Do jumps help in forecasting the density of returns? [*]

Julien Chevallier[†]
Université Paris Dauphine

Florian Ielpo[‡]
Lombard Odier Investment Managers
Université Paris 1

Benoît Sévi[§]
Aix-Marseille School of Economics

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Abstract

The estimation of the jump component in asset pricing has witnessed a considerably growing body of literature. Of particular interest is the decomposition of total volatility between its continuous and jump components. Recent contributions highlight the importance of the jump component in forecasting the volatility at different horizons. In this paper, we extend the methodology developed by Maheu and McCurdy (2011) to measure the information content of intraday data in forecasting the *density* of returns at horizons up to 60 days. We extract jumps as in Andersen, Bollerslev, Frederiksen and Nielsen (2010) to have a measure of the jumps in returns. Then, we estimate a bivariate model of returns and volatilities where the jump component is independently modeled. Our empirical results for S&P 500 futures, WTI crude oil futures, and the USD/JPY exchange rate confirm the importance of considering the continuous/jump decomposition for density forecasting.

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[†]*Corresponding Author*. CGEMP/LEDa, Place du Maréchal de Lattre de Tassigny, 75775 Paris Cedex 16, France. Tel: +33(0)144054464. Fax: +33(0)144054484. julien.chevallier@dauphine.fr

[‡]Lombard Odier Asset Management, rue de la Corraterie 11, 1204 Geneva, Switzerland and University Paris 1 Panthéon - CERMSEM, Maison des Sciences Economiques, 106 Boulevard de l'Hôpital, 75013 Paris, France. Email: florian.ielpo@ensae.org

[§]DEFI, Château La Farge, Route des Milles, 13290 Aix-en-Provence Les Milles, France. Email: benoit.sevi@gmail.com

1 Introduction

When it comes to forecasting the *distribution* of returns for risk management purposes, is the separation between volatility and jumps equally important? In a reference paper, Andersen *et al.* (2007) provide empirical evidence that disentangling jumps from the continuous component significantly help in forecasting the realized volatility at horizons up to 22 days. The explanation for this result is the strong persistence in the continuous component and the absence of autocorrelation in the jump component. Recently, Hansen *et al.* (2011), Shephard and Sheppard (2010), and Maheu and McCurdy (2011) have suggested "complete" models of returns and volatility. In particular, Maheu and McCurdy (2011) propose a bivariate specification of returns and volatility to obtain density forecasts at horizons up to 60 days. They confirm, in the density context, numerous previous findings that intraday data improve forecasts. The estimation of a multivariate models possesses interesting characteristics compared to the separate estimation of an univariate model. Namely, multivariate models allow to obtain densities forecasts.[1] Maheu and McCurdy (2011) allow for a rich underlying distribution in the return equation, by using mixture of normals (see Bertholon *et al.*, 2006). We merge these two strands of this recent literature to investigate whether the separation between the continuous and the jump components is of central importance in predicting the density of returns. Our results strongly argue in favor of separating the two components when forecasting the density of returns up to 60 days. Indeed, disentangling jumps from the continuous component help in forecasting the density of returns.

Econometric methods used to disentangle jumps and volatility are unveiling new empirical questions. While recent developments in financial econometrics allow to derive better forecasts of return densities (see Corradi and Swanson (2006) for a recent survey), the issue of the inclusion of the jump component and its information content for such a purpose has not been investigated to date. Hence, in this paper, we examine whether this refinement to use a clear differentiation between jumps and continuous volatility is empirically worth the trouble in a *density* forecasting exercise. We conduct such an analysis at various horizons (up to 60 days), thus allowing a very detailed analysis of the effects we are interested in.

Forecasting density is essential in empirical finance applications such as portfolio choice, risk management activities or derivatives pricing. Each activity requires indeed a full specification of the return distribution. Forecasts of the future values of economic variables are used widely in decision making. Point forecasts, the often traditional focus, are better seen as the central points of ranges of uncertainty. Consequently, to provide a complete description of the uncertainty associated with the point forecast many professional forecasters now publish density forecasts, or more popularly fan charts. In contrast to interval forecasts, which give the probability that the outcome will fall within a stated interval, density forecasts provide a complete description of the uncertainty associated with a forecast. They can thus be seen to provide information on

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[1]It has also advantages when residuals of univariate models are contemporaneously correlated as highlighted in Bollerslev *et al.* (2009). We do not consider here the possible correlation between errors in single equations, which is left for further research.

all possible intervals. In conjunction with the increased use of density forecasts by professional forecasters and central banks, the academic literature has also devoted increased emphasis to density forecasting (for surveying methods for predictive density evaluation, see among others Tay and Wallis (2000), Clements (2005), Timmermann (2006) and Wallis (2007)).

This paper adopts the parsimonious specification of the Heterogeneous Autoregressive Model of the Realized Volatility (HAR-RV) model by Corsi (2009) to capture the well-known long-memory dependence in volatility. We also proceed with the detection of jumps following Huang and Tauchen's (2005) statistical test relying on the bipower variation (BPV) estimator. We adapt the test statistic to the newly developed median realized volatility (MedRV) following the empirical work by Theodossiou and Zikes (2009) showing the interesting properties of this estimator.

By using intraday data, it is possible to extract jumps as the difference, when statistically significant, between realized volatility and bipower variation/median realized volatility. This decomposition allows to include (or not) jumps for forecasting purposes. Based on the link between the conditional variance and the realized volatility highlighted by Maheu and McCurdy (2011), we estimate two kinds of models: *(i)* EGARCH models based on daily data, and *(ii)* bivariate models based on intraday data including jumps (realized volatility) or not (bipower variation, median realized volatility). The motivation behind considering jump-robust measures for realized volatility is that they simply have better predictive properties than non-jump-robust ones (see Shephard and Sheppard (2011), footnote 3).

As mentioned above, we rely on Maheu and McCurdy's (2011) bivariate model to simultaneously estimate returns and volatilities, while assuming a possible leverage effect. The cornerstone of the model is the link that the authors establish between some realized volatility estimators and the "true" conditional volatility in light of the theory underlying these estimators. Such a model allows to derive density forecasts from intraday data which can be compared between them or to standard EGARCH models. This comparison between EGARCH and bivariate specifications for daily returns and realized volatilities ("naive" and jump-robust estimators) is conducted by using the predictive likelihood of returns (Diebold and Mariano (1995) and Amisano and Giacomini (2007)). We find that intraday data convey most of the asset's informational content, as discussed by Taylor and Xu (1997), among others, for the point forecast case. Recall that the central goal of the paper is to assess the contribution of jumps in forecasting the *density* of returns thereby extending our understanding of the information contained in the different components of the volatility process. The bivariate model is estimated through maximum likelihood with possibly a mixture of normals, which allows to improve significantly the goodness-of-fit of the model.

This paper makes three contributions: *1)* we extend the framework of Maheu and McCurdy (2011) and show how to model jumps in their bivariate framework, *2)* we confirm their findings that intraday data yield to better densities forecasts than daily data for a larger set of financial series, and *3)* we assess the importance of jumps when forecasting the density of returns by

comparing jump-robust and non-jump-robust measures of realized volatilities. Compared to the "naive" measure of realized volatility, considering jumps specifically provides significant improvement on the accuracy of forecasts of return densities. We thus extend the results in Andersen *et al.* (2007) (see also Corsi *et al.* (2010)) in showing the importance of disentangling jumps from the continuous component for forecasting purposes.

The remainder of the paper is organized as follows. Section 2 details our modeling strategy for jumps, the choice of volatility estimators using intraday data and the methodology to estimate the bivariate model. Section 3 discusses the empirical results. Section 4 concludes.

2 Volatility, jumps and discrete time series model specification

In this section, we present first the time series models, second the parameter estimation procedures, and third the density forecast comparison tests.

2.1 Time series models

The aim of our article is to present empirical evidence regarding the interest of disentengling jumps from volatility when it comes to forecasting the density of returns. To do so, we evaluate the relative forecasting performances of different discrete time time series models. These models are selected for their ability to handle non-Gaussian distributions and time varying volatility. Jumps have indeed an effect on both the unconditional distribution of returns and volatility. Their impact on times series models can be of three kinds:

– Jumps can be captured through the conditional distribution chosen in the discrete time series models.

– Jumps can impact the measurement of volatility, as revealed by the now large literature on realized risk measures.

– Jumps can impact the dynamics of volatility, as in most time series models the conditional volatility is computed from past returns or residuals that incorporate a jumps component by essence.

When a model would be able to handle these three aspects in a way that disentangles jumps from volatility components, the question of its superior ability to produce density forecast would still require to be considered. We base our empirical work on the three following time series models:

– A conditionally Gaussian model:

$$r_t = \mu + \sigma_t \epsilon_t, \tag{1}$$

with $\epsilon_t \sim N(0,1)$. This model naturally ignores jumps. The only source of leptokurticity in the returns' process comes from the time varying behavior of volatility.

– A model based on a mixture of two Gaussian distributions for its conditional distribution:

$$r_t = \mu + \sigma_t \epsilon_t, \tag{2}$$

with $\epsilon_t \sim MN(\theta, \mu_1, \sigma_1, \mu_2, \sigma_2)$. This model accomodates jumps in two different ways: first, jumps are captured through ϵ_t that is obtained by mixing two different Gaussian densities. As presented in Bertholon et al. (2006), this distribution is able to span a very large scope of couples of kurtosis and skewness. Possibly, this distribution is consistent with a mixture of a Gaussian distribution and of an extreme-type of jumps (see Section 4.2. in Bertholon et al. (2006)). However for most of the methodologies used here, conditional volatilities σ_t are functions of ϵ_{t-1}. With this modelling approach, the *dynamics* of volatility is a function of past jumps.

– A model mixing a conditionally Gaussian distribution with jumps:

$$r_t = \mu + \sigma_t \epsilon_t + \sum_{i=0}^{N_t} x_{i,t}, \tag{3}$$

with $\epsilon_t \sim N(0,1)$, $x_{i,t} \sim N(\mu_x, \sigma_x)$ and $N_t \sim \mathcal{P}(\lambda)$. With such an approach, the past volatility is no longer a function of past jumps, as ϵ_t has been cleansed from the jump component. This latter component is assumed to be captured by $\sum_{i=0}^{N_t} x_{i,t}$, that is through a separated component.

For each of these models, the structure for the continuous volatility is an Heterogenous Autoregressive Model, as presented in Corsi (2009). We propose to use this specification jointly with different high frequency measures of volatility:

– The realized variance for day d that is given by the sum of squared intraday returns:

$$RV_{d,M} = \sum_{j=1}^{M} r_{d,j}^2, \tag{4}$$

where the $r_{d,j}$ are intraday returns computed as $r_{d,j} = p_{d,j} - p_{d,j-1}$ for $j = 1, ..., M$. $p_{d,j}$ are intraday observations allowing to compute M continuously compounded intraday returns each day.

– Barndorff-Nielsen and Shephard (2004)'s bipower variation (BPV) measure, which is computed as the scaled summation of the product of adjacent absolute returns. Formally, BPV is defined as follows:

$$BPV_{d,m} = \xi_1 \sum_{j=1}^{M-1} |r_{d,j+1}||r_{d,j}| \tag{5}$$

where $\xi_p \equiv 2^{p/2} \Gamma(\frac{1/2(p+1)}{\Gamma(1/2)}) = E(\mid Z \mid_p)$ denotes the mean of the absolute value of standard normally distributed random variable[2], Z. The BPV is a consistent estimator of integrated volatility, and allows to decompose the realized volatility into its diffusive and non-diffusive parts. As the sampling frequency increases, the presence of jumps should have no impact because the return representing the jump is multiplied by a non-jump return which tends to zero asymptotically. This is true in case of rare jumps (one each day) when the probability of two consecutive jumps is negligible.

– Nevertheless, the BPV can be upward or downward biased in empirical applications as the sampling frequency is not high enough to eliminate the influence of jumps (or in presence of zero-return). This has motivated the need for alternative estimators which do not suffer from this weakness. Recently, Andersen et al. (2009) suggested the following median realized volatility (MedRV) estimator:

$$MedRV_N = \frac{\pi}{6 - 4\sqrt{3} + \pi} \left(\frac{N}{N-2} \right) \sum_{i=2}^{N-1} \text{med}(\mid \Delta Y_{i-1} \mid, \mid \Delta Y_i \mid, \mid \Delta Y_{i+1} \mid)^2 \qquad (6)$$

The MedRV estimator has two main advantages: first, the impact of jumps completely vanishes except in the case of two consecutive jumps (which is quite rare at the sampling frequencies used in our empirical application) and second, the estimator is more robust to occurence of zero-returns.[3]

In Section 3, we discuss different empirical key elements related to the use of such volatility measures, such as the intra-day sampling frequency. Using both the BPV and the MedRV, we present in the meantime a way to estimate jumps from intra-day time series. For the time being, we assume that jumps are *observable*.

Now, we discuss the joint dynamics of volatility and returns. Maheu and McCurdy (2011) relate the conditional variance of daily returns σ_t^2 to the realized volatility estimator through a cross-equation restriction. Barndorff-Nielsen and Shephard (2002) and Andersen *et al.* (2003) show that under empirically realistic assumptions, the conditional variance of daily returns should equal the conditional expectation of quadratic variation, or:

$$E_{t-1}(QV_t) = Var_{t-1}(r_t) \equiv \sigma_t^2, \qquad (7)$$

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[2]This notation is used consistently in the paper.

[3]We could use other estimators to obtain measures of integrated variance, such as QRV (Christensen et al., 2009) estimator, which are shown to be more robust in the presence of microstructure noise and zero-returns. A comparison of these estimators and their properties for density forecasting is beyond the scope of this paper and left for future research.

where E_{t-1} stands for the conditional expectation at time $t-1$, QV_t the quadratic variation, $Var_{t-1}(r_t)$ and σ_t^2 the conditional variance of returns. Assuming that RV_t is an unbiased estimator of QV_t, it follows that:

$$\sigma_t^2 = E_{t-1}(RV_t). \tag{8}$$

In other words, the *one-period-ahead* conditional expectation of the realized volatility should equal the "true" conditional volatility assuming the unbiasedness of the realized volatility estimator. Under the assumption of a log-normal distribution for the realized volatility[4], the conditional expectation can then be expressed as:

$$\sigma_t^2 = E_{t-1}(RM_t) = \exp\left(E_{t-1}\log(RM_t) + \frac{1}{2}\mathrm{Var}_{t-1}(\log(RM_t))\right). \tag{9}$$

with RM_t a given realized measure (RV_t, BPV_t, $MedRV_t$).

We now turn to the specification of a predictive model for realized volatility. The HAR-RV model initially developed in Corsi (2009) has been used with success in a number of recent contributions (Andersen et al. (2007), Corsi *et al.* (2008), Liu and Maheu (2009), among others). The economic intuition behind this model is that different groups of investors have different investment horizons, and consequently behave differently (see Muller *et al.* (1997) for the presentation of the H ARCH original model relying on the Heterogeneous Hypothesis). The genuine HAR-RV model is formally a constrained AR(22) model using RV as the realized measures of variance but the HAR can naturally accommodate all realized measures and transformations of these measures.[5] The HAR-RV model using daily, weekly and monthly[6] realized volatility components may be written as follows:

$$\sqrt{RV_t} = \alpha_0 + \alpha_d\sqrt{RV_{t-1}} + \alpha_w(\sqrt{RV})_{t-5:t-1} + \alpha_m(\sqrt{RV})_{t-22:t-1} + u_t. \tag{10}$$

The error term u_t is chosen to fit the distribution of the residuals, but could very well be modeled as a GARCH error (see Corsi *et al.* (2008), Bollerslev *et al.* (2009)). Since the logarithmic transformation exhibits superior forecasting performance (Andersen *et al.* (2007)),

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[4]Empirical evidence of this hypothesis can be found in early contribution such as ABDL (2001a and b, 2003). Similar evidence for foreign exchange rates, futures markets, crude oil futures and the FTSE index may be found in Pong *et al.* (2004), Thomakos and Wang (2003), Wang *et al.* (2008) and Areal and Taylor (2002), respectively.

[5]Forsberg and Ghysels (2007) and Ghysels and Sohn (2009), note that other power transformations may be used to model the dynamics of the realized volatility. These studies show that for a number of stochastic volatility processes used in the financial literature the absolute value of the realized volatility is a better predictor of the future realized volatility, particularly for longer horizons. We do not follow this approach here.

[6]The optimal lag structure for the HAR model has been investigated in Craioveanu and Hillebrand (2010) who find that the genuine structure suggested in Corsi (2009) performs the best.

we retain the following specification:

$$\log(RV_t) = \omega + \phi_1 \log(RV_{t-1}) + \phi_2 \log(RV_{t-5,5}) \tag{11}$$

$$+ \phi_3 \log(RV_{t-22,22}) + \gamma \epsilon_{t-1} + \eta v_t \quad v_t \sim NID(0,1) \tag{12}$$

The error term in the volatility equation is assumed to follow a standard Gaussian, as it is well-known since Andersen *et al.* (2001) that the logarithmic transformation of the realized volatility is normally distributed. The latter specification captures asymmetries coming from two distinct sources: leverage effects (through γ), and unconditional asymmetry (with the mixture of normals).

Densities forecasts using intraday futures will be compared with forecasts obtained with the traditional EGARCH model based on daily data. The EGARCH model considered is the standard specification by Nelson (1991):

$$r_t = \mu + \sigma_t \epsilon_t, \epsilon_t \sim NID(0,1) \tag{13}$$

$$\log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma u_{t-1} + \xi|\epsilon_{t-1}| \tag{14}$$

To make comparisons easier, a leverage term is introduced in the volatility equation. Depending on the standardized return innovation u_t in the return equation, the volatility is impacted through the coefficient γ. The estimated value of this coefficient is generally found to be negative. Indeed, an unexpected fall in returns translates into a positive impact on the level of volatility.[7]

2.2 Parameters estimation of the time series models

The models presented in Section 2.1 are similar to those presented in Maheu and McCurdy (2011). As in their case, the estimation is performed by maximum likelihood: the estimation is possible as both the returns and the volatility are observed.[8] On top of that – and in a similar fashion to Maheu and McCurdy (2011) – we assume that ϵ_t and v_t, that is the disturbances respectively associated to the returns and volatility, are uncorrelated.[9] As it is well-known since Bertholon *et al.* (2006), the mixture of normals yields to estimates by QML which are very close to the true distribution.

We discuss rapidly the estimation of the model presented at equation (3)-(12,13). Let $\tilde{r}_t$ be

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[7]Note that the model does not allow the asymmetry to propagate into future volatility as in the EGARCH model.

[8]As in Maheu and McCurdy (2011), we do not resort to Monte-Carlo simulations, which would be too computationally demanding in terms of numerical implementation, particularly in a rolling window setting that we adopt for the out-of-sample forecasting comparison.

[9]The assumption of conditional independence does not lead to unconditional independence as the two equation are related through the leverage term. A more complete model allowing for conditional dependence has been developed in Bollerslev *et al.* (2009).

the ex-jump return, that is

$$\tilde{r}_t = r_t - \sum_{i=0}^{N_t} x_{i,t}. \tag{15}$$

Let Ω_t be vector containing the following three processes:

$$\Omega_t = (\tilde{r}_t, \sigma_t, J_t), \tag{16}$$

where J_t is the jump component, that is $\sum_{i=0}^{N_t} x_{i,t}$. The estimation of the parameters driving the joint behavior of the three processes can be obtained by maximizing their joint likelihood. The conditional joint density given the past observation of Ω_t can be writen as follow:

$$f(\tilde{r}_t, \sigma_t, J_t | \Omega_{t-1}) = f(\tilde{r}_t, J_t | \Omega_{t-1}, \sigma_t) g(\sigma_t, J_t | \Omega_{t-1}) \tag{17}$$

$$= f(\tilde{r}_t | \Omega_{t-1}, \sigma_t) g(\sigma_t, J_t | \Omega_{t-1}) h(J_t | \Omega_{t-1}, \sigma_t). \tag{18}$$

In equation (20)-(21), $f(.)$, $g(.)$ and $h(.)$ are marginal densities. Equation (21) is obtained as $\tilde{r}_t$ and J_t are assumed to be independant. The maximization of the joint loglikelihood then clearly amounts to maximizing each of its three components independently, given that they do not share common parameters.

Maximum likelihood estimates are thus obtained through the following expressions:

$$\max_{\theta_{\tilde{r}}} \sum_{t=1}^{T} \log f(\tilde{r}_t | \Omega_{t-1}, \sigma_t) \tag{19}$$

$$\max_{\theta_{\sigma}} \sum_{t=1}^{T} \log g(\sigma_t, J_t | \Omega_{t-1}) \tag{20}$$

$$\max_{\theta_J} \sum_{t=1}^{T} \log h(J_t | \Omega_{t-1}, \sigma_t). \tag{21}$$

As jumps are i.i.d. random variables, it is possible to obtain closed form expressions for the model's parameters estimates:

$$\hat{\lambda} = \frac{1}{T} \sum_{t=1}^{T} N_t \tag{22}$$

$$\hat{\mu}_x = \frac{1}{\sum_{t=1}^{T} N_t} \sum_{t=1}^{T} \sum_{i=0}^{N_t} x_{i,t} \tag{23}$$

$$\hat{\sigma}_x^2 = \frac{1}{\sum_{t=1}^{T} N_t} \sum_{t=1}^{T} \sum_{i=0}^{N_t} (x_{i,t} - \hat{\mu}_x)^2. \tag{24}$$

Such a straightforward estimation approach is made possible by two key elements: first, the fact that the volatility σ_t is included within the filtration at time t; second, we assume that $\sum_{i=0}^{N_t} x_{i,t}$ is observable and unrelated to the other components in the dynamics of r_t. With such an approach, we obtain a realistic split between the contributions of jumps and those of volatility to the evolutions of r_t.

2.3 Density forecasting power comparison

To compare the various models presented earlier, we rely on out-of-sample density forecasting exercises. This subsection presents the empirical approach that we retained.

To compare density forecasts between the standard EGARCH model and the bivariate model of daily returns and HAR, we use a criteria known as the predictive likelihood (or logarithmic score). The average predictive likelihood over the out-of-sample observations $t = \tau + k_{max}, \ldots, T - k$ is:

$$D_{M,k} = \frac{1}{T - \tau - k_{max} + 1} \sum_{t=\tau+k_{max}-k}^{T-k} log f_{M,k}(r_{t+k} \mid \Phi_t, \theta), \ k \geq 1 \tag{25}$$

with $f_{M,k}(x \mid \Phi_t, \theta)$ the k-period ahead predictive density for model $\mathcal{M}$, given Φ_t and parameter θ, evaluated at the realized return $x = r_{t+k}$. Therefore, better forecasts will translate into larger $D_{\mathcal{M},k}$.

We use a rolling window scheme to evaluate the predictive power of our forecasts. As in Maheu and MacCurdy (2011), compute the 1 to 60 day-ahead forecasts for each window. We thus obtain $T - \tau - k_{max} + 1$ data blocks for each asset. For each block, we compute the predictive likelihoods and then average over all blocks.

To evaluate the relative accuracy of competing forecasts, we rely on the test statistics developed by Diebold and Mariano (1995) in the context of the comparison of density forecasts (Amisano and Giacomini, 2007). The null hypothesis is that predictive likelihood forecasts of horizon h and models $\mathcal{A}$ and $\mathcal{B}$ have the same performance. The test statistics is given by:

$$t_{\mathcal{A},\mathcal{B}}^k = \frac{(D_{\mathcal{A},k} - D_{\mathcal{B},k})}{\frac{(\hat{\sigma}_{\mathcal{A},\mathcal{B},k})}{\sqrt{T-\tau-k_{max}+1}}} \tag{26}$$

which is asymptotically standard normal. As for the interpretation of the Diebold and Mariano (1995) (DM) test, a significant positive (negative) estimated value rejects the null of equal performance between competing forecasts, and provides evidence in favor of model $\mathcal{A}$ ($\mathcal{B}$).

An additional issue arises when computing the Amisano and Giacomini (2007)'s test statistics when the jumps are explicitly incorporated within the data generating process of r_t. Indeed, as future jumps are unobservable, we need to compute the conditional distribution of r_{t+h} conditionally upon the information available at time t.

$$f(r_{t+h}|\mathcal{F}_t) = \sum_{i=0}^{\infty} f(r_{t+h}|\mathcal{F}_t, N_{t+h} = i) \times P(N_{t+h} = i), \tag{27}$$

where $P(N_{t+h} = i)$ is the probability that the number of jumps between the date $t + h - 1$ and $t + h$ is equal to i, given the estimated parameters. Given the estimated values for the average number of jumps by working day, we approximate the previous quantity by truncating the previous infinite sum. We compute it from $i = 0$ to 10. Beyond this threshold, the results of the numerical tests remain qualitatively unaffected. In the various HAR-based models considered here, $f(r_{t+h}|\mathcal{F}_t, N_{t+h} = i)$ is computed as in Maheu and McCurdy (2011) and in the previously mentioned cases that are not based on an explicit modeling of jumps. We also follow Maheu and McCurdy (2011) for the simulations, computations, and the use of the Newey-West long-run variance (HAC) again to make our results comparable with theirs.

3   Empirical Results

In this section, we present first the data used. Then, we detail the procedures used to find the optimal sampling frequency when using intraday data and to extract jumps. Finally, we comment in details the results obtained for all types of models (with/without jumps), along with the comparison of density forecasts.

3.1   Dataset

In our empirical analysis, we use tick-by-tick data from three different classes of assets: stock index futures, energy futures, and exchange rate. While all assets are very liquid (and are therefore suitable for using realized estimators), we need to remove days where the trading activity has not been sufficient to compute these estimators. To this end, we filter our time series with respect to three parameters: the length of the trading period in the day, the number of zero-returns and the number of transactions. Let us describe below each asset considered in the empirical analysis, and provide more details about the cleaning procedure.

The S&P 500 futures data is a very liquid contract traded on the CME which is a tradable asset in opposition with the underlying S&P 500 cash index. The time span for the S&P 500 futures is from January 1, 1996 to July 31, 2008 (3,192 trading days originally). As is usual, we consider the continuous series of the front month contract using a rollover procedure which selects the

largest volume each day to jump from one contract to the next.[10]. Trading of the S&P 500 futures contract occurs from 8:30 AM to 3:15 PM which should, on normal days, provide 81 intraday returns when using a sampling interval of 5 minutes.[11] We remove days with less than 81 returns which is generally an indication of part closure of the trading place. We check that all accepted days have a sufficient number of transactions, and a limited number of zero-returns as well. After cleaning the data, we obtain a sample of 3,135 days. For these selected days, the average number of trades for the continuous rollover series is 3,090 (the total number of ticks is equal to 9,809,697). We observe a relative stability of the number of transactions each day during the whole period.

Our second asset is the West Texas Intermediate (WTI) light sweet crude oil futures contract, traded on the New York Mercantile Exchange (NYMEX) now a branch of CME. The rollover series is built as explained previously for the S&P 500. The period considered is from October 8, 2001 to January 15, 2010. The WTI contract is one of the most traded futures contract in the world. The total number of ticks for the continuous time series of the front month contract is equal to 52,099,419. The trading period for the WTI futures is from 9:00 AM to 2:30 PM, which should provide 60 intraday returns each day (54 intraday returns for the September 2001/January 2007 period where trading began at 10:00 AM). Similarly to the previous asset, we remove days with less than 60 (54) intraday returns, days with more than 15 zero-returns, and days with less than 700 registered ticks. The number of observations is therefore reduced from 2,140 days to 2,081 days when all these requirements are met. The mean number of trades is equal to 25,035. In contrast to S&P 500 futures, this figure is very different before and after mid-2006, which is mainly due to the launch of electronic trading.[12]

The USD/JPY exchange rate, the third asset, covers the period going from December 30, 1996 to June 1, 2007, *i.e.* a sample of 2,701 days (2,720 initially before cleaning the data) with a total of 22,530,929 transactions (8,342 ticks per day on average). As in previous contributions (see Andersen et al. (2001) among many others), we consider the period going from 9:00 PM to 8:59 PM the next day as a unit period, because the FX market is opened on a 24-hour window.

Insert Figure 1

Figure 1 displays the time-series of S&P 500 futures, WTI futures, and the USD/JPY exchange rate, along with the open-to-close log-returns.[13] As may be seen from these plots, we include in our analysis time-series with very different properties: more or less pronounced volatility clustering, bear and bull markets, extreme variations in a short period of time, etc.

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[10] We do not build our continuous series using a fixed number of days prior to maturity, thus avoiding calendar effects.

[11] We discuss the issue of sampling frequency in the next section.

[12] We have an average number of 2,214 ticks per day during the September 4, 2001 / August 31, 2006 period *vs.* 57,054 ticks for the September 1, 2006 / January 15, 2010 period.

[13] We choose to work with open-to-close returns because overnight returns have shown to follow a very different dynamics. In addition, including overnight returns may alter our analysis when standardizing returns as we work with volatility computed with intraday transaction data.

We also present summary statistics for all time series in Table 1. We observe that the realized volatility and the bi-power variation measures present nonzero skewness and excess kurtosis.[14] These descriptive statistics therefore reveal a "fat tailed" distribution. The logarithm transformations of these quantities are nearly Gaussian, which is a common finding since Andersen *et al.* (2001) among others.[15]

3.2 Optimal sampling frequency and jump detection

This subsection presents empirical technicalities regarding the intra-day volatility and jumps measures: *(i)* the determination of the optimal sampling frequency for each asset, *(ii)* the jump detection, and *(iii)* the sequential jump detection procedures.

3.2.1 Optimal sampling frequency

For these three estimators of realized volatility, theory suggests that returns should be computed at the highest possible frequency, so that estimators converge asymptotically towards the true conditional volatility. However, it is well-known since Andersen and Bollerslev (1997, 1998) and Taylor and Xu (1997) that microstructure noise (due to price discreteness, bid-ask spread, non-synchronous trading, etc.)[16] may impact the realized volatility estimator at high frequency.

To deal with this issue while making our results comparable with the rest of the literature, we follow the 5 minutes 'rule-of-thumb'. As our three series are highly liquid assets, this sampling interval is adequate to make our realized measures not to be impacted by the noise.

Insert Figure 2

Insert Figure 3

We examine further this question for the WTI crude oil futures price series, which did not benefit from such an analysis in previous research. In Figures 2 and 3, we report the volatility signature plot for oil futures. This analysis is crucial as the trading activity dramatically increased in

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[14]Note for a normally distributed random variable skewness is zero, and kurtosis is three.

[15]We come back on this issue when modeling the volatility using the log transformation. Goncalves and Meddahi (2011) suggest a new class of nonlinear transformations based on the Box-Cox transformation which outperform the log transformation in Monte Carlo simulations. We leave as an extension this possible transformation and follow the bulk of the empirical literature by considering the logarithm.

[16]See Hansen and Lunde (2006) for a thorough discussion of this issue and Andersen *et al.* (2011) for a theoretical and empirical analysis of the impact of microstructure noise on the forecast of realized volatility. To deal with this issue, we use staggered versions of BPV and MedRV as advocated in Huang and Tauchen (2005).

September 2006 following the generalization of electronic trading (see Section 3.1) and may result in different noise structure before and after this event. These graphs confirm that the standard 5-minute sampling frequency seems to be appropriate in this case as well.

3.2.2 Jump detection

Once the optimal sampling frequency is determined, realized volatility, bipower variation and median realized volatility estimators are computed. The difference between RV and a jump-robust estimator such as BPV or MedRV provides, when it is statistically significant, an estimate of the sum of squared jumps $\sum_{j=1}^{J(t)} \kappa^2(t_j)$ which have occurred during the period under investigation. Note that a small estimated value for a jump may not be actually a jump but a variation due to the continuous path of the stochastic process and the presence of a jump has thus to be formally tested. BNS (2004, 2006) develop such a testing framework using asymptotic theory on realized variance and multipower variations.[17]

As Andersen *et al.* (2007) put it, "significant" jumps may be identified by comparing realizations of test statistics to a standard normal distribution. They use the test statistic by Huang and Tauchen (2005) to examine the significance of a jump when the chosen jump-robust estimator is the BPV:

$$ZJ_{BPV}(N,d) = \sqrt{N} \frac{(RV_{d,N} - BPV_{d,N})RV_{d,N}^{-1}}{\left((\xi_1^{-4} + 2\xi_1^{-2} - 5)\max\{1, TQ_{d,N}BPV_{d,N}^{-2}\}\right)^{1/2}} \tag{28}$$

with TQ the realized tripower quarticity, which converges in probability to the integrated quarticity. The ratio-statistic in equation (28) has reasonable power against several empirically realistic calibrated stochastic volatility jump diffusion models (Andersen *et al.*, 2007).

The test may be adapted to the MedRV estimator as follows:

$$ZJ_{MedRV}(N,d) = \sqrt{N} \frac{(RV_{d,N} - MedRV_{d,N})RV_{d,N}^{-1}}{\left(0.96\max\{1, MedRQ_{d,N}MedRV_{d,N}^{-2}\}\right)^{1/2}} \tag{29}$$

with $MedRQ$ an estimate of the integrated quarticity obtained by using the same methodology as for $MedRV$. Theodossiou and Zikes (2009) show by means of many simulations and empirical analysis that this test has better properties in the presence of jumps of finite or even infinite activity and zero-returns.[18] If, as we will demonstrate, disentangling jumps from the

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[17]Veraart (2010) studies the limit theory of these estimators in the presence and absence of jumps.

[18]Several other estimators for identifying jumps in the series, such as QRV (Christensen *et al.*, 2010) estimator, which are shown to be more robust in the presence of microstructure noise and zero-returns, may be used. Theodossiou and Zikes (2009) provide a very complete treatment of existing jump detection tests as well as their relative performance in case of microstructure noise and/or jumps. In light of the good properties of the MedRV-based test, we focus on this alternative estimator.

continuous component help in forecasting the density of returns, we are particularly interested in investigating whether the better properties of the test base on MedRV compared to BPV will translate in an improvement in density forecasting as well. A preliminary analysis[19] shows that for both samples "jumpy days" are similar using MedRV instead of BPV, but that the magnitude of jumps is slightly different.

Insert Figure 4

Insert Figure 5

Insert Figure 6

Figures 4 to 6 show the realized volatility, the BPV and the jump component from BPV, as well as the MedRV and the jump component from MedRV for our four series. A preliminary analysis, not reproduced here to conserve space, but available upon request to the authors, shows that for both samples "jumpy days" are quite similar using MedRV instead of BPV, but that the magnitude of jumps is slightly different.

Insert Table 2

More interestingly, Table 2 provides statistics about the contribution of jumps to the total realized volatility for our four series and for different level of significance of the test.[20] These values are in line with results in Huang and Tauchen (2005) and Andersen et al. (2007) for S&P 500 and FX. Our results for WTI are new but very similar to the S&P 500 futures case. Overall, these results point to the fact that jumps contribute to a significant part of the total return variation. Because it is well-known that jumps are not persistent while the continuous component is, considering jumps independently is likely to be rewarded in a forecasting exercise.

Insert Table 3

Table 3 gives the Kolmogorov-Smirnov statistics for different distributions. Results indicate that the Gaussian distribution does not provide a good fit to the series of returns standardized by realized volatility, bipower variation, and median realized volatility. Given these results (where we do not specifically model the jumps), we assume that the conditional distribution of returns is a mixture of Gaussian distributions, in a manner closely related to Maheu and McCurdy (2011).

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[19]Not reproduced here to conserve space, but available upon request to the authors.

[20]The "no threshold" case corresponds to the case measure of jump contribution in Huang and Tauchen (2005) where the difference between RV and BPV is taken directly.

3.2.3 Sequential jump detection

For the purpose of modeling the jump component independently from the continuous component, we need a precise measure of jumps including their size along with their sign. We obtain such a measure using the sequential jump detection procedure developed in Andersen *et al.* (2010).[21] The idea behind this procedure is simple. We first use the standard test presented below using either BPV or MedRV on intraday returns for day d. If we do not reject the existence of a jump for day d, we consider that the largest return in absolute value is a jump and remove it from intraday returns. We replace this "jump" return with the mean of remaining intraday returns and run the jump test again to check the presence of another jump in the same day.

Insert Figure 7

Insert Figure 8

One may wonder whether, in light of the rarity of jumps, sequential detection is useful? To answer this question we first plot in Figures 7 and 8 transactions for S&P 500 and WTI and FX respectively. For each asset, the left panel plots a day with exactly one jump while the right panel plots a day with two or more jumps identified with the sequential procedure.

Insert Table 4

As a second, and more rigorous evidence, we provide in Table 4 statistics about the number of jumps we detect each day for all series. These statistics support the view that days with more than a single jump are quite common.

Using the sequential detection procedure with both the BPV and the MedRV procedure, we obtain for each asset a series of jumps along with their sign and size which will be used in the econometric estimation below. As a rapid check of our results, we compare squared jumps with the squared component resulting from the difference between RV and the jump-robust measure (when significant) and confirm the importance of the sequential procedure.

3.3 Estimation Results

As noted above, we estimate the models on a rolling window of 1,260 daily observations for all series. Concerning the average volatility level for each asset class, we obtain the expected result that the BPV and MedRV estimators are less volatile than the "naive" RV estimator (Table 1). For the S&P500, we find that the logarithmic transformations of the volatility measures

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[21]A quite similar procedure is developed earlier in Andersen *et al.* (2007b).

present a distribution with thin tails. For the WTI futures and the USD/JPY exchange rate, we find evidence of leptokurticity (but globally *less* for $log(BPV_t)$ and $log(MedRV_t)$). Besides, the ranking of volatilities is coherent with our economic intuition: the WTI futures contract is found to be more volatile than the S&P500, and finally the USD/JPY exchange rate.

Concerning the Kolmogorov-Smirnov statistics for daily open-to-close standardized returns, we uncover the adequation to various distributions. The returns are clearly non-Gaussian when looking at the skewness and kurtosis statistics in Table 3. In all models, we unambiguously reject the adequation to the Gaussian law. Due to the asymmetric patterns present in the data, the Student-t distribution is rejected as well. Interestingly, we accept various distributions which account for asymmetry such as the Generalized Hyperbolic and the Mixture of Gaussian distributions. In the estimation of the bivariate models, we focus on the Mixture of Normals because its parameters are easily interpretable in economic terms. In addition, its estimation is easier numerically and thus more adapted to a rolling estimation scheme.

Concerning the presence of jumps, we find for all assets a contribution of jumps to the volatility level (Tables 2 and 4). The contribution of jumps is the highest for the USD/JPY exchange rate (14.45% with no threshold for the detection as in Huang and Tauchen (2005)), followed by the WTI futures contract (6.88%), and finally the S&P500 (5.24%). In terms of frequency, the number of days with jumps is quite important, and strictly positive for all assets. The most common situation is when we observe the occurence of one jump per day. Furthermore, we note that the WTI futures contract is characterized by the highest frequency of jumps (as the percentage of days with jumps is superior to 11%). Finally, it seems that for the USD/JPY the jumps are less frequent than for other assets (6.78%). However, the USD/JPY and the S&P 500 are characterized by the highest number of days with several jumps within the same day. The S&P 500 records 82 days with more than one jump, which is roughly equal to 28% of the number of days with jumps. These ratios are equal to 29% and 44% for, respectively, the WTI futures contract and the USD/JPY exchange rate.

Insert Table 5

Insert Table 6

Insert Table 7

Moving to the estimates of the bivariate models in Tables 5 to 7, we focus our comments on the most interesting parameters. The leverage effect varies depending on the estimates. For the WTI futures contract and the S&P 500, γ is closer to zero when relying on the MedRV or BPV estimators. For the USD/JPY exchange rate, we find that γ is statistically equal to zero based on the MedRV and BPV measures. When looking at the persistence of the HAR components, we find that the ϕ_1, ϕ_2 and ϕ_3 coefficients vary depending on the distributions and the measures of volatility. In the case of the WTI futures contract for instance, the ϕ_1 and ϕ_3 components

lose in persistence when one moves from the RV estimator to the BPV and MedRV estimators. However, the ϕ_2 component gains relatively in persistence. In the case of the S&P 500, if we compare the HAR-MedRV-Mixture with the HAR-MedRV-Jump, we find that the persistence of the ϕ_1 (ϕ_2) component decreases (increases) when moving to the latter model. These effects can be seen as being very specific to the sample data. In addition, the mixture parameter ϕ also varies depending on the volatility measure used. With the WTI futures contract, we find that $\hat{\phi} = 0.449$ with the RV estimator, and $\hat{\phi} = 0.486$ with the BPV estimator. For the remaining parameters, we can notice a relative stability of the results obtained across Tables 5 to 7.

Note that we can also compare the degree of activity of the jump component, i.e. it is possible to rank the assets depending on the intensity and the volatility level of jumps. In the case of the HAR-BPV-Jump model for instance, the USD/JPY exchange rate records the highest intensity of jumps ($\lambda = 1.19$), while the WTI futures contract ranks second ($\lambda = 0.149$), and the S&P 500 exhibits the lowest intensity ($\lambda = 0.138$). Finally, the WTI futures contract exhibits the highest volatility of jumps ($\sigma_x = 0.010$), followed by the S&P 500 ($\sigma_x = 0.006$) and the USD/JPY exchange rate ($\sigma_x = 0.002$) for the HAR-BPV-Jump model. Thus, we uncover that there are different types of jumps (i.e. either frequent and small jumps, or less frequent and large jumps) specific to each asset class considered in this article.

3.4 Forecast accuracy

Insert Table 8

In terms of forecast accuracy, we find that the bivariate models which take explicity into account the jumps tend to perform better than the models based on a Gaussian distribution. This comment is valid for all assets. For the S&P 500, the BPV and MedRV Gaussian models are dominated by the RV Gaussian model. For instance, in Table 8, the Diebold-Mariano test statistic is equal to 2.878 for the Realized Gaussian vs. Bipower Gaussian models at a 30-day horizon. Besides, we confirm the result by Maheu and McCurdy (2011) that the EGARCH model is dominated by the Realized Gaussian model (with a Diebold-Mariano test statistic equal to -3.050 at the 10-day horizon). More generally, at the 5- and 10-day horizons, all non Gaussian models are equivalent, whatever the volatility measure. In Table 8, it appears interesting to note that the Bipower Jump model beats the MedRV Jump model at all horizons. At the more distant horizons (up to 60-day), we can conclude that the Bipower Jump model dominates all other models.

Insert Table 9

When looking at the WTI futures contract, we uncover the same effects as in Maheu and McCurdy (2011), and regarding the comparison of the BPV/MedRV models compared to the Realized Gaussian model. For instance, the Diebold-Mariano test statistic is equal to 4.717 for

the Realized Gaussian vs. Bipower Gaussian models at a 30-day horizon. At the 5-day horizon, all non Gaussian models either are equivalent or dominate Gaussian models according to the Diebold-Mariano test statistics in Table 9. For the horizons superior to 10-day, the Realized MN model dominates the Bipower MN and MedRV MN models. In addition, we note that the BPV Jump model dominates the MedRV model at all horizons.

Insert Table 10

Finally, the results of the Diebold-Mariano test statistics for the USD/JPY exchange rate are presented in Table 10. They show, by and large, that the Bipower Gaussian and MedRV Gaussian models dominate the Realized Gaussian model, which is equivalent to or preferred to the EGARCH model for any horizon superior to 30-day. The Bipower Jump and MedRV Jump models dominate all other models, while MedRV tends to dominate BPV at all horizons in that latter case.

Insert Figure 9

Insert Figure 10

Insert Figure 11

The same insights can be gathered by looking at the average predictive likelihood for the density of the three assets at various horizons (up to 60-day) in Figures 9 to 11. When looking at these graphs, recall that for the interpretation of the Diebold-Mariano test statistic, a significant positive (negative) estimated value rejects the null of equal performance between competing forecasts, and provides evidence in favor of model $\mathcal{A}$ ($\mathcal{B}$). These graphs confirm the highly superior forecast accuracy of models based on intraday data for all of our series. This was the main result in Maheu and McCurdy (2011), and we confirm their findings for the three assets under investigation. For the S&P 500, we observe the superiority of BPV Jump and MedRV Jump estimators for density forecasting particularly at horizons of 20 to 60 days. BPV and MedRV estimators also have good performance for the WTI in comparison with models using realized volatility estimated using the mixture of normals. This indicates that the jump component either included in the total realized volatility (using the mixture of normals) or modeled separately provides information in forecasting the density of returns. Results are very similar for the USD/JPY. The superiority of the BPV Jump and the MedRV Jump is evident at all horizons.

Overall, the empirical results obtained concerning the forecast accuracy depend on the asset under consideration, and the nature of the activity of the jump component. Departing from Maheu and McCurdy (2011), our results therefore tend to provide a deeper understanding of the effects at stake when decomposing the jump and continuous components of volatility. Namely,

19

modeling explicitly jumps is of primary importance to achieve better performances with bivariate models, while decomposing between jumps and the continuous component volatility appears of secondary importance.

4   Conclusion

Jumps in assets' returns are to be related to strong directional variations in the prices that the market participants agree on given their evaluation of the perspectives for a given financial asset.[22] On the contrary, volatility reflects a certain lack of agreement within financial markets: as they do not lead to strong directional variations but up-and-down swings in prices, they should not be mistaken with jumps. In such cases, jumps and volatility do not receive the attention they deserve from an econometric viewpoint.

This paper has examined the forecasting power of jumps in addition, specifically, to the continuous component when the *density* of returns is the variable of interest. Whilst numerous authors have considered the informational content of continuous *vs.* jump components for volatility forecasting, none have thought to address the particular question of density forecasting. Detection of such information for different classes of assets would indicate the ability of new econometric models to anticipate the evolution of density returns in a fundamentally different way compared to more traditional forecasting models.

Our results unveil new effects regarding the importance of distinguishing between the continuous and jump components of volatility. In this regard, this article specifically extends the findings by Maheu and McCurdy (2011) by considering various bivariate models with/without jumps. The empirical application is devoted to three types of assets: the S&P 500, the WTI futures contract, and the USD/JPY exchange rate. The main results may be summarized as follows. First, we confirm the findings by Maheu and McCurdy (2011) that intraday data yield better densities forecasts than daily data. Second, and more importantly, we assess the importance of jumps when forecasting the density of returns by comparing jump-robust (bipower variation, median realized volatility) and non-robust measures of realized volatilities. Compared to the "naive" measure of realized volatility, considering jumps specifically provides significant improvement on the accuracy of forecasts of return densities.

Two central concluding remarks arise. First, we have shown in this paper that the explicit modelling of jumps is central in the estimation of bivariate models in the fashion of Maheu and McCurdy (2011). Such an explicit modeling task yields better performances, as shown in our empirical application and with the forecast accuracy tests of all competing models. Second, discriminating between the continuous component of volatility and jumps appears comparatively less important across our estimates. From that perspectives, our results can be seen as an extension of the contribution by Maheu and McCurdy (2011).

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[22]Lahaye *et al.* (2011) and Evans (2011) have related news to the occurrence of jumps in intraday data. Nevertheless, some jumps are not related to macro announcements or news and may be the consequence of microstructure effects.

Other jump detection techniques may be used (see Boudt *et al.* (2011), or Christensen *et al.* (2010) among others). Boudt *et al.* (2011) in particular provide very interesting empirical results because their measure take the intraday periodicity into account and thus does not over-detect jumps in low-volatility periods and does not under-detect jumps in periods of high-volatility. Nevertheless, collectively taken, our results are sufficiently strong so that we can believe they would be robust to alternative jump detection methods.

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Figures

Figure 1: Raw time-series (left panel) and open-to-close standardized log-returns (right panel)
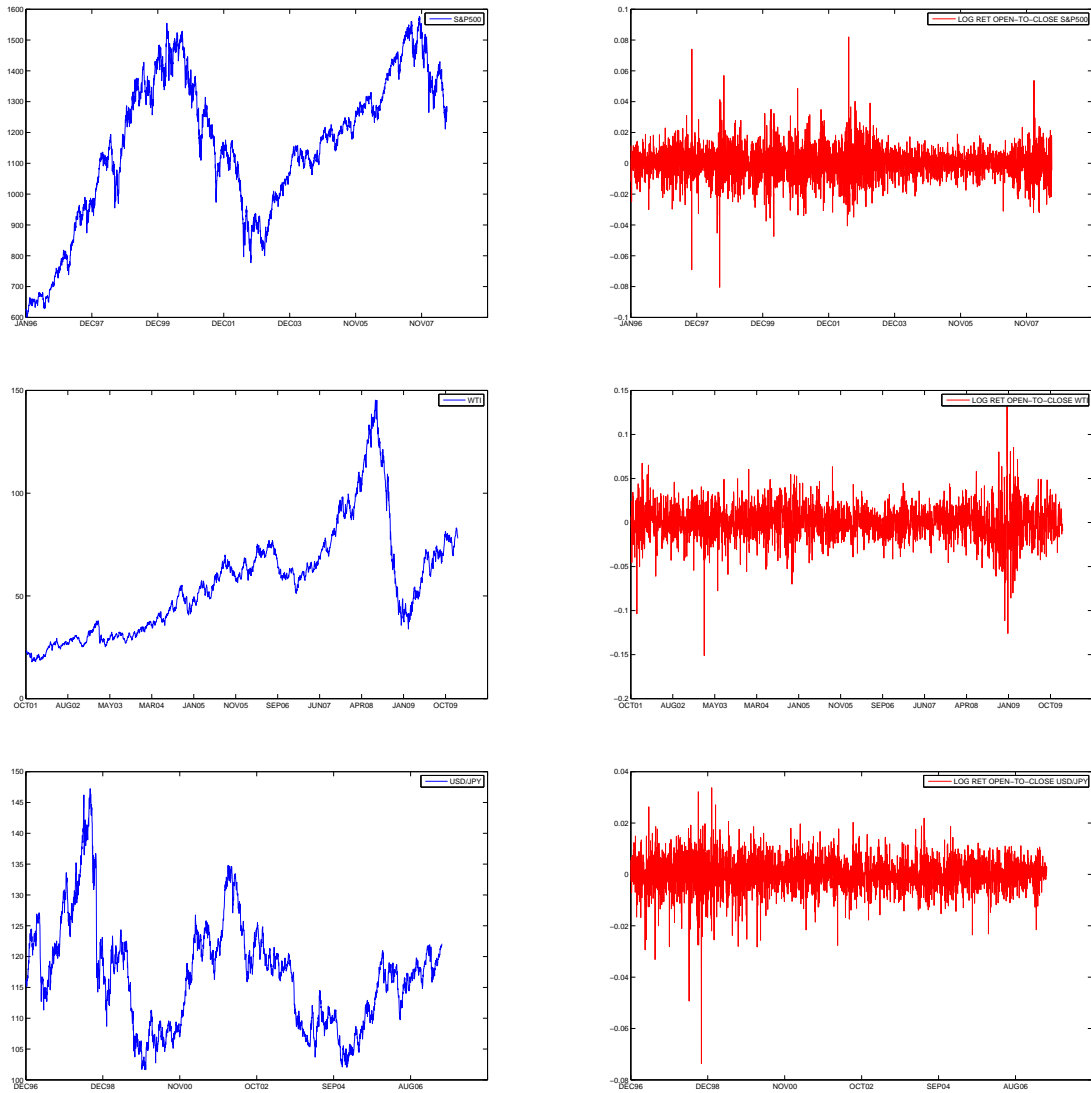
Figure 2: Volatility signature plot for the oil futures contract using front month rollover and the realized volatility, bipower variation and median realized estimators (2001-2006).
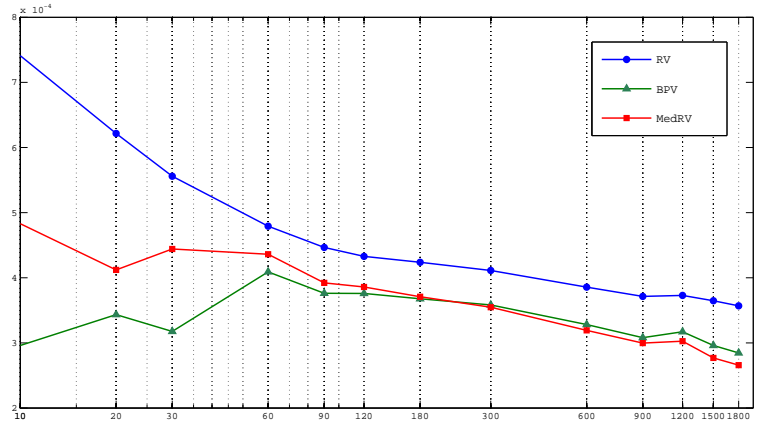
Figure 3: Volatility signature plot for the oil futures contract using front month rollover and the realized volatility, bipower variation and median realized estimators (2006-2010).
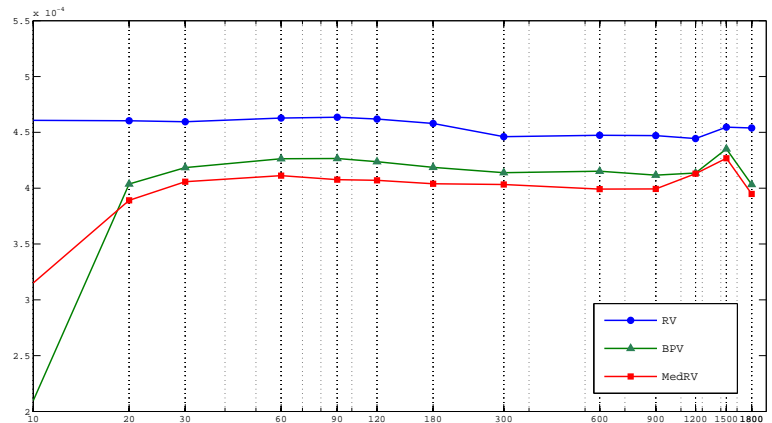
Figure 4: Realized volatility, bi-power variation with jump component, and median realized volatility with jump component for S&P500 futures (from top to bottom and left to right).
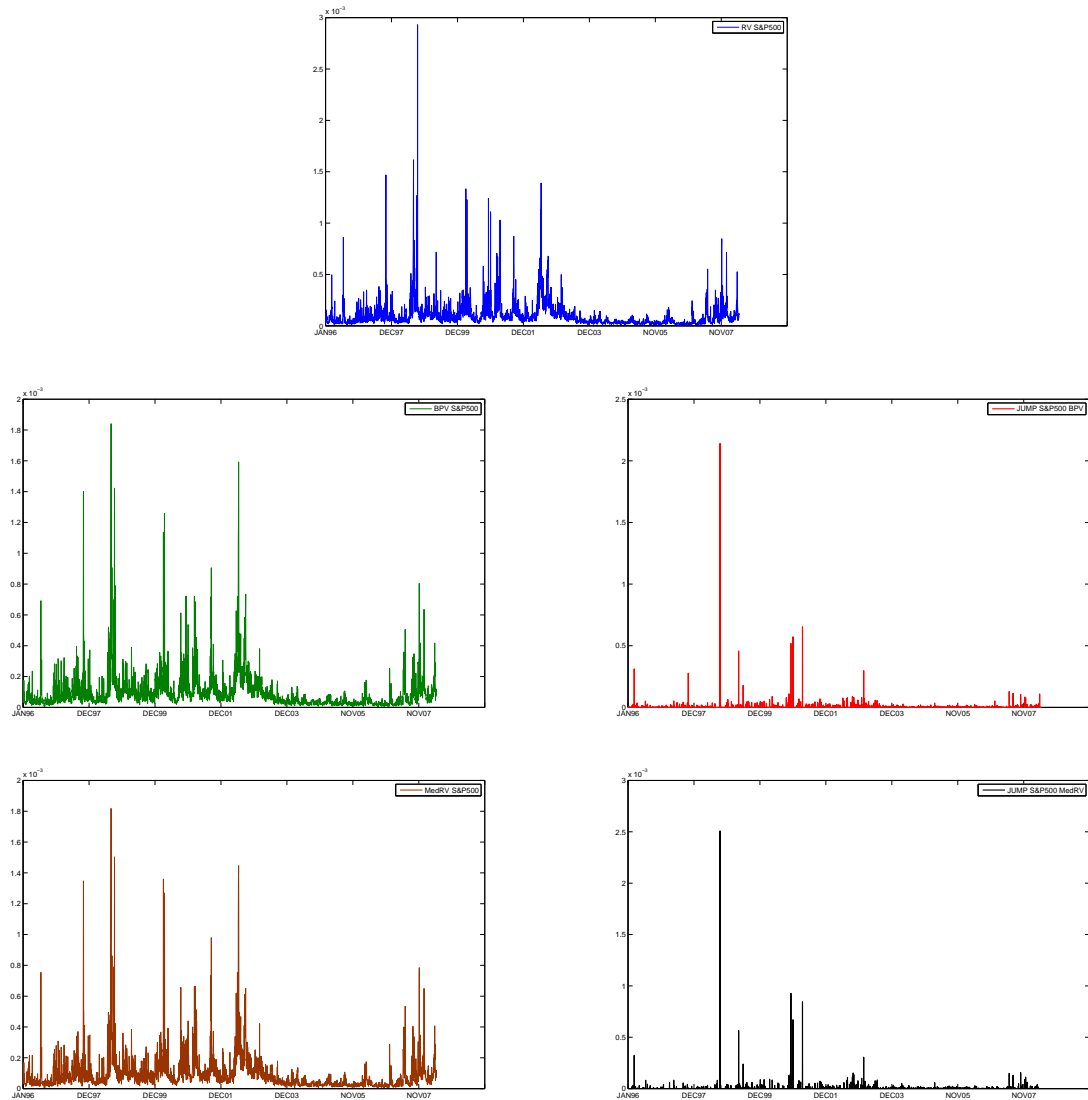
Figure 5: Realized volatility, bi-power variation with jump component, and median realized volatility with jump component for WTI futures (from top to bottom and left to right)
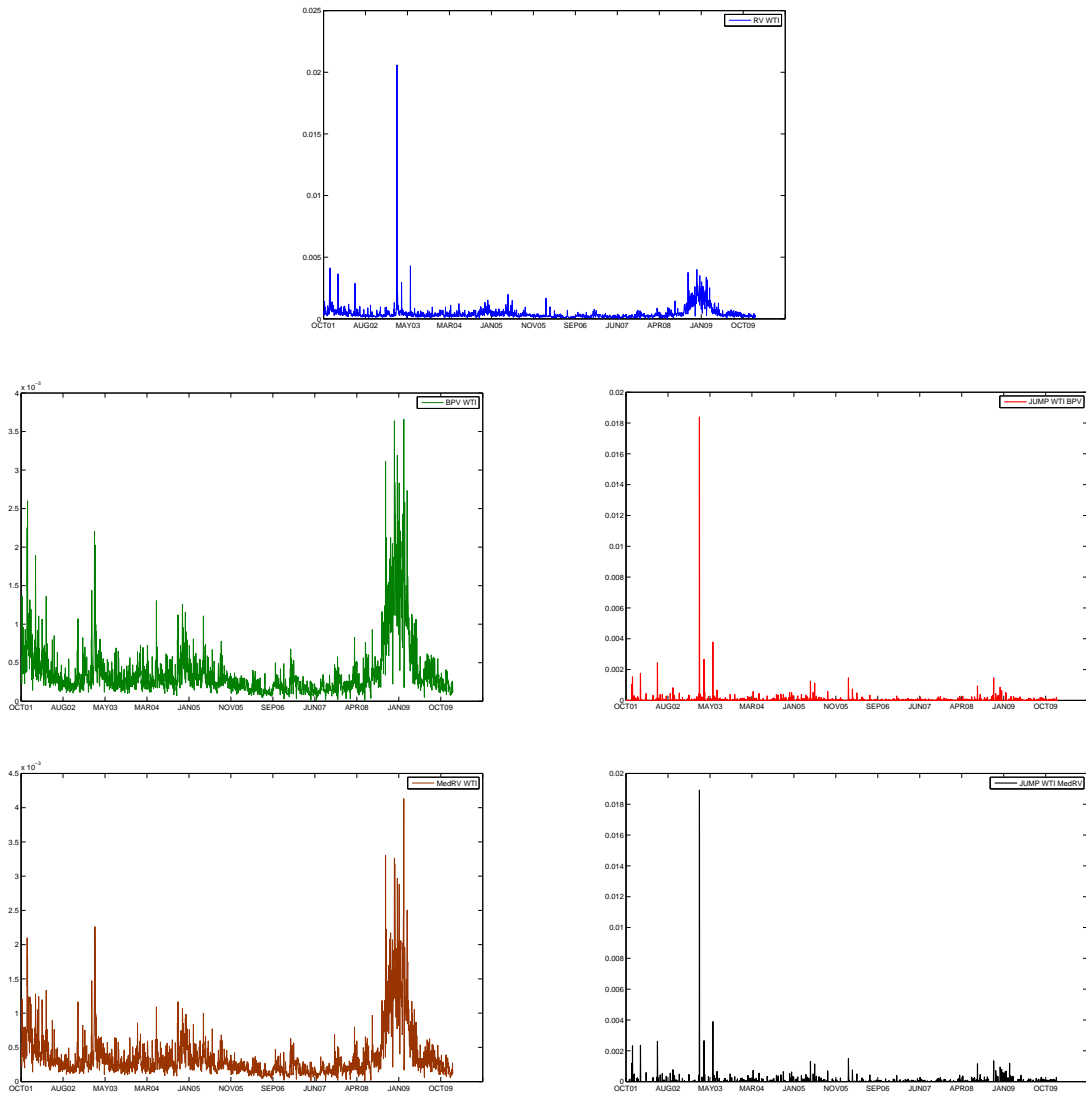
Figure 6: Realized volatility, bi-power variation with jump component, and median realized volatility with jump component for USD/JPY (from top to bottom and left to right).
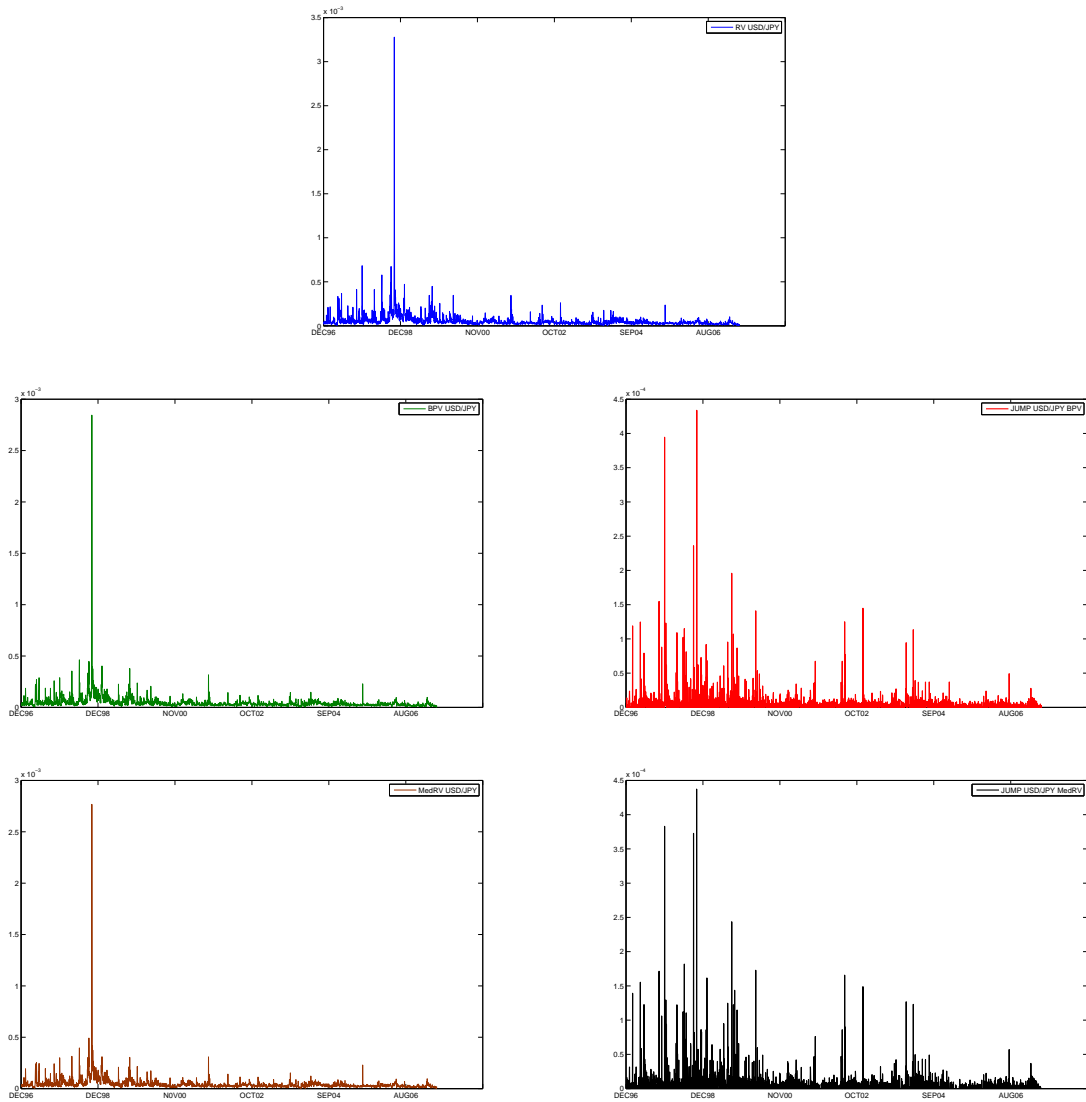
Figure 7: Jumpy days

29
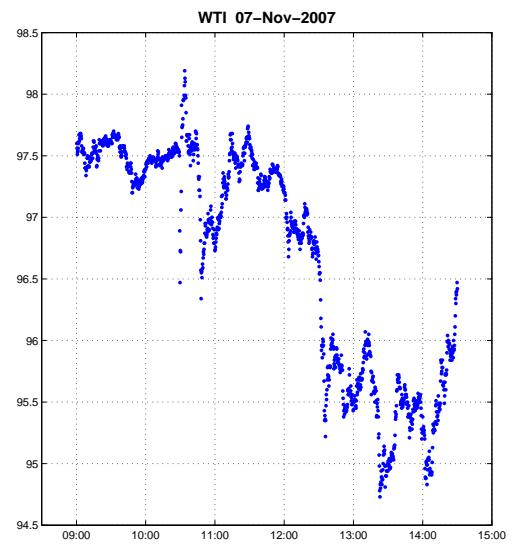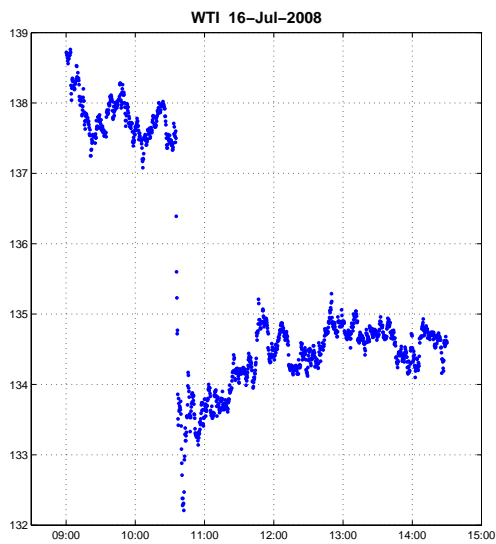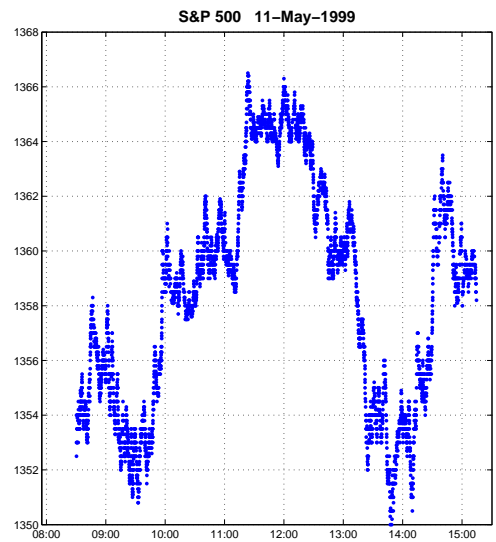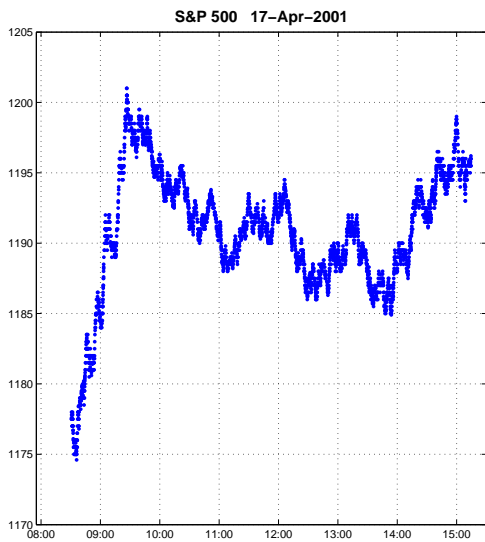
Figure 8: Jumpy days
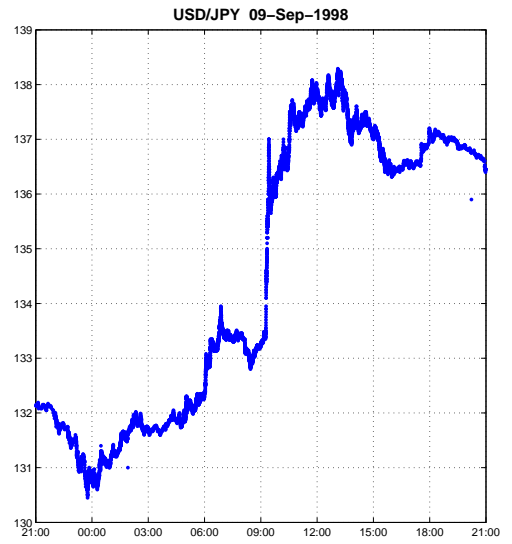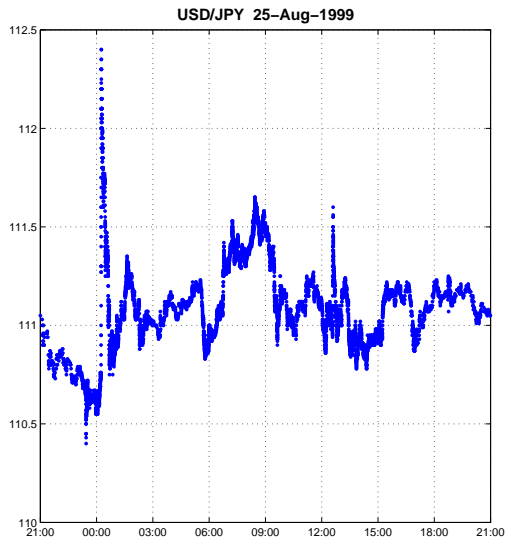
Figure 9: Average predictive likelihood based on Amisano and Giacomini (2007) for the density of S&P 500 futures
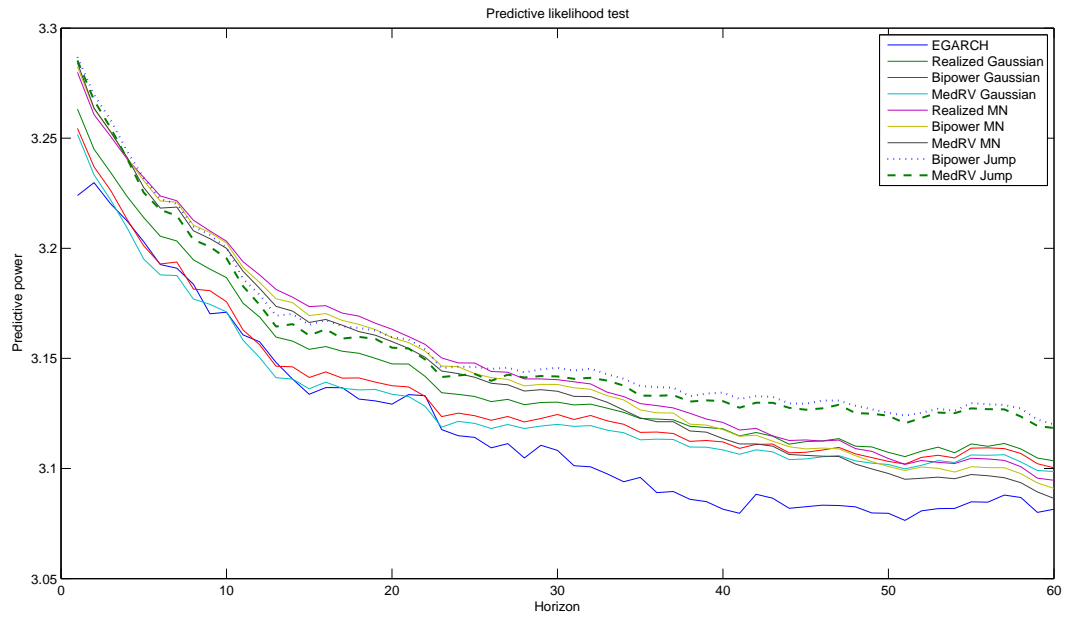
Figure 10: Average predictive likelihood based on Amisano and Giacomini (2007) for the density of WTI futures
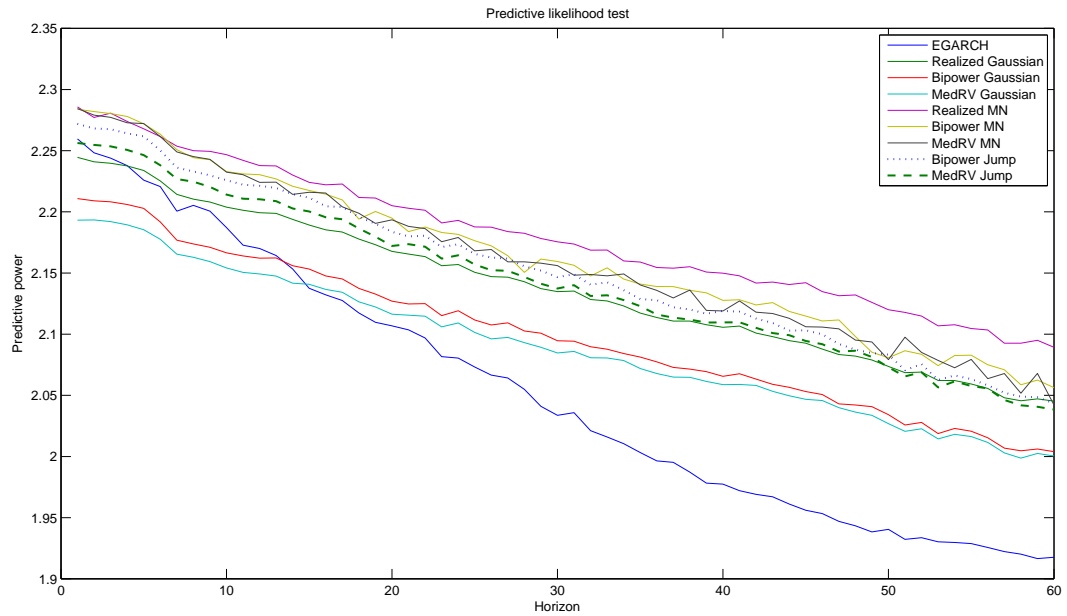
Figure 11: Average predictive likelihood based on Amisano and Giacomini (2007) for the density of USD/JPY
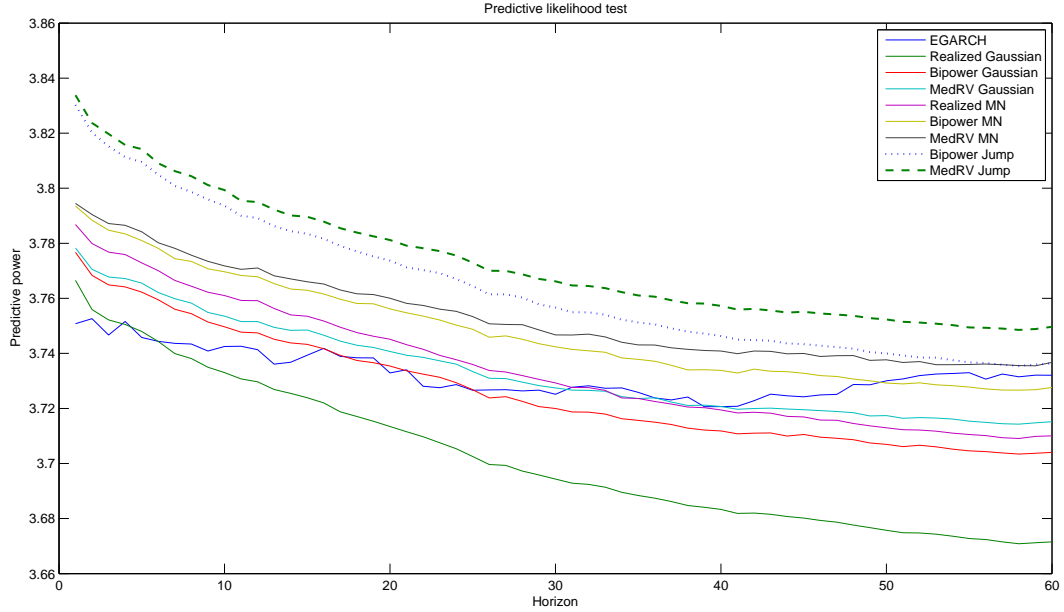
Tables

Table 1: Summary Statistics

S&P500	R_t	RV_t	BPV_t	$MedRV_t$	$\log(RV_t)$	$\log(BPV_t)$	$\log(MedRV_t)$
Mean	0.0515	0.1601	0.1559	0.1544	-9.6046	-9.6671	-9.694
Std. Dev.	0.183	0.0021	0.0019	0.0019	0.8759	0.892	0.9012
Skewness	-0.1334	6.8497	5.2274	5.3659	0.2984	0.2674	0.255
Kurtosis	3.2819	87.9456	44.1957	45.6785	0.0514	-0.0452	-0.0485
Observations	3135						

WTI	R_t	RV_t	BPV_t	$MedRV_t$	$\log(RV_t)$	$\log(BPV_t)$	$\log(MedRV_t)$
Mean	0.1445	0.3061	0.2954	0.2896	-8.1738	-8.2469	-8.2860
Std. Dev.	0.3846	0.0061	0.0057	0.0055	0.6940	0.6971	0.6975
Skewness	-0.1724	3.8354	3.9463	3.8867	0.6875	0.6670	0.6506
Kurtosis	3.2218	19.9500	22.0793	20.8598	0.7653	0.7854	0.7764
Observations	2058						

USD/JPY	R_t	RV_t	BPV_t	$MedRV_t$	$\log(RV_t)$	$\log(BPV_t)$	$\log(MedRV_t)$
Mean	0.0039	0.1155	0.1068	0.1044	-10.1308	-10.2755	-10.3121
Std. Dev.	0.1087	0.0014	0.0011	0.0011	0.6756	0.6706	0.6561
Skewness	-0.8422	22.2219	23.9665	24.9671	0.7511	0.6286	0.6807
Kurtosis	7.343	770.7384	872.5957	924.6845	1.9751	1.95	1.9959
Observations	2703						

Note: Mean values are given as the mean of the annualized squared root values.

Table 2: Relative contribution of jumps in percents computed as the ratio between the difference between RV and BPV, when significant at a given threshold, and RV.

Contribution of jumps	S&P 500	WTI crude oil	USD/JPY
0.1% threshold	2.65%	2.61%	7.79%
0.5% threshold	3.21%	3.37%	9.22%
1% threshold	3.56%	3.81%	9.86%
5% threshold	4.53%	5.06%	11.88%
no threshold	5.24%	6.88%	14.45%

Table 3: Kolmogorov-Smirnov statistics for daily open-to-close standardized returns. The distributions considered are the Gaussian, the symmetric Student-t, the Generalized Hyperbolic (GH), the Normal Inverse Gaussian (NIG), the Hyperbolic (H) and the Mixture of Normals (MN). The values in the table are p-values.

	S&P500			WTI		
	RV_t	BPV_t	$MedRV_t$	RV_t	BPV_t	$MedRV_t$
Gaussian	0	0	0	0	0	0
Student-t	0	0	0	0	0	0
Generalized Hyperbolic	0.93	0.42	0.15	0.94	0.55	0.95
Normal Inverse Gaussian	0.67	0.51	0.15	0.69	0.75	0.95
Hyperbolic	0.97	0.5	0.07	0.26	0.95	0.44
Mixture of Gaussian	0.25	0.45	0.58	0.84	0.95	0.96

	USD/JPY		
	RV_t	BPV_t	$MedRV_t$
Gaussian	0.01	0.38	0.36
Student-t	0	0.21	0.67
Generalized Hyperbolic	0.65	0.28	0.61
Normal Inverse Gaussian	0.28	0.63	0.88
Hyperbolic	NA	NA	NA
Mixture of Gaussian	0.21	0.53	0.67

Table 4: Descriptive statistics about jumps extracted using the sequential procedure in Andersen et al. (2010). The chosen significance threshold is 1%.

Jump descriptive statistics	S&P 500	WTI crude oil	USD/JPY
Total number of jumps	433	329	308
Number of days with jump(s)	297	232	183
Percentage of days with jump(s)	9.47%	11.15%	6.78%
Average duration between two jumps (days)	7.24	6.33	8.76
Average duration between two days with jump(s)	10.56	8.97	14.76
Number of days with exactly 1 jump	215	165	102
Percentage of days with exactly 1 jump	6.86%	7.93%	3.78%
Number of days with exactly 2 jump	50	48	49
Percentage of days with exactly 2 jump	1.59%	2.31%	1.81%
Number of days with exactly 3 jump	18	10	20
Percentage of days with exactly 3 jump	0.57%	0.48%	0.74%
Number of days with exactly 4 jump	6	7	12
Percentage of days with exactly 4 jump	0.19%	0.34%	0.44%
Number of days with exactly 5 jump	8	2	-
Percentage of days with exactly 5 jump	0.26%	0.09%	0%

Table 5: Estimation results for S&P500 Futures.

EGARCH	ω	α	θ	β
Average	-0.328	-0.129	0.078	0.970
Standard Dev.	0.196	0.038	0.027	0.021
Skewness	-0.554	-0.415	-0.097	-0.697
Kurtosis	-0.953	-1.104	0.372	-0.817
5% quantile	-0.681	-0.190	0.033	0.931
95% quantile	-0.087	-0.079	0.123	0.993

HAR-RV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-0.956	0.283	0.432	0.185	-0.105	0.000	0.445
Standard Dev.	0.429	0.045	0.072	0.023	0.013	0.000	0.037
Skewness	-0.260	-0.484	0.374	-0.501	-0.463	0.479	0.611
Kurtosis	-1.327	-1.170	-1.320	-0.846	-1.022	-1.191	-0.698
5% quantile	-1.674	0.210	0.341	0.142	-0.128	0.000	0.397
95% quantile	-0.409	0.341	0.550	0.216	-0.088	0.000	0.520

HAR-BPV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-0.886	0.294	0.436	0.178	-0.101	0.000	0.441
Standard Dev.	0.368	0.043	0.072	0.018	0.014	0.000	0.033
Skewness	-0.229	-0.404	0.232	-0.372	-0.511	0.479	0.836
Kurtosis	-1.337	-1.256	-1.383	-0.577	-1.031	-1.191	-0.328
5% quantile	-1.500	0.227	0.342	0.147	-0.126	0.000	0.402
95% quantile	-0.420	0.349	0.546	0.202	-0.084	0.000	0.512

HAR-MedRV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-0.894	0.297	0.428	0.182	-0.099	0.000	0.450
Standard Dev.	0.356	0.040	0.076	0.017	0.013	0.000	0.031
Skewness	-0.200	-0.447	0.209	0.207	-0.381	0.479	0.848
Kurtosis	-1.366	-1.085	-1.301	-0.262	-1.197	-1.191	-0.302
5% quantile	-1.484	0.227	0.324	0.156	-0.121	0.000	0.414
95% quantile	-0.443	0.349	0.546	0.208	-0.082	0.000	0.518

Table 5 (continued): Estimation results for S&P500 Futures.

HAR-RV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-0.945	0.283	0.433	0.188	-0.104	0.267	-1.071	1.134	0.302	1.098	0.000	0.445
Standard Dev.	0.436	0.042	0.071	0.022	0.011	0.187	0.586	0.215	0.164	0.204	0.000	0.037
Skewness	-0.288	-0.516	0.423	-0.313	-0.293	0.601	-1.289	4.262	0.541	1.204	0.479	0.612
Kurtosis	-1.319	-1.028	-1.254	-0.940	-1.117	-0.828	2.971	21.666	-0.609	0.249	-1.191	-0.697
5% quantile	-1.673	0.211	0.345	0.149	-0.122	0.052	-1.917	0.999	0.104	0.911	0.000	0.397
95% quantile	-0.381	0.338	0.553	0.221	-0.087	0.623	-0.380	1.368	0.625	1.554	0.000	0.520

HAR-BPV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-0.889	0.296	0.436	0.178	-0.099	0.235	-1.143	1.073	0.318	1.007	0.000	0.441
Standard Dev.	0.368	0.040	0.070	0.017	0.012	0.125	0.343	0.096	0.125	0.119	0.000	0.033
Skewness	-0.262	-0.449	0.270	-0.411	-0.388	0.886	-0.087	0.791	0.844	1.398	0.479	0.838
Kurtosis	-1.327	-1.100	-1.325	-0.569	-1.125	-0.226	-0.681	0.247	-0.210	0.906	-1.191	-0.326
5% quantile	-1.503	0.228	0.344	0.150	-0.119	0.094	-1.662	0.957	0.173	0.899	0.000	0.402
95% quantile	-0.429	0.350	0.547	0.202	-0.083	0.500	-0.582	1.273	0.583	1.289	0.000	0.512

HAR-MedRV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-0.902	0.301	0.428	0.181	-0.096	0.236	-1.172	1.068	0.333	0.989	0.000	0.450
Standard Dev.	0.353	0.039	0.074	0.015	0.012	0.114	0.319	0.084	0.119	0.106	0.000	0.031
Skewness	-0.243	-0.557	0.254	0.093	-0.261	0.858	-0.086	0.840	0.804	1.309	0.479	0.850
Kurtosis	-1.351	-0.847	-1.243	-0.424	-1.272	-0.297	-0.558	0.069	-0.294	0.479	-1.191	-0.300
5% quantile	-1.489	0.228	0.327	0.158	-0.114	0.102	-1.655	0.973	0.188	0.890	0.000	0.414
95% quantile	-0.463	0.351	0.546	0.206	-0.081	0.469	-0.653	1.244	0.575	1.228	0.000	0.518

HAR-BPV-Jump	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η	λ	μ_z	σ_z
Average	-0.886	0.291	0.440	0.177	-0.101	0.000	0.441	0.138	0.000	0.006
Standard Dev.	0.376	0.042	0.072	0.019	0.013	0.000	0.033	0.035	0.000	0.002
Skewness	-0.259	-0.410	0.209	-0.190	-0.376	0.514	0.848	0.484	0.399	-0.229
Kurtosis	-1.284	-1.180	-1.407	-0.685	-1.027	-1.122	-0.319	-1.363	0.308	-1.534
5% quantile	-1.524	0.224	0.343	0.144	-0.123	0.000	0.402	0.102	0.000	0.003
95% quantile	-0.402	0.347	0.551	0.204	-0.083	0.000	0.512	0.196	0.001	0.008

HAR-MedRV-Jump	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η	λ	μ_z	σ_z
Average	-0.892	0.296	0.431	0.181	-0.099	0.000	0.450	0.138	0.000	0.006
Standard Dev.	0.362	0.040	0.077	0.019	0.013	0.000	0.031	0.035	0.000	0.002
Skewness	-0.238	-0.443	0.173	0.224	-0.251	0.514	0.863	0.484	0.399	-0.229
Kurtosis	-1.306	-1.034	-1.338	0.012	-1.207	-1.122	-0.287	-1.363	0.308	-1.534
5% quantile	-1.504	0.226	0.325	0.150	-0.119	0.000	0.414	0.102	0.000	0.003
95% quantile	-0.428	0.348	0.549	0.209	-0.081	0.000	0.518	0.196	0.001	0.008

Table 6: Estimation results for WTI Futures.

EGARCH	ω	α	θ	β
Average	-0.563	-0.078	0.104	0.938
Standard Dev.	0.404	0.029	0.014	0.052
Skewness	-0.433	-0.723	-0.525	-0.458
Kurtosis	-1.014	-0.292	0.728	-0.970
5% quantile	-1.240	-0.128	0.075	0.849
95% quantile	-0.149	-0.047	0.123	0.990

HAR-RV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-0.736	0.076	0.543	0.294	-0.041	0.001	0.422
Standard Dev.	0.444	0.007	0.033	0.026	0.003	0.000	0.004
Skewness	-0.072	0.104	-0.299	-0.267	-0.225	0.260	-0.257
Kurtosis	-1.842	0.484	-1.487	-0.954	0.138	-1.016	-0.898
5% quantile	-1.324	0.064	0.490	0.250	-0.047	0.000	0.414
95% quantile	-0.255	0.086	0.582	0.331	-0.036	0.001	0.428

HAR-BPV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-0.698	0.085	0.543	0.289	-0.038	0.001	0.410
Standard Dev.	0.409	0.011	0.051	0.017	0.004	0.000	0.003
Skewness	-0.080	-0.121	-0.089	-0.883	-0.259	0.260	-0.262
Kurtosis	-1.850	-0.099	-1.784	1.708	-0.204	-1.016	-0.106
5% quantile	-1.232	0.066	0.476	0.258	-0.045	0.000	0.405
95% quantile	-0.258	0.101	0.605	0.314	-0.031	0.001	0.414

HAR-MedRV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-0.710	0.089	0.524	0.304	-0.036	0.001	0.413
Standard Dev.	0.419	0.020	0.061	0.018	0.004	0.000	0.003
Skewness	-0.107	0.152	-0.123	-0.347	-0.124	0.260	-0.014
Kurtosis	-1.818	-1.498	-1.813	-0.031	-0.365	-1.016	-0.889
5% quantile	-1.277	0.061	0.442	0.275	-0.042	0.000	0.409
95% quantile	-0.262	0.117	0.595	0.331	-0.029	0.001	0.417

Table 6 (continued): Estimation results for WTI Futures.

HAR-RV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-0.693	0.100	0.524	0.298	-0.038	0.449	-0.763	1.057	0.626	1.238	0.001	0.422
Standard Dev.	0.395	0.027	0.068	0.049	0.005	0.173	0.267	0.119	0.259	0.217	0.000	0.004
Skewness	-0.232	-0.112	-0.493	-0.215	-0.004	-0.654	-1.672	5.545	-0.272	0.828	0.260	-0.242
Kurtosis	-1.403	2.986	1.660	3.405	3.367	-0.245	3.595	32.467	-0.434	1.303	-1.016	-0.851
5% quantile	-1.282	0.060	0.413	0.223	-0.046	0.098	-1.323	1.000	0.160	0.944	0.000	0.414
95% quantile	-0.201	0.140	0.616	0.373	-0.030	0.677	-0.471	1.118	1.011	1.623	0.001	0.428

HAR-BPV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-0.695	0.119	0.540	0.263	-0.032	0.486	-0.757	1.084	0.762	1.325	0.001	0.410
Standard Dev.	0.411	0.041	0.068	0.054	0.009	0.219	0.324	0.178	0.379	0.350	0.000	0.003
Skewness	-0.319	0.464	-0.474	-0.407	-0.182	-0.636	-1.446	3.648	-0.239	1.629	0.260	-0.041
Kurtosis	-0.898	2.343	1.962	2.900	8.250	-0.586	2.359	12.512	-0.572	6.993	-1.016	0.471
5% quantile	-1.348	0.060	0.424	0.175	-0.044	0.046	-1.462	1.001	0.050	0.914	0.000	0.405
95% quantile	-0.144	0.190	0.656	0.344	-0.020	0.769	-0.385	1.576	1.316	1.949	0.001	0.415

HAR-MedRV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-0.666	0.110	0.543	0.273	-0.030	0.518	-0.748	1.054	0.832	1.300	0.001	0.413
Standard Dev.	0.383	0.036	0.062	0.041	0.007	0.164	0.239	0.100	0.299	0.274	0.000	0.003
Skewness	-0.261	0.261	-0.610	0.584	-0.709	-0.841	-1.572	5.561	-0.375	0.972	0.260	-0.049
Kurtosis	-0.989	2.461	3.723	1.194	6.061	0.689	4.999	31.912	0.094	0.898	-1.016	-0.854
5% quantile	-1.247	0.051	0.447	0.215	-0.040	0.186	-1.162	1.007	0.280	0.952	0.000	0.409
95% quantile	-0.161	0.167	0.645	0.346	-0.021	0.743	-0.427	1.066	1.274	1.820	0.001	0.417

HAR-BPV-Jump	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η	λ	μ_z	σ_z
Average	-0.704	0.084	0.544	0.288	-0.038	0.001	0.410	0.149	-0.001	0.010
Standard Dev.	0.412	0.011	0.051	0.018	0.004	0.000	0.003	0.014	0.001	0.000
Skewness	-0.106	-0.072	-0.015	-0.549	-0.256	0.260	-0.260	-0.308	-0.199	0.479
Kurtosis	-1.814	0.113	-1.676	0.548	-0.167	-1.016	-0.115	-0.088	-0.694	-1.117
5% quantile	-1.273	0.065	0.478	0.254	-0.044	0.000	0.405	0.121	-0.003	0.010
95% quantile	-0.261	0.101	0.612	0.315	-0.031	0.001	0.414	0.171	0.000	0.011

HAR-MedRV-Jump	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η	λ	μ_z	σ_z
Average	-0.719	0.088	0.523	0.303	-0.036	0.001	0.413	0.156	-0.002	0.011
Standard Dev.	0.419	0.020	0.060	0.018	0.004	0.000	0.003	0.015	0.001	0.000
Skewness	-0.112	0.129	-0.094	-0.143	-0.143	0.260	-0.013	-0.544	0.302	0.432
Kurtosis	-1.806	-1.473	-1.779	-0.264	-0.274	-1.016	-0.874	0.057	-0.827	-1.277
5% quantile	-1.292	0.062	0.444	0.275	-0.042	0.000	0.409	0.123	-0.003	0.010
95% quantile	-0.265	0.116	0.596	0.333	-0.029	0.001	0.417	0.177	-0.001	0.012

Table 7: Estimation results for USD/JPY.

EGARCH	ω	α	θ	β
Average	-0.734	-0.047	0.103	0.934
Standard Dev.	0.583	0.060	0.060	0.056
Skewness	-1.386	-1.314	1.066	-1.442
Kurtosis	0.729	0.173	-0.669	0.905
5% quantile	-2.098	-0.183	0.056	0.800
95% quantile	-0.160	-0.002	0.214	0.988

HAR-RV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-1.943	0.251	0.286	0.277	-0.050	0.000	0.449
Standard Dev.	0.343	0.030	0.050	0.055	0.046	0.000	0.021
Skewness	-0.061	-0.017	0.085	0.444	-0.353	-0.039	0.984
Kurtosis	-0.856	-1.393	-1.355	-0.154	-1.625	-0.278	0.090
5% quantile	-2.546	0.209	0.216	0.194	-0.116	0.000	0.425
95% quantile	-1.395	0.294	0.363	0.391	-0.003	0.000	0.493

HAR-BPV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-1.743	0.283	0.315	0.237	-0.039	0.000	0.433
Standard Dev.	0.305	0.023	0.044	0.041	0.040	0.000	0.018
Skewness	-0.221	0.105	-0.081	0.684	-0.344	-0.039	1.154
Kurtosis	-0.801	-1.371	-1.370	0.409	-1.638	-0.278	0.637
5% quantile	-2.318	0.249	0.249	0.178	-0.098	0.000	0.413
95% quantile	-1.274	0.316	0.382	0.327	0.003	0.000	0.473

HAR-MedRV-Gaussian	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η
Average	-1.742	0.292	0.319	0.224	-0.035	0.000	0.420
Standard Dev.	0.320	0.020	0.042	0.035	0.037	0.000	0.019
Skewness	-0.105	0.088	-0.194	1.017	-0.358	-0.039	1.170
Kurtosis	-0.930	-1.065	-1.346	1.147	-1.593	-0.278	0.661
5% quantile	-2.306	0.259	0.252	0.175	-0.091	0.000	0.399
95% quantile	-1.242	0.321	0.376	0.308	0.005	0.000	0.463

Table **7** (continued): Estimation results for USD/JPY.

HAR-RV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-1.850	0.249	0.296	0.279	-0.047	0.150	-0.232	1.709	0.037	1.080	0.000	0.449
Standard Dev.	0.375	0.034	0.046	0.049	0.042	0.031	0.134	0.099	0.012	0.023	0.000	0.021
Skewness	-0.009	0.095	0.240	0.596	-0.321	0.601	-1.678	1.121	0.985	0.365	-0.039	0.984
Kurtosis	-1.145	-1.237	-0.926	0.280	-1.653	3.067	1.987	1.728	0.682	3.174	-0.278	0.090
5% quantile	-2.479	0.200	0.231	0.204	-0.107	0.095	-0.537	1.592	0.025	1.037	0.000	0.425
95% quantile	-1.302	0.302	0.374	0.389	-0.002	0.194	-0.125	1.921	0.062	1.111	0.000	0.493

HAR-BPV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-1.669	0.285	0.325	0.233	-0.035	0.159	-0.279	1.600	0.039	1.085	0.000	0.433
Standard Dev.	0.354	0.025	0.043	0.036	0.037	0.050	0.256	0.117	0.017	0.041	0.000	0.018
Skewness	0.134	0.371	0.034	0.607	-0.306	0.341	-2.118	0.844	0.167	-0.001	-0.039	1.155
Kurtosis	-1.025	-0.691	-0.970	0.652	-1.672	1.056	4.770	0.543	0.867	1.323	-0.278	0.637
5% quantile	-2.252	0.249	0.259	0.181	-0.087	0.073	-0.865	1.446	0.002	1.010	0.000	0.413
95% quantile	-1.097	0.324	0.392	0.310	0.004	0.253	-0.006	1.830	0.070	1.157	0.000	0.473

HAR-MedRV-Mixture	ω	ϕ_1	ϕ_2	ϕ_3	γ	ϕ	μ_1	σ_1	μ_2	σ_2	μ	η
Average	-1.671	0.295	0.328	0.220	-0.031	0.158	-0.311	1.586	0.042	1.083	0.000	0.420
Standard Dev.	0.350	0.024	0.042	0.036	0.034	0.051	0.321	0.121	0.017	0.043	0.000	0.019
Skewness	0.232	0.527	0.015	0.600	-0.313	-0.231	-2.493	1.101	0.294	-0.703	-0.039	1.172
Kurtosis	-0.981	-0.001	-0.764	0.813	-1.628	0.200	7.053	0.956	1.056	1.009	-0.278	0.663
5% quantile	-2.206	0.260	0.260	0.164	-0.081	0.063	-1.040	1.441	0.011	0.998	0.000	0.399
95% quantile	-1.091	0.335	0.395	0.301	0.006	0.241	-0.035	1.829	0.074	1.148	0.000	0.463

HAR-BPV-Jump	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η	λ	μ_z	σ_z
Average	-1.791	0.286	0.328	0.218	-0.030	0.000	0.434	1.190	0.000	0.002
Standard Dev.	0.321	0.024	0.038	0.037	0.035	0.000	0.019	0.062	0.000	0.000
Skewness	-0.144	0.046	-0.211	1.009	-0.329	0.044	1.080	0.113	0.250	0.546
Kurtosis	-0.939	-1.278	-1.179	0.988	-1.624	-0.194	0.433	-0.961	0.562	-1.363
5% quantile	-2.357	0.248	0.267	0.167	-0.084	0.000	0.413	1.101	0.000	0.002
95% quantile	-1.307	0.321	0.380	0.307	0.007	0.000	0.475	1.296	0.000	0.003

HAR-MedRV-Jump	ω	ϕ_1	ϕ_2	ϕ_3	γ	μ	η	λ	μ_z	σ_z
Average	-1.797	0.297	0.332	0.202	-0.027	0.000	0.421	1.190	0.000	0.002
Standard Dev.	0.353	0.021	0.038	0.033	0.033	0.000	0.020	0.062	0.000	0.000
Skewness	-0.126	-0.039	-0.277	1.247	-0.325	0.044	1.108	0.113	0.250	0.546
Kurtosis	-1.043	-1.009	-1.110	1.578	-1.586	-0.194	0.485	-0.961	0.562	-1.363
5% quantile	-2.422	0.261	0.268	0.160	-0.076	0.000	0.399	1.101	0.000	0.002
95% quantile	-1.280	0.329	0.385	0.285	0.008	0.000	0.465	1.296	0.000	0.003

Table 8: Average Diebold-Mariano (1995) pairwise test statistic with 5-, 10-, 30-, and 60-day horizon for S&P500 Futures.

5-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-2.638	0.291	1.260	-3.533	-3.002	-2.338	-4.248	-3.054
Realized Gaussian			2.428	2.400	-2.098	-2.609	-2.440	-6.993	-4.469
Bipower Gaussian				2.230	-2.291	-2.764	-26.063	-5.266	-3.731
MedRV Gaussian					-2.320	-2.737	-2.914	-5.581	-5.086
Realized MN						0.420	0.792	0.086	1.757
Bipower MN							1.310	-0.236	1.115
MedRV MN								-0.933	0.450
Bipower Jump									2.177
MedRV Jump									

10-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-3.050	-0.566	-0.013	-3.538	-3.810	-3.312	-3.211	-2.425
Realized Gaussian			2.545	3.263	-6.566	-2.282	-1.784	-2.496	-1.298
Bipower Gaussian				4.663	-3.234	-4.052	-4.813	-4.550	-12.159
MedRV Gaussian					-3.294	-3.951	-4.184	-5.198	-4.384
Realized MN						0.378	1.740	0.296	0.651
Bipower MN							1.961	0.260	0.681
MedRV MN								-0.040	0.481
Bipower Jump									14.063
MedRV Jump									

30-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-2.308	-1.802	-1.256	-2.619	-2.448	-2.092	-4.529	-3.879
Realized Gaussian			2.878	4.669	-1.748	-1.428	-0.829	-5.628	-3.901
Bipower Gaussian				4.094	-2.003	-1.796	-1.340	-3.798	-3.024
MedRV Gaussian					-2.554	-2.366	-1.899	-4.571	-3.894
Realized MN						1.458	2.857	-0.621	-0.164
Bipower MN							4.790	-0.881	-0.411
MedRV MN								-1.214	-0.749
Bipower Jump									3.554
MedRV Jump									

60-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-2.042	-1.974	-1.820	-1.042	-0.727	-0.374	-4.611	-4.577
Realized Gaussian			1.413	1.771	1.782	2.174	2.748	-5.714	-4.682
Bipower Gaussian				2.307	0.772	1.224	1.765	-3.548	-3.243
MedRV Gaussian					0.521	0.977	1.511	-3.809	-3.526
Realized MN						2.136	3.157	-3.106	-2.837
Bipower MN							5.400	-3.440	-3.220
MedRV MN								-3.828	-3.638
Bipower Jump									2.088
MedRV Jump									

Note: this table presents the results of the average Diebold-Mariano (1995) pairwise test statistic with 5-. 10-. 30-. and 60-day horizon. The table reads the following way: a significant positive (negative) estimated value rejects the null of equal performance between competing forecasts, and provides evidence in favor of model A (B).

Table 9: Average Diebold-Mariano (1995) pairwise test statistic with 5-. 10-. 30-. and 60-day horizon for WTI Futures.

5-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		1.924	3.594	4.291	-4.509	-3.733	-3.439	-0.205	-0.633
Realized Gaussian			8.021	8.657	-6.779	-6.743	-10.645	-2.995	-1.144
Bipower Gaussian				9.167	-7.690	-7.806	-11.633	-6.069	-5.520
MedRV Gaussian					-8.080	-8.228	-11.199	-7.170	-9.106
Realized MN						1.139	2.095	2.275	2.724
Bipower MN							2.023	2.227	2.771
MedRV MN								2.588	2.907
Bipower Jump									2.587
MedRV Jump									

10-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		2.535	5.096	6.065	-7.054	-3.328	-12.148	-0.851	0.557
Realized Gaussian			10.306	12.469	-4.452	-3.484	-4.732	-2.734	-1.570
Bipower Gaussian				15.213	-5.976	-2.235	-6.792	-5.897	-9.110
MedRV Gaussian					-6.680	-5.982	-7.684	-7.009	-11.586
Realized MN						1.776	4.001	1.567	2.217
Bipower MN							-1.050	0.973	1.697
MedRV MN								1.337	2.138
Bipower Jump									2.400
MedRV Jump									

30-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-3.874	-2.211	-1.154	-3.355	-3.561	-2.915	-6.019	-4.824
Realized Gaussian			4.717	4.515	-2.381	-2.691	-1.459	-4.998	-0.912
Bipower Gaussian				4.139	-2.934	-3.254	-2.418	-5.760	-7.116
MedRV Gaussian					-3.118	-3.458	-2.741	-6.751	-15.612
Realized MN						0.702	2.984	0.769	1.127
Bipower MN							5.294	0.781	1.159
MedRV MN								-0.121	0.457
Bipower Jump									1.772
MedRV Jump									

60-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-3.572	-2.229	-1.526	-2.647	-3.139	-1.923	-4.842	-3.847
Realized Gaussian			3.009	2.874	-1.087	-1.624	0.204	-2.416	-0.439
Bipower Gaussian				2.540	-1.588	-2.090	-0.767	-23.491	-3.515
MedRV Gaussian					-1.730	-2.189	-1.049	-11.892	-6.148
Realized MN						-0.529	4.604	0.174	0.510
Bipower MN							4.075	0.327	0.688
MedRV MN								-0.888	-0.327
Bipower Jump									1.294
MedRV Jump									

Note: this table presents the results of the average Diebold-Mariano (1995) pairwise test statistic with 5-. 10-. 30-. and 60-day horizon. The table reads the following way: a significant positive (negative) estimated value rejects the null of equal performance between competing forecasts, and provides evidence in favor of model A (B).

Table 10: Average Diebold-Mariano (1995) pairwise test statistic with 5-. 10-. 30-. and 60-day horizon for USD/JPY Futures.

5-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		-0.166	-1.442	-1.761	-1.855	-2.574	-2.860	-4.021	-4.301
Realized Gaussian			-4.087	-3.991	-4.325	-5.569	-5.875	-23.285	-29.566
Bipower Gaussian				-3.191	-1.480	-2.926	-3.442	-12.589	-13.883
MedRV Gaussian					-0.952	-2.280	-2.795	-11.706	-12.657
Realized MN						-4.171	-4.638	-10.633	-12.495
Bipower MN							-12.403	-6.039	-7.003
MedRV MN								-5.236	-6.159
Bipower Jump									-4.210
MedRV Jump									

10-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		1.592	-1.231	-2.051	-4.072	-5.276	-5.861	-3.823	-4.243
Realized Gaussian			-5.097	-4.906	-46.915	-81.027	-41.709	-20.121	-24.834
Bipower Gaussian				-12.436	-1.735	-3.403	-3.880	-11.716	-13.543
MedRV Gaussian					-1.062	-2.610	-3.097	-9.506	-10.980
Realized MN						-4.702	-4.279	-15.814	-33.723
Bipower MN							-5.553	-7.514	-10.237
MedRV MN								-6.541	-9.047
Bipower Jump									-5.545
MedRV Jump									

30-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		2.469	0.470	-0.219	-0.324	-1.456	-1.862	-2.269	-3.015
Realized Gaussian			-9.312	-9.104	-27.744	-32.359	-46.091	-18.215	-22.006
Bipower Gaussian				-8.026	-1.968	-5.125	-42.168	-10.976	-13.711
MedRV Gaussian					-0.349	-3.260	-44.743	-8.542	-10.981
Realized MN						-8.782	-7.907	-7.388	-10.478
Bipower MN							-4.807	-3.345	-5.673
MedRV MN								-2.391	-4.675
Bipower Jump									-10.427
MedRV Jump									

60-day horizon	EGARCH	Realized Gaussian	Bipower Gaussian	MedRV Gaussian	Realized MN	Bipower MN	MedRV MN	Bipower Jump	MedRV Jump
EGARCH		6.140	3.450	2.229	2.309	0.508	-0.551	-0.392	-1.631
Realized Gaussian			-13.920	-13.521	-19.590	-24.772	-24.024	-25.915	-29.580
Bipower Gaussian				-12.057	-8.335	-15.093	-15.630	-13.284	-16.986
MedRV Gaussian					1.376	-12.496	-12.942	-8.305	-12.164
Realized MN						-13.394	-84.304	-8.866	-13.503
Bipower MN							-25.172	-2.801	-6.989
MedRV MN								0.097	-3.579
Bipower Jump									-13.641
MedRV Jump									

Note: this table presents the results of the average Diebold-Mariano (1995) pairwise test statistic with 5-. 10-. 30-. and 60-day horizon. The table reads the following way: a significant positive (negative) estimated value rejects the null of equal performance between competing forecasts, and provides evidence in favor of model A (B).