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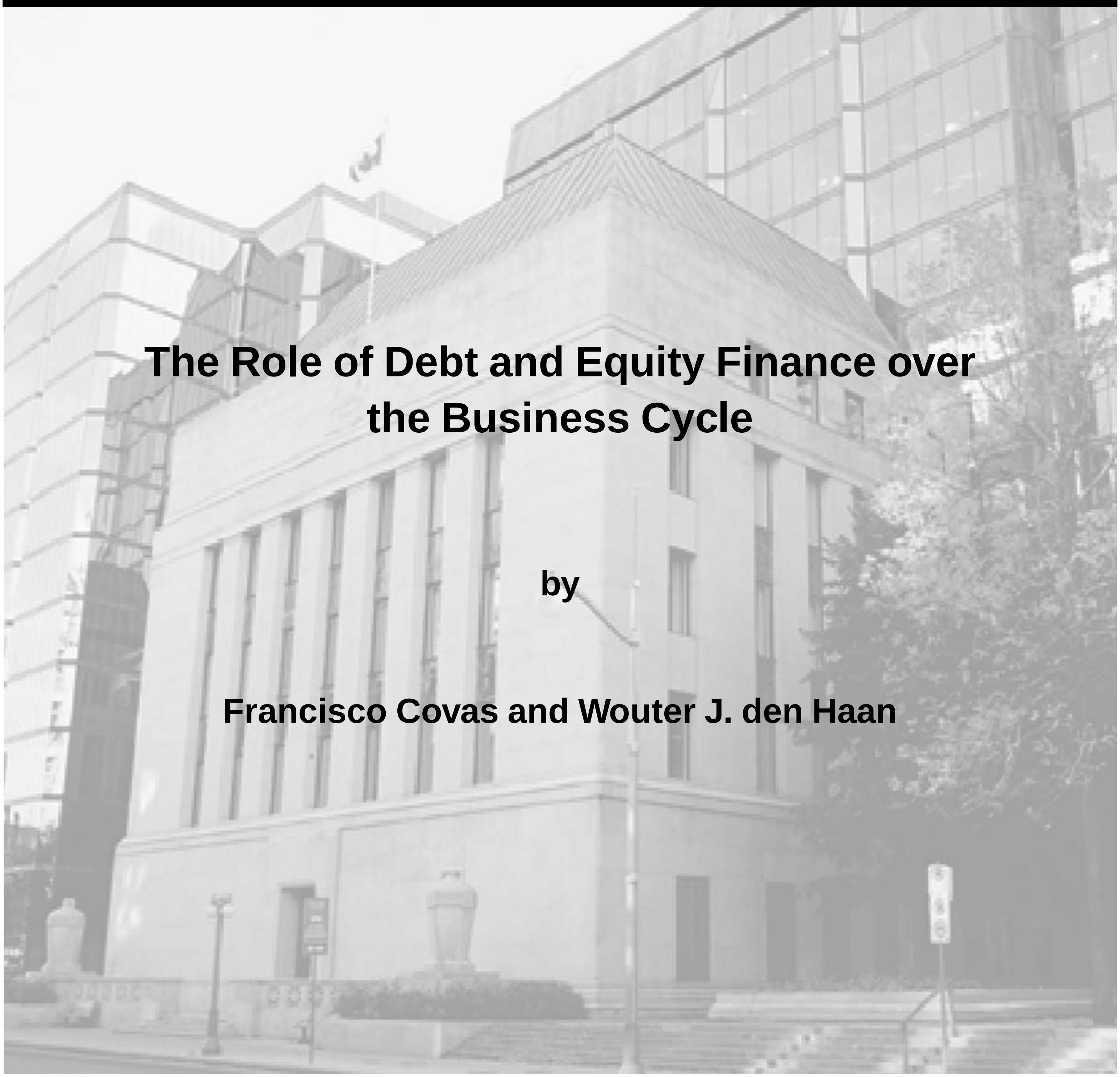

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Francisco Covas[1] and Wouter J. den Haan[2]

[1]Monetary and Financial Analysis Department
Bank of Canada
Ottawa, Ontario, Canada K1A 0G9
fcovas@bankofcanada.ca

[2]Department of Economics
University of Amsterdam
Roetersstraat 11, 1018 WB Amsterdam, The Netherlands
wdenhaan@uva.nl

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Abstract

The authors show that debt and equity issuance are procyclical for most listed U.S. firms. The procyclicality of equity issuance decreases monotonically with firm size. At the aggregate level, however, the authors' results are not conclusive: issuance is countercyclical for very large firms that, although few in number, have a large effect on the aggregate because of their enormous size. If firms use the standard one-period contract, then the shadow price of external funds is procyclical and the cyclicality decreases with firm size. This property generates equity to be procyclical and—as in the data—the procyclicality decreases with firm size. Other factors that cause equity to be procyclical in the model are a countercyclical price of risk and a countercyclical cost of equity issuance. The model (i) generates a countercyclical default rate, (ii) magnifies shocks, and (iii) generates a stronger cyclical response for small firms, whereas the model without equity does the exact opposite.

JEL classification: E3, G1, G3
Bank classification: Financial stability; Business fluctuations and cycles

Résumé

Les auteurs montrent que les activités d'emprunt et d'émission d'actions de la plupart des entreprises américaines cotées sont procycliques, tout en notant que le caractère procyclique de l'émission d'actions s'atténue de façon monotone avec la taille de la firme. À l'échelle de l'ensemble des entreprises, toutefois, les résultats des auteurs ne sont pas concluants : même s'il y a peu de très grandes entreprises, le comportement anticyclique de l'émission d'actions dans leur cas pèse lourd au final en raison de leur énorme taille. Si les firmes ont recours à un emprunt classique d'une période, le prix implicite du financement externe évolue dans le même sens que l'activité économique et son caractère procyclique varie en raison inverse de la taille de l'entreprise. Cette propriété confère à l'émission d'actions un caractère procyclique, qui décroît, tout comme dans les données, avec la taille de la firme. Les autres facteurs à l'origine du comportement procyclique de l'émission d'actions dans le modèle sont le prix du risque (anticyclique) et le coût d'émission d'actions (anticyclique également). Le modèle i) génère un taux de défaillance anticyclique; ii) amplifie les chocs; et iii) engendre une réaction cyclique plus forte parmi les petites entreprises, alors que le modèle sans émission d'actions produit exactement les résultats contraires.

Classification JEL : E3, G1, G3
Classification de la Banque : Stabilité financière; Cycles et fluctuations économiques

1. Introduction

The empirical objective of this paper is to document the cyclical behaviour of firms' external and internal financing sources. In recent papers, Fama and French (2005) and Frank and Goyal (2005) document that firms frequently issue equity. It is therefore important to include equity in such a study. A few papers have studied the cyclical behaviour of aggregate debt and equity finance, but their conclusions differ.[1] In this paper, we use disaggregated data and obtain not only a robust set of results, but also an explanation for the ambiguous findings with aggregate data. Our results can be summarized as follows[2]:

- Debt and equity issuance are procyclical for the majority of publicly listed firms in our sample.
- The procyclicality of equity issuance decreases with firm size.
- Debt and equity issuance are countercyclical for the top 1 per cent of firms. The opposite behaviour for the very largest firms can explain the ambiguous results for aggregate data, because quantitatively these firms are very important.[3]

Existing business cycle models typically assume that net worth can increase only through retained earnings and that external finance occurs through one-period debt contracts.[4] We build a model in which firms can obtain external finance through one-period debt contracts as well as equity. The debt contract specifies a fixed interest payment, which is a tax-deductible expense. If the firm does not make that payment, then the lender gets the remaining resources in the firm minus the bankruptcy costs. We think of default as a distress state in which the reduction in firm value because of bankruptcy costs cannot be avoided through, for example, renegotiation of the contract. These bankruptcy costs imply that the interest rate paid on debt has a premium, which depends on firm characteristics. External finance through the equity contract avoids the bankruptcy premium, but raising equity entails equity issuance costs.

[1] Choe, Masulis, and Nanda (1993) and Korajczyk and Levy (2003) find that equity issuance is procyclical, whereas Jermann and Quadrini (2006) find that equity issuance (minus dividend payments) is countercyclical. Choe, Masulis, and Nanda (1993) find debt issuance to be countercyclical, whereas Jermann and Quadrini (2006) find it to be procyclical. Korajczyk and Levy (2003) find book value leverage to be countercyclical. A more extensive discussion is given in Appendix C.

[2] In Covas and den Haan (2006), we show that the results are very similar when Canadian data are used.

[3] The top 1 per cent of firms cover 18 per cent of gross stock sales, 28 per cent of sales, and 34 per cent of assets in the Compustat data set.

[4] See, for example, Carlstrom and Fuerst (1997), and Bernanke, Gertler, and Gilchrist (1999).

The possible financing sources resemble the two main forms of observed external finance. Our model does not explain why different types of contracts have come into existence. The literature on optimal contracts does exactly this, but it is not well suited to generate predictions about the cyclical behaviour of debt and equity issuance. Biais et al. (2006) derive an optimal contract and show how to implement it with a combination of cash reserves, debt, and equity.[5] The decomposition, however, is not unique and the optimal contract can therefore be implemented with different combinations of cash reserves, debt, and equity. Since our main purpose is to understand the role of debt and equity for business fluctuations, we impose that firms use these two types of contracts.

Besides having debt as well as equity, the models considered have the following characteristics. Firms are ex ante identical, but face a different sequence of idiosyncratic shocks. Firms that default are replaced by new firms with zero assets. Thus, young firms are typically also firms with fewer assets. Firm behaviour is size dependent, because we relax the standard assumption of linear technology. We also avoid the common but unappealing assumption that frictions in obtaining firm finance are present only in the sector that produces investment commodities.

Our starting point is a model in which the one-period debt contract is the only form of external finance. In this framework, shocks are *dampened* and the default rate is *procyclical*. That is, the increase in aggregate productivity induces firms to expand at the cost of a higher default rate. We show that, with diminishing returns in the production function, the increase in net worth that follows the increase in aggregate productivity has a dampening effect on this increase in the default rate, but quantitatively this effect is small. Consequently, the default rate in this model is procyclical, which is counterfactual.[6]

Next, we allow firms to issue equity as well as debt. The friction in obtaining equity finance is characterized by a quadratic function that relates the cost of issuing equity to the amount of equity raised. Equity is procyclical in this framework. The procyclicality of equity is a consequence of a key property of the debt contract. As was mentioned earlier, the expansion following an increase in aggregate productivity goes together with an increase in the default rate. We show that this increases the shadow price of external funds and that this, in turn,

[5] In Townsend (1979), debt is the optimal type of contract. This suggests that equity should not be used, which is counterfactual. In quantitative studies with one-period debt as the only form of external finance, the calibrated bankruptcy cost parameters are based on much more than verification costs (e.g. Carlstrom and Fuerst 1997; Bernanke, Gertler, and Gilchrist 1999), because agency costs have little impact if bankruptcy costs are to be small. With the alternative interpretation of bankruptcy costs, however, the model no longer predicts that debt should be the only form of finance.

[6] The countercyclical behaviour of the default rate is described in Appendix C.

increases the amount of equity issuance. Moreover, this effect is stronger for small firms. Thus, this very simple framework provides an explanation for the observed procyclicality of equity issuance and the dependence on firm size. In our numerical analysis, we show that allowing for equity issuance strongly diminishes the dampening and the countercyclical behaviour of the default rate observed in the model with only debt finance. It cannot, however, overturn them; i.e., there is no magnification and no countercyclical default rate.

Although equity issuance is procyclical in this simple model, its fluctuations are much less volatile than those observed in the data. Retained earnings are also not volatile enough. We add two features to the model that have been emphasized in the literature—a countercyclical price of risk and a countercyclical cost of issuing equity—and we show that these features are effective in generating sufficient volatility in equity issuance and retained earnings. We then show that the model (i) generates a countercyclical default rate, (ii) magnifies shocks, and (iii) generates a stronger cyclical response for small firms for both equity issuance and output. The main shortcoming of the model is that the procyclical response for debt decreases with firm size, whereas in the data the procyclical behaviour of debt is similar across firm categories, except for the group consisting of the very largest firms.

The organization of this paper is as follows. In the next section, we show how the firms' financing sources move over the business cycle. In section 3, we discuss the static version of our model, which is simple enough to derive some analytic results. In section 4, we discuss the dynamic model and document the properties of the model. Section 5 offers some conclusions.

2. Cyclical Properties of Financing Sources

2.1 Data set and methodology

The data set consists of annual Compustat data from 1971 to 2004 for publicly listed firms, except for financial firms and utilities. To study the importance of firm size, we rank firms using the last period's end-of-period book value of asset. We then construct J firm categories and examine the cyclical behaviour of debt and equity for each group $j \in \{1, \ldots, J\}$. A firm group is defined by a lower and an upper percentile. Our firm groups are [0,25%], [0,50%], [0,75%], [0,99%], [90%,95%], [95%,99%], and [99%,100%]. The behaviour of the very largest firms is different from that of the other firms. To understand which large firms behave differently, we consider several groups in the top decile.

Table 1 provides a set of summary statistics for each of these groups. In particular, we find that smaller firms have lower leverage and exhibit higher asset growth. Smaller firms finance a much larger fraction of asset growth with equity, whereas larger firms finance a larger fraction with debt and retained earnings.[7]

In this section, we report results for sale of stock, change in (the book value of) equity,[8] gross issuance of long-term debt, change in liabilities, profits, retained earnings, and dividends.

Our measures for real activity are real GDP and the real value of the group's assets. We use two procedures to construct a cyclical measure for firm finance. In the *flow approach*, the period t observation is the amount of funds raised in period t divided by a trend value of the assets of the group considered.[9] We do not divide by the actual asset value, because this is also affected by cyclical fluctuations and we would lose information by doing so. For example, an observed decrease in the ratio of equity *relative* to assets is consistent with a decrease as well as an increase in the amount of equity.

According to the flow approach, the value for firms that are in group j in period t would be equal to

$$F_t^E(j) = \frac{\sum_{i \in j_{t-1}} S_{i,t}^{\$}/p_t}{A_t^{\$,T}(j)}, \tag{1}$$

where $A_t^{\$,T}(j)$ is the trend of the real asset value of firms in group j, p_t is the producer price level in year t,[10] and $S_{i,t}^{\$}$ is the financing variable considered. For example, $S_{i,t}^{\$}$ could be the gross sale of stock during period t or the change in the book value of equity, $E_{i,t}^{\$} - E_{i,t-1}^{\$}$. A disadvantage of the flow approach is that some series are quite volatile. In particular, the series frequently display sharp changes that are reversed in the next period. Therefore, we also construct a cyclical measure of firm finance using the *level approach* that puts less emphasis on the high-frequency movements of the data. For equity issuance measures, the initial value is set equal to

$$L_1^E(j) = \frac{\sum_{i \in j_1} E_{i,1}^{\$}}{p_1} \tag{2}$$

[7]These results are consistent with those reported in Frank and Goyal (2005).

[8]Change in equity is defined as in Fama and French (2005). As shown in Appendix C, we obtained very similar results with the alternative definition of Baker and Wurgler (2002).

[9]Scaling by the trend asset value is not enough to render the constructed series stationary, presumably because of long-term shifts in firm financing. We remove the remaining trend using the HP filter, but very similar results are obtained when a linear trend is used.

[10]We deflate with producer prices because we want to measure the purchasing power of the funds raised.

and subsequent values are defined using

$$L_t^E(j) = L_{t-1}^E(j) + \frac{\sum_{i \in j_{t-1}} S_{i,t}^{\$}}{p_t}. \tag{3}$$

For debt issuance measures, $E_{i,1}^{\$}$ in equation (2) is replaced by total liabilities in period 1. This variable is then logged and the cyclical component is obtained by applying the HP filter. $L_t^E(j)$ thus measures the accumulated value of the (deflated) amount of funds raised through a particular financing form.

We also consider a modified approach that corrects for possible changes in $L_t^E(j)$ caused by changes in the average firm size of group j. The results are similar to the results reported here and they are discussed in Appendix C, which also contains results for the net sale of stock,[11] the change in equity as defined by Baker and Wurgler (2002), net issuance of long-term debt, and change in total debt.

2.2 Empirical results

In this section, we discuss the cyclical behaviour of equity issuance, debt issuance, profits, retained earnings, and dividends, as well as the correlation between debt and equity issuance.

2.2.1 Cyclical behaviour of equity

Results for equity issuance are reported in Tables 2 and 3. Table 2 uses the level approach and Table 3 uses the flow approach. The top half of each table uses GDP as the real activity variable and the bottom half uses the book value of assets. Each panel reports results for two equity series: the sale of stock and the change in equity.

Correlation between equity finance and GDP. At the aggregate level, the coefficients are small and not even the sign is robust. For the sale of stock, the correlation coefficient is equal to 0.20 and -0.001 for the level and the flow approach, respectively. For the change in equity, the corresponding coefficients are -0.07 and 0.07.

Although the cyclical behaviour of *aggregate* equity depends on the particular definition

[11] We prefer the gross series over the net sale of stock because, as pointed out by Fama and French (2001, 2005), firms often repurchase stock and then reissue it to the sellers of an acquisition, to employee stock ownership plans, and to executives who exercise their stock options. The reissued stock does not show up as a sale of stock, since it does not lead to cash flow. The repurchases, however, do show up. Thus, although these transactions leave equity unchanged, they would cause a reduction in sales minus repurchases.

and methodology used, a robust pattern emerges at the disaggregate level. For both definitions and both approaches, equity behaviour is procyclical for all firm groups considered that exclude the top 5 per cent of firms. For the level approach, several coefficients are significant at the 5 per cent (or lower) level using a one-sided test. For the flow approach, fewer coefficients are significant.[12] The correlation coefficients are higher for the gross series than for the net, which makes sense, since one can expect repurchases to be procyclical.

In contrast, the correlation of the top 1 per cent of firms is negative for both definitions and approaches. For the level approach, the significance levels (using a one-sided test) are 6.3 per cent for the change in equity and less than 1 per cent for the sale of stock. No robust picture emerges for the sign of the correlation for the group of firms between the 95th and the 99th percentile. Although the top 1 per cent of firms comprise a very small number (only 29, on average), they are important for aggregate behaviour, since the distribution of firm size has an extremely fat right tail.

The positive correlation coefficients for the different firm groups indicate that equity is procyclical, but they do not indicate for which group equity issuance moves the most over the cycle. To answer this question we plot the cyclical components. Figure 1 plots the cyclical component of the sale of stock (level approach) and GDP for several firm categories that all exclude the top 1 per cent of firms.[13] The following observations can be made. First, the positive co-movement between equity issuance and real activity is clear.[14] Second, cyclical movements are stronger for smaller firms. Third, the lead-lag structure seems to change over time. For example, equity issuance leads GDP slightly in the second half of the eighties, but it lags GDP slightly in the second half of the nineties; both are periods in which important fluctuations occur. This means that the magnitude for the correlation coefficients may very well underestimate the extent to which equity issuance and GDP are correlated.

Correlation between equity finance and assets. The bottom panels in Tables 2 and 3 report the co-movement of equity issuance and assets.[15] The pattern of results is very similar, but the observed positive correlation is stronger and more significant. For example, for the sale of stock, the correlation coefficients for the bottom 25 per cent of firms (75 per cent)

[12]The lower significance is not surprising, given the stronger emphasis on higher frequencies.

[13]Details on the time-series behaviour of the top 1 per cent of firms are given in Appendix C.

[14]There is one exception. In the early seventies, the cyclical components of equity and GDP move together and, in particular, they both decline during the oil crisis. When the cyclical component of GDP recovers, however, the equity components continue to decline until the recessions of the early eighties, after which they again move closely with GDP.

[15]The asset variable is constructed by setting $S^{\$}_{i,t}$ equal to $A^{\$}_{i,t} - A^{\$}_{i,t-1}$ in equations (1) and (3), for the flow and the level approach, respectively, and by replacing $E^{\$}_{i,t}$ by $A^{\$}_{i,t}$ in equation (2).

are equal to 0.91 (0.65) and 0.80 (0.76) for the flow and level approach, respectively, and the coefficients are highly significant. Even for the top 1 per cent of firms, we find some positive and significant coefficients.

2.2.2 Cyclical behaviour of debt

In this section, we examine the correlation of real activity with long-term debt issuance and the change in total liabilities. Tables 4 and 5 report the results for the level and the flow approach, respectively.

Correlation between debt finance and GDP. At the aggregate level, the correlation between debt and GDP is positive and significant for at least the 5 per cent level (one-sided test), for both debt measures and for both the level and the flow approach. As with equity, the results with aggregate data hide heterogeneous behaviour across the different firm groups. In particular, whereas the correlation coefficients for firms in the bottom 25 per cent, bottom 50 per cent, bottom 75 per cent, and even the bottom 99 per cent are positive and significant, the correlation coefficient for the top 1 per cent is insignificant, small, and for the level approach even negative.

Figures 2 and 3 plot the cyclical component of GDP, together with the cyclical components of long-term debt issuance and the net change in total liabilities, respectively. The level approach is used to construct the financing variables. It shows that the cyclical component for firms in the bottom 25 per cent, the bottom 50 per cent, and the bottom 99 per cent move together closely for both debt definitions. The figures make clear that the issuance of long-term debt and the change in liabilities lag the cycle, which is also made clear by the higher correlation coefficients of the debt variables with lagged GDP.

Figures 2 and 3 provide no reason to believe that changes in debt issuance over the business cycle are quantitatively more important for smaller firms. The one episode where a much sharper increase and subsequent decrease are observed for groups that exclude the larger firms is in the first half of the seventies. Here, debt issuance lags output, however, so that debt is still increasing while GDP is already contracting.

Correlation between debt finance and assets. As with equity, the differences between the different firm categories are smaller when assets are used as the real activity variable. For long-term debt issuance, it is still the case that the correlation coefficients are smaller for the larger firms, but they are always positive, even for the top 1 per cent of firms (although not significant for the flow approach). Interestingly, a very uniform pattern of high and significant correlation coefficients is observed for the change in total liabilities. That is, the correlation coefficients are above 0.9 for both approaches, even for the top 1 per cent of firms.

2.2.3 Co-movement of equity and debt

Table 6 reports the correlation between the gross equity and the gross debt measure (i.e., the change in equity and the change in liabilities), as well as the correlation between the net equity and the net debt measure (i.e., the sale of stock and long-term debt issuance). The correlation coefficients are almost all positive for different firm categories, definitions, and approaches. Several coefficients are significant. The only negative contemporaneous coefficient is found for the [95%,99%] size category using the gross measures and the flow approach.

We have shown that the cyclical behaviour of equity and debt issues is quite different for firms in the top 1 per cent. Nevertheless, the correlation of the two external financing sources for the top 1 per cent has the same sign as the coefficients for the smaller firms (i.e., positive). Several coefficients for the top 1 per cent are highly significant. This result, combined with the fact that debt and equity for the top 1 per cent are positively correlated with assets, suggests that part of the difference between small and large firms is the cyclical behaviour of assets.[16] Below, we show that the differential cyclical behaviour of profits and retained earnings is also important.

Using the flow-of-funds data from the Federal Reserve Board, Jermann and Quadrini (2006) find that aggregate equity issuance is countercyclical, aggregate debt issuance is procyclical, and aggregate equity and aggregate debt are negatively correlated. For some measures, we also find equity issuance to be countercyclical at the aggregate level. The positive correlation between equity and debt, however, is a robust finding when Compustat data are used.[17] This suggests that there is a difference between Compustat data and the flow-of-funds data used by Jermann and Quadrini (2006). The flow-of-funds series are net, so leveraged

[16]In fact, the correlation coefficient (t-statistic) for the cyclical components of assets and GDP is equal to 0.39 (2.54) and 0.47 (3.59) for firms in the bottom 25 per cent and bottom 75 per cent, respectively, while it is -0.02 (-0.08) for firms in the top 1 per cent.

[17]In Appendix C, we consider alternative series and find one exception. Using the flow approach and net sale of stock, we find a negative significant correlation between debt and equity issuance. As pointed out by Fama and French (2001, 2005), however, net sale of stock does not deal correctly with reissues of stock. This measurement error works in the direction of making the series less procyclical. More details are given in Appendix C.

buyouts could be behind the negative correlation between equity issuance and debt issuance. Indeed, Baker and Wurgler (2000) argue that the merger waves in the eighties and nineties are quantitatively important for fluctuations in the flow-of-funds net equity and net debt series.

A reduction in equity because of a leveraged buyout does not show up in our equity series.[18,19] One could argue, however, that one should not clean the data for the effects of leveraged buyouts when trying to discover the cyclical behaviour of debt and equity issuance. Although leveraged buyouts do occur in concentrated waves, they occur when economic conditions are very favorable; that is, one could argue that they are procyclical. Note, however, that although this question is important for the cyclicality of the aggregate series, it is not important for the cyclicality of the majority of firms, since mainly the largest firms are affected by mergers.

2.2.4 Cyclical behaviour of retained earnings, profits, and dividends

In Table 7, we report the cyclical behaviour of retained earnings, profits, and dividends. We report results only for the flow approach.[20] There is again a striking difference between the results for small and large firms. Whereas retained earnings are procyclical and significant for large firms, they are countercyclical (but insignificant) for small firms. The countercyclicality for the bottom 25 per cent, 50 per cent, and 75 per cent is due to firms in the bottom 25 per cent. For firms between the 25th and the 50th percentile, the correlation is 0.20 with a t-statistic of 1.24. For firms between the 50th and the 75th percentile, the correlation is 0.29 and significant with a t-statistic of 2.56. The cyclical behaviour of profits mimics that of retained earnings; that is, countercyclical and insignificant for small firms, but significantly procyclical for large firms. One possible explanation for the countercyclical behaviour of profits for small firms is the stronger procyclical behaviour of assets.[21] When assets are used as the real activity measure, then both the countercyclical behaviour of retained earnings

[18] Also, Jermann and Quadrini (2006) analyze the correlation between GDP and aggregate net equity payouts as a fraction of GDP. Equity payouts are dividends *minus* net equity issuance. With this measure, it is more likely to attain a countercyclical equity issuance, since it is net of dividend payments and is expressed as a fraction of GDP.

[19] A reduction in equity obviously would not show up in the gross series. It also would not show up in the net series, since a firm that disappears from the sample because of a merger is not used in the construction of the set of firm observations in that period. Firms involved in major mergers (Compustat footnote code AB) are eliminated from the sample.

[20] The level approach takes the log of retained earnings. For those among the smallest firms, retained earnings are persistently negative. This means that accumulated earnings at some point become negative and one cannot take the log anymore.

[21] See footnote 16.

and profits for small firms and the procyclical behaviour of large firms become stronger.

The correlation coefficients for dividends are typically positive and often significant. The correlation is stronger when GDP is used instead of assets, especially for firms in the bottom 25 per cent. Thus, dividends typically increase during good times, but more so when good times are characterized as increases in overall activity than by increases in overall firm assets. This is to be expected, since the higher investments are likely to put pressure on dividends.

3. Static Model

In this section, we develop a one-period version of the model. The simplicity will be helpful in understanding some undesirable implications of the standard debt contract, such as dampening of shocks and procyclicality of the default rate. More importantly, the analysis will bring to light one important reason why equity issuance is procyclical: namely, the procyclical behaviour of the shadow price of external funds.

3.1 Debt contract

3.1.1 Description of firm financing problem

Technology is given by

$$\theta\omega k^{\alpha} + (1-\delta)k, \tag{4}$$

where k stands for the amount of capital, θ for the aggregate productivity shock (with $\theta > 0$), ω for the idiosyncratic productivity shock (with $\omega \geq 0$ and $\mathrm{E}(\omega) = 1$), and δ for the depreciation rate. The value of θ is known at the beginning of the period when the debt contract is written, but ω is observed only at the end of the period.

It is standard to assume that (i) agency problems are present only in the sector that produces investment commodities, and (ii) technology in this sector is linear (that is, $\alpha = 1$). The linearity assumption is convenient for computational reasons, since it means that agency costs do not depend on firm size and a representative firm can be used. Neither the assumptions nor the implication that firm size does not matter is appealing. Therefore, we use a standard non-linear production function, and agency problems are present in all sectors.[22]

[22]Chari, Kehoe, and McGrattan (2006) show that financial frictions in the investment sector correspond to "investment wedges," and they argue that these have played at best a minor role in several important economic downturns.

The firm's net worth is equal to n and debt finance occurs through one-period contracts. That is, the borrower and lender agree on a debt amount, $(k-n)$, and a borrowing rate, r^b. The firm defaults if resources in the firm are not enough to pay back the amount due. That is, the firm defaults if ω is less than the default threshold, $\overline{\omega}$, where $\overline{\omega}$ satisfies

$$\theta\overline{\omega}k^{\alpha} + (1-\delta)k = (1+r^b)(k-n). \tag{5}$$

If the firm defaults, then the lender gets

$$\theta\omega k^{\alpha} + (1-\delta)k - \mu\theta k^{\alpha}, \tag{6}$$

where μ represents bankruptcy costs, which are assumed to be a fraction of expected revenues.[23] In an economy with $\mu > 0$, defaults are inefficient and would not happen if the first-best solution could be implemented. Bankruptcy costs are assumed to be unavoidable, however, and the borrower and the lender cannot renegotiate the contract. The idea is that the situation in which firms do not have enough resources to pay the contractually agreed upon payments is like a distress state, involving, for example, loss of confidence, loss of sales, distress sales of assets, and loss of profits.[24]

Using (5), the firm's expected income can be written as

$$\theta k^{\alpha}F(\overline{\omega}) \text{ with } F(\overline{\omega}) = \int_{\overline{\omega}}^{\infty} \omega d\Phi(\omega) - (1-\Phi(\overline{\omega}))\overline{\omega}, \tag{7}$$

and the lender's expected revenues as

$$\theta k^{\alpha}G(\overline{\omega}) + (1-\delta)k \text{ with } G(\overline{\omega}) = 1 - F(\overline{\omega}) - \mu\Phi(\overline{\omega}), \tag{8}$$

where $\Phi(\omega)$ is the cumulative distribution function (CDF) of the idiosyncratic productivity shock, which we assume to be differentiable.

The values of $(k, \overline{\omega})$ are chosen to maximize the expected end-of-period firm income

[23] The results in this section occur if bankruptcy costs are a fraction of actual output, $\theta\omega k^{\alpha}$, or a fraction of the interest payments.

[24] In the framework of Townsend (1979), bankruptcy costs are verification costs and debt is the optimal contract. It is not clear to us, however, that verification costs are large enough to induce quantitatively interesting agency problems. Indeed, Carlstrom and Fuerst (1997) include estimates for lost sales and lost profits, and set μ equal to 0.25 in their calibration. Under this alternative interpretation of bankruptcy costs, debt would no longer be the optimal contract. Convenience and history, however, may also be important reasons behind the dominant use of debt contracts in obtaining external finance.

subject to the constraint that the lender must break even. Thus,

$$\begin{aligned} w(n;\theta) = \max_{k,\overline{\omega}} \min_{\zeta} \; & \theta k^{\alpha} F(\overline{\omega}) + \zeta \left[\theta k^{\alpha} G(\overline{\omega}) + (1-\delta)k - (1+r)(k-n)\right] \\ & \text{s.t. } \zeta \geq 0, \end{aligned} \tag{9}$$

where ζ is the Lagrange multiplier corresponding to the bank's break-even constraint. Rewriting the break-even condition for the bank gives

$$\frac{\theta k^{\alpha} G(\overline{\omega})}{\delta + r} = k - \frac{(1+r)n}{\delta + r}. \tag{10}$$

This equation makes clear the role of the depreciation rate. Incomplete depreciation (i.e., $\delta < 1$) allows the firm to leverage its net worth. That is, the lower the depreciation rate, the larger the share of available resources that is not subject to idiosyncratic risk. For this reason, the bank can lend out a positive amount (i.e., $k > n$), even if the firm always defaults; i.e., even if $\overline{\omega} = G(\overline{\omega}) = 0$.

For an interior solution, the optimal values for k and $\overline{\omega}$ satisfy the break-even condition of the bank (10) and the first-order condition:

$$\frac{\alpha\theta k^{\alpha-1} F(\overline{\omega})}{\delta + r - \alpha\theta k^{\alpha-1} G(\overline{\omega})} = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})}. \tag{11}$$

The Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone, and is always greater or equal to one. That is,

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1 - \mu\Phi'(\overline{\omega})/(1 - \Phi(\overline{\omega}))} \geq 1. \tag{12}$$

3.1.2 Properties of the default rate

Assumption A

- *The maximization problem has an interior solution.*[25]

[25]This is not necessarily the case. For example, if aggregate productivity is low, depreciation is high, bankruptcy costs are high, and/or the CDF of ω has a lot of mass close to zero, then $k = n$ may be the optimal outcome.

- *At the optimal value of $\overline{\omega}$, the CDF satisfies*

$$\frac{\partial\left(\Phi'(\omega)/(1-\Phi(\omega))\right)}{\partial\omega} > 0. \tag{13}$$

This inequality is a weak condition and is satisfied if the density, $\Phi'(\omega)$, is non-zero and non-decreasing at $\overline{\omega}$.[26] The following proposition characterizes the behaviour of the default rate.

Proposition 1 *Suppose that Assumption A holds. Then,*

$$\begin{aligned}
\frac{d\overline{\omega}}{dn} &= 0 \text{ when } \alpha = 1, \\
\frac{d\overline{\omega}}{dn} &< 0 \text{ when } \alpha < 1, \text{ and} \\
\frac{d\overline{\omega}}{d\theta} &> 0 \text{ when } n > 0. \\
\frac{d\overline{\omega}}{d\theta} &= 0 \text{ when } n = 0, \text{ and } \alpha < 1.
\end{aligned}$$

The proofs of the proposition are given in the Appendix A. The first two parts of the proposition say that an increase in the firm's net worth has no effect on the default rate when technology is linear (i.e., $\alpha = 1$), but reduces the default rate when technology exhibits diminishing returns (i.e., $\alpha < 1$). This is an interesting result, since it makes clear that, for the case considered in the literature (i.e., the case with $\alpha = 1$), an increase in net worth, which is the key variable of the net-worth channel, does not lead to a reduction in the default rate. The last part of the proposition says that an increase in aggregate productivity increases the default rate, except when $n = 0$.[27] That is, an increase in θ changes the firm's trade-off between expansion (higher k) and less defaults (lower $\overline{\omega}$) in favour of expansion. More intuition is provided in Appendix A.

With $\alpha = 1$, an increase in θ therefore leads to an increase in the default rate and any subsequent increase in net worth would not effect it. With $\alpha = 1$ and without further modifications, dynamic models with the standard debt contract would, thus, generate a

[26] Such an assumption is standard in the literature. For example, Bernanke, Gertler, and Gilchrist (1999) assume that $\partial\left(\omega d\Phi(\omega)/(1-\Phi(\omega)\right)/\partial\omega > 0$, which would be the corresponding condition if bankruptcy costs are—as in Bernanke, Gertler, and Gilchrist (1999)—a fraction of actual (as opposed to expected) revenues.

[27] The last part of the proposition imposes that $\alpha < 1$, because when $n = 0$ the problem is not well defined for $\alpha = 1$.

procyclical default rate, which is counterfactual.[28] With $\alpha < 1$, the increase in n that follows an increase in θ does have a downward effect on the default rate, but we never find this effect to be large enough to generate a countercyclical default rate in a model with only debt.

3.1.3 Dampening frictions

Cochrane (1994) argues that there are few external sources of randomness that are very volatile. The challenge for the literature is therefore to build models in which small shocks can lead to substantial fluctuations. The debt contract has the unfortunate property that it dampens shocks. That is, the responses of real activity and capital in the model with the debt contract are actually less than the responses when there are no frictions in obtaining external finance. This is summarized in the following proposition. Let y be aggregate output and let y^{net} be aggregate output net of bankruptcy costs. Also, let $\widetilde{k}$ and $\widetilde{y}$ be the solution to capital and aggregate output in the model without frictions, respectively.

Proposition 2 *Suppose that $n > 0$ and Assumption A holds. Then,*

$$\frac{d\ln k}{d\ln\theta} < \frac{d\ln\widetilde{k}}{d\ln\theta} = \frac{1}{1-\alpha}\,d\ln\theta,\ \textit{and} \tag{14}$$

$$\frac{d\ln y^{net}}{d\ln\theta} < \frac{d\ln y}{d\ln\theta} < \frac{d\ln\widetilde{y}}{d\ln\theta} = \frac{\alpha}{1-\alpha}\,d\ln\theta. \tag{15}$$

To understand this proposition, it is important to understand that net worth, n, is fixed when aggregate productivity, θ, changes. For example, consider an enormous drop in θ. Suddenly, n becomes very large relative to θ, but this means that frictions no longer matter. The disappearance of the agency problem implies that the effect of the drop in θ is less. Therefore, it is key that $n > 0$. The proof in Appendix A makes it clear that if $n = 0$, then there is no such increase in n/θ when θ decreases. Consequently, the percentage change in capital and output is equal to that of the frictionless model if $n = 0$.

[28] To alleviate this problem, Bernanke, Gertler, and Gilchrist (1999) assume that aggregate productivity is not known when the contract is written. Dorofeenko, Lee, and Salyer (2006) generate a countercyclical default rate by letting idiosyncratic risk decrease with aggregate productivity.

3.1.4 Tax advantage and optimal leverage

Applying the envelope condition to (9) gives

$$\frac{\partial w(n;\theta)}{\partial n} = \zeta(\overline{\omega})(1+r). \tag{16}$$

Equation (12) implies that the Lagrange multiplier, $\zeta(\overline{\omega})$, is strictly bigger than 1 as long as defaults are non-zero. Consequently, adding a unit of net worth to the firm increases end-of-period firm value by more than $1+r$, and firms have the incentive to drive debt down to the point where $\overline{\omega}$ is equal to zero. That is, in the model described so far, there is no benefit of debt to balance bankruptcy costs.

The trade-off theory of corporate finance argues that the deductibility of interest payments provides such a benefit and leads to an optimal leverage ratio at which defaults are still relevant.[29] In the dynamic model discussed below, we assume that taxes are a fraction of corporate profits. Here, we assume that after-tax cash on hand is simply a fixed fraction of before-tax cash on hand, which simplifies the analysis without affecting the point we want to make. In particular, the advantage of this less realistic way to model taxes is that the problem is almost unchanged, except that the objective of the firm and the Lagrange multiplier are multiplied by $(1-\tau)$. The expression for the value of an extra unit of net worth (16) is then equal to

$$\frac{\partial w(n;\theta)}{\partial n} = \zeta(\overline{\omega})(1+r) = \frac{(1-\tau)(1+r)}{1-\mu\Phi'(\overline{\omega})/(1-\Phi(\overline{\omega}))}. \tag{17}$$

For a high enough level of net worth, $\overline{\omega}=0$, $\zeta<1$, and the internal rate of return is less than $1+r$.[30] When $n=0$, the internal rate of return exceeds $1+r$, as long as the tax rate is not too high. Continuity then implies that there is a level of net worth, n^*, such that the internal rate of return is equal to $1+r$. Equation (17) then implies that, at this level of net worth, $\overline{\omega}>0$.

If the owner could attract external equity and transact at the market rate r, then the firm's net worth would always be equal to n^*. The owner would attract equity when $n<n^*$ (i.e., when the internal rate of return exceeds r), and would take money out of the firm when $n>n^*$ (i.e., when the internal rate of return is less than r). In other words, the optimal

[29]Graham (2000) finds that the tax benefits of debt are, on average, equivalent to 10 per cent of the value of the firm, and therefore quite substantial.

[30]With $\delta<1$, the point at which $\overline{\omega}=0$ is possible with positive debt levels. In particular, equation (10) implies that, at $n=(\delta+r)\widetilde{k}/(1+r)$, the first-best solution of the capital stock, $\widetilde{k}$, can be implemented and $\overline{\omega}=0$.

leverage ratio is equal to $(k^* - n^*)/k^*$, where k^* is the optimal level of capital corresponding to $n = n^*$.[31]

3.2 Equity contract

A key theoretical question we want to answer is what the cyclical behaviour of equity is if we modify the model in which the firm can obtain funds only through the standard debt contract by allowing for equity issuance. We use a reduced-form approach to model the friction associated with obtaining equity financing. The simplicity is helpful to highlight the channel we identify.

3.2.1 Costs of issuing equity

We follow Cooley and Quadrini (2001), using a reduced-form approach and assuming that equity costs are increasing with the amount of equity raised. Whereas Cooley and Quadrini (2001) assume that the cost of issuing equity is linear, we assume that these costs are quadratic; that is, $\lambda(e) = \lambda_0 e^2$ for $e > 0$.[32] Because of these costs, the net worth of firms does not jump instantaneously to the optimal level, n^*. Instead, for any level $n < n^*$, some equity will be issued to reduce the gap. Since there are no costs to issue dividends, a firm can reduce its level of net worth to n^* instantaneously.

Equity issuance costs in our model are like underwriting fees, and it does not matter whether the current or the new owners pay them. Alternatively, one could interpret the equity issuance costs as a reduced-form representation for losses due to an adverse-selection problem that firms face when convincing others to become co-owners. The question arises as to whether such an adverse-selection problem should not affect the debt problem. To some extent it probably should, and it would be worthwhile to construct a framework that analyzes the effect of different frictions on different types of contracts.

[31] Business cycle models that incorporate frictions typically assume that the discount rate of the entrepreneur exceeds the market interest rate. This also assumes that, at some point, the entrepreneur prefers to take funds out of the firm. Incorporating the tax advantage allows us to do this without relying on such an assumption, which is hard to verify.

[32] This avoids a non-differentiability when zero equity is being issued. Jermann and Quadrini (2006) also assume a quadratic cost of issuing equity. Hansen and Torregrosa (1992), and Altinkiliç and Hansen (2000), show that underwriting fees do indeed display increasing marginal costs.

3.2.2 Description of the equity issuance problem

At the beginning of the period, the firm chooses equity, e, and debt issuance, $k - n = k - (e + x)$. A lender that buys equity (debt) does not obtain any information that is helpful in alleviating the friction of the debt (equity) contract. Recall that $w(n;\theta)$ is the expected end-of-period value of a firm that starts with net worth equal to n. The equity issuance decision is represented by the following maximization problem:

$$\begin{aligned} v(x;\theta) &= \max_{e,s}\,(1-s)\tfrac{w(x+e;\theta)}{1+r} \\ \text{s.t.}\quad e &= s\left(\tfrac{w(x+e;\theta)}{1+r}\right) - \lambda(e), \end{aligned} \tag{18}$$

where s is the ownership fraction that the providers of new equity obtain in exchange for e. In this specification, it is assumed that the equity issuance costs are paid by the outside investor, but this is irrelevant.[33]

The expected rate of return for equity providers is equal to

$$\frac{\alpha w(x+e,\theta) - (e+\lambda(e))}{e+\lambda(e)} = \frac{(1+r)\,(e+\lambda(e)) - (e+\lambda(e))}{e+\lambda(e)} = r.$$

That is, providers of equity financing obtain the same expected rate of return as debt providers.

The first-order condition for the equity issuance problem is given by

$$\frac{1}{1+r}\frac{\partial w(x+e;\theta)}{\partial e} = 1 + \frac{\partial \lambda(e)}{\partial e}. \tag{19}$$

That is, the marginal cost of issuing one unit of equity, $1 + \partial\lambda/\partial e$, has to equal the expected benefit. Since $\partial\lambda/\partial e$ is equal to zero at $e = 0$, the firm will issue equity whenever $\partial w/\partial e > 1 + r$. Since $\partial\lambda/\partial e > 0$ for $e > 0$, however, the firm does not increase equity up to the point where $\partial w/\partial e = 1 + r$.

3.2.3 Cyclicality of equity issuance

In this section, we address the question of how equity issuance responds to an increase in aggregate productivity. Clearly, when aggregate productivity is high, the need for external finance increases. This suggests that equity issuance should increase during a boom. But

[33] The maximization problem in (18) and the problem in which issuance costs are paid by the firm correspond to maximizing $w(x+e;\theta)/(1+r) - e - \lambda(e)$ with respect to e.

since another form of finance is possible, it may also be the case that there is a substitution out of equity into debt. The following proposition shows that the latter is not the case in our model.[34]

Proposition 3 *Suppose that Assumption A holds. Then,*

$$\frac{de}{d\theta} > 0 \text{ for } n > 0. \tag{20}$$

That is, when aggregate productivity increases, firms that issue equity will issue more, and firms that issue dividends ($e < 0$) will reduce dividends and possibly even issue equity. This result is driven by the result of Proposition 1 that the shadow price of external funds and the default probability increase with aggregate productivity (for a given value of net worth, $n = x + e$).[35] Even though the firm could obtain more debt financing without additional equity, the rise in the default rate increases the Lagrange multiplier of the bank's break-even condition and therefore increases the value of additional equity. Empirical evidence for this channel is provided by Gomes, Yaron, and Zhang (2006), who show that the shadow cost of external funds exhibits strong cyclical variation. Livdan, Sapriza, and Zhang (2006) also generate a procyclical shadow price of external funds. In their model, this result is driven by the assumption that the discount factor is countercyclical, which leads to a strong demand for investment. In our model, the result is caused by the properties of the standard debt contract.

4. Dynamic Model

In this section, we first discuss the prototype dynamic model, which is a straightforward modification of the static model. We then discuss the benchmark model, which includes two additional features to generate procyclical equity issuance.

[34]Levy and Hennessy (2006) develop a model in which equity is procyclical and debt is countercyclical, whereas Jermann and Quadrini (2006) develop a model in which equity is countercyclical and debt is procyclical. See section 5 for a further discussion.

[35]For low n, the magnitude of $de/d\theta$ increases with firm size, but at some point the relationship reverses and equity issuance decreases as net worth increases. The reason is as follows. Above, we showed that $d\overline{\omega}/d\theta = 0$ if $n = 0$. Consequently, $de/d\theta = 0$ if $n = 0$. For n close to zero, the response will be close to zero. For large enough n, frictions do not matter and $d\overline{\omega}/d\theta$ will be small as well. In our quantitative work, we find that $de/d\theta$ decreases with firm size for most observed values for n. This is partly due to the fact that, with an endogenous equity decision, small values of n are not chosen.

4.1 Prototype dynamic model

4.1.1 Technology

In addition to making firms forward looking, the dynamic prototype model has some features that are not present in the static model. All are related to technology. The first is the specification of the law of motion for productivity. Second, we introduce two minor changes in technology that are helpful in letting the model match some key statistics, such as leverage and the fraction of firms that pay dividends. In particular, we introduce stochastic depreciation and a small fixed cost.

Productivity. The law of motion for aggregate productivity, θ_t, is given by

$$\ln(\theta_{t+1}) = \ln(\bar{\theta})(1-\rho) + \rho \ln(\theta_t) + \sigma_\varepsilon \varepsilon_{t+1}, \tag{21}$$

where ε_t is an identically, independently distributed (i.i.d.) random variable with a standard normal distribution.

Stochastic depreciation. For typical depreciation rates, firms default only for very low realizations of the idiosyncratic shock, because undepreciated capital provides a safety buffer. This generates high leverage. An important reason behind observed defaults is that the value of firm assets has deteriorated over time; for example, because the technology has become outdated. To capture this idea, we introduce stochastic depreciation, which makes it possible to generate reasonable default probabilities while keeping the *average* depreciation rate unchanged. In particular, depreciation depends on the same idiosyncratic shock that affects production, and is equal to

$$\delta(\omega_t) = \delta_0 \exp(\delta_1 \omega_t). \tag{22}$$

Fixed costs. For realistic tax rates, profits are high, which in turn implies that a high fraction of firms pay out dividends. We introduce a fixed cost, η, so that the model can match the observed fraction of dividend payers. Given the importance of internal funds, it is important to match data on funds being taken out of the firm.

4.1.2 Debt and equity contract

At the beginning of the period, aggregate productivity, θ_t, and the amount of cash on hand, x_t, are known. After θ_t is observed, each firm makes the dividend/equity decision and at the same time issues bonds. In the dynamic version, a firm takes into account its continuation value and maximizes its expected end-of-period value, instead of end-of-period cash on hand. Firms default when cash on hand is negative.[36] The debt contract is therefore given by

$$w(n_t;\theta_t) = \max_{k_t,\overline{\omega}_t,r_t^b} \mathrm{E}\Big[\int_{\overline{\omega}_t}^{\infty} v(x_{t+1};\theta_{t+1})d\Phi(\omega) + \int_0^{\overline{\omega}_t} v(0;\theta_{t+1})d\Phi(\omega)|\theta_t\Big] \tag{23}$$

s.t.

$$x_{t+1} = \theta_t\omega_t k_t^\alpha + (1-\delta(\omega_t))k_t - (1+r_t^b)(k_t-n_t) - \tau[\theta_t\omega_t k_t^\alpha - \delta(\omega_t)k_t - r_t^b(k_t-n_t)],$$

$$0 = \theta_t\overline{\omega}_t k_t^\alpha + (1-\delta(\overline{\omega}_t))k_t - (1+r_t^b)(k_t-n_t) - \tau[\theta_t\overline{\omega}_t k_t^\alpha - \delta(\overline{\omega}_t)k_t - r_t^b(k_t-n_t)],$$

$$\int_0^{\overline{\omega}_t}[\theta_t\omega_t k_t^\alpha + (1-\delta(\omega_t))k_t - \mu k_t^\alpha]d\Phi(\omega) + (1-\Phi(\overline{\omega}_t))(1+r_t^b)(k_t-n_t) = (1+r)(k_t-n_t).$$

Note that taxes are a constant fraction of taxable income, which is defined as operating profits net of depreciation and interest expense. The specification of the equity contract is still given by equation (18), but $w(\cdot)$ is now given by equation (23).

4.1.3 Number of firms

Our model has a fixed number of heterogeneous firms. A firm that defaults on its debt is replaced by a new firm that starts with zero cash on hand.[37]

4.1.4 Supply of funds

Our data set covers a subset of all firms. It does not include financial firms, utilities, and firms that are not publicly listed.[38] Although firms outside this group also obtain external financing,[39] we think that our model is most suited to describe firms in the corporate sector;

[36] This would be the correct default cut-off if firms could default and restart with zero initial funds. We also analyzed the model under the assumption that firms default when $v(x_{t+1};\theta_{t+1}) < 0$. Since $v(0;\theta_{t+1}) > 0$, this means that firms default only when cash on hand is *sufficiently* negative. The model with the alternative specification is more difficult to solve, but generates very similar results.

[37] See Covas (2004) for a model in which the number of firms is determined by a free-entry condition.

[38] We have employment numbers for 94 per cent of our firms. Total employment for these firms is equal to 35 million, which is roughly one quarter of total U.S. employment.

[39] See Berger and Udell (1998).

i.e., publicly listed firms and closely held firms. In the prototype version of the model, we assume that investors who provide funds to this sector through debt or equity earn an expected rate of return equal to r. The rate that firms pay for external finance is equal to this constant rate plus the external finance premium, which varies with net worth and aggregate conditions. In the benchmark model, discussed below, the required rate of return on equity varies according to an exogenously specified process. Using an exogenous process for the required rate of return has the advantage that the model remains tractable and generates cyclical properties for the required rates of return of risky assets that are consistent with the data.

Without an exogenously given process for the required rate of return for investors, it would be difficult to solve the model, because it would require keeping track of the cross-sectional distribution of firms' net worth levels. We have made no attempt to try to solve such a model. Algorithms to solve models with heterogeneous households (and homogeneous firms) have only recently been developed, and adding a cross-sectional distribution for our already complex setting would be quite a challenge.[40] Moreover, to generate realistic pricing kernels would require a lot more than just adding a risk-averse household to the model.[41]

4.2 Benchmark model

In the prototype model discussed so far, equity issuance is cyclical for the same reason that $\partial e/\partial\theta > 0$ in the static model. That is, the desire to expand when θ increases leads to an increase in the default rate, which makes the break-even constraint of the bank more binding and increases the value of additional net worth. In this section, we describe the benchmark model, which modifies the prototype model in two aspects. Both modifications provide reasons for equity issuance to be procyclical in addition to the reason identified with the prototype model.

A countercyclical price of risk. The risk premium on risky investments varies countercyclically.[42] This means that the end-of-period value of the firm in (18) should be discounted at a lower rate during good times, which in turn leads to an additional increase in the amount

[40] See den Haan (1996, 1997), Krusell and Smith (1997), and Algan, Allais, and den Haan (2006).

[41] Boldrin, Christiano, and Fisher (2001) are quite successful in replicating key asset-price properties, but they use preferences that display habit formation, investment that is subject to adjustment costs, multiple sectors, and costs to move resources across sectors.

[42] For empirical evidence on the countercyclical price of risk, see Fama and French (1989), Schwert (1989), and Perez-Quiros and Timmermann (2000).

of equity being issued. To capture the cyclical variation in the required rate of return, we assume that equity providers discount firms' future payoffs with

$$M_t = \frac{\theta_t^{\gamma}}{1+r}. \tag{24}$$

Countercyclical issuance costs. One reason for the issuance cost is the concern that a firm has an incentive to issue equity when it has private information that it is overvalued by the market. According to Choe, Masulis, and Nanda (1993), this concern is countercyclical, for the following reason. Firm value is affected by idiosyncratic and aggregate factors. The concern that the firm is exploiting private information is most likely to be related to the idiosyncratic component. Consequently, if aggregate conditions improve, then the idiosyncratic component becomes less important and reduces the concern of investors to buy overvalued equity. To capture this mechanism, we allow the equity issuance cost to vary with aggregate productivity, and set

$$\lambda(e_t;\theta_t) = \lambda_0 \theta_t^{-\lambda_1} e_t^2. \tag{25}$$

4.3 Results for the prototype model

This section reports results for the prototype version, in which the equity issuance cost and the discount factor for firms' dividends do not vary over the business cycle. The parameters used are identical to the calibrated parameter values of the benchmark model discussed below, except that $\lambda_1 = \gamma = 0$.

We report results for the bottom tercile (small firms) and top tercile (large firms). This gives a good idea about the heterogeneity in our model economy. The data exhibit more heterogeneity. This is also true if we exclude firms in the right tail of the distribution, for which we found deviating cyclical behaviour. One reason for the limited heterogeneity in the model is that dividend-paying firms reduce their net worth to the same optimal level and are, thus, identical until the next idiosyncratic shock is realized. These firms account for roughly half the firms in our artificial sample. We discuss this issue further in section 5.

For a typical firm in the bottom tercile, financial frictions are quantitatively important, and additional equity issuance helps to reduce them. In contrast, for a firm in the top tercile, financial frictions may still be present, but they are less important. In particular, the tax advantage of debt often outweighs the remaining bankruptcy costs and dividends are therefore important for firms in this category.

Figure 4 shows how output and the default rate respond to a one-standard-deviation positive shock to aggregate productivity. In addition to the responses for the prototype model, it also shows the responses for the frictionless model and the model with only debt as external finance. The results are discussed in the remainder of this section.

The model without equity issuance. Figure 4 shows that, in the "only debt" model, the default rate increases sharply when aggregate productivity increases, which is counterfactual. Even for firms in the top tercile, there is a small increase in the default rate, but the counterfactual movement of the default rate is much more important for small firms. The stronger countercyclical movement in the default rate for small firms corresponds with a weaker procyclical output response for small firms, which is also counterfactual.

Dampening in the different models. When we examine the output responses, we find that the differences between the different models are most pronounced for small firms. For example, in the model without equity issuance, output increases by less than output in the frictionless version of the model. In particular, the first-period response of output in the "only debt" model is 15.3 per cent less than the response in the frictionless version. In the model with equity issuance, the response of output is still less, but the first-period response is only 6.6 per cent less than the response in the frictionless model.

The model with equity issuance. In the prototype model, equity issuance increases in response to a positive productivity shock, and the subsequent increase in net worth ensures that there is no longer a sharp increase in the default rate of small firms. Recall that the non-linearity in the production function plays a key role, because with a linear production function the increase in net worth would have had no effect on the default rate. The inflow of external equity causes the first-period response of output for small firms in the prototype model to exceed the response in the "only debt" model by 10.2 per cent. For large firms, the model even generates a small decrease in the default rate, since, with positive tax rates, $\overline{\omega} > 0$ at n_t^* (the level of n at which a firm takes money out of the firm). Even large firms face some (small) probability of default. When aggregate productivity increases, the value of n_t^* increases, which implies that large firms reduce dividends and the higher net worth levels correspond with lower default rates. The effect is very small, however, since agency problems are not very important for large firms.

The default rate does not go down in the prototype model, unless the firm is very large and $x_t > n_t^*$. The reason is that—unless equity issuance is increased because of the increase in n_t^*—equity increases because the desire to expand leads to an increase in the default rate.

4.4 Calibration of the benchmark model

The model period is one year, which is consistent with the empirical analysis. For the discount factor, $\beta = (1+r)^{-1}$, the tax rate, τ, the persistence of the aggregate shock, ρ, and the curvature parameter in the production function, α, we use values that are used in related studies. Its values, together with a reference source, are given in the top panel of Table 8. The benchmark value of α is equal to 0.70. It is standard to use higher values of α in models without labour.[43] We will also discuss the results based on a much lower value of α.

The other parameters are chosen to match some key first- and second-order moments that our model should satisfy. The parameter values and the moments we target are given in the bottom panel of Table 8. Although the parameters determine the values of the moments simultaneously, we indicate in the discussion below which parameter is most influential for a particular moment. In the table, this parameter is listed in the same row as the corresponding moment. The set of targeted first-order moments is as follows:

- The ratio of investment to assets, which is pinned down by the parameter that controls average depreciation, δ_0.
- The fraction of firms that pay dividends, which is pinned down by the fixed cost, η. Note that the fixed cost affects profitability and, thus, the rate of return on internal funds. The fixed cost is equal to 17.1 per cent of average aggregate output.
- The default rate, which is pinned down by the bankruptcy cost, μ. Our value of μ is equal to 0.15, which implies that bankruptcy costs are, on average, 2.9 per cent of the value of the firm, $v(\omega\theta k^\alpha + (1-\delta(\omega))k)$.
- The default premium and leverage, which are pinned down by the volatility of the idiosyncratic shock, σ_ω, and the parameter that controls the volatility of depreciation,

[43] Cooper and Ejarque (2003) use a value equal to 0.7; Hennessy and Whited (2005) estimate α to be equal to 0.551; Hennessy and Whited (2006) estimate α to be equal to 0.693 for small firms and equal to 0.577 for large firms; and Pratap and Rendon (2003) estimate α to be between 0.53 and 0.60. It is easy to show that a problem in which technology is given by $k^{\alpha_k} l^{\alpha_l}$ and the wage is constant is equivalent to a problem in which technology is given by k^α with $\alpha = \alpha_k / (1-\alpha_l)$. When the original production function satisfies diminishing returns (for example, because of a fixed factor), then $\alpha < 1$.

δ_1. The higher are σ_ω and δ_1, the less certainty exists about the amount of available funds within the firm, and the higher the premium on debt finance.

- Change in equity to assets, which is pinned down by the parameter that controls the cost of issuing equity, λ_0.

The set of targeted second-order moments is as follows:

- The volatility of aggregate asset growth, which is pinned down by the standard deviation of the innovation to productivity, σ_ε.
- The volatility of change in equity, which is pinned down by the parameter that controls the variation in the cost of issuing equity, λ_1.
- The volatility of retained earnings, which is pinned down by the parameter that controls the variations in the price of risk, γ.

Time-varying discount factor versus time-varying issuance cost. The volatility of equity issuance and the volatility of retained earnings are controlled by the two features that distinguish the benchmark from the prototype model; i.e., the countercyclical variation in the cost of issuing equity and a countercyclical price of risk. Both increase the response of equity issuance to a positive productivity shock for firms that already issue equity. They differ, however, in how they affect firms that issue dividends and, thus, differ in how they affect retained earnings. For a firm that does not issue equity, a reduction in the cost of issuing equity has no direct effect. It still affects the firm indirectly, because it may be hit by some bad shocks in the future, in which case equity finance does become relevant again. Since the firm is forward looking, it would take this into account. In contrast, an increase in the discount factor does have a direct effect on firms that issue dividends. For a firm that issues dividends, it must be the case that

$$M_t \left.\frac{\partial w(x+e)}{\partial e}\right|_{e=0} = \frac{\theta_t^\gamma}{1+r} \left.\frac{\partial w(x+e)}{\partial e}\right|_{e=0} < 1 + \left.\frac{\partial \lambda(e)}{\partial e}\right|_{e=0} = 1. \tag{26}$$

But an increase in the discount factor increases the left-hand side of the inequality. Consequently, an increase in the discount factor implies that firms start paying dividends for higher levels of x_t. This means that firms that issue dividends will issue less and some of them will even start issuing equity.

Generated changes in discount factor and issuance cost The prototype model provides a theoretical reason why equity issuance should be procyclical: the properties of the standard debt contract cause the shadow price of external funds to be procyclical. In the benchmark model, there are two additional features that reinforce the cyclical behaviour of equity: a countercyclical price of risk and a countercyclical cost of issuing equity. The cyclical behaviour of these two features is controlled by two parameters: γ and λ_1. Parameters are calibrated to match the unconditional standard deviation of equity issuance and retained earnings. The model does a good job in generating procyclical equity issuance and retained earnings, *on average*. Note that we use the same discount factor and the same equity issuance cost function for all firms. Nevertheless, we will show below that the cyclical variation in equity issuance is much stronger for small firms. It may very well be that the sensitivities of the discount factor and equity issuance cost to the business cycle are stronger for small firms. In fact, Perez-Quiros and Timmermann (2000) provide evidence that the required rate of return is more cyclical for small firms than for large firms. Allowing for the cyclicality in the discount factor and the equity issuance cost to be firm-dependent would only strengthen the prediction that equity issuance is more cyclical for small firms.

In the remainder of this section, we discuss the magnitude of the implied changes in the discount factor and equity issuance costs. The standard deviation of the required rate of return is equal to 0.16 percentage points. Perez-Quiros and Timmermann (2000) do not report numbers for the cyclical changes in the expected excess rates of return on equity, but their graphs make clear that, even for large firms, expected excess returns increase by several percentage points during NBER recessions. Our results are clearly not generated by unrealistically cyclical changes in the required rate of return.

Kim, Palia, and Saunders (2005) report an average underwriting spread of 7.6 per cent for initial public equity offerings (IPOs), and 5.1 per cent for seasoned public equity offerings (SEOs). Using the difference between the closing and the offer price to construct an estimate of indirect costs, Kim, Palia, and Saunders (2005) report an average of 31.2 per cent for IPOs and 2.6 per cent for SEOs. They also report a wide range of different values. When the lowest and highest 5 per cent are ignored, then the indirect cost varies from -6 per cent to 156 per cent for IPOs, and from -4.7 per cent to 13.1 per cent for SEOs. Similarly, Loughran and Ritter (2002) report that \$9.1 million "is left on the table" for the average IPO, which corresponds to three years of operating profits.

The average equity issuance cost in our model is equal to 4.4 per cent of equity raised, which is reasonable given the estimates found in the literature. The standard deviation of average equity issuance cost is equal to 3.6 per cent. Unfortunately, there is no empirical evidence on the magnitude of the cyclical changes in average equity issuance costs. Given the extent of underpricing, there is clearly room for the magnitude of the cyclical changes we consider. Moreover, given that the cyclical variation in the required rate of return is still quite low, one can to some extent lower the cyclical variation in the equity issuance cost and raise the cyclical variation in the required rate of return. We discuss this further in the next section.

4.5 Results for the benchmark model

In this section, we investigate whether (i) the model does a good job in replicating the cross-sectional pattern of cyclical changes for debt and equity finance documented in our empirical work, (ii) the model can generate a substantially stronger cyclical response for smaller firms, (iii) the model can generate a countercyclical default rate, and (iv) the model with equity issuance can substantially magnify shocks.

Output and default rates. Figure 5 plots the impulse-response functions for output and the default rate when aggregate productivity is hit by a positive one-standard-deviation shock. It also plots the responses in the prototype model. The figure shows that the model can generate a countercyclical default rate and that shocks are strongly magnified. In particular, the first-period response of output for small firms in the benchmark model is 84 per cent higher than the response in the prototype model. In the latter, the output response is slightly below the response of the frictionless model. The increase in equity issuance not only has a direct effect on output by increasing the amount of net worth, it also increases the amount of debt the firm can borrow and it reduces the default rate. For aggregate output, there is also a considerable amount of magnification; the first-period response of output in the benchmark model is 45 per cent higher than the response in the prototype model.

For small firms, the average default rate drops by 118 basis points in the first period and continues to drop until it is 162 basis points below the pre-shock value in the third period. Even at the aggregate level the drop in the default rate is substantial: it drops by 39 basis points in the first period and the maximum reduction is 56 basis points.[44]

[44] Levin, Natalucci, and Zakrajsek (2004) estimate the external finance premium for firms at the 25th, 50th, and 75th percentile of the sales-weighted distribution, and find noticeable cyclical changes in the external finance premium for large firms as well.

Debt and equity. The two top panels of Figure 6 plot the responses of equity and debt for small firms, large firms, and the aggregate. The bottom panels of Figure 6 plot the responses of net worth for the three firm categories, and plot at the aggregate level retained earnings and dividends. First, consider the responses for large firms. In the first period, net worth for large firms increases. The main reason is that the reduction in the price of risk induces dividend-paying firms to reduce dividends, although there also is a small increase in equity issuance. The increase in retained earnings goes together with an increase in debt financing.

Small firms respond to the positive productivity shock by sharply increasing equity. Debt also increases in the first couple years after the shock, but it increases by less than equity. After some time, the impulse-response function even turns negative. Even though debt is monotonically increasing in the aggregate shock, it is—except at low net-worth levels—decreasing in net worth. During the first couple of periods, the direct effect of the increase in productivity dominates and debt increases. After some time, the shift in the cross-sectional distribution towards larger firms implies a (small) reduction in debt levels relative to the pre-shock levels. The increase in average firm size following the productivity shock also explains that, at some point, the response of retained earnings becomes slightly negative and the response of dividends becomes slightly positive.

Table 9 reports the cross-correlations between equity issuance and GDP, debt issuance and GDP, and debt and equity issuance for simulated and actual data. The coefficients have the same sign as their empirical counterpart. That is, both equity and debt issuance are procyclical. Correlation coefficients implied by the model are, however, higher than their empirical counterparts. This is not very surprising, since the model has only one aggregate shock.

Figure 7 shows the counterpart of the observed cyclical equity component plotted in Figure 1, and the counterpart of the observed cyclical debt component plotted in Figure 2 using long-term debt issuance and in Figure 3 using the change in total liabilities. The top panel gives a typical simulation of equity issues for the bottom 25 per cent, the bottom 50 per cent, and the bottom 99 per cent of firms. As in Figure 1, equity issuance displays much larger cyclical swings for smaller firms. The bottom panel of Figure 7 plots the cyclical behaviour of debt issuance for the same size classes. As in the data, the differences in debt issuance over the cycle across firm categories are smaller than for equity. In the simulated data, however, the cyclical movements for debt issues by small firms are still larger, whereas in the data that is observed only in the seventies.

4.6 Alternative parameter values

The role of α. Our benchmark value of α is equal to 0.7, and here we discuss the effects of lowering α to 0.4. A value of α less than one plays a key role in our analysis. If α is equal to one—which is a common assumption in the literature on agency problems—then the increase in net worth (either because of an increase in retained earnings or because of an increase in equity) would have no effect on the default rate. This does not mean, however, that the lower the value of α the more countercyclical the default rate, because α also affects firm profitability. At lower values of α, firms quickly reach a level of net worth at which default rates are small. One can control for this by increasing the fixed cost.

In particular, for our lower value of α, we increase the fixed-cost parameter from 0.0975 to 0.14. All other parameters are kept the same. This version generates similar responses. For example, the first-period output response for small firms is 47 per cent higher than the response in the frictionless model, whereas it is 64 per cent higher for the benchmark parameters.

Lower cyclical variation in equity issuance costs. Cyclical variation in equity issuance costs and the required rate of return plays an important role in generating procyclical equity issuance. Our calibration exercise implies a certain amount of cyclical variation in equity issuance and the required rate of return. As noted earlier, the standard deviation of the required rate of return is much lower than the numbers reported in Perez-Quiros and Timmermann (2000). We also analyze what happens if the cyclical variation in the required rate of return is increased by raising the value of γ from 0.138 to 0.3 and the cyclical variation in equity issuance cost is decreased by lowering the value of λ_1 from 125 to 60. This combination leaves the volatility of equity issuance (of all firms) unchanged. The standard deviation of equity issuance cost is now 1.52 per cent instead of 3.6 per cent, and the standard deviation of the required rate of return is 0.35 percentage points instead of 0.16. For these parameter values, the initial output response for small firms is 29 per cent above the frictionless response, so equity issuance still helps to magnify shocks. This magnification is less than in the benchmark specification, however, where the corresponding number is equal to 64 per cent. Moreover, by shifting from cyclical variation in equity issuance cost to cyclical variation in the required rate of return, the output response of small and large firms becomes more similar: changes in the required rate of return affect the net worth of small and large firms, whereas changes in equity issuance affect firms that issue equity (i.e., small firms). If the price of risk for small firms is more cyclical than the price of risk for large firms, then changes in the required rate of return would also have a bigger effect on small firms.[45]

The numerical experiments described in this paper show that cyclical variation in equity issuance cost and the price of risk is very effective in explaining the pattern of equity issuance observed across firms. More empirical information on the cyclical variation in equity issuance cost and the price of risk would be helpful in distinguishing between these two factors.

5. Conclusions

Most quantitative studies of the importance of financial frictions for aggregate fluctuations assume that firms can obtain external financing only through the one-period debt contract. But it is clear that firms use other forms of financing and that, in particular, they rely on long-term debt and equity. A proper study of the role of financial frictions should take this into account and it is therefore important that theoretical challenges to study the more complex environment be overcome. In this paper, we allow firms to also raise external funds through equity and analyze three reasons for equity to be procyclical: (i) the property of the one-period debt contract, which makes the shadow price of external funds procyclical, (ii) a countercyclical price of risk, and (iii) a countercyclical cost of issuing equity. With these three channels, the model can replicate the empirical findings of equity finance for small and large firms, generate a countercyclical default rate, and magnify shocks.

Levy and Hennessy (2006) and Jermann and Quadrini (2006) also develop theoretical models to study the cyclical behaviour of debt and equity over the business cycle. They identify different channels. In particular, in both models the substitution plays an important role, although the models have opposite implications for which form of external finance becomes more attractive during an expansion. In the numerical example of Levy and Hennessy (2006), equity is procyclical for all firms, debt is procyclical for firms with more stringent financing constraints, and debt is countercyclical for firms with less stringent financing constraints. Key in deriving this result is that the constraint on equity financing is always binding for all firms, but the constraint on debt financing is not binding for firms with less stringent financing constraints; i.e., firms for which resource and asset diversion is more costly. Consequently, as aggregate conditions weaken, external equity issuance diminishes, but for those firms with a slack constraint on debt financing, the reduction in external equity financing can be partly replaced by debt financing. In our framework, however, the constraint on debt

[45] Note that Perez-Quiros and Timmermann (2000) do find larger cyclical variation in the required rate of return for small firms.

financing is always binding[46] and the costs associated with raising external equity are present only initially. That is, after the new shares have been issued there is no longer a difference between new and old shareholders.

In contrast, equity issuance is countercyclical in Jermann and Quadrini (2006). They allow firms to borrow through one-period debt contracts, but there is no default. Consequently, they do not have the procyclical shadow price of external funds. Nor do they have the cyclical changes in equity issuance cost, or cyclical changes in agents' risk tolerance. Key in their paper is the constraint that links the amount of debt to the value of the firm. An aggregate shock that reduces the value of the firm has such a large impact on the available amount of debt financing that it induces firms to issue more equity.

The relevance of different channels may very well change over time, and may differ by type of firm. Empirical work that could distinguish between the different empirical channels would be of interest. There are several important extensions for the theoretical analysis as well. Two limitations of this paper are that it does not allow for long-term debt and idiosyncratic shocks are not persistent. With multi-period debt contracts, there is an additional reason why equity is procyclical. Equity issuance is a wealth transfer from the equity providers to the holders of long-term debt, since the additional equity reduces the probability of default. But this effect is likely to be less important during a boom, since the probability of default is (or should be) smaller. With persistent idiosyncratic shocks, a much richer cross-sectional distribution could be generated and, in particular, this would add heterogeneity in the top half of the distribution.

[46]Because of the tax advantage of debt, firms do not build up enough net worth to fully finance the first-best capital stock with internal funds.

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Table 1: Summary statistics for different size classes

Size classes	**# of firms**	**% assets**	**Averages of** $\frac{\mathbf{L}}{\mathbf{A}}$	$\frac{\mathbf{\Delta A}}{\mathbf{A}}$	$\frac{\mathbf{\Delta L}}{\mathbf{\Delta A}}$	$\frac{\mathbf{\Delta E}}{\mathbf{\Delta A}}$	$\frac{\mathbf{\Delta RE}}{\mathbf{\Delta A}}$	$\frac{\mathbf{\Delta S}}{\mathbf{\Delta A}}$	$\frac{\mathbf{\Delta D}}{\mathbf{\Delta A}}$
$[0, 25\%]$	715	0.006	0.410	0.307	0.348	0.637	0.014	0.526	0.287
$[0, 50\%]$	1415	0.026	0.448	0.214	0.417	0.471	0.111	0.366	0.471
$[0, 75\%]$	2118	0.089	0.498	0.164	0.487	0.328	0.188	0.248	0.631
$[0, 99\%]$	2807	0.657	0.579	0.112	0.589	0.165	0.253	0.146	0.705
$[90\%, 95\%]$	144	0.132	0.586	0.109	0.611	0.129	0.263	0.122	0.717
$[95\%, 99\%]$	117	0.301	0.603	0.092	0.626	0.104	0.279	0.112	0.695
$[99\%, 100\%]$	29	0.343	0.601	0.079	0.630	0.091	0.284	0.116	0.531
All firms	2836	1	0.587	0.101	0.600	0.144	0.261	0.138	0.659

Notes: The data set consists of annual Compustat data from 1971 to 2004. Leverage, $\frac{L}{A}$, equals liabilities divided by assets. Asset growth, $\frac{\Delta A}{A}$, equals the change in the book value of assets from period $t-1$ to t divided by the current value of assets. Change in liabilities, ΔL, equals the change in the book value of total liabilities. Change in equity, ΔE, equals the change in stockholders' equity minus retained earnings. Retained earnings, ΔRE, is the change in the balance-sheet item for retained earnings. Sale of stock, ΔS, equals sale of common and preferred stock, and ΔD is issuance of long-term debt. For further details on the data series used, see Appendix B.

Table 2: Cyclical behaviour of equity issuance: level approach

Size classes	Sale of stock and			Change in equity and		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.02	0.24	**0.29**	0.03	0.26	**0.26**
	(-0.05)	(1.02)	(2.16)	(0.07)	(1.16)	(2.25)
$[0, 50\%]$	0.10	**0.33**	**0.31**	0.16	**0.32**	**0.23**
	(0.29)	(1.78)	(2.32)	(0.45)	(1.89)	(1.79)
$[0, 75\%]$	0.18	**0.35**	**0.30**	0.21	**0.28**	0.15
	(0.63)	(1.91)	(1.84)	(0.67)	(1.81)	(1.06)
$[0, 99\%]$	0.22	**0.36**	**0.33**	0.12	0.12	0.02
	(0.71)	(1.78)	(1.82)	(0.36)	(0.67)	(0.12)
$[90\%, 95\%]$	**0.42**	**0.45**	0.21	0.23	0.10	-0.12
	(2.59)	(5.45)	(1.61)	(0.75)	(0.62)	(-0.79)
$[95\%, 99\%]$	-0.03	0.12	**0.28**	-0.06	-0.09	-0.09
	(-0.07)	(0.49)	(2.48)	(-0.19)	(-0.47)	(-0.53)
$[99\%, 100\%]$	-0.26	**-0.43**	**-0.44**	-0.10	-0.36	**-0.42**
	(-0.93)	(-2.54)	(-3.94)	(-0.26)	(-1.53)	(-4.14)
All firms	0.12	0.20	0.16	0.04	-0.07	-0.15
	(0.34)	(0.83)	(0.93)	(0.12)	(-0.28)	(-1.17)

Size classes	Sale of stock and			Change in equity and		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.37**	**0.80**	**0.73**	**0.40**	**0.83**	**0.73**
	(4.31)	(7.95)	(7.67)	(3.95)	(9.30)	(5.52)
$[0, 50\%]$	**0.37**	**0.78**	**0.69**	**0.45**	**0.82**	**0.64**
	(2.81)	(7.28)	(4.16)	(3.09)	(9.73)	(2.44)
$[0, 75\%]$	**0.40**	**0.76**	**0.65**	**0.51**	**0.78**	0.55
	(2.59)	(7.04)	(2.86)	(2.82)	(9.45)	(1.57)
$[0, 99\%]$	0.23	0.47	**0.45**	0.47	**0.50**	0.23
	(0.69)	(1.39)	(1.89)	(1.19)	(2.28)	(1.32)
$[90\%, 95\%]$	0.24	**0.45**	**0.40**	0.45	**0.41**	0.10
	(1.11)	(3.10)	(2.88)	(1.54)	(2.39)	(0.59)
$[95\%, 99\%]$	0.12	0.25	0.20	0.43	0.37	-0.01
	(0.43)	(0.52)	(0.54)	(1.24)	(1.24)	(-0.06)
$[99\%, 100\%]$	**0.69**	**0.24**	**-0.23**	**0.80**	**0.62**	**0.05**
	(6.43)	(3.25)	(-2.50)	(8.76)	(4.66)	(0.37)
All firms	0.29	0.39	0.23	**0.59**	**0.51**	0.11
	(0.82)	(1.24)	(0.91)	(2.36)	(2.53)	(0.82)

Notes: All series are logged and HP filtered. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 3: Cyclical behaviour of equity issuance: flow approach

Size classes	**Sale of stock and**			**Change in equity and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	-0.11	0.13	0.20	-0.03	0.19	0.20
	(-0.42)	(0.50)	(1.20)	(-0.13)	(0.75)	(1.43)
$[0, 50\%]$	-0.10	0.15	**0.22**	0.03	0.23	**0.20**
	(-0.43)	(0.63)	(1.66)	(0.15)	(1.17)	(2.08)
$[0, 75\%]$	-0.12	0.13	**0.24**	0.07	0.22	**0.17**
	(-0.58)	(0.56)	(1.88)	(0.35)	(1.18)	(2.04)
$[0, 99\%]$	-0.21	0.05	0.30	0.06	0.15	0.10
	(-1.20)	(0.22)	(1.35)	(0.28)	(0.63)	(0.93)
$[90\%, 95\%]$	-0.07	**0.31**	**0.31**	0.18	**0.25**	**0.09**
	(-0.47)	(2.56)	(3.18)	(1.05)	(1.79)	(1.90)
$[95\%, 99\%]$	**-0.28**	-0.29	0.08	0.04	-0.06	-0.14
	(-1.81)	(-1.10)	(0.30)	(0.18)	(-0.22)	(-0.83)
$[99\%, 100\%]$	0.08	-0.13	-0.23	**0.32**	-0.08	**-0.23**
	(0.46)	(-0.90)	(-0.76)	(4.07)	(-0.51)	(-1.83)
All firms	-0.14	-0.00	0.17	0.17	0.07	-0.01
	(-0.74)	(-0.00)	(0.58)	(1.03)	(0.30)	(-0.08)
Size classes	**Sale of stock and**			**Change in equity and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.22**	**0.91**	**0.35**	**0.31**	**0.91**	**0.28**
	(5.96)	(13.66)	(6.38)	(7.96)	(14.91)	(4.85)
$[0, 50\%]$	**0.16**	**0.81**	**0.28**	**0.26**	**0.80**	**0.15**
	(1.72)	(6.39)	(5.04)	(3.63)	(8.02)	(3.04)
$[0, 75\%]$	0.07	**0.65**	**0.33**	0.21	**0.63**	0.12
	(0.38)	(3.90)	(3.38)	(1.13)	(4.69)	(1.49)
$[0, 99\%]$	-0.13	0.15	**0.34**	0.15	0.23	-0.06
	(-0.41)	(0.54)	(2.81)	(0.48)	(0.84)	(-0.61)
$[90\%, 95\%]$	-0.11	**0.35**	**0.29**	0.03	**0.28**	**-0.18**
	(-0.39)	(2.87)	(2.76)	(0.14)	(2.10)	(-1.66)
$[95\%, 99\%]$	-0.08	-0.18	0.09	**0.31**	0.16	**-0.28**
	(-0.93)	(-0.71)	(0.32)	(2.03)	(0.51)	(-3.63)
$[99\%, 100\%]$	0.33	-0.03	**-0.24**	**0.36**	**0.48**	**-0.39**
	(1.26)	(-0.16)	(-1.96)	(2.38)	(3.06)	(-5.30)
All firms	-0.01	0.07	0.08	0.28	0.28	**-0.25**
	(-0.04)	(0.21)	(0.41)	(1.02)	(1.15)	(-2.69)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 4: Cyclical behaviour of debt issuance: level approach

Size classes	LT debt issues and			Change in liabilities and		
	GDP$_{t-1}$	GDP$_t$	GDP$_{t+1}$	GDP$_{t-1}$	GDP$_t$	GDP$_{t+1}$
$[0, 25\%]$	**0.27**	**0.30**	0.11	**0.49**	**0.44**	0.10
	(3.01)	(3.94)	(0.87)	(4.86)	(3.53)	(0.57)
$[0, 50\%]$	**0.29**	**0.30**	0.08	**0.62**	**0.49**	0.04
	(3.45)	(4.12)	(0.63)	(8.06)	(3.77)	(0.18)
$[0, 75\%]$	**0.38**	**0.35**	0.08	**0.69**	**0.52**	-0.00
	(5.08)	(4.03)	(0.65)	(9.28)	(3.60)	(-0.01)
$[0, 99\%]$	**0.50**	**0.31**	0.06	**0.84**	**0.43**	-0.15
	(3.84)	(2.07)	(0.50)	(21.86)	(3.04)	(-0.81)
$[90\%, 95\%]$	**0.51**	**0.36**	0.13	**0.81**	**0.50**	-0.04
	(3.38)	(2.16)	(1.30)	(20.53)	(4.53)	(-0.27)
$[95\%, 99\%]$	**0.47**	**0.19**	0.02	**0.78**	**0.26**	**-0.24**
	(2.34)	(1.28)	(0.17)	(12.48)	(1.65)	(-2.35)
$[99\%, 100\%]$	-0.05	-0.13	**-0.26**	**0.35**	-0.05	**-0.52**
	(-0.23)	(-0.82)	(-1.91)	(3.60)	(-0.44)	(-5.97)
All firms	**0.41**	**0.23**	-0.02	**0.71**	**0.26**	**-0.33**
	(3.36)	(1.77)	(-0.14)	(10.52)	(2.11)	(-2.43)

Size classes	LT debt issues and			Change in liabilities and		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.44**	**0.77**	**0.57**	**0.64**	**0.90**	**0.63**
	(3.82)	(7.68)	(7.61)	(7.06)	(21.44)	(13.30)
$[0, 50\%]$	**0.40**	**0.71**	**0.58**	**0.67**	**0.92**	**0.67**
	(3.81)	(6.40)	(7.71)	(6.86)	(33.65)	(13.28)
$[0, 75\%]$	**0.42**	**0.69**	**0.58**	**0.69**	**0.94**	**0.68**
	(5.14)	(8.67)	(11.70)	(8.33)	(67.39)	(9.25)
$[0, 99\%]$	**0.47**	**0.60**	**0.54**	**0.68**	**0.93**	**0.65**
	(5.06)	(7.74)	(5.25)	(9.77)	(61.14)	(9.28)
$[90\%, 95\%]$	**0.54**	**0.67**	**0.57**	**0.70**	**0.94**	**0.69**
	(4.99)	(7.68)	(6.49)	(8.64)	(59.81)	(12.98)
$[95\%, 99\%]$	**0.40**	**0.49**	**0.45**	**0.59**	**0.90**	**0.61**
	(2.56)	(4.31)	(3.69)	(8.93)	(38.83)	(4.19)
$[99\%, 100\%]$	**0.29**	**0.26**	0.11	**0.70**	**0.94**	**0.62**
	(2.93)	(2.02)	(0.49)	(11.18)	(78.84)	(10.04)
All firms	**0.35**	**0.55**	**0.54**	**0.68**	**0.93**	**0.67**
	(4.06)	(8.93)	(7.47)	(12.12)	(65.71)	(10.14)

Notes: All series are logged and HP filtered. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 5: Cyclical behaviour of debt issuance: flow approach

Size classes	**LT debt issues and**			**Change in liabilities and**		
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
$[0, 25\%]$	0.10	**0.45**	0.29	0.19	**0.56**	0.27
	(0.48)	(6.57)	(1.16)	(1.13)	(6.54)	(0.96)
$[0, 50\%]$	0.17	**0.53**	**0.30**	**0.21**	**0.62**	**0.24**
	(1.11)	(4.74)	(2.40)	(2.13)	(12.09)	(1.80)
$[0, 75\%]$	0.24	**0.59**	**0.40**	**0.25**	**0.69**	**0.27**
	(1.59)	(6.62)	(2.90)	(3.56)	(18.31)	(1.98)
$[0, 99\%]$	**0.52**	**0.44**	0.36	**0.54**	**0.74**	0.24
	(5.75)	(1.91)	(1.09)	(7.21)	(11.53)	(0.88)
$[90\%, 95\%]$	**0.40**	**0.39**	0.36	**0.44**	**0.74**	0.35
	(5.21)	(1.78)	(1.24)	(5.09)	(29.00)	(1.20)
$[95\%, 99\%]$	**0.47**	0.20	0.21	**0.66**	**0.61**	0.11
	(3.81)	(0.59)	(0.81)	(9.53)	(4.14)	(0.34)
$[99\%, 100\%]$	0.18	0.02	-0.13	**0.57**	**0.56**	0.02
	(1.15)	(0.12)	(-1.58)	(10.70)	(9.40)	(0.10)
All firms	**0.52**	**0.40**	0.29	**0.60**	**0.73**	0.16
	(6.05)	(1.97)	(1.01)	(12.29)	(10.60)	(0.67)
Size classes	**LT debt issues and**			**Change in liabilities and**		
	ΔA_{t-1}	ΔA_t	ΔA_{t+1}	ΔA_{t-1}	ΔA_t	ΔA_{t+1}
$[0, 25\%]$	**0.23**	**0.41**	0.26	**0.26**	**0.63**	**0.31**
	(1.93)	(2.70)	(1.62)	(2.59)	(13.29)	(1.89)
$[0, 50\%]$	**0.34**	**0.53**	**0.25**	**0.29**	**0.76**	**0.24**
	(4.83)	(2.92)	(3.03)	(3.34)	(11.16)	(1.95)
$[0, 75\%]$	**0.33**	**0.65**	**0.33**	**0.32**	**0.88**	**0.25**
	(4.81)	(5.93)	(3.94)	(4.21)	(20.44)	(2.04)
$[0, 99\%]$	**0.59**	**0.39**	0.26	**0.39**	**0.94**	**0.34**
	(7.99)	(3.57)	(1.29)	(5.94)	(33.45)	(2.37)
$[90\%, 95\%]$	**0.60**	**0.39**	0.19	**0.35**	**0.94**	**0.32**
	(6.57)	(2.55)	(1.13)	(3.81)	(36.70)	(2.07)
$[95\%, 99\%]$	**0.56**	**0.24**	0.05	**0.43**	**0.90**	**0.38**
	(4.92)	(1.95)	(0.30)	(13.90)	(31.19)	(3.18)
$[99\%, 100\%]$	**0.30**	0.05	-0.06	0.15	**0.94**	**0.18**
	(2.31)	(0.70)	(-0.48)	(1.57)	(91.58)	(1.71)
All firms	**0.52**	**0.35**	0.28	**0.31**	**0.94**	**0.32**
	(8.93)	(3.10)	(1.53)	(4.29)	(34.89)	(2.02)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 6: Co-movement between debt and equity

Size classes	Level Approach						Flow Approach					
	Sale of stock and LT debt issues			**Change in equity and change in liab.**			**Sale of stock and LT debt issues**			**Change in equity and change in liab.**		
	ΔD_{t-1}	ΔD_t	ΔD_{t+1}	ΔL_{t-1}	ΔL_t	ΔL_{t+1}	ΔD_{t-1}	ΔD_t	ΔD_{t+1}	ΔL_{t-1}	ΔL_t	ΔL_{t+1}
$[0, 25\%]$	0.10	**0.39**	**0.52**	0.20	**0.56**	**0.64**	0.23	0.05	0.12	0.28	**0.28**	0.12
	(1.15)	(4.46)	(4.78)	(1.41)	(4.78)	(6.26)	(0.97)	(0.85)	(1.16)	(1.37)	(3.14)	(1.41)
$[0, 50\%]$	0.11	**0.39**	**0.49**	**0.31**	**0.58**	**0.57**	0.08	0.05	**0.23**	0.15	**0.23**	0.10
	(1.24)	(3.81)	(4.33)	(1.80)	(7.08)	(3.47)	(0.49)	(0.86)	(3.29)	(0.61)	(2.54)	(0.98)
$[0, 75\%]$	0.08	**0.40**	**0.51**	**0.39**	**0.56**	**0.46**	0.05	**0.18**	**0.30**	0.12	**0.22**	0.07
	(0.71)	(3.30)	(4.22)	(1.83)	(5.60)	(2.22)	(0.24)	(2.12)	(3.15)	(0.48)	(2.22)	(0.65)
$[0, 99\%]$	-0.02	0.25	**0.35**	0.22	0.25	0.13	0.29	0.19	0.11	0.09	-0.01	-0.00
	(-0.06)	(0.75)	(3.79)	(0.48)	(0.74)	(0.93)	(1.60)	(1.13)	(0.86)	(0.35)	(-0.06)	(-0.02)
$[90\%, 95\%]$	0.27	**0.40**	**0.53**	0.30	0.27	0.06	0.23	0.24	**0.23**	0.09	0.13	-0.08
	(1.57)	(3.58)	(3.19)	(0.91)	(1.13)	(0.77)	(1.09)	(1.57)	(1.78)	(0.45)	(1.53)	(-1.06)
$[95\%, 99\%]$	-0.09	0.00	-0.02	0.12	0.07	-0.14	**0.37**	0.19	-0.24	0.22	-0.07	-0.20
	(-0.17)	(0.00)	(-0.10)	(0.25)	(0.15)	(-0.53)	(2.11)	(1.04)	(-0.77)	(1.61)	(-0.26)	(-1.48)
$[99\%, 100\%]$	0.13	**0.26**	-0.02	**0.69**	**0.55**	0.10	**0.20**	**0.58**	0.06	**0.34**	**0.34**	**-0.32**
	(1.10)	(1.96)	(-0.13)	(4.71)	(2.94)	(0.71)	(2.84)	(11.06)	(0.54)	(2.45)	(2.51)	(-4.96)
All firms	-0.08	0.14	0.11	0.37	0.29	0.01	0.26	0.21	0.00	0.20	0.07	**-0.18**
	(-0.19)	(0.34)	(0.62)	(1.05)	(0.89)	(0.06)	(1.43)	(1.30)	(0.01)	(0.83)	(0.33)	(-1.71)

Notes: For the level approach, all series are logged and HP filtered. For the flow approach, real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 7: Cyclical behaviour of retained earnings, profits, and dividends: flow approach

Size classes	**Retained earnings and**			**Profits and**			**Dividends and**		
	GDP_{t-1}	GDP_{t}	GDP_{t+1}	GDP_{t-1}	GDP_{t}	GDP_{t+1}	GDP_{t-1}	GDP_{t}	GDP_{t+1}
$[0, 25\%]$	-0.15	-0.17	**-0.25**	-0.11	-0.17	**-0.31**	**0.59**	**0.47**	-0.11
	(-1.02)	(-0.59)	(-2.17)	(-0.60)	(-0.62)	(-3.06)	(5.95)	(3.58)	(-0.56)
$[0, 50\%]$	-0.18	0.03	-0.02	-0.17	0.01	-0.08	**0.31**	-0.03	-0.21
	(-0.73)	(0.10)	(-0.17)	(-0.61)	(0.03)	(-0.94)	(3.51)	(-0.10)	(-1.49)
$[0, 75\%]$	-0.16	0.18	0.08	-0.15	0.24	**0.13**	**0.36**	0.28	0.05
	(-0.69)	(0.69)	(1.29)	(-0.55)	(0.91)	(2.85)	(3.10)	(1.26)	(0.30)
$[0, 99\%]$	0.09	**0.46**	**0.17**	0.08	**0.58**	**0.27**	**0.13**	**0.28**	**0.38**
	(0.41)	(3.41)	(2.18)	(0.39)	(4.91)	(2.84)	(2.01)	(3.27)	(7.01)
$[90\%, 95\%]$	0.03	**0.39**	0.12	0.10	**0.55**	**0.29**	**0.17**	**0.22**	**0.29**
	(0.11)	(2.05)	(1.45)	(0.42)	(4.03)	(3.53)	(3.88)	(3.81)	(5.33)
$[95\%, 99\%]$	0.16	**0.48**	**0.18**	0.10	**0.60**	**0.27**	-0.10	**0.19**	**0.45**
	(1.23)	(7.61)	(2.33)	(0.90)	(6.26)	(2.63)	(-1.18)	(2.10)	(5.24)
$[99\%, 100\%]$	0.33	**0.38**	0.10	0.17	**0.39**	0.07	0.09	0.19	0.23
	(1.05)	(4.79)	(0.38)	(0.88)	(4.01)	(0.36)	(0.52)	(1.39)	(1.10)
All firms	0.22	**0.46**	0.14	0.12	**0.53**	**0.20**	0.14	**0.30**	**0.39**
	(0.80)	(4.04)	(1.19)	(0.59)	(5.12)	(1.73)	(1.23)	(3.03)	(5.62)

Size classes	**Retained earnings and**			**Profits and**			**Dividends and**		
	ΔA_{t-1}	ΔA_{t}	ΔA_{t+1}	ΔA_{t-1}	ΔA_{t}	ΔA_{t+1}	ΔA_{t-1}	ΔA_{t}	ΔA_{t+1}
$[0, 25\%]$	-0.30	**-0.60**	**-0.26**	-0.26	**-0.57**	**-0.30**	0.04	0.05	**-0.23**
	(-1.03)	(-2.22)	(-5.49)	(-0.75)	(-1.92)	(-6.15)	(0.29)	(0.19)	(-3.60)
$[0, 50\%]$	**-0.37**	-0.20	0.12	-0.34	-0.17	0.11	0.06	0.06	**0.13**
	(-1.68)	(-0.66)	(1.20)	(-1.19)	(-0.58)	(1.31)	(0.55)	(0.26)	(3.12)
$[0, 75\%]$	-0.22	0.10	**0.26**	-0.24	0.21	**0.34**	0.01	0.12	0.16
	(-1.18)	(0.29)	(1.86)	(-0.93)	(0.62)	(2.69)	(0.08)	(0.44)	(0.72)
$[0, 99\%]$	0.02	**0.71**	**0.37**	-0.02	**0.77**	**0.48**	-0.00	**0.19**	**0.39**
	(0.12)	(13.94)	(7.98)	(-0.13)	(10.35)	(9.90)	(-0.01)	(2.36)	(4.96)
$[90\%, 95\%]$	-0.00	**0.60**	**0.24**	0.03	**0.71**	**0.44**	0.15	**0.17**	**0.34**
	(-0.01)	(4.65)	(4.91)	(0.21)	(8.68)	(12.58)	(1.47)	(2.19)	(7.96)
$[95\%, 99\%]$	0.09	**0.77**	**0.53**	-0.01	**0.77**	**0.59**	**-0.14**	0.09	**0.41**
	(0.58)	(30.97)	(11.14)	(-0.05)	(15.72)	(8.02)	(-2.04)	(0.89)	(5.27)
$[99\%, 100\%]$	-0.00	**0.71**	**0.39**	0.03	**0.65**	**0.29**	-0.02	0.21	**0.34**
	(-0.04)	(10.73)	(5.58)	(0.33)	(5.74)	(5.46)	(-0.07)	(1.62)	(4.11)
All firms	0.06	**0.75**	**0.43**	-0.02	**0.77**	**0.47**	-0.12	**0.20**	**0.48**
	(0.32)	(13.04)	(6.61)	(-0.16)	(9.63)	(11.73)	(-0.62)	(2.17)	(9.86)

Notes: Real GDP is logged and HP filtered. Other series are already expressed as a rate and are HP filtered only. For further details, see the text and Appendix B. The standard errors are computed using the VARHAC procedure in den Haan and Levin (1997), and t-statistics are in parentheses. The correlation coefficients statistically different from zero at the 5 per cent significance level are highlighted in bold.

Table 8: Calibration

Parameter		Source		
β	1.022^{-1}	Zhang (2005)		
α	0.70	Cooper and Ejarque (2003)		
τ	0.296	Graham (2000)		
ρ	0.95^4	Cooley and Hansen (1995)		
Parameter		**Moment**	**Data**	**Model**
σ_ϵ	0.0074	Volatility of asset growth	0.039	0.037
σ_ω	0.31	Default premium	119bp	105bp
δ_0	0.082	Investment to assets	0.133	0.134
δ_1	-2.72	Leverage	0.587	0.532
η	0.0975	Fraction of dividend payers	0.469	0.429
μ	0.15	Default rate	0.022	0.020
λ_0	0.30	Change in equity to assets	0.015	0.011
λ_1	125	Volatility of change in equity	0.254	0.221
γ	0.138	Volatility of retained earnings	0.342	0.397

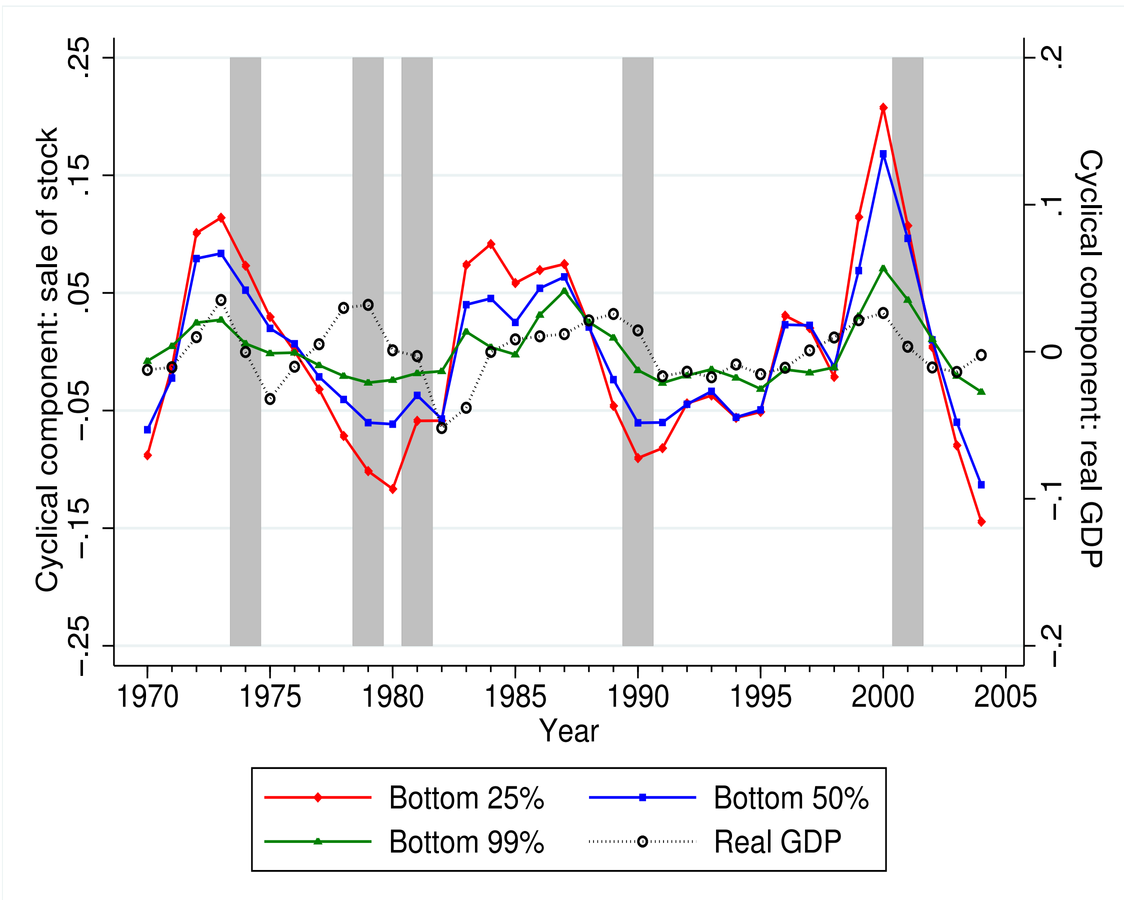
Notes on the model: The parameter β is the discount factor, α the curvature of technology, τ the tax rate, and ρ the persistence of the aggregate shock. The parameter σ_ϵ is the standard deviation of the aggregate shock, σ_ω the standard deviation of the idiosyncratic shock, δ_0 the depreciation rate, and δ_1 the stochastic depreciation parameter. The parameter η is the fixed cost, μ the bankruptcy cost, and λ_0 the equity issuance cost. Finally, λ_1 controls the time-varying cost of equity and γ the variability of the firm's discount factor. The moments in the model are obtained by simulating an economy with 5,000 firms for 5,000 periods and discarding the first 500 observations. **Notes on the data:** Asset growth is the growth rate of the book value of assets. The default premium is the estimated default spread on corporate bonds taken from Longstaff, Mithal, and Neis (2005). Investment includes capital expenditures, advertising, research and development, and acquisitions. Leverage equals liabilities divided by the book value of assets. Dividends is dividends per share by ex-date multiplied by the number of common shares outstanding. Change in equity equals the change in stockholders' equity minus retained earnings. The default rate is the average of annual default rates for all corporate bonds. Finally, retained earnings is the change in the balance-sheet item for (accumulated) retained earnings. The volatilities of asset growth, change in equity, and change in liabilities are from the flow approach. The latter are expressed as a fraction of the volatility of asset growth. The sample period is from 1971 until 2004, except for the default rate series, which is from the period between 1986 and 2004. For further details on the data series used, see Appendix B.

Table 9: Cyclical behaviour of debt and equity in the model

Size classes	**Data**			**Model**		
	Equity issues and GDP					
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
Bottom tercile	-0.04	0.19	0.19	0.12	0.79	0.43
Top tercile	0.19	0.001	-0.10	-0.03	0.28	0.15
All firms	0.17	0.07	-0.01	0.09	0.75	0.41
	Debt issues and GDP					
	GDP_{t-1}	GDP_t	GDP_{t+1}	GDP_{t-1}	GDP_t	GDP_{t+1}
Bottom tercile	0.20	0.61	0.24	-0.10	0.69	0.48
Top tercile	0.60	0.70	0.12	0.04	0.23	0.15
All firms	0.60	0.73	0.16	-0.09	0.66	0.45
	Debt and Equity issues					
	E_{t-1}	E_t	E_{t+1}	E_{t-1}	E_t	E_{t+1}
Bottom tercile	0.10	0.21	0.14	-0.02	0.65	0.40
Top tercile	0.24	0.04	-0.27	-0.15	-0.07	0.15
All firms	0.20	0.07	-0.18	-0.04	0.55	-0.38

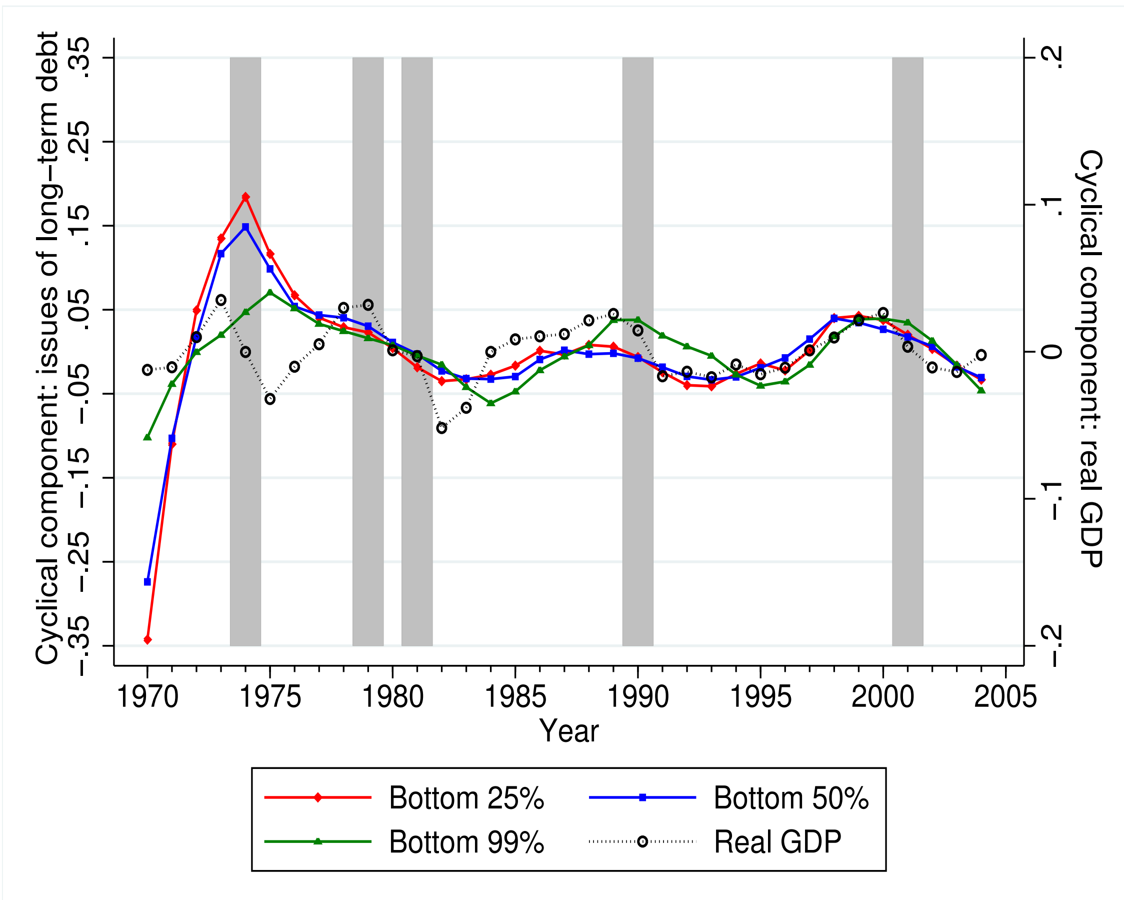
Notes: For the data, the series selected are change in equity and change in liabilities following the flow approach. For the model, we examine the average of equity, e_t, and debt, $(k_t - n_t)$, for the three different size classes.

Figure 1: Cyclical behaviour of sale of stock for different size classes



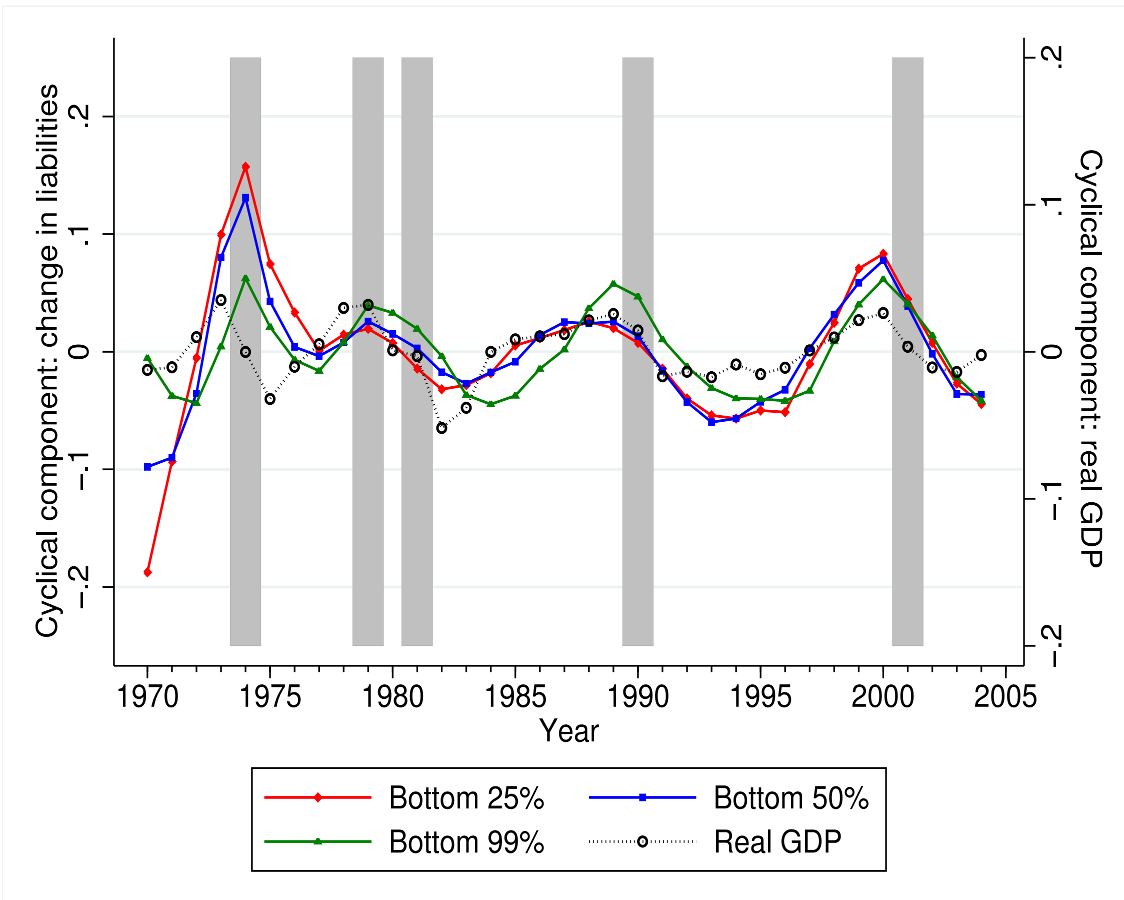
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details, see the text and Appendix B.

Figure 2: Cyclical behaviour of issuance of long-term debt for different size classes



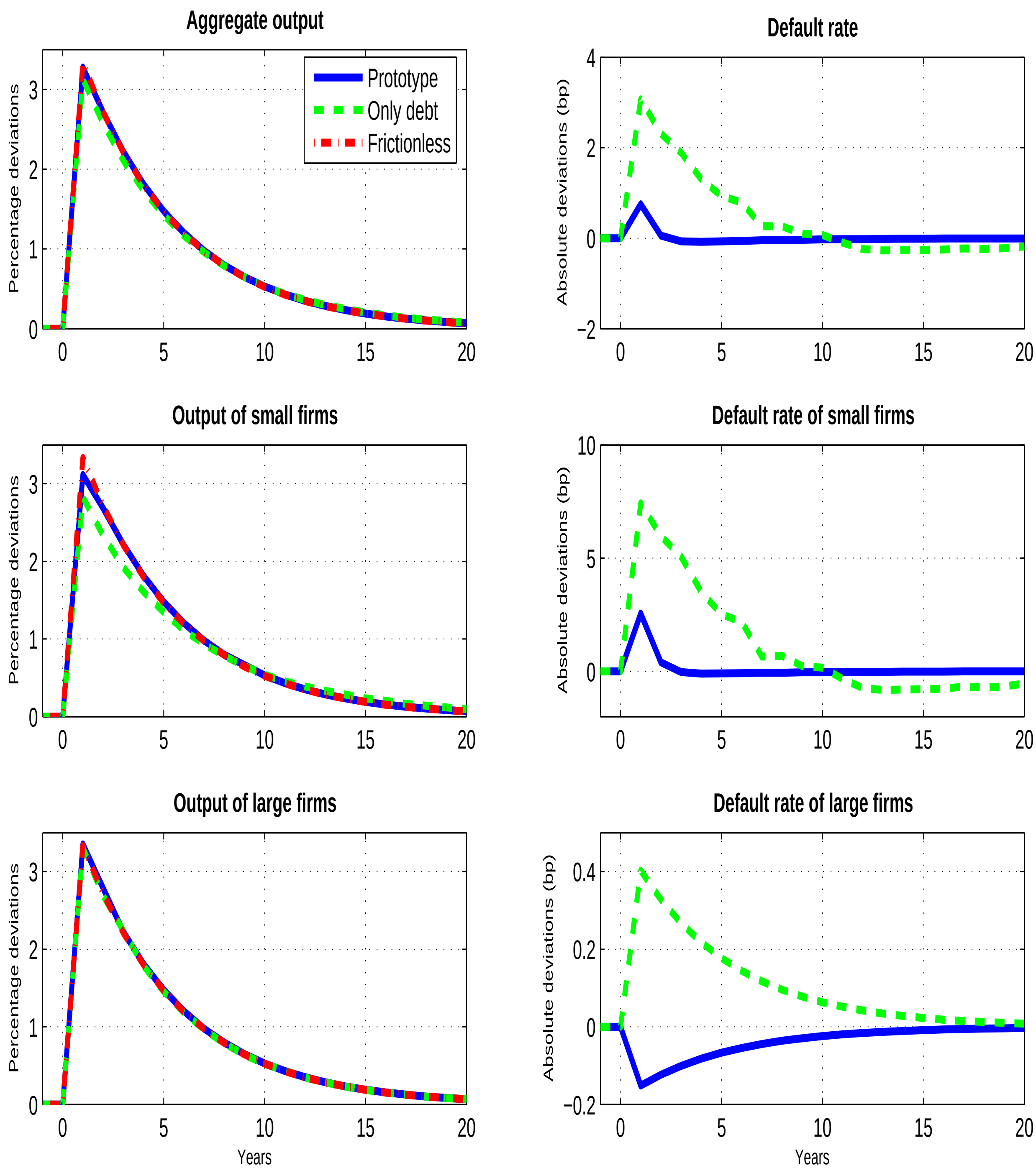
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details, see the text and Appendix B.

Figure 3: Cyclical behaviour of change in liabilities for different size classes



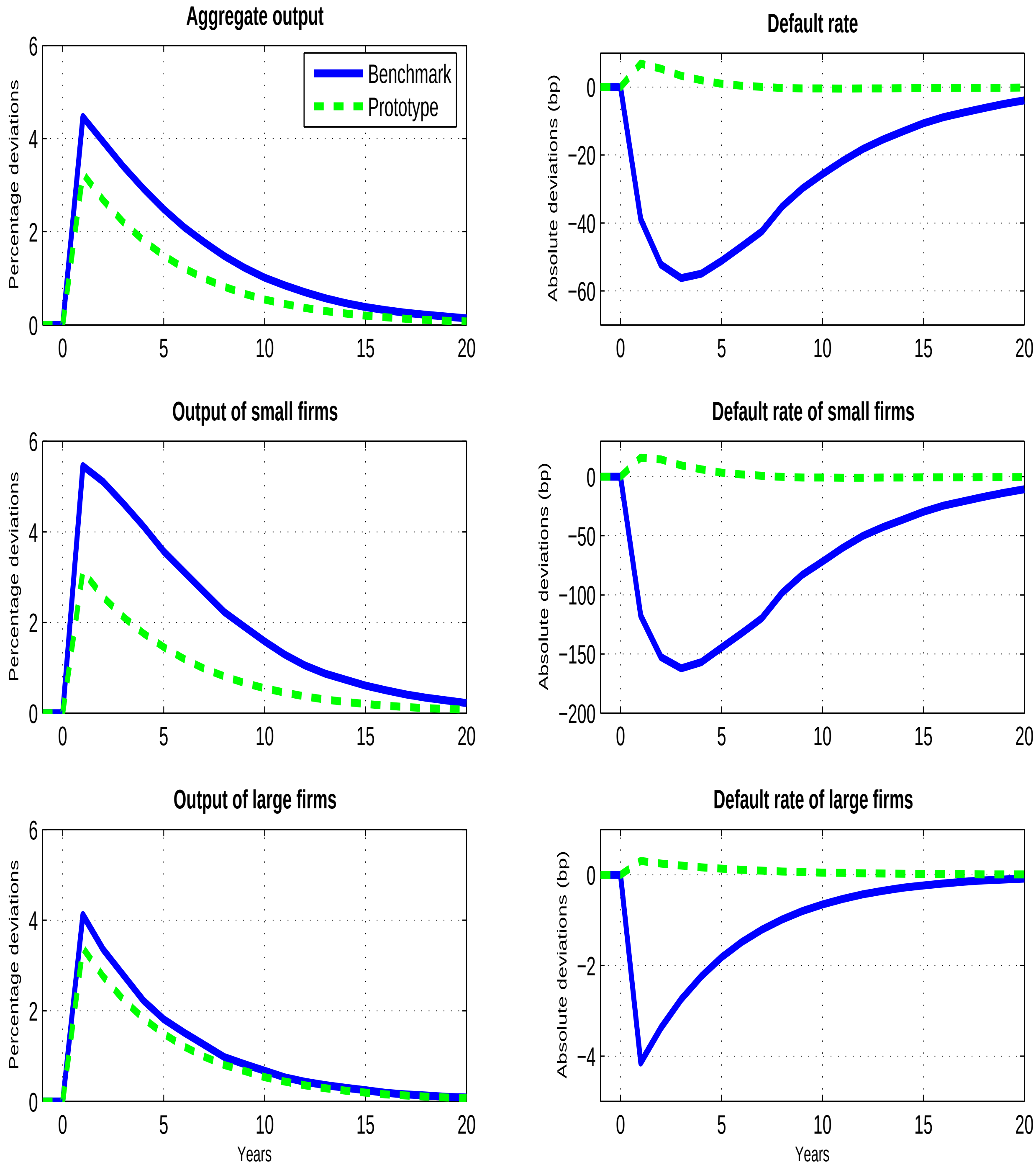
Notes: All series are logged and HP filtered. The shaded areas are NBER dates for recessions. For further details, see the text and Appendix B.

Figure 4: Responses of output and the default rate to positive shock in prototype model



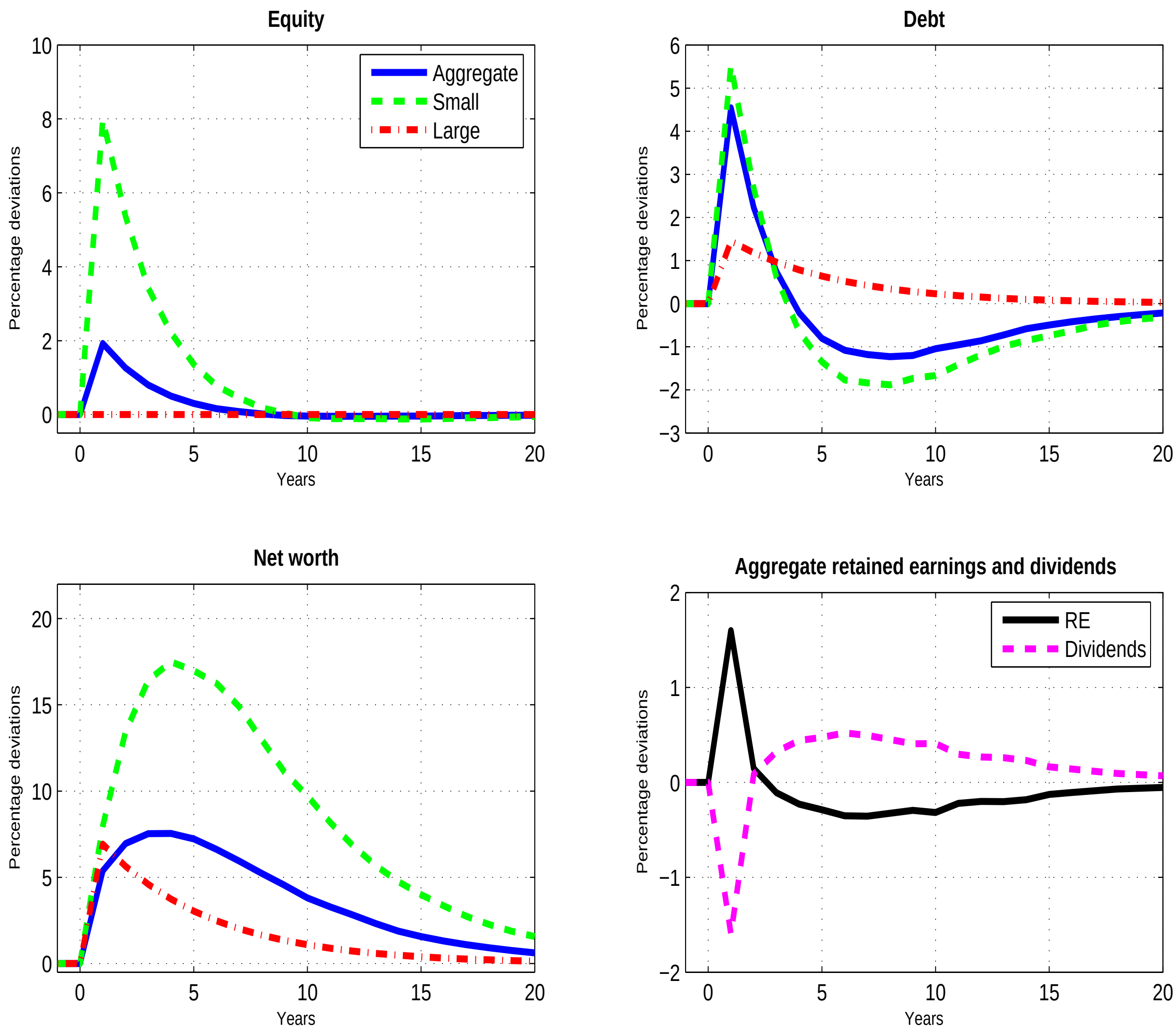
Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 5: Responses of output and the default rate to a positive shock



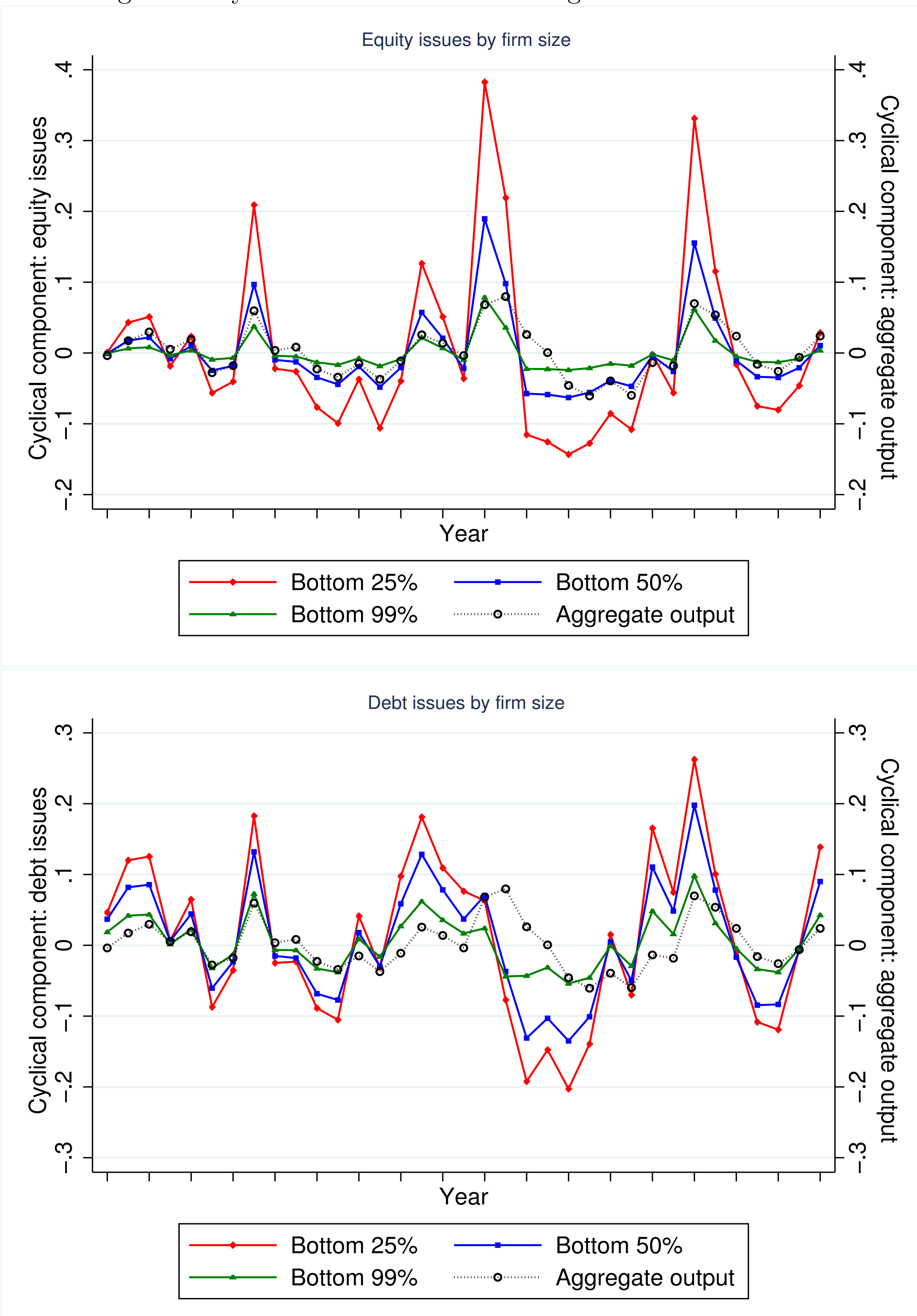
Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 6: Responses of debt, equity, net worth, retained earnings, and dividends to a positive shock



Notes: Small firms are simulated firms at the bottom tercile in terms of the book value of assets. Similarly, large firms are at the top tercile of assets.

Figure 7: Cyclical behaviour of financing sources in the model



Appendix A: Proofs of Propositions

Preliminaries. Before we give the proofs of the propositions, we give the formulas for the derivatives and present a lemma.

The first and second derivatives of $F(\overline{\omega})$ are given by

$$\begin{aligned} F'(\overline{\omega}) &= -(1-\Phi(\overline{\omega})) \leq 0 \text{ and} \\ F''(\overline{\omega}) &= \Phi'(\overline{\omega}) \geq 0. \end{aligned}$$

The first and second derivatives of $G(\overline{\omega})$ are given by

$$\begin{aligned} G'(\overline{\omega}) &= -F'(\overline{\omega}) - \mu\Phi'(\overline{\omega}) \text{ and} \\ G''(\overline{\omega}) &= -F''(\overline{\omega}) - \mu\Phi''(\overline{\omega}). \end{aligned}$$

The signs of the two derivatives of $G(\overline{\omega})$ are not pinned down. For example, there are two opposing effects of an increase of $\overline{\omega}$ on $G(\overline{\omega})$. First, an increase in $\overline{\omega}$ reduces $F'(\overline{\omega})$; i.e., the share that goes to the borrower. This corresponds to an increase in lending rates and, thus, an increase in revenues from firms that do not default. Second, an increase in $\overline{\omega}$ implies an increase in bankruptcy costs. For internal optimal values for $\overline{\omega}$, however, we know that $G'(\overline{\omega}) \geq 0$. If not, then the bank could increase its own and firm profits by reducing $\overline{\omega}$. We summarize this result in the following lemma.

Lemma 1 *For internal optimal values of $\overline{\omega}$, $G'(\overline{\omega}) \geq 0$.*

The following lemma documents a straightforward implication of Assumption A.

Lemma 2 *Under Assumption A,*

$$-\frac{F'(\overline{\omega})}{G'(\overline{\omega})} < 0.$$

To make the algebra less tedious, we set without loss of generality $\delta = 1$ and $r = 0$.

Intuition for proposition 1. Both an increase in k and a reduction in $\overline{\omega}$ lead to an increase in firm profits, and both lead to a reduction in bank profits, at least in the neighborhood of the optimal values for k and $\overline{\omega}$.[1] To satisfy the bank's break-even condition, the firm, thus, faces a trade-off between a higher capital stock and a lower default rate.

If $\alpha = 1$, then the problem is linear and an increase in n simply means that the scale of the problem increases. Consequently, an increase in n does not affect the default rate, but simply leads to a proportional increase in k. When $\alpha < 1$, the decreasing returns imply that an increase in k is not as attractive anymore, and the firm will substitute part of the increase in k for a reduction in $\overline{\omega}$ when n increases.

Next, consider what happens if aggregate productivity increases. For the firm, the relative benefit of a higher capital stock versus a lower default rate does not change.[2] An increase in θ means, however, that the break-even condition for the bank becomes steeper; that is, because the bank's revenues in case of default increase, capital becomes cheaper relative to $\overline{\omega}$. In other words, when aggregate productivity is high, that is a good time for the firm to expand, even when it goes together with a higher default rate.[3]

Proof of proposition 1. The result that $d\overline{\omega}/dn = 0$ when $\alpha = 1$ follows directly from the first-order condition (11). Next, consider the case when $\alpha < 1$. Rewriting the first-order condition gives

$$\frac{1}{\alpha\theta k^{\alpha-1}} = -\frac{G'(\overline{\omega})}{F'(\overline{\omega})}F(\overline{\omega}) + G(\overline{\omega}) \tag{A1}$$

$$= \left(1 - \frac{\mu\Phi'(\overline{\omega})}{(1-\Phi(\overline{\omega}))}\right)F(\overline{\omega}) + G(\overline{\omega}). \tag{A2}$$

Assumption A, together with Lemma 1, implies that the right-hand side decreases with $\overline{\omega}$. Suppose, to the contrary, that $d\overline{\omega}/dn > 0$. Then, equation (A2) implies that an increase in net worth must lead to a decrease in capital. But an increase in $\overline{\omega}$ and a decrease in k

[1] At very low levels of k, the marginal product of capital is very high and bank profits may be increasing in k. Such low levels of k are clearly not optimal, since an increase in k would improve both firm and bank profits.

[2] That is, the iso-profit curve does not depend on aggregate productivity.

[3] In itself, this may not be an implausible or undesirable outcome, but it would be if it leads to procyclical default rates, which are counterfactual. With $\alpha = 1$, that would indeed happen. With $\alpha < 1$, an increase in net worth reduces the default rate. Consequently, it is possible that subsequent increases in net worth through retained earnings (which would occur in the dynamic version of the model) would compensate for the upward pressure on the default rate caused by the increase in aggregate productivity. In our numerical experiments, however, we find that the direct effect of the increase in aggregate productivity is substantially stronger.

reduces expected firm profits, and this can never be optimal, because the old combination of $\overline{\omega}$ and k is still feasible when n increases. Similarly, $d\overline{\omega}/dn = 0$ is not optimal; according to equation (A2), it implies that $dk/dn = 0$, but the zero-profit condition of the bank makes an increase in k feasible. Consequently, $d\overline{\omega}/dn < 0$.

We next show that $d\overline{\omega}/d\theta > 0$. By combining equations (10) and (11), we obtain the following expression that does not depend on θ:

$$-\frac{G'(\overline{\omega})}{F'(\overline{\omega})}\frac{F(\overline{\omega})}{G(\overline{\omega})} = \left(\frac{1}{\alpha(1-\frac{n}{k})} - 1\right). \tag{A3}$$

This equation immediately proves the last part of the proposition that $d\overline{\omega}/d\theta = 0$, when $n = 0$. Using Lemmas 1 and 2 together, with the result that $F'(\overline{\omega}) \leq 0$, implies that the left-hand side is decreasing in $\overline{\omega}$. The right-hand side is decreasing in k. Thus, k has to move in the same direction as $\overline{\omega}$. A decrease in $\overline{\omega}$ and k, however, is not consistent with (A2).[4] ■

Proof of proposition 2. Let $\widetilde{k}$ be the solution of capital when there are no frictions. This capital stock is given by

$$\widetilde{k} = \left(\frac{1}{\alpha\theta}\right)^{1/(\alpha-1)}, \tag{A4}$$

which gives

$$\frac{d\widetilde{k}}{\widetilde{k}} = \frac{1}{1-\alpha}\frac{d\theta}{\theta}.$$

From the break-even condition of the bank we get

$$k^{\alpha}G\left(\overline{\omega}\right)d\theta + \theta\alpha k^{\alpha-1}G\left(\overline{\omega}\right)dk + \theta k^{\alpha}G'\left(\overline{\omega}\right)d\overline{\omega} = dk. \tag{A5}$$

Using the break-even condition, this can be written as

$$\frac{k-n}{\theta}d\theta + \alpha\frac{k-n}{k}dk + \frac{k-n}{G\left(\overline{\omega}\right)}G'\left(\overline{\omega}\right)d\overline{\omega} = dk, \quad \text{or} \tag{A6}$$

$$\frac{d\theta}{\theta} + \alpha\frac{dk}{k} + \frac{G'\left(\overline{\omega}\right)}{G\left(\overline{\omega}\right)}d\overline{\omega} = \frac{k}{k-n}\frac{dk}{k}, \quad \text{or} \tag{A7}$$

[4] An increase in θ and a reduction in k lead to a decrease in the left-hand side, while a reduction in $\overline{\omega}$ leads to an increase in the right-hand side.

$$\frac{dk}{k} = \frac{\frac{d\theta}{\theta} + \frac{G'(\overline{\omega})}{G(\overline{\omega})} d\overline{\omega}}{\frac{k}{k-n} - \alpha}. \tag{A8}$$

First, suppose that $n = 0$. The denominator is then equal to the denominator in the expression for the case without frictions. From proposition 1, we know that $d\overline{\omega}/d\theta = 0$ if $n = 0$. Consequently, the percentage change in capital in the model with frictions is equal to the percentage change in the model without frictions. When $n > 0$, there are two factors that push in opposite directions. The denominator is now larger than $1 - \alpha$, which dampens the increase in capital relative to the increase in the frictionless model. The increase in $\overline{\omega}$, however, implies an increase in $G(\overline{\omega})$, which makes capital more responsive relative to the increase in the frictionless model. We will next show that the first effect dominates. The first-order conditions are given by

$$\zeta(\overline{\omega}) = \frac{\alpha\theta k^{\alpha-1} F(\overline{\omega})}{1 - \alpha\theta k^{\alpha-1} G(\overline{\omega})}, \tag{A9}$$

$$\zeta(\overline{\omega}) = -\frac{F'(\overline{\omega})}{G'(\overline{\omega})} = \frac{1}{1 - \mu\Phi'(\overline{\omega})/(1 - \Phi(\overline{\omega}))}. \tag{A10}$$

Let

$$X(\theta, k) = \alpha\theta k^{\alpha-1}. \tag{A11}$$

From (A9) we get

$$\begin{aligned} F dX + X F' d\overline{\omega} &= \zeta' d\overline{\omega} - X\zeta G' d\overline{\omega} - X G \zeta' d\overline{\omega} - \zeta G dX, \\ (F + \zeta G) dX &= (1 - XG)\zeta' d\overline{\omega} + X(1 - \Phi - \zeta(1 - \Phi - \mu\Phi')) d\overline{\omega}, \end{aligned}$$

$$(F + \zeta G) dX = (1 - XG)\zeta' d\overline{\omega} + 0. \tag{A12}$$

Lemma 2 implies that $\zeta' > 0$. From (A9) we know that $(1 - XG) > 0$. Equation (A12) then implies that dX and $d\overline{\omega}$ must have the same sign. From proposition 1, we know that $d\overline{\omega}/d\theta > 0$. Thus, according to equation (A12), $dX/d\theta > 0$. In the model without frictions, $dX/d\theta = 0$, since without frictions $X = \alpha\theta k^{\alpha-1}$ is constant. But $dX > 0$ implies that $dk/d\theta < d\widetilde{k}/d\theta$.■

Proof of proposition 3. Key in proving this proposition is the first-order condition of the equity-issuance problem, equation (19). Since equity issuance costs do not depend on aggregate productivity, equity issuance decreases (increases) in response to an increase in aggregate productivity, θ, when $\partial w/\partial e$ decreases (increases) with θ. The marginal value of an extra unit of equity in the firm, $\partial w/\partial e$, is equal to $\zeta(\overline{\omega})(1 + r)$. From equation (12) we know that the Lagrange multiplier, ζ, can be expressed as a function of $\overline{\omega}$ alone. Moreover, the regularity condition in Assumption A guarantees that $\zeta(\overline{\omega})$ is increasing in $\overline{\omega}$, which means that the marginal value of an extra unit of equity, $\partial w/\partial e$, is increasing in $\overline{\omega}$. Since $\overline{\omega}$ is increasing with aggregate productivity, $\partial w/\partial e$ is increasing with aggregate productivity, which means that equity issuance is increasing. Thus, an increase in θ increases the default rate, which increases the value of an extra unit of net worth in the firm, $\partial w/\partial e$, which, in turn, increases equity issuance.$\blacksquare$

Appendix B: Data Sources

Output and deflator. Real GDP is defined as real gross domestic product, chained 2000 billions of dollars. The source is the U.S. Department of Commerce, Bureau of Economic Analysis. The PPI is the producer price index for industrial commodities. The source is the U.S. Department of labour, Bureau of labour Statistics. We deflate financing sources with PPI because we want to measure the purchasing power of the funds raised for firms.

Compustat. The Compustat data set consists of annual data from 1971 to 2004. It includes firms listed on the three U.S. exchanges (NYSE, AMEX, and Nasdaq) with a non-foreign incorporation code. We exclude financial firms (SIC codes 6000-6999), utilities (4900-4949), and firms involved in major mergers (Compustat footnote code AB) from the whole sample. We also exclude firms with a missing value for the book value of assets, and firm-years that violate the accounting identity by more than 10 per cent of the book value of assets. Finally, we eliminate the firms most affected by the accounting change in 1988, namely GM, GE, Ford, and Chrysler (for details see Bernanke, Campbell, and Whited 1990). Assets, A, is the book value of assets (Compustat data item 6). Net change in total liabilities, ΔL, is the change in Compustat data item 181 between period t and $t-1$. Retained earnings, ΔRE, is the change in the balance-sheet item for (accumulated) retained earnings (36). Change in the book value of equity, ΔE, equals the change in stockholders' equity (216) minus retained earnings. Sale of stock, ΔS, equals the sale of common and preferred stock (108), and ΔD equals issuance of long-term debt (111). Leverage, L/A, equals liabilities (181) divided by assets. Dividends equals dividends per share by ex-date (26) multiplied by the number of common shares outstanding (25). Operating income equals operating income before depreciation (13). Investment equals capital expenditures (30) plus advertising (45) plus research and development (46) plus acquisitions (129).

Default rate and premium. The annual default rate is from Moody's (mnemonic USMD-DAIW in Datastream), and it is for all corporate bonds in the United States. The default premium is the estimated default spread on corporate bonds taken from Longstaff et al. (2005).

Appendix C: Robustness and Extensions

We have written an extensive appendix in which we do the following:

- We report the robustness of our results by using an alternative methodology to construct the cyclical components of our preferred debt and equity series. The alternative methodology corrects for composition effects in the firm categories.
- We consider alternative equity and debt variables from the Compustat universe: namely, net sale of stock, the change in equity as defined by Baker and Wurgler (2002), net issues of long-term debt, and the change in total debt.
- We discuss the cyclical behaviour of leverage using the Compustat data set.
- We use series from the Federal Reserve Bulletin and the Flow of Funds. The disadvantage of these two series is that they are available only at the aggregate level, but the advantage is that they are available for a longer time period.
- We discuss in detail empirical studies that analyze the cyclical behaviour of debt and equity finance.
- We consider in more detail the time-series behaviour of the debt and equity series of firms in the top 1 per cent of the size distribution.
- We document that the default rate is countercyclical.

The extensive appendix can be downloaded from `http://www.bankofcanada.ca/` `/ec/fcovas/cyclical.pdf`.

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