An Innovative Approach for Modeling Crop Yield Response to Fertilizer Nutrients

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ABSTRACT

Fertilizer recommendations seldom account for agro-climatic conditions, which are important factors that determine the response to fertilizer and the optimal rate of fertilizer. The nitrogen fertilizer response to open pollinated and hybrid canola types will also impact optimal nitrogen rates. This study used quantile regression to model canola yield response to nitrogen fertilizer. Quantile regression can apply different weights to the residuals, facilitating a response estimation where the agro-climatic conditions are not limiting and the yield response is due to the variable of interest. The economically optimal levels of fertilizers were calculated using the proposed and the conventional least squares procedures of the two canola types in western Canada. Results showed that the effects of nitrogen fertilizer on yield depended on the canola type and on the estimation procedure. Optimal levels of nitrogen for open-pollinated canola were estimated as 91, 115, and 134 kg ha^{-1} *for severe, moderate and low levels of agro-climatic constraints. Hybrid had a higher yield potential, and also required more nitrogen fertilizer (137, 142, and 158 kg ha*$^{-1}$*). Unlike conventional approach, proposed approach could benefit producer by recommending less (more) fertilizer when the crop response to fertilizer is expected to be low (high) due to agro-climatic conditions.*

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INTRODUCTION

Fertilizer is a substantial part of the crop production expenditures, but is also an essential nutrient for profitable yields. An increasing fertilizer price and growing awareness on environmental impact of excess fertilizer use has increased interest in the optimal use of fertilizer for crop production. Excess fertilizer application will have negative economic and environmental consequences. Inadequate fertilizer application will result in an opportunity cost from the lost yield potential.

Several studies have been conducted to estimate optimal rates of fertilizer. The conventional approach to determining the optimal fertilizer rate has relied on modeling the relationship between one or more fertilizer inputs and the conditional mean of the crop yield response. Several studies have used least squares estimators to derive a single optimal fertilizer rate (Baker et al., 2004; Makowski, 2005; Beckie and Brandt, 1996). Often data in these studies have outliers and the yield distribution is not normal. Least squares estimators are susceptible to outliers and non-normality. In addition, yield data for fertilizer is often heteroskedastic with higher yield variability associated with higher fertilizer rates. There are several methods to correcting for heteroskedasticity, but the final result will be a single fertilizer rate. Agro-climatic inputs not included in the model also have the potential to interact with included inputs. The conventional approach does not allow for use of this information that was not included in the model. A unique fertilizer rate from the conventional modeling approach does not provide flexibility for a decision maker to use prior information about agro-climatic conditions when selecting fertilizer rates.

Recent ecological studies have raised concern over the adequacy of the estimated response functions using the conventional approach (Thomson et al., 1996; Cade, Terrell and Schroeder, 1999). Thomson et al. (1996) commented that ecological studies are in conflict with the correlation because ecological theory embodies limiting factor and correlation looks for controlling factors. Cade, Terrell and Schroeder (1999) suggested that changes near the maxima, rather than at the centre of the response distribution, are better estimates of effects expected when observed factors are the actively limiting constraints.

The objectives of this study were: 1) to estimate optimal fertilizer rates when the interaction with agro-climatic inputs that are not included in the model impose different levels of constraints, 2) to estimate fertilizer rates using the conventional ordinary least squares (OLS) approach and to compare the results with proposed approach, and 3) to assess the impact of technology on fertilizer use for canola production in the Prairie Provinces of Canada.

CONCEPTUAL MODEL

The issue of interest is knowing the optimal level of a single fertilizer input, X, for a crop with yield Y. Generally, the data on Y and X are from multi-year field experiments with several levels of input X. To illustrate, a scatter plot of Y on X is shown on Figure 1. This scatter plot shows the variability in Y conditional on X. Variability in Y is expected due to variation in agro-climatic condition over years, and variability in the response to the input.

Larger variability associated with higher levels of nitrogen (N) rates has been documented (Babcock 1992; Smith, McKenzie and Grant 2003). For any level of *X* the data points showing higher yields are the result of favourable agro-climatic conditions. As severity of the constraints imposed by agro-climatic condition increases, yield declines. Ecological literature suggests the true relationship between *Y* and *X* can only be measured where factor *X* is limiting. This occurs at the upper layer, or near the frontier, of data points in Figure 1.

The conventional modeling approach is to estimate the relationship between *Y* and *X* as:

$$Y = X'\beta + \varepsilon \tag{1}$$

where $Y = (y_1,...,y_n)'$ is the $(n \times 1)$ vector of yield responses, $X' = (x_1,...,x_n)'$ is the $(n \times p)$ regressor matrix, $\beta = (\beta_1,...,\beta_n)'$ is the $(p \times 1)$ vector of unknown parameters, and $\varepsilon = (\varepsilon_1,...,\varepsilon_n)'$ is the $(n \times 1)$ vector of unknown errors. The error is generally assumed to be normally distributed with mean 0 and variance $k\sigma^2$. The k is constant which could be higher or lower than 1. The β is efficient when $k = 1$. The conventional approach estimates the relationship of *Y* on *X* at the mean level. The conditional expectation function is specified as:

$$\overset{\min}{\beta \in \Re^p} \sum_{i=1}^{n} (y_i - \mu(x_i, \beta))^2 \tag{2}$$

where $\mu(x_i, \beta)$ is a parametric function. A single optimal fertilizer level, based on this average relationship, may be inadequate for decision making. Econometrically, estimated parameters are not robust to outliers and non-normality of Y reduces the efficiency of the parameters.

An alternative approach is to model the yield relationship using quantile regression. The conditional quantile function is estimated (Koenker and Bassett, 1978) using:

$$\min_{\beta \in \Re^p} \sum_{i=1}^{n} \rho_\tau (y_i - \xi(x_i, \beta)) \tag{3}$$

where ρ_τ is the loss function which assigns a weight τ to positive residuals and weight of *1*- τ to negative residuals, $\tau \in (0,1)$. When $\xi(x_i, \beta)$ is formulated as a linear function of parameters, the minimization problem can be solved very efficiently using linear programming. It should be noted that when the data are homoskedastic, the conditional quantile function at each point of the dependent variable's distribution will be identical with each other and with the slope parameter estimated from the conventional approach. When the data are heteroskedastic, which is usually the case for fertilizer response, estimating conditional quantile at various points of the yield distribution will allow tracing out the marginal response of the fertilizer. In addition, the conditional quantile regression estimator is relatively robust to outliers and more efficient under non-normality than a least squares estimator.

EMPIRICAL METHOD

Production economics theory does not specify any specific functional form for yield response, but diminishing marginal productivity is a necessary condition to determine an optimum. For this study, the impact of fertilizer on crop yield is estimated using a quadratic production function. The quadratic production function can exhibit diminishing marginal productivity, and the third stage of production. When experiments are conducted at several locations, productivity differences across locations are expected. Location dummies are a convenient way to take into account these productivity differences in modeling the relationship. In this multi-location study where multiple fertilizers are applied, the functional form is specified as:

$$Y_n = \alpha + \sum_{l=1}^{L-1} D_{\ln}\delta_l + \sum_i \beta_i X_{in} + \sum_i \sum_k \gamma_{ik} X_{in} X_{ik} + \varepsilon_n, \forall i \neq k \tag{4}$$

where Y_n denotes the average crop yields from experiment plots across replication for a treatment, $n = 1, 2, \ldots N$. The unknown parameters α, δ, β, and γ are to be estimated. There are L different locations with fertilizer inputs X_i. For j^{th} location, the dummy, D, receives:

$$D_{jn} = \begin{cases} 1 & \text{if } j=l \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

With this specification, the yield response to inputs is assumed to be the same across locations. The marginal productivity for i^{th} fertilizer input, X_i, is then estimated as:

$$MP_i = \beta_i + \sum_k 2\gamma_{ik} X_{ik} \tag{6}$$

The optimal fertilizer input for a profit maximizing farm is computed by equating the MP to the fertilizer:crop price ratio, and solving for X_i. The approach of estimating the effect of fertilizer at the conditional mean is a convenient choice. However, the relationship at the conditional mean might not be the one of greatest interest. The part of the distribution of greatest interest will depend on the nature of the crop yield distribution and the potential implication of different parameters at different points of the distribution. As suggested in ecological literature, finding the effects at the tails of the distribution is likely to be of more interest than the conditional mean of crop yield.

Quantile Regression

The regression quantile, τ, ($0 < \tau < 1$) is defined (Koenker and Bassett, 1978) as:

$$\hat{\beta}(\tau) = \hat{\beta} \overset{\min}{\in} \Re^{p} \left[\sum_{i \in \{n: y_n \geq x_n' \hat{\beta}_n\}} \tau \left(\left| y_n - \sum_{j=0}^{p} x_n' \hat{\beta}_\tau \right| \right) + \sum_{i \in \{n: y_n < x_n' \hat{\beta}_n\}} (1-\tau) \left(\left| y_n - \sum_{j=0}^{p} x_n' \hat{\beta}_\tau \right| \right) \right] \quad (7)$$

where X_n denotes a *p* x *1* vector of fertilizer inputs, $\hat{\beta}_\tau$ is the corresponding vector of parameters, the absolute value of the error term is in the rounded brackets.

The quantile regression for $\tau = 0$ weighs the positive and negative residuals equally when determining the minimum of the function in equation 7. The difference from OLS is the quantile regression does not square the error term. For the 0.75 quantile regression (Figure 1), the positive residuals are weighted by a factor of 0.75 and the negative residuals by 0.25. As a result, the estimated relationship will be above that of the OLS to reduce the size of the positive errors. This case has been proposed to be more representative of when the input of interest, fertilizer, is the limiting factor of production. The 0.25 quantile weights the positive residuals by 0.25 and the negative residuals by 0.75, hence the estimated response will be below the OLS estimate.

Several algorithms are available for solving this minimization problem. The commonly used algorithms are: simplex (Barrodale and Roberts, 1973), interior point

(Lusting, Marsden and Shanno, 1992), smoothing (Madsen and Nielsen, 1993) algorithms. Each algorithm has its own advantages and disadvantages. The simplex algorithm is chosen for this study because of its stability (Buchinsky, 1998). The simplex algorithms by Barrodale and Roberts (1973) are extended by Koenker and d'Orey (1993) for quantile regression of any given quantile. Although simplex is slower than the interior point and smoothing algorithms for a large data set, this is not an issue for a moderate data set as in this study.

Several alternative procedures exist to compute confidence intervals for the regression quantile parameters, including sparsity (Bassett and Koenker, 1982), inversion rank tests (Gutenbrunner and Jureckova, 1992), and resampling (He and Hu, 2002). The sparsity method is sensitive to the assumption the errors are iid. The resampling method is instable for small data sets, as in this study. The inversion rank tests method is used in this study because it does not require the assumption the errors are iid, and is suitable for the small sample case.

The entire quantile process for the interval ($0.25 \leq \tau \leq 0.75$) is estimated using the QUANTREG procedure of SAS (SAS, 1999). Although it is tempting to consider τ at its maximum $(\tau = 1)$ as the best possible estimate for the limiting relation, it is not used because the asymptotic variance of the rank score statistic is 0 (Cade et al., 1999). A 95% confidence interval is estimated for the quantile process. A lower density of observation towards the tail of the distribution of yields may result in more sampling variation for estimates at the extreme higher and lower quntiles. Therefore, optimal fertilizer and marginal product analysis were restricted to $0.25 \leq \tau \leq 0.75$ for this analysis.

APPLICATION AND DATA

Canola (*Brassica napus* L., *B. rapa* L.) is an important oil seed crop in Canada. Hybrid canola types now dominate those planted in western Canada. The distinction between hybrids and inbred open pollinated cultivars is important (Harker et al 2003; Karamanos, Goh and Poisson 2006). The yield response to management for these two types of canola are reported to vary (Harker et al 2003).

Data for this analysis comes from field experiments were conducted over five years (1999 to 2003) to primarily assess the effect of N on the productivity of canola cultivars over several locations in the Prairie Provinces. Other treatments included levels of phosphorous (P) and sulfur (S). However, S in this analysis was excluded because the yield response was not significant due to relatively S rich soils. Total nutrients were considered in this study to take into account the variation in soil nutrients across time and space. Several commercial cultivars are used during experiments, but are either “hybrid” (HY) or “open-pollinated” (OP) as shown in Table 1. Seed rate varies with the cultivar, location, and year but remains constant for a treatment in any specific experiment. Experiment details of location, years, prior crop, treatments, seeding and harvesting dates are in Table 2.

A set of experiments (N, N*P and N*S) conducted during1999 to 2001 used twelve rates of fertilizer N (0 to 220 kg ha^{-1} in equal increments) as treatments. In addition, the N*P experiments also received 0, 20, or 40 P (kg ha^{-1}) and a blanket application of 50 K (kg ha^{-1}) and 17 S (kg ha^{-1}). Similarly, the NS1 experiment (Table 2) also received 0, 20, or 40 S (kg ha^{-1}) treatments along with a blanket application of 30 P (kg ha^{-1}) and 50 K (kg ha^{-1}). Experiments N:P1 to N:P4 received a combination of five N treatments (0, 50, 100, 150, or 200 kg ha^{-1}) and five P treatments (0, 20, 40, 60, or 80 kg ha^{-1}). The S and K nutrients were applied at 17 and 51 kg ha^{-1}, respectively, on all plots. The N:S experiments (N:S1 to N:S6) used six N treatments (0, 40, 80, 120, 160, or 200 kg ha^{-1}) and different levels of S to maintain the N:S ratio at 1.5:1, 6:1 or 12:1. The P and K fertilizers were blanket applied at the rate of 25 and 30 kg ha^{-1}, respectively. Details on these experiments are described by Karamanos, Goh and Poisson (2004; 2005).

RESULTS AND DISCUSSION

Yield Response to Fertilizers and Confidence Interval

Yield response to fertilizers, estimated using quantile regression for three selected quantiles and using OLS, are presented separately for HY and OP canola in Table 3-6. All N and P relationships exhibited yield increasing at a decreasing rate, with a potential maximum yield.

Table 3 reports the estimated parameters and their confidence intervals for OP and HY canola at the 0.25 yield quantile. The negative parameter estimates for the location dummy variables indicated factors other than N and P fertilizers were responsible for lower yields at those locations. For HY at Red Deer, the yield intercept was not significantly different from Ellerslie with 95% confidence. All linear and quadratic parameters were significant for both canola types.

Tables 4 and 5 report the estimated parameters and their confidence interval for OP and HY canola at the 0.50 and 0.75 quantile, respectively. Like the results for the 0.25 quantile, the expected yield is higher at Ellerslie compared to all other locations except at

Red Deer for HY. All parameter estimates for N and P were statistically significant at 95% confidence interval.

Table 6 reports the OLS estimated parameters and their level of significance (P values) for OP and HY canola cultivars. Consistent with quantile regression results, the expected yield is higher at Ellerslie except at Red Deer for HY. All parameter estimates for N and P were statistically significant at the 95% level. Of special interest is the comparison of results between the quantile regression at 0.50 and OLS. The results are expected to be similar if the level of N fertilizer input does not affect the yield variability and there are few outliers in the data.

Marginal Productivity

Figures 2 and 3 show the marginal yield response for OP and HY canola to fertilizer N rates at three yield quantiles (0.25, 0.5, 0.75) and at the mean yield (OLS). As expected, the marginal products declined with the increased level of N. Yield response to N for the higher quantiles was higher and this could be due to the more suitable agro-climatic conditions, including soil moisture, growing degree days, and soil organic matter content. The marginal product was higher at the higher quantiles, especially when measured near the current input to output price ratio (5.0). The MP and optimal N for the quantile regression at 0.5 and the OLS regression were similar near the current price ratio for both OP and HY canola types.

Canola types differed in their marginal product response to fertilizer N across different agro-climatic stress condition (Figure 2 and 3). For the OP canola, the difference in marginal products were greater at lower levels of N. The benefits of additional N fertilizer for the 0.75 quantile was greater than for the other quantiles (Figure 2). Beyond 170 kg N ha^{-1} the marginal products were similar. In contrast, the HY response across quantiles was similar at lower N rates (Figure 3). These MPs were similar in value to the OP at 0.75 the quantile. The HY canola demanded higher rates of N fertilizer than the OP, even under less than ideal agro-climatic conditions.

Optimal Nitrogen

Figures 4 and 5 illustrate the optimal N rate along the yield quantiles ($0.25 < \tau < 0.75$). A total of 340 and 527 parameters estimates were generated, respectively, for OP and HY canola for yield quantiles between 0.25 and 0.75. The different number of estimates by canola type was due to the differences in the sample size. Parameters for yield quantiles below 0.25 and above 0.75 were not used for optimal N computation because fewer observations at the tails of yield distribution increases the confidence interval and thus lowers the confidence in the results. The optimal N was based on \$ 1.38 kg^{-1} N and \$ 276 t^{-1} canola, a ratio of 5.0. The optimal N trended positive for both canola cultivars from 0.25 to 0.75 quantile. This shows that optimal level N was higher for a more favourable agro-climatic conditions.

The optimal level of N was lower for the OP canola, but it had a higher positive trend and more variability around the trend than the HY canola (Figures 4 and 5). For the OP canola, the optimal N was 91 kg ha^{-1} for the 0.25 quantile and 134 kg ha^{-1} for the 0.75 quantile. For HY canola, the optimal N was 138 kg ha^{-1} at the 0.25 quantile and 157 kg ha^{-1} at the 0.75 quantile. The economically optimal level of fertilizer N was higher and more stable across agro-climatic conditions for HY than for OP. The optimal N varied across different agro-climatic stress conditions for OP canola.

Yield at Optimal Nitrogen

Canola yield was location dependent and predicted yield was higher at higher quantiles (Figure 6 and 7). As expected, predicted yields using optimal N trended positive from lower to higher yield quantiles. The OP canola yields were lower than HY, except for Ellerslie, which reflected the actual yield data obtained from the field experiments. The yield increase over quantiles also tended to trend up more at the higher quantiles (0.50 to 0.75), especially noticeable for the OP canola. The increasing quantile pushed the estimated yield response towards the yield frontier where agro-environmental factors were less limiting, and the response to N was greater.

CONCLUSION

The quantile regressions illustrate the impact that unfavourable and favourable agro-climatic conditions will have on the optimal rate of nitrogen fertilizer to apply to canola. For open pollinated canola types, optimal N was about 20% lower for unfavourable conditions and 20% higher for favourable conditions, compared with the average. For hybrid canola, the differences were not as large (8 and 12%), but optimal N rates were higher for hybrid canola. As expected, the OLS and 0.50 quantile regressions estimated similar responses to N fertilizer. Outliers with larger error terms will affect the OLS more than the quantile because OLS minimizes the square of the errors, while the quantile minimizes weighted absolute error to estimate the parameters. The hybrid cultivar was higher yielding than open-pollinated cultivar. The improved technology with higher yield potential also demanded higher fertilizer N. At the current N fertilizer:canola price ration, the conventional OLS approach estimates the optimal rate of soil plus fertilizer N at 113 kg ha^{-1} for open-pollinated and 144 kg ha^{-1} for hybrid canola. The quantile process determined that for OP canola, the optimal N rates were 91, 115, and 134 kg ha^{-1} for severe, moderate and low levels of agro-climatic constraints, and that for HY canola, the optimal N rates were 137, 142, and 158 kg ha^{-1} for severe, moderate and low levels of agro-climatic constraints. Unlike the conventional approach that generates one rate, there is a potential for producers to benefit from the proposed approach by fertilizing based on expected agro-climatic conditions.

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Table 1. Canola varieties by locations

Location	Varieties	
	Hybrid Cultivar	Open-pollinated Cultivar
Ellerslie, AB	SW Rider	Q2
Fortsaskatchewan, AB	45H21, InVi 2573	46A65
Irricana, AB	InVi2273, InVi2153, SW Rider, Hyola 401	Innovator, Nex 500
Miami, MB	45H21, InVi 2573	46A65
RedDeer, AB	SW Rider	Q2
Rose bank, MB	45H21, InVi 2573	46A65, Conquest
Sylvania, SK	AP Admire, InVi 2573, 45H21	SP Armada, Conquest

Table 2. Experiments and their characteristics

				No. of Fertilizers Treatments[z]					
Expt	Location	Year	Prior Crop	N	P	S	Reps	Seeding	Harvest
N	Irricana, AB	1999	Wheat	12	0	0	1	10-Jun	24-Sep
NP1	Irricana, AB	2000	Barley	12	3	0	1	30-Apr	13-Sep
NP2	Red Deer, AB	2001	Wheat	12	3	0	1	4-May	17-Sep
NP3	Ellerslie, AB	2001	Barley	12	3	0	1	16-May	25-Sep
NS1	Red Deer, AB	2001	Wheat	12	0	3	1	4-May	17-Sep
N:P1	Ft. Sask, AB	2002	Wheat	5	5	0	4	27-May	28-Aug
N:P2	Sylvania, SK	2002	Barley	5	5	0	4	19-May	3-Oct
N:P3	Miami, MB	2002	Wheat	5	5	0	4	27-May	28-Aug
N:P4	Rosebank, MB	2002	Wheat	5	5	0	4	2-Jun	20-Sep
N:S1	Ft. Sask, AB	2002	Wheat	6	0	15	4	28-May	7-Oct
N:S2	Sylvania, SK	2002	Barley	6	0	15	4	28-May	7-Oct
N:S3	Miami, MB	2002	Wheat	6	0	15	4	25-May	28-Aug
N:S4	Rosebank, MB	2002	Wheat	6	0	15	4	1-Jun	20-Sep
N:S5	Sylvania, SK	2003,4	Wheat	6	0	15	4	11-May	4-Sep
N:S6	Rosebank, MB	2003	Wheat	6	0	15	4	13-May	20-Aug

[z] Treatments for each fertilizer were based on total (soil available plus applied fertilizer) nitrogen (N), phosphorus (P) and sulfur (S) nutrients.

Table 3. Estimated parameters and their confidence interval for two canola types at quantile = 0.25

	OP			HY		
	Estimate	95% CI		Estimate	95% CI	
Intercept	2246	1724	2493	1366	848	2697
d1[Z]	-1361	-1539	-1092	176	-162	722
d3	-2205	-2345	-1940	-944	-1282	-505
d4	-1206	-1419	-1029	-703	-972	-188
d5	-2423	-2552	-1801	-1699	-2010	-1285
d6	-3193	-3330	-2997	-1709	-1995	-1320
d7	-2323	-2470	-2088	-1993	-2335	-1514
N	9.04	6.61	12.23	12.95	10.02	15.96
P	8.90	4.83	13.39	10.35	6.03	16.05
N^2	-0.0221	-0.0310	-0.0158	-0.0290	-0.0407	-0.0218
P^2	-0.0346	-0.0567	-0.0120	-0.0384	-0.0713	-0.0152

[Z] d1 = Red Deer, d2 = Irricana, d3= Fort Saskatchewan, d4 = Sylvania, d5 = Miami and d6 = Rosebank. The intercept denotes Ellerslie.

Table 4. Estimated parameters and their confidence interval for two canola types at quantile = 0.5

	OP			HY		
	Estimate	95% CI		Estimate	95% CI	
Intercept	2127	1854	2545	1863	1508	2189
d1[Z]	-839	-1395	-473	133	-109	490
d3	-2112	-2525	-1872	-1112	-1298	-786
d4	-1317	-1718	-1080	-755	-980	-373
d5	-2406	-2779	-2183	-1897	-2076	-1606
d6	-3268	-3951	-3062	-1993	-2175	-1650
d7	-2442	-2831	-2114	-2326	-2486	-2015
N	10.89	7.94	13.19	12.81	10.16	15.41
P	13.50	8.32	17.29	10.23	4.23	14.47
N^2	-0.0257	-0.0314	-0.0168	-0.0275	-0.0348	-0.0209
P^2	-0.0611	-0.0829	-0.0410	-0.0483	-0.0752	-0.0186

[Z] d1 = Red Deer, d2 = Irricana, d3 = Fort Saskatchewan, d4 = Sylvania, d5 = Miami and d6 = Rosebank. The intercept denotes Ellerslie.

Table 5. Estimated parameters and their confidence interval for two canola types at quantile = 0.75

	OP			HY		
	Estimate	95% CI		Estimate	95% CI	
Intercept	2672	1867	3501	1890	1608	2555
d1[Z]	-945	-1222	-450	229	-172	377
d3	-2295	-2654	-1877	-922	-1237	-736
d4	-1491	-1883	-1144	-599	-1008	-43
d5	-2573	-2946	-1523	-1749	-2120	-1594
d6	-3633	-3836	-3140	-2036	-2233	-1743
d7	-2390	-2631	-1990	-2302	-2589	-1849
N	16.04	5.10	20.68	12.69	8.61	15.73
P	8.65	-7.39	23.03	13.64	1.24	21.60
N^2	-0.0412	-0.0527	-0.0124	-0.0244	-0.0369	-0.0105
P^2	-0.0569	-0.0876	0.0148	-0.0668	-0.1031	-0.0294

[Z] d1 = Red Deer, d2 = Irricana, d3 = Fort Saskatchewan, d4 = Sylvania, d5 = Miami and d6 = Rosebank. The intercept denotes Ellerslie.

Table 6. OLS estimated parameters and their P-values for two canola types

	Open-pollinated		Hybrid	
	Estimate	P-value	Estimate	P-value
Intercept	2259	<.0001	1722	<.0001
d1[Z]	-960	<.0001	198	0.1161
d3	-2160	<.0001	-999	<.0001
d4	-1267	<.0001	-540	<.0001
d5	-2253	<.0001	-1706	<.0001
d6	-3257	<.0001	-1860	<.0001
d7	-2283	<.0001	-1915	<.0001
N	11.58	<.0001	13.70	<.0001
P	11.97	0.0053	10.41	0.0012
N^2	-0.0290	<.0001	-0.0302	<.0001
P^2	-0.0587	0.017	-0.0524	0.0032

[Z] d1 = Red Deer, d2 = Irricana, d3 = Fort Saskatchewan, d4 = Sylvania, d5 = Miami and d6 = Rosebank. The intercept denotes Ellerslie.

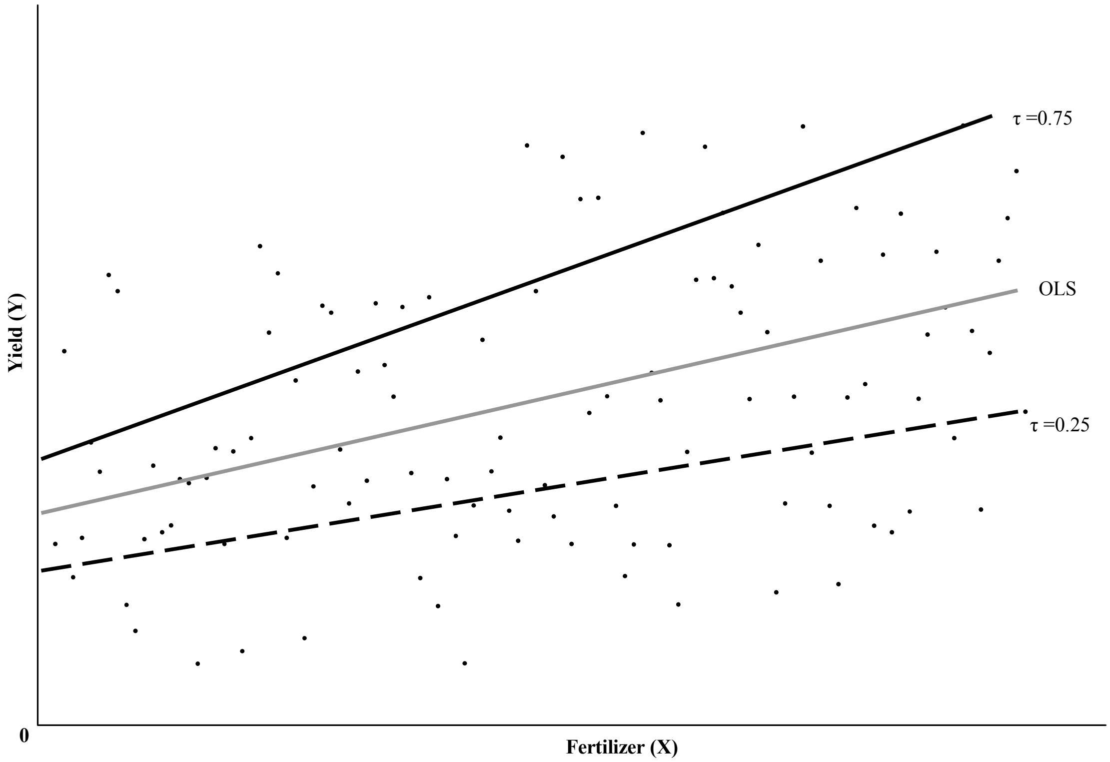

Figure 1. An illustration of estimated responses for OLS and quantile regressions of 0.25 and 0.75

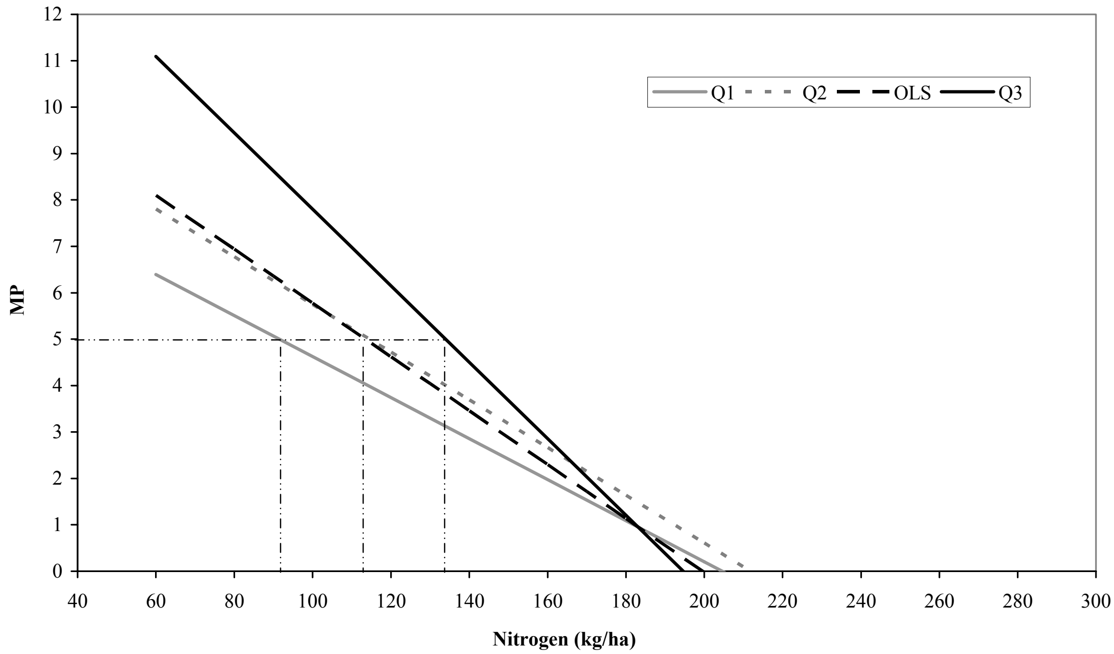

Figure 2. Marginal product (MP) of canola yield for open pollinated canola.

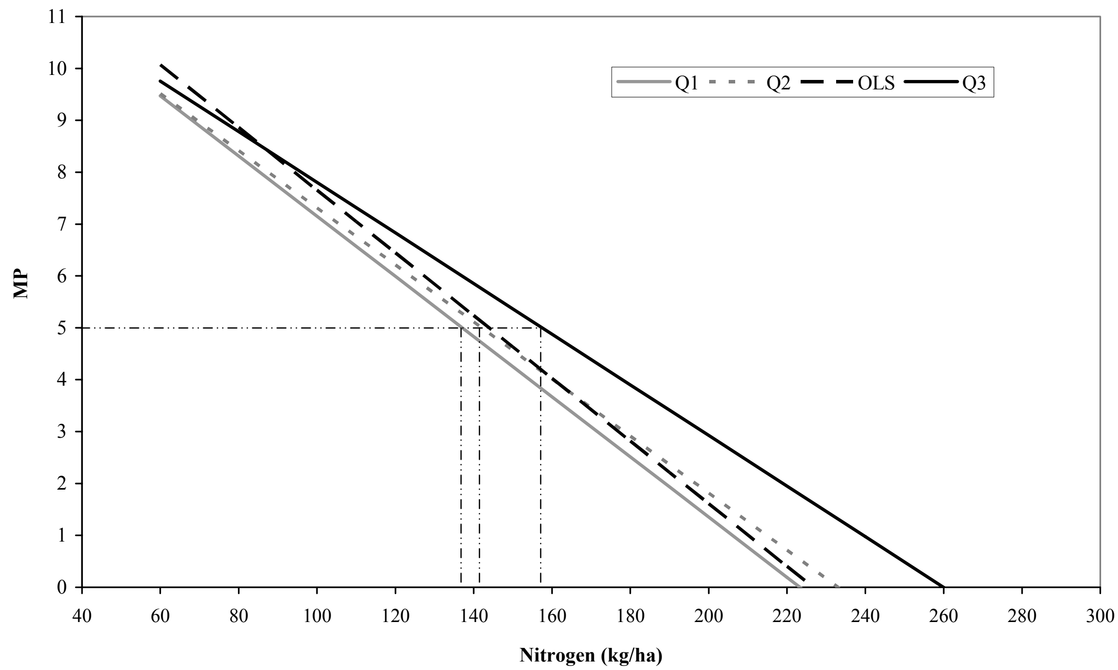

Figure 3. Marginal product (MP) for canola yield for hybrid canola.

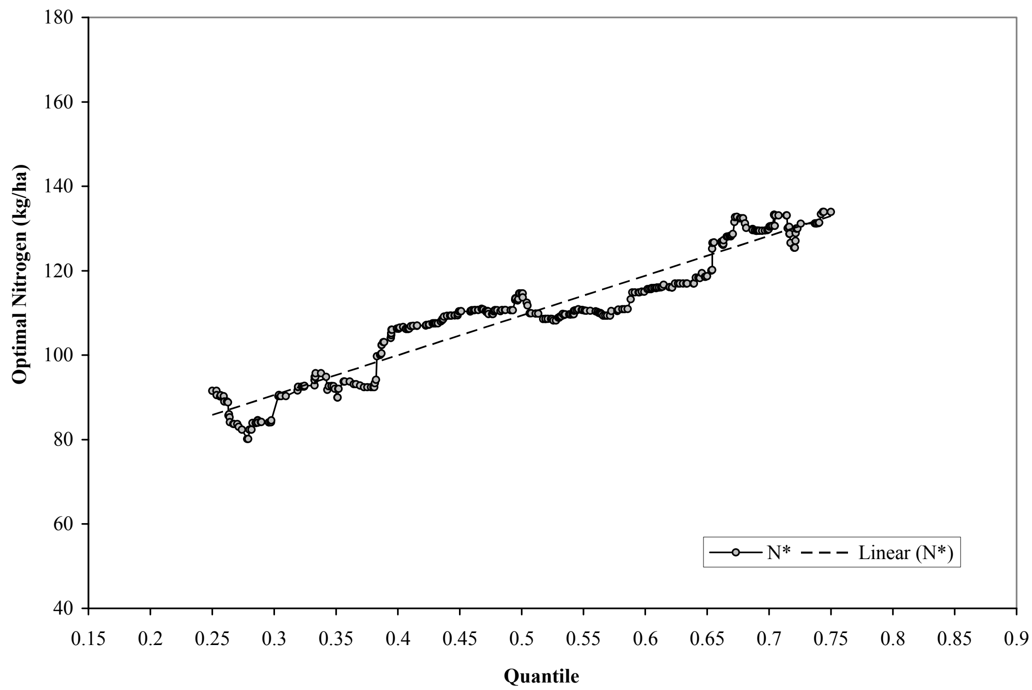

Figure 4. Quantile process for optimal N for open-pollinate canola

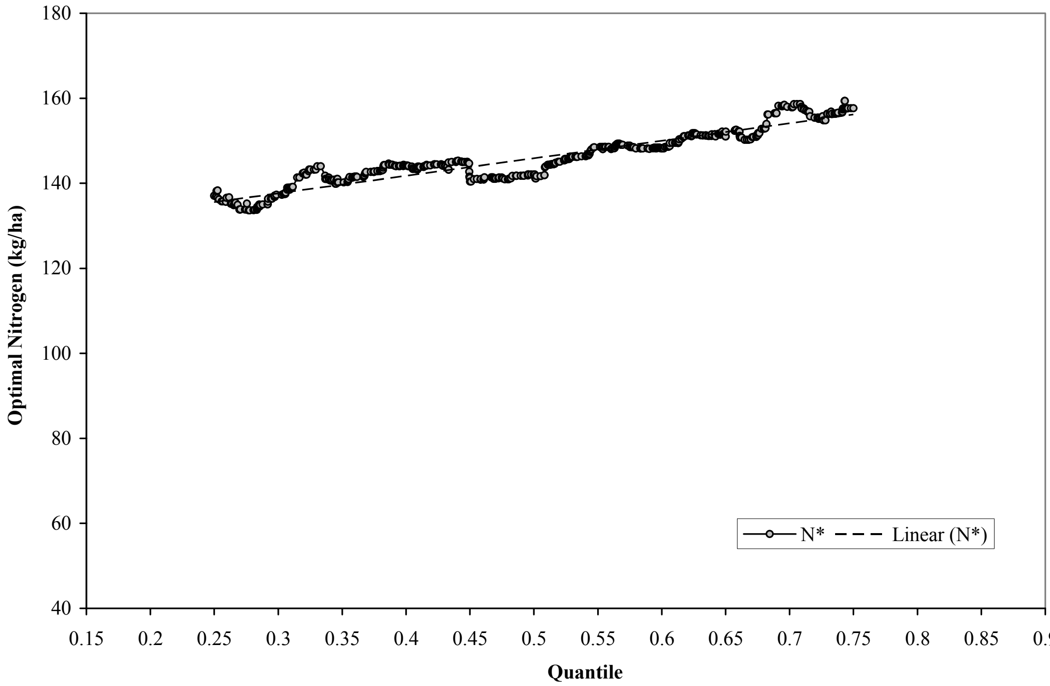

Figure 5. Quantile process for optimal N for hybrid canola

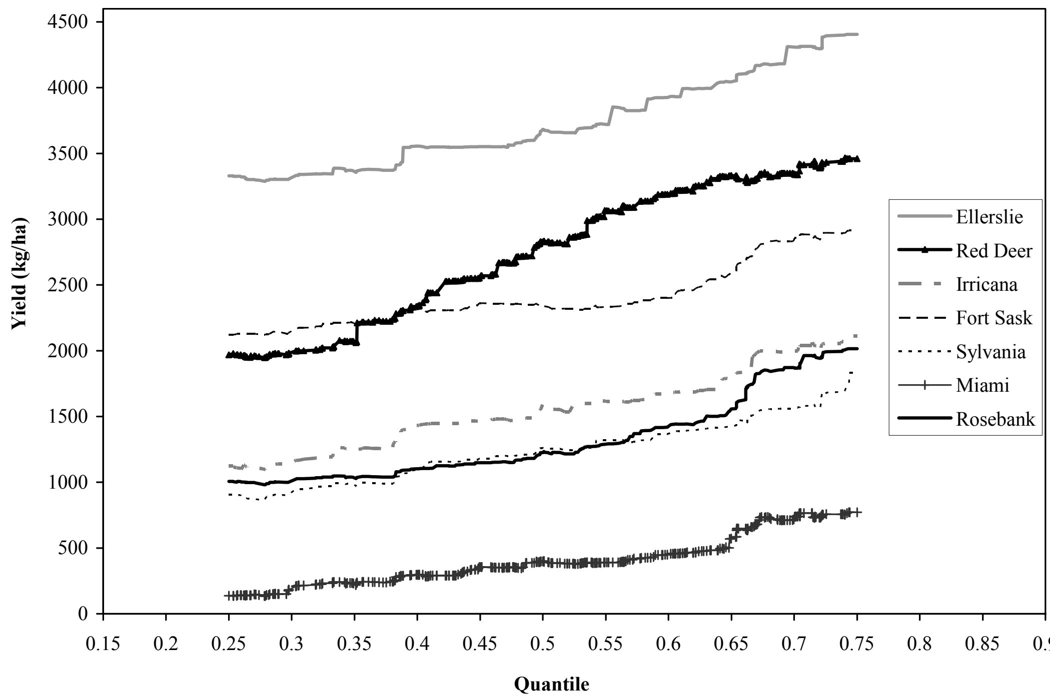

Figure 6. Predicted yields for open-pollinated canola using optimal N at different locations in the Prairie Provinces.

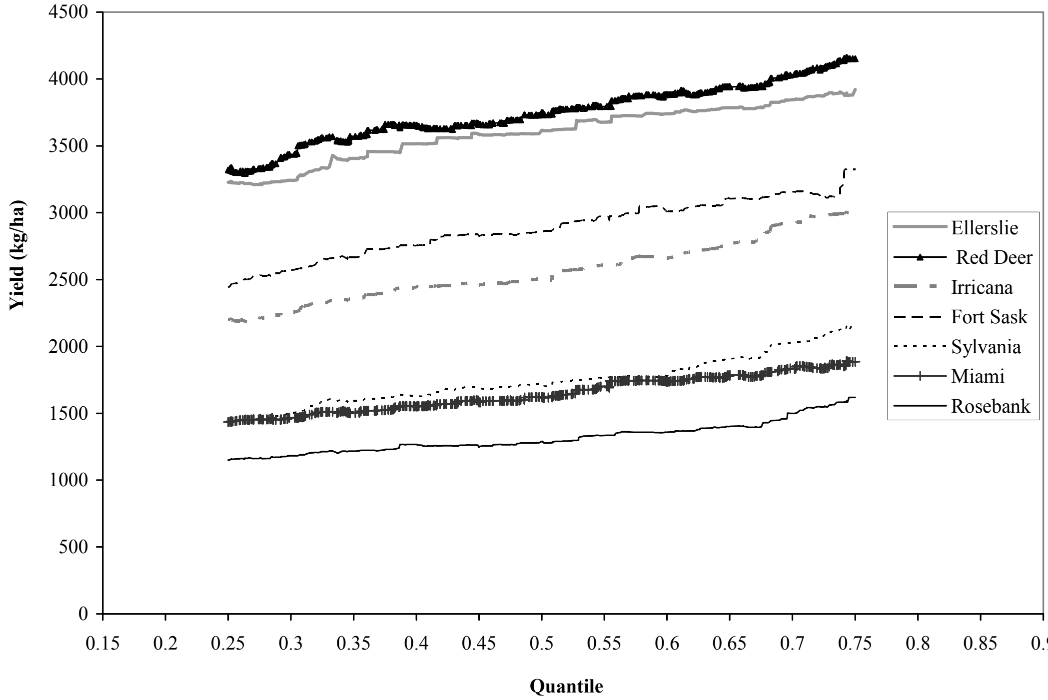

Figure 7. Predicted yields for hybrid canola using optimal N at different locations in the Prairie Provinces.