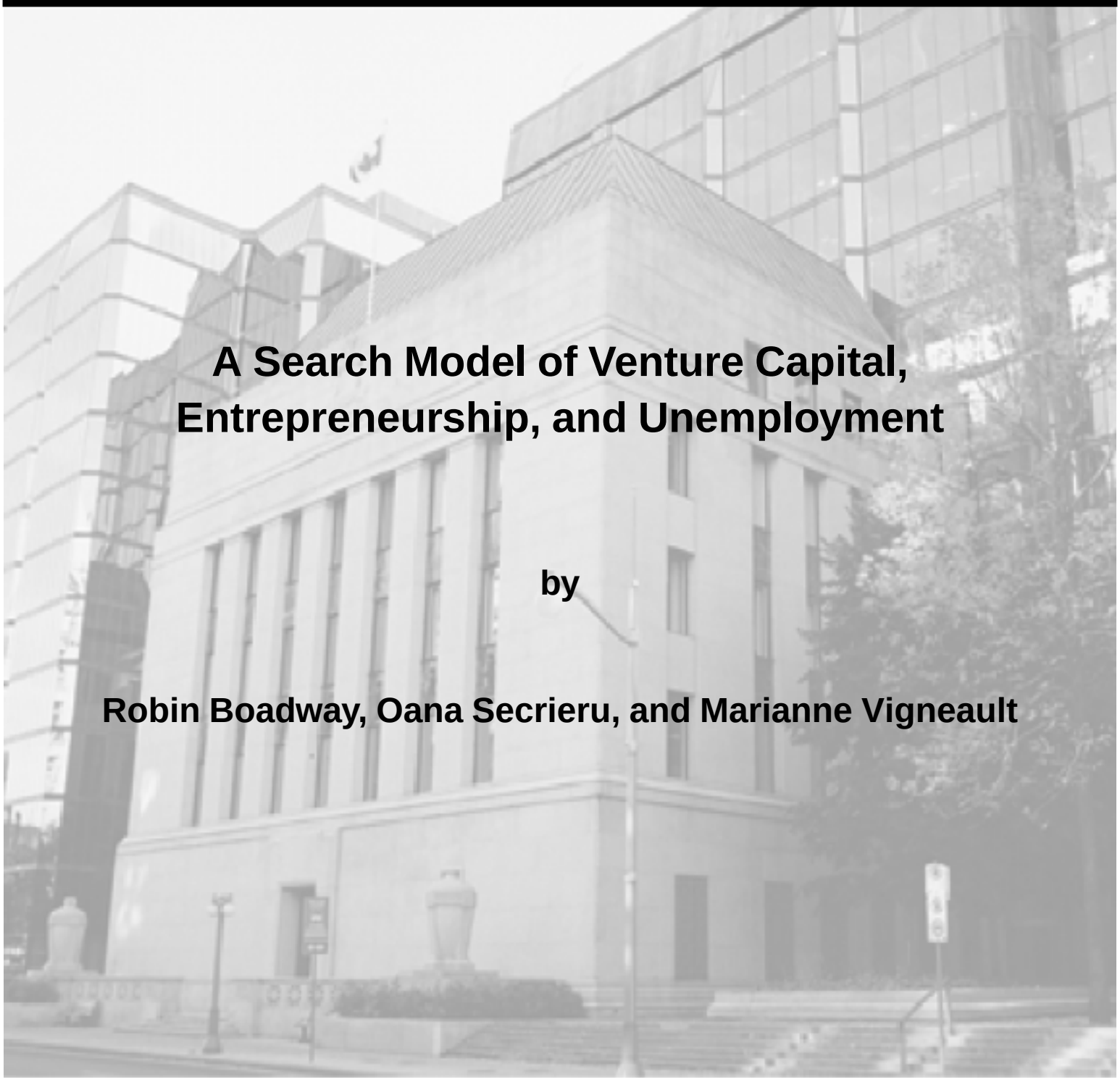
Bank of Canada Banque du Canada

Working Paper 2005-24 / Document de travail 2005-24



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ISSN 1192-5434

Printed in Canada on recycled paper

Bank of Canada Working Paper 2005-24

September 2005

A Search Model of Venture Capital, Entrepreneurship, and Unemployment

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Contents

Acknowledgements

We would like to thank seminar participants at the Bank of Canada for their useful suggestions. Any errors or omissions are ours.

Abstract

The authors develop a search model of venture capital in which the number of successful matches of entrepreneurs and venture capitalists (VCs) at any moment in time is a function of the number of entrepreneurs searching for funds, the number of VCs searching for entrepreneurs, and the number of vacancies posted by each VC. The authors extend the literature by incorporating search unemployment and they explicitly model the occupational choice of individuals to become workers or entrepreneurs. Their analysis shows that, in the market equilibrium, the level of advice VCs offer is inefficiently low compared with the social optimum. Furthermore, the number of vacancies, the level of employment, and the number of potential entrepreneurs are generally either too low or too high relative to their socially optimal level. Policy to achieve the social optimum consists of a capital gains subsidy, an employment tax or subsidy, and an investment tax or subsidy.

JEL classification: D82, G18, G24, H21, J64
Bank classification: Financial markets; Fiscal policy; Labour markets

Résumé

Les auteurs élaborent un modèle de recherche de capital de risque dans lequel le nombre de jumelages réussis entre entrepreneurs et sociétés de capital de risque à n'importe quel moment est fonction du nombre d'entrepreneurs à la recherche de financement, du nombre de sociétés de capital de risque à la recherche d'entrepreneurs et du nombre d'ouvertures dans chacune de ces sociétés. Les auteurs vont au-delà des études existantes en tenant compte du chômage de recherche d'emploi et ils modélisent de façon explicite le choix fait par les personnes de devenir employé ou entrepreneur. Leur analyse montre qu'en situation d'équilibre du marché, le niveau de conseils offert par la société de capital de risque est trop bas comparativement à l'optimum social. De plus, le nombre d'ouvertures, le niveau de l'emploi et le nombre d'entrepreneurs potentiels sont, en général, soit trop bas soit trop élevés par rapport à leur niveau optimal sur le plan social. Une politique visant l'atteinte de l'optimum social devrait comporter une subvention relative aux gains en capital, un impôt sur l'emploi ou une subvention à l'emploi, et un impôt sur l'investissement ou une subvention à l'investissement.

Classification JEL : D82, G18, G24, H21, J64
Classification de la Banque : Marchés financiers; Politique budgétaire; Marchés du travail

1 Introduction

Venture capitalists (VCs) specialize in screening and monitoring projects in particular industries, and in financing small entrepreneurs in those industries. VCs have become important providers of capital to small and young firms, which typically have difficulty raising capital because they have little collateral and operate in an environment fraught with uncertainty and high risk. Furthermore, entrepreneurs' lack of business experience places them at higher risk of failing to turn a good idea into a profitable enterprise. VCs offer valuable advice to these small and young firms.

The relationship between entrepreneurs and VCs is subject to informational asymmetries that can take two forms: adverse selection and moral hazard. Adverse-selection problems arise when entrepreneurs are better informed about the probability of success of their projects than are outside investors. In this case, an entrepreneur may have an incentive to hide the true probability of success. Since investors cannot distinguish between high- and low-quality projects, all must be offered the same financial terms, and an inefficient number of projects may be funded in the pooling equilibrium. As de Meza and Webb (1987) show, either too many or too few projects may be funded, depending on the distribution of project types within the pool. Moral-hazard problems arise when one party to a transaction cannot observe the actions taken by the other party. An important example of this is when an entrepreneurial firm's outside investors cannot observe the entrepreneur's effort. Importantly, problems caused by asymmetric information are less severe in the case of large and established firms, because those firms can use their assets as collateral and can also provide a track record to outside investors. Small and young firms have neither collateral nor a track record.

The specialized screening and monitoring abilities of VCs makes them informed investors and enables them to reduce the agency costs between entrepreneurs and outside investors (Chan, 1983; Admati and Pfleiderer, 1994; Amit, Brander, and Zott, 1998; Casamatta, 2002). Moreover, the presence of informed investors (VCs) in the market increases welfare by helping entrepreneurs to offer high-return projects (Chan, 1983). Indeed, the existing evidence suggests that firms backed by VCs are more innovative, speed up the time to market, and grow faster than their industry counterparts (Hellmann and Puri, 2000; Kortum and Lerner, 2000; Hirukawa and Ueda, 2003; Keuschnigg, 2002; OECD, 1996).

The VCs' superior knowledge of particular industries also plays a certification role for

firms that decide to go public. The VCs can certify that the offering price of the issue reflects all available and relevant information (Megginson and Weiss, 1991). The existing evidence also shows that the capital markets recognize the quality of VCs' monitoring services by requiring less underpricing for issues with higher quality VCs (Barry, Muscarella, Peavy III, and Vetsuypens, 1990; Gompers, 1996), because experienced VCs successfully time initial public offerings: they are more likely to take companies public when their valuations are at their absolute and short-run peaks (Lerner, 1994). Venture capital financing also seems to prevail in high-risk equilibria characterized by a high degree of uncertainty about project quality, which exemplifies the advantage VCs have over banks, for example. Bank financing, on the other hand, prevails in low-risk equilibria where there is little uncertainty about project outcomes (Chan, 1983; Dietz, 2002; Bernhardt and Krasa, 2003; Landier, 2003). While preferred equity is the optimal choice in the presence of high levels of uncertainty regarding project quality (Trester, 1998), debt financing is the optimal choice if the intellectual property rights of the entrepreneur are not securely protected: the VC can steal the entrepreneur's idea, but the bank cannot (Ueda, 2004).[1] At the same time, larger projects tend to receive equity financing, whereas smaller ones receive debt financing (Ueda, 2004) or angel financing (Casamatta, 2002).

The evidence also shows that a large VC firm can receive up to 1,000 investment proposals each year, but it ends up financing only about a dozen of them (Sahlman, 1990). This indicates that experienced venture capital is scarce, possibly due to the slow entry of experienced VCs. Despite this phenomenon, the literature on venture capital financing initially developed using the assumption that any entrepreneur with a viable business idea meets a VC who finances the entrepreneurs' project.[2] This stream of literature typically examines the relationship between an entrepreneur—the agent—who has a high-risk business idea and no funds and a VC—the principal—who can provide both funds and managerial advice in exchange for a share of the entrepreneur's profits. This relationship is characterized by a double moral-hazard problem, because neither the entrepreneur nor the VC can observe the effort/advice level of the other party. In equilibrium, the entrepreneur and the VC underinvest effort and advice, respectively. Each of them would like to be the full residual claimant

[1]There is also a literature that suggests convertible shares are an efficient way of dealing with information asymmetries. See Dewatripont, Legros, and Matthews (2002).

[2]See, for example, Keuschnigg and Nielsen (2003a,b, 2004), and Kanniainen and Keuschnigg (2003).

of profits. Since each is entitled to only a share of profits while bearing the full cost of effort, both parties underinvest. In the case where the VC is allowed to finance and advise more than one entrepreneurial firm, there is a trade-off between the number of firms in the VC's portfolio and the extent of managerial advice offered to each of the firms (Kanniainen and Keuschnigg, 2003; Cumming, 2001). Taxes and subsidies play an important role in restoring efficiency in these models. One additional drawback of the principal-agent approach is that the VC has all the bargaining power, which implies that the VC gets all the rents. This is no longer the case in our paper, where rents are shared between the entrepreneur and the VC according to their bargaining power.

We assume in this paper that informed capital—that is, venture capital—is in limited supply. Therefore, an entrepreneur who has a business idea but no funds may or may not find a VC to screen and invest in the project. To capture this idea, we employ a simple stylized search model of venture capital where the number of successful matches of entrepreneurs and VCs at any moment in time is a function of the number of entrepreneurs searching for funds, the number of VCs searching for entrepreneurs, and the number of vacancies posted by each VC.[3] Our model is a static analogue to the usual dynamic search model, which simplifies the analysis considerably without sacrificing the basic insight of search models.[4] The model is closest to Inderst and Müller (2004), Michelacci and Suarez (2002), and Keuschnigg (2003), who also consider search models of venture capital, although their focus is different than ours. Michelacci and Suarez (2002) develop a search model of venture capital and focus on the relationship between informed capital and the decision of a firm to go public. A firm that is matched with a VC has to decide at which stage to go public. In deciding when to go public, young firms face a trade-off between the liquidity, diversification, and recycling gains of going public and the costs due to being listed before maturity. Firms go public the sooner informed capital is "recycled" for the financing of new firms. The equilibrium is subject to the standard search inefficiencies identified by Hosios (1990), which depend on the balance of bargaining power between the entrepreneur and the VC. If VCs' bargaining power dominates, this leads to inefficiently low entry into entrepreneurship and translates into underdevelopment of the stock market, because not enough capital is made available

[3]See, for example, Pissarides (2000) for the standard labour market search model.

[4]See Johnson and Layard (1986) for the static search model. Boadway, Cuff, and Marceau (2004) have shown that the static model is analogous to the steady-state version of the dynamic model.

for the financing of new start-ups.

Keuschnigg (2003) provides a rich policy analysis of venture capital-backed entrepreneurship using a search model of venture capital. In his paper, optimal policy consists of: (i) subsidies to basic research spending, to reduce the entry costs of potential entrepreneurs; (ii) output subsidies to successfully established firms, to correct for the externalities due to monopolistic power of innovative firms; (iii) revenue subsidies to entrepreneurs and VCs, and a tax on start-up investment spending, to address the underinvestment due to the double moral hazard in the relationship between entrepreneurs and VCs; and (iv) an entry subsidy either to entrepreneurs or to VCs, to correct for search externalities.

Inderst and Müller (2004) provide a short- and long-run analysis of VC entry in a search market. They, much like Keuschnigg (2003), identify inefficiencies due to search, and inefficiencies due to an imbalance in profit sharing relative to the bargaining powers of entrepreneurs and VCs. Inderst and Müller, however, do not derive the policy implications of their model.

While the papers by Michelacci and Suarez (2002), Keuschnigg (2003), and Inderst and Müller (2004) model a search environment for venture capital, they do not explore the implications of imperfect matching for the level of employment and the level of frictional unemployment. These are important issues in the debate on the role of government's involvement in providing incentives for entrepreneurial activity. The rationale for government intervention is that entrepreneurship has been identified as a key component in an economy's ability to grow and alleviate high unemployment. Our paper extends the literature on entrepreneurship in three ways. First, it includes search unemployment. Second, it explicitly models the occupational choice of individuals to become workers or entrepreneurs. Third, it models the more realistic setting wherein entrepreneurs do not have an informational advantage over VCs regarding a project's probability of success. This setting explicitly incorporates the role of VCs as informed investors who have superior knowledge of a particular industry and have superior screening skills. These qualities provide an important motivation for potential entrepreneurs to engage a VC. Optimal policy in our model involves correcting for inefficiencies created by entrepreneurs' employment decisions, VCs' decisions in providing advice and choosing the number of vacancies, and the occupational choices of individuals to become workers or entrepreneurs.

The remainder of this paper is organized as follows. Section 2 describes the model. In sections 2.1 and 2.2, we analyze the entrepreneur's and VC's problems, respectively. In section 2.3, we examine the Nash bargaining problem between the entrepreneur and the VC. In section 2.4, we consider the number of vacancies created by a VC, and in section 2.5 we analyze individuals' decisions to enter entrepreneurship. We examine the social optimum in section 3 and government policies to achieve it in section 4. Section 5 offers some conclusions.

2 The Model

The economy comprises F VCs and I individuals. Both F and I are fixed, and I is a large number. Individuals can become entrepreneurs, workers, or unemployed. Entrepreneurs have no initial wealth; if they become entrepreneurs, they need external financing to start a project. The project requires an initial investment, k. A VC provides the initial investment and business advice in exchange for a share, α, of the new business' profits. As workers, individuals are identical, but if an individual becomes an entrepreneur, then the individual is one of two types, 1 or 2, that differs according to the project's probability of success. There are a fixed proportion, z, of type 2 individuals. Projects undertaken by type 1 entrepreneurs are assumed, for simplicity, to have zero probability of success. The probability of success of a type 2's project, p, depends on the managerial advice provided by the VC, a; that is, $p = p(a)$, and is increasing and concave. Advice is critical for the success of the project, and thus $p(0) = 0$. If successful, the entrepreneur of type 2 employs labour according to the production technology, $f(\ell)$, with $f'(\cdot) \geqslant 0$ and $f''(\cdot) < 0$, and workers are paid the wage w. Projects that are unsuccessful produce no output for simplicity, and their workers receive no wages.

An important assumption of the model is that individuals do not know their type prior to making their occupational choice. This assumption captures the notion that the VC has superior knowledge of the industry, and that as a result the entrepreneur engages a VC to screen a project in order to determine whether it is worthwhile. In this setting, we denote by P the number of the I individuals who decide to become potential entrepreneurs. VC financing is scarce, and so not all potential entrepreneurs are "matched" with a VC. We denote by v the "vacancy" rate of financiers. Creating a vacancy is costly, because it entails the screening of potential entrepreneurs. The screening process is assumed to be perfect,

and so only potential entrepreneurs of type 2 are taken on by the VC. Costly screening thus creates frictions in the VC market, and these frictions are captured by a matching function, $x(P, vF)$, which is increasing, concave, continuously differentiable in both arguments, and homogeneous of degree one (constant returns to scale). The matching function gives the number of matchings that results per unit time.

The probability that a vacancy is matched with a potential entrepreneur is:

$$\frac{x(P, vF)}{vF} = x\Big(\frac{P}{vF}, 1\Big) \equiv q(\theta), \tag{1}$$

where

$$\theta = P/vF \tag{2}$$

is a measure of the scarcity of the venture capital. Similarly, the probability that a potential entrepreneur is matched with a vacancy is:

$$\frac{x(P, vF)}{P} = \frac{vF}{P} x\Big(\frac{P}{vF}, 1\Big) = \frac{1}{\theta} q(\theta). \tag{3}$$

We denote the elasticity of $q(\theta)$ by $\eta(\theta)$. By the properties of the matching function, $q'(\theta) \geqslant 0$ and $0 \leqslant \eta(\theta) \leqslant 1$.

The VC incurs three separate costs. The first is the cost of creating a vacancy, δ, associated with the screening of projects, and is increasing and strictly convex in the number of vacancies that are successfully matched and screened; that is, $\delta'(q(\theta)v) > 0$ and $\delta''(\cdot) > 0$. The VC also incurs a cost when advising a type 2 entrepreneur. A linear advice cost function, ga, is assumed for simplicity. Note that advice is private information and is non-verifiable, so it cannot be contracted upon. Profits, however, are observable ex post. The final cost is the financing cost of the initial investment, k, at the exogenous interest rate, r.

In the market equilibrium, the sequence of events is as follows:

- Stage 1: Occupational choice – Individuals choose whether to become potential entrepreneurs or workers.

- Stage 2: Matching and screening – VCs choose the number of vacancies. Venture capitalists and potential entrepreneurs get matched according to a matching function and screening takes place. Potential entrepreneurs who do not find a match and those who do find a match, but are screened to be of type 1, become unemployed.

- Stage 3: Bargaining – Following a successful match and screening, the VC and the entrepreneur bargain over the division of profits.
- Stage 4: Choice of advice – The VC chooses the level of managerial advice.
- Stage 5: Hiring – The success of the projects is revealed and entrepreneurs hire labour.

The equilibrium concept we use for solving this game is that of subgame perfection. Therefore, we begin by first solving for stage 5.

2.1 Stage 5: The entrepreneur's choice of labour

In the final stage of the game, recall that only type 2 entrepreneurs have survived the screening process. We assume that if entrepreneurs are not successful, they become unemployed and receive zero revenues. In this case, their workers are laid off and receive no pay. At this stage, a, α, w, v, and θ have been determined in the previous stages. A representative entrepreneur of type 2 chooses labour to maximize expected profits. Recall that, with probability p, the project is successful and the entrepreneur obtains a share $(1-\alpha)$ of profits. With probability $(1-p)$ the project fails and the entrepreneur receives zero revenues.[5] The entrepreneur's problem in selecting labour is to:

$$\max_{\ell} \quad (1-\alpha)p(a)[f(\ell) - w\ell]. \tag{4}$$

The solution to the entrepreneur's problem solves the first-order condition:

$$f'(\ell) = w. \tag{5}$$

That is, workers are paid the marginal product of labour. Condition (5) determines $\ell(w)$, with $(\partial\ell/\partial w) < 0$, as expected. Substituting $\ell(w)$ into the entrepreneur's objective function defines the profit function, $\pi^E(a, w, \alpha)$.[6]

[5] Allowing individuals to receive an exogenous outside income in the event of failure does not alter our results. Consequently, to simplify the notation, we assume that individuals earn zero income if entrepreneurs are unsuccessful.

[6] The properties of $\pi^E(\cdot)$ are provided in Appendix A.

2.2 Stage 4: The VC's choice of advice

At this stage, the VC chooses the amount of advice, a, for each of the entrepreneurs in their portfolio to maximize profits, taking α, w, v, and θ as given. Given the homogeneity of type 2 entrepreneurs, we know that the equilibrium involves the symmetric treatment of all entrepreneurs taken on by the VC. Furthermore, the assumption of a linear advice cost function implies that we can examine the VC's choice of advice for a representative type 2 entrepreneur. The VC's problem is therefore to:

$$\max_a \alpha p(a)(f(\ell) - w\ell) - (1+r)k - ga - \delta(q(\theta)v). \tag{6}$$

The first-order condition,

$$\alpha p'(a)(f(\ell) - w\ell) - g = 0, \tag{7}$$

determines the optimal advice function, $a(\alpha, w)$.[7] The second-order condition for a maximum

$$D = \alpha p''(a)(f(\ell) - w\ell) < 0 \tag{8}$$

is satisfied. Substituting $a(\alpha, w)$ into the VC's objective function defines the profit function, $\pi^V(\alpha, w, v, k, \theta)$, for a representative project.

Proposition 1 *The optimal advice function, $a(\alpha, w)$, is increasing in α and decreasing in w.*

Proof: Total differentiation of eq. (7) gives:

$$\frac{\partial a}{\partial \alpha} = -\frac{p'}{\alpha p''} > 0; \tag{9}$$

$$\frac{\partial a}{\partial w} = \frac{p'\ell}{p''(f(\ell) - w\ell)} < 0. \tag{10}$$

Q.E.D.

The intuition for Proposition 1 is straightforward. A higher share of profits provides the VC with incentives to supply more advice. Higher wages reduce the VC's payoff and, thus, the VC's incentives to supply advice.

[7] The comparative statics properties are provided in Appendix A.

2.3 Stage 3: Nash bargaining

At this stage, the VC bargains individually with each entrepreneur, given w, v, and θ. The VC and the entrepreneur anticipate $a(\alpha, w)$ and $\ell(w)$ determined at stages 4 and 5, respectively. The Nash bargaining problem is:

$$\max_{\alpha}[\pi^E]^{\beta}[\pi^V - k]^{1-\beta}, \tag{11}$$

where π^E is determined in stage 5 and π^V is determined in stage 4. The entrepreneurs' bargaining power is denoted by $\beta \in (0, 1)$. The threatpoint for the entrepreneur is zero because the entrepreneur obtains zero profits in the event that bargaining with the VC is unsuccessful. The threatpoint for the VC is the initial investment, k, because the VC gets to keep the initial investment in the event that bargaining with the entrepreneur is unsuccessful.

The optimal equity share, $\alpha(w, v, \theta)$, solves the following first-order condition[8]:

$$\pi^E = \beta\Phi + \beta[\pi^V - k](1 - \alpha)\frac{(p')^2}{\alpha p''}, \tag{12}$$

where $\Phi \equiv \pi^E + \pi^V - k$ is the total surplus to be divided between the entrepreneur and the VC. Equation (12) determines the profit-sharing rule, the comparative statics properties of which are ambiguous. The second term in (12) is negative by the properties of the probability function $p(a)$, which gives rise to the following proposition:

Proposition 2 *In equilibrium, the entrepreneur obtains a share of the surplus that is smaller than their bargaining power, β.*

The reasoning behind this proposition is straightforward. From (9), providing the VC with a larger share of the surplus induces an increase in the VC's provision of advice, which benefits both the VC and the entrepreneur. Thus, the entrepreneur is willing to accept a smaller share of the surplus, because doing so increases the expected size of the surplus.

2.4 Stage 2: Choice of vacancies

The VC chooses the number of vacancies so as to maximize expected profits, taking as given θ and w and anticipating $a(\alpha, w)$, $\ell(w)$, and $\alpha(w, v, \theta)$, which are determined at the later

[8]The proof is provided in Appendix B.

stages. The VC's problem is to:

$$\max_{v} \quad q(\theta)v(1-z)[\alpha p(a)(f(\ell)-w\ell)-ga-(1+r)k]-\delta(q(\theta)v). \tag{13}$$

The optimal number of vacancies, $v(w,\theta)$, solves the first-order condition:

$$(1-z)[\alpha p(a)(f(\ell)-w\ell)-ga-(1+r)k]+v(1-z)p(a)(f(\ell)-w\ell)\frac{\partial\alpha}{\partial v}-\delta'=0. \tag{14}$$

The comparative statics properties of $v(w,\theta)$ are ambiguous.

2.5 Stage 1: Occupational choice

Individuals choose whether to become workers or entrepreneurs by comparing the expected payoffs in each situation. In doing so, individuals anticipate $\ell(w)$, $a(\alpha,w)$, $\alpha(w,v,\theta)$, and $v(w,\theta)$, determined at later stages. An individual who decides to become a worker obtains wages, w, with probability p. An individual who decides to become an entrepreneur obtains profits, π^E, provided that the individual obtains a match with a VC and is screened to be of type 2. In equilibrium, an individual is indifferent between becoming a worker or an entrepreneur. This implies that the following occupational-choice equilibrium condition must hold:

$$pw=\frac{1}{\theta}q(\theta)(1-z)\pi^E. \tag{15}$$

Equation (15) determines the wage rate, $w(\theta)$, which can be either increasing or decreasing in θ.

2.6 The market equilibrium

Solving stages 1 through 5 backwards provides the recursive solution for the market equilibrium values of ℓ, a, α, v, and w as functions of θ. If we denote by E the total number of entrepreneurs, E must be equal to the number of VCs times the number of vacancies screened and filled by each VC:

$$E=(1-z)q(\theta)Fv. \tag{16}$$

Furthermore, equilibrium in the labour market requires that individuals become either entrepreneurs or workers:

$$P+\ell E=I. \tag{17}$$

Equations (16) and (17), and the definition of θ given in (2), determine the market equilibrium values for θ, E, and P.

2.7 Unemployment

We denote by $\hat{U}$ the number of ex ante unemployed. We define ex ante unemployment as the level of unemployment before it becomes known which entrepreneurs' projects are successful. Ex ante unemployment therefore results only from matching frictions in the market for venture capital. Since the total number of potential entrepreneurs, P, is equal to the total number of entrepreneurs, E, plus the number of unemployed, $\hat{U}$, it follows that

$$\hat{U} = P - E. \tag{18}$$

Ex post unemployment, on the other hand, also includes workers and entrepreneurs who become unemployed due to the failure of entrepreneurs' projects. If we denote ex post unemployment by $\overline{U}$, the number of ex post unemployed is given by:

$$\overline{U} = \hat{U} + (1-p)(1+\ell)E. \tag{19}$$

3 The Social Optimum

Since all agents are risk-neutral and care only about expected income, we can abstract from redistributive motives and treat aggregate output or GDP as an index of social welfare. GDP is given by:

$$Y = F\{(1-z)q(\theta)v[p(a)f(\ell) - ga - (1+r)k] - \delta(q(\theta)v)\}. \tag{20}$$

The social optimum is said to be constrained Pareto efficient when ℓ, a, and v maximize GDP subject to (16), (17), and (2). From (17), the optimal choices of ℓ and v determine the optimal division of individuals between entrepreneurs and workers. That is, (17) determines the optimal number of potential entrepreneurs, P.

For the social optimum, equations (16), (17), and (2) can be solved for the scarcity of

venture capital function, $\theta(\ell, v)$, the properties of which are:

$$\frac{\partial \theta}{\partial \ell} = -\frac{E}{\Delta} < 0, \tag{21}$$

$$\frac{\partial \theta}{\partial v} = -F\frac{\theta + (1-z)q(\theta)\ell}{\Delta} < 0, \tag{22}$$

where $\Delta = Fv\{1 + \ell(1-z)q'(\theta)\} > 0$. The intuition for these properties is as follows. Equation (21) shows that an increase in the number of workers, ℓ, by reducing the number of potential entrepreneurs makes venture capital less scarce. Equation (22) shows the effect of an increase in the number of vacancies on θ. An increase in v increases the fraction of entrepreneurs who fail each period, and decreases the number of potential entrepreneurs who find a match each period. Venture capital becomes less scarce as a result.

The planner's problem can be treated as an unconstrained one by using the function $\theta(\ell, v)$ obtained above from (16), (17), and (2). For this problem, the first-order conditions for the social optimum are:

$$\frac{\partial Y}{\partial \ell} = F\left\{(1-z)q(\theta)vpf'(\ell) + [\Psi(1-z) - \delta']vq'\frac{\partial \theta}{\partial \ell}\right\} = 0, \tag{23}$$

$$\frac{\partial Y}{\partial a} = F(1-z)qv[p'f(\ell) - g] = 0, \tag{24}$$

$$\frac{\partial Y}{\partial v} = F\left\{(1-z)q\Psi - \delta' q + [\Psi(1-z) - \delta']vq'\frac{\partial \theta}{\partial v}\right\} = 0, \tag{25}$$

where $\Psi = p(a)f(\ell) - ga - (1+r)k$. These first-order conditions determine the socially optimal values ℓ^*, a^*, and v^*. Then, from (16) and (17), we obtain the optimal number of potential entrepreneurs P^*, and, from (18) and (19), we obtain the optimal levels of ex ante and ex post unemployment.

A comparison of the first-order conditions for the social optimum with those of the no-intervention or laissez-faire market equilibrium given in (5), (7), and (14) gives rise to the following proposition:

Proposition 3 *Employment, advice, the number of vacancies, the supply of entrepreneurship, and unemployment in the laissez-faire market equilibrium are inefficient.*

To see this, we evaluate the derivatives of GDP with respect to ℓ, a, and v given in (23), (24), and (25) at the laissez-faire market equilibrium. Doing so gives:

$$\left.\frac{\partial Y}{\partial \ell}\right|_{mkt} = F\left\{(1-z)qvw + \big(\Psi(1-z) - \delta'\big)vq'\frac{\partial \theta}{\partial \ell}\right\} \gtreqless 0, \tag{26}$$

$$\left.\frac{\partial Y}{\partial a}\right|_{mkt} = F(1-z)qv[(1-\alpha)p'f + \alpha p'w\ell] > 0, \tag{27}$$

$$\left.\frac{\partial Y}{\partial v}\right|_{mkt} = F\left\{q(1-z)\left((1-\alpha)pf+\alpha pw\ell-vp(f-w\ell)\frac{\partial \alpha}{\partial v}\right)+\big(\Psi(1-z)-\delta'\big)vq'\frac{\partial \theta}{\partial v}\right\} \gtreqless 0. \tag{28}$$

Equation (26) reflects the effect of an increase in ℓ, starting from the market equilibrium, on social welfare. Beginning with the market's selection of employment, entrepreneurs choose labour to maximize their own profits, taking the wage as a cost and ignoring the effects of their choice of employment on workers' welfare and total expected output via the cost of additional vacancies of VCs. The first effect is positive; that is, an increase in the level of employment increases workers' welfare and total output. The latter effect is a matching externality, in that an increase in employment, by reducing the number of potential entrepreneurs, decreases the scarcity of venture capital. In equilibrium, the VCs respond by increasing vacancies. The second term in (26) is, thus, negative. The total effect of an increase in ℓ on social welfare is, as a consequence, ambiguous. If the effect of an increase in ℓ on workers' welfare dominates the matching externality effect, the right-hand side of (26) is positive. That is, an increase in ℓ increases welfare and the employment level is inefficiently low compared with the social optimum. The reverse is true if the matching externality effect dominates. The employment level is, thus, inefficiently low or high, depending on which effect dominates.

Equation (27) shows the effect of an increase in the level of advice, starting from the market equilibrium, on social welfare. The VCs' choice of advice considers only their own share of profits, which includes labour costs, and is thus inefficiently low compared with the social optimum. An increase in the level of advice above the market equilibrium increases welfare.

Equation (28) reflects the effect of an increase in the number of vacancies, starting from the market equilibrium, on social welfare. The VCs select the number of vacancies by taking into account the effect of v on their own profits and ignoring the effect on the entrepreneurs' share of profits and on the cost of additional vacancies of all VCs. The latter is a matching externality, in that an increase in v increases vacancies by (i) increasing the flow out of entrepreneurship, and (ii) decreasing the scarcity of venture capital.

Given that ℓ and v are inefficient in the market equilibrium, so too are $P(= I - \ell E)$ and $E = (1-z)q(\theta)Fv$. In particular, the number of potential and actual entrepreneurs may be

inefficiently low or high, depending on the magnitudes of the various externalities described above. Similarly, given that ℓ, E, and P are inefficient in the market equilibrium, the levels of ex ante and ex post unemployment defined in (18) and (19) are also inefficient. Thus, unemployment as well may be too high or too low in the market equilibrium, compared with the social optimum.

4 Optimal Policy

The social optimum can be achieved in the decentralized market setting if the government has at its disposal an appropriate set of policy instruments. The set we consider comprises an employment tax, τ, levied on entrepreneurs; a capital gains tax, t, levied on VCs; and an investment tax, σ, levied on VCs. An optimal policy must be such that the first-order conditions for ℓ, a, and v for the market are equivalent to the first-order conditions for the social optimum given by (23), (24), and (25). Note that for ℓ, a, and v to be chosen optimally, the tax rates must be chosen such that the Nash bargaining solution determines the "optimal" division of profits and the occupational choice condition determines the "optimal" wage rate; that is, the wage at which the optimal number of individuals choose to enter the entrepreneurship lottery.

With the set of policy instruments defined above, entrepreneurs' profits are written as:

$$(1-\alpha)p(a)[f(\ell)-(w+\tau)\ell]. \tag{29}$$

Similarly, a VC's expected profits before screening has taken place can be written as:

$$q(\theta)(1-z)\big\{(1-t)[\alpha p(a)(f(\ell)-w\ell)-(1+r+\sigma)k]-ga\big\}\delta(q(\theta)v). \tag{30}$$

With the inclusion of taxes, the market first-order conditions for the selection of ℓ, a, and v are given by:

$$f'(\ell)=w+\tau, \tag{31}$$

$$(1-t)\alpha p'(f(\ell)-w\ell)-g=0, \tag{32}$$

$$q(\theta)(1-z)(1-t)[\alpha p(a)(f(\ell)-w\ell)-(1+r+\sigma)k]+q(\theta)v(1-z)p(a)(f(\ell)-w\ell)\frac{\partial\alpha}{\partial v}-\delta' q(\theta)=0. \tag{33}$$

A comparison of the first-order conditions (23), (24), and (25) with (31), (32), and (33) yields the results for optimal policy described in sections 4.1–4.3.

4.1 Employment taxes

Proposition 4 *The optimal employment tax is given by:*

$$\tau^* = \{[1-(1-z)q(\theta^*)v^*p(a^*)]f'(\ell^*) - w^*\} + \big(\Psi(1-z) - \delta'(q(\theta^*)v^*)\big)v^*q'(\theta^*)\frac{\partial\theta}{\partial\ell} \gtreqless 0. \quad (34)$$

Intuitively, the employment tax is chosen so as to internalize the externalities caused by the entrepreneurs choosing the employment level without taking into account the effect of their choice on the total expected output and the cost of additional vacancies of VCs. The first term in (34) is of ambiguous sign and the second term is positive according to (21). Since the expression on the right-hand side of (34) is of ambiguous sign, it follows that the government can tax or subsidize employment to achieve the social optimum. The government chooses to tax/subsidize employment such that the optimal tax/subsidy closes the gap between the employment level chosen by the entrepreneur and the socially optimal employment level:

$$\tau^* = \frac{\partial\pi^E}{\partial\ell}\bigg|_{\tau=0} - \frac{\partial Y}{\partial\ell}. \quad (35)$$

4.2 Capital gains taxes

Proposition 5 *The optimal capital gains tax is negative and given by:*

$$t^* = 1 - \frac{f(\ell^*)}{\alpha^*(f(\ell^*) - w^*\ell^*)} < 0. \quad (36)$$

The intuition for Proposition 5 is straightforward. In the market equilibrium, VCs provide a level of advice that is too low compared with the social optimum. In order to induce a higher level of advice, it is optimal for the governement to subsidize capital gains. It is straightforward to show that the optimal capital gains subsidy closes the gap between the level of advice chosen by the VC and the socially optimal level of advice:

$$t^* = \frac{1}{\alpha p'(a)(f - w\ell)}\left\{\frac{\partial\pi^V}{\partial a}\bigg|_{t=0} - \frac{\partial Y}{\partial a}\right\}. \quad (37)$$

4.3 Investment taxes

Proposition 6 *The optimal investment tax is:*

$$\sigma^* = \frac{1}{q(\theta^*)k}\Bigg\{q(\theta^*)\alpha^* p(a^*)(f(\ell^*) - w^*\ell^*) + \frac{1}{(1-t^*)}(1-z)\big(\Psi - \delta'(q(\theta^*)v^*)v^* q'(\theta^*)\frac{\partial\theta}{\partial v} - \frac{q(\theta^*)p(a^*)f(\ell^*)}{(1-t^*)} + \frac{q(\theta^*)t^*(1+r)k}{(1-t^*)} + q(\theta^*)v^* p(a^*)(f(\ell^*) - w^*\ell^*)\frac{\partial\alpha}{\partial v}\Bigg\} \gtreqless 0. \tag{38}$$

As expected, the optimal investment tax can be negative or positive. The government chooses the optimal investment tax in order for VCs to internalize the externalities that arise from their choice of vacancies. Thus, the optimal investment tax closes the gap between the number of vacancies chosen by the market and the socially optimal one:

$$\sigma^* = \frac{1}{kq(\theta^*)(1-z)(1-t^*)}\left\{\frac{\partial\pi^V}{\partial v}\,\Big|_{\sigma=0} - \frac{\partial Y}{\partial v}\right\}. \tag{39}$$

From our discussion in the previous section, the optimal tax rates correct for the fact that both entrepreneurs and venture capitalists (i) do not take into account their choices on expected total output, (ii) include the wage as a cost, and (iii) do not take into account the matching externalities. We showed in the previous section that the level of advice in the market equilibrium is too low relative to the social optimum. Optimal policy therefore involves a capital gains subsidy (i.e., $t^* < 0$). The signs of the optimal employment tax and investment tax depend on the relative strengths of these three factors. Thus, it may be optimal to tax or subsidize employment and investment.

The set of three policy instruments we considered are sufficient to restore the inefficiencies arising in the market equilibrium. With the employment tax, capital gains tax, and the investment tax chosen optimally, the levels of employment, advice, and the number of vacancies become efficient. As a result, the number of entrepreneurs and potential entrepreneurs, and the levels of ex ante and ex post unemployment, are all efficient.

5 Conclusions

In this paper, we have developed a matching model of venture capital and entrepreneurship to capture the idea that informed capital is scarce. This is in contrast with the standard model of venture capital, which considers an isolated setting with one VC and one entrepreneur. In a matching model, the number of successful matches of entrepreneurs and VCs at any moment in time is a function of the number of entrepreneurs looking for funds, the number of VCs looking for entrepreneurs, and the number of vacancies posted by each VC. One feature that distinguishes our model from other venture capital search models is that we explicitly model individuals' decisions to become potential entrepreneurs versus workers in a setting where they have no prior knowledge of their innate ability to succeed as entrepreneurs. An individual who decides to become a potential entrepreneur forgoes wage income and faces the risk of not being matched with a VC, in which case the individual becomes unemployed. A potential entrepreneur becomes an entrepreneur only if they find a match and their project is screened to be worthwhile. The project's probability of success depends on the level of advice provided by the VC. If the entrepreneur's project fails, the entrepreneur becomes unemployed and receives no income. This set-up allows us to generate unemployment in the model and examine the implications of imperfect matching for the level of employment and the level of frictional unemployment.

Our analysis shows that, in the market equilibrium, the level of advice provided by the VC is inefficiently low compared with the social optimum, because the VC considers only their own share of profits when selecting the level of advice. At the same time, the VC's portfolio size is inefficient from a social viewpoint, because the VC ignores the additional effort cost on the part of entrepreneurs and on the cost of additional vacancies of all VCs (the matching externality). These two effects work in opposite directions, and therefore the size of the portfolio in the market equilibrium can be inefficiently low or high. The entrepreneur's choice of employment is also inefficiently low or high in the market equilibrium. The entrepreneur maximizes their own profits, and thus ignores the effect of their choice of employment on total expected output and on the cost of additional vacancies of all VCs. This implies that the number of potential entrepreneurs and the levels of ex ante and ex post unemployment are also inefficient.

The optimal policy to achieve the social optimum consists of: (i) a negative capital gains

"tax" to achieve the socially optimal level of advice, (ii) an employment tax (or subsidy) on entrepreneurs to achieve the socially optimal level of employment, and (iii) an investment tax (or subsidy) to achieve the socially optimal number of vacancies. The optimal employment and investment taxes (or subsidies) ensure that the number of potential entrepreneurs, ex ante and ex post unemployment, are restored to their efficient levels.

One simplification of our model is the assumption that only type 2 entrepreneurs' projects have any probability of success. An interesting extension would be to allow more than one type of entrepreneur to be taken on by the VC. In such a setting, tax policy could result in shifts in the types of entrepreneurs that would receive VC financing. A further simplification of our model is that entrepreneurs seek financing only from VCs. An interesting extension would be to allow entrepreneurs access to both venture capital and bank financing. Such an analysis could provide a more interesting environment for screening undertaken by VCs. In particular, a different form of externality in addition to the search externalities identified in this paper could arise if banks could free ride on screening undertaken by VCs and offer entrepreneurs better financing terms that might entice them away from a VC. Incorporating both VC and bank financing into our model could also help identify factors that make VC financing more attractive than bank financing.

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Appendices

A Stages 4 and 5: Comparative Statics

The Envelope Theorem gives the properties of the entrepreneur's expected profit function, $\pi^E(a, w, \alpha)$ and the properties of the VC's expected profit function $\pi^V(\alpha, w, v, \theta, k)$:

$$\frac{\partial \pi^E}{\partial a} = p'(1-\alpha)(f(\ell) - w\ell) > 0; \tag{A.1}$$

$$\frac{\partial \pi^E}{\partial w} = -(1-\alpha)p\ell < 0; \tag{A.2}$$

$$\frac{\partial \pi^E}{\partial \alpha} = -p(f(\ell) - w\ell) < 0. \tag{A.3}$$

$$\frac{\partial \pi^V}{\partial \alpha} = p(a)(f(\ell) - w\ell) > 0; \tag{A.4}$$

$$\frac{\partial \pi^V}{\partial w} = -\alpha p(a)\ell < 0; \tag{A.5}$$

$$\frac{\partial \pi^V}{\partial v} = -\delta' q < 0; \tag{A.6}$$

$$\frac{\partial \pi^V}{\partial \theta} = -\delta' v q' < 0; \tag{A.7}$$

$$\frac{\partial \pi^V}{\partial k} = -(1+r) < 0. \tag{A.8}$$

B Proof of Equation (12)

The Nash bargaining problem is:

$$\max_{\alpha}[\pi^E]^{\beta}[\pi^V - k]^{1-\beta}, \tag{B.1}$$

where

$$\pi^E = (1-\alpha)p(a)(f(\ell) - w\ell), \tag{B.2}$$

$$\pi^V = \alpha p(a)(f(\ell) - w\ell) - (1+r)k - ga - \delta(q(\theta)v), \tag{B.3}$$

The first-order condition is:

$$\beta[\pi^E]^{\beta-1}[\pi^V - k]^{1-\beta}\frac{d\pi^E}{d\alpha} + (1-\beta)[\pi^E]^{\beta}[\pi^V - k]^{-\beta}\frac{d\pi^V}{d\alpha} = 0, \tag{B.4}$$

with

$$\begin{aligned}\frac{d\pi^E}{d\alpha} &= \frac{\partial \pi^E}{\partial \alpha} + \frac{\partial \pi^E}{\partial a}\frac{\partial a}{\partial \alpha} \\ &= -p(a)(f(\ell) - w\ell)\left[1 + (1-\alpha)(f(\ell) - w\ell)\frac{(p')^2}{D}\right],\end{aligned} \tag{B.5}$$

and

$$\frac{d\pi^V}{d\alpha} = \frac{\partial \pi^V}{\partial \alpha} + \frac{\partial \pi^V}{\partial a}\frac{\partial a}{\partial \alpha} = p(a)(f(\ell) - w\ell). \tag{B.6}$$

Gathering like terms, the first-order condition can be written as (12).

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