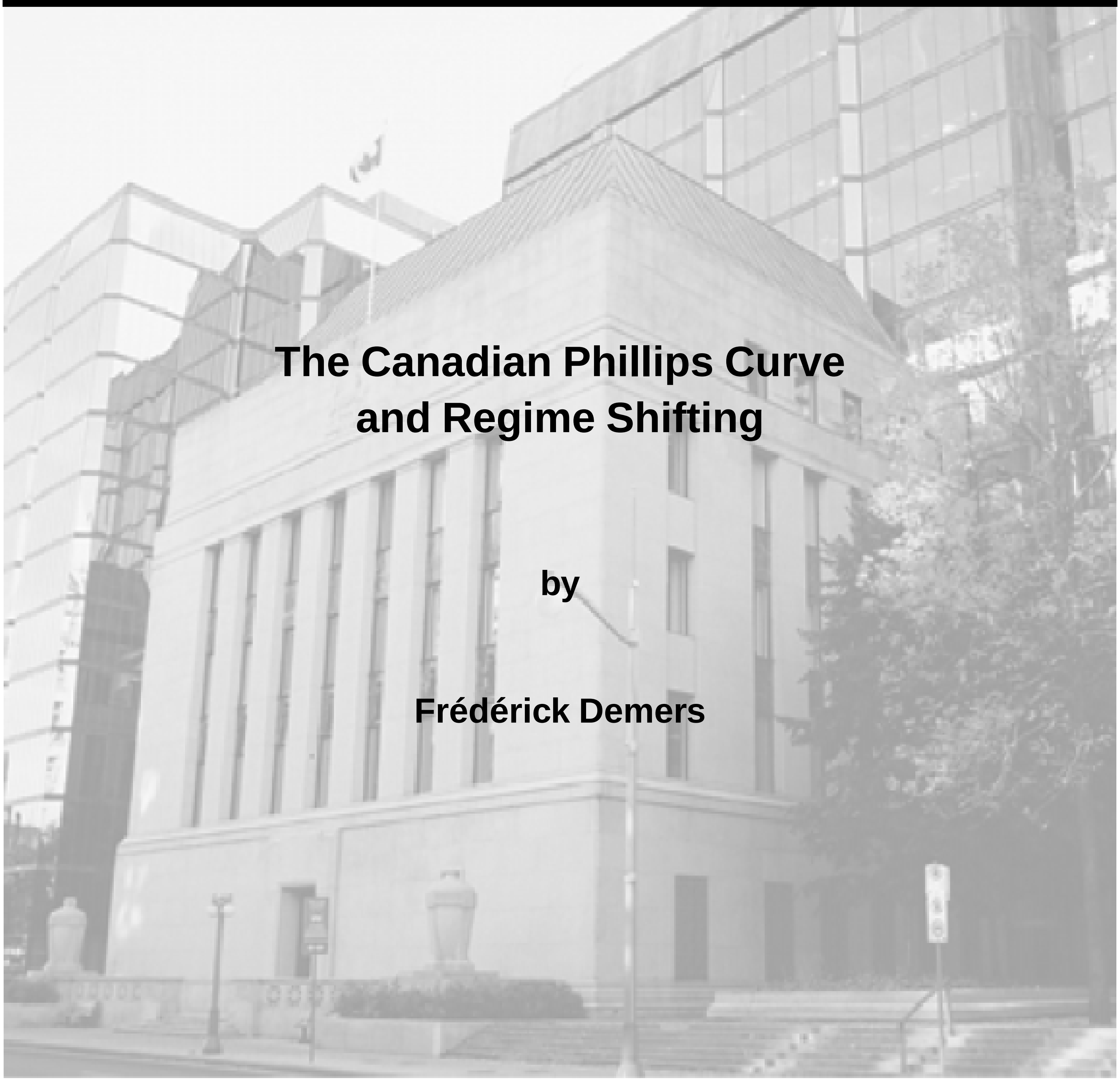
Bank of Canada Banque du Canada

Working Paper 2003-32 / Document de travail 2003-32



The Canadian Phillips Curve and Regime Shifting

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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2003-32

October 2003

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Contents

Acknowledgements

Special thanks for the useful conversations and comments from Allan Crawford, Richard Dion, Paul Fenton, Maral Kichian, René Lalonde, Richard Luger, and Gabriel Rodríguez. Thanks to seminar participants at the Bank of Canada, the Australasian Meeting of the Econometric Society (Sydney, July 2003), and the XXXVII annual meeting of the Canadian Economic Association (Ottawa, June 2003) for their comments.

Abstract

Phillips curves are generally estimated under the assumption of linearity and parameter constancy. Linear models of inflation, however, have recently been criticized for their poor forecasting performance. The author investigates the linearity and constancy assumptions of a standard reduced-form Phillips curve for Canada using two different techniques: (i) the methodology proposed by Bai and Perron (1998), which allows for an unknown number of breaks at unknown dates, and (ii) a three-regimes Markov-switching regression model. Both methodologies strongly reject the linearity and parameter constancy assumptions. The author finds that the output-inflation relationship does not hold under the current monetary policy of inflation targeting, with its low and stable inflation. Since the inflation-control targets were adopted, inflation expectations appear to be more forward looking and well anchored at 2 per cent, the official target rate. Core inflation exhibits very low persistence and there do not appear to be significant asymmetries in the inflation response to output-gap shocks within regimes. Generalized impulse responses are computed to illustrate some properties of the Markov-switching Phillips curve model.

JEL classification: C52, E31
Bank classification: Econometric and statistical methods; Inflation and prices

Résumé

On estime en général les courbes de Phillips sans remettre en question leur linéarité et la stabilité de leurs paramètres. Toutefois, les modèles linéaires de l'inflation ont récemment été critiqués pour l'imprécision de leurs prévisions. S'appuyant sur le cas standard d'une courbe de Phillips de forme réduite estimée pour le Canada, l'auteur explore les postulats de linéarité et de stabilité des paramètres à l'aide de deux techniques différentes : i) la méthodologie de Bai et Perron (1998), qui permet d'intégrer un nombre indéterminé de ruptures à des dates inconnues; ii) un modèle de régression markovien comportant trois régimes. Ces deux méthodologies rejettent d'emblée les postulats de linéarité et de stabilité des paramètres. L'auteur constate que la relation entre la production et l'inflation ne tient plus depuis que la politique monétaire est axée sur la poursuite de cibles de maîtrise de l'inflation et que celle-ci est maintenue à un niveau bas et stable. Depuis l'instauration de cibles en la matière, les attentes semblent plus prospectives et bien ancrées autour du taux cible officiel de 2 %. Les fluctuations de l'inflation mesurée par l'indice de référence ont très peu d'effets persistants, et la réaction de l'inflation aux variations de l'écart de production, à l'intérieur de chacun des régimes, ne présente pas d'asymétries significatives. Pour illustrer certaines propriétés du modèle de Markov relatif à la courbe de Phillips, l'auteur simule certains chocs afin de générer des profils de réaction généralisés.

Classification JEL : C52, E31
Classification de la Banque : Méthodes économétriques et statistiques; Inflation et prix

1 Introduction

Recently, statistical backward-looking Phillips curves have been strongly criticized for their poor forecasting performance. This paper investigates this shortcoming by using various approaches to analyze the parameter constancy and the symmetry assumptions of the Canadian Phillips curve. As Clements and Hendry (1999) comment, omitted structural changes and/or any other types of non-linearities can lead to a poor forecast performance.

Fillion and Léonard (1997) address the issue of non-linearity as a potential source of structural changes by introducing into the Phillips curve exogenous breaks obtained from a Markov-switching autoregressive (AR) model of inflation. Similarly, Dupasquier and Ricketts (1998b) consider a time-varying parameter model with threshold effects for the output-gap coefficient to examine the response of inflation to positive/negative output-gap shocks. They find that it is less costly to reduce inflation when the economy is in excess demand than when it is in excess supply. Huh and Lee (2002), using a vector autoregressive (VAR) model with threshold effects, and modelling inflation as an integrated process, find that the response of the change in inflation to a change in output growth depends on the state of the economy, the magnitude of the change in inflation, and whether monetary policy aims for a reduction of inflation or simply for containment of inflation. Khalaf and Kichian (2003) investigate the nature of the instability of the Canadian Phillips curve using exact testing methods and a time-varying parameter model. They report that there is evidence in favour of two (linear) breaks in addition to stochastic and continuous breaks in the inflation dynamics. Using various techniques and specifications, non-linearities in the U.S. Phillips curve have recently been investigated by Clark and McCracken (2003), Fauvel, Guay, and Paquet (2002), Hooker (2002), Hamilton (2001), Eliasson (1999), and Hillman (1998), among others.

In this paper, the possible presence of non-linearities and asymmetries in the Phillips curve is reconsidered, with a focus on the inflation-output relationship. The aim is to extend the work of Fillion and Léonard (1997) by (i) estimating a Phillips curve with an unknown number of endogenous structural changes following the methodology of Bai and Perron (1998); (ii) building upon (i) and using a more flexible methodology that consists of estimating the Phillips curve in a Markov-switching framework; and (iii) incorporating threshold effects into the Markov-switching model. For the Markov-switching approach, a three-regimes model is used that allows for the population parameters, including the coefficients on the exogenous variables, to switch according to an unobserved state variable that is governed by a first-order Markov-chain process. The results, using either method, show clear evidence of both instability and switching behaviour, and show that the inflation-output short-run relationship does not hold under the current monetary policy of inflation targeting. Since the adoption of the inflation-control targets, however, inflation expectations appear to be more forward looking and well anchored at 2 per cent, the official target rate. As for the threshold effects on the output gap — or the convexity of the Phillips curve — the statistical evidence in favour of such asymmetries is rather weak, despite the relatively large coefficients that are obtained. Finally, the high degree of persistence commonly found in Phillips curve estimates appears to be generated by a number of discrete shifts in mean, suggesting, as Khalaf and Kichian (2003) argue, that the AR process of inflation is far from the unit root hypothesis once structural changes are accounted for.

This paper is organized as follows. Section 2 identifies the data used in this study. Section 3 describes the estimation and some diagnostic tests that result from a benchmark model. Section 4 reports the estimation results using the methodology proposed by Bai and Perron (1998). Section 5 provides the results based on a Markov-switching Phillips curve, and section 6 reports the results for threshold effects in the inflation-output gap relationship. Section 7 performs some simulations and computes the generalized impulse-response function of the Markov-switching model. Section 8 offers some conclusions and suggestions for further research.

2 Data

Inflation, denoted as π_t, is constructed[1] as follows: from 1964Q1 to 1984Q1, π_t is the log difference (multiplied by 100) of the consumer price index (CPI) measure, which excludes the eight most volatile components, as defined by the Bank of Canada; from 1984Q1 to 2002Q1, π_t is the log difference of the core CPI currently used at the Bank. This core measure, explained in detail by Macklem (2001), excludes the eight most volatile components and the effect of variations in indirect taxes. The output gap, denoted as y_t, is the measure estimated by the Bank's staff (Butler 1996). To address the issue of pass-through effects, a measure of imported inflation is used, defined as an eight-quarter moving average of the change in the Canada-U.S. exchange rate plus the underlying rate of inflation in the United States (i.e., the CPI excluding food and energy prices), denoted as π_t^*. The first difference of the effective indirect tax rate is also used, denoted as τ_t. Figure 1 depicts π_t and y_t (normalized data) over time.

3 Benchmark Model

This section presents the estimation results of a simple linear backward-looking Phillips curve, defined as

$$\pi_t = c + \phi(L)\pi_t + \beta(L)y_t + \gamma(L)\pi_t^* + \lambda(L)\tau_t + u_t, \qquad (1)$$

[1] This construction is due to limited data availability.

where $\phi(L)$, $\beta(L)$, $\gamma(L)$, and $\lambda(L)$ are k_i'th order polynomials in the lag operator; u_t is the innovation that is orthogonal to all available information. The sample period is from 1964Q4 to 2002Q1. Table 1 reports the ordinary least squares estimates of (1), with $k = 2$ for π_t, and $k = 1$ for the exogenous variables.[2] Since the subsequent sections consider a version of (1) where γ and λ are restricted to be equal to 0, Table 1 also reports the results of a restricted version of (1) for comparison. Moreover, to assess the presence of a possible non-linear response of inflation to output-gap shocks, Table 1 also reports results from a linear model with threshold effects. The unrestricted model is therefore labelled as (1.1), the restricted linear model is labelled as (1.2), and the threshold model is labelled as (1.3).[3]

The results from (1.1) and (1.2) show that the point estimate for the coefficient on the output gap, at 0.23, is close to that found by Fillion and Léonard. The long-run response of inflation to a 1 percentage point output-gap shock, given by $\beta/(1 - \phi_1 - \phi_2)$, is 1.3. Moreover, the sum of the AR coefficient is high (0.83), and shows that inflation is quite persistent when a linear model is considered. The p-value of rejecting the null hypothesis that $\gamma = \lambda = 0$ is 0.096, which suggests that pass-through effects or changes in the indirect tax rate have (jointly) little explanatory power for core inflation. Although the Lagrange multiplier (LM) test for serial correlation suggests that the residuals are free of serial correlation (at the 5 per cent significance level), important autoregressive conditional heteroscedasticity (ARCH) effects still remain according to the LM test. Furthermore, the stability assumption (for all the parameters of the model) is strongly rejected by Andrews' (1993) sup$F$ test. This suggests that the Canadian Phillips curve is not generated by a linear relationship and that some kind of non-linearities could be present.

4 Models of Multiple Structural Change

This section draws on the methodology proposed by Bai and Perron (1998, hereafter BP), which can be used to estimate models with multiple unknown structural breaks simultaneously. In general, models ignore the possibility that the dynamics of the data or the relationship between a set of variables may have changed over time. Until recently, econometric theory was developed only for the presence of a single known break (Chow 1960). Following the earlier work of Quandt (1960), Banerjee, Lumsdaine, and Stock (1992), Zivot and Andrews (1992), and Andrews (1993), among others, show the importance of treating the break point as unknown, rather than arbitrarily fixing the date. The BP methodology explicitly treats the number of break points and their location as

[2] To select k, a t-sig rule is used (based on the 10 per cent significance level) in a general-to-specific framework. For the estimation, it is assumed that $\pi_t, \pi_t^* \sim I(0)$. In fact, this can be confirmed using Zivot and Andrews' (1992) unit root test.

[3] Results from (1.3) are discussed in section 6. Standard errors are adjusted using the parametric heteroscedastic autocorrelation consistent (HAC) based on a VAR proposed by den Haan and Levin (1996).

unknown, endogenous to the data.[4] In doing so, it avoids the important issue of data mining that comes with imposing the break point upon the data.

Most economic models today still assume parameter constancy, which makes them subject to the Lucas critique by imposing the view that economic agents respond in the same manner to shocks over time, irrespective of changes in policy regime. Obviously, omitted parameter inconstancies can generate serious consequences for policy-making if the relationships are over/underestimated. The goal in this paper is to test the hypothesis that the Canadian Phillips curve has undergone a number of structural changes since the 1960s, possibly as a result of monetary policy changes.

Consider the following model[5]:

$$\begin{aligned} \pi_t &= c_1 + \beta_1 y_{t\text{-}1} + \phi_1 \pi_{t\text{-}1} + \gamma_1 \pi^*_{t\text{-}1} + \lambda_1 \tau_{t\text{-}1} + e_t, \quad t = 1, ..., T_1 \\ &\vdots \\ \pi_t &= c_m + \beta_m y_{t\text{-}1} + \phi_m \pi_{t\text{-}1} + \gamma_m \pi^*_{t\text{-}1} + \lambda_m \tau_{t\text{-}1} + e_t, \quad t = T_m + 1, ..., T, \end{aligned} \tag{2}$$

where e_t is white noise but does not need to be homoscedastic across regimes. There can be m break points, denoted as $\{T_j\} = (T_1, ..., T_m)$, which minimize the squared sum of the residuals over the whole sample period. To apply the test, the sample size must be trimmed by some factor, say α, which implicitly defines the minimal length of each segment: $q = \text{integer}[\alpha T]$. Hence, each segment has a minimal length of q and q observations are removed from each end of the sample. The BP procedure allows for various values for α, ranging from 5 to 30 per cent. Given the sample size, α is set equal to 0.2, which yields a q of 30. Hence, up to four regimes can generate the Phillips curve. Under the parameterization given by (2), where lagged values of inflation are used as regressors, the break does not occur immediately (innovative), but appears only gradually (additive). Standard errors are calculated on the assumption that the residuals are heteroscedastic across regimes.

The BP estimation procedure is based upon least-square principles. To determine the number of break points, the sequential procedure suggested by BP is used. Using the supremum of the F-statistic, labelled as sup$F(l+1|l)$, the first step is to test the null hypothesis that there is $l = 0$ break; if the null of l break is rejected in favour of the $l+1$ breaks alternative, the test is applied to each subsample, and so on, until rejection fails. This is known as the "sequential procedure." Appropriate asymptotic critical values, which depend on α and on the number of regressors, are tabulated in BP. Although the break could be partial (i.e., only a subset of θ, the set of population parameters, could be shifting), only the alternative hypothesis that θ is shifting at time T_j is considered.

[4] The term *endogenous* strictly implies data dependence.

[5] Note that the indexes of the parameters do not relate to the lag polynomial but to the regime.

4.1 Empirical results

Because of the low significance of the imported inflation measure and the change in the indirect tax rate, a restricted version of (2) is estimated, where $\gamma_i = \lambda_i = 0$. Hence, the unrestricted version is labelled (2.1) and the restricted version is labelled (2.2). Table 2 reports estimation results.[6]

For model (2.1), the break points are located in 1973Q2, 1982Q2, and 1991Q2 with the following calculated sup$F(l+1|l)$ statistics: sup$F(1|0) = 51.16$, sup$F(2|1) = 19.64$, and sup$F(3|2) = 44.62$.[7] Note that the first break point coincides with the initial stages of the first oil-price shock, while the third break point captures the Bank's adoption of the inflation target in February 1991. For the first regime (1964Q2 to 1973Q2), only the constant and the output-gap coefficient are statistically significant. The long-run response of inflation to an output-gap shock is low, at 0.58. The results are somewhat similar for the second regime (1973Q3 to 1982Q2), with a noticeable exception: the persistence of inflation, measured by ϕ, increases to 0.329 and is significant at the 1 per cent significance level. Meanwhile, the long-run response of π_t to $y_{t\text{-}1}$ remains low at 0.41, but it is only about two-thirds of the response in regime 1. For the third regime (1982Q3 to 1991Q2), only the constant and the lagged inflation are significant. During the fourth regime (1991Q3 to 2002Q1), only a constant of 1.7 helps to forecast inflation. In fact, since the adoption of an inflation target, the average quarterly (annualized) growth rate of core inflation has been hovering around 1.8 per cent.

Under the restriction that $\gamma_i = \lambda_i = 0$ (i.e., model (2.2)), the results are very similar, including the location of the break points (1973Q2, 1982Q1, and 1991Q2), with the following calculated statistics: sup$F(1|0) = 65.98$, sup$F(2|1) = 20.98$, and sup$F(3|2) = 49.06$. For both models, diagnostic tests are performed on each segment: the residuals are free of serial correlation and ARCH effects, which suggests that the ARCH effects found in the linear model are spurious. Although the variance of the forecast error is not serially correlated, it is not constant across regimes and, as Ball and Cecchetti (1990) argue, it increases with the level of inflation.

In general, the results implied by the BP methodology, contrary to those suggested by (1), indicate that inflation exhibits very moderate persistence after structural changes are accounted for. Furthermore, the effect of output-gap shocks matters only until 1982; there appears to be very little correlation between inflation and the output gap afterwards. Lastly, there is no statistical evidence that pass-through effects and second-round effects of changes in the indirect tax rate can affect core inflation significantly.

[6] The Gauss codes of BP are used for the estimation.

[7] A single break is detected at the 1 per cent significance level, 1973Q2. Note that the results are unchanged if the trimming is set to 0.15 instead of 0.2. For comparison, the breaks found by Khalaf and Kichian (2003) are located in 1984Q1 and 1991Q2. The location of the break points used by Fillion and Léonard (1997) and by Dupasquier and Ricketts (1998a,b) are also close to the ones reported here.

5 Inflation and Non-Linearity

Following the work of Hamilton (1989), Markov-switching (hereafter MS) models have been used in various scenarios, generally in AR models. An interesting feature of MS models is that the filtered probabilities can be interpreted as the agents' beliefs that the economy is in either one of the possible states that describe the economy. In the BP framework, however, agents are always in one state, with probability 1. Therefore, one of the advantages of the MS approach is that agents do not need to assign a probability of 1 that the economy is evolving in a particular state. They can in fact assign a probability of being in each of the possible states, conditional upon the state in which they were at time $t-1$. For the case considered in this paper, the filtered probabilities are interpreted as the expectations of agents that the central bank will conduct the monetary policy with some mean value for the rate of inflation as the target rate.[8] Similarly, as Ruge-Murcia (1995) suggests, under rational expectations the filtered probabilities are a measure of the credibility of the central bank's monetary policy.

This paper extends the estimation of Ricketts and Rose (1995), who estimate an MS(3)-AR(1) model with the restriction that inflation is characterized by a unit root process when it is in a high-inflation regime. However, such an extreme restriction on the AR coefficient is not imposed, and exogenous variables are included. Hence, MS-AR and MS-ARX models are estimated to illustrate the (possible) differences in the coefficients and in the transition probabilities that can arise only by using a wider set of information.

Various studies have shown that the three-regime MS-AR model can best describe inflation in Canada since the 1960s (e.g., Demers and Rodríguez 2001; Ricketts and Rose 1995). Hence, a three-regime MS-AR model is directly estimated in this paper, and a statistical support is provided for a non-linear Phillips curve (i.e., a MS-ARX model). For parsimony, an AR(1) specification is used:

$$\begin{aligned} \pi_t &= c_{s_t} + \phi_{s_{t-1}} \pi_{t-1} + v_t, \\ v_t &\sim i.i.d.\ N(0, \sigma^2_{v,s_t}). \end{aligned} \tag{3}$$

The unobserved regime-generating stochastic process, s_t, is an ergodic Markov chain of order one where m, the number of states, is defined according to the transition probabilities:

$$p_{ij} = \Pr[s_t = i \mid s_{t-1} = j], \ \sum_{i=1}^{m} p_{ij} = 1 \qquad i, j = 1, ..., m, \tag{4}$$

[8] This interpretation is also done by Ricketts and Rose (1995).

with transition matrix **P**. A logistic form for the p_{ij}'s is assumed:

$$p_{ij} = \exp(\xi_{ij}) / \left(1 + \sum_{j=1}^{m} \exp(\xi_{ij})\right), \tag{5}$$

where the ξ_{ij}'s are real valued; (5) also implies that $0 \leq p_{ij} \leq 1$. The class of model described by (3) corresponds to the one considered by Hansen (1992), where the constant, the AR parameters, and the variance can shift according to the unobservable state variable, s_t.

Additional information, such as indirect tax changes or imported inflation, is also considered. Although this extra information is rich and theoretically pertinent, it provides no significant improvement in the value of the likelihood function.[9] The analysis conducted in this study suggests that only the output gap appears to improve the likelihood value significantly. In effect, as seen in section 4, the coefficients on these variables are generally small and statistically insignificant.[10] For this reason, only the estimation results with the output gap are reported, yielding the following specification:

$$\begin{aligned} \pi_t &= c_{s_t} + \phi_{s_{t-1}} \pi_{t-1} + \beta_{s_{t-1}} y_{t-1} + u_t, \\ u_t &\sim i.i.d.\ N(0, \sigma^2_{u,s_t}). \end{aligned} \tag{6}$$

To initialize the algorithm for the estimation of (3) and (6), a grid search is performed for the vector of population parameters, θ, over some parameter space, $\widetilde{\boldsymbol{\theta}}$.[11] Then, the values are used to obtain the argmax$\{\mathcal{L}\}$, where $\mathcal{L}$ is the log of the likelihood function.

5.1 Empirical results

To reduce the number of free parameters, a number of restrictions are imposed. First, the matrix **P** is set such that

$$\mathbf{P} = \begin{bmatrix} p_{11} & 1 - p_{11} & 0 \\ (1 - p_{22})/2 & p_{22} & (1 - p_{22})/2 \\ 0 & 1 - p_{33} & p_{33} \end{bmatrix}. \tag{7}$$

This parameterization of (7) implies that the switch from the second regime to the third and first regimes is symmetrical and that inflation must go through regime 2 before reaching regime 1 or 3. This set-up reduces the number of free parameters by 3 and has almost no impact on the likelihood value. Also, after some experimentation, it is imposed that inflation is simply a white-noise process

[9] From the perspective of numerical optimization, more variables imply a heavier computational burden and they are thus less appealing.

[10] This is in line with the results of Khalaf and Kichian (2003).

[11] See Kim and Nelson (1999) for details on the maximization of the likelihood function.

with mean c_3 in the third regime, which reduces the number of free parameters by another 2, leaving a total of 13 free parameters to estimate.[12] ML estimation results for various specifications are reported in Table 3.

Since departure from linearity must be supported by the data, Table 3 also reports ML results for a model with two states, labelled (6.1), to test against an alternative three-states model. However, this model does not provide a satisfactory fit of the data and is rejected against the (preferred) alternative MS(3) model (see section 5.2), labelled (6.3). An MS(4) model is not estimated, since the computational cost is high and the MS(3) model provides a satisfactory fit of the data.[13]

From the ML estimation results obtained by estimating (6.2), it is interesting to note that the output gap is significant only in the regime that corresponds to the period of high inflation (regime 1). This fact suggests that the trade-off between output and inflation exists only when the economy is in a state of high and volatile inflation, and that the same inflation-output relationship is not observed when inflation is low and stable.[14] The long-run response of inflation to an output-gap shock is 0.61 during regime 1, whereas it is 0.25 during regime 2, much lower than the estimated response of 1.3 provided by (1.1) or (1.2).

Using the ML results of (6.3), the unconditional rates of inflation are 9.0, 3.8, and 1.8 per cent, respectively. The typical duration of each regime, given by the formula $1/(1 - p_{ii})$, is 32.3, 43.5, and 90.9 quarters for regimes 1, 2, and 3, respectively.

Figure 2 compares the respective transition probabilities of the MS models (3) and (6.2). Interestingly, the filtered probabilities are almost identical, with one exception. In effect, the AR model, (3), signals that the high-inflation regime ends roughly six quarters earlier than suggested by the Phillips curve model, (6.2). In general, it appears that to identify the regimes in π_t, the use of either model gives a good approximation of the process, although conditioning on a *complete* set of information is more appropriate.

Because the output-gap estimate is generated from a two-sided filter, the ML estimates in this study could suffer from the endogeneity of y_t. As a sensitivity analysis, (6.2) is reestimated using a recursive output gap-estimate based on a polynomial deterministic function (up to the fourth order).[15] Based on this alternative measure of y_t, β_1 and β_2 are quite lower (0.21 and 0.14,

[12] This parameterization is consistent with the results reported in Table 2.

[13] In their investigation of U.S. monetary policy, Sims and Zha (2002) argue that most of the improvement in fit is obtained by allowing the variance to change over time and that the improvement in fit from time-varying parameters is rather marginal. To address this issue, models in this study are estimated where a single component can shift, either c, ϕ, β, or σ. For instance, the results of this study (unreported) indicate that the model with a shifting intercept fits the data more appropriately than an MS heteroscedastic model.

[14] This is in line with the results of Dupasquier and Ricketts (1998b).

[15] Results are not reported but are available from the author. To start the recursion of the output-gap estimates, the GDP data from 1961Q1 are used. In general, output-gap estimates based on this method provide a plausible approximation of Canada's business cycle over that period.

respectively) than suggested by the official ex-post measure, and they are both insignificant at standard levels.[16]

A comparison of the results from the linear Phillips curve specification (Table 1) with those of the non-linear specification (Table 3) shows that the persistence of inflation, contrary to the linear specification, is quite low within regimes, which confirms the results obtained by using the BP methodology reported in section 4. The error variances (a proxy for inflation uncertainty) are positively related to the mean of π_t across regimes.

5.2 Testing for linearity

To provide statistical support for the departure of the Phillips curve from the linearity hypothesis, the null hypothesis of linearity must be tested against a non-linear alternative. The first step is to test whether the null hypothesis of linearity can be rejected against an alternative MS(2) specification; then, upon rejection, one can test the null hypothesis that the model is an MS(2) against an alternative that it is in fact an MS(3). Standard likelihood-ratio tests are invalid in this case, however, since some parameters are unidentified under the null hypothesis (e.g., p_{ij}), which causes the information matrix to be singular. Therefore, the upper bound test suggested by Davies (1987) is used to test for the null of m regimes against an alternative hypothesis of $m+1$ regimes.[17]

Thus, using the results from (1.2) and testing them against an alternative (i.e., (6.1)) with two regimes, the null hypothesis of linearity is strongly rejected, since the p-value is 0.0. Similarly, when testing the null of two regimes against an alternative of three regimes (i.e., (6.3)), the null is again rejected with a p-value of 0.048. Hence, according to Davies' test, clear empirical evidence exists that the Canadian Phillips curve is subject to regime switching.

6 Threshold Effects

So far, π_t has been shown to be subject to regime switching, but one can also suppose that the response of inflation to an output-gap shock will be asymmetric within regimes, depending on the sign and the magnitude of the shock. In other words, the Phillips curve could be convex, as many authors have recently argued (e.g., Dupasquier and Ricketts 1998b; Laxton, Rose, and Tambakis 1999). To accommodate potential asymmetries, threshold effects are considered in addition to the Markov-switching behaviour of the Phillips curve. The coefficient on the output gap is allowed to be different, depending on the state of the inflation process and depending on whether y_t is above/below some threshold level, say z.[18] In addition to the non-linearities across regimes, this

[16]These results are in line with the findings of Orphanides and van Norden (2001) who argue that the predictive power of the output gap is overstated when ex-post (or two-sided filters) output-gap estimates are considered.

[17]For an overview of the testing procedure, see Garcia and Perron (1996).

[18]It is assumed that z is common to all regimes; however, this assumption could easily be relaxed.

specification allows for various degrees of asymmetries within regimes. The general specification of the threshold model is defined as

$$\pi_t = c_{s_t} + \phi_{s_{t-1}} \pi_{t-1} + \beta_{s_{t-1}} y_{t-1} + \mathbf{1}(y_{t-1} > z)\psi_{s_{t-1}} y_{t-1} + u_t, \tag{8}$$

where the ψs are also allowed to switch according to s_t; $\mathbf{1}(\cdot)$ is an indicator function such that $\mathbf{1}(\cdot) = 1$ if $y_{t-1} > z$, else $\mathbf{1}(\cdot) = 0$.[19] The threshold parameter, z, is fixed to 0.[20] This implies that inflation has a higher response when the output gap is positive than otherwise.

For comparison, the linear Phillips curve specification is first estimated with threshold effects (labelled (1.3), in Table 1), but the statistical evidence in favour of such non-linearity appears to be weak, judging by the low t-statistic on the threshold parameter (Table 1). Table 4 reports the ML estimation results for two specifications of (8). The first specification, labelled (8.1), is unconstrained; for the second specification, labelled (8.2), the restriction that $\beta_2 = \beta_3 = 0$ is imposed. Results from two additional models are reported to investigate the sensitivity of the results. Hence, in one model $\beta_2 = \beta_3 = \psi_1 = 0$ and in another model there is the additional restriction that $\psi_3 = 0$. These two models are labelled, respectively, (8.3) and (8.4).

For (8.1), the point estimates for the threshold parameters are interestingly high for regimes 1 (0.445) and 2 (0.412), but remain low for regime 3 (0.088). These estimates are not very accurate, however, and they suggest a high degree of uncertainty, judging by the large standard errors attached to them. Even when testing under the (invalid) assumption that the alternative hypothesis is nested under the null, it is impossible to reject the null hypothesis that $\psi_1 = \psi_2 = \psi_3 = 0$ using standard asymptotics. Other coefficients remain basically unchanged from (6.2).

Because the addition of threshold effects yields no significant improvement in the log likelihood value, it can be concluded that non-linearities in the Phillips curve arise essentially through regime switching and not through threshold effects in the response of inflation to output-gap shocks. To the extent that the results of this study are comparable, they are in line with those reported by Dupasquier and Ricketts (1998b), who argue that the inflation-output trade-off increases with the level of inflation because of the presence of menu costs, although, according to our specification, asymmetries appear to be more moderate — and insignificant — than those reported by Dupasquier and Ricketts (1998b).[21]

[19] As an alternative, the indicator function is also defined in terms of $\Delta y_{t-1} > 0$, but the results (unreported) are almost identical. Of course, in the regime of low inflation, regime 3, the number of observations where $y_t > 0$ is quite small relative to the total number of observations in regime 3. This could potentially increase the uncertainty around the estimates of that parameter.

[20] For the selection of z, a grid search is also performed over the interval $[0, 2]$ with an increment of 0.025, but the likelihood value is the same for all values of z. Threshold effects on the measure of imported inflation are analyzed, but they are not reported because the coefficients are also close to 0 and highly insignificant.

[21] For an excellent discussion of menu costs, see Ball and Mankiw (1994).

7 Dynamic Simulations and Impulse-Response Functions

This section presents dynamic simulations based on (1.2), the traditional linear Phillips curve, and on (8.3), the non-linear alternative representation estimated above, to illustrate the appropriateness of the MS Phillips curve over the linear specification. Furthermore, to illustrate the fact that the Phillips curve described by (6.2) or (8.3) does not allow for a *free lunch*, a simple shock analysis is performed using various scenarios. In other words, it is shown that the rate of inflation is not tied to a particular level and could move towards another level if agents came to believe, upon observing the inflation process, that the central bank was changing the inflation target without communicating with the public its change in policy.

7.1 Dynamic simulations

Figure 3 depicts the dynamically simulated fit of (1.2) and (8.3). The ability of (8.3) to mimic well the actual path of inflation is striking compared with the poor forecast obtained with (1.2). The respective root-mean-squared errors (RMSEs) of 2.77 for (1.2) and of 1.56 for (8.3) illustrate well the different forecasting capabilities of the two models.[22] This reinforces the view that the Canadian Phillips curve is indeed subject to regime shifts and that a high degree of persistence does not provide a good approximation of Canada's core inflation rate.

7.2 Impulse-response functions

This section computes the impulse-response function of inflation to an innovation. In non-linear models, however, traditional impulse responses are not valid, since the response to a shock, say δ, is generally asymmetric and depends on a particular history, say $\omega_{t\text{ -}1}$. Here, the generalized impulse (denoted hereafter as GI) function introduced by Koop, Pesaran, and Potter (1996) is used. Using a notation similar to Koop, Pesaran, and Potter, denote the GI for π_t as

$$GI_\pi\left(n, u_t, \omega_{t\text{ -}1}\right) = E\left[\pi_{t+n} | u_t = \delta, \omega_{t\text{ -}1}\right] - E\left[\pi_{t+n} | \omega_{t\text{ -}1}\right], \tag{9}$$

for $n = 1, 2, ..., N$. The conditional expectations of inflation in (9) are estimated as the mean of R realizations of π_{t+n} that are obtained by iterating on the model, with and without using δ to compute the realization of π_t. Then, $(N+1) \times R$ randomly sampled residuals are used to obtain the R realization of π_{t+n} and δ; history- and shock-specific GIs are computed over the interval $\delta/\sigma_{u,s_t} = \pm 3, \pm 2.7, ..., \pm 0.3$.

The GI is also defined over some subset to distinguish between shocks that are generated under the various states of the Markov chain that describe inflation. The GI can be defined as

[22] This study also looks at the forecast from a Phillips curve with four AR lags and four lags on the output gap; the RMSE is 2.17.

$GI_{\pi,i}(n, \mathcal{S}_i, \mathcal{H}_i)$, where $\mathcal{S}$ is the set of all negative or positive shocks that are drawn from either one of the three regimes, whereas $\mathcal{H}$ is the set of shocks that occur while agents believe that inflation is in a particular regime. Hence, there are six different shocks (regime 1, 2, or 3, positive and negative), while the impulse responses are computed over three regimes. To determine the regime in which the expectations of agents are located at the time of the shock, a simple rule is used: if the filtered probability of being in regime i is at least 50 per cent, then the shock is said to occur in regime i. In total, average impulse responses are computed for 18 different scenarios. Finally, N is fixed to 16 and R to 1000.

Figure 4 depicts the GI for the case of positive shocks. If agents believe they are in the regime of high inflation, regime 1, a positive shock has only temporary effects and inflation returns rapidly — in less than six quarters — to the regime 1 unconditional mean of 9 per cent. However, if π_t is in either regime 2 or 3 and the positive shock is generated by regime 1 or 2, then these typical positive shocks result in a regime switch and, therefore, a permanent increase in π_t. If, instead, the shock is generated from regime 3, the effects are temporary in all regimes. Another interesting result is that the responses to positive shocks that are generated by regime 1 have proportionally a greater impact on inflation expectations. For instance, about 50 per cent of the shock is reversed when it is generated by regime 1, whereas 60 per cent is reversed when it is generated by regime 2; virtually all of the shock dissipates when it is generated by regime 3.

Similarly, when the shocks are negative (Figure 5), inflation expectations are not affected and there are no permanent effects when the shock occurs in either regime 2 or 3. In regime 1, negative shocks that are generated from regimes 1 and 2 are reversed by about 60 per cent in both cases, whereas shocks generated by regime 3 are almost fully reversed.

These results are in line with the analyses of Andolfatto and Gomme (2001) in that if the "monetary control error is fairly small" and/or the shocks are sufficiently large, an agent's beliefs will adjust quite rapidly to a change in regime. Ricketts and Rose (1995) report similar results from a simulation exercise that suggests that the response of inflation to a shock depends largely on the probability assigned by agents of being in any of the possible regimes.

8 Summary and Conclusion

This paper has estimated a Phillips curve for Canada, allowing for asymmetries and non-linearities. Using two very different techniques, strong statistical evidence has been found that the process that generates inflation has undergone dramatic changes in Canada since the 1960s, particularly around the year of the first oil-price shock, the 1981-82 recession, and the adoption of an official inflation target in February 1991. It has been shown that, to properly model inflation, it is not sufficient to take into account only mean shifts; a complete shift in the variance-covariance matrix must be allowed to replicate the behaviour of the Canadian core rate of inflation over time. In effect, the results, summarized below, show that from 1992 until 2002 inflation was well anchored at 2 per cent and was almost unaffected by output-gap shocks, even in the rare occasions when excess demand was high. These results should not be interpreted, however, to mean that inflation is not positively affected by demand and supply conditions. On the contrary, the results of this study can be interpreted as further evidence of successful inflation targeting.[23] In other words, inflation expectations have been firmly anchored since the application of the inflation target in Canada and, as the shock analysis shows, innovations generated by the regime of low inflation do not affect inflation expectations permanently, on average. Moreover, the policy instrument, the overnight rate of interest, has been moved in such a way that excess demand has not translated into actual inflation.

Furthermore, having accounted for structural changes in the inflation process, this study has not found evidence that foreign inflation passes through to Canadian core inflation, or that variations in the indirect tax rate help to improve forecasts of core inflation. Empirical evidence suggests that a simple constant was sufficient to forecast inflation during the 1992-2002 period. As Hillman (1998) argues, these results could perhaps differ if the model selection is done on an out-of-sample basis. More work is needed in this area. For future research, it would be interesting to build a small model where the Phillips curve, the IS curve, and the central bank's reaction function are estimated simultaneously in an MS framework. This would allow one to assess the degree of non-linearities present in these three fundamental relationships of the economy.

Summary of Results

Equation	1.2	2.2 (Bai-Perron)				6.3 (Markov-switching)		
Regime	–	1	2	3	4	1	2	3
Response to y_t	1.30	0.67	0.35	0.03	-0.03	0.61	0.25	—
Inflation persistence	0.83	-0.05	0.32	0.54	0.01	0.40	0.18	—
Variance of forecast error	2.56	3.61	3.28	0.51	0.47	3.14	2.37	0.48

[23]Laxton, Rose, and Tambakis (1999) argue that identifying convexity could be difficult when monetary policy is successful. Rowe (2002) also provides an interesting discussion on the optimal monetary policy and the absence of correlation between inflation and indicators when monetary policy is optimal and successful.

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Table 1: Benchmark Model (1964Q4 to 2002Q1)*

Parameter	1.1	1.2	1.3
c	0.646^a (0.228)	0.761^a (0.197)	0.656^a (0.266)
β	0.228^a (0.071)	0.209^a (0.071)	0.120 (0.088)
ϕ_1	0.464^a (0.079)	0.483^a (0.083)	0.480^a (0.074)
ϕ_2	0.362^a (0.082)	0.376^a (0.080)	0.363^a (0.073)
λ	0.272 (0.420)	–	–
γ	0.046^b (0.028)	–	–
β_z	–	–	0.330 (0.303)
$\sup F$	51.16	65.98	–
z	–	–	1.04
σ^2	2.569	2.604	2.560
$\mathcal{L}^{**}$	-285.146	-287.491	-284.822
$\bar{R}^2$	0.753	0.755	0.755

a and b : significant at 1 and 10 per cent, respectively

*HAC standard errors in parentheses

**$\mathcal{L}$ denotes the log of the likelihood value

Table 2: Models of Multiple Structural Change*

Parameters	2.1		2.2	
c_1	3.958^a	(0.670)	3.540^a	(0.574)
β_1	0.681^b	(0.346)	0.703^b	(0.349)
ϕ_1	-0.183	(0.179)	-0.048	(0.137)
γ_1	-0.141	(0.118)	–	
λ_1	0.253	(1.045)	–	
σ_1^2	3.497		3.606	
c_2	6.115^a	(1.316)	6.311^a	(1.336)
β_2	0.276^c	(0.152)	0.242	(0.197)
ϕ_2	0.329^a	(0.132)	0.318^a	(0.136)
γ_2	0.068	(0.062)	–	
λ_2	0.233	(0.52)	–	
σ_2^2	3.126		3.280	
c_3	2.223^a	(0.543)	1.840^a	(0.317)
β_3	0.011	(0.058)	0.013	(0.039)
ϕ_3	0.442^a	(0.133)	0.539^a	(0.193)
γ_3	0.011	(0.043)	–	
λ_3	0.035	(0.420)	–	
σ_3^2	0.496		0.506	
c_4	1.699^a	(0.330)	1.695^a	(0.279)
β_4	-0.019	(0.069)	-0.029	(0.067)
ϕ_4	-0.008	(0.136)	0.013	(0.129)
γ_4	0.012	(0.033)	–	
λ_4	0.268	(0.193)	–	
σ_4^2	0.461		0.471	
$\bar{R}^2$	0.810		0.816	

a, b, and c: significant at 1, 5, and 10 per cent, respectively

For 2.1, breaks are located in: 1973Q2, 1982Q2, and 1991Q2

For 2.2, breaks are located in: 1973Q2, 1982Q1, and 1991Q2

*HAC standard errors are in parentheses

Table 3: ML Estimation Results*

Parameters	6.1	6.2	6.3	3	3.1
p_{11}	0.986^a (0.014)	0.972^a (0.027)	0.972^a (0.027)	0.969^a (0.027)	0.969^a (0.027)
p_{22}	0.987^a (0.013)	0.977^a (0.018)	0.977^a (0.018)	0.977^a (0.018)	0.977^a (0.018)
p_{33}	–	0.989^a (0.014)	0.989^a (0.014)	0.989^a (0.034)	0.989^a (0.034)
ϕ_1	0.658^a (0.066)	0.404^a (0.119)	0.404^a (0.119)	0.389^a (0.117)	0.389^a (0.117)
ϕ_2	0.729^a (0.068)	0.185 (0.122)	0.184 (0.121)	0.252^a (0.117)	0.251^a (0.115)
ϕ_3	–	-0.001 (0.165)	–	0.019 (0.233)	–
c_1	2.441^a (0.538)	5.337^a (1.127)	5.338^a (1.125)	5.714^a (1.134)	5.717^a (1.133)
c_2	0.722^a (0.229)	3.085^a (0.496)	3.088^a (0.491)	2.852^a (0.495)	2.858^a (0.487)
c_3	–	1.743^a (0.293)	1.789^a (0.107)	1.749^a (0.419)	1.784^a (0.105)
σ_1^2	5.091^a (0.879)	3.143^a (0.747)	3.143^a (0.443)	3.626^a (0.845)	3.624^a (0.844)
σ_2^2	0.706^a (0.155)	2.364^a (0.443)	2.367^a (0.213)	2.293^a (0.424)	2.293^a (0.424)
σ_3^2	–	0.474^a (0.105)	0.477^a (0.105)	0.471^a (0.102)	0.47^a (0.102)
β_1	0.124 (0.133)	0.363^a (0.143)	0.364^a (0.142)	–	–
β_2	0.034 (0.051)	0.202 (0.177)	0.206 (0.177)	–	–
β_3	–	-0.025 (0.07)	–	–	–
$\mathcal{L}$	-265.9	-259.1	-259.1	-261.2	-261.6

a: significant at 1 per cent

*Standard errors are in parentheses

Table 4: ML Estimation Results (Threshold Models)*

Parameters	8.1	8.2	8.3	8.4
p_{11}	0.971^a (0.025)	0.969^a (0.027)	0.971^a (0.025)	0.971^a (0.025)
p_{22}	0.977^a (0.018)	0.978^a (0.017)	0.977^a (0.018)	0.977^a (0.018)
p_{33}	0.989 (0.014)	0.989^a (0.014)	0.989 (0.014)	0.989 (0.014)
ϕ_1	0.415^a (0.118)	0.347^a (0.151)	0.415^a (0.118)	0.415^a (0.118)
ϕ_2	0.166 (0.132)	0.219 (0.149)	0.17 (0.128)	0.17 (0.128)
ϕ_3	–	–	–	–
c_1	5.293^a (1.129)	5.605^a (1.163)	5.289^a (1.129)	5.289^a (1.13)
c_2	2.924^a (0.536)	2.767^a (0.574)	2.917^a (0.509)	2.916^a (0.506)
c_3	1.6^a (0.236)	1.772^a (0.114)	1.771^a (0.115)	1.783^a (0.107)
σ_1^2	3.232^a (0.776)	3.264^a (0.798)	3.229^a (0.771)	3.229^a (0.771)
σ_2^2	2.323^a (0.435)	2.264^a (0.414)	2.338^a (0.437)	2.339^a (0.437)
σ_3^2	0.465^a (0.101)	0.472^a (0.103)	0.474^a (0.103)	0.475^a (0.103)
β_1	0.301^b (0.165)	-0.132 (1.07)	0.304^b (0.160)	0.304^b (0.160)
β_2	-0.009 (0.108)	–	–	–
β_3	-0.074 (0.09)	–	–	–
ψ_1	0.445 (0.32)	0.612 (1.324)	–	–
ψ_2	0.412 (1.725)	0.386 (0.276)	0.432 (0.278)	0.433 (0.28)
ψ_3	0.088 (1.716)	0.082 (0.313)	0.082 (0.319)	–
$\mathcal{L}$	-258.78	-258.98	-259.22	-259.24

a and b: significant at 1 and 5 per cent, respectively

*Standard errors are in parentheses.

Figure 1: Inflation vs Output Gap

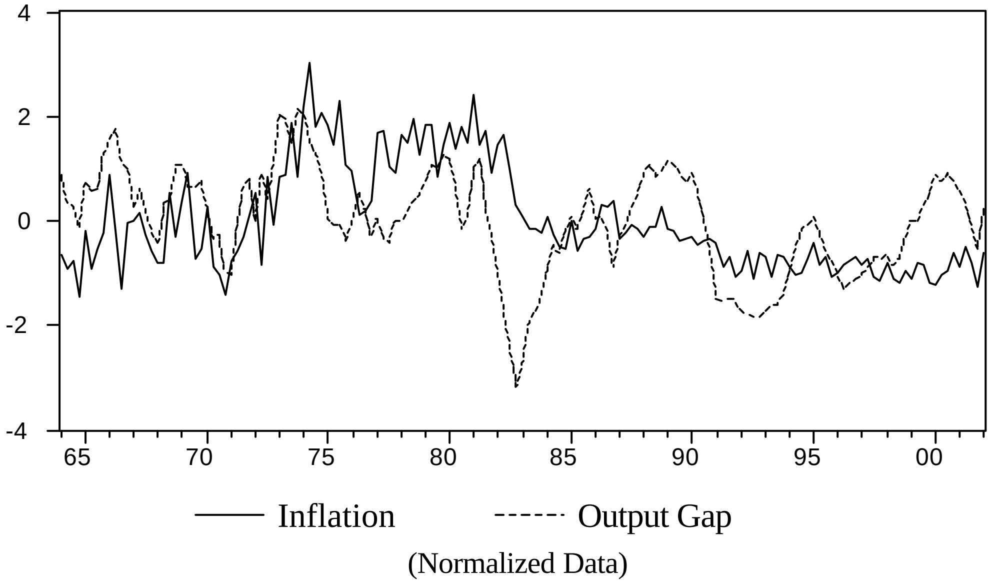


Figure 2: Filtered Probabilities of being in Regime 1, 2, or 3, respectively: 1964Q4 to 2002Q1

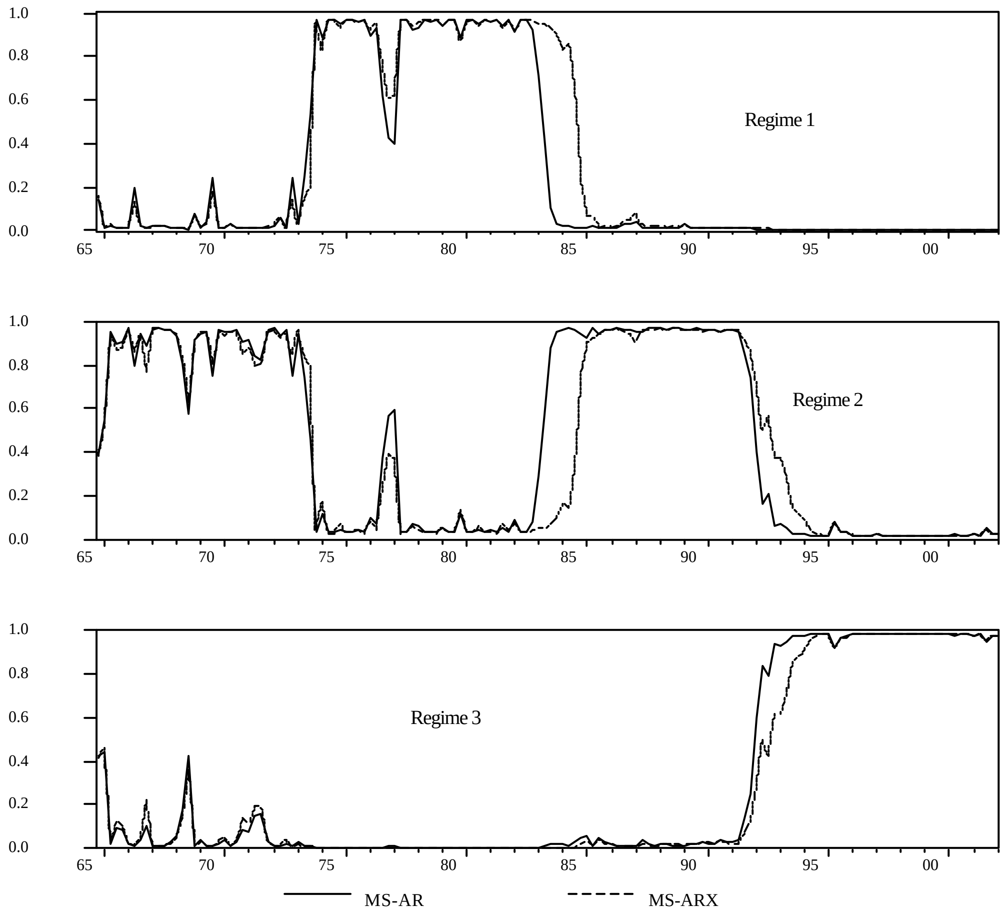


Figure 3: Dynamic Simulations and Actual Inflation

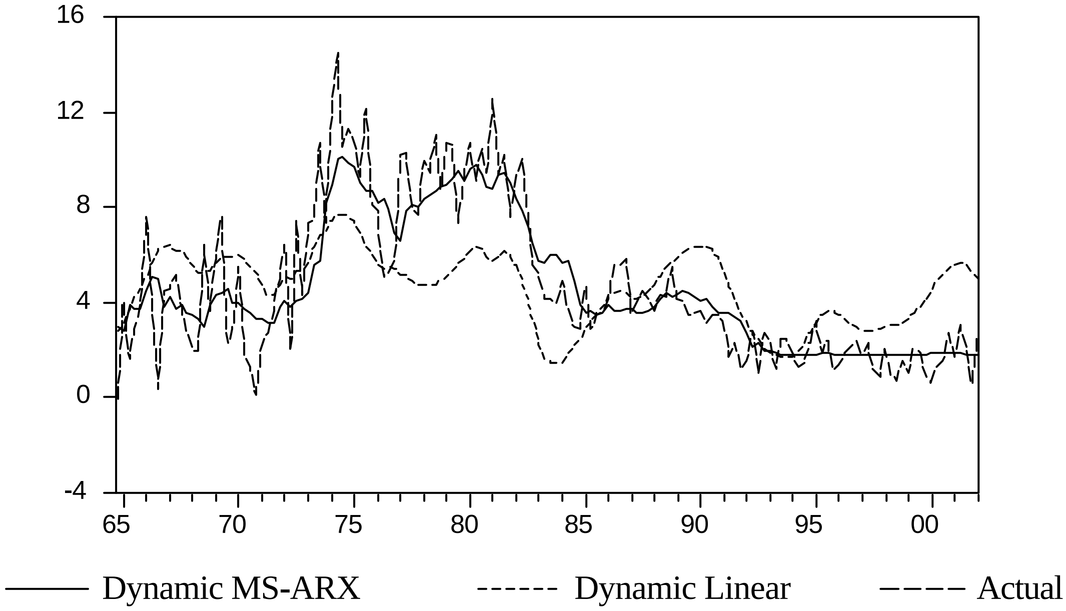


Figure 4: Generalized Impulse Responses to Positive Shocks

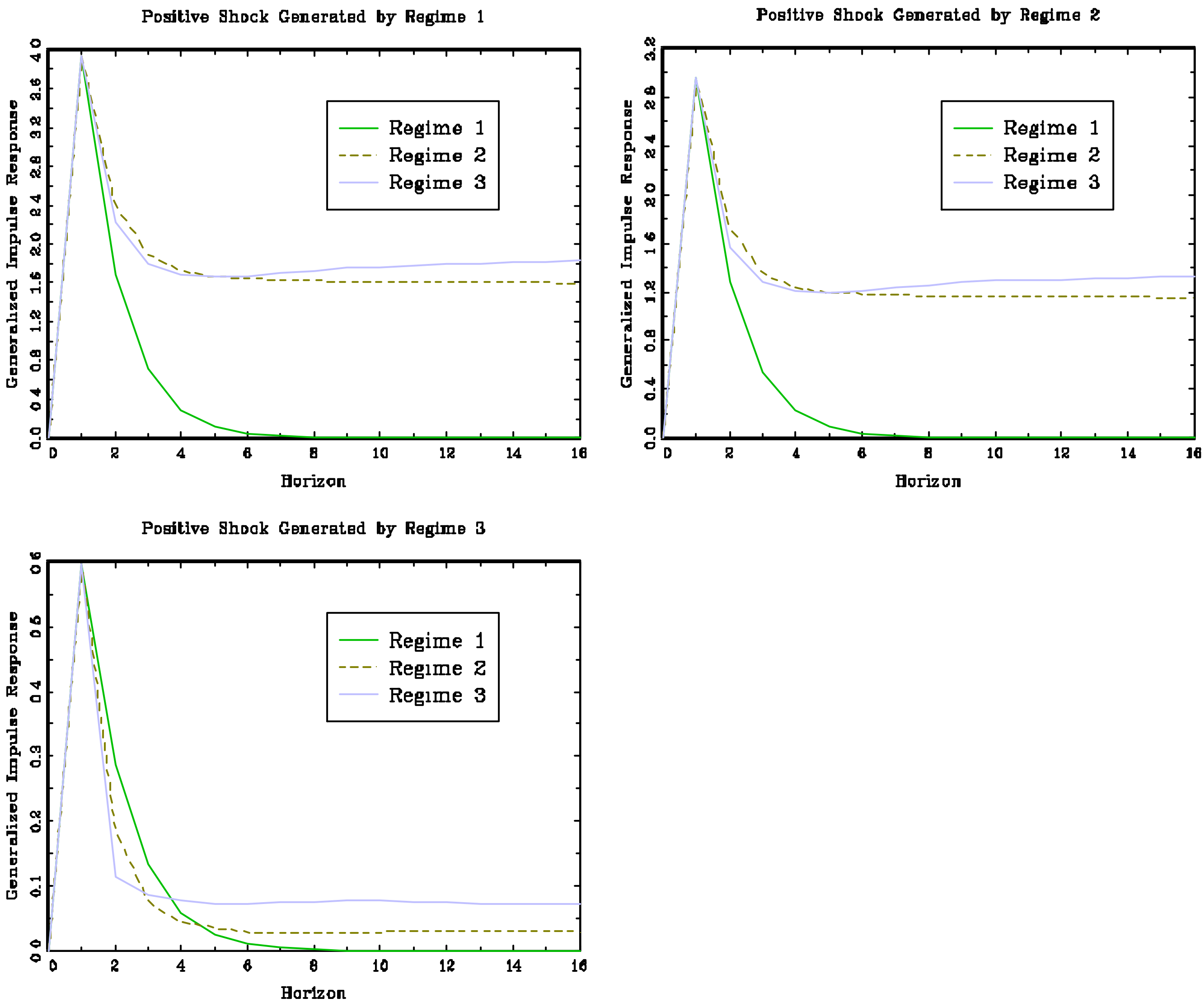
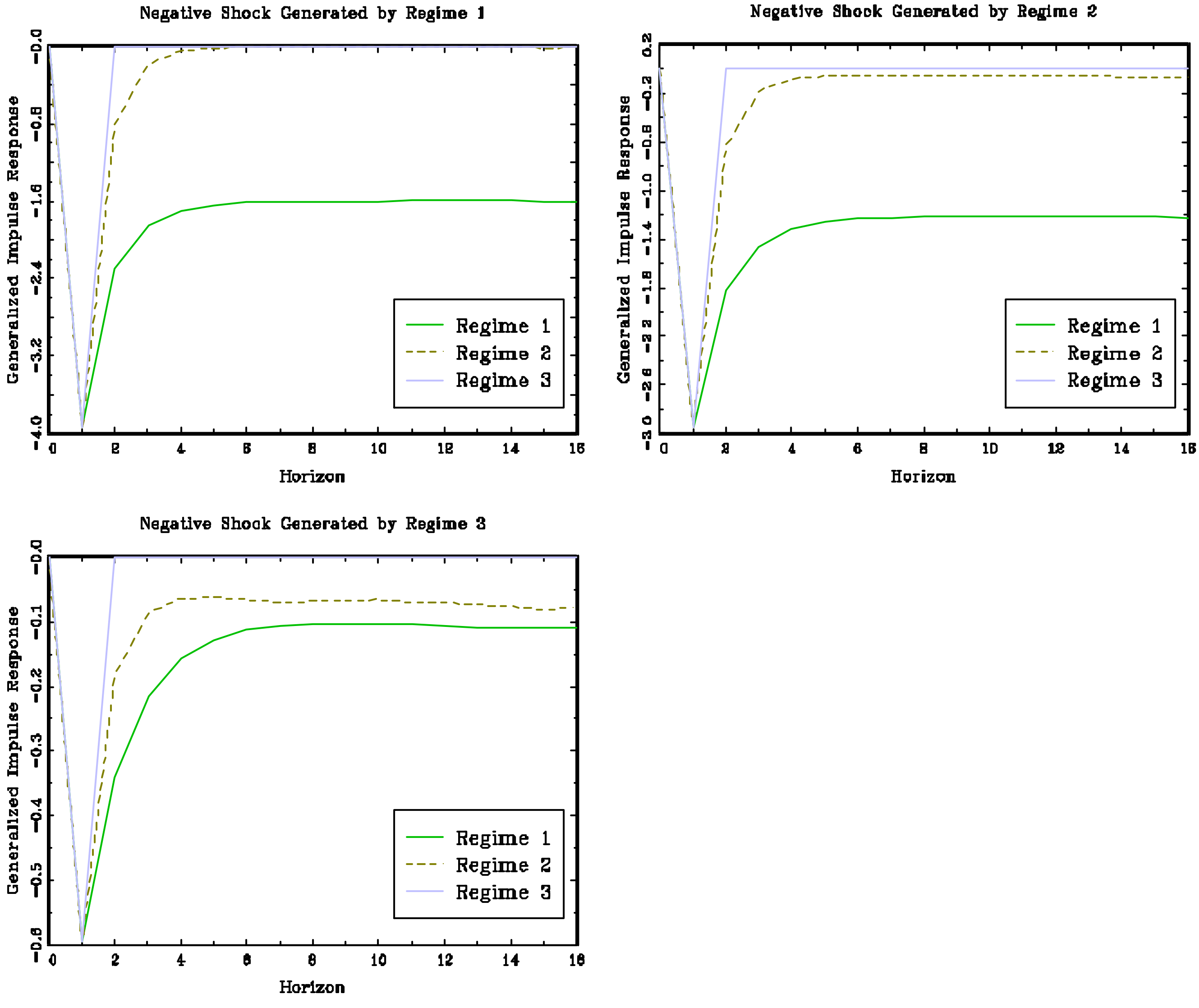


Figure 5: Generalized Impulse Responses to Negative Shocks



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