1.9)$$

The corresponding volume of employment (l_k) then follows from

$$\bar{y} = f(k, l_k). \quad (2.1.10)$$

Under *Classical Unemployment* ($\lambda_y = 0, \lambda_i > 0, \lambda_l = 0$) employment (l_d) follows from

$$(1 - \tau_y)f_l(k, l_d) = \frac{W}{P}. \quad (2.1.11)$$

The corresponding volume of output is

$$y_s = f(k, l_d). \quad (2.1.12)$$

In case of *Repressed Inflation* ($\lambda_y = 0, \lambda_i > 0, \lambda_l > 0$) the constraint on labour supply is binding and employment is equal to

$$l = \bar{l}. \quad (2.1.13)$$

As a result, output is then constrained to

$$y_i = f(k, \bar{l}). \quad (2.1.14)$$

Investment demand follows from eqs. (2.1.6) and (2.1.7). Under Classical Unemployment and Repressed Inflation investment demand determined in this way is *notional* demand. In both cases all buyers are rationed in the goods market, which implies that *actual* investment equals the constraint: $i = \bar{i}$ (and $j = \bar{j}$). Notional investment demand depends on profitability, as can be shown by integrating eq. (2.1.7) for $\lambda_y = 0$ subject to the transversality condition given in footnote 2:

$$q_t = \int_i^{\infty} \left[(1 - \tau_y)(1 - \tau_z)f_k(k, l) + (1 - \tau_j) \left(\frac{i}{k}\right)^2 h' \left(\frac{i}{k}\right) \rho_v \right] dv. \quad (2.1.15)$$

Investment demand depends on q , which is equal to the present value of marginal profits. The second term on the RHS of eq. (2.1.15) is of minor importance since it shows the reduction in installation costs from an increase of capital with one unit. The first term on the RHS represents the after-tax marginal product of capital in the usual sense. If $\lambda_y > 0$, we must have $\lambda_l = 0$. In that case the first term on the RHS of (2.1.15) changes into

$$(1 - \tau_z) \frac{W f_k(k, l)}{P f_l(k, l)}.$$

Under Keynesian Unemployment marginal profits depend on the possibility of minimizing production costs at a given level of output. For instance, if real wages rise entrepreneurs may reduce costs by increasing the capital intensity of production [cf. Blanchard (1983), Blanchard and Sachs (1982), Malgrange and Villa (1984)].

2.2. The behaviour of households

The welfare of households depends on consumption of goods (c) and of leisure and on the amount of real cash balances (M/P) held. Denoting the maximum available time by l_m we can write the instantaneous utility function as

$$u = u \left(c, (l_m - l), \frac{M}{P} \right). \quad (2.2.1)$$

It will be assumed subsequently that leisure is (weakly) separable from goods and real cash balances.³

Households take as given the time paths of prices, wage rates, real interest rates and dividends paid by firms. There may be a constraint on the amount

³See Deaton and Muellbauer (1980) for a discussion of the consequences of separability.

of goods households can buy: $c \leq \bar{c}$, or the amount of labour actually supplied may have an upper bound: $l \leq \bar{l}$. It should be recalled that the values of endogenous variables at which constraints become binding are endogenous to the model. The value of the constraint on labour supply is indicated by a double bar over the variable l to mark the difference with the constraint on labour demand introduced in section 2.1.

Given a constant (utility) rate of time preference (ν) the decision problem of the representative household is to choose a time path of consumption, leisure and real cash balances that maximizes the present value of utility:

$$U = \int_0^{\infty} u\left(c, (l_m - l), \frac{M}{P}\right) e^{-\nu t} dt, \quad (2.2.2)$$

subject to the inequality constraints mentioned above and the dynamic budget constraint:

$$\dot{A} = rA + \pi + (1 - \tau_l)l \frac{W}{P} - \left(r + \frac{\dot{P}}{P}\right) \frac{M}{P} - c - \frac{T}{P}, \quad (2.2.3)$$

where A stands for the sum of interest bearing bonds issued by firms and non-interest bearing real cash balances, and π indicates dividends paid by firms to households. The symbol τ_l represents a proportional tax rate on labour income, whereas T stands for the amount of lump-sum taxation.

The first three terms on the RHS of eq. (2.2.3) give the income flows, i.e. real interest payments received by households, real dividends and after-tax real wages. The last three terms relate to expenditures, i.e. the opportunity cost of holding money, real consumption expenditures and real lump-sum taxes paid by households.

It does not matter how firms are financed as long as the conditions of the Modigliani–Miller theorem hold. Blanchard and Sachs (1982) assume that all investment is financed from retained earnings, but that firms have outstanding (real) debt. Another possibility is that only replacement investment is financed out of retained earnings and net investment by issuing bonds [cf. Abel and Blanchard (1983)]. In all possible cases the resulting dividend is paid out to households.

Again the standard solution technique can be applied to obtain as necessary conditions for an optimum:⁴

$$u_c\left(c, \frac{M}{P}\right) = x + \lambda_c, \quad (2.2.4)$$

⁴The transversality condition can, in this case, be written as: $\lim_{t \rightarrow \infty} e^{-\nu t} x_t A_t = 0$ [cf. d'Autume (1982)].

$$u_{(l_m - l)}(l) = (1 - \tau_l) \frac{W}{P} x - \lambda_h, \quad (2.2.5)$$

$$u_m \left(c, \frac{M}{P} \right) = \left(r + \frac{\dot{P}}{P} \right) x, \quad (2.2.6)$$

$$\dot{x} = (v - r)x, \quad (2.2.7)$$

together with the Kuhn–Tucker conditions:

$$\begin{aligned} \lambda_c (\bar{c} - c) &= 0, \quad \lambda_c \geq 0, \\ \lambda_h (\bar{l} - l) &= 0, \quad \lambda_h \geq 0. \end{aligned} \quad (2.2.7)$$

The symbol x stands for the costate variable associated with the dynamic budget constraint. The symbols λ_c and λ_h are Lagrange multipliers associated with the inequality constraints. The LHS of eqs. (2.2.4), (2.2.5) and (2.2.6) relate respectively to the marginal utility of consumption, the marginal utility of leisure and the marginal utility of holding real cash balances. The specifications take account of the assumed separability with regard to leisure.

Under *Keynesian Unemployment* ($\lambda_c = 0, \lambda_h > 0$) households are rationed in the labour market. The constraint on labour supply is binding and actual employment is given by

$$l = \bar{l}. \quad (2.2.8)$$

Demand for consumption goods (c_d) then follows from

$$u_c \left(c_d, \frac{M}{P} \right) = x. \quad (2.2.9)$$

As observed by Deaton and Muellbauer (1980), if a consumer has no choice over hours worked and if goods are weakly separable from leisure the spending of income or wealth is explicable without reference to the number of hours actually worked.

Under *Classical Unemployment* ($\lambda_c > 0, \lambda_h > 0$) households are rationed in the market for goods and in the labour market. In this case both constraints are binding:

$$c = \bar{c}, \quad l = \bar{l}. \quad (2.2.10)$$

In the case of *Repressed Inflation* ($\lambda_c > 0, \lambda_h = 0$) households are only rationed in the goods market:

$$c = \bar{c}. \quad (2.2.11)$$

The supply of labour (l_s) can be derived from

$$u_{(l_m-l)}(l_s) = (1 - \tau_l) \frac{W}{P} x. \quad (2.2.12)$$

Eq. (2.2.12) can also be used to determine *notional* supply of labour in the case when households are rationed with regard to hours worked. In the case of rationing in the goods market, *notional* consumption demand follows from eq. (2.2.9). Demand for real cash balances is governed by eq. (2.2.6). If consumers are rationed they need, *ceteris paribus*, less money than in the case of unconstrained demand.

In Walrasian equilibrium the marginal rate of substitution between real money balances and consumption equals the nominal rate of interest ($r + \dot{P}/P$), whereas the marginal rate of substitution between leisure and consumption equals the after-tax real wage rate ($(1 - \tau_l)W/P$).

2.3. Market clearing

In the preceding subsections we analysed the behaviour of the representative firm and the representative household. We now tie together the different pieces by considering market clearing and price formation.

Wages and prices are fixed in the short run. Consequently, at each moment the short side of the market determines actual output and actual employment. The constraints on demand and supply introduced in subsections 2.1 and 2.2 ought to be explained in this way. If sellers (firms) are constrained in the market for goods, total demand ($y_d = c_d + j_d$) falls short of total supply (y_s). On the other hand, if buyers (households and firms) are constrained in the goods market total supply (y_s or y_i) falls short of total demand (y_d).⁵ In the latter case there are two possibilities. Firms may be constrained in the labour market ($y_l < y_s$) or there may be no binding constraint on the demand for labour ($y_s < y_l$). Market clearing with regard to goods can now be summarized by the following equation for actual output (y):

⁵These rationing rules imply that investment demand is constrained if and only if consumption demand is constrained. As an alternative one could assume that investment is allowed to be carried out by firms as in Neary and Stiglitz (1983). In this case there would also be a regime of Underconsumption in the terminology of Muellbauer and Portes (1978).

$$y = \min [y_d, y_s, y_l]. \tag{2.3.1}$$

The corresponding equation for clearing the labour market is

$$l = \min [l_k, l_d, l_s]. \tag{2.3.2}$$

Ignoring borderline cases eqs. (2.3.1) and (2.3.2) give rise to six possible short-run equilibria, which are presented in table 1.

Table 1
Possible regimes.

Keynesian Unemployment	Classical Unemployment	Repressed Inflation
(1) $y_d < y_s < y_l$ $l_k < l_d < l_s$	(3) $y_s < y_d < y_l$ $l_d < l_k < l_s$	(5) $y_l < y_d < y_s$ $l_s < l_k < l_d$
(2) $y_d < y_l < y_s$ $l_k < l_s < l_d$	(4) $y_s < y_l < y_d$ $l_d < l_s < l_k$	(6) $y_l < y_s < y_d$ $l_s < l_d < l_k$
$y = y_d$ $l = l_k$	$y = y_s$ $l = l_d$	$y = y_l$ $l = l_s$

Prices and wages react to excess demand (supply). However, as observed by Malinvaud (1980), the definition of what is meant by excess demand or excess supply is not unambiguous in the theory of fix-price equilibria. Honkapohja (1979) relates changes in nominal prices and wages to the difference between the ex-post quantity and the unsatisfied demand and supply. This suggestion is captured by

$$\frac{\dot{P}}{P} = \beta_p (y_d - \min [y_s, y_l]) \tag{2.3.3}$$

and

$$\frac{\dot{W}}{W} = \beta_w (\min [l_d, l_k] - l_s), \tag{2.3.4}$$

where β_p and β_w are parameters. For a long-run stationary equilibrium we must have $\dot{P} = \dot{W} = 0$. Combining eqs. (2.3.3) and (2.3.4) with the possibilities presented in table 1, it appears that the stationary equilibrium will be Walrasian ($y = y_d = y_s = y_l$ and $l = l_k = l_d = l_s$) in cases (1), (3), (4) and (6). In cases (2) and (5) the long-run solution is characterized by $y = y_d = y_l < y_s$ and $l = l_k = l_s < l_d$.

The long-run solution for cases (2) and (5) is illustrated in fig. 1. The upper part shows the production function for a given capital stock. In the lower part of the figure notional labour demand and supply are shown. The long-run equilibrium points are indicated by A ($y = y_d = y_l$) [resp. A ($l = l_k = l_s$)]. The resulting real wage rate $(W/P)^*$ is not a market clearing rate. There is still a possibility of mutual advantageous trade. In the present model it seems natural to assume that agents will exploit this potential. For that reason eq. (2.3.4) will be changed into

$$\frac{\dot{W}}{W} = \beta_w(l_d - l_s). \tag{2.3.5}$$

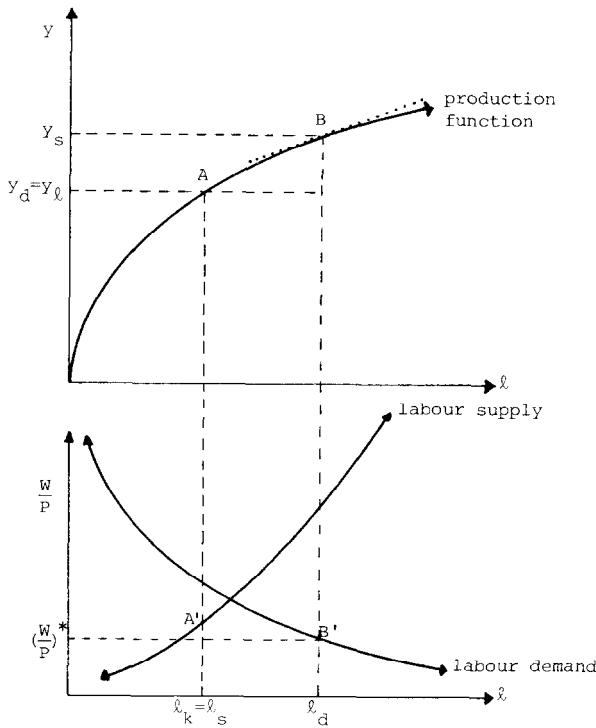


Fig. 1

Eqs. (2.3.3) and (2.3.5) taken together guarantee that in the long run the economy exhibits a Walrasian equilibrium. In the short run, it may be possible that the nominal wage rate rises despite (Keynesian) unemployment. This case is illustrated in subsection 3.3 below. In such a situation firms know that unemployment will not last for ever and that they have to compete for labour now in order to exploit profitable opportunities later on.

In this sense wage formation is linked to perfect foresight. The assumption that wages and prices are not fully flexible is maintained, because there are a number of good reasons for price inertia, as discussed for instance in Nadiri (1983).

3. Tax incidence

The incidence of several tax instruments will be analysed assuming equal yields in the final, Walrasian steady state. Tariffs in subsections 3.1–3.4 are based on a yield of 5 percent in terms of initial GNP. In these cases the proceeds of the tax will be redistributed back to individuals by means of lump-sum payments. In subsection 3.5 changes between taxes other than lump-sum will be briefly considered.

3.1. A sales tax

The effect of a 5.3 percent balanced budget tax increase on sales ($\tau_y = 0.52891$ and $\tau_{y,y} = T/P$) is presented in table 2. Variables are measured as percentage deviations from the initial steady state path. The last column relates to the new steady state (SS).

If the savings behaviour in the economy can be described in terms of an infinitely-lived representative household, then in the *long run* the after-tax rate of return on capital does not depend on the tax rate [cf. Atkinson and Stiglitz (1980), Abel and Blanchard (1983)]. The relevant equation for the long run is:

$$\frac{(1-\tau_z)(1-\tau_y)}{(1-\tau_j)} f_k(k, l) = (v + \delta)(1 + h(\delta) + \delta h'(\delta)) - \delta^2 h'(\delta). \quad (3.1.1)$$

To restore the after-tax rate of return the capital stock must fall, but the new long-run equilibrium of capital cannot be derived from eq. (3.1.1). In our model the supply of labour is endogenous. Therefore the long-run results depend on the amount of leisure households take. As appears from table 2, leisure increases. This is the result of opposing forces. The decline of the capital stock induces a negative (wealth) effect. The fall in the real wage leads to a higher demand for leisure. As shown by Atkinson and Stiglitz (1980), the labour supply curve slopes upward if the utility function is logarithmic (as assumed in the appendix) and households have positive non-labour income besides their wage income. On balance the effect of the decline in real wages dominates and labour supply diminishes.

The fall in real wages can be explained by a decline in the demand for labour. A lower level of the capital stock leads to less employment. In order to restore full employment the real wage rate must decline substantially. As a consequence the sales tax is shifted from capital to labour.

Table 2
A sales tax.

t	0	1	5	10	25	50	SS
c_d	1.35	2.21	1.60	-0.51	-3.50	-4.48	-4.60
i_d	-16.60	-11.89	-7.22	-7.14	-7.74	-7.97	-8.00
j_d	-20.56	-14.58	-8.13	-7.60	-7.85	-7.98	-8.00
y_d	-4.21	-2.06	-0.87	-2.31	-4.61	-5.37	-5.46
y_s	-1.99	-2.74	-3.88	-4.45	-5.19	-5.43	-5.46
y_i	-0.17	-1.17	-3.04	-3.99	-5.08	-5.42	-5.46
l_k	-9.50	-3.06	3.75	2.49	-0.54	-1.58	-1.71
l_d	-4.62	-4.64	-3.78	-2.95	-2.03	-1.75	-1.71
l_s	-0.40	-0.95	-1.73	-1.81	-1.75	-1.72	-1.71
c	1.35	1.38	-2.03	-3.09	-4.20	-4.56	-4.60
i	-16.60	-12.11	-8.16	-7.80	-7.92	-7.99	-8.00
j	-20.56	-14.85	-9.31	-8.44	-8.08	-8.01	-8.00
y	-4.21	-2.74	-3.88	-4.45	-5.19	-5.43	-5.46
l	-9.50	-4.64	-3.78	-2.95	-2.03	-1.75	-1.71
r	0.01	0.01	-0.01	-0.00	-0.00	-0.00	0.00
q	-7.38	-4.76	-1.51	-0.77	-0.19	-0.02	0.00
x	-2.02	-3.25	-2.91	-0.41	3.37	4.66	4.82
k	0.00	-1.33	-3.96	-5.50	-7.34	-7.93	-8.00
P	0.00	-0.06	0.95	2.27	4.13	4.74	4.82
W	0.00	-1.55	-4.60	-6.10	-7.63	-8.09	-8.15
U	-0.40	-2.70	-7.67	-10.49	-13.95	-15.07	-14.77
Regime	K.U.	Cl.U.	Cl.U.	Cl.U.	Cl.U.	Cl.U.	

In our numerical example the substitution effect dominates the wealth effect with regard to leisure in the long run. Therefore, the supply of labour decreases. This result reinforces the downward movement of the capital stock.

In the *short run* ($t=0$) notional investment demand decreases substantially. The sequence of marginal profits declines as a result of the tax. This is reflected in the downward jump of the non-predetermined variable q . Despite a loss of wealth in the long run anticipated by households, notional consumption demand increases. This can be explained by the fact that households anticipate also the rationing in the market for goods, which already appears at $t=1$. As shown by d'Autume and Michel (1985), there will be no anticipatory buying of investment goods. On the contrary, investors will buy less today if they anticipate future constraints on the quantity of goods they can buy.

The fall in investment demand leads to a situation of Keynesian Unemployment (K.U.) at $t=0$. It should be remarked that a tax on sales has an important impact on after-tax real labour costs, which induces substitution of labour for capital. As a result notional labour demand (l_d) and notional supply of goods (y_s) decrease from the very beginning. As soon as notional

demand for goods (y_d) recovers there results a situation of excess demand. In fact from $t=1$ until the new long-run equilibrium is approached asymptotically the regime of Classical Unemployment (Cl.U.) prevails.

It can be concluded that in case of short-run price rigidity a sales tax is shifted to wages in the long run, but that the adjustment process is characterized by (mainly classical) unemployment. In the long run, lifetime utility of the representative agent (U) decreases by 14.8 percent.

3.2. A profit tax

The results of a balanced-budget tax on profits of about 9 percent ($\tau_z = 0.089241$ and $\tau_z[y - l(W/P)] = T/P$) are presented in table 3. As appears from formula (3.1.1), the *long-run* effect on the after-tax rate of return is larger than in the case of a sales tax. As a consequence the capital-labour ratio decreases more in the present case of tax on profits. The change in the long-run supply of labour is not much different, as a comparison of tables 2 and 3 shows. An equal yield tax on profits therefore leads to a lower level of capital in the long run.

The decline in labour supply is again the result of opposing forces. The accumulation effect with regard to leisure is positive, whereas the real-wage effect has a negative sign. Both effects are now stronger, but the real-wage effect still dominates.

The adjustment process shows a picture similar to that of table 2. If a profit tax is imposed, the regime of Keynesian Unemployment lasts somewhat longer. But as capital decumulates a switch towards the regime of Classical Unemployment becomes unavoidable, despite a decline of real wages. As a matter of fact the switch is realized at $t=2$.

It should be observed that Keynesian Unemployment is caused by a substantial fall in notional investment demand. In contrast with this, notional consumption demand is even higher than in the case of a sales tax. Anticipatory buying of consumption goods is higher because future rationing is more stringent. The notional supply of goods decreases strongly as the stock of capital falls towards its new long-run equilibrium value.

The explanation of results is complicated not only because of the usual circular causation problems but also because of 'bootstraps phenomena', as they are called by Neary and Stiglitz (1983). The bootstraps phenomenon implies that the prevalence of a certain regime today is more likely if it is expected to prevail tomorrow. The switch towards Classical Unemployment at $t=2$ can be explained, among other things, by the fact that agents expect this regime to prevail in the future.

The nominal wage rate decreases more in the case of a sales tax. The reason is that in our specification of the model the nominal wage rate changes under the influence of notional excess demand (supply) instead of

Table 3
A profit tax.

<i>t</i>	0	1	5	10	25	50	SS
c_d	1.40	2.74	3.21	0.06	-4.68	-6.28	-6.50
i_d	-29.19	-19.44	-10.57	-10.61	-11.69	-12.08	-12.14
j_d	-35.09	-23.49	-11.79	-11.26	-11.87	-12.10	-12.14
y_d	-7.87	-3.93	-0.60	-2.82	-6.50	-7.76	-7.94
y_s	0.00	-2.18	-5.07	-6.20	-7.46	-7.88	-7.94
y_l	-0.17	-1.66	-4.27	-5.59	-7.28	-7.86	-7.94
l_k	-16.93	-6.15	8.12	5.93	0.68	-1.15	-1.40
l_d	0.00	-2.07	-3.65	-3.16	-1.92	-1.46	-1.40
l_s	-0.41	-0.81	-1.67	-1.60	-1.45	-1.41	-1.40
c	1.40	2.74	-2.19	-4.02	-5.83	-6.42	-6.50
i	-29.19	-19.44	-11.97	-11.66	-11.99	-12.12	-12.14
j	-35.09	-23.49	-13.55	-12.59	-12.24	-12.15	-12.14
y	-7.87	-3.93	-5.07	-6.20	-7.46	-7.88	-7.94
l	-16.93	-6.15	-3.65	-3.16	-1.92	-1.46	-1.40
r	0.02	0.02	-0.01	-0.01	-0.00	-0.00	0.00
q	-12.97	-7.81	-2.13	-1.13	-0.31	-0.04	0.00
x	-2.09	-3.83	-5.17	-1.55	4.47	6.65	6.96
k	0.00	-2.26	-6.07	-8.28	-11.08	-12.01	-12.14
P	0.00	-0.45	0.69	2.74	5.76	6.81	6.96
W	0.00	-0.67	-2.25	-3.61	-5.85	-6.65	-6.75
U	-0.90	-4.24	-11.85	-16.11	-21.67	-23.58	-23.03
Regime	K.U.	K.U.	Cl.U.	Cl.U.	Cl.U.	Cl.U.	

actual unemployment. However, the real wage rate declines more in case of a profit tax, because labour becomes relatively more abundant compared with the example of a sales tax.

A profit tax with the same yield as a sales tax puts a heavier burden upon the economy. This is reflected in the long-run value of the lifetime utility of the representative individual, which falls by 23 percent.

3.3. An investment tax

A tax on investment expenditure with equal yield has rather dramatic effects, as shown in table 4. Such a tax can be conceived as an investment tax credit with the sign reversed. In order to realize the same yield, the tariff has to be rather high ($\tau_j = -0.28655$ and $\tau_{jj} = T/P$).

There is no need to discuss the outcomes at length. The results are qualitatively equivalent to those in the case of a tax on profits. The only difference is that all figures are blown up. Again the drain on investment results in Keynesian Unemployment at the beginning, but the regime of Classical Unemployment takes over at $t=2$. Ultimately the investment tax is shifted to labour and real wages decline substantially.

Table 4
An investment tax.

<i>t</i>	0	1	5	10	25	50	SS
<i>c_d</i>	3.69	7.18	8.81	0.44	-12.91	-18.05	-18.97
<i>i_d</i>	-67.04	-49.06	-27.07	-26.69	-29.69	-31.07	-31.33
<i>j_d</i>	-73.35	-55.78	-29.92	-28.14	-30.07	-31.12	-31.33
<i>y_d</i>	-15.88	-8.82	-1.03	-6.83	-17.27	-21.37	-22.11
<i>y_s</i>	0.00	-5.30	-13.35	-16.69	-20.43	-21.85	-22.11
<i>y_l</i>	-0.47	-4.13	-11.39	-15.11	-19.95	-21.78	-22.11
<i>l_k</i>	-31.06	-13.10	29.76	24.82	5.22	-2.85	-4.31
<i>l_d</i>	0.00	-5.09	-10.18	-9.44	-6.10	-4.58	-4.31
<i>l_s</i>	-1.12	-2.23	-4.97	-4.86	-4.50	-4.34	-4.31
<i>c</i>	3.69	7.18	-6.06	-11.47	-16.72	-18.63	-18.97
<i>i</i>	-67.04	-49.06	-31.13	-29.84	-30.67	-31.21	-31.33
<i>j</i>	-73.35	-55.78	-34.77	-32.02	-31.31	-31.31	-31.33
<i>y</i>	-15.88	-8.82	-13.35	-16.69	-20.43	-21.85	-22.11
<i>l</i>	-31.06	-13.10	-10.18	-9.44	-6.10	-4.58	-4.31
<i>r</i>	0.04	0.06	-0.02	-0.02	-0.01	-0.00	0.00
<i>q</i>	-9.68	2.28	20.83	24.70	27.57	28.50	28.65
<i>x</i>	-5.48	-9.96	-14.52	-5.60	12.96	21.72	23.40
<i>k</i>	0.00	-5.46	-15.51	-21.24	-28.33	-30.88	-31.33
<i>P</i>	0.00	-0.93	2.28	8.29	18.33	22.61	23.40
<i>W</i>	0.00	-1.38	-4.82	-8.35	-15.03	-17.75	-18.23
<i>U</i>	-10.44	-17.09	-37.71	-50.34	-67.59	-74.51	-72.69
Regime	K.U.	K.U.	Cl.U.	Cl.U.	Cl.U.	Cl.U.	

There are two points that deserve further comment. First, one would perhaps expect that long-run labour supply would be in line with the results in tables 2 and 3. Instead, there is a greater decrease in the case of an investment tax. This is caused by a more than proportional decline in the real wage rate compared with the decline in real wealth. The fall in real wages depends upon the value of the elasticity of factor substitution, which amounts to 0.5 in our simulation runs. Second, the long-run value of the shadow price q increases by 28.65 percent, which is exactly equal to the tariff imposed. This result follows immediately from eq. (2.1.6). For values of q lower than the long-run solution investment is depressed.

It should be stressed that adjustment paths are not symmetrical if the impulse is reversed. In the perhaps more relevant case of an investment tax credit the regime of Repressed Inflation prevails for some time, because investment is boosted by the tax credit. Afterwards there is a switch to Keynesian Unemployment as capital accumulation induces excess supply of goods.

3.4. A wage tax

The effect of a compensated wage tax with equal yield [$\tau_l=0.121953$ and $\tau_l l(W/P)=T/P$] is presented in table 5. Households are compensated for the loss of income by a lump-sum rebate. As appears from the *long-run* results, there is no possibility of shifting the burden of the tax. The demand for labour being infinitely elastic in the long run the full impact is on disposable real wages. The decline in after-tax real wages induces a fall in labour supply. Since the rate of return on capital does not change, the stock of capital must decline at the same rate as labour input. Consequently, all volume variables decrease by the same percentage.

Table 5
A wage tax.

t	0	1	5	10	25	50	SS
c_d	1.56	1.15	0.24	-0.59	-1.79	-2.16	-2.20
i_d	-0.85	-1.20	-1.53	-1.79	-2.11	-2.19	-2.20
j_d	-1.09	-1.49	-1.76	-1.96	-2.16	-2.19	-2.20
y_d	0.89	0.48	-0.27	-0.94	-1.88	-2.17	-2.20
y_s	0.00	-0.50	-1.43	-1.79	-2.10	-2.19	-2.20
y_l	-1.37	-1.32	-1.44	-1.66	-2.06	-2.18	-2.20
l_k	2.14	1.38	0.37	-0.55	-1.79	-2.16	-2.20
l_d	0.00	-0.97	-2.40	-2.57	-2.32	-2.22	-2.20
l_s	-3.21	-2.88	-2.42	-2.28	-2.21	-2.20	-2.20
c	-1.16	-1.02	-1.17	-1.62	-2.05	-2.19	-2.20
i	-1.55	-1.75	-1.89	-2.05	-2.18	-2.20	-2.20
j	-1.98	-2.20	-2.22	-2.29	-2.24	-2.20	-2.20
y	-1.37	-1.32	-1.44	-1.79	-2.10	-2.19	-2.20
l	-3.21	-2.88	-2.42	-2.57	-2.32	-2.22	-2.20
r	-0.01	-0.00	-0.00	-0.00	-0.00	-0.00	0.00
q	-0.38	-0.46	-0.36	-0.26	-0.08	-0.01	0.00
x	-2.33	-1.83	-0.75	0.24	1.73	2.19	2.25
k	0.00	-0.16	-0.72	-1.21	-1.94	-2.17	-2.20
P	0.00	0.20	0.76	1.29	2.00	2.22	2.25
W	0.00	1.15	2.78	2.92	2.46	2.28	2.25
U	-0.58	-0.82	-1.80	-2.68	-3.94	-4.35	-4.40
Regime	R.I.	R.I.	R.I.	Cl.U.	Cl.U.	Cl.U.	

In the *short run* ($t=0$), there is a somewhat larger impact on labour supply compared with the long-run result. The corresponding fall in output leads to excess demand for goods. The regime of Repressed Inflation prevails. The excess demand for goods is reinforced by anticipatory buying by consumers. Along with this households take more leisure (and supply less labour), because consumption of goods and leisure are linked through the jump

variable x . It appears that there are bootstraps effects all around. The only force in the opposite direction is the decline in notional investment demand, caused by the fall in the marginal productivity of capital. As the supply of labour diminishes, capital becomes more abundant.

The situation of Repressed Inflation lasts for some time, but at $t=6$ there is a shift to Classical Unemployment. This is due to a substantial decline in notional labour demand (l_d) caused by two factors. First, as the stock of capital falls less labour is needed. Secondly, higher real wages induce a change towards more capital-intensive techniques of production. The regime of Classical Unemployment prevails until a new equilibrium is attained asymptotically.

In the long run a wage tax falls entirely on labour. Disposable real labour income declines *pari passu* with the change in the tariff. However, in the case of a tax on wages, welfare is the least affected. Lifetime utility of the representative individual decreases by only 4.4 percent in the long run.

3.5. Changes between taxes

If yields are equal, taxes may be changed. The possible implications of such changes can be illustrated by giving an example. Table 6 reports the results with regard to the regimes prevailing in the case when a sales tax is changed in favour of a tax on wages ($\tau_y \rightarrow \tau_l$) and also for the opposite case of substituting a tax on wages for a tax on sales ($\tau_l \rightarrow \tau_y$). Both exercises show the result of a combined impulse: one tax is abolished and the other is imposed at $t=0$.

In the first mentioned case investment demand increases and labour supply decreases, which results in excess demand for goods and labour (Repressed Inflation). Under Repressed Inflation prices and wages rise. As real cash balances decline consumption demand decreases and there is a switch to Keynesian Unemployment at $t=4$. In this respect the situation differs from that in table 5, where the switch is towards Classical Unemployment. The difference is caused by the role of investment and capital accumulation. If a tax on wages is imposed, the stock of capital falls. If at the same time a sales tax is abolished, profits go up and capital is accumulated. Therefore the supply of goods increases in the latter case.

If a sales tax is imposed instead of a tax on wages, the results are less spectacular. There is a close resemblance to the results presented in table 2. There is Keynesian Unemployment in the short run, but after a while there is a switch to Classical Unemployment. A comparison of both cases shows that in the case of a tax change the supply of goods falls at a lower rate. This can be explained by the fact that labour supply increases if the tax on wages is reduced.

In the long run, the results are approximately symmetrical as might be

Table 6
Changes between a sales tax and a tax on wages.

Change	$\tau_y \rightarrow \tau_l$				$\tau_l \rightarrow \tau_y$			
	0	1	5	10	0	1	5	10
y_d	6.16	2.82	0.17	0.39	-4.31	-2.55	-0.90	-1.55
y_s	1.81	1.79	1.21	1.07	-1.99	-2.31	-2.60	-2.79
y_l	-1.59	-0.56	0.61	0.92	1.03	0.07	-1.70	-2.44
Regime	R.I.	R.I.	K.U.	K.U.	K.U.	K.U.	Cl.U.	Cl.U.

expected. Output increases by 3.45 percent in the first case ($\tau_y \rightarrow \tau_l$) and decreases by 3.34 percent in the second case ($\tau_l \rightarrow \tau_y$).

4. Epilogue

Building on a seminal article by Blanchard and Sachs (1982) this paper takes a further step towards the integration of microeconomics and macroeconomics. Although the emphasis lies on an analysis of tax incidence, there are several other features that deserve attention.

First, the regime of Repressed Inflation must be taken into account if shocks in different directions are allowed for. In fact, it is a logical consequence of assuming that households want to supply certain amounts of labour in every period of time. Secondly, there is a need for a theory of price inertia that fits well into the concept of rational decision-making. Without such a theory price formation remains somewhat ambiguous, as we have indicated. Thirdly, models of the scope and size tackled in this paper must be solved numerically. This may be considered a disadvantage. On the other hand, as our numerical experiments show, there are certain tendencies that come to the foreground quite clearly. The bootstraps phenomenon discussed above is an excellent example.

Appendix: Specification of the model

For the production function we maintain the CES specification proposed by Blanchard and Sachs (1982):

$$y = \gamma[\alpha k^{-\rho_1} + (1-\alpha)l^{-\rho_1}]^{-1/\rho_1}.$$

The elasticity of substitution is then $\sigma_1 = 1/(1+\rho_1)$. Installation costs are a linear function of the ratio of investment over the stock of capital:

$$h = bi/k.$$

The utility function is of the following form:

$$u = \ln [\xi c^{-\rho_2} + (1 - \xi)(M/P)^{-\rho_2}]^{-1/\rho_2} + \phi \ln(l_m - l).$$

The (partial) elasticity of substitution between consumption and real cash balances is $\sigma_2 = 1/(1 + \rho_2)$. There is a positive relationship between real cash balances and the marginal utility of consumption. An increase in prices reduces, ceteris paribus, real cash balances and therefore also the marginal utility of consumption. To restore the old situation, consumption must rise. The specification of the utility function in this way implies that the price system has a direct stabilizing effect on the economy. The term for leisure is added in an additively separable way. The model is recursive, which facilitates computation to a large extent.

We are now in a position to present the complete model as it is applied in numerical exercises. The transversality conditions are not repeated here, because they are applied in an indirect manner in two-point boundary value problems:

$$l_d: l_d = \{1/\alpha [P(1 - \alpha)(1 - \tau_y)\gamma]/W\}^{\rho_1/(1 + \rho_1)} - (1 - \alpha)/\alpha\}^{1/\rho_1} k \quad (\text{A.1})$$

$$y_s: y_s = \gamma[\alpha k^{-\rho_1} + (1 - \alpha)l_d^{-\rho_1}]^{-1/\rho_1}, \quad (\text{A.2})$$

$$i_d: i_d = ([q/(1 - \tau_j) - 1]k)/2b, \quad (\text{A.3})$$

$$j_d: j_d = i_d(1 + bi_d/k), \quad (\text{A.4})$$

$$c_d: (\xi c_d^{-(1 + \rho_2)})/[\xi c_d^{-\rho_2} + (1 - \xi)(M/P)^{-\rho_2}] = x, \quad (\text{A.5})$$

$$l_s: l_s = l_m - \phi \left/ \left(x \frac{W}{P} (1 - \tau_l) \right) \right., \quad (\text{A.6})$$

$$y_t: y_t = \gamma[\alpha k^{-\rho_1} + (1 - \alpha)l_s^{-\rho_1}]^{-1/\rho_1}, \quad (\text{A.7})$$

$$y_d: y_d = c_d + j_d, \quad (\text{A.8})$$

$$l_k: l_k = \{(1 - \alpha)/[(\gamma/y_d)^{\rho_1} - \alpha(1/k)^{\rho_1}]\}^{1/\rho_1}, \quad (\text{A.9})$$

$$y: y = \min[y_s, y_d, y_t], \quad (\text{A.10})$$

$$l: l = \min[l_d, l_k, l_s], \quad (\text{A.11})$$

$$c: c = c_d - a(y_d - y), \quad (\text{A.12})$$

$$j: j = j_d - (1 - a)(y_d - y), \quad (\text{A.13})$$

$$i: i = [(-1 + \sqrt{(1 + 4bj/k)})/2b]k, \quad (\text{A.14})$$

$$r: [(1 - \xi)(M/P)^{-1 + \rho_2}]/[\xi c^{-\rho_2} + (1 - \xi)(M/P)^{-\rho_2}] = x(r + \dot{P}/P), \quad (\text{A.15})$$

$$l_{ds}: l_{ds} = \min[l_d, l_s], \quad (\text{A.16})$$

$$T: \frac{T}{P} + \tau_j j = \tau_y y + \tau_z \left((1 - \tau_y)y - l \frac{W}{P} \right) + \tau_l l \frac{W}{P}, \quad (\text{A.17})$$

$$k: \dot{k} = i - \delta k, \quad (\text{A.18})$$

$$x: \dot{x} = (v - r)x, \quad (\text{A.19})$$

$$q: \dot{q} = (r + \delta)q - (1 - \tau_z)(1 - \tau_y)\gamma [\alpha k^{-\rho_1} + (1 - \alpha)l_{ds}^{-\rho_1}]^{-(1 + \rho_1)/\rho_1} \\ \times l_{ds}^{-(1 + \rho_1)\alpha} (l/k)^{(1 + \rho_1)} - (1 - \tau_j)b(i/k)^2, \quad (\text{A.20})$$

$$P: \dot{P}/P = \beta_p [y_d - \min(y_s, y_l)], \quad (\text{A.21})$$

$$W: \dot{W}/W = \beta_w (l_d - l_s) + \varepsilon \dot{P}/P. \quad (\text{A.22})$$

The equations are explained in the text. Two additional observations are to be made. First, where Blanchard and Sachs (1982) replace minimum functions by a CES function with very low elasticity we apply the minimum rule exactly. This facilitates the interpretation of the simulation results. Secondly, eq. (A.20) captures all possibilities in a single relationship. The second term on the RHS of eq. (A.20) is derived from eqs. (2.1.5) and (2.1.7). Under Keynesian Unemployment ($\lambda_y > 0$, $\lambda_l = 0$) this term can be written as:

$$(1 - \tau_z) \frac{W f_k(k, l)}{P f_l(k, l)}.$$

Notional labour demand (l_d) then follows from:

$$(1 - \tau_y) f_l(k, l_d) = \frac{W}{P}.$$

Substitution of the latter expression in the former expression results in

$$(1 - \tau_z)(1 - \tau_y)f_l(k, l_d) \frac{f_k(k, l)}{f_l(k, l)}, \quad (a)$$

with $l = l_k$ in the present case. Under Classical Unemployment ($\lambda_y = 0, \lambda_l = 0$) the second term on the RHS of (A.20) should be equal to:

$$(1 - \tau_z)(1 - \tau_y)f_k(k, l),$$

with $l = l_d$. Therefore, we may write

$$(1 - \tau_z)(1 - \tau_y)f_l(k, l_d) \frac{f_k(k, l)}{f_l(k, l)}. \quad (b)$$

Under Repressed Inflation ($\lambda_y = 0, \lambda_l > 0$) applying the same procedure as in the former case gives

$$(1 - \tau_z)(1 - \tau_y)f_l(k, l_s) \frac{f_k(k, l)}{f_l(k, l)}, \quad (c)$$

with $l = l_s$. Combination of (a), (b) and (c) leads to

$$(1 - \tau_z)(1 - \tau_y)f_l(k, l_{ds}) \frac{f_k(k, l)}{f_l(k, l)},$$

where $l_{ds} = \min [l_d, l_s]$.

There is, however, a minor problem in the case of Keynesian Unemployment if at the same time $l_k < l_s < l_d$. In this situation the opportunity costs of labour used in eq. (A.20) exceed the real wage rate.

Endogenous variables:

c = actual consumption

c_d = notional consumption demand

i = actual investment

i_d = notional investment demand

j = total investment spending

j_d = notional total investment spending

k = real capital stock

l = employment

l_d = notional labour demand

l_k = Keynesian labour demand

l_s = notional labour supply

P = nominal price level

q = shadow price of investment goods

r = real interest rate

W = nominal wage rate

x = shadow price of consumption goods

y = real production

y_d = notional demand for goods

y_l = labour constrained supply of goods

y_s = notional supply of goods

Calibration of the model is to a large extent based on the parameter values presented in Blanchard and Sachs (1982). Parameter values are chosen to be acceptable compared with the results of empirical studies. In addition, the set of parameter values should generate a reasonable initial situation. The parameter values used in the simulations are given below:

$a = 0.9$	$\delta = 0.1$	$M = 0.25$
$\alpha = 0.25$	$\varepsilon = 1$	$v = 0.10$
$b = 4.0$	$\phi = 0.09946$	$\rho_1 = 1$
$\beta_p = 0.1$	$\gamma = 0.23717$	$\rho_2 = 6$
$\beta_w = 0.05$	$l_m = 9.0$	$\xi = 0.95$

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