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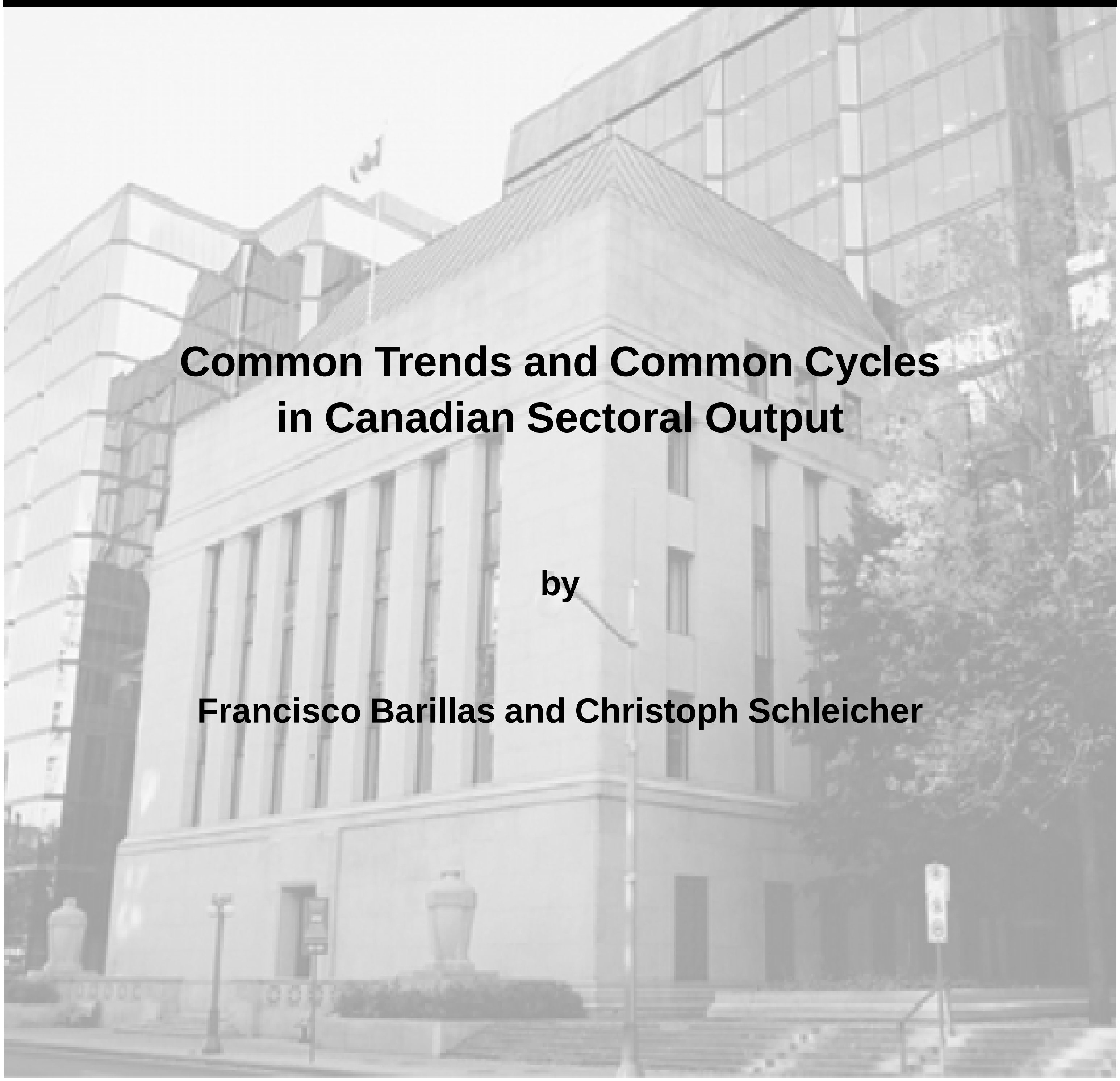

Common Trends and Common Cycles in Canadian Sectoral Output

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Abstract

The authors examine evidence of long- and short-run co-movement in Canadian sectoral output data. Their framework builds on a vector-error-correction representation that allows them to test for and compute full-information maximum-likelihood estimates of models with codependent cycle restrictions. They find that the seven sectors under consideration contain five common trends and five codependent cycles and use their estimates to obtain a multivariate Beveridge-Nelson decomposition to isolate and compare the common components. A forecast error variance decomposition indicates that some sectors, such as manufacturing and construction, are subject to persistent transitory shocks, whereas other sectors, such as financial services, are not. The authors also find that imposing common feature restrictions leads to a non-trivial gain in the ability to forecast both aggregate and sectoral output. Among the main conclusions is that manufacturing, construction, and the primary sector are the most important sources of business cycle fluctuations for the Canadian economy.

JEL classification: C15, C22, C32, E32
Bank classification: Business fluctuations and cycles; Econometric and statistical methods

Résumé

Les auteurs cherchent à établir le degré de covariation à court et à long terme dans les chiffres sectoriels de la production au Canada. Leur cadre d'analyse s'appuie sur un modèle vectoriel à correction d'erreurs assorti de contraintes de codépendance des cycles, qu'ils estiment et testent au moyen de la méthode du maximum de vraisemblance à information complète. Ils constatent que les sept secteurs considérés présentent cinq tendances communes et autant de cycles codépendants; à partir de leurs estimations, ils calculent une décomposition de Beveridge-Nelson multivariée en vue d'isoler les composantes communes et de les comparer. Une décomposition de la variance des erreurs de prévision révèle que certains secteurs, comme la fabrication et la construction, sont soumis à des chocs transitoires dont l'effet est persistant, alors que d'autres, comme les services financiers, ne le sont pas. Les auteurs observent par ailleurs que l'imposition de contraintes en matière de caractéristiques communes améliore de façon tangible la capacité de prévoir la production au niveau tant global que sectoriel. L'une de leurs principales conclusions est que le secteur primaire, le secteur de la fabrication et celui de la construction contribuent dans une large mesure aux fluctuations cycliques de l'économie canadienne.

Classification JEL : C15, C22, C32, E32
Classification de la Banque : Cycles et fluctuations économiques; Méthodes économétriques et statistiques

1 Introduction

To conduct good fiscal and monetary policy, a clear understanding of the working of the economy – and especially of the factors that drive the business cycle – is necessary. In recent decades, economists have focused mainly on understanding movements in aggregate output and on explaining the persistence of aggregate economic activity. To do this, they have relied upon, among other things, dynamic general-equilibrium (DGE) models, which focus on the self-interested responses of economic agents to disturbances. Although these models have become a helpful tool, they are based on the implicit assumption that aggregate shocks affect all sectors of the economy equally. Empirical evidence, however, suggests that this is in fact too strong an assumption. For instance, Long and Plosser (1987), using a simple factor analysis on the innovations of a vector autoregression (VAR), show that approximately half of the variance in U.S. industrial production is explained by a more diverse set of independent disturbances, rather than by a common aggregate shock. In addition, typical DGE models are generally less concerned with understanding the prevalent synchronized nature of the business cycle across sectors, which is typically referred to as co-movement.

The aforementioned abstractions of typical DGE models have been addressed in the recent theoretical literature on the business cycle. Consequently, great progress has been made in understanding sectoral fluctuations and their importance for aggregate movements, from a theoretical standpoint. Seminal research by Long and Plosser (1983) shows that, in a multisector real business cycle model, even when productivity shocks are independent across sectors, agents' choices cause co-movement of activity measures across different sectors. More recently, Horvath (1998 and 2000) has developed a multisector DGE model in which aggregate fluctuations are driven by independent sectoral shocks. Building on important linkages between sectors, this model can capture the qualitative features of macroeconomic fluctuations without relying on implausible aggregate shocks. Other authors have also focused on sector-specific shocks that might explain the observed co-movement as well as the mechanism behind the propagation of shocks throughout sectors. For example, reallocation of labour and capital across sectors as a result of sectoral shocks may be an important mechanism in generating the persistence of aggregate fluctuations (Davis and Haltiwanger 1999 and Campbell and Kuttner 1996). Similarly, shocks can be propagated among sectors through the buildup and unwinding of inventory imbalances (Cooper and Haltiwanger 1990).

Notwithstanding the advances in the theoretical literature on business cycles, few empirical studies have looked at the dynamics and co-movement among sectoral data. Exceptions are Engle and Issler (1995) and Harvey and Mills (2002), who study sectoral output dynamics for the United States and the United Kingdom, respectively. To our knowledge, no such study has hitherto been conducted for Canada.

Our empirical model is based on a VAR, which allows for dynamic feedback between the individual sectors without imposing any a priori restrictions. Following the literature on cointegration, long-run co-movement is characterized by common stochastic trends, leading to a vector-error-correction model (VECM) representation. Our study of short-run co-movement builds on Vahid and Engle's (1993, 1997) notion of common and codependent cycles. Common cycles are stationary components that are synchronized in phase but that can differ in amplitude. The concept of codependence is more general, in that it allows for non-synchronized co-movement. Codependent variables are characterized by impulse functions that become collinear after a certain number of periods. The length of this initial heterogeneous adjustment can be interpreted as a measure of structural frictions or of adjustment costs. We depart from the existing literature in that we use (full-information) maximum-likelihood estimates of restricted VECMs to test for the number of cofeature combinations. The estimated models can then be used to obtain a trend-cycle decomposition (following the method proposed by Proietti 1997) and to compute a variance decomposition to assess the relative importance of transitory and permanent shocks for each sector.

In addition, we are interested in whether the disaggregated nature of our data set can provide superior forecasts of aggregate output. For this purpose, we conduct an out-of-sample forecasting exercise that also serves as a test for the hypothesis that the imposition of short-run restrictions leads to overall efficiency gains.

This paper is organized as follows. Section 2 gives an overview of the empirical framework and describes the concepts of common cycles and codependence. In section

3 we discuss the multivariate Beveridge-Nelson decomposition with long- and short-run restrictions and illustrate how it can be computed from reduced-form VECM parameters. Section 4 presents the data used in our empirical analysis, as well as the results of the cointegration and common cycle tests, the trend-cycle decomposition, and the results of a variance decomposition of permanent and transitory shocks to the data. Section 5 describes our out-of-sample forecasting exercise. Section 6 contains a discussion and concludes.

2 Vector-Error-Correction Models with Common Short-Run Features

We base our empirical model on the assumption that the data can be described by a finite-order VAR of order p:

$$y_t = \Pi_1 y_{t-1} + \Pi_2 y_{t-2} + ... + \Pi_p y_{t-p} + u_t, \tag{1}$$

where y_t is a vector of N $I(1)$ variables and u_t is a vector of Gaussian white noise disturbances. This can be written more compactly as

$$\Pi(L) y_t = u_t, \tag{2}$$

where $\Pi(L) \equiv I_N - \Pi_1 L - \Pi_2 L^2 - ... - \Pi_p L^p$. Since $y_t \in I(1)$, the roots of $|\Pi(z)| = 0$ fall on or outside the unit circle; i.e., $|z| \geq 1$. This prevents explosive processes, but allows the VAR to have unit roots. The VAR in levels can be reparameterized to yield the interim multiplier representation (see Banerjee et al. 1993)

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + ... + \Gamma_{p-1} \Delta y_{t-p+1} + u_t, \tag{3}$$

where $\Gamma_j = -\sum_{i=j+1}^{p} \Pi_i$ and $\Pi = \sum_{i=1}^{p} \Pi_i - I_N = -\Pi(1)$. Elements of y_t are cointegrated if there exists a linear combination that is stationary. Engle and Granger (1987) show that if there are cointegrating relationships, the rank of Π equals $r < N$, such that Π can be factored as the product of two $N \times r$ matrices ($\Pi = -\beta\alpha'$). Here, α includes the r cointegrating vectors that span the cointegration space, while β is called the matrix of adjustment coefficients that are the factor loadings in the VECM:

$$\Gamma(L) \Delta y_t = -\beta z_t + u_t, \tag{4}$$

where $z_t \equiv \alpha' y_t$ is the error-correction term. The common-trends assumption imposes cross-equation restrictions on the VAR, as shown by Engle and Granger (1987). Since the VAR in levels in equation (2) parsimoniously encompasses the VECM in equation (4), we can reduce the number of parameters of the dynamic representation by estimating the VECM, which takes these restrictions into account. In this case, the VAR has $N^2 p$ parameters and the VECM has only $N^2(p-1)+2Nr-r^2$ parameters in the conditional mean after accounting for free parameters in the cointegrating vector.

2.1 Common cycles

Similar to the definition of cointegration, we may ask whether the stationary components of the data share common elements. This question underlies the concepts of codependence (Gourioux and Peaucelle 1992) and common features (Engle and Kozicki 1993). The idea behind codependence is that a linear combination of the data exists that is of lower moving-average order than the individual series themselves. In its strongest form, a linear combination of the data will annihilate any serial correlation. This is Engle and Kozicki's definition of a serial correlation common feature (SCCF), which renders cyclical components that are completely synchronized. Vahid and Engle (1993) show that, for $I(1)$ series, the same linear combination that eliminates serial correlation in the differences of the data will also eliminate common cycles in the levels. We can therefore define an $N \times s$ matrix $\tilde{\alpha}$ of rank s, such that $\tilde{\alpha}' \Delta y_t$ is unpredictable (white noise). The s linear combinations contained in $\tilde{\alpha}$ are the cofeature vectors and the space spanned by $\tilde{\alpha}$ is called the cofeature space. Since any cofeature combination of the data in levels is a random walk, the cofeature vectors need to be linearly independent of the cointegrating vectors. Therefore, the number of common trends and common cycles cannot exceed the dimension of the system ($r + s \leq N$).

Vahid and Engle (1993) show that the existence of common cycles places additional cross-equation restrictions on the VECM, which yield efficiency gains if correctly imposed. To include the common cyclical restrictions in our VECM framework, recall that premultiplication by the cofeature matrix eliminates all serial correlation in Δy_t. It is possible to rotate $\tilde{\alpha}$ in such a way as to have an s dimensional identity submatrix, since the cofeature vectors are identified only up to an invertible transformation:

$$\tilde{\alpha} = \begin{bmatrix} I_s \\ \tilde{\alpha}^*_{(N-s)\times s} \end{bmatrix}.$$

We can then consider $\tilde{\alpha}'\Delta y_t = \tilde{\alpha}' u_t$ as a system of s equations. Adding to this the unconstrained VECM equations for the remaining $N-s$ elements, we obtain the pseudo-structural model or constrained VECM:

$$\begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ 0_{(N-s)\times s} & I_{N-s} \end{bmatrix} \Delta y_t = \begin{bmatrix} & 0_{s\times(np+r)} & & \\ \Gamma_1^* & \ldots & \Gamma_{p-1}^* & \beta^* \end{bmatrix} \begin{bmatrix} \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix}, \tag{5}$$

where Γ_i^* and β^* represent the partitions of Γ_i and β that correspond to the bottom $N-s$ reduced-form VECM equations. The error term in equation (5) is given by

$$v_t = \begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ 0_{(N-s)\times s} & I_{N-s} \end{bmatrix} u_t. \tag{6}$$

The constrained VECM has $s(np+r) - s(N-s)$ fewer parameters than the unconstrained VECM, and therefore potentially produces more efficient estimates. The rows of zeros on the right-hand side of the VECM are exclusion restrictions that result from common cycles. The parameters of this reduced-rank VECM can be consistently estimated by simultaneous equation estimation techniques, such as two-stage least squares (2SLS) or full-information maximum-likelihood (FIML). The implied reduced-form VECM and its innovations can be recovered by pre-multiplying the pseudo-structural form by the inverse of

$$\begin{bmatrix} I_s & \tilde{\alpha}^{*\prime} \\ 0_{(N-s)\times s} & I_{N-s} \end{bmatrix}.$$

To carry out the estimation of the model outlined above, several tests are required. Before testing for cointegration, one has to determine whether all the variables are $I(1)$ by employing standard unit-root tests. It is also important to determine the required number of lags, p, in the VECM that adequately capture the dynamics of the system, either by using an information criterion or a sequence of likelihood-ratio tests. Conditional on these settings, the cointegrating rank, r, can be determined by employing Johansen's (1988, 1991) technique, which estimates the number of linearly independent cointegrating vectors.

Having chosen r, the number of common cycles can be determined using Engle and Vahid's (1993) approach. This test involves searching for linear combinations of the first differences of y_t whose correlation with the elements of the relevant past information set, determined as the dependent variables in the VECM representation of the system, are zero. This can be done by computing the canonical correlations between the first differences of the variables and the right-hand side of the VECM. The canonical correlations that are insignificantly different from zero represent linear combinations of Δy_t that are uncorrelated with the relevant history of the variables, and thus give the number of independent cofeature vectors, s. The statistic to test for the null hypothesis that the dimension of the cofeature space is at least s can be found using standard distribution theory, as in Tiao and Tsay (1985), and is given by:

$$C(s) = -(T-p-1)\sum_{i=1}^{s}\log(1-\lambda_i^2), \tag{7}$$

where $\lambda_1^2, ..., \lambda_s^2$ are the s smallest squared canonical correlations between Δy_t and the right-hand side of the VECM, $W_t = (\alpha' y_{t-1}, \Delta y_{t-1}, ..., \Delta y_{t-p+1})$. Under the null, this statistic has a χ^2 distribution with $s(Np+r) - s(N-s)$ degrees of freedom.

2.2 Codependent cycles

The common cycle framework discussed in section 2.1 assumes that different economic variables are affected by an exogenous shock in a synchronous fashion, such that their impulse responses are collinear. This may be an unrealistic assumption, since it is often believed that different variables adjust to a shock with different speeds. In the case of our sectoral data set, for example, this heterogeneity in adjustment may be explained by structural differences such as labour-market rigidities, adjustment costs, degree of openness to trade with other economies, and dependence on raw materials. In this section, we therefore follow Vahid and Engle (1997), who extend the common cycle framework to the more general case where impulse responses are allowed to be linearly independent for the first q periods.

Consider a stationary N-dimensional time series, x_t, with Wold representation:

$$x_t = \varepsilon_t + \sum_{j=1}^{\infty} C_j u_{t-j}. \tag{8}$$

We say that x_t has $N-1$ codependent cycles of order q, if there exists a vector $\tilde{\alpha}_{q\cdot}$ such that

$$\tilde{\alpha}'_{q\cdot} C_j \begin{cases} \neq 0 & \text{if } j = q \\ = 0 & \text{if } j > q \end{cases} . \tag{9}$$

In other words, a linear combination of the data exists that has an MA(q) representation. More generally, let us assume that there are s_j linearly independent vectors, $\tilde{\alpha}_{j\cdot}$, that are collected in the $N \times s_j$ matrix $\tilde{\alpha}_j$, where $j = 0, ..., \bar{q}$. Then $\tilde{\alpha}'_j x_t$ is a VMA(j) and x_t has $s = s_0 + ... + s_{\bar{q}}$ cofeature combinations and $N - s$ codependent cycles. Since all cofeature vectors form an s-dimensional basis in $\mathbb{R}^N$, the matrix $\tilde{\alpha} \equiv [\tilde{\alpha}_0, ..., \tilde{\alpha}_{\bar{q}}]$ is defined only up to an invertible transformation and therefore contains $s(N-s)$ parameters after normalization.

Vahid and Engle (1997), building on earlier work by Tiao and Tsay (1989), call a structure that satisfies equation (9) a scalar component model of order $(0, q)$, denoted as SCM$(0, q)$.[1] Note that the case of SCCF discussed in section 2.1 is the special case when $s = s_0$, such that x_t contains s SCM$(0, 0)$ and the cofeature combination of the data becomes an innovation.

Existing tests for codependence exploit the condition that the cofeature combination $\tilde{\alpha}'_j x_t$ is uncorrelated with lagged information beyond x_{t-j}. Vahid and Engle use this orthogonality condition to construct a generalized method of moments (GMM) estimator that contains a generalization of Tiao and Tsay's test (7) as a special, albeit suboptimal, case. The null hypothesis of these tests is H_0: $q \geq j$. Therefore, a failure to reject an SCM$(0, q_1)$ implies that any SCM$(0, q_2)$ $(q_2 > q_1)$ will not be rejected

[1] Tiao and Tsay consider the more general class of scalar component models, SCM$(\bar{p}, \bar{q})$, that have an ARMA$(\bar{p}, \bar{q})$, representation. For the purpose of this paper we restrict ourselves to the case where $\bar{p} = 0$.

either. As a result, Vahid and Engle suggest an incremental model selection scheme that starts with a test for SCM$(0,0)$ and proceeds to SCM$(0,1)$ and so forth.

A potential problem of this approach is that it does not provide an upper bound, $\bar{q}$, on the order of codependence. This issue is addressed by Schleicher (2003), who shows that, for finite-order VAR models and VECMs, the maximum order of codependence is restricted by the dimension of the VAR system, as well as by the number of cointegrating relationships. These results are summarized in the following theorem,[2] which may be interpreted as an extension of Vahid and Engle's (1993) Theorem 1, in which they show that the sum of common trends and common cycles needs to be greater than or equal to the dimension of the system.

Theorem 1 *Let y_t be an N-vector of I(1) variables that satisfy a finite-order VECM with r linearly independent cointegrating vectors $(r \leq N)$, and let s_j be the number of linearly independent vectors $\tilde{\alpha}_{j\cdot}$, such that $\tilde{\alpha}'_{j\cdot}\Delta y_t$ is an SCM$(0,j)$ $(0 \leq j \leq \bar{q})$. Then it must be that*

$$\sum_{j=0}^{\bar{q}} s_j(j+1) \leq N - r. \tag{10}$$

Two corollaries of this theorem are that (i) there can be at most $(N-r)/(q+1)$ linearly independent cofeature vectors that yield SCM$(0,q)$, and (ii) the maximum possible order of an SCM cofeature is $\bar{q} = N - r - 1$. These results place strong limitations on the relevance of codependent cycles in applied research. Consider, for example, the trivariate system that consists of output, consumption, and investment, which has been very popular in the related literature (see, for example, King et al. 1991, Proietti 1997, and Issler and Vahid 2001). Since it is widely agreed that this system contains one common stochastic trend, exemplified by the "great ratios" between consumption and output and investment and output, we have the condition that $(q+1)s \leq N - r = 1$. This excludes any form of codependence, except the SCCF.

We specify our empirical model as a VECM with Gaussian errors, and are therefore in a position to estimate the joint likelihood of the complete system subject to constraints

[2] See Schleicher (2003) for a detailed proof.

imposed by the assumption of codependence. We believe that this full-information approach has two advantages. First, Monte Carlo experiments by Schleicher (2003) indicate that likelihood-ratio tests based on FIML estimation are considerably more powerful than the GMM-based tests, and possess good size properties at samples of 100 or more observations. Second, we need parameter estimates of the implied reduced-form VECM to compute the permanent-transitory decomposition, discussed in section 3. To compute the χ^2 critical values of the LR tests, we need to know the exact number of cross-equation restrictions. Schleicher (2003) shows that this number is given by

$$sNp + \sum_{j=0}^{\bar{q}} [s_j(j+1)r] - s(N-s), \tag{11}$$

such that, contrary to the VAR scenario discussed by Vahid and Engle, the number of cross-equation restrictions actually increases with the order of codependence. In Appendix A, we explicitly derive these restrictions for a VECM with two lags and SCMs up to order two.

3 Trend-Cycle Decomposition

In this section, we review the restrictions that common trends and common cycles impose on the multivariate Beveridge-Nelson decomposition. We also demonstrate a state-space approach that enables us to compute a trend-cycle decomposition from the reduced-form parameters of the VECM discussed in section 2.

Because we are assuming that $y_t \in I(1)$, its first difference is $I(0)$ and it has a Wold representation

$$\Delta y_t = C(L)u_t, \tag{12}$$

where $C(L) \equiv I + C_1 L + C_2 L^2 + \ldots$. Using the factorization $C(L) = C(1) + (1-L)C^*(L)$, the Wold representation can be rewritten as

$$\Delta y_t = C(1)u_t + \Delta C^*(L)u_t, \tag{13}$$

where $C_j^* = -\sum_{i>j} C_j$ for all $i > 1$ and $C_0^* = I_N - C(1)$. Integrating both sides, we obtain

$$y_t = C(1)\sum_{s=0}^{\infty} u_{t-s} + C^*(L)u_t = T_t + C_t. \tag{14}$$

Equation (14) is the multivariate version of the Beveridge-Nelson (1981) trend-cycle representation. The series y_t is represented as the sum of a random-walk part, which in this context is interpreted as the stochastic trend, and a stationary part or "cycle." Stock and Watson (1988) show that, if $C(1)$ has full rank, then the trend is a linear combination of N random walks, and that, as a result, the variables are not cointegrated, since there is no linear combination of the elements of y_t that is stationary. If the rank of $C(1)$ is $k < N$, then the trend part can be reduced to linear combinations of k random walks and $C(1)$ can be expressed as the product of two rank k matrices, as follows:

$$\begin{aligned} y_t &= \gamma\tau_t + C_t \\ \tau_t &= \tau_{t-1} + \delta' u_t, \end{aligned} \tag{15}$$

where γ and δ are both of rank $k = N - r$, $\tau_t = \delta' \sum_{s=0}^{\infty} u_{t-s}$, and $C_t = C^*(L)u_t$. Equation (15) expresses the trend as a linear combination of k common trends plus some stationary "cyclical" components, C_t.

In a similar manner, we can generalize the Beveridge-Nelson decomposition (14) to include common cyclical components. Analogous to the case of common trends, common cycles arise whenever $C^*(L)$ is of reduced rank. From the definition of common cycles, we know that $\tilde{\alpha}' C^*(L) = 0$. We therefore can decompose $C^*(L)$ as $C^*(L) = \tilde{\gamma}\tilde{C}^*(L)$, where $\tilde{\gamma}$ is an $N \times (N - s)$ matrix that lies in the left null-space of $\tilde{\alpha}$, such that $\tilde{\alpha}'\tilde{\gamma} = 0$ and $\tilde{C}^*(L)$ is an $(N - s) \times N$ matrix in the lag operator.[3] Then the second term in (14) can be expressed as

$$C^*(L)u_t = \tilde{\gamma}\tilde{C}^*(L)u_t = \tilde{\gamma}c_t, \tag{16}$$

where c_t is an $(N - s) \times 1$ vector of common cycles.[4]

[3] For a more detailed discussion of this result, see Vahid and Engle (1993).

[4] The extension to the case of codependent cycles is

$$C^*(L)u_t = C_0^* u_t + C_1^* u_{t-1} + ... + C_q^* u_{t-q} + \tilde{\gamma} \sum_{j=q+1}^{\infty} \tilde{C}_j^* u_{t-j}; \tag{17}$$

As a result, we can restrict the multivariate decomposition to include both common trends and common cycles in the following way:

$$y_t = \gamma \tau_t + \tilde{\gamma} c_t. \tag{18}$$

There is a crucial theoretical connection between the cointegrating space and the cofeature space that is given by the fact that the cofeature vectors, $\tilde{\alpha}$, must be linearly independent from the cointegrating vectors. An intuitive explanation for this result is that $\tilde{\alpha}' y_t \in I(1)$, while $\alpha' y_t \in I(0)$. As a result, if there are r linearly independent cointegrating vectors, there can be at most $N - r$ linearly independent cofeature vectors. This implies that $r + s \leq N$ (which is a special case of Theorem 1 in section 2.2). Vahid and Engle (1993) show that, if the cointegrating rank, r, and the cofeature rank, s, add up to the number of variables in the VAR system, there exists a unique and computationally simple trend-cycle decomposition of the data. Since, by definition, every element of the cointegrating space eliminates the stochastic trends and every element of the cofeature space eliminates the cycles, we can stack the cointegrating and cofeature matrices to obtain the following system:

$$\begin{bmatrix} \underset{s \times N}{\tilde{\alpha}'} \\ \underset{r \times N}{\alpha'} \end{bmatrix} y_t = \underset{N \times N}{A} y_t = \begin{bmatrix} \tilde{\alpha}' T_t \\ \alpha' C_t \end{bmatrix}. \tag{19}$$

Because the cointegrating and cofeature vectors are linearly independent and $r + s = N$, the matrix A will have an inverse that can be partitioned as $A^{-1} = \begin{bmatrix} \underset{N \times s}{\tilde{\alpha}^-} & \underset{N \times r}{\alpha^-} \end{bmatrix}$. The trend-cycle decomposition can then be recovered as simple linear combinations of the data y_t:

$$y_t = A^{-1} A y_t = \tilde{\alpha}^- \tilde{\alpha}' y_t + \alpha^- \alpha' y_t = T_t + C_t. \tag{20}$$

This decomposition applies only in the very special case when $r + s = N$. It is also possible to include both common trends and common cycle restrictions and decompose the data in the general case when $r + s \leq N$. To do this, we follow the methodology outlined in Proietti (1997) and Hecq, Palm, and Urbain (2000). This entails writing

that is, the C_j^* have full rank for $j \leq q$ and reduced rank for $j > q$.

the reduced-form VECM in (4) in state-space form, as follows:

$$\Delta y_t = Z f_t \tag{21}$$
$$f_t = m + T f_{t-1} + Z' u_t, \tag{22}$$

where f_t is the $(N(p-1)+r)$-dimensional state vector

$$f_t = \begin{bmatrix} \Delta y_t \\ \Delta y_{t-1} \\ \vdots \\ \Delta y_{t-p+1} \\ \alpha' y_{t-1} \end{bmatrix},$$

T is the $(N(p-1)+r) \times (N(p-1)+r)$ transition matrix

$$T = \begin{bmatrix} \Gamma_1 + \beta\alpha' & \Gamma_2 & \dots & \Gamma_{p-1} & \beta \\ I_N & 0_{N\times N} & \dots & 0_{N\times N} & 0_{N\times r} \\ 0_{N\times N} & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ \alpha' & 0_{r\times N} & \dots & \dots & I_r \end{bmatrix},$$

$Z = [I_N, 0_{N\times N}, ..., 0_{N\times r}]$ is an $N \times (N(p-1)+r)$ matrix, and $m' = [\mu', 0_{1\times N}, ..., 0_{1\times N}, 0_{1\times r}]$ is a vector of dimension $N(p-1)+r$.

The trend of the Beveridge-Nelson decomposition ($y = \tau_t + c_t$) can be defined as the forecast of the time series, adjusted for the mean growth rate, as the forecast horizon approaches infinity:

$$\tau_t = y_t + \lim_{k\to\infty} \sum_{i=1}^{k} \left[\Delta\tilde{y}_{t+i|t} - E(\Delta y_t)\right], \tag{23}$$

where $\Delta\tilde{y}_{t+i|t}$ is the i-th step best linear predictor of Δy_t based on information at time t. The cyclical component, c_t, is then given by

$$c_t = -\lim_{k\to\infty} \sum_{i=1}^{k} \left[\Delta\tilde{y}_{t+i|t} - E(\Delta y_t)\right]. \tag{24}$$

If we assume that the constant, μ, in the VECM is zero, the best linear predictor of Δy_{t+i} is given by

$$\Delta \tilde{y}_{t+i|t} = ZT^i f_{t|t}, \tag{25}$$

where $f_{t|t}$ is the contemporaneous Kalman-filter estimate of the state vector. Given the stability condition that all eigenvalues of T lie inside the unit disk, the sum of the geometric series $\sum_{i=1}^{k} T^i$ converges to $(I-T)^{-1}T$ as $k \to \infty$. Since all components of f_t are observed at time t, $f_{t|t} = f_t$ and the cyclical component of the time series can be computed as

$$c_t = -Z(I-T)^{-1}T f_t. \tag{26}$$

In the general case, when the constant, μ, does not equal zero, we can express the mean growth rate as $m^* = \sum_{i=0}^{\infty} T^i m = (I-T)^{-1}m$ and transfer the drift term from the transition equation into the observation equation:

$$\Delta y_t = Z f_t^* + Z m^* \tag{27}$$

$$f_t^* = T f_{t-1}^* + Z' u_t, \tag{28}$$

where $f_t^* \equiv f_t - m^*$. The cyclical component of the Beveridge-Nelson decomposition is then given by

$$c_t = -Z(I-T)^{-1}T f_t^*. \tag{29}$$

This method can be applied to the estimated parameters of unrestricted VECM, as well as to the implied reduced form of the restricted VECM.

4 Empirical Evidence

The data set used in this study consists of quarterly (log) Canadian sectoral GDP from 1961Q1 to 2001Q2,[5] so that there are 162 observations. We use per-capita series, since most multi-sector real business cycle models are based on a representative agent. Ideally, we would like to examine the data at a fairly low aggregation level, but our analysis is restricted to examining seven sectors, which in their aggregate

[5] The data are obtained from CANSIM and expressed in constant 1992 dollars.

comprise total private sector GDP. This is adequate, given that even VARs of moderate dimensions can be subject to considerable estimator bias (see Abadir, Hadri, and Tzavalis 1999). Private sector GDP[6] is thus divided into the following seven sectors: agriculture, fishing, logging, and mining (PRIMARY); construction (CONST); manufacturing (MANUF); retail and wholesale trade (TRADE); finance, insurance, and real estate (FIRE); transportation (TRANS); and other services (SER).[7] Figure 1 shows the per-capita logarithms of the seven sectoral output series.

In the procedures for common trend–common cycle analysis, all inferences in both the cointegration and the common cycles stages are conditional on the data being $I(1)$ and on the number of lags chosen. Augmented Dickey-Fuller and Phillips-Perron tests show that the series are $I(1)$ processes, and, as a result, the VECM framework described in section 2 is an appropriate modelling environment. We employ a series of tests (LR tests and information criteria) to determine the lag-length of our vector-autoregressive system, and conclude that a VAR(3) model provides the best fit for the data.

In our next step, we use the cointegration test of Johansen (1988) to determine the number of common trends among the sectoral output series. A constant term is included in the VECM and critical values are extracted from Osterwald-Lenum (1992). The results of this test (Table 1) reject the hypotheses of less than two cointegrating relationships at the 1 per cent level and less than three cointegrating relationships at the 5 per cent level. After experimenting with different lag-lengths of the VAR polynomial, however, and considering several subsamples, we conclude that our system is better characterized by five common trends ($r = 2$) than by four ($r = 3$), as this result is more robust to changes in specification. In the remainder of our analysis, we therefore keep the number of cointegrating equations fixed at two. The fact that the number of common secular components is relatively high compared with the dimension of the system is an intuitive result, because if common stochastic trends arise from technology shocks, very heterogeneous sectors should not

[6] The public sector is excluded.

[7] Communication and other utility industries, business services industries, education, health and social service industries, accommodation, food and beverage, and other service industries formed the SER group.

share the same trends. This point was first raised by Durlauf (1989), who notes that a technological improvement in agriculture should not imply technical change for FIRE.

We then search for possible short-run features shared by the various sectors; that is, we look for SCCFs and codependence in the form of SCMs. As Vahid and Engle (1997) show, the presence of codependence of order q in the differenced data (the VECM) corresponds to codependent cycles of order $q-1$ in levels. We first use Tiao and Tsay's (1985 and 1989) canonical correlation-based test and Vahid and Engle's GMM test,[8] to assess the hypothesis that there are at least s_q cofeature combinations that satisfy an SCM(0,q). As the results in Table 2 indicate, at the 5 per cent level Tiao and Tsay's test does not reject the hypotheses that $s_0 \geq 1$, $s_1 \geq 4$, and $s_2 \geq 4$. That is, we would have four cofeature combinations, one of which constitutes an SCCF, and three an SCM(0,1). The restrictions implied by Theorem 1, however, stipulate that we can have at most five SCCFs, two SCM(0,1), and one SCM(0,2). This follows, because the cofeature vectors must be independent of the two cointegrating vectors, and higher-order SCMs place additional restrictions on the long-run impact matrix of the VECM. The GMM test (Table 3) is more conservative, in that it allows for only one cofeature combination, in the form of an SCM$(0,1)$.[9]

Because we specified our model in a VAR framework with Gaussian errors, we are in a position to directly estimate the full system under the restrictions imposed by the SCMs. This estimation is done by maximizing the concentrated likelihood function of the implied reduced form. As stated earlier, there are two important reasons why we prefer this approach over the standard limited information-based analysis. First, a primary motive of this paper is to obtain a trend-cycle decomposition. For this, we need reduced-form parameters of the VECM. Second, Monte Carlo experiments indicate that likelihood-ratio tests based on FIML estimation of the restricted VECMs have considerably higher power than the GMM and Tiao-Tsay tests. We therefore compute all possible combinations of SCMs up to order 2 that are permitted under

[8] We use an iterative updating GMM estimator instead of the two-step method proposed by Vahid and Engle (1997).

[9] Monte Carlo experiments by Schleicher (2003) indicate that the iterative and two-step GMM estimators tend to significantly over-reject even at sample sizes of 200 observations, when the true data-generating process contains an SCM$(0,1)$ and a cointegrating relationship.

Theorem 1 and not encompassed by rejected models. Since most of these models are non-nested alternatives, we compare the individual models with the unrestricted VECM and then aim to select the most parsimonious variation that is not rejected by the likelihood-ratio statistic.[10] Table 4 shows the results. In each case, the p-value is calculated from a χ^2-distribution, with the number of degrees of freedom equal to the number of cross-equation restrictions given by Proposition 1. We first note that all three possible cases of one cofeature ($s_0 = 1$, $s_1 = 1$, and $s_2 = 1$)[11] are clearly supported by the data. When we move to the next level, however, and consider different combinations with two cofeatures, we find that the model with two SCCFs is rejected at the 1 per cent level, which agrees with the results of the LIML tests. The two models with the most compelling test statistics are the combination of one SCCF and one SCM(0,2) ($p \sim 0.19$), and the combination of two SCM(0,1) ($p \sim 0.09$). Both models impose 26 non-linear cross-equation restrictions, such that the number of effective parameters in the conditional mean is reduced from $pN^2 + rN = 112$ to 88 (a 23 per cent decrease). We choose the first model, $(s_0, s_1, s_2) = (1, 0, 1)$, as the reference model for further analysis. In levels, this model yields cycles with impulse-response functions that have rank 6 and rank 5 after two periods. Table 5 reports the cointegration vectors α and the cofeature vectors $\tilde{\alpha}$.

We then decompose the series into trend and cycle components, to separate transitory phenomena from the long-run behaviour of sectoral output.[12] Given that the number of common trends and cycles does not exactly add up to the number of variables in the system, the computationally simple decomposition proposed by Vahid and Engel (1993) cannot be performed. Consequently, as outlined in section 3, we write the estimated VECM in state-space form and follow Proietti (1997) in obtaining the multivariate Beveridge-Nelson decomposition of the system. The resulting transitory and permanent components are plotted in Figures 2 and 3. To better understand these figures, one can compare stochastic trend and cyclical components with actual anecdotal recessions.[13] We also report standard deviations and contemporaneous cor-

[10] We also compare this approach with LR tests between nested submodels and find that these two approaches are consistent with each other.

[11] We also consider SCM$(0, 3)$ and SCM$(0, 4)$; however, these are rejected.

[12] The measure of the stochastic trend in this paper should not be interpreted as potential output.

[13] In the United States, the NBER officially dates the turning points of the economy, whereas in

relations of the cyclical components in Table 6. An immediately noticeable feature of our decomposition is that the cycles of the three sectors that comprise the primary industries, construction, and manufacturing are significantly more volatile than those of the remaining sectors. The standard deviation of the most volatile cycle (construction) is more than four times as large as the one of the least volatile cycle (transportation). A similar statement can be made about the trend components. The primary sector, construction, and manufacturing undergo large and persistent long-run fluctuations, while the other sectors grow at a more steady rate.[14]

From our plots we further observe that the transitory components of the primary industries, manufacturing, and trade have very similar shapes that are procyclical, in that they exhibit downward movement during each recession. The cycles of these three sectors are also highly correlated among themselves. On the other hand, FIRE and transportation have cycles that are significantly positively correlated only among themselves, and negatively correlated with most other sectors. We therefore characterize these sectors as being acyclical. Construction is positively correlated with the primary industries, other services, and trade, but not with manufacturing. Overall, these results match the notion that Canadian cyclical fluctuations are driven mainly by construction, manufacturing, and the primary industries.

It is interesting to compare the behaviour of different sectors during the prolonged downturn at the beginning of the 1990s. During this episode, all series except the primary industries, FIRE, and other services undergo a severe downturn. When we look at the decompositions for construction and manufacturing, we observe that the trend components of both series are declining. However, their transitory components move in different directions. While the cycle of manufacturing decreases slightly, the cycle of construction increases sharply. In terms of our permanent-transitory decomposition, we may interpret these observations as an asynchronous adjustment to a permanent shock, which drives the trend temporarily below actual output. In the construction sector, actual output adjusts slowly to the new secular level; therefore,

Canada there are no official recession dates. Cross (2001), however, sets reference cycle dates for Canada, which are shaded in our graphs.

[14]To better assess the importance of transitory and permanent shocks, we conduct a forecast error variance decomposition in the next section.

its transitory component remains positive during this period. On the other hand, in the manufacturing sector, output reverts quickly to the new trend level.

4.1 Variance decomposition

Our first impressions of our trend-cycle decomposition indicate that individual sectors behave rather differently during periods that are generally classified as economic downturns. In this section, we extrapolate from this idea by computing a forecast error variance decomposition to assess the relative importance and persistence of permanent and transitory shocks. In particular, we are interested in whether cycle innovations explain a significant proportion of the total forecast error over business-cycle horizons. Consider, therefore, the innovation u_t, which can be expressed as the sum of its trend and cycle components:

$$u_t = u_{trend,t} + u_{cycle,t}. \tag{30}$$

However, since the trend and cycle innovation will be correlated in most cases, it is first necessary to orthogonalize $u_{trend,t}$ and $u_{cycle,t}$. Following Issler and Vahid (2001), we assume that both innovations have the structure

$$\begin{bmatrix} u_{trend,t} \\ u_{cycle,t} \end{bmatrix} \sim N\left(0, \begin{bmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{bmatrix} \right). \tag{31}$$

It is then possible to decompose the variance of ε_t into that of two orthogonal components, $\mu_{trend,t}$ and μ_{cycle_t}, in the following way:

$$\begin{aligned} VAR(u_t) &= VAR(\mu_{trend,t}) + VAR(\mu_{cycle,t}) \\ &= \left\{ \left(1 + \frac{\sigma_{12}}{\sigma_{11}}\right)^2 \sigma_{11} \right\} + \left\{ \sigma_{22} + \frac{\sigma_{12}^2}{\sigma_{11}} \right\}. \end{aligned} \tag{32}$$

This orthogonalization procedure is comparable to a Cholesky factorization and, as a result, is sensitive to the ordering of the variables. Although there is no consensus on what innovation should be placed first in the orthogonalization procedure, we put trend innovations first, because in real business cycle models trend shocks cause both trend and cyclical activity. We find that our results are hardly affected by the ordering, however, as the covariances between the trend and cycle innovations were effectively zero for each sector. Consequently, in Table 7 we show only the results for trend innovations preceding cyclical innovations in the orthogonalization.

We obtain one-step-ahead innovations for the trends by taking first differences of the estimated trends. For longer horizons, we accumulate one-step-ahead trend innovations. First-quarter cycle innovations are the residuals from a regression of the estimated cycles on the right-hand side of the VECM (information set). For cyclical h-step-ahead innovations, we shift the information set backwards.

The results show that transitory movements are most important for manufacturing, construction, and the primary industries. It is in these sectors that the benefits of smoothing cyclical fluctuations are greater, because transitory shocks are very significant and persistent. For manufacturing, transitory shocks account for 71 per cent of the variance at the shortest horizon and 14 per cent after two years. For the primary industries and construction, the proportion of transitory shocks is 55 per cent at the shortest horizon and 14 per cent and 34 per cent, respectively, after two years. We conclude that, while transitory shocks have the strongest initial impact on manufacturing, their effect is most persistent for construction.

Permanent shocks explain the bulk of the variance for FIRE, trade, transportation, and other services. In these sectors, transitory shocks account for less than a quarter of output variation at the one-quarter horizon and their effect vanishes rapidly. After one year, the proportion of transitory variance is around 10 per cent for FIRE and other services, and only 3 per cent for trade and transportation.

It is important to note that, because we are using real variables, these results may understate the role of some sources of transitory shocks, such as monetary policy. For example, in a similar exercise in a VAR with output, consumption, and investment, King et al. (1991) find that, when monetary variables are included in the VAR, transitory shocks become more important.

5 Out-of-Sample Forecasts

In this section, we compare the out-of-sample forecasting performance of our restricted VECM with those of competing models. There are two main motives for this exercise. First, the out-of-sample forecasts act as a model-specification test and thus provide an idea of whether the data support the more parsimonious representation implied by the codependent cycle restrictions. Second, we are interested in whether the disaggregated nature of our data set, together with the long-run and short-run restrictions, enhance our capability to forecast aggregate output. This second question is of particular relevance for institutions like central banks, whose contemporaneous policy decisions affect the real economy with a lag of several quarters.

We divide our sample of 162 observations into an estimation window (1961Q1-1987Q4, two-thirds of the sample) and a forecasting window (1988Q1-2001Q2, one-third of the sample). We then use information available at 1987Q4 to select the specifications of our VECM following the procedures outlined in section 4. Based on a VAR with 3 lags, the Johansen test yields a VECM with five common trends ($r = 2$). We then perform likelihood-ratio tests among different SCMs and find that a variation with five codependent cycles of order one – $(s_0, s_1, s_2) = (0, 2, 0)$ – is the most parsimonious presentation that is clearly supported by the data.[15] This model is similar in structure to the $(s_0, s_1, s_2) = (1, 0, 1)$ model we use for our entire sample, and also imposes 26 cross-equation restrictions on the VECM.

Besides the restricted VECM, we compute forecasts using the unrestricted VECM and the unconditional mean (time trend) and use an ARIMA(1,1,0) model to compute forecasts for aggregate private GDP.[16] To obtain aggregate GDP forecasts from the VECMs, we take the logarithm of the sum of the exponentials of the individual sector forecasts.

[15]Our test results are available upon request.

[16]Stock and Watson (1998) compare several linear and non-linear forecasting models (autoregressive, artificial neural network, smooth-transition autoregression, and exponential smoothing models) and find that the autoregressive model has the best forecasting performance within a set of 200 macroeconomic time series. We find that the specification with only one autoregressive lag minimizes the mean-squared forecast error over most horizons.

Tables 8 and 9 show root-mean-squared errors (RMSEs) for horizons from 1 to 10 quarters. For the forecasts of the individual sectors, we also provide the determinant of the RMSE matrices as a measure of overall forecasting performance. Figure 4 shows the results for this metric and the RMSE of the GDP forecast. For the aggregate GDP forecasts, we find that the restricted VECM is the best performer over all horizons beyond $h = 3$, followed by the unrestricted VECM and the ARIMA model. The efficiency gains are most pronounced over horizons between 6 and 9 quarters. The unrestricted VECM performs marginally better for the first three quarters. At the one-year horizon, the RMSE of the restricted VECM is 1 per cent smaller than that of the unrestricted VECM, 15 per cent smaller than that of the ARIMA model, and 22 per cent smaller than that of the unconditional mean. This result indicates that the VECMs are indeed able to extract predictable dynamics from the sectoral data set lost in the aggregate series.

We use White's (2000) reality check test, based on 10^6 bootstrap resamples of our forecast errors, to assess the validity of our results (see Table 10). We find that, over all horizons, the restricted VECM outperforms the ARIMA model in more than 90 per cent of all cases. The unconditional mean forecasts further allow us to make statements about the content horizon[17] of our competing models. Both VECMs outperform the unconditional mean in 95 per cent of all cases over all horizons. The ARIMA model has a 95 per cent content horizon of 8 quarters.

For the individual sectors, the forecasts are considerably less precise. With some exceptions (construction and other services), the unconditional mean outperforms each of the two VECMs. This fact is evident when we look at the determinant of the RMSE matrix of the unconditional mean forecast, which is about the same as that of the unrestricted VECM and consistently lower than that of the restricted VECM. Our overall impression from this exercise is that, while the VECMs are not very accurate in predicting individual sectors, they provide a useful tool for forecasting the aggregate series.[18]

[17] Following Galbraith (2003), we define the δ-level content horizon as the maximal forecast horizon at which a model outperforms the unconditional mean forecast with probability δ.

[18] Despite the fact that the restricted VECM is not the best model for forecasting all of the individual series, it is still the best for forecasting the aggregate series, because for construction and

6 Conclusions

This paper confirms the prediction of several real business cycle models that sectoral outputs share both common trends and common cycles. For our Canadian data set, these common components are characterized by two cointegrating relationships and two codependent cycle cofeatures, one of which represents synchronous and the other asynchronous short-run co-movement. In contrast to existing studies, we employ a full-information maximum-likelihood approach to test for and estimate these cofeatures.

Using a multivariate version of the Beveridge-Nelson decomposition, we find that the temporary components of manufacturing, trade, and the primary sector are very similar and procyclical. Other sectors, such as construction and FIRE, have a distinctive idiosyncratic cycle. We also encounter a wide variation in cyclical volatility, with construction being the most volatile and transportation being the least volatile sector. In addition, our findings indicate that the permanent components (stochastic trends) of the data are less homogeneous than their temporary counterparts.

A variance decomposition reveals that the primary sector, construction, and manufacturing are driven mainly by persistent temporary shocks, while for the remaining sectors permanent shocks are relatively more important even in the short run. We conclude that manufacturing, the primary industries, and construction are important sources of fluctuations for the Canadian economy, based on the fact that they follow the aggregate cycle of the economy and are subject to persistent transitory shocks. Although the initial effect of transitory shocks is strongest for the manufacturing sector, their effect is most persistent for construction.

Modern macroeconomic theory places very strong emphasis on the distinction between permanent and transitory phenomena, as well as the importance of adjusting policy decisions accordingly. In this respect, we argue that the empirical model discussed in this paper provides a very useful tool for policy-makers. Compared with other trend-cycle decompositions like those based on the HP filter or bandpass filters, the trend-cycle decomposition described in section 3 has the additional advantage of

other services, which are two large sectors in the economy, it produces the most accurate forecasts.

being the optimal signal-extracting device at the end of the sample,[19] which makes it particularly valuable for current analysis.

An out-of-sample forecasting exercise establishes that the imposition of common cycle constraints results in a small but non-negligible gain in efficiency. Moreover, we find that forecasts of aggregate private GDP based on the individual sectors are superior to those based on a univariate ARIMA model. This finding suggests that the disaggregate nature of our data set enables the VECMs to extract predictable dynamics that are lost by examining solely aggregate series.

One question of considerable interest is whether the common short-run features are stable over time, or whether there are structural breaks. For a subsample of our data (the first two-thirds of all observations), we find that the number of cofeatures remains unchanged, although their composition changes slightly. We believe that a more systematic and exhaustive approach to this question would be a very interesting avenue of future research, as it could provide a useful framework to test for and estimate changes in structural rigidities.

[19] Koopman and Harvey (1999) discuss this point.

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Table 1: Cointegration Test (Johansen 1988)

Eigenvalue stat. $-T\ \ln\,(1-\lambda_j)$	Trace stat. $-T\sum_{j\leq i}\ln(1-\lambda_i)$	5 per cent critical value	1 per cent critical value	Null hypothesis
0.30	181.53 **	124.24	133.57	$r=0$
0.27	123.82 **	94.15	103.18	$r\leq 1$
0.20	74.49 *	73.24	76.07	$r\leq 2$
0.12	39.78	47.21	54.46	$r\leq 3$
0.08	19.30	29.68	35.65	$r\leq 4$
0.04	6.48	15.41	20.04	$r\leq 5$
0.00	0.22	3.76	6.65	$r\leq 6$

*(**) denotes rejection of the null hypothesis at the 5 per cent (1 per cent) level.
Note: Critical values are taken from Osterwald-Lenum (1992).

Table 2: Canonical Correlation Test for SCM

SCM(0,0):		SCM(0,1):		SCM(0,2):		Null
$C(s,j)$	p-value	$C(s,j)$	p-value	$C(s,j)$	p-value	hypothesis
10.74	0.38	6.14	0.80	7.19	0.71	$s\geq 1$
42.53	0.01 *	19.86	0.59	20.98	0.52	$s\geq 2$
80.04	0.00 **	40.42	0.28	38.90	0.34	$s\geq 3$
124.75	0.00 **	68.04	0.07	67.01	0.08	$s\geq 4$
175.23	0.00 **	120.42	0.00 **	103.95	0.01 *	$s\geq 5$
275.07	0.00 **	164.40	0.00 **	135.85	0.00 **	$s\geq 6$
396.66	0.00 **	220.35	0.00 **	169.05	0.00 **	$s=7$

*(**) denotes rejection of the null hypothesis at the 5 per cent (1 per cent) level.

Table 3: GMM Test for SCM

SCM(0,0):		SCM(0,1):		SCM(0,2):		Null
J-stat.	p-value	J-stat.	p-value	J-stat.	p-value	hypothesis
19.63	0.03 *	6.43	0.78	8.01	0.63	$s \geq 1$
58.13	0.00 **	44.34	0.00 **	36.39	0.03 *	$s \geq 2$
102.36	0.00 **	86.12	0.00 **	56.67	0.02 *	$s \geq 3$
145.14	0.00 **	107.22	0.00 **	91.49	0.00 **	$s \geq 4$
198.41	0.00 **	136.22	0.00 **	116.83	0.00 **	$s \geq 5$
259.50	0.00 **	176.44	0.00 **	142.00	0.00 **	$s \geq 6$
313.97	0.00 **	207.82	0.00 **	169.16	0.00 **	$s = 7$

*(**) denotes rejection of the null hypothesis at the 5 per cent (1 per cent) level.

Table 4: Likelihood-Ratio Tests

No. of cofeatures							
	s_0	s_1	s_2	Log-lik.	LR	DoF	p-value
$s = 0:$	0	0	0	3344.26	reference	-	-
$s = 1:$	1	0	0	3338.88	10.76	10	0.3765
	0	1	0	3338.82	10.88	12	0.5392
	0	0	1	3337.60	13.32	14	0.5015
$s = 2:$	2	0	0	3322.70	43.12	22	0.0046 **
	1	1	0	3326.80	34.92	24	0.0696
	1	0	1	3328.15	32.22	26	0.1859
	0	2	0	3326.09	36.34	26	0.0856
	0	1	1	3317.01	54.50	28	0.0020 **
$s = 3:$	1	2	0	3308.11	72.30	40	0.0013 **

*(**) denotes rejection of the null hypothesis at the 5 per cent (1 per cent) level.
Note: We exclude models like $(s_0, s_1, s_2) = (0, 0, 2)$ that violate Theorem 1, and models that nest rejected models (e.g., (4,0,0) nests (3,0,0) and (2,0,0)).

Table 5: Cointegration and Cofeature Vectors

		PRIM	CONS	FIRE	MANU	SERV	TRAD	TRAN
$\alpha_1\prime$	EC_1	1.000	0.000	0.405	3.718	1.105	-1.629	-2.582
				(0.88)	*(1.78)*	*(2.66)*	*(-1.70)*	*(-2.33)*
$\alpha_2\prime$	EC_2	0.000	1.000	0.552	-6.239	-1.362	1.901	3.425
				(0.80)	*(-1.98)*	*(-2.18)*	*(1.32)*	*(2.05)*
$\tilde{\alpha}_1\prime$	SCCF	1.000	0.000	1.133	0.562	0.144	-3.075	0.142
				(1.22)	*(1.06)*	*(0.14)*	*(-2.20)*	*(0.27)*
$\tilde{\alpha}_2\prime$	$SCM(0,2)$	0.000	1.000	3.906	1.097	-7.888	0.029	-0.578
				(1.81)	*(1.14)*	*(-2.16)*	*(0.02)*	*(-0.79)*

Note: Pseudo t-values are printed in italics.

Table 6: Standard Deviations and Correlations of Cycles (Restricted VECM)

	St.D.	PRIM	CONS	FIRE	MANU	SERV	TRAD	TRAN
PRIM	*0.023*	1.00	0.56	-0.57	0.78	0.55	0.96	-0.42
CONS	*0.046*	0.56	1.00	0.30	0.06	0.90	0.55	-0.12
FIRE	*0.011*	-0.57	0.30	1.00	-0.82	0.13	-0.52	0.51
MANU	*0.040*	0.78	0.06	-0.82	1.00	0.07	0.85	-0.22
SERV	*0.012*	0.55	0.90	0.13	0.07	1.00	0.49	-0.21
TRAD	*0.012*	0.96	0.55	-0.52	0.85	0.49	1.00	-0.21
TRAN	*0.010*	-0.42	-0.12	0.51	-0.22	-0.21	-0.21	1.00

Table 7: FEVD: Permanent Component of Restricted VECM

Forecast horizon (quarters)	PRIM	CONS	FIRE	MANU	SERV	TRAD	TRAN
$h = 1$	45	45	81	29	72	82	89
$h = 4$	75	52	90	74	91	97	97
$h = 8$	86	66	96	86	97	99	99
$h = 12$	90	76	98	91	99	99	99
$h = 16$	92	81	99	94	99	100	99
$h = 20$	94	84	99	95	99	100	100
$h \to \infty$	100	100	100	100	100	100	100

Table 8: Out-of-Sample Forecasts: Root-Mean-Squared Errors (RMSE)

		h=1	h=2	h=3	h=4	h=6	h=8	h=10	
GDP:	VECM_{R}	0.48	0.95	1.41	1.89	2.96	4.08	5.14	(10^{-2})
	VECM_{UR}	0.46	0.91	1.39	1.91	3.02	4.14	5.17	(10^{-2})
	ARIMA	0.53	1.09	1.67	2.22	3.29	4.34	5.33	(10^{-2})
	UCM	0.70	1.32	1.89	2.43	3.47	4.46	5.40	(10^{-2})
|RMSE|:	VECM_{R}	0.00	0.06	0.22	0.65	1.89	3.52	4.08	(10^{-11})
	VECM_{UR}	0.00	0.05	0.16	0.46	1.28	2.42	2.40	(10^{-11})
	UCM	0.00	0.04	0.14	0.54	1.21	2.61	3.44	(10^{-11})

Note: UCM = unconditional mean forecast.

Table 9: Out-of-Sample Forecasts: Root-Mean-Squared Errors (RMSE)

		h=1	h=2	h=3	h=4	h=6	h=8	h=10	
PRIM:	VECM_{R}	1.90	2.28	2.96	3.34	3.88	4.12	3.86	(10^{-2})
	VECM_{UR}	1.91	2.38	3.10	3.50	4.13	4.49	4.38	(10^{-2})
	UCM	1.58	2.05	2.33	2.59	2.80	3.20	3.29	(10^{-2})
CONS:	VECM_{R}	1.73	2.77	3.26	3.77	4.73	6.21	7.90	(10^{-2})
	VECM_{UR}	1.70	2.69	3.20	3.74	4.84	6.45	8.20	(10^{-2})
	UCM	1.94	3.33	4.55	5.72	7.70	9.63	11.52	(10^{-2})
FIRE:	VECM_{R}	0.85	1.31	1.51	1.72	1.98	2.12	2.24	(10^{-2})
	VECM_{UR}	0.82	1.24	1.25	1.27	1.34	1.35	1.37	(10^{-2})
	UCM	0.59	0.81	0.92	1.10	1.20	1.32	1.28	(10^{-2})
MANU:	VECM_{R}	1.66	3.08	4.26	5.51	7.81	9.53	10.76	(10^{-2})
	VECM_{UR}	1.61	2.97	4.20	5.55	7.97	9.73	10.88	(10^{-2})
	UCM	1.65	2.88	3.90	4.87	6.66	8.35	9.81	(10^{-2})
SERV:	VECM_{R}	0.72	1.35	1.93	2.54	3.88	5.32	6.79	(10^{-2})
	VECM_{UR}	0.73	1.37	2.01	2.71	4.10	5.54	6.99	(10^{-2})
	UCM	0.85	1.63	2.39	3.15	4.68	6.24	7.82	(10^{-2})
TRAD:	VECM_{R}	1.54	2.51	3.52	4.51	6.29	7.94	9.42	(10^{-2})
	VECM_{UR}	1.49	2.36	3.39	4.41	6.20	7.84	9.28	(10^{-2})
	UCM	1.54	2.46	3.42	4.37	6.06	7.63	9.05	(10^{-2})
TRAN:	VECM_{R}	1.92	3.25	4.43	5.36	6.56	7.33	8.34	(10^{-2})
	VECM_{UR}	1.92	3.19	4.30	5.21	6.37	7.14	8.09	(10^{-2})
	UCM	1.47	2.39	3.30	4.04	5.19	6.22	7.42	(10^{-2})

Note: UCM = unconditional mean forecast.

Table 10: Bootstrap Tests for Out-of-Sample Forecasts

		RMSE of GDP forecast			|RMSE|	
		VECM_{UR}	ARIMA	UCM	VECM_{UR}	UCM
$h = 1$	VECM_{R}	0.204	0.973	1.000	0.136	0.064
	VECM_{UR}		0.961	1.000		0.161
	ARIMA			1.000		
$h = 2$	VECM_{R}	0.318	0.997	1.000	0.100	0.030
	VECM_{UR}		0.996	1.000		0.129
	ARIMA			1.000		
$h = 3$	VECM_{R}	0.597	1.000	1.000	0.004	0.055
	VECM_{UR}		1.000	1.000		0.342
	ARIMA			1.000		
$h = 4$	VECM_{R}	0.840	1.000	1.000	0.001	0.224
	VECM_{UR}		0.999	1.000		0.741
	ARIMA			1.000		
$h = 6$	VECM_{R}	0.970	0.999	1.000	0.000	0.012
	VECM_{UR}		0.991	0.998		0.357
	ARIMA			0.996		
$h = 8$	VECM_{R}	0.988	0.996	0.999	0.000	0.090
	VECM_{UR}		0.968	0.986		0.616
	ARIMA			0.960		
$h = 10$	VECM_{R}	0.952	0.991	0.996	0.000	0.228
	VECM_{UR}		0.963	0.974		0.945
	ARIMA			0.825		

Note: Entry (x, y) denotes the probability that model x outperforms model y.

Figure 1: Per-Capita Sectoral Outputs (logs)

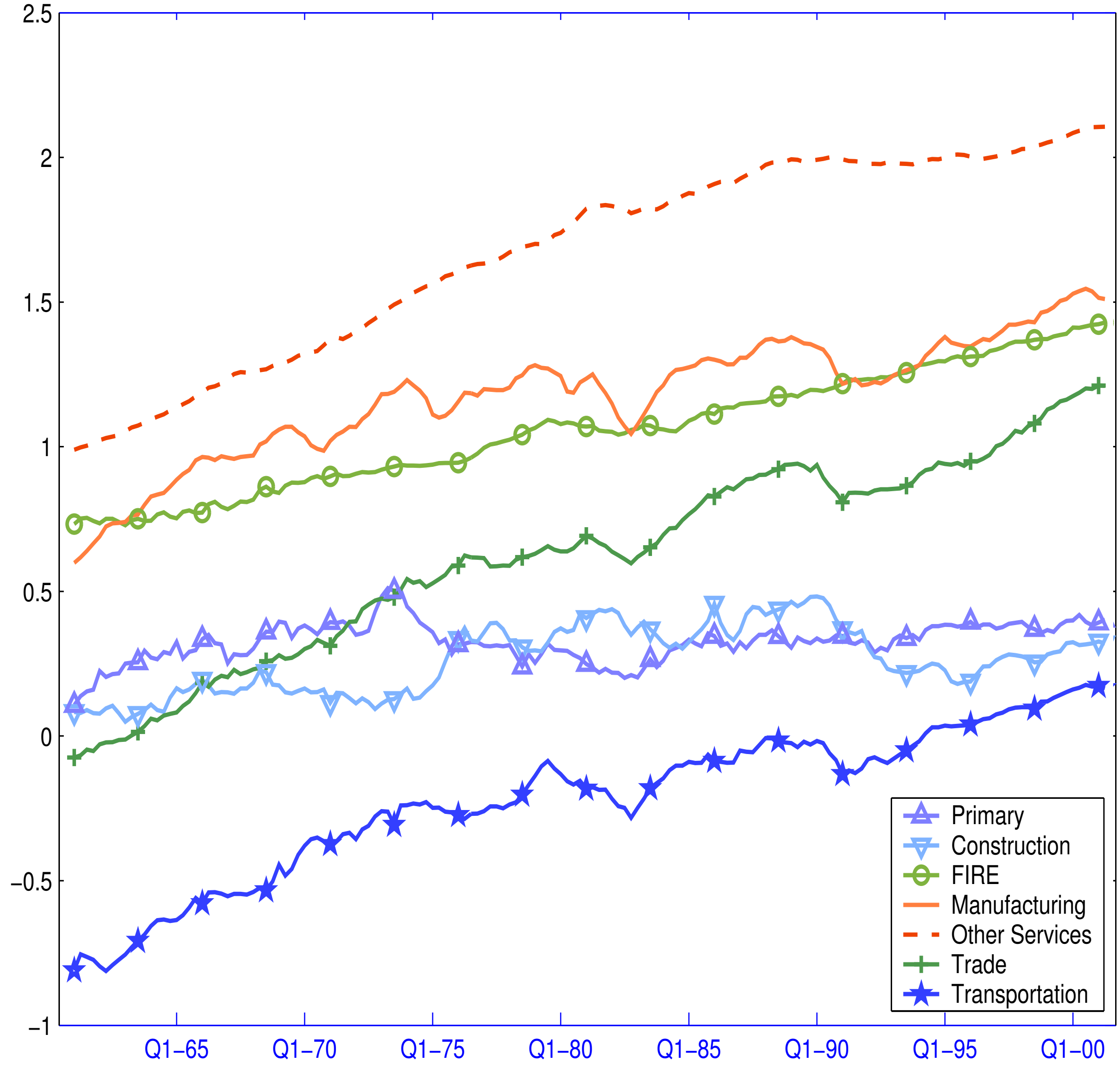


Figure 2: Temporary Components of the Restricted VECM

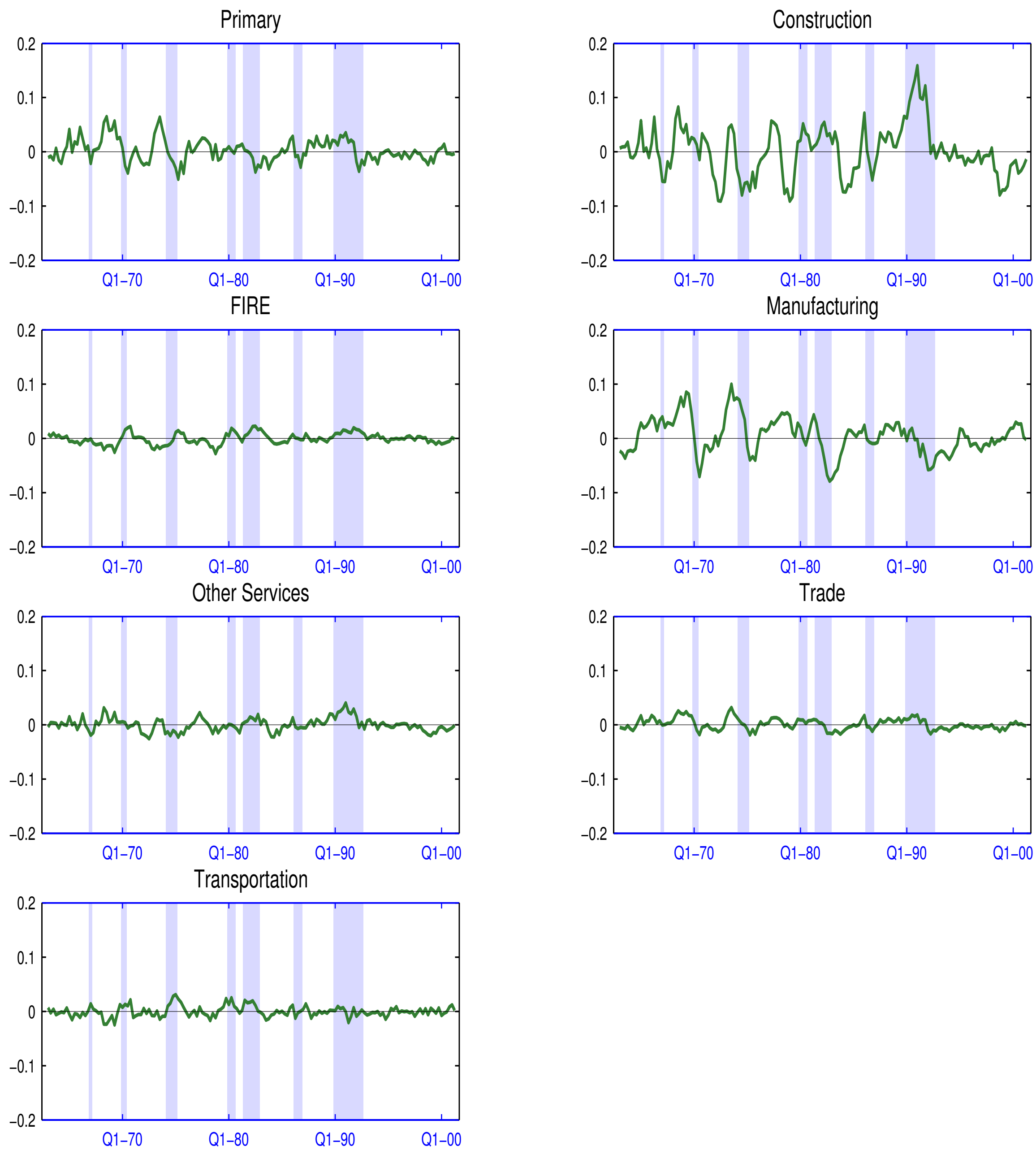


Figure 3: Permanent Components of the Restricted VECM

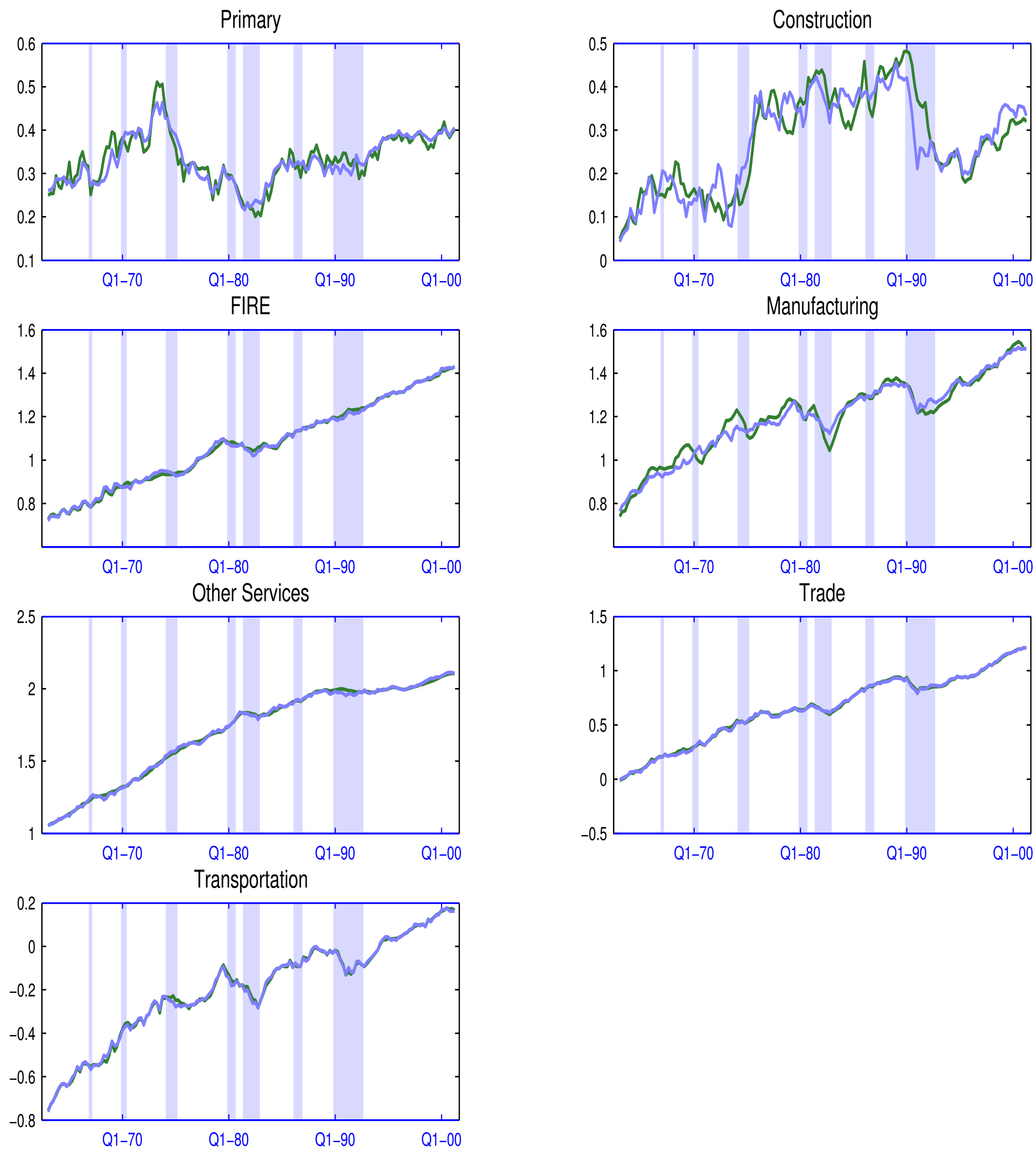
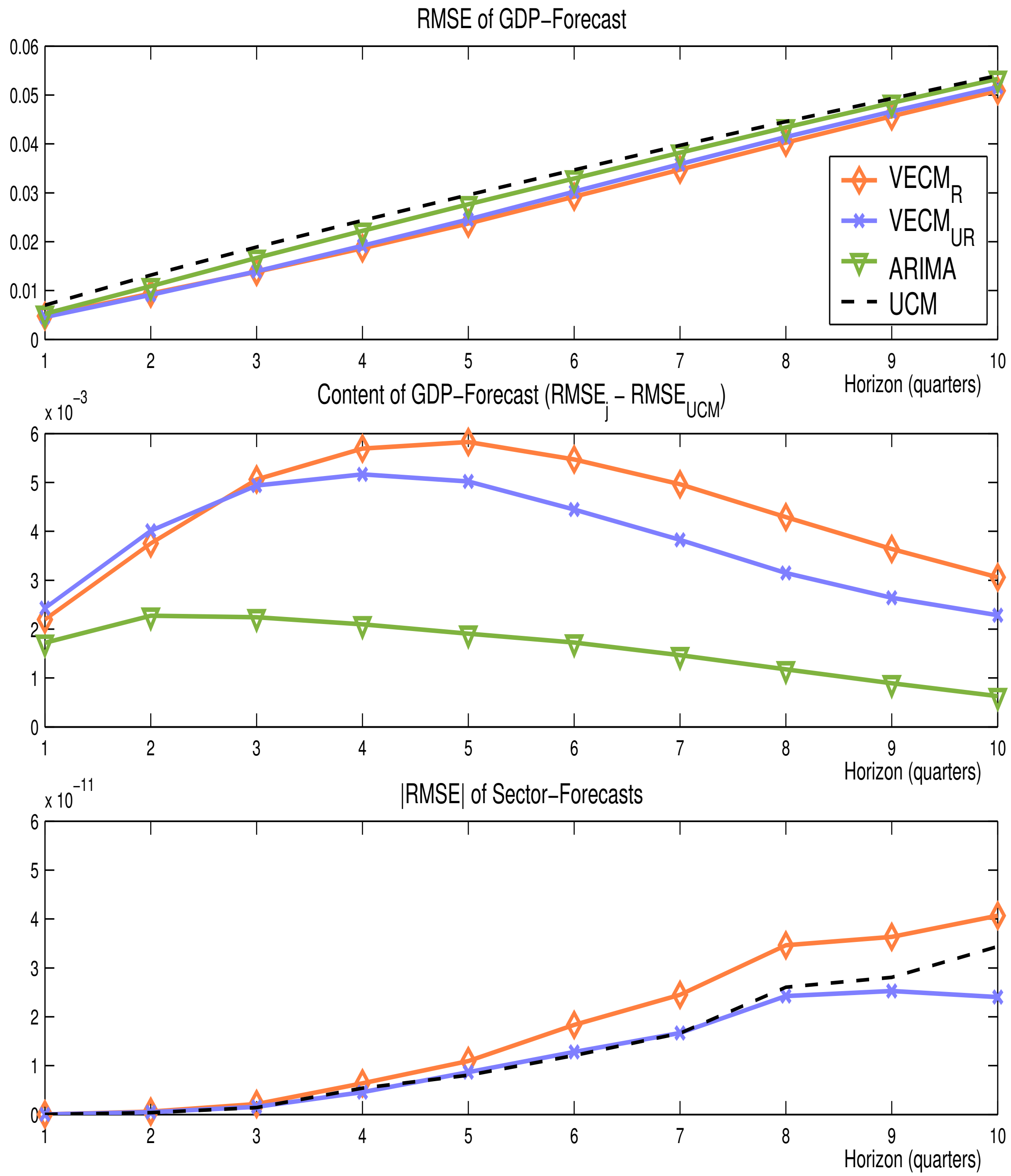


Figure 4: Root-Mean-Squared Errors (RMSE) of Out-of-Sample Forecasts (UCM is the forecast based on the unconditional mean)



Appendix A: Restrictions Implied by SCM$(0, q)$

This appendix illustrates the restrictions codependent cycles impose on VECMs. For simplicity, we limit ourselves to the case with two lags from our empirical analysis; the generalization to p lags is discussed in Schleicher (2003). The basic model is given by[20]:

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Delta y_{t-1} + \Gamma_2 \Delta y_{t-2} + u_t, \tag{A-1}$$

where Π can be factored as $-\beta\alpha'$. The s_q-dimensional scalar component model SCM$(0, q)$ satisfies

$$\begin{aligned} \tilde{\alpha}_q' \Delta y_t &= \tilde{\alpha}_q' \Pi y_{t-1} + \tilde{\alpha}_q' \Gamma_1 \Delta y_{t-1} + \tilde{\alpha}_q' \Gamma_2 \Delta y_{t-2} + u_t \\ &= \tilde{\alpha}_q' u_t + \sum_{j=1}^{q} \tilde{\alpha}_q' \Theta_j u_{t-j}, && \text{(A-2)} \\ \tilde{\alpha}_q' \Theta_q &\neq 0. && \text{(A-3)} \end{aligned}$$

For the SCM$(0, 0)$, which is identical to the SCCF, these restrictions are given by

$$\begin{aligned} \tilde{\alpha}_0' \beta &= 0 \qquad (s_0 r \text{ equations}) \\ \tilde{\alpha}_0' \Gamma_1 &= 0 \qquad (s_0 N \text{ equations}) \\ \tilde{\alpha}_0' \Gamma_2 &= 0 \qquad (s_0 N \text{ equations}). \end{aligned}$$

Because we introduce $s_0(N - s_0)$ additional parameters in the cofeature vectors (after normalization), we have a net loss of $s_0(pN + r) - s_0(N - s_0)$ degrees of freedom.

To obtain the restrictions for SCM$(0, 1)$, we substitute the right-hand side of Δy_{t-1} into (A-1) to obtain

$$\Delta y_t = \Pi y_{t-1} + \Gamma_1 \Pi y_{t-2} + (\Gamma_1^2 + \Gamma_2) \Delta y_{t-2} + \Gamma_1 \Gamma_2 \Delta y_{t-3} + u_t + \Gamma_1 u_{t-1}. \tag{A-4}$$

The linear combination $\tilde{\alpha}_1 \Delta y_t$ will be a VMA(2) (condition (A-2)) if (and only if)

$$\tilde{\alpha}_1' \beta = 0 \qquad (s_1 r \text{ equations})$$

[20] The constant is not affected by codependence restrictions; therefore, it is omitted for ease of exposition.

$$
\begin{aligned}
\tilde{\alpha}_1'\Gamma_1\beta &= 0 \qquad (s_1 r \text{ equations}) \\
\tilde{\alpha}_1'(\Gamma_1^2+\Gamma_2) &= 0 \qquad (s_1 N \text{ equations}) \\
\tilde{\alpha}_1'\Gamma_1\Gamma_2 &= 0 \qquad (s_1 N \text{ equations}).
\end{aligned}
$$

If $\tilde{\alpha}_1'\Gamma_1 = 0$, this collapses to the SCM$(0,0)$ scenario. If, on the other hand, $\tilde{\alpha}_{1,j}'\Gamma_1 \neq 0$ (condition (A-3)), we have $s_1(pN+2r)$ additional restrictions, while gaining $s_1(N-s_1) - 2s_0s_1$ additional parameters in the cofeature vectors.

Similary, we can obtain restrictions for SCM$(0,2)$ by substituting the right-hand side of Δy_{t-2} into (A-4) to obtain

$$
\begin{aligned}
\Delta y_t \;=\; & \Pi y_{t-1} + \Gamma_1 y_{t-2} + (\Gamma_1^2+\Gamma_2)\Pi y_{t-3} + (\Gamma_1^3+\Gamma_2\Gamma_1+\Gamma_1\Gamma_2)\Delta y_{t-3} \\
& +(\Gamma_1^2\Gamma_2+\Gamma_2^2)\Delta y_{t-4} + u_t + \Gamma_1 u_{t-1} + (\Gamma_1^2+\Gamma_2)u_{t-2}. \qquad \text{(A-5)}
\end{aligned}
$$

The linear combination $\tilde{\alpha}_2\Delta y_t$ will be a VMA(3) (condition (A-2)) if (and only if)

$$
\begin{aligned}
\tilde{\alpha}_2'\beta &= 0 \qquad (s_2 r \text{ equations}) \\
\tilde{\alpha}_2'\Gamma_1\beta &= 0 \qquad (s_2 r \text{ equations}) \\
\tilde{\alpha}_2'(\Gamma_1^2+\Gamma_2)\beta &= 0 \qquad (s_2 r \text{ equations}) \\
\tilde{\alpha}_2'(\Gamma_1^3+\Gamma_2\Gamma_1+\Gamma_1\Gamma_2) &= 0 \qquad (s_2 N \text{ equations}) \\
\tilde{\alpha}_2'(\Gamma_1^2\Gamma_2+\Gamma_2^2) &= 0 \qquad (s_2 N \text{ equations}).
\end{aligned}
$$

If $\tilde{\alpha}_2'\Gamma_1 = 0$ or $\tilde{\alpha}_2'(\Gamma_1^2+\Gamma_1) = 0$, this set of restrictions corresponds to the SCM$(0,0)$ and SCM$(0,1)$ case, respectively. If, on the other hand, $\tilde{\alpha}_{2,j}'\Gamma_1 \neq 0$ and $\tilde{\alpha}_{2,j}'(\Gamma_1^2+\Gamma_2) \neq 0$ (condition (A-3)), we have $s_2(pN+3r)$ additional restrictions, while gaining $s_2(N-s_2) - 2s_2(s_0+s_1)$ additional parameters in the cofeature vectors.

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