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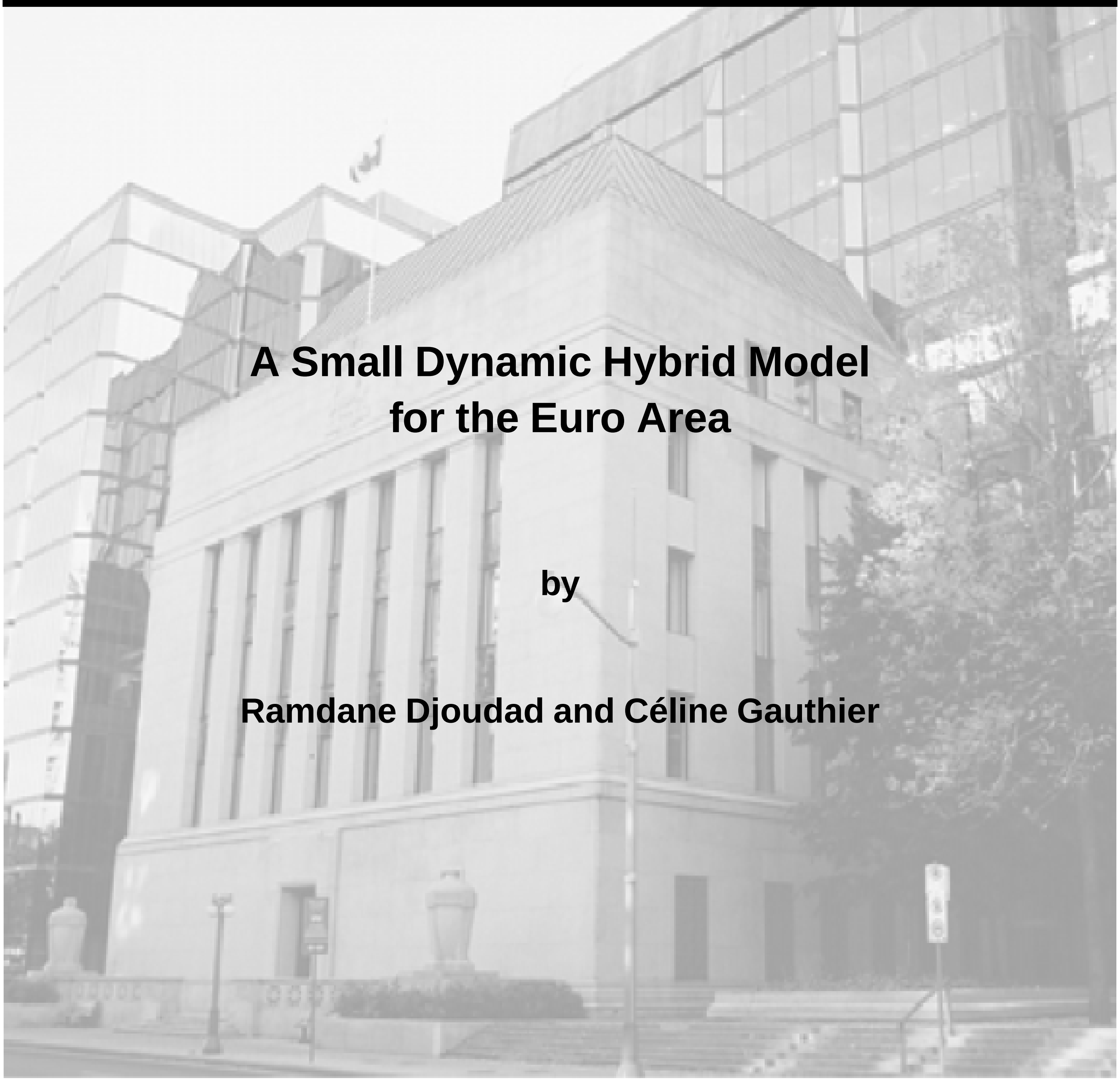

A Small Dynamic Hybrid Model for the Euro Area

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Abstract

The authors estimate and solve a small structural model for the euro area over the 1983–2000 period. Given the assumption of rational expectations, the model implies a set of orthogonality conditions that provide the basis for estimating the model's parameter by generalized method of moments. The authors' main results are: (i) the impulse-response functions implied by the model are consistent with the standard stylized facts about the dynamic effects of monetary policy, (ii) evidence suggests that flexibility in Europe has increased since the adoption of the Maastricht Treaty, and (iii) the inflation expectations captured by the model might explain the European Central Bank's reluctance to ease monetary conditions in 2000.

JEL classification: E31
Bank classification: Transmission of monetary policy

Résumé

Les auteurs estiment et résolvent un petit modèle structurel applicable à la zone euro pour la période comprise entre 1983 et 2000. Partant de l'hypothèse d'attentes rationnelles, le modèle impose un ensemble de conditions d'orthogonalité qui permettent d'en estimer les paramètres à l'aide de la méthode des moments généralisés. Les auteurs en concluent essentiellement que i) les profils de réaction générés par le modèle cadrent avec les faits stylisés relatifs aux effets dynamiques de la politique monétaire; ii) la flexibilité semble s'être accentuée en Europe depuis l'entrée en vigueur du Traité de Maastricht; iii) les attentes d'inflation représentées dans le modèle peuvent expliquer l'hésitation de la Banque centrale européenne à assouplir les conditions monétaires en 2000.

Classification JEL : E31
Classification de la Banque : Transmission de la politique monétaire

1. Introduction

With the adoption of the euro in January 1999, a euro-area model has become a sine qua non condition by which to evaluate the monetary policy of the European Central Bank (ECB). An initial question for us was whether a model with strong theoretical foundations but possibly limited forecasting ability was preferable to a good forecasting model with an undefined structure. Given the availability of indicator models that can forecast fairly well, we decided to put more weight on analytical rigour in designing our model. We chose a New Keynesian (NK) approach, which has solid theoretical foundations based on optimizing rational consumers and firms in a highly aggregated framework.

To our knowledge, few models have been built so far for the euro area. Coenen, Levin, and Wieland (2001) assess the role of money to forecast output in a small rational-expectations framework. Coenen, Levin, and Wieland (2000) estimate a small model of the euro area "used as a laboratory for evaluating the performance of alternative monetary policy strategies." Fagan, Henry, and Mestre (2001) manage to build a medium-sized model detailed enough to include agents' behaviour separately. The goals of this paper are to estimate a small NK model, extract the inflation expectations it captures, and search for evidence of increased flexibility in Europe since the adoption of the Maastricht Treaty.

The NK approach is widely used in monetary policy analysis, since it is designed to describe the behaviour of economy-wide variables that enter into most monetary policy discussions. The baseline macroeconomic framework is a dynamic general-equilibrium model with money, nominal price rigidities, and rational expectations. In this model, monetary policy affects the real economy in the short run as in the traditional Keynesian IS/LM framework. The NK approach is appealing because the aggregated behavioural equations are the outcome of optimizing households and firms, in keeping with the most recent advances in modern macroeconomics.

According to the consensus view among central bankers and monetary economists, a contractionary monetary shock raises unemployment, at least temporarily, and leads to a delayed and gradual fall in inflation. Mankiw (2000) discusses the inability of NK models both to generate the degree of inflation persistence observed in the data and to replicate empirically plausible impulse-response functions to monetary policy shocks. We follow Amato and Laubach (2001), allowing for a fraction of firms to use backward-looking rules-of-thumb to set prices. This leads to endogenous persistence in inflation and estimated impulse-response functions that replicate the consensus view fairly well.

Typically, in the empirical applications of these models, the forward-looking part is solved by assuming perfect foresight, using consensus forecasts (e.g., Rudebusch 2000), or using forecasts derived from a multivariate vector autoregression (VAR) (Sbordone 2000; Galí and Gertler 1999). We solve our forward-looking model as a *rational-expectations* model following Fair and Taylor (1983). This allows us to examine the impacts of various shocks via changes on agents' expectations, which, to our knowledge, has not yet been done in the empirical literature.

Other contributions of this paper are: (i) we find evidence to suggest that flexibility in Europe has increased since the adoption of the Maastricht Treaty, (ii) in contrast with some recent empirical work based on a forward-looking model, we find that excess demand (as measured by the output gap) leads to higher inflation, and (iii) the inflation expectations captured by our model might explain the recent reluctance of the ECB to ease monetary conditions.

Section 2 briefly describes the NK approach. In section 3, we expose how the model is estimated and solved. We also present the results and the model fit, and analyze the impulse responses. Section 4 concludes.

2. An Overview of the New Keynesian Approach

In the NK approach, the baseline macroeconomic framework is a dynamic general-equilibrium model with money, nominal price rigidities, and rational expectations. In this model, households maximize their expected utility under a budgetary constraint. Production is divided into two sectors: a perfectly competitive final-good sector, which aggregates all intermediate goods and converts them into a homogeneous final good, and a monopolistically competitive intermediate-goods sector. Given the demand from the final-good sector, each firm in the intermediate-goods sector produces a unique product optimally.

For monetary policy to have a role, price rigidities are required. They are introduced by assuming frictions to price adjustment on the part of imperfectly competitive firms. Since firms know that their prices may be sticky in future periods, they will consider future market conditions when they set their current prices.

We don't explicitly derive these optimal behavioural equations, borrowing specifications from the existing literature.[1] Instead, we present a baseline model that, despite its simplicity, contains the main ingredients of richer frameworks that have been used for policy analysis. It consists of two

1. Clarida, Galí, and Gertler (1999), and particularly King (2000), give an excellent exposition of the NK perspective, from which this section is inspired.

optimal behavioural equations: a Phillips curve that links inflation to the output gap, and an IS curve that relates output inversely to the real interest rate. The model is closed by specifying a reaction function for the monetary authorities.

2.1 The Phillips curve

The traditional backward-looking Phillips curve is probably less relevant in today's world, in which monetary authorities increasingly have credible inflation targets. Nevertheless, many studies find support for a strictly backward-looking equation. For example, Rudebusch and Svensson (1999) find that a traditional Phillips curve can explain most of the inflation that occurred in the United States over the 1960–99 period. Galí, Gertler, and López-Salido (2001) obtain similar results for the euro area over the period 1970–98. Despite its apparent empirical success, however, the traditional Phillips curve seems to have had a tendency recently to over-predict inflation (as noted in particular by Galí, Gertler, and López-Salido 2001), possibly because it lacks a forward-looking component.

In the NK framework, the non-neutrality of monetary policy results from assuming frictions to price adjustment on the part of imperfectly competitive firms. In these models, pricing decisions are optimal given the assumed frictions to price adjustment (based on overlapping contracts in Taylor 1979, convex costs of adjusting prices, or a probability of price adjustment in Calvo 1983). At the aggregate level, this framework provides a relationship between current inflation, the current output gap, and expected future inflation.

The supply side of the economy is thus summarized by a Phillips curve as described by equation (1), where π_t is the inflation rate and x_t is the output gap. The equation represents a loglinear approximation around the steady-state aggregation of individual firms' pricing decisions[2]:

$$\pi_t = \beta E_t \pi_{t+1} + \alpha_\pi x_t . \tag{1}$$

The larger is α_π, the stronger is the adjustment of prices to deviations of output from its potential (or the more flexible are prices).[3]

2. Equation (1) can also be described as an aggregate supply curve by replacing the current inflation rate with its definition in terms of the change in prices between period t and t-1. This yields an equation that explains how the aggregate supply depends on current prices and other factors.

3. We should expect that a rigid labour market in which wage-setting mechanisms are not market-driven will be reflected in a low α. In section 3.1, we do rolling regressions to determine whether there is evidence that the euro area has become more flexible in recent years with the convergence efforts and the subsequent adoption of the euro. Using a measure of real marginal cost instead of the output gap in their Phillips curve specification, Galí, Gertler, and López-Salido (2001) find that prices are more flexible in the U.S. economy than in the euro area over the 1970–98 period.

These models are distinguished by the absence of lagged variables in the structural equations. Inflation dynamics are entirely explained by the current output gap and expected future inflation. This might account for the difficulty in using these models to replicate the strong serial correlation typically found in both output-gap and inflation data.

Amato and Laubach (2001) allow for the possibility that a fraction of firms use backward-looking rules-of-thumb to set prices. This leads to endogenous persistence in inflation:

$$\pi_t = \beta_f E_t \pi_{t+1} + \beta_b \pi_{t-1} + \alpha_\pi x_t \ . \tag{2}$$

By iterating equation (2) forward, we easily see that inflation depends entirely on past inflation and on current and expected future economic conditions.

In the steady state, inflation is constant. We can thus, from (2), express output as a function of steady-state inflation as follows:

$$y = \bar{y} + \frac{(1-\beta_f-\beta_b)}{\alpha_\pi}\pi \ , \tag{3}$$

where y is real output and $\bar{y}$ is potential output. The long-run slope of the Phillips curve, $\frac{(1-\beta_f-\beta_b)}{\alpha_\pi}$, measures the response of output to changes in the long-run rate of inflation, after the economy has made a transition from one inflationary steady state to another. With $\beta_f + \beta_b$ close to unity, equation (3) implies that there is a negligible long-run trade-off between inflation and output. As expected by the mainstream theory, King and Wolman (1996) suggest that the long-run effect of inflation on output is very small.[4] We will therefore assume in our model that there is no long-run trade-off between real output and inflation; i.e., that $\beta_f + \beta_b = 1$. The Phillips curve will be specified as:

$$\pi_t = \mu_\pi E_t \pi_{t+1} + (1-\mu_\pi)\pi_{t-1} + \alpha_\pi x_t + u_t \ , \tag{4}$$

where $0 \le \mu_\pi \le 1$. For $0 < \mu_\pi < 1$, actual inflation is a function of the current output gap, as well as expected and past inflation.

There is no consensus in the literature over the degree of "forward-lookingness" in the determination of inflation. It is possible to derive an equation with $\mu_\pi = 1$ from the models of

4. The mainstream assumption of long-run monetary neutrality is questioned by Mankiw (2000), but it remains very plausible.

price-setting behaviour (see Roberts 1995), but many authors assume that there is some inertia in inflation, so μ_π should be less than 1 (Svensson 1997; Fuhrer and Moore 1995; Fuhrer 1997). Rudebusch (2000) gives empirical evidence that the value of μ_π for the United States lies between 0 and 0.6. This represents quite well the range of estimates provided by various researchers. For example, Fuhrer (1997) finds that backward-looking behaviour explains more than 75 per cent of the variation in U.S. inflation, but he also does not reject the possibility that the forward-looking component is statistically significantly different from zero. Some others find higher estimates of μ_π (see, for example, the estimates of Chadha, Masson, and Meredith 1992; Brayton et al. 1997; and Gagnon and Khan 2001).

2.2 The IS curve

The demand side of the economy is obtained by linearizing the Euler consumption equation that results from the households' optimal saving decision in a closed economy with a government but no investment.[5] The standard NK IS curve is illustrated in equation (5), where x_t is the output gap, π_t the inflation rate, r the equilibrium real interest rate, and i_t the 3-month nominal interest rate:

$$x_t = -\gamma[i_t - E_t\pi_{t+1} - r] + \zeta E_t x_{t+1}. \tag{5}$$

Amato and Laubach (2001) consider both forward-looking and rule-of-thumb consumers. Rule-of-thumb consumers are also forward looking, but act with a delay of one period (i.e., they base their forward-looking expectations on information available in the previous period). Such behaviour leads to endogenous persistence in output:

$$x_t = -\gamma[i_t - E_t\pi_{t+1} - r] + \zeta_b x_{t-1} + \zeta_f E_t x_{t+1}. \tag{6}$$

Equation (6) implies a negative relationship between real ex ante interest rates and the current output gap, everything else being equal.[6] The difference with the traditional IS curve is that the current output now depends both on past and expected output.

The negative effect of real interest rates on consumption reflects intertemporal substitution. Many economists would argue that the long-term interest rate is more relevant for aggregate demand

5. A closed economy is a reasonable approximation, given that the domestic economy is about 80 per cent of European GDP.

6. The Fisher equation is implicitly imposed in (6), because the real interest rate is defined as the nominal interest rate minus the rate of inflation that is expected to prevail between t and t+1. This specification of the Fisher equation omits any inflation-risk premium in the nominal interest rate (see McCallum and Nelson 1999b for a discussion of this point).

than the short-term interest rate. The NK IS curve can accommodate this view. By iterating equation (5) or (6) forward, it is easy to show that output is a function of the sum of current and expected short-term interest rates, which will be reflected in the long-term interest rate under the pure term-structure hypothesis. The coefficient associated with the long-term interest rate will be greater than γ, since it includes the influence of expected future output. This explains why the long-term interest rate is more important than the short-term interest rate in a traditional IS curve that omits expected output.

2.3 The monetary policy reaction function

The NK IS/LM model differs from Hick's original model in that it makes the price level an endogenous variable. Therefore, the model can no longer be solved without specifying a monetary policy rule.[7] Two different approaches have been adopted in the literature: (i) either specify money-demand and money-supply equations, where the money-supply process contains a systematic monetary policy component, or (ii) specify an interest rate rule for monetary policy. We follow the latter approach, which is becoming increasingly popular, and where money, being demand-determined at the interest rate set by the monetary authority, plays no role.

A typical Taylor rule formulation in this literature is:

$$i_t = r + \pi^c + \varphi\,(\pi_t - \pi^c) + \alpha_i x_t\,, \tag{7}$$

where i_t is the nominal short-term interest rate set by the central bank period to period, r the equilibrium real interest rate (assumed to be constant), π^c the inflation target, and x_t the output gap. What is immediately noticeable in this formulation is that φ must be greater than 1 for the *real* interest rate to be raised when the deviation of inflation from its target increases. It can also be shown (King 2000) that φ must be greater than 1 to obtain a unique stable equilibrium. Taylor (1993) also suggests that the central bank should lower the nominal interest rate when output is below capacity, thus implying a positive value for α_i.

One can also derive forward-looking specifications of the Taylor-type monetary policy rule. Following Clarida, Galí, and Gertler (1998), we assume that within each operating period the central bank has a target for the nominal short-term interest rate, i_t', that is based on the state of

7. Many studies concentrate on evaluating the optimal reaction function related to a specific model. Our goal is rather to estimate the policy rule that the monetary authority has conducted in the past.

the economy.[8] The target is then a function of the expected future inflation[9]:

$$i_t' \quad = r + \pi^c + \varphi\ (E_t \pi_{t+1} - \pi^c) + \alpha_i x_t\ . \tag{8}$$

Again, necessary conditions for unicity of the equilibrium is that $\varphi > 1$. This specification of the monetary target seems to be more appropriate, given the medium-term inflation objective of many industrialized countries.[10] The monetary authority would not react to an actual deviation from the target as long as it expected inflation to return to its target in the near future.

The policy reaction function described in (8) is still incomplete, because it assumes an immediate adjustment of interest rates, and thus ignores the tendency of central banks to smooth changes in interest rates.[11] To take this into consideration, we add the following relationship:

$$i_t = \rho i_{t-1} + (1-\rho) i_t + v_t\,, \tag{9}$$

where i_t is the monetary authorities' actual interest rate set in period t; v_t is an exogenous random shock to the interest rate and $0 \le \rho \le 1$. The higher is ρ, the higher is the degree of interest rate smoothing. Substituting (8) into (9), we obtain (10)[12]:

$$i_t = \rho i_{t-1} + (1-\rho)[r + \pi^c + \varphi(E_t \pi_{t+1} - \pi^c) + \alpha_i x_t] + v_t\ . \tag{10}$$

This is the reaction function that Clarida, Galí, and Gertler (1998) find characterizes German monetary policy after 1979. They also show that this specification works well against various alternatives, including a backward-looking specification. Gerlach and Schnabel (2000) find the same kind of support for the euro area over the 1990–98 period.

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8. The approach proposed by Clarida, Galí, and Gertler (1998) allows for the possibility that, within an operating period, a central bank targets a reserve aggregate, as long as the target for reserves is based on an implied objective for the expected short-term interest rate.
9. It is perfectly possible for current output to be unavailable at the moment the central bank chooses its target interest rate. However, the GMM estimation methodology takes this into account.
10. Clarida, Galí, and Gertler (1998) and Gerlach and Schnabel (2000) use a one-year horizon for the inflation forecasts, while other studies adopt our specification (for example, Christiano and Rostagno 2001). It would be interesting in further research to analyze the robustness of the results to different horizons for expected inflation. Nevertheless, Clarida, Galí, and Gertler (1998) state that, since the forecasts over near-term horizons are highly collinear, the results should not be sensitive to small changes in assumed horizons.
11. See Rudebusch (1995) for evidence on the serial correlation of interest rate changes. One explanation for this smoothing is fear of disrupting financial markets (Goodfriend 1991); another is uncertainty about the effects of interest rate changes.
12. Notice that the stability condition for φ is the same with or without smoothing. With partial adjustment, however, the condition $\varphi > 1$ no longer guarantees that the real interest rate goes up when the expected inflation is rising. It only guarantees that it will eventually go up.

It is also possible to use the estimated value of φ and ρ to compute a central bank's implicit inflation target, π^c, over the sample. Since the constant term in (10) is $c = (1-\rho)(r + \pi^c - \varphi\pi^c)$, we easily obtain the following expression for the inflation target in terms of *r*, φ, and ρ:

$$\pi^c = \frac{[c - (1-\rho)r]}{(1-\rho)(1-\varphi)}. \tag{11}$$

Naturally, this implies that an assumption is made about the real long-run interest rate equilibrium (*r*).[13] To make this assumption, we simply use the average of the ex post real interest rate over the sample.[14]

3. Estimation and Results

Given the assumption of rational expectations, equations (4), (6), and (10) imply a set of orthogonality conditions, which provide the basis for estimating the model's parameter by GMM (Hansen 1982), with optimal weighting matrices that account for possible serial correlation in the residuals. To the extent that the dimension of the vector of instruments exceeds the number of parameters being estimated, these orthogonality conditions imply some overidentifying restrictions that can be tested to assess the validity of our specification as well as the set of instruments used.

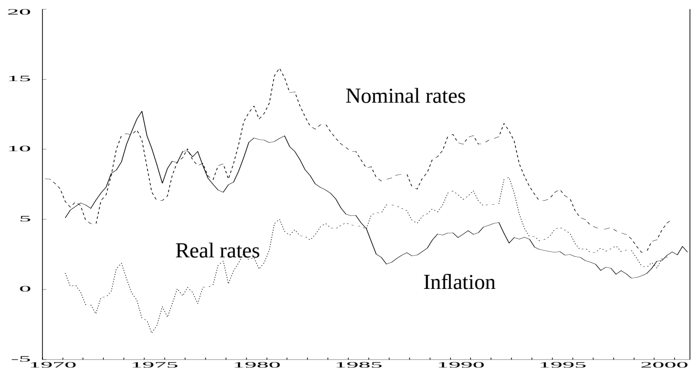
Once the model is estimated, we want to analyze the impulse-response functions of the variables. A tricky feature of forward-looking models is that not only the endogenous variables respond to shocks, but under rational expectations the expectations on those variables should also be affected. Thus, we need to solve the model as a *rational-expectations* model, such that expectations of future endogenous variables are conditional forecasts based on the model itself (see Appendix A).

The specification retained and estimated results are reported in Appendixes B and C, respectively. The equations are estimated separately with GMM using lagged variables as instruments, over the 1983–2000 period, on a quarterly basis, based on data published by the ECB.[15] We begin our sample in the early 1980s because there is evidence of a change in the ECB's conduct of monetary

13. We could also use the one provided by the IS curve estimation. Normally, they shouldn't differ too much if our model is well specified.
14. As noted in Clarida, Galí, and Gertler (1998), by using the average real interest to proxy *r*, the estimate we find for π^c should not differ too much from the average of π over the sample we use.
15. The data base is constructed by the ECB (see Fagan, Henry, and Mestre 2001). Appendix D gives the data descriptions.

policy at that time: it seems that the control of inflation became a major focus of monetary policy (Figure 1). We can then identify the features of monetary policy that prevailed during a period when a policy-making commitment to reduce inflation was considered effective. Figure 1 plots the rate of inflation, π_t, versus nominal and real short-term interest rates, i_t and r_t, respectively (where we use $r_t = i_t - \pi_t$).

Figure 1: Inflation and Interest Rates for the Euro Area



Note: The figure shows annual inflation; the ex post real interest rate is computed with annual inflation.

Inflation is measured as the annualized quarterly increase in the harmonized prices index. The output gap represents the difference between real output and its Hodrick-Prescott (HP) filtered series. The real interest rate is the nominal 3-month interest rate in period *t* minus expected inflation in *t*+1. Most of the estimated coefficients are of the expected sign and they are all (except for the constants of the Phillips curve) significant at the 10 per cent level. Based on Hansen's (1982) overidentifying J-test, we cannot reject the orthogonality conditions, except in the case of the IS equation.

3.1 The Phillips curve

The equation is estimated using as instruments past values of the output gap (one lag), inflation (three lags), and the interest rate (seven lags). The coefficients on the forward-looking and backward-looking components of inflation are constrained to sum to one, as are the coefficients on the lagged variables for inflation ($\sum_{j=1}^{4} \mu_{j\pi} = 1$). This latter condition implies the natural rate hypothesis. The specification we estimate is:

$$\pi_t = c_\pi + \mu_\pi E_t \pi_{t+1} + (1-\mu_\pi) \sum_{j=1}^{4} \mu_{j\pi} \pi_{t-j} + \alpha_\pi x_t + u_t, \tag{12}$$

where $\mu_{4\pi} = 1 - \mu_{1\pi} - \mu_{2\pi} - \mu_{3\pi}$ and c_π is a constant.

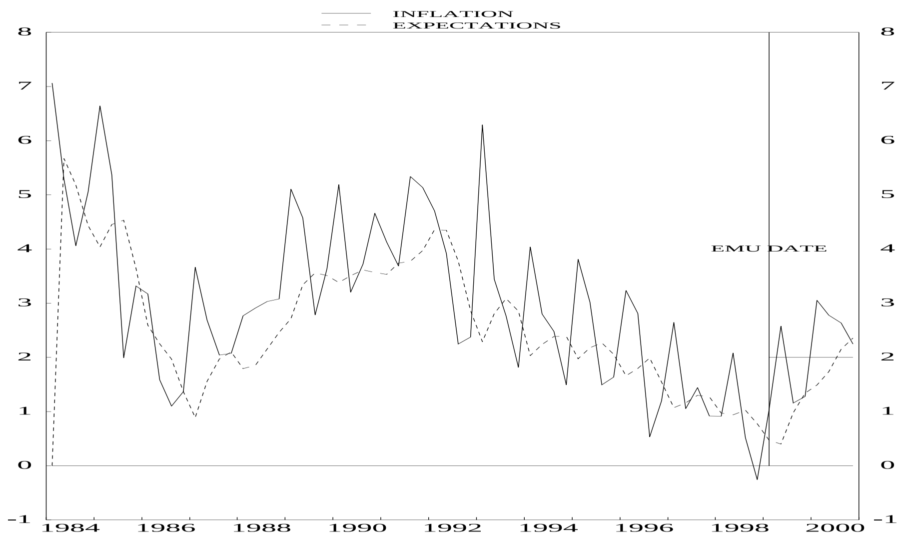
The size of the estimated coefficient on the forward-looking component is consistent with previous results reported in Rudebusch (2000), which have ranged from 0 to 0.6.[16] Our estimation is on the higher side, which suggests that the forward-looking component is important. A 1 per cent increase in expected inflation leads to an increase in current inflation of 0.38 per cent. While Galí, Gertler, and López-Salido (2001) find a stronger expected inflation effect ($\mu_\pi = 0 \cdot 92$), their model does not allow for backward-looking expectations. The output-gap coefficient is significant and, more importantly, of the right sign, in contrast to some earlier findings (Galí and Gertler 1999; Galí, Gertler, and López-Salido 2001).[17]

To determine the robustness of our results, we estimated the Phillips curve equation over alternative configurations that considered one to seven lags for the three instrumental variables (inflation, interest rate, and output gap), and kept only the outcomes where the coefficients associated with expected inflation and the output-gap terms were significant, providing that the overidentifying restrictions were not rejected. In all cases, the value of the output-gap coefficient was positive, lying between 0.09 and 0.19, and the expected inflation coefficient hovered between 0.35 and 0.59. We conclude that there is no general support for the counterintuitive effect of the output gap on inflation that was found in previous work.

The model is solved numerically following Fair and Taylor (1983), and consistent expectations are calculated for the 1985Q1–2000Q4 period[18] (Figure 2). Note that recent expectations are inconsistent with the ECB's target of below 2 per cent. The inflation expectations exceed 2 per cent at the fourth quarter of 2000, which explains, in a sense, why the ECB was late to lower its target interest rate after the easing by major central banks.

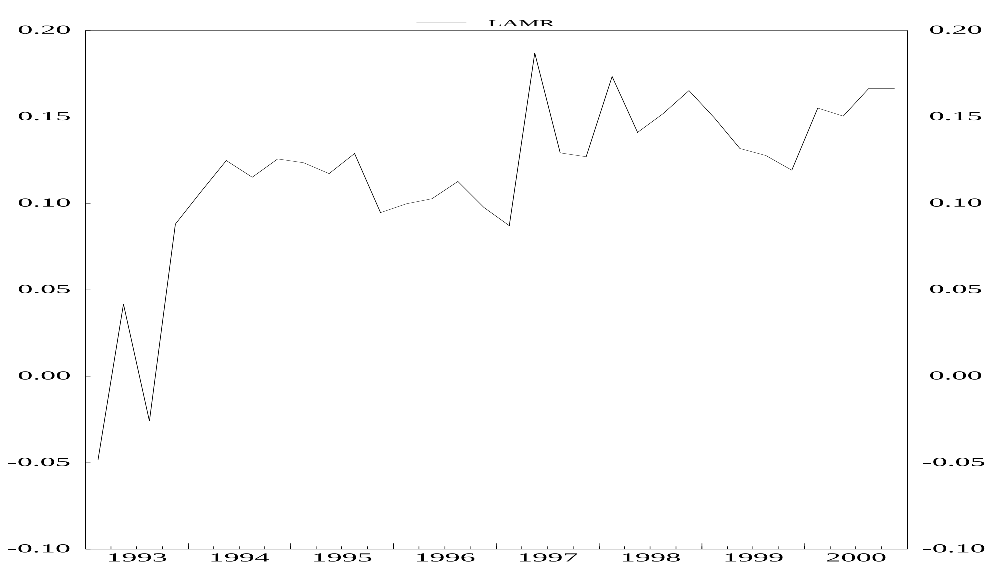
16. See Appendixes A and B for the estimation results.
17. Some other results (Gagnon and Khan 2001) suggest that the sign of the coefficient related to the output gap depends on whether a backward-looking component is included.
18. The model is solved simultaneously.

Figure 2: Inflation and Inflation Expectations for the Euro Area



We also performed rolling regressions over the 1992–2000 period to assess the possibility of increasing price flexibility following the signing of the Maastricht Treaty (1991). In a free-market economy, prices signal excess demand or supply. We should thus expect that, the less regulated an economy, the clearer the correlation between the output gap and prices. Figure 3 shows the value of the estimated coefficient related to the output gap in the Phillips curve that comes out of a rolling regression over the period 1993–2000. This coefficient follows an upward trend, which indicates an increasing response of prices to the output gap, which in turn suggests an increasingly flexible economy.

Figure 3: Coefficient α_π Over the Sample Period 1993–2000 (rolling regressions)



3.2 The IS curve

To ensure stability, we impose that the coefficients of the forward- and backward-looking variables on the output gap sum to one. For this equation, the instruments used for the estimation include the output gap lagged two periods, inflation lagged four periods, and the real wage lagged two periods. The specification estimated is the following:

$$x_t = c_x + \theta_{x1} E_t x_{t+1} + \theta_{x2} x_{t-1} + \theta_{x3} x_{t-2} - \gamma_x [i_t - E_t \pi_{t+1}] + \nu_{xt} , \tag{13}$$

with $\theta_{x3} = 1 - \theta_{x1} - \theta_{x2}$ and $c_x = \gamma_x r$. Hence, the interest rate at equilibrium is consistent with a close output gap.

While the interest rate coefficient is significant and of the right sign, it is rather small. A 1 per cent increase in the real interest rate will lower the output gap by only 0.03 per cent. This finding seems to be common in the literature (Rudebusch 2000). The traditional interest rate channel seems controversial, as Bernanke and Gertler (1995) point out: empirical studies have great difficulty in identifying significant interest rate effects on output, perhaps because monetary policy operates through other channels (e.g., asset prices, exchange rate, credit, wealth effect) than the short-term interest rate. Gauthier, Graham, and Liu (2003) find evidence to support this view.[19]

Following equation (13), the equilibrium real interest rate can be defined as $r = c_x / \gamma_x$. This gives an estimated level of the equilibrium real interest rate of 4.33 per cent, which seems a bit high. This is a consequence of the low estimated value for γ_x . Alternatively, if we compute the average real interest rate over the sample, assuming an ex post real interest rate ($r_t = i_t - \pi_t$), we get an interest rate value similar to our finding. It is also consistent with the values that Clarida, Galí, and Gertler (1998) find, of 3.76 per cent for the Bundesbank from 1979 to 1994, 6.01 per cent for the Bank of France from 1983 to 1989, and 6.94 per cent for the Bank of Italy from 1981 to 1989.[20] These high levels seem to have been caused by the fact that, during those periods, central banks were trying to reduce inflation by pushing up the real short-term interest rate. An implication of those policies of high real short-term interest rates is their persistence over the period of low

19. Moreover, the rejection of the overidentifying restrictions in the IS curve may be a sign of mis-specification. This is left for further work.

20. Because, prior to the formation of the Economic and Monetary Union (EMU), the Bundesbank had a strong influence on the monetary policy of France and Italy, the values of the equilibrium real rate are computed with the inflation target of the Bundesbank implied (i.e., 2 per cent). For the Bundesbank, we computed the average value of the real ex post interest rate from 1979 to 1994.

inflation (Figure 1), in contrast to the end of the 1970s, when the real short-term interest rate of the euro area was close to zero. The equilibrium real interest rate would probably be lower in the future.

3.3 The monetary policy reaction function

Our main goal in this section is to estimate a representative reaction function for the monetary authority, rather than determine the optimal response that would fall out of the model. We estimate the monetary policy reaction-function equation using as instruments four lags for inflation, two lags for the output gap, and one lag for the interest rate. The equation follows Gerlach and Schnabel (2000) and Clarida, Galí, and Gertler (1998). The estimated equation is as follows:

$$i_t = c_i + \rho i_{t-1} + \delta_i E_t \pi_{t+1} + \beta_i x_t + v_{i_t}, \tag{14}$$

with $c_i = (1-\rho)(\tilde{i} - \varphi\pi^c)$, $\delta_i = (1-\rho)\varphi$, and $\beta_i = (1-\rho)\alpha_i$.

The equation fits well in the sample. The central bank raises the nominal interest rate sufficiently to increase the real interest rate when inflation deviates from its target, since the coefficient associated with expected inflation (corrected for the presence of smoothing) is greater than 1 ($\delta_i/(1-\rho) = 1.4$) and statistically significant at the 1 per cent level. An increase in expected annualized inflation of 1 per cent leads the monetary authority to raise the real interest rate by 0.4 per cent (very close to what Clarida, Galí, and Gertler 1998 find for Germany over the 1973–94 period, but much smaller than the estimate of Gerlach and Schnabel 2000 for the ECB over the period 1990–98). We also find that the central bank responds to excess demand or supply pressures: a 1 per cent increase in the output gap leads the monetary authority to raise the real interest rate by 0.32 per cent.[21]

Using the estimate of r, φ, c_i, and ρ, and using expression (11) from section 2.3, we find an estimate of the central bank's inflation target equal to 4.5. Remember from equation (14) that this estimate requires a value for the long-run real interest rate equilibrium. The value we use (the sample average real rate) is therefore 4.60. The estimated inflation target is a bit high, since the ECB's medium-term inflation target is below 2 per cent and the Bundesbank's official target was 2 per cent before EMU. Note, however, that the sample average value of inflation is also quite large (3.2 per cent).

21. This is somewhat higher than the Bundesbank's estimate of 0.25 per cent that Clarida, Galí, and Gertler (1998) find, and higher than the 0.23 per cent estimate that Gerlach and Schnabel (2000) find for the ECB.

3.4 Model fit

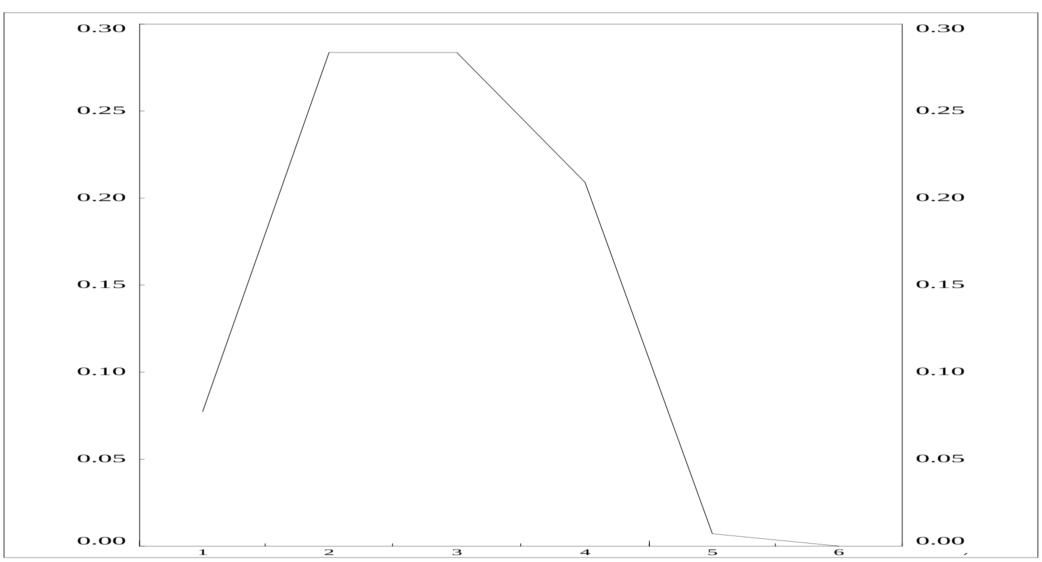
Using the estimated parameters and the expected values for all the variables, we calculate the forecasted series in sample. The results are shown in Appendix E. The model tracks the historical data rather well, though predicted inflation is less volatile than history. The monetary policy reaction function gives very good results, irrespective of whether smoothing is incorporated.[22]

3.5 Impulse-response functions

Appendix F shows the impulse-response functions of each variable in the system to structural shocks. Qualitatively, the results are similar to those of Rudebusch (2000), who uses survey expectations data to proxy inflation expectations. A 1 per cent shock to the output gap leads to a 0.18 per cent increase in inflation in the first period and inflation that dies off 9 to 10 periods later. The interest rate rises initially, peaking between 4–5 quarters, before settling at its long-run value after around 20 quarters. This high degree of persistence is a reflection of the large (0.90) interest-rate-smoothing parameter.

A 1 per cent positive shock to inflation increases inflation expectations by as much as 0.3 per cent for 3 quarters, before falling back to equilibrium (Figure 4).

Figure 4: Response of Expectations to an Inflation Shock



22. The graphs are based on the reaction function, which incorporates interest rate smoothing.

Although the response of output to an inflation shock is of the right direction, it is small in magnitude. Interest rates rise in response to an inflation shock for 5 to 6 quarters before easing. A 1 per cent inflation shock widens the output gap by only 0.01 per cent in the second and third quarters, which then falls precipitously, undershooting its return to equilibrium for a while. Interest rates rise in response to an inflation shock for 5 to 6 quarters before easing.

A 1 per cent shock to the interest rate has a negative though small impact on the output gap and on inflation, as expected. The limited effect is common in structural models, in contrast to simple VARs, perhaps because we don't allow for interest rate dynamics in the IS curve and Phillips curve equations. To assess monetary policy effects implied by the model, it is important to examine the model's empirical impulse-response functions, more specifically the effects of monetary policy shocks to output and inflation. A monetary policy shock has an immediate effect on output that peaks at 4 to 5 quarters, and a subsequent effect on inflation that is largest at 8 to 9 quarters. Mankiw (2000) notes that empirical NK models fail to capture these dynamics. To reconcile NK models with the data, he suggests replacing rational expectations with adaptive expectations in model specifications to get more realistic dynamic responses to monetary policy actions. We have shown, however, that it is possible to find plausible impulse-response functions while maintaining the assumption of rational expectations by using model-consistent expectations.

4. Conclusion

We have proposed, estimated, and solved a small structural hybrid model for the euro area. Although the euro area did not formally exist before 1999, our aggregate estimates fit the data in the sample relatively well.

Our main findings are that: (i) the impulse-response functions implied by the model, which combines forward- and backward-looking expectations, are consistent with the standard stylized facts about the dynamic effects of monetary policy, and (ii) since the adoption of the Maastricht Treaty, there has been evidence that prices in Europe have increased in flexibility.

In future research, it would be interesting to use a longer sample while accounting for different inflation targets over time in the monetary policy rule. Further work might also be done to improve the model's specification by including government expenditures, the exchange rate, and other financial variables through which monetary policy may operate.

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Appendix A: Fair and Taylor's (1983) Methodology

To solve the model as a *rational-expectations* model, so that expectations of future endogenous variables are conditional forecasts based on the model itself, we follow the methodology suggested by Fair and Taylor (1983). This method uses dynamic structure to solve rational and perfect-foresight models. Suppose we have the following model:

$$\pi_t = \mu\pi_{t+1} + x_t , \qquad \text{(A1)}$$

with the condition:

$$-\infty < \lim_{t \to \infty} \pi_t < \infty . \qquad \text{(A2)}$$

The variable π in (A1) is bounded and x is an exogenous variable. We look for π_0 that solves (A1) and satisfies (A2). The solution is

$$\pi_0 = \sum_{t=0}^{\infty} \mu^t x_t . \qquad \text{(A3)}$$

The Fair-Taylor method fixes T and gives a value for T+1 (assume that $\pi_{T+1}= 0$, for simplicity). For this terminal value at (T+1), we have a system of (T+1) equations in the (T+1) unknowns, π_T.

$$\pi_t = \mu\pi_{t+1} + x_t, \qquad t = 0, 1, \ldots, T . \qquad \text{(A4)}$$

To solve the preceding system, Fair and Taylor assume that each initial guess is zero for $\pi_{T,t}{}^0$. The algorithm creates a sequence of approximations for those guesses:

$$\pi^{j+1}{}_{T,t} = \mu\pi^{j}{}_{T,t+1} + x_t, \qquad t = 0, 1, \ldots, T . \qquad \text{(A5)}$$

New guesses for $\pi_{T,t}{}^{j+1}$ are calculated by applying (A1) to the old guess of π_{t+1}, $\pi_{T,t}{}^{j}$. For each j, j=1,.., T, we repeat (A1). The guesses $\pi_{T,t}{}^{j+1}$ are based only on the $\pi_{T,t}{}^{j}$. The Fair-Taylor method stops when every $|\pi_{T,t}{}^{j+1} - \pi_{T,t}{}^{j}|$ is less than a prespecified convergence criteria. Note that the solution does not depend on the T chosen. For details, see Fair and Taylor (1983) and Judd (1998).

Appendix B: Equations

B.1 Phillips curve

$$\pi_t = c_\pi + \mu_\pi E_t \pi_{t+1} + (1-\mu_\pi)(\mu_{1\pi}\pi_{t-1} + \mu_{2\pi}\pi_{t-2} + \mu_{3\pi}\pi_{t-3} + \mu_{4\pi}\pi_{t-4}) + \alpha_\pi x_t + u_t, \quad \text{(B1)}$$

with $\mu_{4\pi} = 1 - \mu_{1\pi} - \mu_{2\pi} - \mu_{3\pi}$ and c_π as a constant.

π_t : Inflation in period t;

x_t : a measure of the output gap in period t.

$E_t\pi_{t+1}$: the period t expectation of inflation in period t+1;

u_t : a structural error in the Phillips curve.

B.2 IS curve

$$x_t = c_x + \theta_{x1} E_t x_{t+1} + \theta_{x2} x_{t-1} + \theta_{x3} x_{t-2} - \gamma_x [i_t - E_t \pi_{t+1}] + \nu_{xt}, \quad \text{(B2)}$$

with $\theta_{x3} = 1 - \theta_{x1} - \theta_{x2}$ and where the constant represents: $c_x = \gamma_x r$.

r : The long-run equilibrium real interest rate;

i_t : the nominal short-run interest rate;

ν_{xt} : a structural error in the IS curve.

B.3 Reaction function

$$i_t = c_i + \rho i_{t-1} + \delta_i E_t \pi_{t+1} + \beta_i x_t + v_{it}, \quad \text{(B3)}$$

with $c_i = (1-\rho)(\tilde{i} - \varphi \pi^c)$, $\delta_i = (1-\rho)\varphi$ and $\beta_i = (1-\rho)\alpha_i$.

v_{it} : A structural error in the reaction function;

$\tilde{i}$: the long-run equilibrium nominal interest rate.

Appendix C: Estimation Results[1]

Table C1: Phillips Curve

Variable	Coefficient	*p*-value
c_π	-0.07	0.46
$E_t\pi_{t+1}$	0.38	0.00
π_{t-1}	0.33	0.00
π_{t-2}	-0.20	0.05
π_{t-3}	-0.25	0.33
x_t	0.17	0.08
J-Stat	4.94	0.66
R^2	0.53	

Table C2: IS Equation

Variable	Coefficient	*p*-value
c_x	0.13	0.08
E_ty_{t+1}	0.48	0.00
i_t-$E_t\pi_{t+1}$	-0.03	0.00
x_{t-1}	0.45	0.00
J-Stat	23.36	0.00
R^2	0.83	

Table C3: Reaction-Function Equation

Variable	Coefficient	*p*-value
c_i	0.28	0.00
$E_t\pi_{t+1}$	0.14	0.00
x_t	0.32	0.06
i_{t-1}	0.90	0.00
J-Stat	7.46	0.59
R^2	0.97	

1. The estimation sample is from 1983Q1–2000Q4.

Appendix D: Data Descriptions

All data are quarterly time series. The data for the period from 1983Q1 to 1998Q4 are taken from the data base constructed by the ECB (Fagan, Henry, and Mestre 2001). We extend the sample to 2000Q4 using the associated recent data published in the ECB's monthly bulletin. Note that historical revisions of the data were incorporated in the sample.

Price inflation is measured as the annualized quarterly growth rate of the harmonized index of consumer prices (HICP):

$$\pi_t = 100 \cdot (\ln(HICP_t) - \ln(HICP_{t-1})) \cdot 4 .$$

The models have been estimated using non-seasonally adjusted data. One of the referees has asked that an HICP be used, seasonally adjusted; however, none were available. Using a year-over-year series would have meant altering the calculations of the real interest rate, which is quarterly, and the impulse responses would have been hard to interpret because there would have been an MA process embedded in the residuals.

Wage inflation is measured as the annualized quarterly growth rate of compensation by employees:

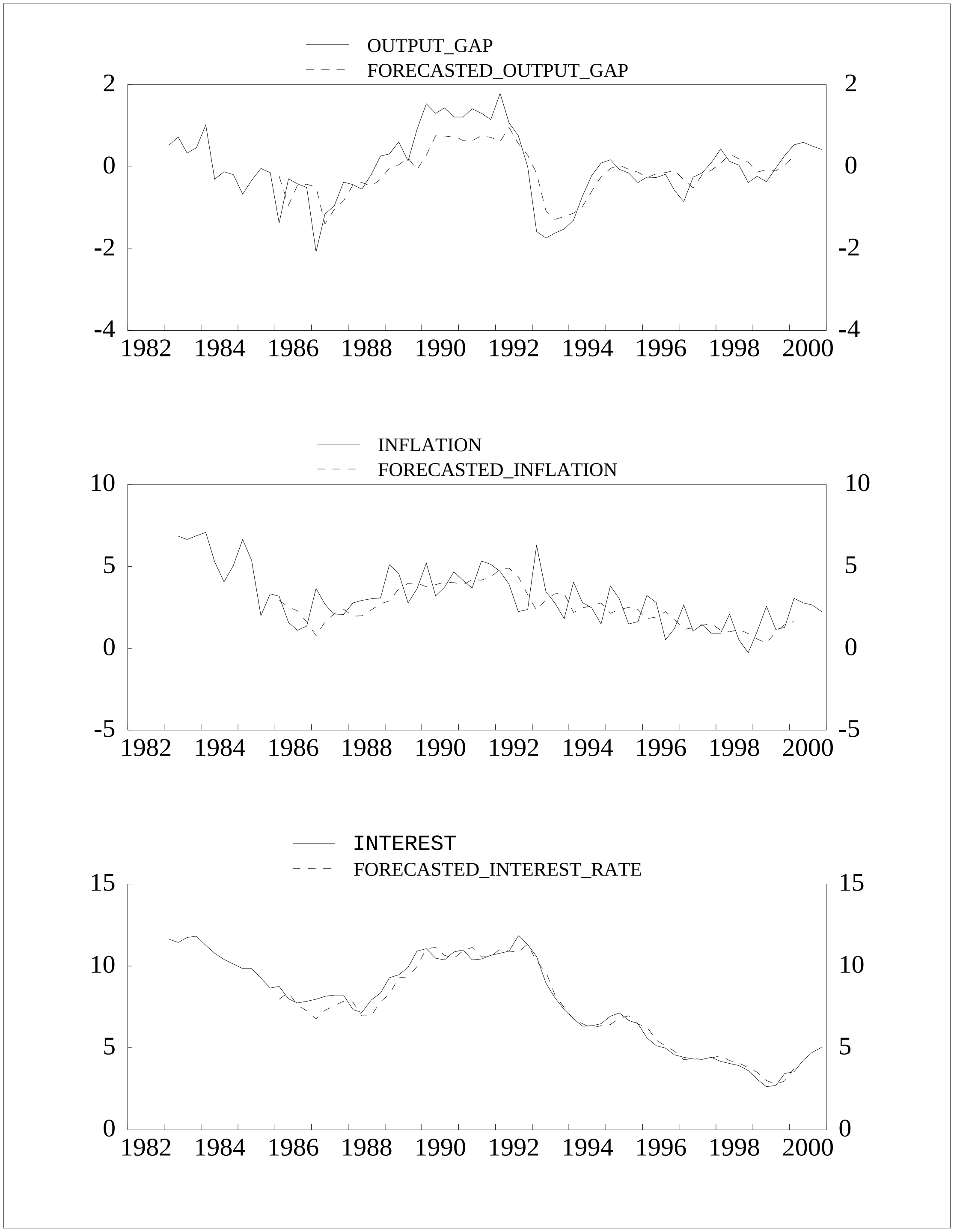
$$w_t = 100 \cdot (\ln(W_t) - \ln(W_{t-1})) \cdot 4 \text{ with } W_t = \frac{WIN_t}{LNN_t} .$$

Real GDP is $y_t = 100 \ln(YER_t)$.

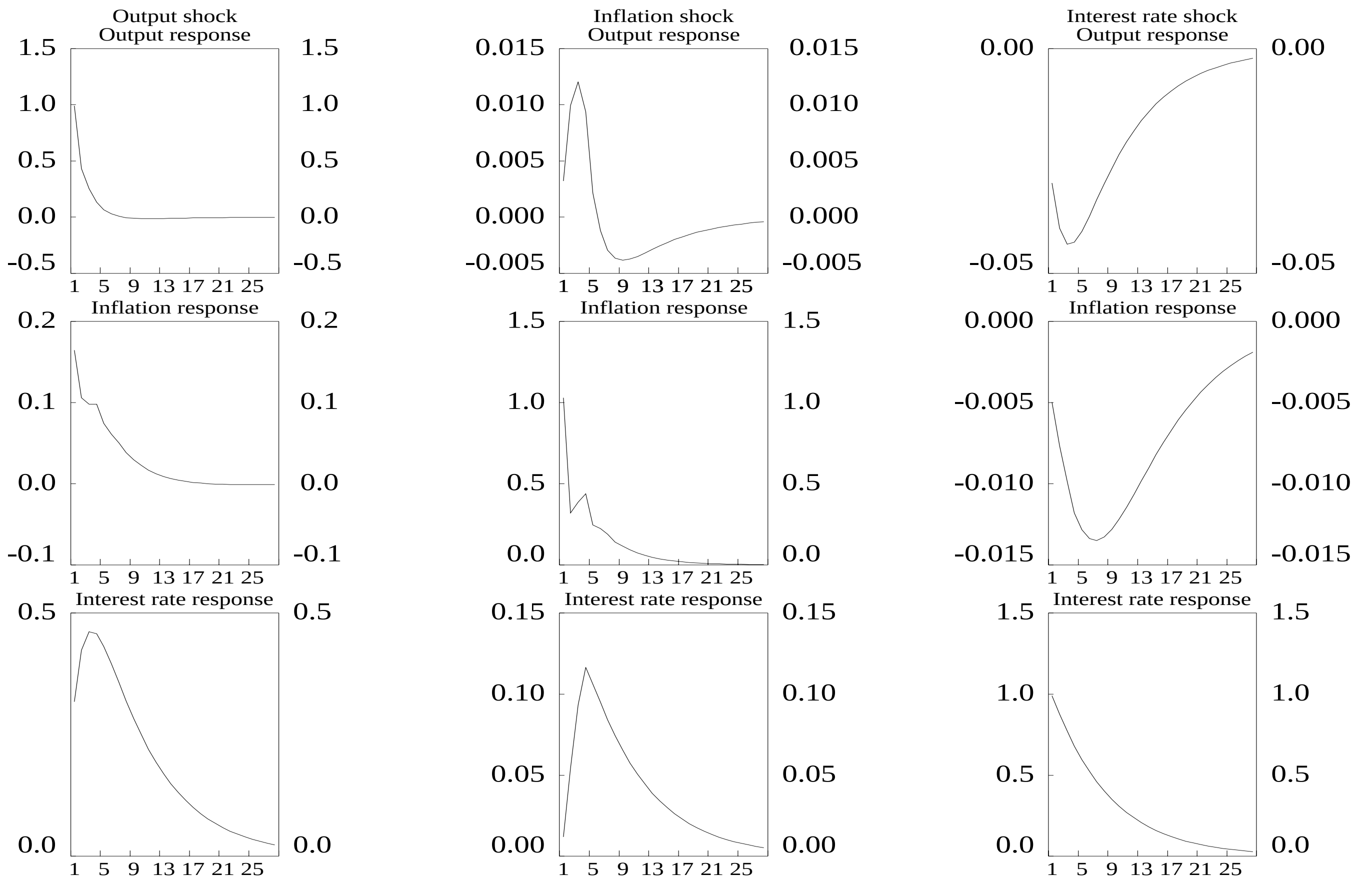
The output gap is measured as the real GDP deviating from its potential (HP filter with a lambda equal to 1600): $x_t = y_t - \bar{y}_t$.

The nominal short-run interest rate is $i_t = 100 \ln(STN_t)$.

Appendix E: Observed and Forecasted Series



Appendix F: Impulse-Response Functions



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