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An Economic Theory of Glass Ceiling*

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Keywords: glass ceilings, promotions, career options

JEL Classification: J16, D82

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An Economic Theory of Glass Ceiling*

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1. Introduction

The glass ceiling is one of the most controversial and emotive aspects of employment in organizations. The term appears to have originated only in the mid 1980s but became so rapidly sealed in the lexicon that by 1991 the US had created a Federal Glass Ceiling Commission with the Secretary of Labor as its chair. When setting up the Glass Ceiling Commission in 1991 the US Department of Labor defined the concept as "those artificial barriers based on attitudinal or organizational bias that prevent qualified individuals from advancing upward in their organizations into management-level positions". It added that these barriers reflect "discrimination ... a deep line of demarcation between those who prosper and those who are left behind." One only has to look at the casual empirical evidence to see why the issue remains topical and heated.

Women form a disproportionately small group in senior management positions. For example, Figure 1 provides the proportion of females in employment

*We thank Dan Bernhardt, Martin Cripps, Spyros Dendrinos, Bart Lipman, Martin Hellwig, Ian Jewitt, Glenn Loury and Andy McLennan for useful conversations and comments. We also thank seminar participants at Boston, Brown, Korea and Kyoto Universities and the Universities of Bonn, Bristol, SUNY at Albany, and Queensland, and the ESEM 2008 in Milan. Any remaining errors are our own. The authors thank the Leverhulme Trust for funding this research project. (Emails: p.a.grout@, i.park@, s.sonderegger@bristol.ac.uk)

amongst US professions and the proportion of female within those employees working as officials and managers (US Equal Employment Opportunity Commission). Women constitute just over half of all professions but little more than a third of all officials and managers. In the US Fortune 500 women account for only 15.6% of all corporate officer positions of any type. Furthermore, not only do women form a minority of employees at senior levels, they also receive lower remuneration than men. This disparity is reflected throughout senior management. Figure 2 shows the relative salary of educated women (according to highest education attainment) to equivalent educated men in the US from 1990 to date. The ratio for both women with bachelors degree and those with an advanced qualification (i.e., higher than bachelors) are relatively constant and very similar, with mean values below 0.6 across the period.

In the current debate, and to a lesser extent in the academic literature, there appear to be two strongly held and opposing interpretations of all this evidence. One interpretation is that this is the result of real gender differences. The other interpretation is that what is observed is only consistent with discrimination by gender. This paper provides an economic theory of the glass ceiling. One of the main insights emerging from our analysis is that in some real sense these two views are not in conflict with each other. A critical assumption of the model is that there is more diversity in women with regard to job commitment than men[1] (we discuss this later in this section) who, as a result, have less potential for private information. In this sense, consistent with the first interpretation, gender differences are at the heart of the story. However, in our model, employers know that if female workers were offered the same promotion contracts as men, all promoted women would display the same job commitment as men, and so this private information would become irrelevant. Thus, if any difference persists between male and female promotion contracts in equilibrium nonetheless, it cannot be justified by invoking differences in the two genders' propensity to leave their high-rank jobs. In spite of this, we show that in equilibrium firms choose to offer promotion contracts that differ between the two genders, even when they compete *à la* Bertrand for employees. So, in a real sense the second interpretation is also correct. The model allows us to understand the ways in which the discrimination interpretation is correct and gives insight into the design of anti-discrimination policies.

There are papers that find pure gender discrimination. Goldin and Rouse

[1] This is backed for instance by recent evidence by Bertrand, Goldin and Katz (2009), who have tracked the careers of MBAs who graduated between 1990 and 2006 from a top U.S. business school. They show that the presence of children is the main contributor to greater career discontinuity and shorter work hours for female MBAs relative to their male counterparts.

(2000) look at the effect of symphony orchestras choosing to use "blind" auditions that conceal the musician. They find blind auditions increase the probability that women are hired. Similarly, Neumark et al (1996), using matched groups of men and women, find evidence of gender discrimination in hiring in restaurants. On the other hand, there is also a considerable literature on gender differences and associated economic disparities both within economics and from outside the discipline. For example, Babcock and Laschever (2003) argue that women are poor negotiators and generally dislike the process of negotiating. Browne (1995, 1998) suggests that men are more interested in striving for status in hierarchies and "engage in risk taking behavior that is often necessary to reach the top of hierarchies." Kanazawa (2005) uses General Social Survey data to show that men rank financial reward and power positions much higher in their preferences for employment, concluding that since men covet and strive for such positions they are the ones who are more likely to succeed in achieving them, whereas "women have better things to do." Some recent economic experiments are related to this literature. Gneezy et al (2003) find that men perform significantly better than women in a more competitive, tournament environment, while there is no gap in a non-competitive, piece rate environment. When subjects can choose between a piece rate and a tournament environment, Niederle and Vesterlund (2007) find a significant gender gap in choice, with 35% of women and 73% of men selecting the tournament. Fryer et al (2008) report that the introduction of financial incentives (as opposed to increased competition) exacerbates the gender gap by enhancing the performance of men significantly, while it increases the stress levels of women more than men, in their experiment.

Economists have studied differential treatments in labor market from the perspectives of discrimination theories, pioneered by Becker (1959), Arrow (1973) and Phelps (1977), and further developed by other authors, e.g., Coate and Loury (1993). The current paper, however, explains the glass ceiling as a competitive equilibrium outcome of agents dealing with informational problems. Hence, our approach is more closely related to Lazear and Rosen (1990) who study gender differential treatment by non-discriminating employers. Similar to them, we study an environment where women have non-market opportunities that are on average better than those for men. There are, however, important differences. In Lazear and Rosen, the fact that women face worse promotion prospects than men is somewhat "justified" since promoted women do have on average a higher propensity to leave or take a career break than men. By contrast, in our approach firms know that they can offer contracts that would *remove* such gender differential conditional on promotion. Specifically, in our equilibrium the promotion offer made to men would dominate the non-market option for *all* promoted women, rendering them identical to promoted men because they would never leave. This implies that there is no *a priori* reason why women should be made promotion offers differently from men. Nonetheless, we show that women are consistently offered inferior promotion deals than men, earning a lower wage in high-rank jobs. This differential treatment is "unjustified" since promoted women are no worse than men in productivity. Relative to Lazear and Rosen, our model shows that the glass ceiling may emerge as an equilibrium phenomenon of the labor market competition, even though it is not grounded in productivity differences.

The formal model we consider is loosely as follows. There are two technologically identical firms competing for employees.[2] Employees are hired to entry-level positions and each firm has need for one senior management slot for every two entry-level positions. The amount of effort (human capital investment) that employees put in at entry-level positions affects their productivity in senior-level posts if they get promoted, but has no impact on their output in the second period otherwise.

The firms compete *à la* Bertrand in the labor market at an initial hiring stage by offering contracts that specify their terms of promotion. We analyze what happens when contracts can be made gender specific with a view to understanding the impact on the equilibrium when various possible legal anti-discrimination restrictions are placed on the market. Employees approach the firm that offers the best contract for them, and then take up the other offer if they are not chosen by that firm.

Firms select their senior level managers from the intake at the lower level in the previous period based on the effort levels they exerted and the contracts offered. Since we assume that these effort levels are not observable by the rival firm, promotions take place internally within the firm in equilibrium. Ex-post negotiations are allowed between a firm and workers promoted in the rival firm, but it does not happen in equilibrium for it would necessitate reneging on a contract with some other worker. Note that even if the effort levels were observable by rival firms, there are good reasons why ex-post negotiations may not matter. For instance, it would be so if we introduced a small firm-specific element in human capital investment into the model.

We assume that women differ in terms of the non-market options available to them. Whether these options are worthwhile to pursue when they arise, however, is endogenously determined. The equilibrium has two main features.

The first is that promoted women end up being paid less than men, even though they are equally productive. This is a striking results since, as discussed above, this differential treatment cannot be justified by invoking gender differences in productivity. Employers know that offering the male promotion contract to female workers would be sufficient to erase all gender differences

[2]We think of the two firm market being sustained by entry costs that limit the number of firms that can be accommodated in the market.

among promoted workers. Hence, in our model, the differences in the promotion offers made to male and females do not stem from productivity differences, but from other forces.

The second feature is that the differential treatment suffered by women in their promotion deals persists even when firms compete Bertrand-style for employees. Moreover, we show that even though firms are ex-ante identical and follow the same strategy, the equilibrim is asymmetric ex-post, in the sense that one firm is more female friendly than the other. Hence, in equilibrium one firm will employ more women[3] and extract a smaller surplus from them than the other firm.

What is the basic intuition why, even in the presence of Bertrand competition, promoted women do not receive full reward for their effort? It starts from the observation that the option available for the workers who forgo promotion is worse for women than men due to the competitive labor market, because the compensation fully reflects the gender differences in commitment for those workers. This means that women with low non-market options are willing to accept worse promotion deals than men, since their market alternatives are worse than those of men. Hence, a firm can, at least initially, promote women more "cheaply" than men. But, promoting more women is gradually more costly, since it requires inducing women with better and better non-market alternatives to apply for promotion, and therefore it requires the promotion deals offered to women to become gradually more attractive. Bertrand competition raises firms' bids for women workers and reduces the extra surplus that promoted women generate. However, this competition stops short of erasing the extra surplus *completely*, because at some point it pays for a firm to abandon the bidding war and instead, extract the maximum surplus from the residual women workforce.[4] This also explains why one firm is more female friendly than the other in equilibrium.

Thus the model provides an explanation for the glass ceiling phenomenon that, although not grounded in real productivity differences between men and women in high-rank posts, is the outcome of a competitive process. We are able to show that this equilibrium does not depend on being able to offer gender specific contracts. However, if promotion rules (such as minimum promotion ratios for women) are imposed on the organization, then the equi-

[3]It is worth noting that separation of black and white workers is taking place between firms in Lang, Manove and Dickens (2005). However, their model is one of monopolistic competition in labor market without human capital investment and promotion decisions and thus, their focus and analysis are different from ours.

[4]Astute readers will have notice that this is not a full description of the equilibrium in a simultaneous move game because if one firm abandons the bidding war, the other firm's bid would not be optimal. For this reason, firms use a mixed strategy in equilibrium. In addition, see the discussion following Theorem 2 in Section 4.

librium can change to improve the position of those women who have a low propensity for career breaks. A central feature of our approach that separates our conclusions from other discrimination models is that these restrictions cannot be temporary incursions into the labor market to shift the equilibrium to a more favorable one in a multi-equilibrium environment.[5] In our model, the interventions would have to be "permanent."

However, we also show that there are significant differences between promotion rules, in particular, between rules that set lower bounds on the fraction of women to be promoted and rules on the fraction of senior posts that must be filled by women. In the former it is necessary to choose exactly the right bound (which is a measure zero event) to eliminate the glass ceiling problem, but in the latter setting any bound in an interval will do.

Before moving to the main body of the paper it is useful to highlight the potential applicability of the model. Although we have focused on gender, we see the model as being relevant for other observationally distinct groups as well. An immediate broad analogy is the treatment of immigrant employees (particularly where the culture of the country they have left is very different from the host country), since these foreign employees may wish to return to their home country at some point.[6]

The next section describes how we model the environment as a game. Section 3 analyzes the game when there is a monopoly employer, which is extended in Section 4 to characterize the unique equilibrium outcome under Bertrand competition between employers. Section 5 addresses policy implications. Section 6 contains some concluding remarks.

2. Model

We consider a labor market with two populations of workers, male and female, of the same measure 2. There is an initial period of their employment, period 1, during which they decide on human capital investment. Depending on their investment decision, they may get one of two kinds of jobs in period 2, referred to as upper-tier and low-tier jobs as explained below. In the middle of period 2 all workers may encounter a non-market option with the same probability p, at which point separation may occur as detailed below. We assume that female

[5] Although many models have the feature that affirmative action only needs to be temporary, this is not always the case (see Coate and Loury (1993)).

[6] An alternative anecdotal example (encountered by one of the authors) concerns a full time employee who enjoyed writing novels in his spare time. He claimed that he always felt disadvantaged relative to his colleagues since it was always implied that he was not as committed to the career as others because "surely he would really prefer to be a full time author" and was only waiting for the opportunity. He claimed this question arose to differing degrees of directness at every appraisal he ever had, and that he thus felt obliged to display greater commitment to the cause than other employees.

workers are heterogenous in the value of their non-market options. Precisely, each female worker has a private type θ drawn from a commonly known cdf $D(\theta)$ on $\mathbb{R}_+$, where θ is the value of her outside, non-market option she may encounter while on the job. Men are much less varied in this dimension. In particular, for simplicity, we assume that all male workers are homogeneous in the sense that the value of their non-market option is zero (i.e., they do not encounter such options). For expositional ease, we assume that $p = 1$ and that θ is uniformly distributed over an interval $[0, \frac{1}{\alpha}]$, i.e., $D(\theta) = \alpha\theta$, where $\alpha \in (0, 1/4)$ to ensure that the outside option is significant for a sufficiently large fraction of female workers. Our results extend to large enough $p < 1$ and to a wide class of single-peaked distributions D, but exposition becomes more complex.

There are two firms, A and B, that are ex ante identical: each firm has a measure 2 of entry-level positions to fill in period 1, and a measure 1 of managerial positions, called *upper-tier* jobs, to fill in period 2. Note that it is not possible for both firms to hire only female (or only male) workers and, in particular, female workers are relatively scarce in this sense. This aspect is important for our results. However, the assumption that the total measure of the labor force is equal to total vacancies is not essential, and nor is that the ratio of upper-tier to entry-level posts is 1/2, and therefore, they can be relaxed but at a cost of expositional complication.

All workers (male and female) are assumed to have the same productivity in entry-level posts, which we normalize to 0. However, as indicated they can make a human capital investment/effort, $e \in \mathbb{R}_+$ in period 1, that would increase their productivity in the next period if they get promoted to an upper-tier post. The worker's cost of making effort is quadratic[7], $c(e) = \frac{1}{2}e^2$, and is incurred in period 1. The exerted effort level is observable only by the firm that he/she works for. Female workers learn their private types, θ, early in period 1, in particular, before their effort decisions.

In period 2, workers may get *promoted* to upper-tier posts. A promoted worker generates a flow revenue of $y(e) = 1 + e$ for the firm during period 2 (of length 1), where e is the effort exerted in period 1. However, at the midpoint of period 2, female workers encounter a shock that increases their outside option value from 0 to θ, in which case they choose whether (i) to remain in the job and get the contracted wage, or (ii) to leave the job (forfeiting the wage for the second half of period 2) and get the newly available outside option, which generates a flow (monetary equivalent) utility of θ for the remainder of period 2. We assume that if a worker leaves an upper-tier post then no revenue is generated by that post for the remainder of the period unless the worker is replaced by another worker who has held an upper-tier job in either firm.

[7] Our results extend straightforwardly to strictly convex cost functions.

All workers that do not get promoted can get a *low-tier* job in period 2, where they generate a constant flow revenue of $1 + \kappa$ after retraining which costs $\kappa > 0$ for the employer. The interpretation is that these jobs require "run of the mill" operations that can be carried out by anyone who has worked in an entry-level position. Since, unlike the upper-tier posts, all workers are equally productive while on low-tier jobs, competitive employers pay a flow wage that leaves them with zero expected profit: $w_m = 1$ for men and $w_f < 1$ for women. Here, $w_f < 1$ reflects the market's expected loss in revenue due to the prospect of departure by female workers for the non-market option of θ (if $\theta > w_f$) in the middle of period 2. For ease of exposition, we treat w_f as exogenous, although it can be determined endogenously in equilibrium, taking into account the types of women who get low-tier jobs, without affecting the main results. We present our analysis for $w_f \in (0.9, 1)$ to stress that our results hold even when w_f is arbitrarily close to w_m (which is the case when κ is arbitrarily small), but our results extend to a wider range of w_f at the cost of expositional complication.

Given that the competitive wages for low-tier jobs leave zero profit for the employer, it is inconsequential for our analysis whether such jobs are available in the firms that hired the workers initially or in a different section of the labor market. For simplicity, we present our analysis presuming the latter. The upper-tier posts are different in the sense that each firm has a comparative advantage of identifying more qualified workers among its own employees.

Therefore, prior to period 1, the two firms compete, *à la* Bertrand, in attracting workers by offering more favorable (for the workers) contracts than their rivals. First, the two firms, $i = A, B$, publicly and simultaneously announce their *contracts* $\mathbf{s}^i = (s^i_f, s^i_m)$, consisting of one flow salary level $s^i_g \in \mathbb{R}_+$ for each gender $g \in \{m, f\}$ to be paid to workers promoted to upper-tier posts in period 2. We assume that the court enforces that no promoted worker gets paid less than the contracted salary rate s^i_g at any point in period 2 while on the job. Thus, firm i's "contract (offer) strategy" is represented by a probability measure F^i on $\mathbb{R}^2_+$. Note that a contract does not specify a salary in period 1, which we assume is equal to the productivity, 0. This assumption is for ease of exposition and, as explained in Section 4, our main results extend to the case that a contract specifies period 1 salary as well.

After the contracts are offered, the workers are matched with the firms in an "allocation" stage. A precise modeling of this process, such as initial application and selection procedures and the second matching process of unfilled posts and residual labor supply, would involve nontrivial ad hoc assumptions. Hence, we take an alternative approach of directly postulating workforce allocation rules based on the fundamental principle that the firm offering a more favorable contract gets the first pick in hiring decisions. Here allocations are represented by measures, μ^i_g, of gender $g \in \{m, f\}$ workers hired by firm $i \in \{A, B\}$, such that $\mu_m^i + \mu_f^i = 2$ for $i = A, B$, and $\mu_g^A + \mu_g^B = 2$ for $g = f, m$. We denote $\boldsymbol{\mu}^i = (\mu_m^i, \mu_f^i)$.

If a firm, say i, has offered a contract $\mathbf{s}^i$ and hired $\boldsymbol{\mu}^i = (\mu_m^i, \mu_f^i)$ of workers, the continuation game has a nonempty set of equilibria (Lemma 6), denoted by $E(\mathbf{s}^i, \boldsymbol{\mu}^i)$. Let $E(\mathbf{s}^i) = \cup_{\boldsymbol{\mu}^i} E(\mathbf{s}^i, \boldsymbol{\mu}^i)$ and $\Pi^*(\mathbf{s}^i)$ be the maximum of the firm's equilibrium profit levels in $E(\mathbf{s}^i)$.

If firm i were to take the first pick in hiring, it would hire so as to maximize its profit in the continuation equilibrium. If there is a unique equilibrium in $E(\mathbf{s}^i)$ that generates $\Pi^*(\mathbf{s}^i)$ and it is the unique continuation equilibrium following the firm's offer of $\mathbf{s}^i$ and hiring $\boldsymbol{\mu}^i$ of workers, then the firm will indeed hire $\boldsymbol{\mu}^i$. In this case, the value of contract $\mathbf{s}^i$ is the female worker's ex ante utility in this equilibrium. Generally, the *value* of a contract $\mathbf{s}^i$, denoted by $v(\mathbf{s}^i)$, is the ex ante expected utility of a female worker in an equilibrium of $E(\mathbf{s}^i)$ that generates $\Pi^*(\mathbf{s}^i)$ for the firm. We stress that $v(\mathbf{s}^i)$ is uniquely defined in all contracts relevant for our main analysis, and the values of other contracts are inconsequential. Since all female workers prefer to be hired by the firm that has offered a contract with a higher value, we postulate that this firm has priority in hiring women as specified below. Note that $v(\mathbf{s}^i)$ is defined endogenously to ensure that it correctly reflects the value female workers attach to a contract in equilibrium.[8]

The valuation of male contracts is different due to the homogeneity of male workers. Since they can guarantee a utility of 1 by exerting no effort and getting a low-tier job in period 2 (i.e., forgoing promotion), the value of any male contract is at least 1. On the other hand, even if some firm offers a very attractive male salary s_m then as long as this firm hires more than measure 1 of male workers, they would compete for promotion and, as a result, all promoted male workers end up exerting an effort level, say e_m, that restores the equivalence of pursuing promotion and not, i.e., $s_m - e_m^2/2 = 1$. So, if either firm hires more than measure 1 of male workers, male workers know that they will end up obtaining a utility of 1, i.e., no higher than what they could obtain when they are hired by the other firm. This means that either firm is at least as well placed as the other firm in hiring additional male workers up to measure 1.

Based on these observations, we postulate the labor force allocation as[9]:

(H1) The firm, say A, offering a contract with a strictly higher value first hires

[8]Alternatively, one could define the value exogenously albeit ad hoc, e.g., as the level of female salary offered. Our qualitative results remain valid in this alternative definition.

[9]This allocation would ensue if the actual matching process is, for instance, as follows: All workers apply to both firms (at no cost) and the firms have one chance of offering positions and the workers choose among the offered positions (randomly if equivalent offers), given that the firms have to fill all positions to operate.

as large a fraction of women as it wants, and as large a fraction of men as it wants up to measure 1. Then, firm B fills all its posts from the residual labor force. Finally, firm A hires any remaining workers.

(H2) If the two firms offer contracts of the same value, they are allocated measure 1 of each gender. Then, either firm may propose an alternative allocation, which is implemented if accepted by the other firm.

A strategy of each firm in an allocation stage, given the offered contracts $\{\mathbf{s}^A, \mathbf{s}^B\}$, is a hiring decision as per the rule (H1) if $v(\mathbf{s}^A) \neq v(\mathbf{s}^B)$, and a decision as to which alternative allocation to propose and/or to accept as per (H2) if $v(\mathbf{s}^A) = v(\mathbf{s}^B)$.

If $v(\mathbf{s}^i) > v(\mathbf{s}^j)$ we describe firm i as a *leader* (in hiring) and j as a *follower*. Note that this is a slight abuse of terminology since, unlike the Stackelberg setting, the identities of firms are endogenous.

After contracts $\mathbf{s}^i$ are offered and workforce is allocated as $\boldsymbol{\mu}^i$ for $i = A, B$, a "promotion subgame" ensues, comprising periods 1 and 2. During these periods firms cannot fire workers, however workers may leave the firm at any time, forgoing any unpaid flow salary. At the beginning of period 1 all female workers learn their private types θ, and every worker in either firm exerts an effort level $e \in \mathbb{R}_+$, which is observable only by the employer. In period 2, each firm decides who to promote based on gender and exerted effort level, and pays them the salaries specified in the relevant contracts. All unpromoted workers leave the firm and get a low-tier job that pays a flow wage of w_g.

After the firm has completed promotion decisions, we assume that any unpromoted worker may sue his/her employer. There is a case for a court to consider provided that he/she is potentially valuable for the firm to promote in the sense that $1 + e - s_g^i \geq 0$ where e is the exerted level of effort and g is the gender of the worker. If the court verifies either

(Ci) that the firm promoted some worker who has exerted a strictly lower level of effort than the plaintiff, or

(Cii) that a non-zero measure of upper-tier posts are unoccupied,[10]

then the court finds in favor of the plaintiff and a hefty compensation payment must be made by the firm to the plaintiff. We assume a sufficiently high verifiability of effort levels in the court so that firms never leave any scope for a worker to successfully sue the firm.[11]

We note that verifiability of effort, which permits court protection postulated above, is instrumental in obtaining uniqueness of equilibrium in our

[10](Cii) is not necessary for our main result, but simplifies exposition considerably.

[11]Note that we are implicitly setting the cost of going to a court at zero, however, our results would continue to hold when the cost is positive unless it were prohibitively large.

model, but not essential for the glass ceiling phenomenon. That is, our equilibrium (which exhibits a glass ceiling) continues to be an equilibrium, albeit no longer the unique one, even if effort is unverifiable (see Section 4).

At the midpoint of period 2, female workers encounter a non-market option of value θ, in which case they may leave the job to get θ. Here, we assume that when an employee is attracted to a non-market option that is more valuable than the current employment, departure is irreversible and renegotiation of salary is irrelevant at that point. A positive measure of such departures causes damage to the firm. For expositional ease, we capture such damage by simply postulating that the posts vacated by departures cannot be replaced. However, alternative modeling of what may happen to the vacated posts would not change our main results so long as the damage inflicted to the firm is nontrivial.[12]

In a promotion subgame of firm i with contract $\mathbf{s}^i$ and allocation $\boldsymbol{\mu}^i$, each worker's strategy consists of an effort level,[13] decision as to whether to sue the firm or not in the relevant contingencies, and for female workers, the decisions as to whether to leave employment for the non-market option of value θ; and firm i's strategy specifies who to promote based on gender and exerted effort level (contingent on the profile of efforts exerted by all workers). Each worker maximizes the expected value of his/her income stream, including that from the non-market option and compensation from a lawsuit, net of any effort cost. Each firm maximizes its expected profit, i.e., total revenue net of total salary and lawsuit-compensation payments. We assume no discounting for simplicity.

A strategy profile in this promotion subgame, together with a belief profile on the type distribution of female workers contingent on the exerted effort level, constitutes a *(perfect Bayesian) continuation equilibrium* of this subgame if the strategies are mutual best-responses and the belief profile satisfies Bayes rule whenever possible.

A strategy profile of the two firms in offering contracts, a strategy profile of the two firms in the allocation stage for each possible set of offered

[12] For example, the firms may try to fill vacated posts by recruiting workers in upper-tier posts of the other firm by offering a higher salary. Then, a positive measure of departures in either firm would inevitably launch a recruiting war between the two firms, pushing the salaries of all upper-tier post workers up to their productivities (which the other firm can infer correctly in equilibrium), thereby depleting any positive profit of the firm for the remainder of period 2. Foreseeing this, the firms avoid hiring that would lead to departures and consequently, the equilibrium outcome is the same as in our model.

[13] Precisely, male workers' effort choice is represented by a probability measure ξ_m on $\mathbb{R}_+$ and female workers' effort choice by a probability measure ξ_f on $\mathbb{R}_+ \times [0, 1/\alpha]$ where $[0, 1/\alpha]$ is the type space. Each ξ_g may be interpreted either as the common mixed strategy adopted by all workers of gender $g \in \{f, m\}$, or the distribution of pure strategies adopted by measure μ_g^i of gender g workers.

contracts $\{\mathbf{s}^i\}_{i=A,B}$, and a profile of continuation equilibria for every possible promotion subgame, constitute an *equilibrium* of the grand game if, given the profile of continuation equilibria, i) the strategy profile in the allocation stage contingent on $\{\mathbf{s}^i\}_i$ is an equilibrium in the continuation game, and ii) the contract strategies of the two firms are mutual best-responses given the rest of strategies.

An *equilibrium outcome* of the game consists of a pair of (possibly stochastic) contracts $\{\mathbf{s}^i\}_i$, allocations $\{\boldsymbol{\mu}^i\}_i$ contingent on $\{\mathbf{s}^i\}_i$, and the ensuing effort profile of the workers and the promotion decisions, that arise in an equilibrium. We characterize the equilibrium outcome in the next two sections.

3. Analysis of a Monopoly Firm

We start by analyzing the case where firm i is the only employer, because many of the core insights in this simpler environment carry over to the case of Bertrand competition. Then, firm i offers a contract, hires men and women as it wishes and a subsequent promotion subgame ensues (without presence of another firm). Consider a promotion subgame after firm i has offered a contract $\mathbf{s}^i$ and hired measure $\boldsymbol{\mu}^i = (\mu^i_m, \mu^i_f)$ of workers where $\mu^i_m + \mu^i_f = 2$, which we denote by $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgame. Since no promotions would take place if upper-tier salaries are lower than the wage for low-tier jobs, it suffices to consider the cases that $s^i_g \geq w_g$, $g = m, f$, with at least one strict inequality.[14]

Consider a promoted worker in an arbitrary equilibrium of $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgame. If this worker is male, it is clear that the firm would not pay him more than s^i_m because paying more would only incur higher expense without any benefit. If this worker is female and she does not leave in the middle of period 2, the firm realizes at that point that her outside option is no longer relevant and thus would pay no more than s^i_f for the remainder of period 2. Foreseeing this, she would leave for her outside option if and only if $\theta > s^i_f$, i.e., her decision to leave depends only on the contracted salary level s^i_f. Understanding all this, the firm would pay her no more than s^i_f throughout period 2. This establishes that any promoted worker would be paid exactly the contracted salary while on the job. It then follows, as the next lemma states, that all promoted workers of the same gender will have exerted the same level of effort and that no rationing of promotion takes place among workers who extended non-zero effort. The basic idea behind this is that rationing in promotion would be broken by some workers exerting a slightly higher effort level to break the tie and ensure promotion; and no worker would exert a higher effort than any other, indistinguishable worker for promotion in the absence of rationing in promotion.

[14] Allowing other $\mathbf{s}^i$'s only complicates exposition with no additional results or insights.

Lemma 1. *In any equilibrium of* $(\mathbf{s}^i, \boldsymbol{\mu}^i)$*-subgame, all promoted workers of gender* g *will have exerted the same effort level, say* e_g*, and get paid exactly the contracted salary,* s_g^i*, while on the job. In addition, if* $e_g > 0$*, all workers of gender* g *who exert* e_g *get promoted, and all other workers exert* $e = 0$*.*

Proof. It has been shown above that all promoted workers are paid exactly the contracted salary while on the job. To reach a contradiction, suppose there is rationing in promoting workers who exerted a certain effort level, say $e' > 0$. Note that e' is the lowest effort level that workers may get promoted with, for otherwise (Ci) implies that the firm would lose the lawsuits filed by those who are rationed out after exerting e'. This also implies that the measure of workers who exert an effort level strictly above e', if exist, is strictly less than 1. Note that $1 + e' - s_g^i \geq 0$ if workers of gender g are promoted after exerting e', since otherwise the firm would not have promoted them. Thus, a worker who deviates by exerting a slightly higher effort than e' (instead of e'), benefits by warranting him/herself sure promotion owing to (Ci)-(Cii) because $e' > 0$ implies $s_g^i > w_g$. Since this would contradict e' being an equilibrium effort level, we conclude that any rationing, if it exists, would be among workers who exerted $e = 0$ and it would be possible only if they will get the same level of compensation promoted or not (for otherwise they would benefit by exerting small $e > 0$ for the same logic as above). Then, there is no more than one effort level for each gender, say e_g, that leads to promotion, since all workers prefer lower effort conditional on the same salary afterwards. Any worker who does not exert this effort level would not be promoted, so exerts $e = 0$. ∎

In an equilibrium of the $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgame, the effort level that workers of gender g exert for promotion, which we denote by e_g, determines their decisions as to whether to pursue promotion or not, and the firm's surplus from promoting them. For notational ease, we denote $\mathbf{s}^i = (s_m, s_f)$ in this section.

Male workers would pursue promotion only if $s_m - (e_m)^2/2 \geq 1$, with certainty if the inequality is strict, because the payoff they can guarantee themselves from forgoing promotion is 1. Subject to this constraint, the firm's surplus per male promotion, $1 + e_m - s_m$, is uniquely maximized at $e_m = 1$ and $s_m = 1.5$, i.e., when the efficiency is achieved by equating the marginal revenue and the marginal cost of effort. This proves

Lemma 2. *The maximum possible equilibrium surplus of a firm per male promotion is 0.5, which is possible if and only if* $e_m = 1$ *and* $s_m = 1.5$*.*

As will become clear below, these are the equilibrium terms of promotion for men.

Next, we consider female workers whose optimal behavior depends on their private types θ. If a woman pursues promotion, she would get a utility of

$$u_h(\theta|e_f, s_f) := \max\left\{s_f, \frac{s_f + \theta}{2}\right\} - (e_f)^2/2 \tag{1}$$

because she would leave the post for her outside option at midpoint of period 2 if $\theta > s_f$. If she forgoes promotion, she would exert no effort, get a low-tier job for a wage of $w_f < 1$, and would leave the employment for a non-market option at midpoint if $\theta > w_f$, which warrants an expected utility of

$$u_\ell(\theta) := \max\left\{w_f, \frac{w_f + \theta}{2}\right\}. \tag{2}$$

Thus, a female worker of type θ would pursue promotion if $u_h(\theta|e_f, s_f) > u_\ell(\theta)$ and forgo promotion if the reverse inequality holds.

Whether women of types $\theta > s_f$ would pursue promotion is critical for the firm's surplus because any such woman, by leaving at mid-career, would curtail any revenue that might be forthcoming otherwise. To understand when this happens and when not, observe that the graph of $u_\ell(\theta)$ is flat at the level of w_f for $\theta \leq w_f$, then increases with a slope of 1/2 for $\theta > w_f$. Similarly, that of $u_h(\theta|e_f, s_f)$ is flat at the level of $s_f - (e_f)^2/2$ for $\theta \leq s_f$ and increases with a slope of 1/2 for $\theta > s_f$.

First, if $s_f - (e_f)^2/2 < w_f$ in equilibrium, the graph of $u_h(\theta|e_f, s_f)$ lies entirely below that of $u_\ell(\theta)$ because $s_f \geq w_f$, whence no women would pursue promotion. Also, if $e_f = 0$, the firm's expected profit from promoting any woman would be $\frac{1}{2}(1 - s_f)(1 + D(s_f)) = \frac{1}{2}(1 - s_f)(1 + \alpha s_f) < 0.07$ since she would leave if her type exceeds $s_f \geq w_f > 0.9$, rendering women much less productive resource than men who can generate a surplus of 0.5 by Lemma 2. These cases do not arise in equilibrium and have little bearing on our analysis, so will no longer be discussed. Hence, we consider $s_f > w_f$ below.

Next, if $u_h(\theta|e_f, s_f) > u_\ell(\theta)$ for all $\theta \geq 0$ so that the graph of $u_h(\theta|e_f, s_f)$ lies entirely above that of $u_\ell(\theta)$ then women of all types pursue promotion. In this case we define the *threshold* (*type*), which is denoted by θ^c and codes the marginal type women of types below which pursue promotion, to be ∞. Also, if the upward-sloping part of $u_h(\theta|e_f, s_f)$ falls on that of $u_\ell(\theta)$, women of types $\theta < s_f$ definitely pursue promotion (since $s_f > w_f$) whilst women of all other types are indifferent and some of them may pursue promotion (and leave at midpoint). However, the exact subset of types who pursue promotion only to depart later, is inconsequential so long as the measure of such a subset, say $\zeta \geq 0$, is unchanged because they do not affect any other aspect of the equilibrium. Thus, we assume without loss of generality that women pursue promotion if and only their types are below the *threshold* θ^c defined by $D(\theta^c) = D(s_f) + \zeta$ in this case. Note that $s_f < \theta^c$ if $\zeta > 0$.

Finally, the remaining possibility is that the graph of $u_h(\theta|e_f, s_f)$ either crosses that of $u_\ell(\theta)$ from above at exactly one point, or overlaps with it for $\theta \leq w_f$ and lies below it for $\theta > w_f$.[15] In the former case, women pursue promotion if and only if their types are below the crossing point which therefore constitutes the *threshold*, defined by

$$\theta^c(e_f, s_f) = 2s_f - w_f - (e_f)^2. \tag{3}$$

In the latter case, only women whose types are below w_f may pursue promotion out of indifference and exactly who do so is inconsequential for the same reason as above. So, we assume that women pursue promotion if and only their type is below the *threshold* θ^c defined by $D(\theta^c)$ being equal to the fraction of women that pursue promotion in equilibrium.

Note that our notion of threshold type enables us to consider only the "cutoff-style" equilibria of $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgames in which women pursue promotion if and only if their types are below a certain threshold $\theta^c \geq 0$. If $\theta^c > s_f$, some women of types above s_f pursue promotion only to leave afterwards. Such equilibria of $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgames are referred to as equilibria "with departures." However, these do not arise along the equilibrium path. In all other equilibria of $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgames, i.e., with $\theta^c \leq s_f$, any woman who chooses to pursue promotion will never leave the post, which we refer to as "departure-free" equilibria.

We now examine the possible equilibrium surplus of a firm per female promotion. Focusing on departure-free equilibria with threshold $\theta^c \in (w_f, 1/\alpha)$, the firm's surplus per female promotion is bounded above by

$$\max_{e_f \geq 0, s_f \geq \theta^c} \; 1 + e_f - s_f \quad s.t \quad u_\ell(\theta^c) = u_h(\theta^c|e_f, s_f) \tag{4}$$

where the constraint is equivalent to $e_f = \sqrt{2s_f - w_f - \theta^c}$ since $s_f \geq \theta^c > w_f$. It is straightforwardly verified that if $\theta^c \leq 1 + w_f$ then $s_f \geq \theta^c$ does not bind and the solution to (4) is $(e_f, s_f) = (1, \frac{1+w_f+\theta^c}{2})$; If $\theta^c > 1 + w_f$, on the other hand, the constraint $s_f \geq \theta^c$ binds, so the solution is $(e_f, s_f) = (\sqrt{\theta^c - w_f}, \theta^c)$. Therefore, the optimized per-promotion surplus is $1.5 - \frac{w_f+\theta^c}{2}$ for $\theta^c \leq 1 + w_f$ and $1 + \sqrt{\theta^c - w_f} - \theta^c$ for $\theta^c > 1 + w_f$ and thus, it exceed the per-promotion surplus from men, 0.5, if and only if $\theta^c < \bar{\theta} := 2 - w_f \; < \; 1 + w_f$.

If $\theta^c \leq w_f$ in departure-free equilibria, $s_f - (e_f)^2/2 = w_f$ as discussed above. In this case, by solving $\max_{e_f, s_f} \; 1 + e_f - s_f$ subject to $s_f - (e_f)^2/2 = w_f$, we get the maximum per-promotion surplus of $1.5 - w_f$, which is obtainable when $e_f = 1$ and $s_f = 0.5 + w_f$. Since per-promotion surplus is lower in equilibria with departures (as shown in Appendix), we have the next lemma.

[15]The two graphs coincide everywhere if $s_f = w_f$ and $e_f = 0$, a case we dismissed earlier.

Lemma 3. *Fix an arbitrary* $\theta \in (0, \bar{\theta}]$ *where* $\bar{\theta} = 2 - w_f$. *The maximum possible surplus per female promotion in equilibria with threshold* θ *is*

$$\pi_f^*(\theta) := \begin{cases} 1.5 - w_f & \text{for } \theta < w_f, \\ 1.5 - \frac{w_f+\theta}{2} \ \geq \ 0.5 & \text{for } \theta \in [w_f, \bar{\theta}] \text{ with strict inequality if } \theta < \bar{\theta}. \end{cases}$$

This maximum is achieved only in departure-free equilibria in which $e_f = 1$ *and*

$$s_f^*(\theta) := \begin{cases} 0.5 + w_f \ < \ 1.5 & \text{for } \theta < w_f, \\ \frac{1+w_f+\theta}{2} \ \leq \ 1.5 & \text{for } \theta \in [w_f, \bar{\theta}] \text{ with strict inequality if } \theta < \bar{\theta}. \end{cases}$$

Note that $\pi_f^*(\theta) + s_f^*(\theta) = 2$. *In particular,* $\pi_f^*(\theta)$ *strictly decreases in* $\theta \in [w_f, \bar{\theta}]$ *from* $\pi_f^*(w_f) = 1.5 - w_f > 0.5$ *down to* $\pi_f^*(\bar{\theta}) = 0.5$. *If the equilibrium threshold type is* $\theta > \bar{\theta}$, *the per-promotion surplus is less than 0.5.*

Proof. See Appendix. ∎

A graphical illustration is useful. Consider $\theta^c \in (w_f, \bar{\theta})$ depicted in the first diagram of Figure 3. The graph of $u_\ell(\theta^c) + c(e_f)$ in the second diagram is the set of all pairs (e_f, s_f) that would induce the threshold θ^c as per (3). The firm's surplus, $1 + e_f - s_f$, is depicted by the horizontal distance between $1 + e_f$ and $u_\ell(\theta^c) + c(e_f)$ and thus, obtains its maximum value $\pi_f^*(\theta^c) = \frac{3-w_f-\theta^c}{2}$ at $e_f = 1$, i.e., when efficiency is achieved. At the optimum, $s_f^*(\theta^c) = 2 - \pi_f^*(\theta^c) = \frac{1+w_f+\theta^c}{2} > \theta^c$ because $1 + w_f > \bar{\theta} > \theta^c$, i.e., the flat part of the graph of $u_h(\theta|e_f, s_f)$ crosses the upward-sloping part of the graph of $u_\ell(\theta)$ as depicted in the first diagram, hence no promoted women leave their posts mid-career.

[Figure 3 about here]

We are now ready to examine a monopoly firm's optimal hiring between male and female workers. We begin with the following observation.

Lemma 4: *In any continuation equilibrium after a firm offers a contract* $\mathbf{s}^i = (s, s)$ *where* $s > 1.1$, *all promoted workers will have exerted the same effort* $e^* = \sqrt{2s - 2} > 0$ *and will never leave their posts.*

Proof: Since men would pursue promotion only if $s - e_m^2/2 \geq 1$, i.e., $e_m \leq e^* = \sqrt{2s - 2}$, where e_m is the effort level they exert for promotion, $1 + e^* - s$ is the maximum possible surplus per male promotion.

First, consider the case that $1 + e^* - s > 0$, so that $s \in (1.1, 3)$. Let e_f denote the effort level exerted by any promoted women, which is unique if exists by Lemma 1. If $e_f > e^*$ in any equilibrium after $\mathbf{s}^i = (s, s)$ is offered, then i) no men would be promoted since otherwise women could have guaranteed promotion by exerting $e \in (e_m, e_f)$ due to (Ci) given $e_f >$

$e^* \geq e_m$ and thus, ii) all upper-tier posts should be filled by women since otherwise women could have guaranteed promotion by exerting $e \in (e^*, e_f)$ due to (Cii). This would require that at least one half of women get promoted, i.e., $u_\ell(\frac{1}{2\alpha}) \leq u_h(\frac{1}{2\alpha}|e_f, s) \Rightarrow e_f \leq \max\{\sqrt{2s - w_f - \frac{1}{2\alpha}}, \sqrt{s - w_f}\} < e^*$ where the inequality follows from $s > 1.1$, given $w_f \in (0.9, 1)$ and $\alpha \in (0, 1/4)$, contradicting $e_f > e^*$. Thus, we deduce that $e_f \leq e^*$ in any equilibrium after $\mathbf{s}^i = (s, s)$ is offered. Consequently, the maximum possible total surplus of the firm is $1 + e^* - s$, which is feasible only if all promoted workers will have exerted e^* and measure 0 of them leave their posts.

Hence, it suffices to show that a continuation equilibrium exist in which the firm's total surplus is $1 + e^* - s$. To do this, note from (H1)-(H2) that the firm can ensure to hire some measure $\mu^i_m \geq 1$ of men and measure $\mu^i_f = 1 - \mu^i_m$ of women. Then, not all upper-tier posts are filled by women: If they were, we would have $u_\ell(\frac{1}{\alpha}) \leq u_h(\frac{1}{\alpha}|e_f, s)$ which would imply $e_f < e^*$ since $s \in (1.1, 3)$ given $w_f \in (0.9, 1)$ and $\alpha \in (0, 1/4)$, whence men would guarantee promotion by exerting some $e \in (e_f, e^*)$ by (Ci). In light of (Cii), this means that some men get promoted in equilibrium after exerting $e_m \leq e^*$. Thus, women of sufficiently low θ should get promoted as well due to (Ci) because $u_\ell(0) = w_f < 1 \leq u_h(0|e_m, s)$. Given $\mu^i_m \geq 1$, this means that men should be indifferent between pursuing promotion and not, i.e., $e_m = e^*$ and consequently, $e_f = e^*$ since otherwise (Ci) would dictate that some workers can benefit by exerting slightly less effort. Hence, in the unique continuation equilibrium women exert e^* if and only if their types are below $\theta^c(e^*, s) = 2 - w_f < 1.1 < s$ and measure $1 - \mu^i_f D(\theta^c(e^*, s)) < 1$ of men exert e^*. Consequently, the firm's total surplus is $1 + e^* - s$, as desired.

Next, suppose $1 + e^* - s \leq 0$, so that $s \geq 3$. Then, any surplus of the firm would come from female promotion. If $1 + e_f - s > 0$ where e_f is the effort level that promoted women will have exerted, then $2[u_\ell(\frac{1}{2\alpha}) - u_h(\frac{1}{2\alpha}|e_f, s)] = \min\{w_f + \frac{1}{2\alpha} - 2s, w_f - s\} + e_f^2 > \min\{w_f + \frac{1}{2\alpha} - 2s, w_f - s\} + (s-1)^2 > 0$ where the last inequality ensues because $s \geq 3$, $w_f \in (0.9, 1)$ and $\alpha \in (0, 1/4)$, which would imply that less than one half of women pursue promotion. Since men would not exert more than $e^* \leq s - 1 < e_f$, this would contradict e_f being equilibrium effort level because women would guarantee promotion by exerting $e \in (e^*, e_f)$ due to (Ci) or (Cii). Hence, we conclude that $1 + e_f - s \leq 0$ and thus, any promoted worker will have exerted $e = s - 1$. No promoted women would leave because $2[u_\ell(s) - u_h(s|s-1, s)] = w_f + s^2 - 3s + 1 > 0$. ∎

Lemma 4 highlights a key aspect of our framework: Firms know that offering women the same promotion deal as men would be enough to erase *all* differences between the two genders (unless they offer very inefficient deals, i.e., $s < 1.1$, which is easily verified to be suboptimal), in the sense that all promoted women would exert the same effort level as men, and would remain in their posts with certainty throughout period 2. These women would therefore be identical to their male counterparts in all respects. We stress that this result holds for firms engaged in Bertrand competition as well, since the proof does not rely on the firm being a monopolist.

This observation is crucial for interpreting our results. In particular, it shows that, if any difference between male and female promotion contracts persists in equilibrium, this cannot be justified by invoking differences in the two genders' propensity to leave their high-rank jobs. Our model therefore allows us to investigate whether the differential treatment suffered by women with respect to promotion may survive independently of an often-invoked argument, namely that firms are less eager to promote women because women are more likely to leave their posts afterwards.

We characterize a monopoly firm's optimal strategy below, which indeed treats women differently from men. Recall from Lemma 2 that this firm, i, can extract the maximum possible per-promotion surplus of 0.5 from men by offering $s_m = 1.5$ and inducing $e_m = 1$. In fact, a firm can guarantee an equilibrium in which all upper-tier posts generate a surplus of 0.5 by hiring measure 2 of men after offering a male salary $s_m = 1.5$.[16] Thus, the firm may benefit by promoting women only if per-promotion surplus exceeds 0.5 for women. By Lemma 3, this is feasible in equilibria with threshold $\theta \in (0, \bar{\theta})$.

In such an equilibrium of $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgame, women pursue promotion if and only if their types are below the threshold, say θ, and hence, the measure of promoted women is $\mu_f \alpha\theta < 1$ where the inequality follows because $\mu_f \leq 2$, $\alpha \leq 1/4$ and $\bar{\theta} < 2$. The remaining upper-tier posts (of measure $1 - \mu_f\alpha\theta$) may be filled by measure $2 - \mu_f$ of men to the extent possible. Letting π_g denote the equilibrium per-promotion surplus for gender g workers, therefore, the firm's total surplus is no higher than $\pi_f\mu_f\alpha\theta + \pi_m \max\{2 - \mu_f, 1 - \mu_f\alpha\theta\}$ which, in light of the upper bounds of π_m and π_f identified in Lemmas 2 and 3, achieves the maximum value of

$$\Pi_L(\theta) := 0.5 + \frac{D(\theta)(\pi_f^*(\theta) - 0.5)}{1 - D(\theta)} \tag{5}$$

when $\mu_f = 1/(1 - D(\theta)) \in (1, 2)$, $\pi_f = \pi_f^*(\theta)$ and $\pi_m = 0.5$. Indeed, for threshold $\theta \in (w_f, \bar{\theta})$, the firm obtains a total profit equal to this upper bound in the unique equilibrium of the continuation subgame ensuing the firm's offer of a contract $(s_m, s_f) = (1.5, s_f^*(\theta))$, as stated in the next lemma.

Lemma 5. *(a) A monopoly firm i's total surplus in any equilibrium with threshold $\theta \leq \bar{\theta}$ is bounded above by $\Pi_L(\theta)$, which strictly increases in $\theta \leq w_f$*

[16] Exactly measure 1 of men pursue promotion by exerting $e = 1$ in equilibrium: If less than measure 1 were to pursue, deviation by exerting an effort level slightly less than 1 would warrant promotion by court protection (Cii) and thus, beneficial.

and is strictly concave in $\theta \in [w_f, \bar{\theta}]$ with a unique maximum at

$$\hat{\theta} = \arg \max_{w_f \leq \theta \leq \bar{\theta}} \Pi_L(\theta) < \bar{\theta}. \tag{6}$$

(b) For $\theta \in [w_f, \bar{\theta}]$, $(\mathbf{s}^*(\theta), \boldsymbol{\mu})$-subgame has a unique equilibrium if

$$\mu_f \leq \mu_f^*(\theta) := 1/(1 - D(\theta)) \quad \textit{where} \quad \mathbf{s}^*(\theta) := (1.5, s_f^*(\theta)). \tag{7}$$

In this equilibrium, measure $1 - \mu_f D(\theta)$ of men and all women of types below θ exert $e = 1$ and get promoted, so the firm's surplus, $0.5 + \mu_f D(\theta)(\pi_f^*(\theta) - 0.5)$, strictly increases in μ_f from 0.5 when $\mu_f = 0$ to $\Pi_L(\theta)$ when $\mu_f = \mu_f^*(\theta)$.

(c) For $\theta \in [w_f, \bar{\theta}]$, the firm's surplus is strictly lower than $\Pi_L(\theta)$ in any equilibrium of any $(\mathbf{s}^*(\theta), \boldsymbol{\mu})$-subgame if $\mu_f > \mu_f^*(\theta)$.

Proof. See Appendix. ∎

Lemma 5 establishes that a firm optimally extracts a higher surplus from promoting women compared with men, by offering an inferior contact. This happens because the utility that women of a sufficiently low type θ can expect to obtain if they forgo promotion, $u_\ell(\theta)$, is smaller than 1, the utility that a man can obtain in the same contingency. Intuitively, by forgoing promotion and therefore entering the low-tier job market, low-type women would be pooled with high-type women who would depart for their outside option, and would therefore be paid a low wage that reflects the departure risk of the pool. Since the utility that they can expect to earn if they forgo promotion is lower than that of men, low-type women are therefore willing to pursue promotion even if their deal is inferior to that offered to men. Note that this result depends on women's type being their private information. The inability of employers to observe a woman's type is thus a key ingredient of our analysis.

Not also that, faced with an unconstrained labor force, a firm would optimally hire (and promote) a positive mass of both men and women even if the profit that the firm earns from each promoted woman exceeds what it earns from promoting a man. Conditional on a particular threshold type θ, hiring too many women, i.e., more than measure $\mu_f^*(\theta)$ of them, would result in too few workers pursuing promotion: Out of all women hired by the firm, only a fraction $D(\theta) < D(\bar{\theta}) < 0.5$ decide to go for promotion, resulting in some upper-tier posts remain unfilled even after promoting all men. Thus, the firm would benefit by hiring more men so as to be able to fill up all remaining upper-tier posts with men, which happens when $\mu_f = \mu_f^*(\theta)$. On the other hand, if too few women were to be hired, i.e., if $\mu_f < \mu_f^*(\theta)$, then the firm could increase its profits by hiring more women so as to fill a larger fraction of upper-tier posts by "higher-yielding" women.

Lemma 5 leads to the equilibrium outcome of a monopoly firm described in Theorem 1 below, for which we need the following technical result:

Lemma 6. *Any $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgame has an equilibrium.*

Proof. See Appendix. ∎

Theorem 1. *If there is a monopoly firm, in the unique equilibrium outcome the firm maximizes its total surplus by offering $s_m = 1.5$ and $s_f = s_f^*(\hat{\theta})$ and then hiring measure $\mu_f^*(\hat{\theta}) \in (1, 2)$ of women and measure $2 - \mu_f^*(\hat{\theta})$ of men; Women of types below $\hat{\theta}$ and all men exert $e = 1$ and get promoted.*

Proof. In any equilibrium of any $(\mathbf{s}^i, \boldsymbol{\mu}^i)$-subgame, the firm's total surplus is no higher than 0.5 if the threshold exceeds $\bar{\theta}$. Thus, Theorem 1 follows from Lemmas 5 and 6. ∎

It is evident from Theorem 1 that women are treated unfavorably in promotion: They have to work as hard as men to be promoted even if they know that they will be paid strictly less after promotion ($s_f^*(\hat{\theta}) < s_m = 1.5$). This treatment is unjustified since, as shown in Lemma 4, offering $s_f = s_m$ would be sufficient to ensure that promoted women exert the same effort as men and remain in their posts with certainty throughout period 2, making them fully identical to men from the employer's perspective. Part of the rationale for the result is that the monopoly employer capitalizes on its market power to extract a higher surplus from some women who are more desperate than men in pursuing promotion since their alternatives are worse. However, surplus-extraction by a monopoly employer is only part of the story. The next section characterizes the equilibrium offers when firms engage in Bertrand competition for employees. Most surprisingly, we show that the differential treatment suffered by promoted women survives even in this case.

4. Unique Equilibrium under Bertrand Competition

In light of the hiring rules in the presence of a rival firm, (H1) and (H2), each firm's critical concern is whether to try to assume the leader status by offering a contract of a higher value than the rival's. Recall that the value $v(\mathbf{s})$ of a contract $\mathbf{s}$ is the ex ante utility of a female worker in the continuation equilibrium most favored by the firm, following the firm's offer of $\mathbf{s}$. For $\mathbf{s}^*(\theta)$ where $\theta \in [w_f, \bar{\theta}]$, therefore,

$$v(\mathbf{s}^*(\theta)) = v(\theta) := u_\ell(\theta)D(\theta) + \int_{\theta' > \theta} u_\ell(\theta')dD(\theta') \tag{8}$$

because, by Lemma 5, measure $\mu_f^*(\theta)$ of women are hired and they pursue promotion when their types are below θ in the unique equilibrium of the continuation subgame ensuing an offer of $\mathbf{s}^*(\theta)$. Note that, abusing notation slightly, we also use $v(\cdot)$, as a function of θ, to represent ex ante utility of a female worker in a departure-free equilibrium with threshold θ. Clearly,

$v(\theta)$ is constant at $v(w_f)$ for $\theta \leq w_f$ and strictly increases for $\theta \geq w_f$. That $v(\mathbf{s}) \geq v(w_f)$ for all $\mathbf{s}$ is trivial since women get $v(w_f)$ by forgoing promotion.

It is intuitive (and proved in Appendix) that offering a contract of a value larger than $v(\bar{\theta})$ is suboptimal because the maximum possible surplus thereof falls short of 0.5 which can be guaranteed by $\mathbf{s}^*(\theta)$ for $\theta \in (w_f, \bar{\theta})$ according to Lemma 5. Thus, we focus on contracts with values between $v(w_f)$ and $v(\bar{\theta})$.

For $\theta \in [w_f, \bar{\theta}]$, Lemma 5 (b) and (c) establish that the unique equilibrium ensuing a firm's offer of $\mathbf{s}^*(\theta)$, conditional on the firm becoming a leader, delivers $\Pi_L(\theta)$ to the firm, the maximum possible surplus subject to offering a contract of value $v(\theta)$. Furthermore, if an equilibrium with threshold θ generates a surplus of $\Pi_L(\theta)$, the equilibrium contract must be $\mathbf{s}^*(\theta)$ by Lemmas 2 and 3 and the discussion surrounding derivation of (5). Consequently, $\mathbf{s}^*(\theta)$ is the uniquely optimal contract for a firm to offer among all contracts whose value is $v(\theta)$, conditional on the firm becoming a leader.[17] Since $\mathbf{s}^*(\theta)$ is an optimal contract among those with the same value, $v(\theta)$, conditional on the firm becoming a follower as well (as is shown in Appendix), we have

Lemma 7. *In any equilibrium, both firms offer a contract in*

$$\mathbf{S}^* \ = \ \{\, \mathbf{s}^*(\theta) \mid w_f \leq \theta \leq \bar{\theta} \,\}. \tag{9}$$

Proof. See Appendix. ∎

It is clear from Lemma 5 that both firms would have liked to obtain the maximum possible surplus, $\Pi_L(\hat{\theta}) = \max_\theta \Pi_L(\theta)$, by hiring measure $\mu_f(\hat{\theta}) > 1$ of women after offering $\mathbf{s}^*(\hat{\theta})$, but this is not possible because hiring more than half of the total women workforce requires the firm being a leader in hiring, and not both firms can do so. Furthermore, Bertrand competition implies that achieving $\Pi_L(\theta)$ for any other $\theta \neq \hat{\theta}$ by offering $\mathbf{s}^*(\theta)$ and acquiring the leader status, would not be viable in equilibrium unless the other firm can also obtain at least the same payoff as a follower, for otherwise the other firm would snatch the leader status by "overbidding" marginally.

In order to see when this may be the case, from Lemma 5 we derive the surplus of a firm as a follower when it has offered $\mathbf{s}^*(\theta')$, i.e, conditional on the other firm having offered $\mathbf{s}^*(\theta)$ where $\theta \in (\theta', \bar{\theta}]$ and hired measure $\mu_f^*(\theta) > 1$ of women as per Lemma 5, leaving measure $2 - \mu_f^*(\theta)$ of them for the follower:

$$\Pi_F(\theta|\theta') := 0.5 + \frac{1 - 2D(\theta)}{1 - D(\theta)} D(\theta')(\pi_f^*(\theta') - 0.5). \tag{10}$$

[17] However, $\mathbf{s}^*(\theta)$ is not optimal among all contracts such that $s_f = s_f^*(\theta)$. For example, if $\theta > \hat{\theta}$ then increasing s_m may lower the threshold so that $\Pi_L(\theta)$ is higher (hence lower v), and consequently leads to a higher total surplus, albeit it is not optimal for the reduced threshold. This is why we opted against defining $v(\cdot)$ as $v(\mathbf{s}) = s_f$.

It is straightforward to verify that $\Pi_F(\theta|\theta')$ is strictly concave as a function of $\theta' \geq w_f$, and strictly decreases in $\theta' \in (w_f, \bar{\theta})$ because

$$\theta^{**} := \arg \max_{w_f \leq \theta \leq \bar{\theta}} D(\theta)(\pi_f^*(\theta) - 0.5) \ = \ w_f \tag{11}$$

as a routine calculation shows. That $\Pi_F(\theta|\theta')$ peaks at $\theta' = w_f$, which is due to $w_f > 0.9$, is unimportant for our results but facilitates exposition: This implies that the lower is the threshold of the offered contract, $\mathbf{s}^*(\theta) \in \mathbf{S}^*$, the higher is the firm's profit conditional on becoming a follower.

Given $\theta' \in (\theta^{**}, \bar{\theta})$, note from (10) that $\Pi_F(\theta|\theta')$ strictly decreases in $\theta \leq \bar{\theta}$ because $\pi_f^*(\theta') > 0.5$. To examine the relationship with $\Pi_L(\theta)$, note also that $\Pi_L(\theta') > \Pi_F(\theta'|\theta')$ since $\mu_f^*(\theta') > 1$, whilst $\Pi_L(\bar{\theta}) = 0.5 < \Pi_F(\bar{\theta}|\theta')$. In addition, it is a straightforward calculation to verify that $\Pi_L(\theta)$ has a steeper slope than $\Pi_F(\theta|\theta')$ when they intersect in $(\theta', \bar{\theta})$.[18] Letting $\theta^* \in (\theta^{**}, \bar{\theta})$ denote the unique θ that satisfies $\Pi_L(\theta) = \Pi_F(\theta|\theta^{**})$, these observations produce the graphs of $\Pi_L(\theta)$ and $\Pi_F(\theta|\theta^{**})$ in Figure 4. The graph of $\Pi_L(\theta)$ peaks at $\hat{\theta} \in [w_f, \bar{\theta})$. The dotted curve represents $\Pi_F(\theta|\theta')$ for an arbitrary $\theta' \in (\theta^{**}, \bar{\theta})$.

[Figure 4 about here]

As illustrated in Figure 4, for any pair of threshold levels, say θ^A and θ^B in $[\theta^{**}, \theta^*)$, it cannot be an equilibrium that firm i offers a contract $\mathbf{s}^*(\theta^i)$, $i = A, B$, because the firm that were to offer a contract with a lower threshold, say $\theta^A \leq \theta^B$, would do better by offering a contract with a threshold slightly higher than θ^B, say $\theta^B + \epsilon < \theta^*$, which would secure the leader status for firm A and a payoff, $\Pi_L(\theta^B + \epsilon)$, that exceeds the payoff that it can get by offering $\mathbf{s}^*(\theta^A)$, $\Pi_F(\theta^B|\theta^A)$, since $\Pi_F(\theta^B|\theta^A) \leq \Pi_F(\theta^B|\theta^{**}) < \Pi_L(\theta^B)$. In addition, it cannot be an equilibrium that either firm, say B, offers a contract $\mathbf{s}^*(\theta)$ where $\theta \in [\theta^*, \bar{\theta}]$, either, because then firm A's best response would be to offer $\mathbf{s}^*(\theta^{**})$, which in turn implies that firm B should offer $\mathbf{s}^*(\hat{\theta})$, or $\mathbf{s}^*(\hat{\theta} + \epsilon)$ if $\hat{\theta} = \theta^{**}$, rather than $\mathbf{s}^*(\theta^*)$.

This leads us to conclude that the firms use a mixed strategy in contract announcement in equilibrium. The next theorem characterizes a symmetric equilibrium in which the two firms randomize over $\mathbf{s}^*(\theta)$ where θ ranges between θ^{**} and an upper bound $\widetilde{\theta} < \theta^*$, and exhibits the aforementioned features of the glass ceiling. In fact, it is the unique equilibrium outcome.

[18] Intersection means $D(\theta)(\pi_f^*(\theta) - 0.5) = (1 - 2D(\theta))D(\theta')(\pi_f^*(\theta') - 0.5)$. It suffices to show that the derivative of the RHS minus that of the LHS is negative. This subtraction is $\frac{D'(\theta)(\pi_f^*(\theta) - 0.5)}{1 - 2D(\theta)} - \frac{D(\theta)}{2} = \frac{D'(\theta)D(\theta')(\pi_f^*(\theta') - 0.5)}{D(\theta)} - \frac{D(\theta)}{2} \leq D'(\theta)(1 - \frac{w_f + \theta'}{2}) - \frac{D(\theta)}{2} = \frac{\alpha}{2}(2 - w_f - \theta' - \theta) \leq \frac{\alpha}{2}(2 - 3w_f) < 0$, where the first equality follows from the equation above, the first inequality from $D(\theta') \leq D(\theta)$, the second equality from $D(\theta) = \alpha\theta$, the second inequality from $w_f \leq \theta' \leq \theta$, and the final inequality from $w_f > 0.9$.

Theorem 2. *The game described in Section 2 has a unique equilibrium outcome. It is symmetric: both firms offer a contract* $\mathbf{s}^*(\theta)$ *according to a continuous distribution function on* θ *with a support* $[\theta^{**}, \widetilde{\theta}]$ *where* $\widetilde{\theta} \in (\hat{\theta}, \theta^*)$ *and* $\Pi_L(\widetilde{\theta}) < \Pi_F(\theta^{**}|\theta^{**})$. *The firm's ex ante equilibrium payoff is* $\Pi_L(\widetilde{\theta}) > \Pi_L(\theta^*) > 0.5$. *Furthermore,*

(i) The firm, say $i \in \{A, B\}$, *that offers a contract with a higher threshold, say* θ^i, *hires measure* $\mu_f^*(\theta^i)$ *of women and measure* $2 - \mu_f^*(\theta^i)$ *of men, and the other firm hires the residual labor force;*

(ii) The ex ante utility of women is higher when hired by the firm that offers a contract with a higher threshold;

(iii) Every promoted female worker in either firm will have exerted the same effort as men ($e = 1$) *and will never leave the firm, but will be paid strictly less than men.*

The proof primarily consists of showing that there is a unique distribution function on $\mathbf{S}^*$ that satisfies the incentive compatibility conditions for every contract in its support, which gets lengthy and hence is deferred to Appendix.

A few aspects of the equilibrium call for discussion. First, the glass-ceiling features described in Theorem 2 also emerge in an alternative model in which the two firms announce their contracts sequentially in a pre-determined order. In this model a unique equilibrium exists in pure strategies: The first firm offers $\mathbf{s}^*(\theta^*)$ and becomes a leader and the second firm offers $\mathbf{s}^*(\theta^{**})$ and becomes a follower. The mixed strategy in Theorem 2 is necessary because the sequence of pure best responses "cycles." This is reminiscent of a capacity-constrained Bertrand duopoly model of Kreps and Scheinkman (1983, Lemma 6), however the analysis differs due to one significant technical feature, namely, that the threshold of the optimal contract to offer as a follower, θ^{**}, may in our model be strictly below that as a leader, $\hat{\theta}$.

Second, although both firms follow the same equilibrium strategy, the competitive outcome characterized in Theorem 2 is essentially asymmetric: firms differ in the actual contracts they offer to female workers. The firm that offers a better contract (i.e., a higher female salary) faces an unconstrained female workforce, since it is the workers' favorite employer. The other firm faces a constrained female workforce, since its female contract is less attractive. Hence, in equilibrium, more and less "female-friendly" firms coexist.

Third, the equilibrium in Theorem 2 exhibits that women are disadvantaged in compensation but not in the requirements for promotion. This is because efficient promotion requirement is the same for men and women and the firms can freely control the salary level to adjust the attractiveness of promotion package without distorting efficiency. However, one can think of reasons that the core forces of Theorem 2 may cause disadvantage for women in promotion requirements as well as in compensation. For example, if firms are constrained by a lower bound in the upper-tier salary for some reason (e.g., social norm, limits on salary difference for same jobs), the harsh treatment in equilibrium may be partly reflected as tougher promotion criteria. Alternatively, if the exerted effort is observed with some noise, even if the promotion criteria are similar (as in Theorem 2), it is conceivable that women of very low types would exert more effort than men because they face worse consequences in case of non-promotion.

<u>*Extensions*</u>

Effort is unverifiable. We carried out our analysis presuming that the effort level is sufficiently verifiable so that the workers are protected by the court if they do not get promoted "unfairly." This aspect has the effect of pinning down the effort levels that are viable in equilibrium, and guarantees uniqueness of equilibrium. In many circumstances, though, effort levels may be too costly to verify. In the extreme case where it is unverifiable at all, equilibrium effort levels are supported by self-confirming beliefs. It is natural in this case that there exist multiple equilibria, supported by various self-confirming workers' belief profiles over the effort levels required in order to achieve promotion. It is straightforward to verify that the equilibrium in Theorem 2 continues to be an equilibrium in this case, supported by the belief that any women who exert an effort level less than 1 will be perceived to be of a very high type who will leave in the middle of her career, hence will not be promoted. In addition, there are a variety of equilibria, both in pure and mixed strategies, that maintain the core features of the equilibrium identified in Theorem 2, namely that promoted women may end up being paid less and/or exerting more effort than their male counterparts. It may be worth stressing that firms may actively try to influence workers' beliefs–for instance, by taking specific actions, setting precedents, or even by making "cheap-talk" declarations. These actions/declarations may act as devices to coordinate workers' beliefs over the terms under which promotion may be achieved for male/female workers in different firms. Naturally, whenever possible, each firm would find it optimal to shape workers' beliefs so as to maximize its expected profit, taking beliefs over the other firm as given, which if successful would lead to an equilibrium akin to that of Theorem 2.

Contracts also specify wage in period 1. In the main analysis, for ease of exposition we disallowed a contract to specify a wage for period 1. Relaxing this restriction does not change the result. The core logic behind this conclusion is straightforward: Any wage for period 1 simply increases the workers' utility by the same amount regardless of what they do, without any strategic implication, and thus, does not affect workers' decisions as to whether to pursue promotion or not, or their effort decisions. Since the firms compete for women at the hiring stage as our analysis made clear, this means that firms would not offer a positive first-period wage for men.

For women as well, for any given $\theta \in (0, \bar{\theta})$, it is a routine calculation to verify that, conditional on being a leader, a firm obtains a higher surplus by offering a female contract $\mathbf{s}^*(\theta)$ without period 1 wage than it does by offering any other female contract of the same value. The basic reason is that since period 1 wage is paid to *all* women hired, a firm cannot recover that extra expense from the smaller number of women who would pursue promotion without lowering the ex ante value of the contract.[19] The same is also true conditional on the firm being a follower. Hence, firms would not offer a positive wage for period 1 for women, either. As a consequence, Theorem 2 continues to holds when contracts specify a wage for period 1 as well.

More than two firms. Our main insights extend to cases of more than two firms. It is straightforward to verify that, if effort levels are unverifiable, "glass ceiling" equilibria can be sustained by self-confirming beliefs of workers on appropriate promotion strategies that vary across firms, regardless of the number of firms. The situation is less clear-cut if effort levels are verifiable, though, since the complexity of the model rapidly increases with the number of firms competing in the market. However, our analysis has identified a key

[19]For any period 1 female wage $s_1 > 0$, the expected utility of a woman from forgoing promotion is $u_\ell(\theta|s_1) = s_1 + u_\ell(\theta)$. Let $(\mathbf{s}_1, \mathbf{s})$ denote a contract specifying s_1 for period 1 and s_f for period 2 conditional on promotion for women, such that the value of $(\mathbf{s}_1, \mathbf{s})$ is equal to $v(\theta')$ for some $\theta' \in (\theta, \bar{\theta})$. Note that this implies that the expected payoff of a woman in period 2 is strictly lower than θ' under $(\mathbf{s}_1, \mathbf{s})$ and thus, there is a threshold type $\theta'' < \theta'$ who is indifferent between pursuing promotion or not under $(\mathbf{s}_1, \mathbf{s})$. Since the firm would be worse off if $\theta'' = 0$, assume $\theta'' > 0$. Since women of all types $\theta \leq w_f$ would be indifferent if $\theta'' \leq w_f$, we may assume that $\theta'' \in [w_f, \theta')$. Note that θ'' is uniquely determined by θ' and s_1 because $s_1 + v(\theta'') = u_\ell(\theta'')D(\theta'') + \int_{\theta > \theta''} u_\ell(\theta) dD = v(\theta')$ and $v(\theta)$ increases in $\theta \in [w_f, \bar{\theta}]$. Since the maximum possible surplus of firm subject to inducing θ'' is $\Pi_L(\theta'')$ as per Lemma 3, the firm's surplus from offering $(\mathbf{s}_1, \mathbf{s})$ is no higher than $\Pi_L(\theta'') - s_1 \mu_f^*(\theta'')$ conditional on becoming a leader. We now show that $\Delta := \Pi_L(\theta'') - s_1 \mu_f^*(\theta'') - \Pi_L(\theta') < 0$ for all relevant parameter values, i.e., for all $s_1 > 0$, $w_f \in (0.9, 1)$ and $\theta' \in (0.9, 4/3)$. Since $\Delta = 0$ if $s_1 = 0$ by definition, it suffices to show that Δ decreases in $s_1 > 0$. Since a routine calculation verifies that $\frac{\partial \Delta}{\partial s_1} = \Pi_L'(\theta'') \frac{\partial \theta''}{\partial s_1} - \mu_f^*(\theta'') - s_1 \mu_f^{*\prime}(\theta'') \frac{\partial \theta''}{\partial s_1} = 2 \frac{8\sqrt{\alpha}(-2+s_1) - \alpha\sqrt{\alpha}(2+w_f)^2 + (2+\alpha(2+w_f))\sqrt{-16s_1+\alpha(2+w_f+2x)^2}}{\sqrt{-16s_1+\alpha(2+w_f+2x)^2}\left(2+\alpha(2+w_f) - \sqrt{\alpha}\sqrt{-16s_1+\alpha(2+w_f+2x)^2}\right)^2}$, it suffices to show that the top line, which we denote by Δ^{top}, is negative. First, note that $\Delta^{\text{top}}|_{s_1=0} = \sqrt{\alpha}\big(-16 - \alpha(2 + w_f)^2 + (2+\alpha(2+w_f))(2+w_f+2\theta')\big)$, which is negative because $(2+\alpha(2+w_f))(2+w_f+2\theta')$ achieves its maximum value when $(\alpha, w_f, \theta') = (1/4, 1, 4/3)$ and it is smaller than 16. Next, the derivative of Δ^{top} with respect to s_1 is $8\frac{-2-\alpha(2+w_f)+\sqrt{\alpha}\sqrt{-16s_1+\alpha(2+w_f+2\theta')^2}}{\sqrt{-16s_1+\alpha(2+w_f+2\theta')^2}}$, which is also negative because $\sqrt{\alpha}\sqrt{-16s_1 + \alpha(2 + w_f + 2\theta')^2}$ is routinely verified to achieve its maximum value when $(\alpha, w_f, \theta', s_1) = (1/4, 1, 4/3, 0)$ and it is smaller than 2. Therefore, Δ^{top} is negative for all relevant parameter values as desired. The same can be analogously verified for the contingency of the firm becoming a follower.

rationale behind our results, namely that firms do not want to hire too many women at the entry stage. Intuitively, a firm hiring too many women would find it hard to fill all its upper-tier posts and at the same time offer women promotion packages that are inferior to those of men. Consider, for instance, a firm that hires only women. To fill all its upper-tier posts, at least one half of its workforce must pursue promotion. To achieve this, women must be offered sufficiently favorable promotion packages, i.e., more favorable than those offered to men. But then, it is undesirable for the firm to hire only women in the first place, which would bring a contradiction. This argument applies whenever the share of women hired is "too large," and it ensures that there is an upper bound on the share of women that firms will optimally hire. This basic aspect of our framework remains in force when we allow for more than two firms and consequently, our results extend to such cases. To be precise, for any given number of firms, one can find an open set of environments (i.e., parameter values) for which our results hold. On the other hand, fixing an environment, our results may cease to hold if there are too many firms.

5. Policy Analysis

A key result of our analysis is that promoted women are worse off than their male counterparts, since they have to show their determination by working at least as hard as the equivalent male would do, even though they anticipate poorer compensation after promotion. Although this is not a result of discrimination in the conventional sense, it is the case that women who wish to be promoted may have to accept a worse deal because, and only because, they are female. In this sense career women are clearly disadvantaged in the workplace.

As highlighted in the previous section, in our model the disadvantaged treatment suffered by women in upper-tier jobs is essentially a spillover of the wage differential in low-tier jobs. In low-tier jobs, women always leave when encountered by a non-market option (since in this case the value of their non-market option is strictly higher than that from remaining in the job). Hence, on average, the surplus generated by a woman is strictly below that of a man. This results in women earning strictly lower wages than men in low-tier jobs. Note however that although the wage earned by women in the low-tier market reflects their average productivity, some women—those with a low propensity of leaving for the outside option—would find it less than their (expected) productivity. For those women, the existence of more "family-oriented" fellow women imposes a negative externality. This externality is key to the disadvantaged treatment suffered by women in upper-tier jobs.

A natural policy to consider to eliminate the problem is to require that firms offer the same wage to men and women in low-tier jobs. From our earlier analysis, it is clear that if $w_f = w_m$ then a firm would be unable to extract more surplus from promoted women than promoted men. However, it is doubtful that such a policy would be effective. Essentially, women in low-tier jobs *are* on average less productive than men. A policy that obliges firms to pay their male and female employees exactly the same low-tier wage would act as a tax on firms that hire a mixed labor force. A likely outcome is that, to circumvent the policy, firms would "specialize", and employ only female or only male workers in low-tier jobs. Under perfect competition, in each type of firm employees would be paid their expected productivity, but, crucially, wages in firms specialized in female workers would be lower than those in firms specialized in male workers. Thus, imposing gender-free wages in low-tier jobs may not alleviate the glass-ceiling phenomenon in upper-tier posts, since male/female wage differentials in low-tier jobs may persist in spite of the policy.

Below we review a number of policy measures that target firms' promotion decisions more directly, and discuss the extent to which they may alleviate the glass-ceiling phenomenon.

(a) *Gender-free contracts for upper-tier jobs.* Consider a policy requiring that all contracts offered must be gender-free, i.e., the same set of contracts are available for workers of either gender to choose one from and apply for. So long as the belief is viable that women would get treated favorably both in hiring and promotion only when they select the "right" contract, outcomes where promoted women suffer a disadvantage vis-à-vis men may still emerge in equilibrium.

To formalize this general observation, we need to modify the hiring process of our model as follows:

1) Both firms simultaneously announce one menu of post-promotion salaries.
2) Every worker applies to both firms by selecting one salary each from their menus (as the terms of employment to be considered for).
3) Firms simultaneously make employment offers to workers who, if pursued by both firms, decide which one to accept.
4) The promotion subgame ensues in each firm as described in Section 2.

Consider the equilibrium of Theorem 2 embedded in the modified model with the following off-equilibrium belief: neither firm would hire any women who select the "men's salary." Note that this belief is justified in the case of unilateral deviation by a female worker, because the leader firm can achieve its ideal entry-level hiring without including the deviator and the follower firm would be able to fill all its upper-tier posts as it wishes regardless of whether it hires the deviated worker or not (since only a fraction of workers with the "men's salary" would be promoted any way). Thus, unilateral deviation is not beneficial and consequently, the equilibrium of Theorem 2 continues to be an equilibrium under this policy as stated in the next Proposition. Note that this result is valid regardless of whether firms have a continuum of positions to fill or finitely many of them.

Proposition 1. *The following is an equilibrium of the game modified as above: The two firms announce a contract (as a menu of post-promotion salaries without being gender-specific) in the same manner as in Theorem 2; all male workers select* $s = 1.5$ *and all female workers select the other salary for both firms; the two firms hire as in Theorem 2 and all agents behave in the subsequent promotion subgame as in Theorem 2.*

Different salaries that firms offer in this equilibrium can be interpreted as corresponding to potentially different "career-paths" that appear to be available for all in principle, but in practice are intended for different types of workers. In particular, female workers may understand that, if they wish to achieve promotion, then they should follow the path that was intended for them. Thus, it is possible for firms to implement harsher promotion terms for women than for men and consequently, the glass ceiling equilibrium described in Theorem 2 still emerges, supported by the belief that the firms would not hire any women who indicate that they intend to pursue the "wrong" career path (namely, that intended for men). This effect is even more prominent when effort level is not verifiable, because in that case firms would be at more liberty to mistreat those women who choose the wrong path. Therefore, although there are other equilibria, including one in which all workers get promoted on the same terms, putting this policy in place does not warrant transition to a more equitable promotion outcome.

(b) *A certain fraction of women must be promoted.* Now consider an alternative policy that requires each firm to promote at least a fixed fraction, say $x \in (0, 1)$, of its female employees. Under this policy, if a firm hires a measure $\mu_f \in [0, 2]$ of women, then they must promote at least measure $x \cdot \mu_f$ of women. Clearly, this requires that all women of type $\theta < D^{-1}(x)$ pursue promotion. Hence, this policy effectively imposes restrictions on the contracts that firms may offer: the marginal female type who would pursue promotion must be no lower than $D^{-1}(x)$. If $x > D(\theta^{**})$, therefore, this policy would improve the ex ante welfare of women. Nevertheless, the effect of this policy falls short of removing gender differential so long as $x < D(\bar{\theta})$, and the main features of the glass ceiling will remain (albeit to a lesser degree) in the ensuing equilibrium. If $x > D(\bar{\theta})$, on the other hand, a reversed glass ceiling phenomenon arises: men work as hard as women to get promoted, yet they are paid less than their female counterparts. Only when $x = D(\bar{\theta})$, such a policy would correct the glass ceiling phenomenon. Furthermore, such remedy would be temporary in nature unless the policy were to remain in place permanently, for the competitive equilibrium would reemerge when the policy is lifted.

Proposition 2. *Suppose that a policy requires firms to promote at least a fixed fraction x of their female employees. If $x \leq D(\theta^{**})$, Theorem 2 continues to hold. If $D(\theta^{**}) < x < D(\bar{\theta})$, any equilibrium conforms to the equilibrium described in Theorem 2 with the following modification: θ^{**} is replaced by $\theta_x = D^{-1}(x)$ and $\widetilde{\theta}$ by an appropriate $\widetilde{\theta}' \in (\hat{\theta}, \bar{\theta})$ such that $\Pi_L(\widetilde{\theta}') < \Pi_F(\theta_x|\theta_x)$. If $x = D(\bar{\theta})$, in equilibrium both firms offer the same salary, 1.5., to both men and women. If $x > D(\bar{\theta})$, men are offered a contract that is inferior to that offered to women.*

Proof. See Appendix. ∎

(c) *A certain fraction of upper-tier positions must be held by women.* Consider now a variant policy, namely, that a certain fraction, say y, of all upper-tier posts should be occupied by women in both firms.[20] Although policies (b) and (c) may at first glance appear similar – they both impose quota requirements on the firms – there are important qualitative differences between the two. To see this, note that the requirement imposed by policy (b) has the same bite, independently of the number of women present in a firm. By contrast, the requirement imposed by policy (c) may be more or less stringent, depending on the number of female workers available. Clearly, if a firm has a small female workforce then the requirements of policy (c) may be fulfilled only if a large fraction of those women actually get promoted, whilst it can be satisfied more easily if a firm has a large female workforce. In contrast with policy (b), therefore, policy (c) lowers a follower's payoff *more* than it does a leader's. This raises the relative value of obtaining a "leader" position in the market, and thus strengthens the competitive pressure for firms to attract female workers. As a result, the "no-glass-ceiling" outcome can be sustained more easily than under policy (b). Proposition 3 makes this point precise.

Proposition 3: *There is $y_0 < D(\bar{\theta})$ such that if a policy requires the firms to fill a fraction $y \in [y_0, D(\bar{\theta})]$ of upper-tier posts with women, then in any equilibrium both firms offer the same salary, 1.5., to both men and women and all women of types $\theta < \bar{\theta}$ exert $e = 1$ and get promoted.*

Proof. See Appendix. ∎

Proposition 3 highlights how, under policy (c), the "no-glass-ceiling" equilibrium may be achieved for a "wide" range of y values. By contrast, under policy (b) this is only possible if x is *precisely* equal to $D(\bar{\theta})$ – something

[20]This policy has been adopted for instance by Norway, where, from 1 January 2008, it has become compulsory for companies to allocate 40% of their management board seats to women. Similarly, Spain has recently passed legislation obliging some large companies to increase the number of women on corporate boards.

that is virtually impossible to achieve if policy-makers are less than perfectly informed. In this sense, therefore, policy (c) appears more promising in eliminating the glass-ceiling phenomenon, since it imposes less stringent informational requirements on planners. Note however that, similar to policy (b), policy (c) offers only a temporary remedy and may generate a "reversed glass ceiling" phenomenon if $y > D(\bar{\theta})$.

6. Concluding Remarks

Beyond policy interventions, another often-invoked vehicle for eliminating discriminatory outcomes is competition. As argued by Becker (1957), in the presence of taste-based discrimination, sufficiently strong competition between firms should eliminate any type of differential treatment of employees. In our framework, however, competition has limited effect: firms compete Bertrand-style for female employees but they are nonetheless able to hire women at remuneration levels that do not reflect their productivity in upper-tier jobs. Intuitively, even though promoted women are more profitable than promoted men, this does not imply that firms benefit from hiring as many women as possible. As discussed earlier, a monopolist would not find it optimal to hire only women, since this would either generate vacancies in upper-tier posts, or it would result in the firm having to offer very generous promotion packages to induce "family-oriented" women (i.e., with a high θ) to pursue promotion. An implication of this is that in a "women only" world women would be better off than they currently are, since they would not suffer from internal competition from male employees.

The motivations that lead a monopolist to refrain from hiring only women also carry over to an environment where firms compete for employees, ensuring that the incentives to "overbid" one's rival to gain a leader position are relatively weak. As a result, competition for women at the hiring stage is never too strenuous, and does not eliminate the differential treatment suffered by promoted women in our equilibrium.

An alternative way in which competition between firms may operate is in ex-post stage, i.e., after employees have made their human-capital investment/effort choice in period 1. Although such competition may in principle lower the firms' ability to extract greater surplus from female employees, there are at least two reasons for why this effect may not be big.[21] First, since an employee's effort choice is not readily observable to outsiders (as in our model), poaching employees from a rival firm before promotion decisions involves a

[21] This is backed for instance by Baker, Gibbs, and Holmstrom (1994), who provide a detailed (and rather unique) analysis of twenty years of personnel data from one firm. They find that the percentage of outside hires to total hires in a position is declining in the position's seniority (and reaches only 10% for the three highest tiers in the organization).

risk, as poached employees could turn out to have low human capital. Second, if employees acquire human capital that is partly firm-specific (see e.g., Becker, 1962) competition between firms is weakened even after human-capital investment choice is observed, e.g., after promotion decisions are made.

To summarize, in this paper we provide an economic model that explains the glass ceiling phenomenon as a natural equilibrium outcome of Bertrand competition in the labor market. Our findings suggest that, because of the specific nature of the problem at hand, competition between firms falls short of eliminating this phenomenon. The implications are also less than promising as to what can be done to improve things for career women. Insisting that contracts cannot be gender-specific may not change the situation. Imposing a quota for women in senior positions, is subject to problems, too. One is that, to get the proportion right, the planner may need to have a great deal of information, although to a lesser degree when the quota is imposed relative to available posts. Perhaps more importantly, a short-lived intervention will not solve the problem since the discriminatory practice would re-emerge once it is lifted, unlike in most statistical discrimination models. In terms of the application of our results, as explained earlier, they may be more widely applicable than gender, e.g., to other observationally distinct groups such as immigrant employees.

Appendix

Proof of Lemma 3. Given the discussion preceding Lemma 3, it only remains to verify that per-promotion surplus in any equilibrium with departures, i.e., with threshold $\theta > s_f$, is less than $\pi^*_f(\theta)$ for $\theta \le \bar{\theta}$ and less than 0.5 for $\theta > \bar{\theta}$.

We start with the cases that $\theta \neq \infty$, i.e., $\theta \in (s_f, 1/\alpha)$, so that $u_h(s_f|e_f, s_f) = u_\ell(s_f)$ which implies that $e_f = \sqrt{s_f - w_f}$. Thus, the per-promotion surplus is

$$\begin{aligned}\tilde{\pi}(s_f|\theta) &:= (1 + \sqrt{s_f - w_f} - s_f)\frac{D(s_f) + (D(\theta) - D(s_f))/2}{D(\theta)} \\ &= \frac{s_f + \theta}{2\theta}\Big(1 + \sqrt{s_f - w_f} - s_f\Big) \qquad (12)\end{aligned}$$

and its first and second derivatives with respect to s_f are

$$\begin{aligned}\tilde{\pi}'(s_f|\theta) &= \Big(1 - 2s_f - \theta + \frac{3s_f - 2w_f + \theta}{2\sqrt{s_f - w_f}}\Big)\Big/(2\theta), \\ \tilde{\pi}''(s_f|\theta) &= \frac{1}{\theta}\Big(\frac{3s_f - \theta - 4w_f}{8(s_f - w_f)^{3/2}} - 1\Big). \qquad (13)\end{aligned}$$

Since it is routinely calculated that $\tilde{\pi}''(s_f|\theta) < 0$ and $\tilde{\pi}'(\theta|\theta) > 0$ for $w_f \le s_f \le \theta \le \bar{\theta} = 2 - w_f$ and $w_f \in (0.9, 1)$, it follows that $\tilde{\pi}(s_f|\theta)$ increases in $s_f \in (w_f, \theta)$ if

$\theta \le \bar{\theta}$. Since, in addition, $\pi_f^*(\theta) - \tilde{\pi}(\theta|\theta)$ decreases in $\theta < \bar{\theta}$ with a positive value at $\theta = \bar{\theta}$, we have $\pi_f^*(\theta) > \tilde{\pi}(\theta|\theta) \ge \tilde{\pi}(s_f|\theta)$ for all $s_f \in [w_f, \theta)$ if $\theta \le \bar{\theta}$.

Next, consider $\theta \in (\bar{\theta}, 1/\alpha)$. For $s_f < \bar{\theta}$, since (12) decreases in θ, we have $\tilde{\pi}(s_f|\theta) \le \tilde{\pi}(s_f|\bar{\theta}) < \tilde{\pi}(\bar{\theta}|\bar{\theta}) < 0.5$ because $\tilde{\pi}(s_f|\bar{\theta})$ has been verified above to increase in $s_f \in (w_f, \bar{\theta})$ and $\tilde{\pi}(\bar{\theta}|\bar{\theta}) = 1 + \sqrt{2 - 2w_f} - (2 - w_f) < 0.5$. For $s_f \in [\bar{\theta}, \theta)$, we have $\tilde{\pi}(s_f|\theta) \le \tilde{\pi}(s_f|s_f) < 0.5$ because $\tilde{\pi}(s_f|s_f) = 1 + \sqrt{s_f - w_f} - s_f$ is routinely verified to be strictly concave in $s_f > 0$ with a maximum of $5/4 - w_f < 0.5$ at $s_f = (1 + 4w_f)/4 > \bar{\theta}$ for $w_f \in (0.9, 1)$.

Finally, if the threshold of an equilibrium is ∞ then $e_f \le \sqrt{s_f - w_f}$ and thus, per-promotion surplus is $(\alpha s_f + 1)(1 + e_f - s_f)/2 \le (s_f + 4)(1 + \sqrt{s_f - w_f} - s_f)/8 < 0.5$ where the last inequality is easily verified for $w_f \in (0.9, 1)$ and $s_f \ge w_f$. ∎

Proof of Lemma 5: (a) Fix a threshold type $\theta \le \bar{\theta}$. As per the explanation preceding Lemma 5, the firm's total surplus is no higher than $\pi_f \mu_f \alpha\theta + \pi_m \max\{2 - \mu_f, 1 - \mu_f \alpha\theta\}$ where $\pi_m \le 0.5$ and $\pi_f \le \pi_f^*(\theta)$. Thus, it is bounded above by

$$\mu_f \alpha\theta(\pi_f^*(\theta) - 0.5) + 0.5 \text{ if } \mu_f < \frac{1}{1 - \alpha\theta}, \text{ and } \mu_f \alpha\theta \pi_f^*(\theta) + (2 - \mu_f)0.5 \text{ otherwise.}$$

This upper bound increases in μ_f for $\mu_f < \frac{1}{1-\alpha\theta}$, but decreases for $\mu_f > \frac{1}{1-\alpha\theta}$ since $\alpha\theta\pi_f^*(\theta) < 0.5$ by Lemma 3 because $\alpha < 1/4$, $\bar{\theta} = 2 - w_f$ and $w_f > 0.9$. Therefore, it is highest when $\mu_f = 1/(1 - \alpha\theta) > 1$, producing $\Pi_L(\theta)$ defined in (5).

Next, it is routinely calculated that $\Pi_L'(\theta) = \frac{\alpha(1-w_f)}{(1-\alpha\theta)^2} > 0$ for $\theta \le w_f$ and that $\Pi_L''(\theta) = -\frac{\alpha(1-\alpha(2-w_f))}{(1-\alpha\theta)^3} < 0$ for $\theta \in [w_f, \bar{\theta}]$, establishing that $\Pi_L(\theta)$ achieves a unique maximum at a point, say $\hat{\theta} \in [w_f, \bar{\theta}]$. That $\hat{\theta} < \bar{\theta}$ follows from $\Pi_L'(\bar{\theta}) = \frac{\alpha(1-\bar{\theta}+(\alpha\bar{\theta}^2-w_f)/2)}{(1-\alpha\bar{\theta})^2} < 0$.

(b) Fix $\theta \in [w_f, \bar{\theta}]$ and consider $(\mathbf{s}^*(\theta), \boldsymbol{\mu})$-subgame with $\mu_f \le \mu_f^*(\theta)$. Since $\theta = \theta^c(1, s_f^*(\theta))$, the following is straightforwardly verified to be an equilibrium: women pursue promotion if and only if of types below θ by exerting $e = 1$ and measure $1 - \mu_f D(\theta)$ of men pursue promotion by exerting $e = 1$. Thus, firm's surplus is $0.5 + \mu_f D(\theta)(\pi_f^*(\theta) - 0.5)$, which strictly increases in μ_f from 0.5 when $\mu_f = 0$ to $\Pi_L(\theta)$ when $\mu_f = \mu_f^*(\theta)$. That this is the unique equilibrium of $(\mathbf{s}^*(\theta), \boldsymbol{\mu})$-subgame follows from the following somewhat more general result.

[A] In $(\mathbf{s}, \boldsymbol{\mu})$-subgame with $v(\mathbf{s}) = v(\theta)$ for some $\theta \in [w_f, \bar{\theta}]$, suppose there is a departure-free equilibrium, called the "core" equilibrium, in which the firm's total surplus exceeds 0.5, the female worker's ex ante utility is $v(\theta)$. Then, this is the unique equilibrium of $(\mathbf{s}, \boldsymbol{\mu})$-subgame.

Since we already showed that the firm's surplus cannot exceed 0.5 from promoting workers of only one gender, both genders get promoted in the core equilibrium, so that $s_m - (e_m)^2/2 \ge 1$ and $s_f - (e_f)^2/2 \ge w_f$ where e_g is the promotion effort level for gender g in the equilibrium. Moreover, the per-promotion surplus for female must exceed 0.5 due to Lemma 2, and that for male must be positive (since total surplus from female promotions cannot exceed 0.5), i.e., $\pi_m = 1 + e_m - s_m > 0$ and $\pi_f = 1 + e_f - s_f > 0.5$. In light of (Ci), this in turn implies that $e_f = e_m = e^* > 0$.

To reach contradiction, suppose there is another equilibrium, in which gender g exert e'_g for promotion (if they get promoted). It is not possible that gender g workers exert e'_g such that $1+e'_g-s_g=0$ for then, more workers of gender g pursue promotion than in the core equilibrium, whence less of the other gender should pursue promotion by exerting $e' \geq e^*$ for there to be no rationing in promotion (Lemma 1), but then the latter gender would benefit by exerting $e \in (e'_g, e')$ due to (Ci)–(Cii). The only exception to this logic is when $e'_g = 0$ and $s_g = w_g$, which is feasible only for $g = m$, i.e., $s_m = 1$, which would imply that men would exert $e > 0$ in no circumstances, contradicting the existence of the core equilibrium. Therefore, we deduce that $1 + e'_g - s_g > 0$ for $g = m, f$, and $e'_f = e'_m = e'$ due to (Ci).

Recall that $s_m - (e^*)^2/2 \geq 1$, i.e., men weakly prefer pursing promotion in the core equilibrium. Thus, any equilibrium with $e' > e^*$ is not viable because then a strictly lower measure of workers would pursue promotion than in the core equilibrium, leaving some upper posts unfilled; Any equilibrium with $e' < e^*$ is not either because then more than measure 1 of workers would pursue promotion (note that it is not viable that no worker of one particular gender pursues promotion in this case because some worker of that gender, the lowest type if female, would benefit by exerting e^*, guaranteeing promotion). This prove [A].

(c) Fix $\theta \in [w_f, \bar{\theta}]$ and consider $(\mathbf{s}^*(\theta), \boldsymbol{\mu})$-subgame with $\mu_f > \mu^*_f(\theta)$. For the same reason as above, $e_m = e_f = e^* > 0$. Then, $e^* \geq 1$ is not viable because $\mu_f\alpha\theta + 2 - \mu_f < 2 - \mu^*_f(\theta)(1-\alpha\theta) \leq 1$, i.e, some upper posts are unoccupied so that deviation of exerting slightly less effort level pays off due to (Cii). If $e^* < 1$, all men get promoted, and all upper posts must be filled since otherwise $e^* > 0$ would not be viable due to (Cii). Thus, $\mu_f\alpha\theta^c(e^*, s^*_f(\theta)) + 2 - \mu_f = 1$, i.e., $\mu_f\alpha(2s^*_f(\theta) - w_f - (e^*)^2) = \mu_f - 1 \Leftrightarrow \mu_f\alpha(1+\theta-(e^*)^2) = \mu_f - 1$. Consequently, firm's total surplus is

$$\begin{aligned}
\Pi(\mu_f) &= \mu_f\alpha(2s^*_f(\theta) - w_f - (e^*)^2)(1+e^*-s^*_f(\theta)) + (2-\mu_f)(1+e^*-1.5) \\
&= (\mu_f - 1)(1+e^*-s^*_f(\theta)) + (2-\mu_f)(1+e^*-1.5) \\
&= (\mu_f - 1)(1-s^*_f(\theta)) + (2-\mu_f)(1-1.5) + e^* \\
&= (\mu_f - 1)(1 - \frac{1+w_f+\theta}{2}) - 1 + \mu_f/2 + \sqrt{1+\theta - \frac{\mu_f - 1}{\mu_f\alpha}} \\
&= (1-\mu_f)\frac{1+w_f+\theta}{2} - 2 + 3\mu_f/2 + \sqrt{1+\theta-\frac{\mu_f-1}{\mu_f\alpha}},
\end{aligned}$$

and its derivative is

$$\Pi'(\mu_f) = 1 - \frac{\theta + w_f + \Lambda}{2} \quad \text{where} \quad \Lambda = \frac{\sqrt{1 - \frac{1}{\alpha} + \frac{1}{\alpha\mu_f} + \theta}}{\mu_f(1-\mu_f(1-\alpha-\alpha\theta))}. \tag{14}$$

It is routine calculation to verify that Λ is quasi-convex in $\mu_f > 0$ with a minimum value of $\Lambda_0 = 16(1-\alpha-\alpha\theta)\sqrt{1/\alpha - 1 - \theta}/(3\sqrt{3})$ at $\mu_f = \frac{3}{4(1-a\alpha-\alpha\theta)}$. It is then straightforwardly verified that $\Pi'(\mu_f)$ increase in $\alpha > 0$, decrease in $w_f > 0$, and that conditional on $\alpha = 1/4$ and $w_f = 0.9$ it increases in $\mu_f \in [\mu^*_f(\theta), 2]$ with a negative value at $\mu_f = 2 > \bar{\theta}$. Therefore, the firm's total surplus strictly decreases in $\mu_f \geq \mu_f^*(\theta)$. ∎

Proof of Lemma 6. Fix $\mathbf{s}^i = (s_m, s_f)$ and $\boldsymbol{\mu}^i = (\mu_m, \mu_f)$. For each possible equilibrium effort $e \geq 0$, all men go for promotion if $e < \bar{e}_m := \sqrt{2s_m - 2} \Leftrightarrow s_m - (e)^2/2 > 1$; all men are indifferent if $e = \bar{e}_m$; and none of them go for promotion if $e > \bar{e}_m$. The per-promotion surplus of firm is $1 + e - s_m$ which is positive iff $e \geq \underline{e}_m := 1 - s_m$. Hence, male promotion is feasible only if $E_m := [\underline{e}_m, \bar{e}_m] \neq \emptyset$.

On the other hand, the measure of women who may go for promotion is positive only if $e \leq \bar{e}_f := \sqrt{2(s_f - w_f)}$, and gradually increases as e decreases until $e = \sqrt{s_f - w_f}$, and all women go for promotion if $e < \sqrt{s_f - w_f}$. The firm's per-promotion surplus (conditional on no departure) is $1 + e - s_f$ which is positive iff $e \geq \underline{e}_f := 1 - s_f$. Hence, female promotion is feasible only if $E_f := [\underline{e}_f, \bar{e}_f] \neq \emptyset$.

We prove the Lemma for the case that $E_m \neq \emptyset \neq E_f$ here, because all other cases are straightforward. First, consider the contingency that $\underline{e}_m \geq \underline{e}_f$. Suppose, in addition, that $\bar{e}_m \geq \bar{e}_f$. Then, as e decreases from $\bar{e}_m$, the total measure of workers who pursue promotion, $\mu^p(e)$, is μ_m for $e \in E_m \setminus E_f$; $\mu_m + D(\theta^c(e, s_f))$ for $e \in E_m \cap E_f$; $D(\theta^c(e, s_f))$ for $e \in E_f \setminus E_m$. If $\mu_m \geq 1$, it is an equilibrium that measure 1 of men exert $\bar{e}_m$ and get promoted. If there is e^* such that $\mu^p(e^*) \geq 1$, it is an equilibrium that e^* is the effort for workers pursuing promotion for both genders. Otherwise, it is an equilibrium that all men exert $\underline{e}_m$ and women exert e_f such that $D(\theta^c(e_f, s_f)) = 1 - \mu_m$. On the other hand, if $\bar{e}_m < \bar{e}_f$, then $\mu^p(e)$ decreases in $e \in E_m \cup E_f$ so that there is e^* such that $\mu^p(e^*) \geq 1$ and hence, there is an equilibrium in which e^* is the equilibrium promotion effort level for both genders similarly to above. The argument is analogous for the alternative contingency that $\bar{e}_m < \bar{e}_f$, hence is omitted. ∎

Proof of Lemma 7. <u>Step i: Firms can guarantee a surplus > 0.5.</u> After offering $\mathbf{s}^*(\theta)$ for some $\theta \in (w_f, \bar{\theta})$, a firm, say i, can hire measure $\mu_f^*(\theta)$ of women if it becomes the leader, and ensure to hire measure $\mu_f \leq 1$ of women otherwise by (H1)-(H2). Consequently, it can guarantee a total surplus higher than 0.5 unless the other firm hires measure 2 of women as a leader, in which case firm i's surplus is 0.5. This also implies that neither firm ever hires measure 2 of women as a leader along the equilibrium path: If a firm were to do so, its surplus is strictly less than 0.5 when it becomes a leader as we have shown already and, since its surplus as a follower cannot be any higher, the firm's strategy is dominated. Thus, we proved that both firms can guarantee a surplus exceeding 0.5.

<u>Step ii: Firms only offer $\mathbf{s}$ with $v(\mathbf{s}) \leq v(\bar{\theta})$.</u> Consider $\mathbf{s}$ such that $v(\mathbf{s}) = v(\theta)$ for some $\theta > \bar{\theta}$. If firm i offers such an $\mathbf{s}$, by definition of $v(\mathbf{s})$, its total surplus is strictly lower than 0.5 by Lemma 3 in the contingency of it becoming a leader. Since its total surplus can be no higher when it becomes a follower, offering such $\mathbf{s}$ is strictly dominated by offering $\mathbf{s}^*(\theta)$ for some $\theta \in (w_f, \bar{\theta})$ due to Step i.

<u>Step iii: Leader firms hire more than measure 1 of women.</u> Suppose firm i offered $\mathbf{s}$ such that $v(\mathbf{s}) = v(\theta)$ for some $\theta \in [w_f, \bar{\theta}]$. (Note that $\theta \geq w_f$ since $v(\mathbf{s}) \geq v(w_f)$.) Let $\boldsymbol{\mu} = (\mu_f, 2 - \mu_f)$ and e_f be the hiring and female effort level in the equilibrium most favored by the firm following the firm's offer of $\mathbf{s}$, that generate

$v(\mathbf{s})$, which we refer to as the "v-setting" equilibrium. Since the total surplus of this equilibrium exceeds 0.5 by Step i, the per-promotion surplus must be greater than 0.5 for women, in particular, $\pi_f = 1 + e_f - s_f > 0.5 \geq \pi_m = 1 + e_m - s_m > 0$ where the last inequality follows because a surplus exceeding 0.5 cannot be garnered from women promotions alone. This in turn implies that $e_f = e_m = e^* > 0$ due to (Ci) and that all upper posts are filled due to (Cii).

Let θ^c denote the threshold in this equilibrium, i.e., women pursue promotion iff their types are below θ^c. Suppose $\theta^c < w_f$. Then, $s_f - (e^*)^2/2 = w_f$ and the firm's surplus is $\mu_f D(\theta^c)\pi_f + (1 - \mu_f D(\theta^c))\pi_m$ where $1 - \mu_f D(\theta^c) \leq 2 - \mu_f \Rightarrow \mu_f \leq \frac{1}{1-D(\theta^c)} < \frac{1}{1-D(w_f)} = \mu_f^*(w_f)$. Hence, if the firm hired measure $\mu_f^*(w_f)$ of women then the continuation equilibrium with threshold w_f generates a higher total surplus, $\mu_f^*(w_f)D(w_f)\pi_f + (1 - \mu_f^*(w_f)D(w_f))\pi_m$, contradicting the initial equilibrium being the v-setting one. Hence, $\theta^c \geq w_f$.

Note that $\pi_f \leq \max_{w_f \leq \theta' \leq \bar{\theta}} \pi_f^*(\theta') = 1.5 - w_f \leq 0.6$. If $\pi_m \leq \pi_f/2$, a firm's equilibrium surplus is bounded above by $\max_{0 \leq \mu_f \leq 2} \mu_f D(\bar{\theta})0.6 + (1 - \mu_f D(\bar{\theta}))0.3 = 0.3(1 + \mu_f \alpha\bar{\theta}) \leq 0.3(1 + 2\frac{1}{4}(2 - w_f) < 0.5$, a contradiction to the initial equilibrium surplus exceeding 0.5. Hence, $\pi_m > \pi_f/2$. Suppose departure takes place in the initial equilibrium, i.e., $\theta^c > \theta \geq w_f$. Then, if the firm hired measure $\mu_f - \epsilon$ (instead of μ_f) of women where $\epsilon > 0$ is small, there is an equilibrium in which women pursue promotion with a slightly lower threshold and more men get promoted than in the initial equilibrium, thus generating a higher total surplus, which would contradict the initial equilibrium being the v-setting one. Hence, we conclude that the initial equilibrium must be departure-free.

Thus, there is no other equilibrium in $(\mathbf{s}, \boldsymbol{\mu})$-subgame by the result [A] obtained in the proof of Lemma 6. Consequently, either firm, upon becoming a leader after offering $\mathbf{s}$ such that $v(\mathbf{s}) = v(\theta)$, can guarantee the maximum surplus possible after offering $\mathbf{s}$. Since $\pi_f > \pi_m$ in this equilibrium, it is optimal for a firm to hire measure $\mu_f^*(\theta) > 1$ of women upon becoming a leader after having offered $\mathbf{s}$.

<u>Step iv: Firms do not offer $\mathbf{s} \neq \mathbf{s}^*(\theta)$ such that $v(\mathbf{s}) = v(\theta)$ for some $\theta \in (w_f, \bar{\theta}]$</u>. Consider $S(\theta) := \{\mathbf{s} | v(\mathbf{s}) = v(\theta)\}$ where $\theta \in (w_f, \bar{\theta}]$. Subject to offering $\mathbf{s} \in S(\theta)$ for a fixed $\theta \in (w_f, \bar{\theta}]$, offering $\mathbf{s}^*(\theta)$ is uniquely optimal conditional on being a leader by Lemma 5. We now show that, subject to offering $\mathbf{s}$ with $v(\mathbf{s}) = v(\theta)$, offering $\mathbf{s}^*(\theta)$ is uniquely optimal conditional on being a follower as well.

To reach a contradiction, suppose that a follower with residual female workforce of measure $\mu_f^r < 1$ obtains a surplus exceeding 0.5 after offering $\mathbf{s} = (s_m, s_f) \neq \mathbf{s}^*(\theta)$ such that $v(\mathbf{s}) = v(\theta)$ where $\theta \in (w_f, \bar{\theta}]$. Let $\boldsymbol{\mu}'$ and e' be the hiring and female effort level in the v-setting equilibrium. Since the surplus exceeds 0.5 in this equilibrium, $e' = e_f = e_m$ as before. By definition, $\theta = 2s_f - w_f - (e')^2$. Let $\pi_g' = 1 + e' - s_g$.

First, suppose some men get promoted in the v-setting equilibrium and $e' = \sqrt{2s_m - 2}$ so that all men are indifferent. Then, $s_f = \frac{2s_m - 2 + w_f + \theta}{2}$ and $\theta = \theta^c(s_f, e')$. Thus, the surplus with residual hiring $\mu_f^r \leq 1$ is $(1 + \sqrt{2s_m - 2} - s_m) + \mu_f^r \alpha\theta(s_m - s_f)$ $< 0.5 + \mu_f^r \alpha\theta(1.5 - s_f^*(\theta))$ if $\mathbf{s} \neq \mathbf{s}^*(\theta)$, i.e., offering $\mathbf{s}^*(\theta)$ is strictly better.

Next, suppose some men get promoted in the v-setting equilibrium and $e' < \sqrt{2s_m - 2}$ so that all men get promoted. This cannot be a v-setting equilibrium because slightly higher effort would increase total surplus (by enhancing efficiency). To see this, note that the equilibrium surplus is $(2-\mu'_f)(1+e'-s_m)+\mu'_f\alpha\theta(1+e'-s_f)>0.5$. The last inequality implies $s_f \le s_m$. Then, from $\theta''=\theta^c(e'',s_f)=2s_f-w_f-(e'+\delta)^2=\theta-\delta^2-2\delta e'$ for $e''=e'+\delta>e'$, we have

$$\begin{aligned}
& (2-\mu_f^*(\theta''))(1+e''-s_m)+\mu_f^*(\theta'')\alpha\theta''(1+e''-s_f) \\
= \; & (2-\mu_f^*(\theta''))(1+e'-s_m)+\mu_f^*(\theta'')\alpha\theta''(1+e'-s_f)+\delta \\
= \; & (1+e'-s_m)+\mu_f^*(\theta'')\alpha\theta''(s_m-s_f)+\delta \\
= \; & (1+e'-s_m)+\Big(\mu'_f+\frac{1}{1-\alpha\theta''}-\frac{1}{1-\alpha\theta}\Big)\alpha(\theta-\delta^2-2\delta e')(s_m-s_f)+\delta \\
= \; & (1+e'-s_m)+\mu'_f\alpha\theta(s_m-s_f)+\Big(\frac{1}{1-\alpha\theta''}-\frac{1}{1-\alpha\theta}\Big)\alpha(\delta^2-2\delta e')(s_m-s_f) \\
& -\Big(\mu'_f\alpha e'+\frac{2\alpha^2\theta e'}{(1-\alpha(\theta-\delta^2-2\delta e'))(1-\alpha\theta)}\Big)(s_m-s_f)\delta+\delta
\end{aligned}$$

which can be shown to exceed $(2-\mu'_f)(1+e'-s_m)+\mu'_f\alpha\theta(1+e'-s_f)=(1+e'-s_m)+\mu'_f\alpha\theta(s_m-s_f)$ for sufficiently small $\delta>0$.

Lastly, suppose no men get promoted in the v-setting equilibrium. Then, either $\pi'_f\le 0$ or $\mu'_f\alpha\theta\ge 1 \Leftrightarrow \theta\ge 1/(2\alpha)\ge 2>\bar\theta=2-w_f$, a contradiction.

Thus, we have shown that offering any $\mathbf{s}\neq\mathbf{s}^*(\theta)$ such that $v(\mathbf{s})=v(\theta)$ for some $\theta\in(w_f,\bar\theta]$ is dominated by offering $\mathbf{s}^*(\theta)$.

<u>Step v: Firms do not offer $\mathbf{s}\neq\mathbf{s}^*(w_f)$ such that $v(\mathbf{s})=v(\theta)$ for some $\theta\le w_f$.</u> Any such $\mathbf{s}=(s_m,s_f)\neq\mathbf{s}^*(w_f)$ means $s_f-(e')^2/2=w_f$ so that $e'=\sqrt{2(s_f-w_f)}$, whence $s_m\ge 1+s_f-w_f$ and $1+e'-s_m\le\sqrt{2(s_f-w_f)}-(s_f-w_f)$ which is uniquely maximized when $s_f=s_f^*(w_f)$ and consequently, $1+e-s_m\le 0.5$ and $1+e-s_f\le\pi_f^*(w_f)$ with at least one strict inequality. Thus, offering such an $\mathbf{s}$ is dominated by offering $\mathbf{s}^*(w_f)$ when a follower as well as when a leader. ∎

Proof of Theorem 2. Recall that F^i is a probability measure on $\mathbb{R}^2_+$ that represents firm i's contract strategy. In light of Lemma 7, we treat F^i as a distribution on the threshold type, i.e., $F^i(X)=F^i(\{\mathbf{s}^*(\theta)|\theta\in X\})$ where $X\subset[w_f,\bar\theta]$ is measurable. We also treat F^i as a cdf function so that $F^i(\theta)=F^i([0,\theta])$. Let $supp(\gamma)$ denote the support of a probability measure γ.

The proof consists of a series of lemmas. Lemmas A1-A2 characterize the support of equilibrium cdf's, Lemmas A3-A10 identify an equilibrium by imposing incentive compatibility on the support, and Lemmas A11-A12 prove uniqueness.

Lemma A1. *Let $\widetilde\theta^i=\sup supp(F^i)$, $i=A,B$. Then, $\widetilde\theta^A=\widetilde\theta^B\in[\hat\theta,\theta^*]$ and the equilibrium payoffs of both firms are $\Pi_L(\widetilde\theta)$ where $\widetilde\theta=\widetilde\theta^A=\widetilde\theta^B$.*

Proof. By Lemma 7, by offering $\mathbf{s}^*(\theta^{**})$ either firm can guarantee a payoff of $\Pi_F(\bar\theta|\theta^{**})>0.5$ because the other firm would hire measure $\mu_f^*(\theta)$ of women when it becomes a leader after offering $\mathbf{s}^*(\theta)$ for some $\theta\in(\theta^{**},\bar\theta)$ due to Lemma 5. Consequently, offering $\mathbf{s}^*(\theta)$ with $\theta>\theta^1$ is strictly dominated where θ^1 satisfies $\Pi_L(\theta^1)=\Pi_F(\bar\theta|\theta^{**})$, because the payoff from doing so is bounded above by $\Pi_L(\theta)<\Pi_L(\theta^1)$. Given this, either firm can analogously guarantee a payoff of

$\Pi_F(\theta^1|\theta^{**})$ by offering $\mathbf{s}^*(\theta^{**})$ and thus, offering $\mathbf{s}^*(\theta)$ with $\theta > \theta^2$ is strictly dominated where θ^2 satisfies $\Pi_L(\theta^2) = \Pi_F(\theta^1|\theta^{**})$. Applying analogous arguments, we deduce recursively that offering a contract with threshold $\theta > \theta^n$, $n = 2, \cdots$, is strictly dominated where $\lim_{n\to\infty} \theta^n = \theta^*$. Hence, $\widetilde{\theta}^i \leq \theta^*$ for $i = A, B$.

Without loss of generality, let $\widetilde{\theta}^A \leq \widetilde{\theta}^B \leq \theta^*$. If $\widetilde{\theta}^A < \hat{\theta}$, firm B will be a leader for sure if it offers $\mathbf{s}^*(\hat{\theta} + \epsilon)$ for any sufficiently small $\epsilon > 0$, and thus, can ensure a payoff arbitrarily close to $\Pi_L(\hat{\theta})$. Hence F^B is a Dirac measure at $\hat{\theta}$ because the only way for firm B to get $\Pi_L(\hat{\theta})$ is to offer $\mathbf{s}^*(\hat{\theta})$ by Lemma 3. Then, firm A can guarantee itself a payoff arbitrarily close to $\Pi_L(\hat{\theta})$ by offering $\mathbf{s}^*(\hat{\theta}+\epsilon)$ for the same logic as above, contradicting $\widetilde{\theta}^A < \hat{\theta}$. Hence, $\hat{\theta} \leq \widetilde{\theta}^A$ must hold.

Once again by the same reasoning as above, firm B can guarantee itself a payoff arbitrarily close to $\Pi_L(\widetilde{\theta}^A)$ by offering $\mathbf{s}^*(\widetilde{\theta}^A + \epsilon)$ for a sufficiently small $\epsilon > 0$. Hence, firm B should not offer any contract $\mathbf{s}^*(\theta)$ where $\theta > \widetilde{\theta}^A$ by Lemma 3. Since $\widetilde{\theta}^A \leq \widetilde{\theta}^B$ by supposition, we have established $\widetilde{\theta}^A = \widetilde{\theta}^B = \widetilde{\theta} \in [\hat{\theta}, \theta^*]$. Applying the same argument one more time, we deduce that the equilibrium payoffs of both firms are no lower than $\Pi_L(\widetilde{\theta})$. In addition, $\widetilde{\theta} \in supp(F^i)$ means that the firm's payoff is no higher than $\Pi_L(\widetilde{\theta})$ because $\Pi_L(\theta)$ is an upper bound of firm i's payoff from offering $\mathbf{s}^*(\theta)$ for θ near $\widetilde{\theta}$ by Lemma 3. Hence, it follows that $\Pi_L(\widetilde{\theta})$ is the equilibrium payoff of both firms. ∎

Lemma A2. *There is $\underline{\theta} \in [\theta^{**}, \widetilde{\theta})$ such that $[\underline{\theta}, \widetilde{\theta}] \subset supp(F^i) \subset \{\theta^{**}\} \cup [\underline{\theta}, \widetilde{\theta}]$ for $i = A, B$. F^i is nonatomic on $(\underline{\theta}, \widetilde{\theta}]$ for $i = A, B$, and $\underline{\theta}$ is an atom for at most one F^i. If $\underline{\theta} > \theta^{**}$, $F^A(\theta) \neq F^B(\theta)$ for some $\theta > \underline{\theta}$.*

Proof. We take granted that $supp(F^i) \subset [\theta^{**}, \widetilde{\theta}]$ from (11) and Lemmas 7 and A1, and that either firm, upon becoming a leader after offering $\mathbf{s}^*(\theta)$, $\theta > \theta^{**}$, hires measure $\mu_f^*(\theta)$ of women and obtains $\Pi_L(\theta)$ from Lemma 5.

First consider the case that both $supp(F^A)$ and $supp(F^B)$ are convex sets. Let $\underline{\theta}_A = \min supp(F^A) \geq \underline{\theta}_B = \min supp(F^B) \geq \theta^{**}$. Note that $\underline{\theta}_A = \underline{\theta}_B = \underline{\theta}$ since otherwise we would have $(\underline{\theta}_B, \underline{\theta}_A) \not\subset supp(F^B)$ because offering $\mathbf{s}^*(\theta^{**})$ is dominant by (11), contradicting convexity of $supp(F^B)$. F^A and F^B are nonatomic for $\theta \in (\underline{\theta}, \widetilde{\theta}]$ for otherwise $supp(F^j)$ would not be convex due to:

[B] If $F^i(\{\theta'\}) > 0$ where $\theta' \in (\theta^{**}, \bar{\theta})$, then $F^j([\theta'', \theta']) = 0$ for some $\theta'' < \theta'$ where $j \neq i$,

because one can find a sufficiently small $\epsilon > 0$ such that $F^i((\theta', \theta' + \epsilon))$ is arbitrarily small and thus, firm $j \neq i$ would prefer offering $\mathbf{s}^*(\theta' + \epsilon)$ to $\mathbf{s}^*(\theta)$ with θ in a small interval $[\theta'', \theta']$, since the former increases the chance of becoming a leader by a discrete amount while marginally reducing the profit in the contingency of becoming a follower.

In addition, $\underline{\theta}$ cannot be an atom for both F^A and F^B: If it were either agent would benefit by offering $\mathbf{s}^*(\underline{\theta} + \epsilon)$ for sufficiently small $\epsilon > 0$ by the same reason as above. Consequently, $\underline{\theta} = \theta^{**}$ since if $\underline{\theta} > \theta^{**}$, given $F^i(\underline{\theta}) = 0$ for some $i = A, B$, firm $j \neq i$ would strictly prefer offering $\mathbf{s}^*(\theta^{**})$ to offering $\mathbf{s}^*(\underline{\theta})$. This proves the lemma when both $supp(F^A)$ and $supp(F^B)$ are convex sets.

To analyze the alternative case, we let $V^i(\theta)$, $i = A, B$, denote firm i's expected payoff from offering $\mathbf{s}^*(\theta)$ for $\theta \in (\theta^{**}, \widetilde{\theta}]$, conditional on firm $j \neq i$ using the equilibrium strategy F^j of contract announcement. Define an auxiliary function

$$V_0^i(\theta|\hat{t}) := \Pi_L(\theta) \lim_{z \uparrow \hat{t}} F^j(z) + \int_{z \geq \hat{t}} \Pi_F(z|\theta) dF^j, \; j \neq i. \tag{15}$$

Note that $V_0^i(\theta|\hat{t}) = V^i(\theta)$ for $\theta < \hat{t}$ if $\lim_{z\uparrow\theta} F^j(z) = \lim_{z\uparrow\hat{t}} F^j(z)$.

Observe that $V_0^i(\theta|\hat{t})$ is strictly concave in $\theta < \hat{t}$ because both $\Pi_L(\theta)$ and $\Pi_F(z|\theta)$ are strictly concave in θ. Suppose that F^j is continuous at $\theta = \hat{t}$, i.e., $F^j(\{\hat{t}\}) = 0$, there exists an interval $(t, \hat{t})$ such that $(t, \hat{t}) \cap supp(F^j) = \emptyset$, and $\frac{\partial V_0^i(\hat{t}|\hat{t})}{\partial\theta} \geq 0$. Then, for $\theta < \hat{t}$ with $F^j(\hat{t}) - F^j(\theta) = \delta \geq 0$, we have

$$\begin{aligned} V^i(\theta) \;&\leq\; \Pi_L(\theta)F^j(\theta) + \int_{z>\theta} \Pi_F(z|\theta)dF^j \\ &=\; V_0^i(\theta|\hat{t}) - \Big(\delta\Pi_L(\theta) - \int_{\theta<z<\hat{t}} \Pi_F(z|\theta)dF^i\Big) \\ &\leq\; V_0^i(\theta|\hat{t}) - \Big(\delta\Pi_L(\theta) - \int_{\theta<z<\hat{t}} \Pi_F(\theta|\theta)dF^i\Big) \\ &\leq\; V_0^i(\theta|\hat{t}) \;<\; V_0^i(\hat{t}|\hat{t}) \;=\; V^i(\hat{t}) \;\leq\; \Pi_L(\widetilde{\theta}), \end{aligned} \tag{16}$$

where the first inequality follows because $\Pi_L(\theta)$ is an upper bound of i's payoff when both firms offer $\mathbf{s}^*(\theta)$, the second because $\Pi_F(z|\theta)$ decreases in z, the third from $\Pi_L(\theta) > \Pi_F(\theta|\theta)$, the fourth because $V_0^i(\theta|\hat{t})$ is strictly concave and $\frac{\partial V_0^i(\hat{t}|\hat{t})}{\partial\theta} \geq 0$, the next equality because $F^j(\{\hat{t}\}) = 0$, and the last inequality from $\Pi_L(\widetilde{\theta})$ being the equilibrium expected payoff. Since (16) implies that $\theta \notin supp(F^i)$, we have shown

[C] if $(t, \hat{t}) \cap supp(F^j) = \emptyset$, $F^j(\{\hat{t}\}) = 0$ and $\frac{\partial V_0^i(\hat{t}|\hat{t})}{\partial\theta} \geq 0$, then $\lim_{\theta\uparrow\hat{t}} F^i(\theta) = 0$.

We consider the case that there is an interval (t, t') such that $[t, t'] \cap supp(F^i) = \{t, t'\}$ for some $i = A, B$. Then, since $V^j(\theta) = V_0^j(\theta|t')$ for $\theta \in (t, t')$ and $V_0^j(\theta|t')$ is strictly concave as shown above, $(t, t') \cap supp(F^j)$ is either empty or singleton. If it is a singleton, i.e., $(t, t') \cap supp(F^j) = \{t''\}$, then $V^j(t'') = \Pi_L(\widetilde{\theta})$ and $\frac{\partial V_0^j(t''|t'')}{\partial\theta} = 0$. By [C] with firms' roles switched, $t'' = \min supp(F^j)$ and consequently, $t = \theta^{**}$. However, if $F^j(\{t'\}) = 0$, then $\frac{\partial V_0^i(t'|t')}{\partial\theta} \geq 0$ for otherwise offering $\mathbf{s}^*(t' - \epsilon)$ would be better than offering $\mathbf{s}^*(t')$ for sufficiently small $\epsilon > 0$, contradicting $t' \in supp(F^i)$ and consequently, [C] would imply that $t \notin supp(F^i)$, a contradiction. If $F^j(\{t'\}) > 0$, on the other hand, $F^i(\{t'\}) = 0$ by [B] and thus, a symmetric argument would imply that $t'' \notin supp(F^j)$, a contradiction.

Hence, consider the case that $(t, t') \cap supp(F^j) = \emptyset$. If $F^j(\{t'\}) = 0$, the same argument as above would imply $t \notin supp(F^i)$, a contradiction. If $F^j(\{t'\}) > 0$ so that $F^i(\{t'\}) = 0$ by [B], as before we would have $t' = \min supp(F^j)$ and $t = \theta^{**}$. Note that in this case $[t', \widetilde{\theta}] \subset supp(F^k)$ for $k = A, B$, because the argument up to now have established that any open interval I with $I \cap supp(F^k) = \{\inf I, \sup I\}$

necessitates $\inf I = \theta^{**}$. Hence, F^k, $k = A, B$, is nonatomic for $\theta \in (t', \widetilde{\theta}]$ by [B] and thus, for all $\theta \in (t', \widetilde{\theta})$,

$$V^k(\theta) = \Pi_L(\theta)F^\ell(\theta) + \int_{z>\theta} \Pi_F(z|\theta)dF^\ell, \quad \ell \neq k, \quad \text{and}$$

$$\frac{dV^k(\theta)}{d\theta} = \frac{\partial V_0^k(\theta|\theta)}{\partial\theta} + \Big(\Pi_L(\theta) - \Pi_F(\theta|\theta)\Big)\frac{dF^\ell(\theta)}{d\theta} = 0 \tag{17}$$

$$\implies \frac{\partial V_0^k(\theta|\theta)}{\partial\theta} \leq 0, \quad \forall\theta \in (t', \widetilde{\theta}). \tag{18}$$

Note that we must have $\frac{\partial V_0^i(t'|t')}{\partial\theta} < 0$ for $t = \theta^{**} \in supp(F^i)$ to hold as presumed. If $F^A(\theta) = F^B(\theta)$ for all $\theta > t'$ so that $\frac{\partial V_0^1(\theta|\theta)}{\partial\theta} = \frac{\partial V_0^2(\theta|\theta)}{\partial\theta}$ for $\theta > t'$ by (17), then $\frac{\partial V_0^j(t'|t')}{\partial\theta} = \frac{\partial V_0^i(t'|t')}{\partial\theta} < 0$ by continuity, contradicting $t' = \min supp(F^j)$. This completes the proof of the Lemma. ∎

By Lemmas A1 and A2, in equilibrium firm $i \in \{A, B\}$ offers contracts $\mathbf{s}^*(\theta)$ according to a continuous cdf F^i defined on $\mathbb{R}_+$ with $F^i(\underline{\theta}) = 0$ and $F^i(\widetilde{\theta}) = 1$ for some $\underline{\theta} \in [\theta^{**}, \widetilde{\theta})$. Thus, the following incentive compatibility condition must hold:

$$\Pi_L(\widetilde{\theta}) = \Pi_L(\theta)F^j(\theta) + \int_\theta^{\widetilde{\theta}} \Pi_F(\cdot|\theta)dF^j, \quad j \neq i, \quad \forall\theta \in (\underline{\theta}, \widetilde{\theta}] \tag{19}$$

$$\geq \Pi_L(\theta)F^j(\theta) + \int_\theta^{\widetilde{\theta}} \Pi_F(\cdot|\theta)dF^j, \quad j \neq i, \quad \forall\theta \notin supp(F^i). \tag{20}$$

At this point, a change of variable proves useful: define $x = \bar{\theta} - \theta$ and

$$x^* = \bar{\theta} - \theta^*, \ \hat{x} = \bar{\theta} - \hat{\theta}, \ x^{**} = \bar{\theta} - \theta^{**}. \tag{21}$$

Also define functions $P, Q : [0, \bar{\theta}] \to \mathbb{R}_+$ and $\beta : [0, \bar{\theta}] \to [0, 1]$ as:

$$P(x) := \Pi_L(\bar{\theta}-x)-0.5, \ Q(x) := \alpha(\bar{\theta}-x)\Big(\pi_f^*(\bar{\theta}-x)-0.5\Big), \text{ and } \beta(x) := \frac{1-2\alpha(\bar{\theta}-x)}{1-\alpha(\bar{\theta}-x)}. \tag{22}$$

Recall from (6) and (11) that $\theta^{**} \leq \hat{\theta}$, whence $\hat{x} \leq x^{**}$. We present the proof presuming that $\hat{x} < x^{**}$, because the proof is only simpler if $\hat{x} = x^{**}$.

Lemma A3: (a) *$P(x)$ is a strictly concave function and peaks at $\hat{x}$.*
(b) *$Q(x) = (1 - \alpha\bar{\theta} + \alpha x)P(x)$ is strictly concave and $Q'(x^{**}) = 0$ if $x^{**} < \bar{\theta} - w_f$.*
(c) *$\beta(x)$ is a strictly increasing and strictly concave function.*
(d) $Q(x)\beta(z) = \Pi_F(z|x) - 0.5$.
(e) $P(x) > Q(x^{**})\beta(x)$ $\forall x \in (x^*, x^{**}]$.
(f) $P(x^*) = Q(x^{**})\beta(x^*) < Q(x^{**})\beta(x^{**})$.

Proof. Part (a) follows because $\Pi_L(\theta)$ is a strictly concave function and peaks at $\hat{\theta}$ by Lemma 3. Parts (b) and (d) follow from (5), (11), (10) and (22). Part (c) holds because $\beta'(x) > 0$ and $\beta''(x) < 0$ for $x \in [0, \bar{\theta}]$. Parts (e) and (f) follow from the relationship between Π_L and Π_F illustrated in Figure 4. ∎.

Let $\widetilde{\theta} = \max supp(F^i)$, $i = A, B$, as per Lemma A1, and $a = \bar{\theta} - \widetilde{\theta} \in [x^*, \hat{x}]$ and $b = \bar{\theta} - \underline{\theta} \in [\hat{x}, x^{**}]$. Define $G_a(x) := 1 - F^j(\bar{\theta} - x)$ and $g_a(x) := \frac{dG_a(x)}{dx}$ on $I \subset [a, b)$ for which $\frac{dG_a(x)}{dx}$ is well defined. Since G_a is continuous and increasing, $[a, b] \setminus I$ consists of at most countably many points. Then, (24) implies

$$P(x) \cdot (1 - G_a(x)) + Q(x) \int_a^x \beta(z) g_a(z) dz \;=\; P(a) \qquad \forall x \in [a, b). \tag{23}$$

Note that (23) holds at x even if $g_a(x)$ is not defined since $\int_a^x \beta(z) g_a(z) dz$ is fully determined by the values of g_a at which it is defined. Differentiation of (23) yields

$$P'(x) \cdot (1 - G_a(x)) + Q'(x) \int_a^x \beta(z) g_a(z) dz + \Big(Q(x)\beta(x) - P(x)\Big) g_a(x) = 0 \tag{24}$$

for $x \in I$. Rearranging (24), we have

$$g_a(x) = \frac{\int_a^x K(x, z) g_a(z) dz + P'(x)}{P(x) - Q(x)\beta(x)} \quad \text{where } K(x, z) := Q'(x)\beta(z) - P'(x) \tag{25}$$

for $x \in I$. Note, however, that if g_a satisfies (25) except for countably many points, then (25) determines g_a for all $x \in [a, b)$ uniquely and continuously. Hence, finding an equilibrium involves finding a function g_a that satisfies (25), one for each firm.

Lemma A4: *In equilibrium, $F^A = F^B$; $G_a(x) = 1 - F^i(\bar{\theta} - x)$ is a nonatomic distribution with support $[a, x^{**}]$ where $a \in (x^*, \hat{x})$, and its derivative, g_a, solves (25).*

Proof. The equation in (25) is a Volterra integral equation which has a *unique* continuous solution $g_a(\cdot)$ on $[a, b)$ by Proposition 5.7.4 of Marsden-Hoffman (1993). Since $F^j(\bar{\theta} - x) = 1 - G_a(x) = 1 - \int_a^x g_a(z) dz$ by construction, this means that $F^A(\theta) = F^B(\theta)$ for all $\theta \in (\underline{\theta}, \widetilde{\theta}]$ in equilibrium and by Lemma A2, therefore, $\underline{\theta} = \theta^{**}$ and F^i is nonatomic on $[\theta^{**}, \widetilde{\theta}]$, $i = A, B$. Thus, the lemma is proved by applying the Proposition 5.7.4 of Marsden-Hoffman again. ∎

Consequently, any equilibrium is symmetric and finding an equilibrium comes down to identifying when, i.e., for which values of a, the solution g_a is consistent with G_a being a nonatomic cdf on $[a, x^{**}]$. To do this, below we examine the properties of the function G_a that solves (24), or equivalently, (25), such that $G_a(a) = 0$ and $G_a(x^{**}) = 1$. Define $\tilde{x}$ and $\bar{x} < \bar{\theta}$ to be the values that satisfy, respectively,

$$P(\tilde{x}) = Q(x^{**})\beta(x^{**}) \quad \text{and} \quad P(\bar{x}) = P(x^{**}), \quad \text{so that} \quad \tilde{x} < \bar{x} \leq \hat{x}, \tag{26}$$

where $\bar{x} = \hat{x}$ holds iff $x^{**} = \hat{x} = \bar{\theta} - w_f$.

Lemma A5: *If $a \in [\tilde{x}, x^{**}]$, then $G_a(x) < 1$ for all $x \in [a, x^{**}]$.*

Proof. If, to the contrary, $G_a(x) = 1$ for some $x \in (a, x^{**}]$ then (23) would not hold at x because $Q(x) \int_a^x \beta(z) g_a(z) dz < Q(x^{**}) \int_a^x \beta(x^{**}) g_a(z) dz = Q(x^{**})\beta(x^{**}) = P(\tilde{x}) \leq P(a)$. Hence, $G_a(x) < 1$ for all $x \in (a, x^{**}]$. ∎

Hence, if G_a is a nonatomic cdf on $[a, x^{**}]$, then $a \in (x^*, \tilde{x})$.

Lemma A6: *If* $x' = \min\{x \in [x^*, x^{**}] |\, g_a(x) = 0\} \geq \hat{x}$ *and* $G_a(x') < 1$*, then* $g_a(x) < 0$ *for all* $x \in (x', x^{**})$*.*

Proof. Since the derivative of the first two terms of (24) is

$$P''(x) \cdot (1 - G_a(x)) - P'(x)g_a(x) + Q''(x) \int_a^x \beta(z)g_a(z)dz + Q'(x)\beta(x)g_a(x), \quad (27)$$

which is negative at $x = x'$ because $P''(x'), Q''(x') < 0$ by Lemma A3, we deduce from (24) that g_a is strictly decreasing at x' (because $g_a(x') = 0$). Furthermore, since (27) stays negative for $x > x'$ so long as $\int_a^x \beta(z)g_a(z)dz > 0$, $g_a(x)$ also stays strictly negative for $x > x'$ so long as $\int_a^x \beta(z)g_a(z)dz > 0$ by (24). If $\int_a^x \beta(z)g_a(z)dz \leq 0$ for some $x > x'$, on the other hand, $g_a(x)$ stays negative due to (24). Consequently, $g_a(x) < 0$ for all $x > x'$. ∎

Lemma A7: *For* $a \in [x^*, \tilde{x}]$*, if* $G_a(x) < 1$ *for all* $x \leq x'$ *where* $x' \leq \hat{x}$*, then* $g_a(x) > 0$ *for all* $x \in [a, x']$*.*

Proof. Since g_a that solves (25) is continuous and $g_a(a) > 0$, negation of the conclusion of the lemma would imply that $g_a(x) = 0$ for some $x \in (a, x']$, whence (24) would fail at $x'' = \min\{x|\, g_a(x) = 0\}$. ∎

Lemma A8: $G_{x^*}(x) = 1$ *for some* $x \in [x^*, x^{**}]$*,*

Proof. Suppose to the contrary that $G_{x^*}(x) < 1$ for all $x \in [x^*, x^{**}]$. By Lemma A7, $g_{x^*}(x) > 0$ on $[x^*, \hat{x}]$. Since the LHS of (23) would be strictly larger than the RHS at $x = x^{**}$ if $g_{x^*}(x) \geq 0$ for all $x \in [x^*, x^{**}]$, there exists $x' = \min\{x|\, g_{x^*}(x) = 0\} \in (\hat{x}, 1)$. Then $G_{x^*}(x') < 1$ by (24) and thus, $g_{x^*}(x) < 0$ for all $x > x'$ by Lemma A6. Then, the LHS of (23) becomes

$$P(x)\cdot(1-G_{x^*}(x'))+Q(x) \int_{x^*}^{x'} \beta(z)g_{x^*}(z)dz+\int_{x'}^{x} \Big(Q(x)\beta(z)-P(x)\Big)g_{x^*}(z)dz, \quad (28)$$

which is greater than $P(x^*)$ at $x = x^{**}$ by Lemma A3 (c) and (f), failing (23). ∎

Let $a^* = \max\{a \leq \tilde{x}|\, G_a(x) = 1 \text{ for some } x \in [x^*, x^{**}]\}$, which exists by Lemma A8 and continuity of the solution to (25).

Lemma A9: $G_{a^*}(x) < 1$ *for all* $x \in [x^*, x^{**})$ *and* $G_{a^*}(x^{**}) = 1$*.*

Proof. If $x' = \min\{x < x^{**} | G_{a^*}(x) = 1\}$ exists, then $\int_{a^*}^{x'} \beta(z)g_{a^*}(z)dz > 0$ by (23) and thus, $g_{a^*}(x') > 0$ by Lemma A3 and (24), which in turn implies that $G_{a^*}(x' + \epsilon) > 1$ for small $\epsilon > 0$. This would contradict the continuity of the solution to (25) because the solution $G_a(\cdot)$ is bounded above by 1 for all $a > a^*$ by the definition of a^* and Lemma A5. ∎

Lemma A10: G_{a^*} *is a cdf with* $g_{a^*}(x) > 0$ *for all* $x \in [a^*, x^{**})$*.*

Proof. By Lemmas A7 and A9, $g_{a^*}(x) > 0$ for all $x \in [a^*, \hat{x})$. If $x' = \min\{x \in [\hat{x}, x^{**}) |\, g_{a^*}(x) = 0\}$ existed, then $g_{a^*}(x) \leq 0$ for all $x \geq x'$ by Lemma A6, i.e., G_{a^*} would be non-increasing for $x > x'$. This would contradict Lemma A9. ∎

To prove uniqueness, we suppose that G_a is a cdf for some $a < a^*$ and then derive a contradiction to $G_a(x^{**}) = 1$ in the next two lemmas. Let $G^*(x) = G_{a^*}(x)$ and $g^*(x) = g_{a^*}(x)$. Note that $g^*(x^{**}) = g_a(x^{**}) = 0$ from (24) and $Q'(x^{**}) = 0$.

Lemma A11: *If $a < a^*$, then $g_a(a^*) < g^*(a^*)$ and $g_a(x) > g^*(x)$ for some $x \in (a^*, x^{**})$.*

Proof. Suppose, to the contrary, that $g_a(a^*) \geq g^*(a^*)$. Then, $x' = \min\{x|\, g_a(x) = g^*(x)\} \in [a^*, x^{**})$ should exist for $G_a(x^{**}) = G^*(x^{**})$ to hold. If $x' > \hat{x}$, (25) would imply that $g_a(x') > g^*(x')$ because $K(x', z) > 0$. Hence, $x' \leq \hat{x}$. Then, by (25), $(g_a(x') - g^*(x'))(P(x') - Q(x')\beta(x')) = \int_a^{x'} K(x', z)g_a(z)dz - \int_{a^*}^{x'} K(x', z)g^*(z)dz = \int_a^{a^*} K(x', z)g_a(z)dz + \int_{a^*}^{x'} K(x', z)(g_a(z) - g^*(z))dz > 0$ because i) $g_a(a^*) \geq g^*(a^*)$ and (25) imply $\int_a^{a^*} K(a^*, z)g_a(z)dz \geq 0$ and ii) $K(x, z)$ increases not only in z but in $x \in (x^*, \hat{x})$ as shown below:

$$K_1(x, z) = Q''(x)\beta(z) - P''(x) > Q''(x) - P''(x) > 2\alpha P'(x) - \alpha(\bar{\theta} - x)P''(x) > 0 \tag{29}$$

where the second inequality follows from differentiaion of $Q(x) = P(x)(1-\alpha(\bar{\theta}-x))$. Hence, we must conclude that $g_a(a^*) < g^*(a^*)$.

Then, (25) implies that $\int_a^{a^*} K(a^*, z)g_a(z)dz < 0$ and consequently, $\int_a^x K(x, z)g_a(z)dz < 0$ for $x \in (a, a^*)$ by (29). Hence, $g_a(x) < g_x(x)$ for $x \in (a, a^*)$ and consequently, if $g_a(x) \leq g^*(x)$ for all $x \in (a^*, x^{**})$, from (23) we would have

$$\begin{aligned}
P(a^*) - P(a) &= Q(x^{**})\Big[\int_{a^*}^{x^{**}} \beta(z)g^*(z)dz - \int_a^{x^{**}} \beta(z)g_a(z)dz\Big] \\
&= Q(x^{**})\Big[\int_{a^*}^{x^{**}} \beta(z)\Big(g^*(z) - g_a(z)\Big)dz - \int_a^{a^*} \beta(z)g_a(z)dz\Big] \\
&< Q(x^{**})\Big[\beta(x^{**})\Big(1 - \int_{a^*}^{x^{**}} g_a(z)dz\Big) - \int_a^{a^*} \beta(z)g_a(z)dz\Big] \\
&= Q(x^{**})\int_a^{a^*} \Big(\beta(x^{**}) - \beta(z)\Big)g_a(z)dz \\
&< Q(x^{**})\int_a^{a^*} \Big(\beta(x^{**}) - \beta(z)\Big)g_z(z)dz \qquad (30) \\
&= \int_a^{a^*} P'(z)\Phi(z)dz \quad \text{where } \Phi(z) := Q(x^{**})\frac{\beta(x^{**}) - \beta(z)}{2\alpha P(z)(\bar{\theta} - z)},
\end{aligned}$$

where the last equality follows from $P'(z) = \Big(P(z) - Q(z)\beta(z)\Big)g_z(z)$ by (25) and $Q(z) = P(z)(1 - \alpha(\bar{\theta} - z))$ and $\beta(z) = \frac{1-2\alpha(\bar{\theta}-z)}{1-\alpha(\bar{\theta}-z)}$. Since $\Phi(z) < 1$ for $x \in (a, a^*)$ as shown below, it follows that $P(a^*) - P(a) < \int_a^{a^*} P'(z)dz$, a contradiction.

To show $\Phi(z) < 1$, differentiate $\Phi(z)$ and rearrange to get

$$\Phi'(z) = \frac{-Q(x^{**})}{2\alpha P(z)^2(\bar{\theta} - z)^2}\left[\frac{\alpha P(z)(\bar{\theta} - z)}{(1 - \alpha(\bar{\theta} - z))^2} + \Big(\beta(x^{**}) - \beta(z)\Big)\Big(P'(z)(\bar{\theta} - z) - P(z)\Big)\right]$$

$$= \frac{-Q(x^{**})}{2\alpha P(z)^2(\bar\theta - z)^2}\left[P(z)\Big[\frac{\alpha(\bar\theta - z)}{(1-\alpha(\bar\theta - z))^2} - \beta(x^{**}) + \beta(z)\Big] + \Big(\beta(x^{**}) - \beta(z)\Big)P'(z)(\bar\theta - z)\right]$$
$$< \ 0$$

since $\frac{\alpha(\bar\theta-z)}{(1-\alpha(\bar\theta-z))^2} - \beta(x^{**}) + \beta(z) > \frac{\alpha(\bar\theta-z)}{(1-\alpha(\bar\theta-z))^2} - \beta(\bar\theta) + \beta(z) = \frac{\alpha(\bar\theta-z)(1-\alpha(\bar\theta-z)+\alpha^2(\bar\theta-z)^2)}{(1-\alpha(\bar\theta-z))^2} > 0$. Hence, it suffices to show that $\Phi(x^*) < 1$. Since $P(x^*) = Q(x^{**})\beta(x^*)$ by Lemma A3 (f),

$$\Phi(x^*) = \frac{\beta(x^{**}) - \beta(x^*)}{2\alpha\beta(x^*)(\bar\theta - x^*)} < \frac{\beta(x^{**}) - \beta(0)}{2\alpha\beta(0)\bar\theta} \tag{31}$$

because $\frac{\beta(x^{**})-\beta(x)}{2\alpha\beta(x)(\bar\theta-x)}$ is routinely shown to decrease in $x \in (0, \bar\theta)$.[22] Since

$$\bar\theta = 2 - w_f \quad \text{and} \quad \theta^{**} = \max\{w_f, \frac{2-w_f}{2}\} = w_f \tag{32}$$

where the last equality is from $w_f > 0.9$, after rearrangement we have

$$2\alpha\beta(0)\bar\theta - \beta(x^{**}) + \beta(0) = \frac{2\alpha(1 + 2\alpha^2(2-w_f)^2 w_f - \alpha(8 - 6w_f + w_f^2))}{1 - 2\alpha + \alpha^2(2-w_f)w_f}. \tag{33}$$

It is straightforward calculation [see GC09Aug05.nb] to verify that (33) is positive for all $w_f \in (0.9, 1)$ and all $\alpha \in (0, 1/4)$, which establishes that $\Phi(x^*) < 1$ by (31) as desired. ■

Lemma A12: *Let* $x' = \max\{x |\, g_a(z) \le g^*(z)\ \forall z \in [a^*, x]\}$. *Then,* $G_a(x') > G^*(x')$ *and* $G_a(x^{**}) > 1$.

Proof. Note that $x' \in [a^*, x^{**})$ by Lemma A11 and that $g_a(x') = g^*(x')$ by continuity of g_a and g^*. For any $y \in [a^*, x^{**})$ such that $g_a(y) = g^*(y)$, (23) and (24) imply, respectively,

$$\frac{P(y)}{Q(y)}(G^*(y) - G_a(y)) \ < \ \int_{a^*}^{y} \beta(z)g^*(z)dz - \int_{a}^{y}\beta(z)g_a(z)dz \tag{34}$$
$$\frac{P'(y)}{Q'(y)}(G^*(y) - G_a(y)) \ = \ \int_{a^*}^{y} \beta(z)g^*(z)dz - \int_{a}^{y}\beta(z)g_a(z)dz. \tag{35}$$

These two relationships are incompatible at $y = x'$ if $G^*(x') \ge G_a(x')$, because

$$\frac{P(x)}{Q(x)} = \frac{1}{1-\alpha(\bar\theta - x)} > \frac{P'(x)}{P'(x)(1-\alpha(\bar\theta-x)) + \alpha P(x)} = \frac{P'(x)}{Q'(x)}. \tag{36}$$

Thus, $G_a(x') > G^*(x')$ must hold. Consequently, $G_a(x^{**}) > G^*(x^{**}) = 1$ follows provided that $g_a(x) \ge g^*(x)$ for all $x \in (x', x^{**})$, a condition we verify below.

To reach a contradiction, suppose $g_a(x) < g^*(x)$ for some $x \in (x', x^{**})$. Then, since $g_a(x)$ cuts $g^*(x)$ at $x = x'$ from below by definition of x', by continuity, there

[22] The derivative of $\frac{\beta(x^{**})-\beta(x)}{2\alpha\beta(x)(\bar\theta-x)}$ is $\frac{-(1-2\alpha(\bar\theta-z))^2+\beta(x^{**})(1-4\alpha(\bar\theta-z)+2\alpha^2(\bar\theta-z)^2)}{2\alpha(\bar\theta-z)^2(1-2\alpha\bar\theta+2\alpha z)^2}$, the top line of which can be shown to be negative for all $x \in (0, \bar\theta)$ given $0 < \beta(x^{**}) < 1$.

is $y \in (x', x^{**})$ such that $g_a(y) = g^*(y)$ and $g_a(x) \geq g^*(x)$ for all $x \in (x', y)$ where the last inequality is strict for a subset of positive measure of (x', y). Differentiating (24) and rearranging, we obtain

$$g'(x) = \frac{P''(x) + \int_a^x [Q''(x)\beta(z) - P''(x)]g(z)dz + [2K(x,x) + Q(x)\beta'(x)]g(x)}{P(x) - Q(x)\beta(x)}. \tag{37}$$

Let $a' \in (a, a^*)$ such that $\int_{a'}^{x'} g_a(z)dz = G^*(x')$. Then, G^* restricted to $[a^*, x']$ first-order stochastically dominates the cdf obtained from g_a restricted to $[a', x']$ and consequently, $\int_{a'}^{x'}[Q''(y)\beta(z) - P''(y)]g_a(z)dz > \int_{a^*}^{x'}[Q''(y)\beta(z) - P''(y)]g^*(z)dz$ since $Q''(y)\beta(z) - P''(y)$ decreases in z. If $y \leq \hat{x}$, then $\int_a^{a'}[Q''(y)\beta(z) - P''(y)]g_a(z)dz > 0$ and $\int_{x'}^{y}[Q''(y)\beta(z) - P''(y)]g_a(z)dz > \int_{x'}^{y}[Q''(y)\beta(z) - P''(y)]g^*(z)dz$ since $Q''(x)\beta(z) - P''(x) > 0$ for $x \leq \hat{x}$ by (29). Thus, by comparing (37) evaluated for $g = g_a$ at $x = y$ and that evaluated for $g = g^*$ and $a = a^*$ at $x = y$, we deduce that $g_a'(y) > g^{*\prime}(y)$, i.e., $g_a(x)$ cuts $g^*(x)$ at $x = y$ from below. Since this would contradict the definition of y, we presume that $y > \hat{x}$.

Observe that, since $\int_{x'}^{y} \beta(z)\Big(g_a(z) - g^*(z)\Big)dz > 0$,

$$\int_a^{x'} \beta(z)g_a(z)dz - \int_{a^*}^{x'} \beta(z)g^*(z)dz < \int_a^{y} \beta(z)g_a(z)dz - \int_{a^*}^{y} \beta(z)g^*(z)dz. \tag{38}$$

Furthermore, since $G_a(x') > G^*(x')$ as shown above,

$$0 < G_a(x') - G^*(x') < G_a(y) - G^*(y). \tag{39}$$

On the other hand, (35) implies

$$\begin{aligned} 0 &= P'(y) \cdot (G_a(y) - G^*(y)) - Q'(y)\Big[\int_a^{y} \beta(z)g_a(z)dz - \int_{a^*}^{y} \beta(z)g^*(z)dz\Big] \\ &= P'(x') \cdot (G_a(x') - G^*(x')) - Q'(x')\Big[\int_a^{x'} \beta(z)g_a(z)dz - \int_{a^*}^{x'} \beta(z)g^*(z)dz\Big]. \end{aligned} \tag{40}$$

First, consider the case that $x' < \hat{x}$. Since (35) implies $\int_a^{x'} \beta(z)g_a(z)dz - \int_{a^*}^{x'} \beta(z)g^*(z)dz > 0$, the first equality of (40) would fail due to (38) and (39), given that $y \geq \hat{x}$ as verified above. Next, if $x' \geq \hat{x}$, (35) implies that both sides of (38) are negative. Together with $P'(y) < P'(x') < 0$ and $0 \leq Q'(y) < Q'(x')$, this would contradict the second equality of (40). These contradictions necessitate us to conclude that $g_a(x) \geq g^*(x)$ for all $x \in (x', x^{**})$, as desired. ∎

Up to now we have characterized the equilibrium outcome and proved the uniqueness. The properties (i)-(iii) of Theorem 2 follow from the characterization of the equilibrium outcome, thus completing the proof of Theorem 2. ∎

Proof of Proposition 2: If $x \leq D(\theta^{**})$, it is evident that the policy does not have a bite in any part of the proof of Theorem 2 and hence, Theorem 2 holds. If $D(\theta^{**}) < x < D(\bar{\theta})$, it is lengthy yet straightforward (hence, omitted here) to verify that the proof of Theorem 2 works in the same manner to prove the existence of the equilibrium with the role of θ^{**} played by $D^{-1}(x)$, which proves the claim of Proposition 2 for this case. If $x = D(\bar{\theta})$, the maximum possible surplus that a firm may derive from a woman employee is 0.5 by Lemma 3 and hence, in conjunction with Lemma 2, either firm's total profit is bounded above by 0.5. Note that either firm can guarantee a profit level no lower than 0.5 by offering $\mathbf{s}^*(\bar{\theta})$ and hiring at least measure 1 of men (which a firm can do according to (H1) or (H2)). Hence, the equilibrium profit is 0.5 for both firms, which, given $x = D(\bar{\theta})$, is possible only if they both offer a salary of 1.5 for both men and women according to Lemmas 2 and 3. Finally, if $x > D(\bar{\theta})$ then the payoff of a woman pursuing promotion is at least $u_\ell(x) > 0.5$. Note that the equilibrium payoff of a man pursuing promotion cannot exceed 0.5 for otherwise, given that men's payoff from not pursuing promotion is 0.5, the firm can do better by offering a slightly worse contract for men. This proves the claim of Proposition 2 for the case $x > D(\bar{\theta})$. ∎

Proof of Proposition 3: Suppose that the said policy is in place with

$$y \in [y_0, D(\bar{\theta})] \quad \text{where} \quad y_0 = D(\bar{\theta})\frac{1-2D(\bar{\theta})}{1-D(\bar{\theta})} < D(\bar{\theta}). \tag{41}$$

Since Lemma 7 still applies, let $\underline{\theta}^i$ and $\widetilde{\theta}^i$ be the infimum and the supremum, respectively, of the threshold types of the contracts in the support of the equilibrium contract strategy of firm $i = A, B$. Without loss of generality, we assume that $\underline{\theta}^A \leq \underline{\theta}^B$. If $\underline{\theta}^A = \underline{\theta}^B < \bar{\theta}$, then at least one firm, say B, does not offer $\mathbf{s}^*(\underline{\theta}^B)$ with a positive probability for the same reason as given in the proof of Lemma A2.

If $\underline{\theta}^A < \bar{\theta}$, therefore, firm A is a follower with probability 1 upon offering $\mathbf{s}^*(\underline{\theta}^A)$. Since in this case firm B hires a measure $\mu = 1/(1 - D(\theta^B))$ of women by Lemma 3 where $\theta_B > \underline{\theta}^A$, the residual female workforce available to A is $2 - \frac{1}{1-D(\theta^B)} = \frac{1-2D(\theta^B)}{1-D(\theta^B)}$. In order for firm A to be able to fulfil the policy requirement, we need

$$D(\underline{\theta}^A)\frac{1-2D(\theta^B)}{1-D(\theta^B)} \geq y. \tag{42}$$

However, since $x(1-2x)/(1-x)$ increases in $x > 0$, together with (41), we have

$$y_0 = D(\bar{\theta})\frac{1-2D(\bar{\theta})}{1-D(\bar{\theta})} \geq D(\theta^B)\frac{1-2D(\theta^B)}{1-D(\theta^B)} \geq D(\underline{\theta}^A)\frac{1-2D(\theta^B)}{1-D(\theta^B)} \tag{43}$$

where at least one of the inequalities is strict. Since this would contradict (42), we have to conclude that $\underline{\theta}^A \geq \bar{\theta}$ and consequently, that $\underline{\theta}^A = \underline{\theta}^A = \bar{\theta}$. In this case, offering $\mathbf{s}^*(\bar{\theta})$ is optimal for both firms, which completes the proof. ∎

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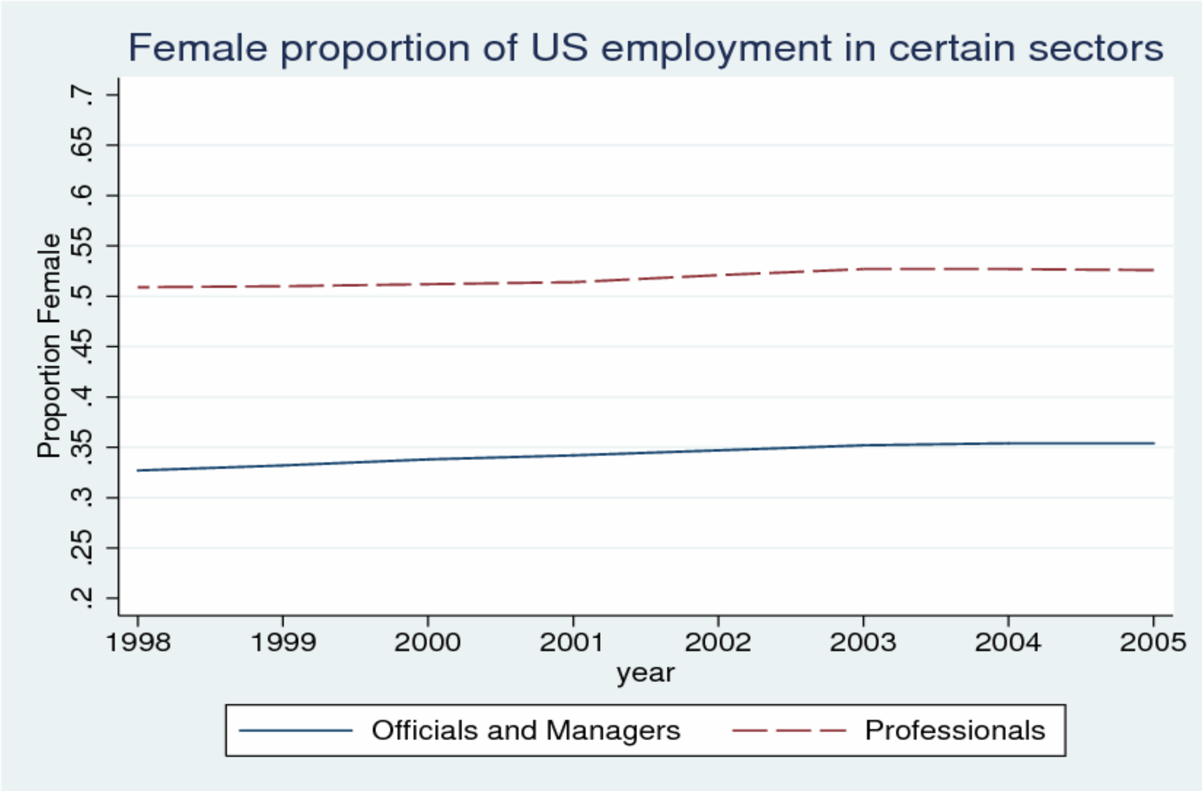
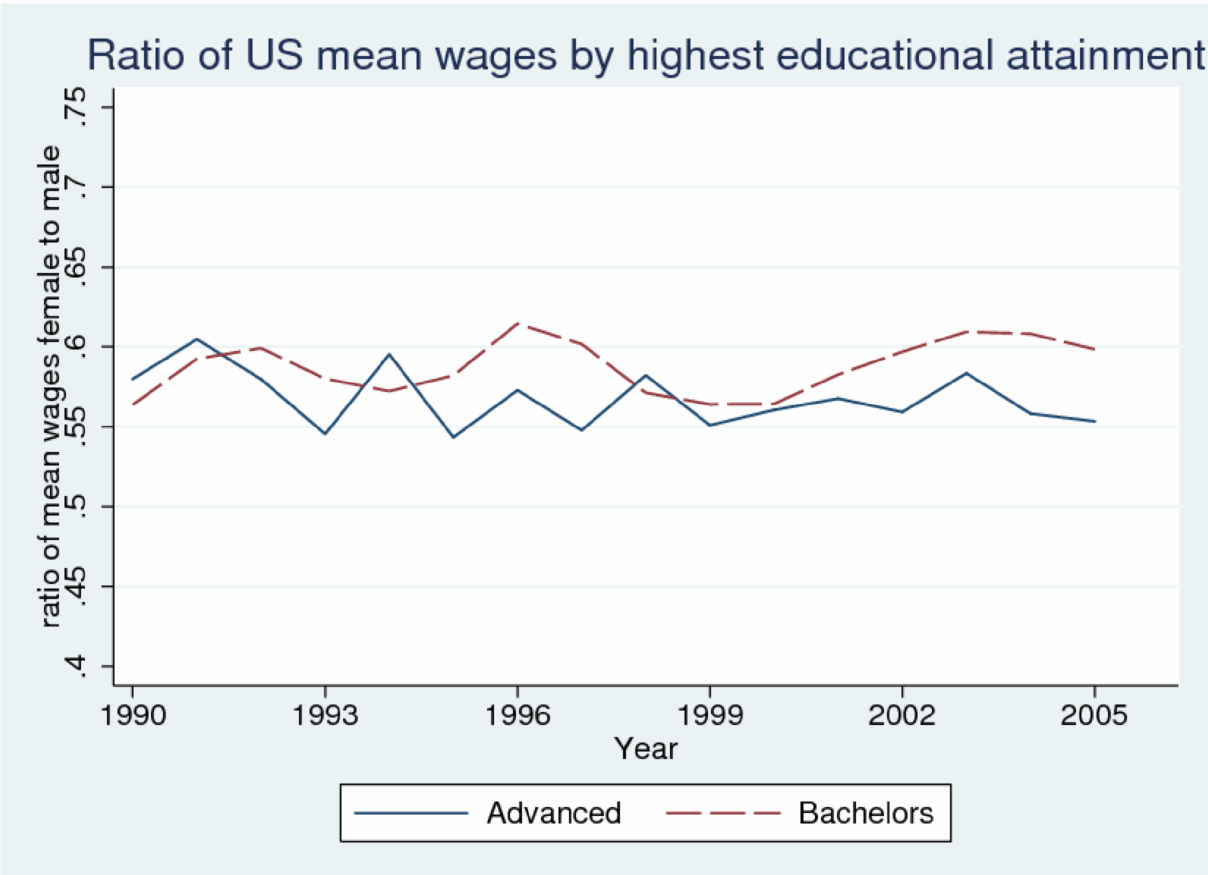
Figure 1.



Source: U.S. Equal Employment Opportunity Commission (EEOC).

Figure 2.



Source: US Census Bureau.

Figure 3

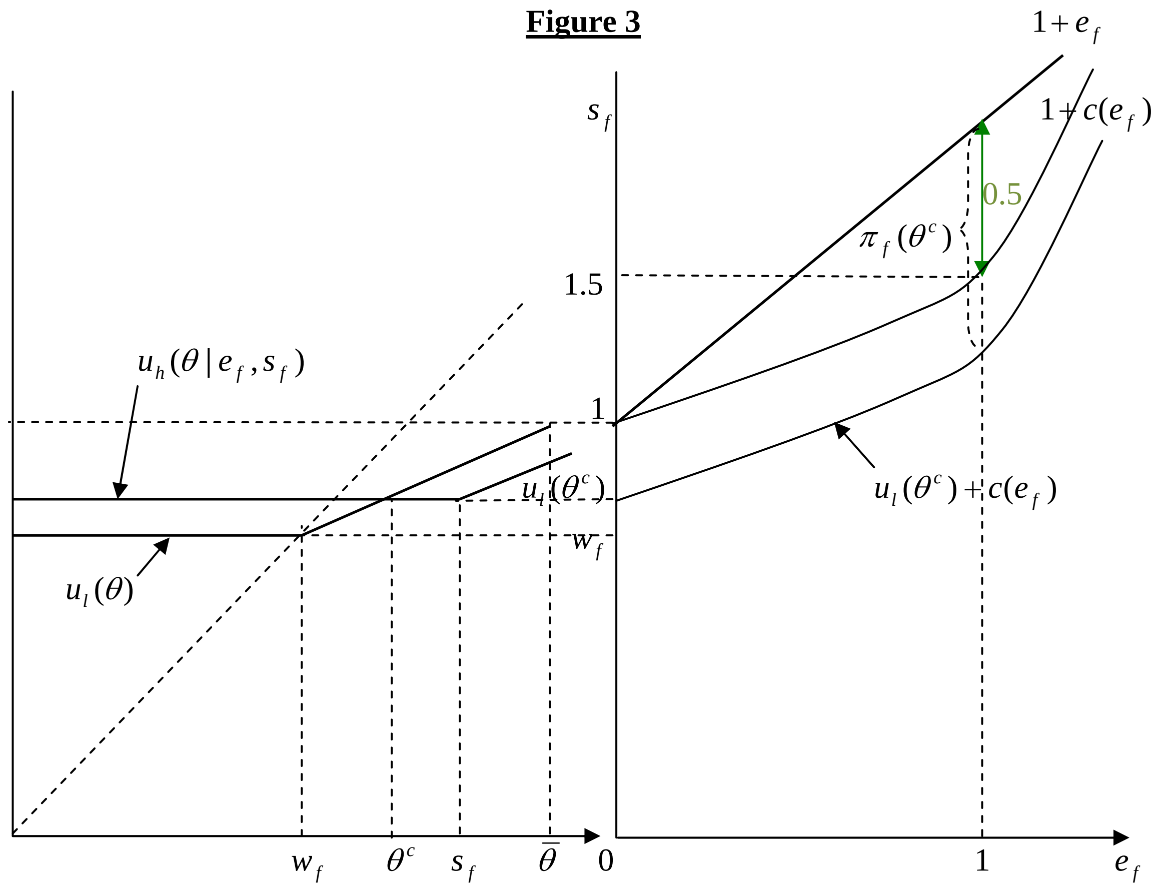


Figure 4