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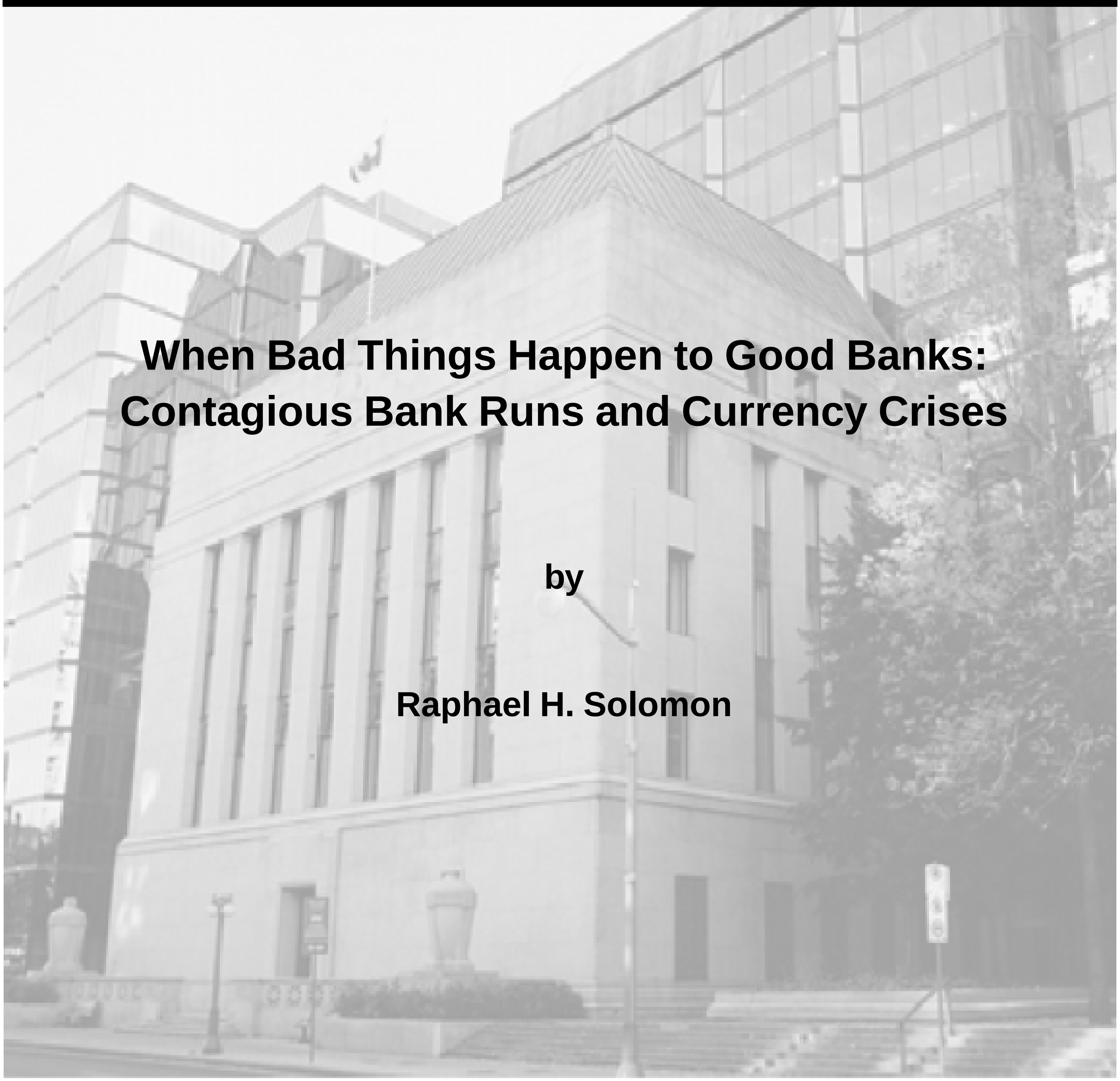

When Bad Things Happen to Good Banks: Contagious Bank Runs and Currency Crises

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Abstract

The author develops a twin crisis model featuring multiple banks. At each bank, domestic and foreign depositors play a banking game. This game has a run and a no-run equilibrium. Bank failures drain reserves in addition to those drained when foreign agents convert domestic currency to foreign. The fixed exchange rate collapses if a threshold number of banks fail. Agents observe sunspots to aid their equilibrium selection. The numerical solution matches somewhat the Turkish financial sector prior to the crisis of 2001. The Turkish exchange rate appears to have exposed the financial system to a 10 per cent risk of collapse.

JEL classification: E58, F30, G21
Bank classification: Exchange rates; Financial institutions

Résumé

L'auteur élabore un modèle des crises jumelles (crise de liquidité bancaire suivie d'une crise de change) où il y a plusieurs banques. Dans chacune d'elles, les déposants nationaux et étrangers se livrent à un jeu bancaire comportant deux équilibres : un équilibre de panique bancaire et un équilibre sans panique bancaire. Si des banques font faillite, les réserves diminuent, tout comme lorsque les non-résidents convertissent dans la devise étrangère leurs fonds en monnaie locale. Le régime de changes fixes s'effondre si le nombre de faillites bancaires dépasse un certain seuil. Les agents sont attentifs à des aléas de type « tache solaire » dans le choix de leur équilibre. La solution numérique obtenue recouvre relativement bien la situation du secteur financier turc d'avant la crise de 2001. La politique de taux de change de la Turquie semble avoir exposé le système financier du pays à un risque d'effondrement de 10 %.

Classification JEL : E58, F30, G21
Classification de la Banque : Taux de change; Institutions financières

1. Introduction

In the 1970s, many developing countries partially liberalized their financial systems by removing restrictions on deposit interest rates, reducing or eliminating reserve requirements and allowing foreign competition in the banking sector. Many of these countries kept one vestige of the previous régime: a fixed exchange rate. The combination of a fixed exchange rate with a liberalized banking sector proved particularly lethal to the financial system. By the early 1980s, many of these same countries experienced pervasive bank runs. In an attempt to stem the tide, central banks bailed out the banks that were experiencing runs, thereby weakening their foreign currency reserve positions. Speculators pounced on the weakened currencies, forcing the abandonment of the fixed exchange rate. Kaminsky and Reinhart (1999) termed this phenomenon a twin crisis, since it begins with a crisis in the banking sector and ends with a currency crisis.

The first component of a twin crisis model is a model of bank runs. Most models of bank runs follow Diamond and Dybvig's classic (1983) paper. In the literature, uncertainty and incomplete information play a significant role. I focus on the few papers that model multiple banks, since those models allow the study of bank-systemic issues and their interaction with a currency peg. But the Diamond-Dybvig multiple-bank literature is disappointing on several counts.[1] First, many of the papers fail to consider explicitly the sequential service constraint[2] (a notable exception is Smith 1991). This failure means that contracts between the banks and their depositors may not be implementable. Second, most of these papers focus on banks trading contingent claims as a way to prevent crisis. The pre-eminent paper of this type is Allen and Gale (2000). In papers with full risk-sharing and no aggregate uncertainty, it is impossible to examine the dynamics of banking crises, since crises do not occur.

The second component of a twin crisis model is a currency crisis model. Currency crisis theorists belong to one of three schools. Models of the first generation follow Krugman (1979); in these models, "bad" macroeconomic

[1] Some recent papers with multiple banks in the Diamond-Dybvig framework are: Allen and Gale (2000), Dasgupta (2001), Freixas, Parigi, and Rochet (2000), and Huang and Xu (2000).

[2] Diamond and Dybvig (1983, 408) explained that "a *sequential service constraint . . .* specifies that a bank's payoff to any [person] can depend only on the agent's place in line and not on future information about [people] behind him in line."

fundamentals cause currency crises. Models of the second generation follow Obstfeld (1986); in those models, random shifts in expectations cause currency crises. Krugman (1999) and others present models that include the financial sector, referred to collectively as third-generation models. My paper marries Krugman's (1999) third-generation approach with the multiple-bank bank-run literature, creating a new perspective on twin crises.

Why do we need another twin crisis model? Many models of twin crises (such as Chang and Velasco 2000a, b, 2001) are deterministic; in those models, every bank run leads to a currency crisis. There are two problems with such models. First, they obscure the difference between a bank run and a bank panic.[3] Second, deterministic twin crises models approximate reality poorly. Sometimes, several banks collapse but the currency peg survives, as in the Overend's crisis of 1866 (Clapham 1944) and the French–Arab crisis of 1988–90 (Gup 1998). At other times, countries experience partial banking collapses but severe currency crises, such as the six developing countries discussed by Sundararajan and Baliño (1991). A concern about determinism thus informs my model design.

I construct a model in which foreign and domestic depositors interact in an economy with a fragile banking system and a fixed exchange rate. N ex-ante identical banks offer a contract to their depositors based on when the depositors arrive, and their country of residence. This contract includes payments in domestic and foreign currency. Risk-averse domestic agents are of two types: impatient and patient. Impatient agents have an immediate need for liquidity; they withdraw from their bank as soon as they discover this need. Patient agents may defer withdrawing from the bank for one period in the hope of collecting a higher return, or they may pool themselves with the impatient agents. Foreign agents have the same choices as patient agents, but they may act differently, since their preferences are approximately risk-neutral.[4] After depositing at the bank, domestic agents learn whether they are patient or impatient. Patient agents and foreign agents then play a post-deposit subgame. The actions of patient and foreign agents at any one

[3] Bhattacharya and Thakor (1993, 26) distinguish runs and panics. "A bank run relates to an individual bank; a panic is a simultaneous run on many banks. A model of banking panics must explicitly address the contagion effects of runs. Neither Diamond-Dybvig nor Chari-Jagannathan model panics."

[4] Another way to justify the risk-neutrality of foreigners is to consider agents' risk preferences for small gambles. I assume that foreign agents deposit a small fraction of their investment portfolio in this bank and are thus risk-neutral over this small investment.

bank determine whether that bank fails. The failures of banks in the system determine whether the currency peg survives, since bank failures cause the central bank to lose foreign exchange reserves.

Sunspot variables break the deterministic link between a banking crisis and a currency crisis. Each depositor observes a "sunspot variable" particular both to their bank and to their country of residence. That is, Nature reveals a two-dimensional sunspot vector for each of the n banks, but each depositor observes exactly one of the $2n$ sunspots. The sunspot vectors are statistically dependent, allowing for the possibility of interbank "contagion." Since I model an unsophisticated banking system, banks may not trade contingent claims on each other's deposits. I make the following standard modelling assumptions. Nature assigns depositors randomly to banks. The proportion of domestic depositors at any bank with immediate liquidity needs ("impatient") is constant across banks and known to all. There is an equal measure of domestic and foreign depositors at each bank.

I calibrate the model to Turkish data, in an attempt to explain the Turkish twin crisis of 2001. In January 2000, the Governor of the Central Bank of the Republic of Turkey, Gazi Erçel, adopted a fixed exchange rate path for the lira in an attempt to control inflation. Depositors lost faith both in Turkish banks and in the lira. Thirteen months later, the severity of the foreign exchange reserve drain forced the Central Bank to float the lira (Eichengreen 2001). During this period, the deposit insurance fund seized control of 10 banks and closed 8 other banks. Fifteen months after the beginning of the float, the 10 banks administered by the deposit insurance fund had also ceased operations (Turkiye Bankalar Birligi Web site: http://www.tbb.org.tr/english/default.html). This paper argues that the Turkish crisis was not preordained.

How appropriate is a sunspot explanation for the Turkish crisis? Boyd et al. (2000, 4) comment that "banking crises are often the outcome of a bad realization in a sunspot equilibrium." They conclude that a crisis is sunspot-driven if there are no significant movements in real GDP growth, inflation, the real value of equity, or aggregate credit extension in a three- to five-year period prior to the crisis. I compute year-on-year changes in real GDP, prices, real value of equity, and real domestic credit for the period 1987 to 2003. None of these variables had extreme observations more frequently than one would expect in a Gaussian distribution.

The role of the interbank market in propagating the Turkish crisis makes a sunspot explanation somewhat more tenuous. As Daníelsson and Saltoğlu

(2003, 6) explain, Turkish banks faced margin calls on off-balance-sheet investments and met them by borrowing from other banks. Overnight borrowing became very expensive, causing banks to liquidate assets. I abstract from these interbank market difficulties for two reasons. First, it would be difficult to meld an interbank market, for which the natural period of time is one day, with the longer-term interpretation given to periods in twin crisis models. Second, ignoring the interbank market allows me to bring the sunspot aspect of the crisis into focus.

The rest of this paper has the following structure. Sections 2 and 3 set out the formal model, modified from Solomon (2003) to take multiple banks into account. Section 4 explains the statistical set-up for the sunspot variables. Section 5 defines a Nash equilibrium of the banking game. In section 6, I describe the results of the numerical model, which is calibrated to Turkish data. Section 7 concludes. Proofs are provided in the appendix.

2. The Model

2.1 Assets, currencies, and goods

There are two assets in which agents and banks may invest: world and productive. The world asset is a storage technology. One unit invested in the world asset returns one unit whenever the investment is liquidated. The productive asset yields R_1 units in period 1 or R_2 units in period 2 per unit invested in period 0. Since $0 < R_1 < 1 < R_2$, the world asset dominates the productive asset if assets are liquidated in period 1, but not if assets are liquidated in period 2.

The lira and the dollar are the two currencies of the model. A fixed exchange rate of unity initially prevails between them. The government can print liras but not dollars, since dollars are foreign currency. The government incurs a liability of one dollar for every lira it prints. The consumption good always costs one dollar.[5] This economy is so open that the composite consumption good is imported and thus priced in dollars. Similarly, since the goods and services underlying the productive asset are exported, liquidating the productive asset yields dollars.

[5] I thus examine twin crises in a zero-inflation environment. If real-world bank runs are motivated by both sunspots and fundamentals, such as inflation, one should view estimates of systemic risk (described in section 6.2) as a lower bound.

2.2 Decision-makers and their decisions

A small, open economy lasts for three periods, denoted 0, 1, and 2. There are five types of decision-makers in the model economy: domestic impatient agents, domestic patient agents, foreign agents, banks, and the government. Nature assigns each agent to a bank (without loss of generality, since all banks are ex-ante identical). I index the domestic agents at each bank along a continuum of unit measure, where the measures of impatient and patient agents are λ and $1 - \lambda$, respectively.[6] I index the foreign agents at each bank along a separate continuum of unit measure. In the aggregate, the measures of domestic impatient, domestic patient, and foreign agents are $n\lambda$, $n(1 - \lambda)$, and n, respectively.

In period 0, agents decide whether to deposit or to receive the autarchic return. If agents deposit, they choose when to withdraw a pre-specified amount – period 1 or period 2. Each bank determines the sizes of the withdrawals available to depositors – the "deposit contract." The government sets the economic environment, fixing tax rates and deciding when to bail out failing banks.

2.3 Objective functions and notation

Impatient agents value consumption only in period 1. Patient agents value consumption in period 2 and the holding of deposits during period 1. A patient agent that pretends to be impatient forgoes the holding of deposits and consumes in period 1. A patient agent that claims to be impatient receives utility as if they were an impatient agent.

Banks do not distribute consumption; they give currency to their clients upon withdrawal, who use the currency to purchase consumption. Domestic agents receive payments in dollars, because they consume the imported composite good. Foreign agents receive payments in liras, for two reasons. First, it allows them to be exposed to exchange rate risk, which is an important component in foreign investment decisions. Second, it separates the interests of the government and the bank. This specification implies that the bank does not care about a devaluation, whereas the government potentially does. It is useful to think of the currency specification as a reduced form; it allows the model to mimic certain aspects of foreign and domestic investment with-

[6] As usual in models in the Diamond and Dybvig (1983) tradition, agents do not discover their type until period 1. The nationality of each agent is public information.

out fully specifying the currency investment functions of foreign and domestic agents. One drawback of the currency specification is that domestic agents do not mind if their currency devalues; since I do not perform a post-crisis welfare analysis in this paper, this consideration diminishes in importance.

Let $c_I(j)$ denote consumption of an impatience-claiming depositor of bank j, $j = 1...n$. Analogously, let $c_P(j)$ and $m(j)$ represent, respectively, consumption of and the value of deposits held by a patience-claiming depositor of bank j. The utility function for impatience-claiming agents is[7]:

$$g\left(c_I\left(j\right)\right) = -\frac{1}{\alpha}\exp[-ac_I\left(j\right)] + \frac{1}{\alpha} + \zeta c_I\left(j\right), \ \alpha > 0, \ \zeta > 0. \tag{1}$$

The utility function for patience-claiming agents is:

$$g\left(A\left[y\left(j\right), m\left(j\right)\right]\right) = g\left(c_P\left(j\right)^{\beta} m\left(j\right)^{1-\beta}\right), \ 0 < \beta < 1. \tag{2}$$

Let $\rho_t\left(j\right)$ denote the gross lira return for foreigners withdrawing in period t. Let $\hat{\rho}_t\left(j\right)$ be foreigners' (unmodelled) income earned in **their** home country in period t. The overall utility function for foreign agents is

$$g(\rho_1\left(j\right), \rho_2\left(j\right)) = \sum_{t=1}^{2} -\tfrac{1}{\alpha}\exp[-\alpha\left(\rho_t\left(j\right) + \hat{\rho}_t\left(j\right)\right)] + \tfrac{1}{\alpha} + \zeta\left(\rho_t\left(j\right) + \hat{\rho}_t\left(j\right)\right). \tag{3}$$

For $\left(\hat{\rho}_1\left(j\right), \hat{\rho}_2\left(j\right)\right) \gg (0,0)$, foreign marginal utility from deposits held in local banks is approximately constant. Accordingly, I model foreign utility by the identity function. In addition, let $\gamma_b(j)$ be the share of the j^{th} bank's deposits invested in the productive asset.

Competitive considerations require that banks maximize the expected utility of their domestic depositors, subject to constraints, including feasibility, individual rationality, and incentive compatibility. For the banks to calculate expected utility, they must consider the possibility of bank runs. The probability of bank runs may, in turn, depend on the contract the banks offer. These issues complicate the banks' problem.

For simplicity, I do not model the government's objective function, treating tax rates as exogenous. I check the sensitivity of the exogenous parameterization to small changes in the policy variables; no major changes emerge. In particular, optimal tax rates cannot prevent a twin crisis.

[7] In Solomon (2003), I detail the properties and motivations for all utility functions.

2.4 A collection of sunspot variables

To take seriously the possibility of a bank run, a bank needs to assess the beliefs of its depositors. I introduce a collection of sunspot variables to make this task tractable. A sunspot variable reveals information unrelated to the fundamentals of the economy; agents may use this information to assist in decision-making and possibly in equilibrium selection. Duffy and Fisher (2002, 4) note that "the semantics of the language of sunspots matters; if it is not immediately clear to all individuals how a sunspot variable realization is to be interpreted, then that sunspot variable is unlikely to play any role in coordinating expectations."

In Solomon (2003), I compare the sunspot variables with interpretations of a newspaper article that announces some change in circumstances unrelated to fundamentals. Gençay and Selçuk (2001, 3) state that

> On February 19, 2001, the day before the [debt] auction, Turkish Prime Minister Bulent Ecevit stormed out of a key meeting of top political and military leaders stating a 'dispute' had arisen between himself and the country's president. He further emphasized that 'of course, this is a serious political crisis' without elaborating the future of the government or the economic program.

A resident of Turkey and a foreigner might interpret this news report differently. These differing interpretations explain why there must be at least two sunspot variables—one for domestic agents and one for foreign agents—but not bank-specific sunspots. Bank-specific sunspot variables can be justified by additional sunspot-style announcements: "bank j could be vulnerable to a run," or "bank i is sound."[8]

Let $s(i, j)$ be the sunspot variable observed by an agent of type i, depositing at bank j, where $i = d$, f (domestic or foreign), and $j = 1...n$. I discuss the correlation between different sunspot variables below.

[8] The announcements must be irrelevant to the fundamentals. Given that all banks offer the same contract in equilibrium, all banks are both sound and vulnerable to a run (simultaneously). But the combination of Ecevit's declaration with an announcement that one equilibrium seems more likely than another could be sufficient (in line with Duffy and Fisher) to induce changes in behaviour.

3. The Game

3.1 Timing of the model

The model unfolds over three periods, as described below.

Period 0:

1. Domestic agents receive an endowment, e_d units of the consumption good. Foreigners arrive with resources for investment, e_f dollars.[9]
2. The government announces the withdrawal tax rate, τ, and the deposit tax rate, $1-\eta$.
3. Banks announce the contracts, $C(c_I(j), c_P(j), \gamma_b(j), \rho_1(j), \rho_2(j))$, they will offer conditional on deposits and government policies.
4. Foreign and domestic agents deposit e_f and e_d dollars, respectively, at their banks.
5. Banks send the deposit tax to the central bank. Banks invest the rest of their deposits in the two assets according to the contracts.

Period 1:

1. Domestic agents learn their type: patient or impatient.
2. Nature draws the sunspot variables $\{s(d,j)\}_{j=1}^{n}$ and $\{s(f,j)\}_{j=1}^{n}$, revealing them to domestic and foreign agents, respectively.
3. Banks open for business. Agents of various types arrive in random order. Banks liquidate their holdings of the world asset. Agents claiming to be impatient receive, net of taxes, $(1-\tau)\,c_I(j)$ dollars, if available. Agents claiming to be patient receive utility services valued at $m(j)$ liras. If any foreign agents arrive, they receive $\rho_1(j)\,e_f$ liras, which banks borrow from the government. Banks remit withdrawal taxes.
4. If every bank serves all its depositors in queue, go to item 5. Some banks may liquidate productive assets to serve as many domestic agents as possible. The government may bail out some banks at this stage.

[9] I state all amounts in this subsection in per-capita terms.

5. Foreigners trade liras for dollars at the central bank.

Period 2:

1. Any remaining investment in the productive asset matures.
2. Banks pay $(1-\tau)\,c_P\,(j)$ dollars to any domestic agent who claimed to be patient in period 1, if available. Banks remit taxes to the government. Banks also pay $\rho_2\,(j)\,e_f$ liras to any remaining foreign agents. Banks that borrowed liras from the government repay $\rho_2\,(j)\,e_f$ dollars, if possible.
3. Foreign agents holding liras trade them for dollars at the central bank. If the quantity of liras in circulation exceeds the dollar reserves of the central bank, a currency crisis occurs.
4. The economy ends.

3.2 The contract and related variables

In period 0, banks offer contracts, $C(c_I\,(j)\,, c_P\,(j)\,, \gamma_b\,(j), \rho_1\,(j)\,, \rho_2\,(j)) \in \Re^5_+$, to which agents respond. Determining which is the optimal contract is the subject of section 5. Notice that I preclude suspension of convertibility, since the payment to domestic agents is not contingent on the history of withdrawals. That is, the bank must continue paying $(1-\tau)\,c_I$ to domestic agents until it runs out of dollars and has liquidated all assets. This restriction is effectively a "strong form" of sequential service.[10]

The value of deposits held by patient agents is not uniquely determined. I propose that the value of deposits held by patient agents at each bank be proportional to the expected dollar payments to domestic agents at that bank, conditional on there not being a run at that bank. That is, $m\,(j) = \kappa\,[\lambda c_I\,(j) + (1-\lambda)\,c_P\,(j)]$.

3.3 The rules of the game

In period 0, banks invest their deposits according to the contract. If agents do not deposit resources at their bank, domestic agents can divide their

[10] Not only does this restriction improve the tractability of the numerical model, but it also allows the comparison of contracts for different parameter values (see section 6).

endowment between the two assets. Foreigners, by contrast, may not invest directly in the productive asset. These considerations define the individual rationality constraints for domestic and foreign agents.

In period 1, all domestic and possibly some foreign agents arrive at their banks. Banks can always accommodate the demands of foreigners by borrowing liras. Agents claiming to be patient present no problems for banks, either; these agents hold deposits. By contrast, banks obtain dollars only by liquidating assets. As agents arrive, claiming to be impatient, banks liquidate assets and pay each agent $(1-\tau)\, c_I\,(j)$ dollars, remitting τc_I to the central bank as taxes.

Banks must determine the amount of the productive asset to liquidate in period 1. Let $a_d\,(j)$ be the measure of domestic agents that claim to be impatient at the j^{th} bank. Let $a_f\,(j)$ be the measure of foreign agents that arrive in period 1 at the j^{th} bank. Then $L\,(a_d\,(j))$ is the amount of the productive asset the j^{th} bank will liquidate in period 1 if $a_d\,(j)$ agents claim to be impatient. From its liquidation of the world asset, the j^{th} bank receives $(1-\gamma_b\,(j))\,[\eta e_d + e_f]$. If this amount does not suffice to pay agents claiming to be impatient, the j^{th} bank will liquidate a portion of the productive asset. The amount liquidated will be $\frac{a_d(j)c_I(j)-(1-\gamma_b(j))\left[\eta e_d+e_f\right]}{R_1}$. Finally, the j^{th} bank cannot liquidate more of the productive asset than its original investment, $\gamma_b\,(j)\,[\eta e_d + e_f]$. Thus,[11]

$$L\,(a_d\,(j)) = \min\left(\gamma_b\,(j)\,[\eta e_d + e_f]\,, \frac{a_d(j)c_I(j)-(1-\gamma_b(j))\left[\eta e_d+e_f\right]}{R_1}\right). \tag{4}$$

Some banks may liquidate all their assets while additional domestic agents remain in their queues claiming to be impatient. Whether this confluence of events will trigger a bank bailout depends on the behaviour of foreign agents. The government bails out banks only if no foreigners are present at those banks in period 1.

The results of this paper turn on this bailout assumption in a critical way; if the presence of foreigners does not affect the bailout, it is impossible to tell how foreigners will react to a domestic run. How realistic is my assumption about bailouts? Boyd et al. (2000, 13) argue that the government chooses the size of bank bailouts. They note that the government has several options, ranging from covering the losses of uninsured depositors to defaulting on

[11] In equilibrium, $L(\cdot)$ takes only two values: 0 or $\gamma_b\,(j)\,(\eta e_d + e_f)$.

its deposit insurance obligations, from recapitalizing the banking system to inflating or depreciating away the real value of its nominal debt. Each of these options may be undertaken partially, wholly, or not at all. My bailout rule is consistent with that of Boyd et al. and with some simple regression results using the dataset of Caprio and Klingebiel (1996).[12]

There are several constraints on the size of the bailout in the model. The government will never bail out a bank by giving more than it needs to give all its agent $c_I(j)$. Since a bank that needs a bailout can obtain $(\eta e_d + e_f)$ $+ (\eta e_d + e_f)(R_1\gamma_b(j) - \gamma_b(j))$ by liquidating all its assets, the typical bank bailout will be of size $a_d(j)c_I(j) - (1 - \gamma_b(j) + R_1\gamma_b(j))(\eta e_d + e_f)$. I assume that the government will not bail out the bank with an amount greater than the government collected in taxes from depositors of that bank, $\tau c_I(j)$. The bailout function is thus:

$$B(a_d(j)) = \min \begin{bmatrix} a_d(j)c_I(j) - (1 - \gamma_b(j) + R_1\gamma_b(j))(\eta e_d + e_f), \\ \tau c_I(j) \end{bmatrix}. \quad (5)$$

After a bailout has occurred, the bailed-out banks distribute $(1 - \tau)c_I(j)$ to each agent in queue, again exhausting their resources.[13] An agent not in queue in period 1 when the bailout occurs receives nothing in period 2.

Foreigners proceed to the central bank after visiting their banks. The central bank compares liras in circulation with dollars in its reserve vault. If the latter quantity exceeds the former one, foreigners exchange each lira for one dollar. If the former exceeds the latter, the exchange rate becomes the ratio of the latter to the former. In that case, the central bank exchanges liras for dollars at the new exchange rate.[14] Since investors have forced this depreciation, I call this event a currency crisis.

In period 2, domestic agents that claimed to be patient return to their banks. Some banks may have liquidated all of their assets in period 1; if

[12] Regression results (details available upon request) show a strong positive association between the cost of a bailout and the presence of a foreign debt problem. The regressions also show an insignificant but positive association between the length of the crisis and a foreign debt problem. Deposits in Turkish banks held by foreigners are foreign debt. When foreigners arrive in this model, they create a foreign debt problem. This foreign debt problem forces the government to delay bailing out the banks (beyond the duration of the model), thus increasing the cost of the eventual bailout. This interpretation links the three-period model to the reality of the regression model.

[13] The bailout policy is public information. I ignore here issues of central bank credibility.

[14] This scheme for paying foreigners satisfies a sequential service constraint for currency exchange.

so, there is nothing with which to pay returning domestic agents. If any bank has assets left over in period 2, those assets mature and yield dollars. These banks use the dollars to pay domestic agents who claim to be patient $(1-\tau)\,c_P\,(j)$, remitting $\tau c_P\,(j)$ to the government as taxes. If some banks cannot serve every domestic agent who claims to be patient, they will not change the amount each agent receives. Even if some banks cannot serve everyone in period 2, the government will not bail them out. In equilibrium, however, a bailout in period 2 is never necessary.

After all assets have matured and banks have paid all agents who claim to be patient, some dollars may remain in the bank vaults. If so, banks use them to repay their loans from the central bank. This guarantees that no bank makes profits. If foreign agents have collected liras, they convert them to dollars at the central bank. The procedure for determining the exchange rate remains the same as the one used in period 1.

4. The Sunspot Variables and Contagion

I model sunspots as Bernoulli random variables. This choice allows for a tractable form of statistical dependence: a first-order Markov chain. The notation is based on Helgert (1970) and Solomon (2003):

$Pr(s(d,0)=1)=p.$[15]
$Pr(s(d,j)=1|s(d,j-1)=0)=p_0.$
$Pr(s(d,j)=1|s(d,j-1)=1)=p_1.$
$Pr(s(f,j)=1|s(d,j)=1)=\pi_2.$
$Pr(s(f,j)=1|s(d,j)=0)=\pi_3$, where $\pi_2>\pi_3$.
$\mathbf{p}=(p,p_0,p_1,\pi_2,\pi_3).$

For each bank j, $j=1...n$, there are four possibilities (Table 1).

Table 1: Set of Possible States for Each Bank

$s(d,j)=1;\ s(f,j)=0$ [state 1]	$s(d,j)=1;\ s(f,j)=1$ [state 2]
$s(d,j)=0;\ s(f,j)=0$ [state 3]	$s(d,j)=0;\ s(f,j)=1$ [state 4]

[15] There is no 0^{th} bank. But Nature performs an initial draw of the random variable before beginning the Markov chain; this is analogous to selecting a conditional distribution.

Let n_h represent the number of occurrences of state h, $h = 1...4$ and let $\mathbf{n} \equiv (n_1, n_2, n_3, n_4)$. Refer to n as the aggregate state. The n_h must obey $0 \leq n_h \leq n$ and $\Sigma_h n_h = n$. I call the distribution of $\mathbf{n}$ Augmented-Helgert (AH). The $AH(n;p)$ distribution is the product of three distributions. Two of these are independent binomial distributions. The third is the Helgert distribution of the sum of Bernoulli variables distributed according to a Markov chain, denoted He.[16] Since there is no simple closed-form expression for $He(\cdot)$, expectations over $AH(\mathbf{n};\mathbf{p})$ need to be computed numerically. Fortunately, Helgert (1970) provides a recursion that simplifies the computations.

5. Equilibrium

I solve the game by backward induction, deriving first the equilibrium of the post-deposit subgame and then the equilibrium of the entire game, assuming subgame perfection.

5.1 Classes of contract

Diamond and Dybvig (1983, 409) show that whether the post-deposit subgame has a run equilibrium depends on the contract the bank offers. Any bank can offer a contract that prevents runs or a contract that allows runs. Contracts that prevent runs are referred to as contracts of the No-Run Class and contracts that allow runs are referred to as contracts of the Run Class.[17]

5.2 Equilibrium of the subgame

The post-deposit subgame consists of Nature's "selecting" the realization of the sunspot vector, followed by the choices made by the two types

[16] Let $Bin(z; z_1, z_2)$ denote the probability that the realization of a binomially distributed random variable is z, assuming z_1 draws and a per-draw probability of z_2. Let $He(z; z_1, z_2, z_3, z_4)$ denote the probability that the realization of a Helgert-distributed random variable is z, assuming z_1 draws, initial probability z_2, and transition probabilities z_3 and z_4. Then, $AH(\mathbf{n};\mathbf{p}) = Bin(n_1; n_1 + n_2, 1 - \pi_2) \cdot Bin(n_3; n_3 + n_4, 1 - \pi_3) \cdot He(n_1 + n_2; n, p, p_0, p_1)$.

[17] I consider only equilibria of the Run Class, since those contracts can lead to a twin crisis. I discuss equilibrium for contracts of the No-Run Class in Solomon (2003).

of agents.[18] A strategy indicates how agents respond to the sunspot variable they see. I consider only type-symmetric pure strategies.[19] A domestic strategy can be written formally as $\sigma_d : s(d,j) \longrightarrow \{CI, CP\}$, where CI means claim to be impatient and CP means claim to be patient. A foreign strategy can be written as $\sigma_f : s(f,j) \longrightarrow \{A1, A2\}$, where $A1$ means arrive in period 1 and $A2$ means arrive in period 2. Each type of agent has four possible strategies; these strategies are listed in Table 2.

Table 2: Strategy Descriptions

Strategy	$s(d,j)=0$	$s(d,j)=1$	Strategy	$s(f,j)=0$	$s(f,j)=1$
$\sigma_{d,1}$	CP	CI	$\sigma_{f,1}$	$A2$	$A1$
$\sigma_{d,2}$	CI	CI	$\sigma_{f,2}$	$A2$	$A2$
$\sigma_{d,3}$	CP	CP	$\sigma_{f,3}$	$A1$	$A1$
$\sigma_{d,4}$	CI	CP	$\sigma_{f,4}$	$A2$	$A1$

Let $\Sigma_d = \{\sigma_{d,1}, \sigma_{d,2}, \sigma_{d,3}, \sigma_{d,4}\}$ and $\Sigma_f = \{\sigma_{f,1}, \sigma_{f,2}, \sigma_{f,3}, \sigma_{f,4}\}$ be the strategy sets for domestic and foreign agents with typical elements σ_d and σ_f, respectively. Domestic and foreign agents have expected payoff functions of $E_s U_d$ and $E_{AH} U_f$, respectively. These functions depend on the strategy played by all other agents of the same nationality, the strategy played by all agents of the opposite nationality, the strategy played by that individual agent, and the contract offered by the banks.

Definition 1 *A pair of strategies (σ_d, σ_f) is a type-symmetric Nash equilibrium of the post-deposit subgame if two conditions hold:*

1. For all patient agents in $[0, 1-\lambda]$ and for all $\tilde{\sigma}_d \in \Sigma_d$, $E_s U_d(\sigma_d, \sigma_f, \sigma_d, \cdot) \geq E_s U_d(\sigma_d, \sigma_f, \tilde{\sigma}_d, \cdot)$[20]

2. For all foreign agents in $[0,1]$ and for all $\tilde{\sigma}_f \in \Sigma_f$, $E_{AH} U_f(\sigma_d, \sigma_f, \sigma_f, \cdot) \geq E_{AH} U_f(\sigma_d, \sigma_f, \tilde{\sigma}_f, \cdot)$

[18] Only patient and foreign agents play this subgame. Impatient agents always report their type honestly.

[19] By type-symmetric strategies, I mean that each type of agent plays a strategy that may differ across banks only insofar as the realizations of the sunspot vector differ.

[20] The suppressed arguments in $E_s U_d$ and $E_{AH} U_f$ stand for the contract vector $C(c_I, c_P, \gamma_b, \rho_1, \rho_2)$.

Even though it is possible to compute $E_s U_d$ and $E_s U_f$ for each of the 16 strategy pairs and all the possible deviations from them, I focus on a particular equilibrium: $(\sigma_{d,1}, \sigma_{f,2})$. The utility calculations are in the appendix. It is possible to reduce these utility comparisons to three simple conditions, as explained in the following two theorems.

Theorem 1 *Regardless of domestic actions, if ρ_1 is 0 (or, by continuity, a small positive), it is optimal for foreign agents to arrive in period 2.*

Proof. See the appendix. ■

Theorem 2 *If three regularity conditions hold, domestic depositors will rationally follow the sunspot signals, running only when they see a bad signal at their bank, $s(d,j) = 1$. Formally, if $A(c_P, m) > c_I$, $(1 - \gamma_b + R_1\gamma_b)(\eta e_d + e_f) \leq c_I$, and $\lambda c_I + (1-\lambda) c_P + \rho_2 e_f = (1 + R_2\gamma_b - \gamma_b)(\eta e_d + e_f)$, a domestic depositor at bank j will run on their bank only when $s(d,j) = 1$.*

Proof. See the appendix. ■

5.3 Aggregation of the equilibrium at the bank level

The banks pay less attention to the actions of their depositors individually than to the cumulative effect of their actions. Refer to $(a_d(j), a_f(j))$ as the bank-level strategy aggregator.

Definition 2 *A domestic bank run occurs at the j^{th} bank when $a_d(j) = 1$*

Definition 3 *A foreign bank run occurs at the j^{th} bank when $a_f(j) = 1$.*

Definition 4 *A currency crisis occurs when the central bank is forced to devalue the lira, because the demand for dollars exceeds the supply of dollars in its reserves.*

By Theorems 1 and 2, for some parameter values, a Nash equilibrium of the subgame is $(\sigma_{d,1}, \sigma_{f,2})$; the bank-level Nash strategy aggregator corresponding to this Nash equilibrium is

$(1,0)$, when $s(d,j) = 1$ and $s(f,j) = 0$, [state 1]
$(1,0)$, when $s(d,j) = 1$ and $s(f,j) = 1$, [state 2]
$(\lambda,0)$, when $s(d,j) = 0$ and $s(f,j) = 0$, [state 3]

$(\lambda, 0)$, when $s(d, j) = 0$ and $s(f, j) = 1$. [state 4]

The aggregate state is a random vector $(n_1, 0, n - n_1, 0)$ distributed according to a conditional $AH(\mathbf{n}; \mathbf{p})$ distribution. Note that, for this equilibrium, sunspots matter, although only for domestic agents.[21] In some situations, domestic agents receive higher expected utility by running on the bank; in other situations, they receive higher expected utility by not running.

5.4 Equilibrium of the game: individual rationality

I assume foreigners may not invest in the productive asset except through a bank; thus, their per-dollar gross autarchic return is 1. Since foreigners are approximately risk-neutral, they deposit so long as the expected return to depositing equals e_f. Domestic agents can split their endowment between the two assets. Let γ_{aut} be the fraction of a typical domestic agent's endowment invested in the productive asset. In autarchy, domestic agents earn $r_{aut}(\gamma_{aut}) = \lambda g(e_d[R_1\gamma_{aut} + 1 - \gamma_{aut}]) + (1 - \lambda) g(e_d[R_2\gamma_{aut} + 1 - \gamma_{aut}])$. There is a unique $\gamma^*_{aut} \in [0, 1]$ that maximizes this expression.[22] For a contract to be admissible, expected per-capita domestic utility must equal or exceed $r_{aut}(\gamma^*_{aut})$ and expected per-capita foreign utility must equal e_f.

5.5 Equilibrium of the game

To solve the game, the banks maximize the utility of their domestic depositors. In so doing, they must take into account not only the behaviour of agents during the post-deposit subgame, but also whether agents would rationally deposit at their banks. These considerations require that the banks constrain their maximization with incentive-compatibility and individual rationality constraints. In addition, since the banks know that runs occur in the post-deposit subgame with positive probability, they must account for this possibility when computing expected utility. Let $U^*_d$and U^*_f denote the values of U_d and U_f, respectively, when the Nash equilibrium $(\sigma_{d,1}, \sigma_{f,2})$ is played during the post-deposit subgame.

One may simplify considerably the problem of determining each bank's optimal contract. I search for a symmetric equilibrium in which each bank

[21] There is another equilibrium of the subgame, one in which patient agents honestly report their type regardless of the sunspots. As is common in the sunspots literature, I assume that if an equilibrium exists where sunspots matter, that equilibrium is selected.

[22] In Solomon (2003), I prove that γ^*_{aut} exists and is unique.

offers the same contract. If there are no profitable deviations from the symmetric contract, that contract is a Nash equilibrium of the game.

Suppose $n-1$ banks offer the contract $C = (c_I, c_P, \gamma_b, \rho_1, \rho_2)$ and one bank offers the contract $\hat{C} = (\hat{c}_I, \hat{c}_P, \hat{\gamma}_b, \hat{\rho}_1, \hat{\rho}_2)$. I now show that $\hat{\rho}_2 = \rho_2$ in equilibrium. If $\hat{\rho}_2 < \rho_2$, then foreign depositors receive fewer liras from the deviating bank than from any other bank. The rules for currency conversion of the model imply that the dollar value of the liras received from the deviating bank is also smaller than the analogous value for other banks. If $\hat{\rho}_2 < \rho_2$, foreigners do not deposit at the deviating bank. Since $\rho_2 \le R_2$, the presence of foreign deposits at any bank does not decrease the expected domestic utility. Accordingly, $E_s U_d^* \left(\hat{C}\right) \le E_s U_d^* (C)$. On the other hand, if $\hat{\rho}_2 > \rho_2$, domestic depositors at the deviating bank lose utility because foreigners collect some payments otherwise paid to domestic agents. In this case, $E_s U_d^* \left(\hat{C}\right) < E_s U_d^* (C)$. Since the expected utility of domestic agents is smaller at the deviating bank, no bank will deviate with respect to ρ_2. That is, $\hat{\rho}_2 = \rho_2$ in equilibrium.

The rules of the game largely insulate individual banks from one another. Only ρ_2 links the banks, as explained above. If ρ_2 is identical across banks, the rest of the contracts are independent of each other. Independence of the contracts holds despite the possibility of bailouts. Since each bank's bailout consists of a rebate of the taxes paid by its depositors, a bailout of every bank in the system is feasible. Furthermore, if $\hat{c}_I$ and $\hat{c}_P$ differ from c_I and c_P, the size of the potential bailout of the deviating bank will differ from the size of the potential bailouts of other banks, but it will not affect the utility of domestic depositors at other banks. Thus, one may consider the decision problem for a representative bank; the contract chosen by the representative bank is chosen optimally by all banks.

Formally, the banks solve this maximization problem as follows:

$$\max_{(c_I, c_P, \gamma_b, \rho_1, \rho_2) \in \Re_+^5} E_s U_d^*, \text{ s.t.}, \tag{6}$$

$$\begin{aligned} \lambda c_I &= (1-\gamma_b)(\eta e_d + e_f), & (7)\\ E_s U_d^* &\ge r_{aut}(\gamma_{aut}^*), & (8)\\ E_{AH} U_f^* &= n e_f, & (9)\\ A(c_P, m) &\ge c_I, & (10)\\ \lambda c_I + (1-\lambda) c_P + \rho_2 e_f &= [1 - \gamma_b + R_2 \gamma_b](\eta e_d + e_f). & (11) \end{aligned}$$

Equation (7) guarantees that, if there is no bank run, the bank need not liquidate any of the productive asset. Inequality (8) and equation (9) are the individual rationality constraints; they guarantee that foreign and domestic agents will deposit their resources at the bank, eschewing autarchy. Inequality (10) is the domestic incentive-compatibility constraint. Equation (11) is the zero-profit constraint for each bank.

As a technical matter, the solution to this problem might not be an equilibrium of the game. The solution to this problem is the optimal contract of the Run Class. The optimal contract of the No-Run Class might yield higher utility, measured with a utility function that does not take into account the possibility of runs. The final step in determining that the solution to the maximization problem of this section is indeed the equilibrium of the game requires this comparison of expected utilities across the optimal contracts of both the Run and No-Run classes.

6. Calibration and Results

The theoretical model described above does not admit a closed-form solution. In particular, the fact that it is possible for banks to choose either a contract that allows runs or one that prevents runs makes the banks' objective functions non-differentiable. There are two advantages to calibrating the model: it enables one to find the optimal contract, and it permits one to comment on Turkish policy.

6.1 Calibration of parameters

Table 3: Calibrated Parameter Values for the Empirical Model

Parameter	Value	Parameter	Value	Parameter	Value
e_d	10	π_1	0.1	ζ	10^{-10}
e_f	10	π_2	0.46	R_1	0.3
η	0.94	π_3	0.29	R_2	1.7
α	4	β	0.98	τ	0.145

Table 3 lists the values of the parameters used in the numerical solution of the model. I calibrate the parameters e_d, e_f, η, R_2, and τ to match Turkish data. In particular, I use data on Turkish tax rates, required reserves, bank deposits, and stock returns. I calibrate the parameters α, β, ζ, and R_1 using some studies based on other countries, since no specific Turkish studies are available. I also use these values in the numerical solution in Solomon (2003); I refer the interested reader there for further discussion.

I set n to 81; this matches the number of banks in Turkey at the beginning of the fixed exchange rate period. The parameter λ (the fraction of impatient agents) does not have a real-world analogue. I solve the model for λ from 0.01 to 0.48.[23] I take π_2 and π_3 from Kaminsky and Reinhart's (1999) seminal article on twin crises. Kaminsky and Reinhart also estimate the unconditional probability of a bank run: $\hat{\pi}_1 = 0.1$. Thus, I match the unconditional moment:

$$E_{AH}\left[\frac{n_1+n_2}{n}\right] = 0.1. \tag{12}$$

In Turkey, about 20 per cent of the banks failed during the crisis. One way to interpret this deviation from the Kaminsky and Reinhart estimate is that Nature chose a "bad" distribution. Accordingly, I match the conditional moment:

$$E_{AH}\left[\frac{n_1+n_2}{n}\,\middle|\, s\left(d,0\right)=1\right] = 0.2. \tag{13}$$

Restrictions (12) and (13) together require $p_0 = 0.01$ and $p_1 \in [0.91, 0.93]$. The "initial" parameter p ranges between 0.01 and 0.13.

6.2 Systemic risk

The challenge for any model of twin crisis is to explain why some banks fail and others do not; further, such a model must determine the threshold where banking sector problems become sufficiently severe to threaten the currency. A model of twin crisis must attempt to quantify "systemic risk."[24] What was the extent of systemic risk in Turkey in January 2000? Did the fixed exchange rate path augment systemic risk in Turkey? Davis (1995) defines systemic risk generally as

[23] Values of λ greater than 0.48 give rise to contracts of the No-Run Class.

[24] For a review of the systemic risk literature, see Davis (1995) and de Bandt and Hartmann (2000).

> a disturbance in financial markets which entails unanticipated changes in prices and quantities in credit or asset markets, which lead to a danger of failure of financial firms and which in turn threatens to spread so as to disrupt the payments mechanism and capacity of the financial system to allocate capital.

González-Hermosillo (1996) argues that economists should model systemic risk; this paper takes up her challenge. In the banking context, she defines systemic risk as the joint probability of a large number of bank failures. Since this paper focuses on twin crises, I define systemic risk as the joint probability of enough bank failures such that there is a currency crisis. In other words, systemic risk is the probability of a twin crisis.

Solving this model numerically yields a numerical estimate of systemic risk. The magnitude of systemic risk varies with λ, p, p_0, and p_1. Decision-makers in the model economy know these four parameters; analysts of the Turkish banking system do not. Varying λ and $\mathbf{p}$ yields estimates of systemic risk between 7.5 per cent and 14.9 per cent.

These values compare favourably with other studies of systemic risk. Mizrach (1996) notes that the risk of devaluing the French franc averaged 14.72 per cent in the five days prior to the realignment on 12 January 1987. Glick and Hutchison's (2001) study of 90 countries computed systemic risk of 20 per cent (on average) for the period 1975–97.

Let the threshold number of banks to fail be n^*. Both $\mathbf{p}$ and λ affect n^*, which ranges between 21 and 25 banks. For fixed $\mathbf{p}$, n^* is a "step" function of λ, since n^* must be an integer. The values of n^* are realistic. In Turkey, 18 banks closed and several other banks were recapitalized (an option not present in my model). This aspect of the model corresponds closely to Turkish experience.

6.3 Optimal contracts

Turkish and foreign data determine 13 of the parameters completely and 3 of the parameters partially. Only the fraction of impatient agents, λ, is free. From the bank's perspective, the optimal contract is a function of the 17 parameters of the economy. Table 4 shows a sample calibration for the free and semi-free parameters. Table 5 displays the results of that calibration.

Table 4: Free and Semi-Free Parameters

Parameter	Value
λ	0.2
p	0.01
p_0	0.01
p_1	0.92

Table 5: Part of the Optimal Contract

Solution	Value
c_I	8.13
c_P	24.99
γ_b	0.92
ρ_2	1.02

I collapse the optimal contract to a scalar function of a single parameter: λ.[25] In what follows, I detail the properties of this function, which I denote $c_I^* (\lambda)$.

A graph of $c_I^* (\lambda)$ (available upon request) shows that c_I^* is an increasing, continuous, convex function of λ. Indeed, a quadratic polynomial fits $c^* (\lambda)$ with R^2 greater than 0.99. To obtain an intuitive explanation of the shape of c_I^*, consider the size of a bank run. Since λ is the proportion of impatient agents in the economy, $1 - \lambda$ is the proportion of domestic agents who may run on their bank. As λ increases, the size of a bank run decreases. The banks can afford to offer more generous terms to impatient agents when the size of a potential run decreases.

To obtain a second explanation for the shape of c_I^*, examine the role of taxation in the model. An increase in c_I and λ increases tax revenue. Tax revenue serves two purposes: as bailouts and as payments to foreigners converting liras to dollars. A simultaneous increase in c_I and λ increases both domestic and foreign utility, so such an increase must be incentive-compatible if it is feasible. Note that $c_I^* (\lambda)$ satisfies the property that all depositors can receive $(1 - \tau) c_I$ during a run, if payments from bailouts are included.[26] At

[25] Details of how I collapse the state-space are provided in the appendix.

[26] Since the bailout function depends on a_d, this does not mean that a bank run ceases to be an equilibrium of the post-deposit subgame for these contracts.

banks at which a run occurs, domestic depositors receive a higher payment. At banks without runs, domestic depositors receive better insurance against the "risk" of being impatient.

7. Conclusion

I have described a theoretical model of twin crisis with contagion in the banking system. The model explains how shifting public opinion can cause a crisis in an apparently healthy banking system, in turn leading to a currency crisis.

The calibrated model describes the Turkish crisis. Turkish residents, responding to "sunspot information," withdrew deposits from some banks. On the heels of these withdrawals, foreign investors liquidated lira-denominated investments, draining central bank reserves. The Turkish government behaved mechanically. It bailed out some banks (using the deposit insurance fund) and paid dollars to foreigners holding liras, maintaining the fixed exchange rate as long as possible.

As measured in the model, Turkish government policy exposed the financial system to systemic risk of about 10 per cent. This is a large risk; policy-makers should calculate the expected benefits of the policy's succeeding and the expected costs of the policy's failing, and compare these with the costs and benefits of other policies that expose the financial system to different levels of systemic risk. An assessment of these costs and benefits falls outside the scope of the model.

The model's implications for government policy differ from those in the rest of the banking literature. Many researchers note that banking systems are inherently fragile because they are susceptible to bank runs. Diamond and Dybvig (1983) suggest that the government could suspend convertibility to prevent bank runs. Indeed, optimal suspension schemes are one focus of the literature that has followed Diamond and Dybvig. But in my model, the government may not suspend convertibility. Anecdotal evidence from the 2001 crisis in Argentina suggests that suspension of convertibility is not a viable option for policy-makers in the middle of a twin crisis. Although the Argentine government suspended convertibility of deposits to cash in December 2001, a federal court ruled in September 2002 that this decree was unconstitutional (La Nacion, 13 September 2002). The inability of the government to suspend convertibility prompts the government to consider other policy options to reduce the fragility of the system. These considerations merit future research.

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Appendix

A.1 Proofs of Theorems in the Main Text

Proof of Theorem 1. Assume that domestic agents run on some banks according to the sunspot signals. Let Res_t be the reserves at the central bank's disposal in period t, $t = 1, 2$. Thus, $Res_1 = n(1-\eta)e_d + (n_3+n_4)\tau\lambda c_I$ and $Res_2 = n(1-\eta)e_d + (n_3+n_4)[\tau\lambda c_I + \tau(1-\lambda)c_P + \rho_2 I]$. Here, n_3 and n_4 are the third and fourth components of $\mathbf{n}$ that are distributed $AH(\mathbf{n};\mathbf{p})$. A foreign agent arriving in period t receives $R_{f,t} = \min\left[\frac{Res_t}{n}, \rho_t e_f\right]$. I need to show that there exists a value of ρ_1 such that $R_{f,2} \geq R_{f,1}$. Let $\rho_1 = 0$. Then, $R_{f,2} > 0$ and $R_{f,1} = 0$. This proof also works in the two boundary cases: no bank runs $(n_3 + n_4 = n)$ and runs on every bank $n_3 = n_4 = 0$. ■

Proof of Theorem 2. This is a modification of the case discussed by Diamond and Dybvig (1983). By Theorem 2, I can ignore the actions of foreigners and focus only on patient agents. Furthermore, since each bank chooses the same contract in equilibrium, I can examine the actions of patient agents at one bank. Suppose that $s(d,j) = 0$, and that no patient agents run on the j^{th} bank. If a single patient agent were to run, that agent would receive c_I dollars or utility of $g(c_I)$. By remaining with the group, the agent receives c_P dollars in period 2 with certainty, as well as the utility services of deposits of m. Since $A(c_P, m) > c_I$, and since g is a strictly positive function, $g(A(c_P,m)) > g(c_I)$; it is not profitable to deviate. Suppose now that $s(d,j) = 1$, and that all patient agents run on the bank. If a single patient agent were to run, that agent would receive 0 with certainty, since $(1-\gamma_b+R_1\gamma_b)(\eta e_d + e_f) \leq c_I$ and the bailout function depends on a_d. If the agent were to run, the agent would receive c_I, with probability $\min\left[1, \frac{(1-\gamma_b+R_1\gamma_b)(\eta e_d+e_f)}{(1-\tau)c_I}\right]$ and 0 with the complementary probability. Since $g(0) = 0$, there is no profitable deviation in this case, either. ■

A.2 Utility Calculations

$$
\begin{aligned}
E_s U_d(\sigma_{d,1}, \sigma_{f,2}, \sigma_{d,1}, \cdot) &= \pi_1 U_{d,1} + (1-\pi_1) U_{d,2} \\
E_s U_d(\sigma_{d,1}, \sigma_{f,2}, \sigma_{d,2}, \cdot) &= g((1-\tau)c_I) \\
E_s U_d(\sigma_{d,1}, \sigma_{f,2}, \sigma_{d,3}, \cdot) &= (1-\pi_1) U_{d,2} \\
E_s U_d(\sigma_{d,1}, \sigma_{f,2}, \sigma_{d,4}, \cdot) &= (1-\pi_1) g((1-\tau)c_I)
\end{aligned}
$$

$$E_{AH}U_f\left(\sigma_{d,1},\sigma_{f,2},\sigma_{f,1},\cdot\right)=\left[1-\pi_1-\pi_3+\pi_1\pi_2+\pi_1\pi_3\right]U_{f,1}$$
$$+\left[\pi_1+\pi_3-\pi_1\pi_2-\pi_1\pi_3\right]U_{f,2}$$
$$E_{AH}U_f\left(\sigma_{d,1},\sigma_{f,2},\sigma_{f,2},\cdot\right)=U_{f,2}$$
$$E_{AH}U_f\left(\sigma_{d,1},\sigma_{f,2},\sigma_{f,3},\cdot\right)=U_{f,1}$$
$$E_{AH}U_f\left(\sigma_{d,1},\sigma_{f,2},\sigma_{f,4},\cdot\right)=\left[1-\pi_1-\pi_3+\pi_1\pi_2+\pi_1\pi_3\right]U_{f,2}$$
$$+\left[\pi_1+\pi_3-\pi_1\pi_2-\pi_1\pi_3\right]U_{f,1}, \text{ where}$$
$$U_{d,1}=\left(\min\left[1,\frac{\left[1-\gamma_b+R_1\gamma_b\right]\left(\eta e_d+e_f\right)}{(1-\tau)c_I}\right]\right)g\left((1-\tau)c_I\right)$$
$$U_{d,2}=\lambda g\left((1-\tau)c_I\right)+(1-\lambda)g\left(A\left[(1-\tau)c_P,m\right]\right)$$
$$U_{f,1}=\min\left[n\rho_1e_f, n(1-\eta)e_d+(n_2+n_3+n_4)\tau\lambda c_I\right]$$
$$U_{f,2}=\min\left[\begin{array}{c} n\rho_2e_f, n(1-\eta)e_d+(n_3+n_4)\left[\tau(\lambda c_I+(1-\lambda)c_P)+\rho_2e_f\right] \\ +(n_1+n_2)\left[(1-\gamma_b+R_1\gamma_b)(\eta e_d+e_f)+(\tau-1)c_I\right]\end{array}\right].$$

The expressions for $U_{d,t}$ and $U_{f,t}$ for $t=1{,}2$ follow from the rules of the game. For example, at a bank where a run occurs, every domestic agent demands c_I. Those whom the bank serves receive utility of $g((1-\tau)c_I)$, since τc_I is paid in taxes. The multiplicative term preceding $g\left((1-\tau)c_I\right)$ in $U_{d,1}$ represents the fraction of domestic agents that can be served. The other three expressions have similar interpretations.

Table A1: Latin Symbols 1

Symbol	Meaning
A	Part of the utility function of the patient
$A1$	Strategy: arrive in period 1
$A2$	Strategy: arrive in period 2
AH	Augmented-Helgert distribution
a_d	Measure of domestic agents claiming to be impatient
a_f	Measure of foreign agents arriving in period 1
B	Bailout function
Bin	Binomial distribution
C	The contract
CI	Strategy: claim to be impatient
CP	Strategy: claim to be patient
c_I	Payment promised to those claiming to be impatient
c_P	Payment promised to those claiming to be patient
e_d	Domestic endowment
e_f	Foreign investment

Table A2: Latin Symbols 2

Symbol	Meaning
g	Part of the domestic utility function
$Helg.$	Helgert distribution for sum of Bernoulli-Markov variables
L	Liquidation function
m	Deposit money balances
n	Number of banks
n^*	Critical number of banks to fail
n_1	Number of banks whose sunspot vector is (1,0)
n_2	Number of banks whose sunspot vector is (1,1)
n_3	Number of banks whose sunspot vector is (0,0)
n_4	Number of banks whose sunspot vector is (0,1)
p	Initial probability for the Markov chain
p_0	Markov transition probability: state 1 to state 0
p_1	Markov transition probability: state 1 to state 1
PRA	Temporary variable used in appendix proofs
PRB	Temporary variable used in appendix proofs
Q	$\mathrm{E}_{AH}\left[n - n_1 - n_2\right]$
R_1	Return on the productive asset by period 1
R_2	Return on the productive asset by period 2
r_{aut}	Autarchic return function
$s\left(d, j\right)$	Domestic sunspot at j^{th} bank
$s\left(f, j\right)$	Foreign sunspot at j^{th} bank
U_d	Domestic utility (generic)
$U_{d,k}$	K^{th} temporary calculation for domestic utility
U_d^*	Domestic utility when Nash equilibrium strategy is played
U_f	Foreign utility (generic)
$U_{f,k}$	K^{th} temporary calculation for foreign utility
U_f^*	Foreign utility when Nash equilibrium strategy is played

Table A3: Greek Symbols

Symbol	Meaning
α	Curvature parameter from the utility function
β	Elasticity of consumption
γ_{aut}	Fraction invested in the productive asset in autarchy
γ_{aut}^*	Optimal γ_{aut}
γ_b	Fraction invested in the productive asset by the bank
ε	Temporary variable used in appendix proofs
η	Reserve requirement
θ	Temporary variable used in appendix proofs
κ	Deposits parameter
λ	Fraction of impatient agents
π_1	Unconditional probability of $s\left(d,j\right)=1$
π_2	Probability of $s(f,j)=1$ conditional on $s(d,j)=1$
π_3	Probability of $s(f,j)=1$ conditional on $s(d,j)=0$
ρ_1	Return to foreign deposits, 1 period
ρ_2	Return to foreign deposits, 2 period
Σ_d	Domestic strategy set
Σ_f	Foreign strategy set
σ_d	Typical domestic strategy
σ_f	Typical foreign strategy
$\sigma_{d,i}$	Domestic strategy “i"
$\sigma_{f,i}$	Foreign strategy “i"
τ	Taxation rate for domestic deposits
χ	Temporary variable used in appendix proofs
ψ	Temporary variable used in appendix proofs

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