

THEO VAN DE KLUNDERT

*Tilburg University**The Netherlands*

*The Energy Problem in a Small Open Economy**

The paper deals with the economic consequences of an oil price shock in a small open economy. The analysis along neoclassical lines is based upon a three-factor nested CES production function. The model takes account of capital accumulation. Analytical solutions for the short- and long-run are presented for a linearized version of the original model, which makes existing results more tractable.

1. Introduction

The energy problem in a country depending for its needs on foreign supplies becomes manifest if the price of energy rises. It is obvious that in such a case its terms of trade deteriorate. On the other hand there may be possibilities of energy conservation which may lighten the burden. If account is taken of energy conservation in industry what will be the effect of a price increase on consumption, production, wages and employment? More specifically, what will be the time paths of the variables compared with the development without a price increase of energy?

These problems are analysed in Bruno (1981a, 1981b), Bruno and Sachs (1979, 1981), Tatom (1979), and Rasche and Tatom (1977, 1981). The present contribution aims at a restatement of existing results applying a linearized version of a model of a small open economy. This may contribute to a better understanding of the main issues involved.

A number of simplifying assumptions are made to focus on what is considered to be essential for the energy issue. It is assumed that the domestic good is a perfect substitute for the world final good. The price is therefore determined on the world market. The same holds true for the rate of interest. The balance of payments on current account is supposed to be in equilibrium all the time.

The plan of the paper is as follows. The details of the model

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in its linear form are discussed in Section 2. In Section 3 solutions for the short and long run are derived. In both cases conclusions will be presented point by point. The original version of the model is given in an appendix.

2. The Model

In this section we specify a macroeconomic model for a small open economy. The following assumptions are made:

1. Domestic and foreign goods are perfectly homogeneous
2. The world market price of goods is fixed
3. Goods are produced by applying labor, capital, and energy; demand for these factors is determined by familiar neoclassical conditions
4. The world market price of energy is given
5. The supply of labor is fixed and the labor market clears instantaneously
6. The stock of capital is given in the short-run
7. Profit maximization by entrepreneurs determines the rate of investment
8. The balance of payments on current account is always in equilibrium
9. The rate of exchange is constant
10. Financial capital is perfectly mobile between countries.

The equations will be written in a linear form by applying the well-known logarithmic transformation.¹ To be more specific, all variables are percentage deviations from a reference path. The variables are equal to zero unless the path is disturbed by an exogenous shock such as, for instance, a rise in the price of energy.

The production function needs proper specification. Engineering analysis as reported in Berndt and Wood (1979) and other evidence suggest that capital–energy substitution can be separated within the production function. It will be assumed that this subfunction can be characterized by a constant elasticity of substitution. Capital and energy combine to form effective capital which is a substitute for labor. Here again we assume a constant but different elasticity of substitution. Furthermore, it will be assumed

¹See for instance Dornbusch (1980). The original nonlinear version of the model is presented in the appendix.

that the weakly separable production function is homogeneous of the first degree.

Gross output, x , is obtained by using labor, ℓ , and effective capital, k_e .²

$$x = \lambda_\ell \bar{\ell} + (1 - \lambda_\ell) k_e \quad (1)$$

where λ_ℓ stands for the production elasticity of labor, which under neoclassical conditions corresponds to the share of wages in gross output. Barred variables are exogenous. Assumptions 3. and 5. imply that labor is always fully employed. Therefore, ℓ can be considered as an exogenous variable.

Effective capital is an aggregate depending on the input of energy, n , and material capital, k :

$$k_e = [\lambda_n / (1 - \lambda_\ell)] n + [(1 - \lambda_\ell - \lambda_n) / (1 - \lambda_\ell)] k \quad (2)$$

where λ_n stands for the production elasticity of energy, which equals the value of energy input in gross production.

The combination of both formulas leads to a traditional three-factor formulation of the production process:

$$x = \lambda_\ell \bar{\ell} + \lambda_n n + (1 - \lambda_\ell - \lambda_n) k. \quad (1')$$

Labor demand follows from the equality of the marginal product of labor and the real wage rate, w . With labor supply fixed, the equilibrium wage level can be expressed as:

$$w = (1/\sigma_1)(x - \bar{\ell}) \quad (3)$$

where σ_1 denotes the elasticity of substitution between labor and effective capital.

Energy is used until the point is reached where the marginal value product of this factor equals its real price, p_n . Because of the weak separability of the production function the marginal product of energy can be found by applying the chain rule of calculus. The linearized form of the equation reads as

$$n = (\sigma_2/\sigma_1)x + [1 - (\sigma_2/\sigma_1)]k_e - \sigma_2 \bar{p}_n. \quad (4)$$

²Time subscripts are omitted for the sake of convenience.

The price of energy is an exogenous variable. The symbol σ_2 indicates the elasticity of substitution between capital and energy.

Capital accumulation depends positively on gross investment and negatively on technical obsolescence. Linearization of the definition equation for accumulation results in

$$\dot{k} = \delta(i - k). \quad (5)$$

A dot over a variable indicates that the first time derivative of that variable is taken. The symbol δ is used for the rate of technical obsolescence. For reason of simplicity it is assumed that the natural rate of growth is zero in the initial situation. All deviations are taken from a stationary state equilibrium.

In a small open economy with perfect capital mobility and static exchange rate expectations the rate of interest must be equal to the interest rate on the world market, r^* , which is an exogenous variable. If the actual rate of return deviates from this value entrepreneurs will adapt the stock of capital. Following Kouri (1979) and Bruno (1981b) it will be assumed that investment is a function of the distance between the actual rate of return and the interest rate and of the costs of adjustment with regard to the installation of capital. If these costs are a rising function of the rate of gross capital formation per unit of capital in place the investment relationship comes down to:

$$i = k + \epsilon(r - \bar{r}^*) \quad (6)$$

where ϵ denotes the elasticity of investment with respect to the rate of return.

The actual rate of return on capital is determined by what is left after the other factors have been paid according to their marginal productivity. The rate of return as a residual follows from the definition equation

$$r = \frac{1}{1 - \lambda_\ell - \lambda_n} x - \frac{\lambda_\ell}{1 - \lambda_\ell - \lambda_n} (\bar{\ell} + w) - \frac{\lambda_n}{1 - \lambda_\ell - \lambda_n} (n + \bar{p}_n) - k. \quad (7)$$

The substitution of Equation (1') into (7) leads to a linear expression

for the well-known factor price frontier (FPF) in the three-factor model as discussed by Bruno (1981b)

$$r = -\frac{\lambda_\ell}{1 - \lambda_\ell - \lambda_n} w - \frac{\lambda_n}{1 - \lambda_\ell - \lambda_n} \bar{p}_n. \quad (7')$$

National income, y , is the sum of value added in the domestic goods sector and the energy sector. The production of energy, h , is considered exogenous, while the relatively small inputs of this sector will not separately be accounted for.

The country may be a net importer or exporter of energy at the given world real market price p_n . If the ratio between the value of the domestic energy supply and gross production of goods is denoted by θ_n , real national income can be written as

$$y = \frac{1}{1 - \lambda_n + \theta_n} x - \frac{\lambda_n}{1 - \lambda_n + \theta_n} (n + \bar{p}_n) + \frac{\theta_n}{1 - \lambda_n + \theta_n} (\bar{h} + \bar{p}_n). \quad (8)$$

Domestic absorption (consumption and investment) equals real income by assumption. As a consequence the balance of payments on current account is always in equilibrium. This rather strong assumption is introduced because we are not interested here in transitory balance-of-payments problems. What a country can spend depends ultimately on its earning capacity. Investment expenditure is already determined by Equation (6). As a consequence of these assumptions consumption, c , has a residual character

$$c = [(1 - \lambda_n + \theta_n)/(\lambda_\ell + \theta_n)] y - [(1 - \lambda_n - \lambda_\ell)/(\lambda_\ell + \theta_n)] i. \quad (9)$$

Equations (1)—(9) can be solved for the nine endogenous variables: x , k_e , n , w , i , r , y , c and k . A short-run solution is obtained by putting $\dot{k} = 0$. Long-run values of the variables can be found by setting $\dot{k} = 0$ for $t \rightarrow \infty$, which implies $i = k$.³ Short-run as well as long-run results will be derived in the next section.

³The homogeneous part of the differential equation in the capital stock takes the form: $\dot{k} = -\{[\delta\epsilon\lambda_\ell(1 - \lambda_\ell)]/[\sigma_1(1 - \lambda_\ell - \lambda_n) + \sigma_2\lambda_n\lambda_\ell]\}k$. Therefore the model is stable as could be expected.

3. Analytical Solutions

It should be observed from the outset that long-run ($t \rightarrow \infty$) solutions are more fundamental than outcomes for the short-run ($t = 1$). In the former case the volume of capital has reached an optimal value, as will be shown below, while in the short run the stock of capital is given. For this reason, the results for $t \rightarrow \infty$ will be considered first. The main point of interest will be the impact on the economy of an energy price shock.

In the *long run*, capital has adjusted to the given rate of interest which implies $r = \bar{r}^* = 0$. The solution for the real wage rate then follows directly from the FPF equation (7')⁴

$$w_x = -\frac{\lambda_n}{\lambda_\ell} \bar{p}_n. \quad (10)$$

The substitution of (10) into (3) and setting $\bar{\ell} = 0$ results in

$$x_x = -(\lambda_n/\lambda_\ell)\sigma_1 \bar{p}_n. \quad (11)$$

In addition we have from (1)

$$k_{e_x} = -(\lambda_n/\lambda_\ell)[\sigma_1/(1 - \lambda_\ell)] \bar{p}_n. \quad (12)$$

Before commenting on these results we shall derive the long-run solutions for the other variables. The demand for energy on long term can easily be found by substitution of the Equations (11) and (12) in equation (4)

$$n_x = -\left[\frac{\sigma_1 - \sigma_2 \lambda_\ell}{1 - \lambda_\ell} \cdot \frac{\lambda_n}{\lambda_\ell} + \sigma_2 \right] \bar{p}_n. \quad (13)$$

Stability of the model implies $\dot{k} = 0$ or, according to Equation (5), $i = k$. This relationship could be used to find the long-term solution for k , but there is a more direct way. Recalling $\bar{\ell} = 0$ Equation (1') can be written in the form

$$n - k = [1/(1 - \lambda_\ell - \lambda_n)][(1 - \lambda_\ell)n - x]. \quad (14)$$

Combination of Equations (11), (13) and (14) leads to

⁴Long-run solutions are indicated by the subscript ∞ as a shortcut for $t \rightarrow \infty$.

$$n_x - k_x = -\sigma_2 \bar{p}_n. \quad (15)$$

This result shows that in the long run the stock of capital is optimally adjusted. The actual solution for k follows from Equations (13) and (15)

$$k_x = - \left[\frac{\sigma_1 - \sigma_2 \lambda_\ell}{1 - \lambda_\ell} \cdot \frac{\lambda_n}{\lambda_\ell} \right] \bar{p}_n. \quad (16)$$

The outcomes can easily be checked. Substitution of the results (13) and (16) into Equation (2) gives the same solution for k , as shown in Equation (12).

Next we turn to income and consumption. Taking account of $\bar{h} = \bar{\ell} = 0$ and the production function (1') expression (8) can be rewritten as

$$\begin{aligned} y &= [(1 - \lambda_\ell - \lambda_n)/(1 - \lambda_n + \theta_n)]k \\ &\quad - [(\lambda_n - \theta_n)/(1 - \lambda_n + \theta_n)] \bar{p}_n. \end{aligned} \quad (17)$$

Substitution of this result and $i = k$ into Equation (9) gives the final outcome for consumption:

$$c_x = [(\lambda_n - \theta_n)/(\lambda_\ell + \theta_n)] \bar{p}_n. \quad (18)$$

The analytical results derived for the long run lead to a number of interesting conclusions:

1. As appears from Equation (13), an increase in the price of energy results in a lower input of energy and, therefore, other things being equal, a lower level of output
2. At constant factor prices of labor and capital, a fall in output implies decreasing factor demand. Full utilization of labor and capital could nevertheless be maintained if the real wage rate and the interest rate decline sufficiently
3. The capital stock is an endogenous variable. Its long-run solution follows from Equation (16). The capital stock declines if $\sigma_1 > \sigma_2 \lambda_\ell$, which will be assumed. If the production function implied by the model is written as $x = f(k, l, n)$, it can be shown that this condition is equivalent to $f_{kn} > 0$; that is, a larger amount of energy in use raises the marginal productivity of physical cap-

- ital.⁵ The opposite case $f_{kn} < 0$ is not theoretically interesting
4. Labor is fully employed and the required fall in the wage rate is given in Equation (10). The outcome with regard to the demand for labor results from opposing forces. Less output means less need for labor. On the other hand, a lower wage rate leads to the application of a relatively more labor-intensive technique. Effective capital decreases as shown in Equation (12). The corresponding decline in production is given in Equation (11). It should be observed that the solution for k_e does not depend on σ_2 . Energy and capital can be meaningful aggregated which is of course a reflection of the assumed separability of the production function
 5. The loss in consumption is caused by two factors. A fall in output as a result of a price increase in energy reduces both income and consumption. An increase in the flow of output to pay for imported energy has a similar negative effect on income and consumption. In the initial situation there may be domestic energy production ($\theta_n > 0$). However, h is taken as an exogenous variable, implying that domestic production of energy is not affected by a price increase in energy.⁶ The (net) change in the flow of output to pay for imported energy is found by subtracting the amount saved by using less energy ($\lambda_n n$) from the rise in the bill caused by the price increase $[(\lambda_n - \theta_n) \bar{p}_n]$. It appears that the amount saved by using less energy equals the output loss $[\lambda_n n]$ mentioned above.⁷ Therefore the change in the volume of consumption ultimately depends on the terms-of-trade

⁵Tatom (1979) proves that the long-run effects of a rise in the price of energy are unambiguous, given that $f_{kn} > 0$. In this case the stock of capital is reduced. The equivalence of the conditions $\sigma_1 > \sigma_2 \lambda_e$ and $f_{kn} > 0$ was brought to my attention by an anonymous referee. It is mentioned in passing by Bruno and Sachs (1981).

⁶This assumption may be acceptable as a first approximation for non-OPEC countries, which have no influence on the world market price of energy. However, one could introduce a supply curve for domestic energy without great difficulty. Note that an autonomous increase in the domestic supply of energy ($h > 0$) leads to a rise in income and consumption. In the present model such a rentier income has no impact on production and accumulation. To show the effect on allocation one needs a Scandinavian type of model, where a distinction is made between sheltered and exposed sectors of the economy.

⁷The exact equality of these magnitudes follows from applying the rules of calculus in the derivation of the linear model. The implications of this procedure can be deduced by comparing the results with those derived graphically by Rasche and Tatom (1981) with regard to income and consumption.

effect as shown in formula (18). For non-OPEC ($\lambda_n > \theta_n$), the effect will be negative.

In the *short run* ($t = 1$) the stock of capital is constant ($k = 0$). The change in the demand for energy in case of a price shock can then be derived from Equations (1), (2) and (4):

$$n_1 = -\frac{\sigma_1 \sigma_2 (1 - \lambda_\ell)}{\sigma_1 (1 - \lambda_\ell - \lambda_n) + \sigma_2 \lambda_\ell \lambda_n} \bar{p}_n. \quad (19)$$

The substitution of (19) into (1'), taking account of $\bar{\ell} = 0$ and $k = 0$, results in

$$x_1 = -\frac{\sigma_1 \sigma_2 (1 - \lambda_\ell) \lambda_n}{\sigma_1 (1 - \lambda_\ell - \lambda_n) + \sigma_2 \lambda_\ell \lambda_n} \bar{p}_n. \quad (20)$$

The corresponding real wage rate can now be deduced from (3):

$$w_1 = -\frac{\sigma_2 (1 - \lambda_\ell) \lambda_n}{\sigma_1 (1 - \lambda_\ell - \lambda_n) + \sigma_2 \lambda_\ell \lambda_n} \bar{p}_n. \quad (21)$$

If the wage rate is known, the rate of profit follows from the FPF function (7'). After some manipulation we get

$$r_1 = -\left[\frac{\sigma_1 - \sigma_2 \lambda_\ell}{\sigma_1 (1 - \lambda_\ell - \lambda_n) + \sigma_2 \lambda_n \lambda_\ell} \right] \lambda_n \bar{p}_n. \quad (22)$$

National income can be found by substituting (19) and (20) in the definition Equation (8):

$$y_1 = -[(\lambda_n - \theta_n)/(1 - \lambda_n + \theta_n)] \bar{p}_n. \quad (23)$$

The volume of consumption can be deduced from Equations (6), (9), (22), and (23):

$$c_1 = -\frac{1}{\lambda_\ell + \theta_n} \left[(\lambda_n - \theta_n) - \frac{\epsilon (\sigma_1 - \sigma_2 \lambda_\ell) (1 - \lambda_\ell - \lambda_n) \lambda_n}{\sigma_1 (1 - \lambda_\ell - \lambda_n) + \sigma_2 \lambda_\ell \lambda_n} \right] \bar{p}_n. \quad (24)$$

The formulas derived for the short run ($t = 1$) lead to the following conclusions:

6. Higher energy prices induce a lower level of demand for energy and a corresponding reduction in output, as appears from Equations (19) and (20)

7. The decline in output implies a lower demand for labor. Therefore, to preserve full employment the wage rate must fall. The necessary reduction in w is given in Equation (21)

8. The rate of profit declines, given that $\sigma_1 > \sigma_2 \lambda_\ell$. As noted before, this condition is equivalent to $f_{kn} > 0$. A lower amount of energy in use reduces the marginal productivity of capital. This implies a lower rate of profit on the short run

9. Real national income depends on the change in the terms of trade. An energy importing country ($\lambda_n > \theta_n$) incurs a loss in income if the price of energy goes up. Formula (23) should be compared with (17). The only difference is the effect of capital accumulation on income, which is not relevant for the short-run ($k = 0$). The explanation of (23) is the same as given under conclusion 5.

10. The outcome for the level of consumption is somewhat ambiguous, as appears from Equation (24). The negative terms-of-trade effect reduces consumption in the case of an energy importing country ($\lambda_n > \theta_n$). The second term in the RHS of (24) relates to investment. If the rate of profit falls, investment will decline which leaves more income for consumption. Eventually the level of consumption will be higher.

If the labor market does not clear instantaneously the short-run results will of course be different.⁸ On the other hand if the Phillips mechanism works and the model is stable there will be no change with regard to the long-run results of a price increase in energy.

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Appendix

In this appendix the model in its original nonlinear form is presented. The variables are written with a hat to distinguish them

⁸Fixing real wages turns an energy price shock into a labor supply shock as well, raising unemployment. As a result real output, demand for energy inputs, the rate of profit, and real income decline even further.

from their counterparts in the main text which relate to percentage deviations of a stationary state solution.

Production Function

$$\hat{x}^{\rho_1} = \delta_\ell \hat{\ell}^{\rho_1} + \delta_e \hat{k}_e^{\rho_1}; \quad (\text{A.1})$$

$$\hat{k}_e^{\rho_2} = \delta_n \hat{n}^{\rho_2} + \delta_k \hat{k}^{\rho_2}. \quad (\text{A.2})$$

Factor Demand

$$\hat{w} = \frac{\partial \hat{x}}{\partial \hat{\ell}} = \delta_\ell \left[\frac{\hat{x}}{\hat{\ell}} \right]^{1-\rho_1}; \quad (\text{A.3})$$

$$\hat{p}_n = \frac{\partial \hat{x}}{\partial \hat{n}} = \frac{\partial \hat{x}}{\partial \hat{k}_e} \cdot \frac{\partial \hat{k}_e}{\partial \hat{n}} = \delta_\ell \delta_n \hat{x}^{1-\rho_1} \hat{k}_e^{\rho_1-\rho_2} \hat{n}^{\rho_2-1}. \quad (\text{A.4})$$

Investment

$$\frac{\hat{i}}{\hat{k}} = \delta \left[\frac{\hat{r}}{\hat{r}^*} \right]^\epsilon. \quad (\text{A.5})$$

Definitions

$$\hat{r} = (\hat{x} - \hat{\ell} \hat{w} - \hat{n} \hat{p}_n) / \hat{k}, \quad (\text{A.6})$$

$$\hat{y} = \hat{x} - \hat{n} \hat{p}_n + \hat{h} \hat{p}_n; \quad (\text{A.7})$$

$$\hat{c} = \hat{y} - \hat{i}. \quad (\text{A.8})$$

Capital Accumulation

$$\dot{\hat{k}} = \hat{i} - \delta \hat{k} \quad (\text{A.9})$$

The elasticities of substitution are equal to: $\sigma_1 = 1/(1 - \rho_1)$ and $\sigma_2 = 1/(1 - \rho_2)$. In the initial situation the system is in a stationary state with $\hat{i} = \delta \hat{k}$ and $\hat{r} = \hat{r}^*$. The amount of labor and the price of energy are then fixed. The solution of the model is not difficult in this case, although the resulting algebraic expressions may be intricate.

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