On price level stability, real interest rates and core inflation[1]

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Abstract

The paper addresses several issues pertaining to the problem of monetary policy, inflation measurement and relative prices. After some preliminary empirical analysis showing that the problem must be relevant, we set out to conduct a mainly theoretical investigation. If the per period utility function is not homothetic in the commodity vector, empirical price aggregates do not have exact theoretical counterparts, and their properties, as well as those that are calculated from them (such as real interest rates), must be interpreted with care. We examine the consequences of goods heterogeneity in the framework of a stochastic dynamic equilibrium model without a steady state. To solve the model we posit specific, though we think plausible, assumptions concerning fiscal policy in a small open economy. Conclusions are obtained with important policy implications to the effect that inflation variability may be tolerated, and that the correct meaning and interpretation of real interest rates may run counter to accepted ideas. Our general conclusion is that neglecting goods heterogeneity may, in certain circumstances, grossly mislead policymakers and analysts.

1. Introduction

We start from a few propositions related to monetary (exchange rate) policies and inflation, which are applied, though not restricted, to transition countries in central and eastern Europe.

1. CPI inflation must be stable and low.
2. While the real exchange rate must be relatively stable, strict pegs should be avoided because they do not give monetary policy the opportunity to control inflation via control of the real interest rate.
3. The conduct and/or evaluation of monetary policy must rely on a core, or underlying, inflation concept.

These propositions are not unanimously upheld, but each of them musters a significant degree of endorsement. The first can be derived from the Maastricht criteria that are to be satisfied by countries wishing to join the European Union in the foreseeable future. The second has acquired wide currency in general, and the third has sneaked into central bank practice and has held and gained ground among financial analysts and in academic circles.

In this paper we develop arguments to the effect that none of the above claims is unquestionable. We focus on transition countries, but many of our arguments have broader validity. The main novelty of our approach lies in discarding the assumption of homothetic consumer preferences. We model an economy where consumers buy several goods (one of them durable) and services, and where their preferences are represented by the addilog utility function. This functional form has been used, for instance, by Clarida (1996) to estimate demand for imported durables, and by Bils and Klenow (1998) to test competing business cycle theories. One important empirical conclusion of these papers is that the non-homothetic preference specification seems to be confirmed. If we take this for granted then

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the observation that large and non-transitory relative price changes occur in reality raises the possibility that this can be important when we interpret CPI inflation as defined by the usual Laspeyres indices. The present study offers an initial contribution in this direction by building a dynamic general equilibrium model for a small open economy.

We are faced with methodological difficulties in carrying out the proposed approach. As the model contains several types of goods and we assume both non-homothetic preferences and permanent productivity improvement, there is neither a non-stochastic steady state nor a balanced growth path in this model. Linear approximation methods, or non-linear techniques that assume stationarity, are therefore not available options. Also, we want to model a wide variety of shocks which make the state space too large for the easy application of discrete non-linear approximation methods. We generate (almost) exact formulas for price indices, which are obtained by making specific behavioural assumptions concerning government (fiscal) policy. Essentially, it is assumed that fiscal policy is set with a view to controlling aggregate demand in a way that ensures the economy's solvency and observes the rationality of individual decisions.

Section 2 reviews the story of relative and absolute price changes in 1990s Hungary. This is then supplemented with evidence from transition and developed countries, showing that the Hungarian experience can be generalised. In Section 3, a stochastic general equilibrium model serving as a framework for the discussion in subsequent sections is set up and solved in two versions. In the light of these solutions, Section 4 addresses the propositions described in the introduction. First, we explore the consequences of our utility specification for the long-term variability of prices. Second, we ask about the consequences of stabilising prices when monetary policymakers are facing different kinds of shocks. Third, the meaning of real interest rates is analysed. Finally, the various methodologies used to calculate core inflation indices are examined. Section 5 summarises and draws conclusions for further research.

2. The experience with inflation and relative prices in Hungary

Inflation in transition has generated a substantial literature.[2] Here we describe some "internal" features of the inflationary process, having to do with relative price developments. We do not concern ourselves with the reasons for the ups and downs or with the speed of disinflation, etc. The focus will be on Hungary and salient features of price formation will be pointed out. Then the more general transition literature will be invoked to show that these salient features are probably not peculiar to Hungary.

2.1 Relative consumer prices in Hungary

This subsection is based on Valkovszky and Vincze (2001). The following is a short summary of relevant findings.[3]

The investigation was based on a database including the price indices of the 160 items making up the Hungarian consumer price index. First, we tried to identify "tradable" prices. We followed a statistical approach and came to the conclusion that all the goods classified as durable could pass as tradables when this concept is defined as being well explained by nominal exchange rate changes in the long run. In other words, the foreign exchange price of these goods seemed to be stable in the longer term. However, significant short-run deviations could be observed, especially when unexpected changes occurred in the path of the nominal exchange rate. Then we proceeded to investigate relative price changes. We could identify six subaggregates. First, there are non-energy administered (regulated) prices, characterised by infrequent changes. Relative administered prices have exhibited a positive trend and have grown substantially. They show a zigzag picture, undergoing short periods of large hikes, and more protracted periods of almost no change. Clearly, the upward trend was due to the initial "underpricing" of public services, but the exact timing and size of the catching-up process was

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[2] For summaries see Begg (1998), Cottarelli and Boyle (1999) and Wyplosz (1999).

[3] On this topic and in general on the Hungarian CPI, see also Ferenczi et al (2000). On the behaviour of relative prices in transition, see Coorey et al (1998) and, in the case of Slovenia, Cufer et al (1998).

politically motivated. Second came energy prices, which have partly been regulated, but also exposed to world price developments, ie supply shocks from our point of view. The relative energy price was declining at a mild pace until 1995, when it rose steeply, followed by a continued upward trend. This shape was due to deliberate policies, which had checked energy price increases before 1995 for social reasons, while the correction of the relative energy price was part of the fiscal adjustment package of that year. The third category consisted of excisable goods. Their relative prices do not show a zigzag pattern, but certain substantial changes have occurred. On the whole they exhibited a slightly negative relative price trend, which was notably reversed for some time in 1995. Again, political motives were apparently behind the timing of changes. The fourth group included non-processed food prices. These seem to be relatively stable in the long term, but have had cycles, exhibiting not just very large short-term fluctuations, but fluctuations that have apparently been persistent. Fifth, we distinguished processed food prices that have been fairly stable in relative terms. Finally, we could see, as a different subcategory, the remaining service prices. Their development almost mirrored that of durables, exhibiting a slight, but consistently positive, upward trend. This trend, and temporary variations, appeared to be related to wage changes. Thus when the foreign exchange value of wages increased, the relative price of services became temporarily higher, suggesting that a Balassa-Samuelson type hypothesis can be invoked to explain the upward trend as well.

Similar features of price formation are very likely to characterise the inflationary processes of other transition economies. In fact, relative administered public utility and energy prices exhibited the very same pattern in the Czech and Slovak Republics, with the difference that an important upward adjustment of almost 50% took place in 1998-99 and 1999-2000 respectively, while in Hungary and Poland relative prices in this group have followed a rather stable 2-3% per annum upward trend throughout the last five years. The excisable goods group revealed large relative price swings all over the central and eastern European countries (CEECs), admittedly in accordance with the hectic fiscal measures taken. In the second half of the 1990s one can observe, however, a slight upward trend in the relative price of alcoholic beverages and tobacco in many of the CEECs. Unfortunately, due to the lack of comparative databases, we were not able to repeat our study exactly with respect to other transition countries. Even if we cannot reproduce the distinction between processed and non-processed foodstuffs, on the basis of the Eurostat database the whole group of foodstuffs featured as a very volatile part of the CPI basket, not only in the short, but also in the long run. In comparison, the relative price of foodstuffs in the United Kingdom has had a variance roughly one third of that in Hungary in the same period. It is important to mention that the relative food price movements of the countries concerned showed remarkable correlations. This suggests that not only (international) energy prices, but also food prices delivered symmetric external shocks to the region.

Though the following cannot do justice to the whole set of observations, the above description can be summarised in five important stylised facts.

1. A substantial part of the CPI in transition economies, and especially those items that are relatively "durable", behaves like "textbook" tradables. There is evidence that short-run unpredictable exchange rate changes are not passed on to domestic prices immediately, suggesting that at least a certain degree of nominal rigidity exists.

2. Prices that contain a substantial non-tradable component and are non-regulated seem to conform with the Balassa-Samuelson hypothesis.

3. Agricultural prices impart substantial volatility to the CPI and are heavily influenced by supply shocks. Supply shocks must be broadly understood, including not just changes in weather conditions, but the actions of politicians as well.

4. Public service and household energy prices have generally showed upward trends, with infrequent large changes. This suggests that both political considerations and the costs of changing prices must have played a substantial role in the catching-up process of these prices.

5. There seems to be evidence that the income elasticities and intertemporal substitutabilities of the different subgroups are different. The expenditure share of durables has shown very large variations.

3. The model

Our goal is to build a model for a small open economy where consumers purchase four types of goods:

1. A tradable and durable good;
2. A non-tradable and non-durable good;
3. A non-durable good with rigid but uncertain supply (called "food" hereafter); and
4. A public service.

These goods have different income elasticities of demand, which, in a growing economy, implies that no convergence to stable ratios, ie stationary solution in growth rates, is available. To study the behaviour of aggregate price indices, real interest rates and possible "core inflation" measures, we must construct a dynamic general equilibrium model. In order to focus on the above-mentioned problems, this model economy contains many simplifications. However, they are probably not crucial to the problems at hand and mainly serve to make the exercise manageable. The simplifications include the following: there is no money or physical capital in the model, foreign demand plays only an indirect role via the terms-of-trade effect, labour is supplied in a fixed quantity, international capital markets are passive, and the supply of "food" is exogenous. We also make a shortcut to fiscal policy determination.

3.1 Household behaviour

Households decide at the beginning of each period on how to divide their wealth among two financial assets (domestic government bonds (BH_t) and foreign bonds (BF_t)) and four consumption goods (non-tradable consumption (CN_t), food (CA_t), a publicly provided service (CPI_t) and a durable good (D_t)). Labour supply is rigid and normalised to 1. Disposable wealth includes assets carried over from the previous period, interest earned on them, profits distributed by firms owned by households, and wages paid out by the non-tradable and tradable sectors.

The representative household has a Houthakker (addilog) type per period utility function

$$U\left(CN_t, CA_t, CP_t, D_t\right) = \frac{CN_t^{1-\vartheta^N}}{1-\vartheta^N} + \frac{D_t^{1-\vartheta^D}}{1-\vartheta^D} + \frac{CA_t^{1-\vartheta^A}}{1-\vartheta^A} + \frac{CP_t^{1-\vartheta^P}}{1-\vartheta^P} \tag{1}$$

Thus we assume that services derived from possessing the durable good are proportional to its stock. We assume that $\vartheta^N, \vartheta^D, \vartheta^A$ and ϑ^P are definitely higher than 1, but that $\vartheta^N, \vartheta^D < \vartheta^A, \vartheta^P$, implying that the public service and foodstuffs have lower income elasticities than non-tradables and durables. This statement is a consequence of a fundamental fact about addilog preferences.

Proposition 1: *If preferences belong to the addilog family, i and j are two goods, φ_i and φ_j are income elasticities of demand, and ϑ_i and ϑ_j are utility function parameters, then $\frac{\varphi_i}{\varphi_j} = \frac{\vartheta_j}{\vartheta_i}$ (see Chari and Kehoe (1999)).*

This is the reason why Bils and Klenow (1998) call $\frac{1}{\varphi_i}$ an index of luxuriousness. It must be noted that their empirical results imply that luxuriousness and durability are positively correlated.

Then the household's programme can be written as

$$\max E_t \left[\sum_{r=t}^{\infty} \beta^{T-t} U\left(CN_t, CA_t, CP_t, D_t\right) \right] \tag{2}$$

subject to

$$BH_t + S_t BF_t + PN_t CN_t + PA_t CA_t + PP_t CP_t + T_t +$$
$$PI_t \left(D_t - \left(1-\delta^D\right) D_{t-1}\right) = I_{t-1} BH_{t-1} + S_t I^*{}_{t-1} BF_{t-1} + W_t + \Pi T_t + \Pi A_t \tag{3}$$

By definition

$$D_t = ID_t + \left(1-\delta^D\right) D_{t-1} \tag{4}$$

where ID_t is the purchase of the durable good.

Let NW_t denote nominal wealth denominated in the home currency in period t. Then one can define the optimal value function in the usual way

$$V(NW_t) = \max_{CN_s, CA_s, CP_s, D_s, BF_s} E_t \left(\sum_{s=t}^{\infty} \beta^s U\left(CN_s, CA_s, CP_s, D_s\right) \right) \tag{5}$$

where

$$NW_t = I_{t-1} NW_{t-1} + \left(S_t I^*{}_{t-1} - I_{t-1}\right) S_{t-1} BF_{t-1} + W_t + \Pi T_t + \Pi A_t +$$
$$\left(1-\delta^D\right) PI_t D_{t-1} - I_{t-1}\left(PN_{t-1} CN_{t-1} + PA_{t-1} CA_{t-1} + PP_{t-1} CP_{t-1} + PI_{t-1}\left(D_{t-1} - \left(1-\delta^D\right) D_{t-2}\right)\right) - T_t \ . \tag{6}$$

Under certain conditions this exists, and the maximum principle can be written as

$$V\left(NW_t, \ldots\right) = \max_{CN_s, CA_s, CP_s, D_s, BF_s} \left(U\left(CN_t, CA_t, CP_t, D_t\right) + \beta E_t\left(V\left(NW_{t+1}, \ldots\right)\right)\right). \tag{7}$$

The set of first-order conditions for this problem can be derived by differentiating (5) with respect to the decision variables, making the first derivatives equal to zero, and using the envelope condition applied to (7).

Let us define the auxiliary variable Λ_t (the marginal indirect utility of nominal wealth in t).[4]

The following is the standard dynamic first-order condition:

$$\Lambda_t = I_t \beta E_t\left(\Lambda_{t+1}\right) \tag{8}$$

It can be proved that

$$\Lambda_t = \frac{CN_t^{-\vartheta^N}}{PN_t} \tag{9}$$

Then we can obtain the following intratemporal first-order conditions:

$$\frac{PN_t}{PA_t} = \frac{CN_t^{-\vartheta^N}}{CA_t^{-\vartheta^A}} \tag{10}$$

$$\frac{PN_t}{PP_t} = \frac{CN_t^{-\vartheta^N}}{CP_t^{-\vartheta^N}} \tag{11}$$

$$\frac{PI_t}{PN_t} = \frac{D_t^{-\vartheta^D}}{CN_t^{-\vartheta^N}} + \beta\left(1-\delta^D\right) E_t\left(\frac{PI_{t+1}}{PN_{t+1}} \frac{CN_{t+1}^{-\vartheta^N}}{CN_t^{-\vartheta^N}}\right). \tag{12}$$

One can also derive, from the optimal choice between domestic and foreign bonds, the portfolio choice equation

[4] This is the partial derivative of the value function associated with the consumer's maximisation problem with respect to nominal wealth.

$$E_t\left(\Lambda_{t+1}\left(I_t - I_t^* \frac{S_{t+1}}{S_t}\right) = 0\right) \tag{13}$$

The transversality condition may be written as

$$\lim_{T\to\infty} E_t\left[\frac{\beta^{T-t}\Lambda_{t+T}}{\Lambda_t} NW_{t+T}\right] = 0$$

for all *T.*

3.2 Production

Production has a hierarchical structure. Differentiated producers produce a continuum of tradable goods using homogenous labour with diminishing returns to scale technology. We assume that tradable producers are distributed on $(0, 1)$ uniformly. This output can be sold domestically or exported. If sold domestically, a competitive sector aggregates these differentiated products into a domestically produced tradable aggregate. Then another competitive sector produces, via a CES technology, an intermediate input-capital good aggregate of this domestic tradable aggregate and of an imported good. Non-tradables are made from this latter good and by homogenous labour via a Cobb-Douglas technology. In addition, the food product is produced randomly and a public good service is provided at zero cost.

3.2.1 The tradable sector

We assume that the production function is

$$QT_{jt} = \theta_t LT_{jt}^{\alpha} \tag{14}$$

where QT_{jt} is the quantity of output, LT_{jt} labour, and θ_t and α are production function parameters.

Output can be sold either domestically or abroad (exports).

$$QT_{jt} = X_{jt} + Z_{jt} \tag{15}$$

where X_{jt} is domestic purchases, and Z_{jt} exports.

First, let us assume that prices are flexible and that firms set period *t* prices based on current information. Firms are assumed to be price-takers in international markets and price-makers at home. A firm entering period *t* + 1 has to decide on how much to produce and what price to charge at home. This assumption implies the ability to price discriminate and leads to a higher domestic price at home than abroad. Thus if the firm produces a certain amount of its product and faces the problem of dividing it between domestic sales and exports, it will first satisfy domestic demand and then export the rest. The possibility of international price discrimination can be explained by the presence of transportation and other transaction costs (tariffs, etc) that make perfect arbitrage impossible. This problem can be written as

$$\max_{PQ_{jt}, LT_{jt},} PX_t\left(\theta_t LT_{jt}^{\alpha} - X_{jt}\right) + PQ_{jt}X_{jt} - W_t LT_{jt} \tag{16}$$

where PQ_{jt} is the domestic price, and W_t the nominal wage. Moreover

$$PX_t = PX_t^* S_t \tag{17}$$

is the (common) export price expressed in domestic currency, PX_t^* is the export price in foreign currency and S_t is the nominal exchange rate. To carry out maximisation the firm must take into account the demand function (see below), which is supposed to be known by the seller and has a price elasticity ϕ which is identical across firms. Then profit maximisation implies the following formula for domestic tradable prices:

$$PQ_{jt} = PX_t \frac{\phi}{\phi - 1}. \tag{18}$$

The role of marginal cost is assumed by the exchange rate, which is the marginal opportunity cost of selling in the domestic market.

Labour demand can be explicitly expressed from the other first-order condition as

$$LT_{jt} = \left(\frac{\alpha \theta_t PX_t}{W_t} \right)^{\frac{1}{1-\alpha}} \tag{19}$$

Now we can consider nominal rigidity in price setting, by assuming that period t + 1 prices must be set based on period t information. Only one equation changes, namely the equation determining home-produced tradable prices. The new formula is the following:

$$PQ_{t+1} = \frac{\phi}{\phi - 1} \frac{E_t \left(S_{t+1} PX^*_{t+1} \Lambda_{t+1} \right)}{E_t (\Lambda_{t+1})} \tag{20}$$

Thus the marginal opportunity cost, the variable to be marked up, is modified by taking uncertainty into account. The effective marginal opportunity cost $\left(\frac{E_t \left(PX^*_{t+1} S_{t+1} \Lambda_{t+1} \right)}{E_t (\Lambda_{t+1})} \right)$ is higher (lower) than the expected export price if the export price and the marginal utility of nominal wealth are positively (negatively) correlated, since

$$\frac{E_t \left(PX^*_{t+1} S_{t+1} \Lambda_{t+1} \right)}{E_t (\Lambda_{t+1})} = E_t \left(PX^*_{t+1} S_{t+1} \right) + \frac{\mathrm{cov}_t \left(\Lambda_{t+1}, PX^*_{t+1} S_{t+1} \right)}{E_t (\Lambda_{t+1})}.$$

As usual, if shocks are small in the sense that $PX^*_{t+1} S_{t+1} \leq PQ_{j,t+1}$ it is surely ex post rational to satisfy domestic demand at the predetermined prices. In the following, this is always assumed to be the case.

Assuming tradable firms are uniformly distributed on the interval (0, 1), the aggregate input demand functions are exactly the same as that of the individual demand functions, with the superscripts omitted.

To derive the demand elasticity we start with the Dixit-Stiglitz technology for the aggregate home tradable good

$$X_t = \left(\int (X_{jt})^v \, dj \right)^{\frac{1}{v}} \tag{21}$$

Then the aggregate domestic tradable price index PQ_t can be written as

$$PQ_t = \left(\int (PQ_{jt})^{\frac{v}{v-1}} \, dj \right)^{\frac{v-1}{v}} \tag{22}$$

and the demand for the jth good as

$$X_{jt} = \left(\frac{PQ_{jt}}{PQ_t} \right)^{\frac{1}{v-1}} X_t \tag{23}$$

Thus $\phi = \frac{1}{v-1}$

To complete our description of the intermediate-capital good (tradable) sector, we define a Cobb-Douglas technology as follows:

$$Y_t = X_t^{\rho} M_t^{1-\rho} \tag{24}$$

Then price and demands can be derived as

$$PI_t = \rho^{-\rho} (1-\rho)^{1-\rho} PM_t^{\rho} PQ_t^{1-\rho} \tag{25}$$

$$X_t = (1-\rho) \frac{PI_t}{PQ_t} Y_t \tag{26}$$

$$M_t = \rho \frac{PI_t}{PM_t} Y_t \tag{27}$$

where

$$PM_t = PM_{t^*} S_t \tag{28}$$

is the import price in domestic, and PM_t^* the import price in foreign currency.

3.2.2 The non-tradable sector

When the Cobb-Douglas technology is written as

$$QN_t = LN_t^{\eta} YN_t^{1-\eta} \tag{29}$$

prices and demand satisfy

$$PN_t = \eta^{\eta} (1-\eta)^{1-\eta} W_t^{\eta} PI_t \tag{30}$$

$$LN_t = (1-\eta) \frac{PN_t}{W_t} QN_t \tag{31}$$

$$YN_t = \eta \frac{PN_t}{PI_t} QN_t \tag{32}$$

3.2.3 Food sector

Here we assume that food products cannot be both exported and imported and that decisions are taken one period in advance, before the realisation of technological uncertainty. (Alternatively, technological uncertainty can be reinterpreted as involving quotas). Output is purely stochastic, and it is equal to consumption.

$$QA_t = CA_t \tag{33}$$

3.2.4 Public service sector

Provision of the public service (QP_t) requires tradable input proportionally, and is supplied elastically to satisfy household demand for it.

$$YP_t = QP_t \tag{34}$$

$$QP_t = CP_t \tag{35}$$

where YP_t is input demand for tradables, and CP_t household demand for the public service.

3.3 Market equilibrium

3.3.1 Goods markets

There is a non-trivial goods market for the intermediate-capital good and a trivial one for the non-tradable good. Market clearing requires

$$Y_t = YN_t + ID_t + YP_t \tag{36}$$

and

$$QN_t = CN_t \tag{37}$$

3.3.2 Labour market

Labour market equilibrium is equivalent to

$$1 = LN_t + LT_t \tag{38}$$

3.4 Exogenous variables

Exogenous variables of the model include the (euro) import price $\left(PM_t^*\right)$, the (euro) export price $\left(PX_t^*\right)$, the (euro) interest rate $\left(I_t^*\right)$, productivity in the home tradable sector (θ_t) and food supply (CA_t).

3.5 Policy

Government policy has three interrelated aspects. Policy is generically defined in the following way:

$$f_P\left(PM_t^*, PX_t^*, I_t^*, \theta_t, CA_t, \Omega_{t-1}\right)$$

where Ω_{t-1} is the set of period t – 1 variables. In other words, the government can set these variables based on past information as well as on the current realisation of shocks.

3.5.1 Pricing of the public service

Efficiency would require setting

$$PP_t = PI_t \tag{39}$$

However, we assume that public service prices must be predetermined and cannot adjust to shocks immediately. We also allow for the possibility that the government may wish prices to deviate from marginal costs for reasons of taxation. Thus we assume the existence of a general relationship such as

$$PP_{t+1} = F_{PP}\left(\Omega_{t-1}\right)E_t\left(PI_t\right) \tag{40}$$

Here $F_{PP}\left(\Omega_{t-1}\right)$ is assumed to be a stationary mark-up. One can see that the government can affect the price level by changing mark-up unless monetary policy strives to dampen it. However, this should not involve conditional variability, ie price level uncertainty, whatever the nominal exchange rate policy.

3.5.2 Demand management

Let us define the marginal utility of nominal euro wealth as

$$\Lambda_t^e = S_t\Lambda_t \tag{41}$$

The government is supposed to be able to set the marginal utility of nominal euro wealth as

$$\Lambda_t^e = f_{PD}\left(\Omega_{t-1}, PM_t^*, PX_t^*, I^*{}_t, \theta_t, CA_t\right) \tag{42}$$

under the implementability constraints:

$$\Lambda_t^e = \beta I_t^* E_t \left(\Lambda_{t+1}^e\right) \tag{43}$$

Our shortcut to policy determination means that via these instruments the government can, to a certain extent, control the marginal utility of euro wealth in the economy. Euro wealth is a real variable for a small open economy and it is generally believed that domestic spending is an important concern for governments in countries where foreign debt is not insignificant. The exogeneity of foreign interest rates makes the debt process of small open economies inherently unstable. Thus some feedback regulation by the government may indeed be necessary (see below for details on the feasibility of this policy formulation).

3.5.3 Monetary (exchange rate) policy

Monetary policy has two varieties:

Fixed exchange rate:

$$S_t = 1, \tag{44}$$

and flexible exchange rates

$$S_t = F_{pM}\left(\Omega_{t-1}, PM_t^*, PX_t^*, I_t^*, \theta_t, CA_t\right) \tag{45}$$

Since, in this model, trends in the nominal exchange rate do not make sense, the exchange rate is supposed to be stationary. Identifying monetary policy with exchange rate policy in this model without money is more attractive than the alternative of identifying it with interest rate policy, since it avoids questions of indeterminacy. Defining exchange rate flexibility simply by allowing for its response to shocks also seems plausible.

3.6 Solution of the model

The endogenous part can be solved in the following order. A first set of equations can be solved from the exogenous and policy processes directly.

$$PM_t = PM_t^* S_t \tag{46}$$

$$PX_t = PX_t^* S_t \tag{47}$$

For the fully flexible price version

$$PQ_t = PX_t \frac{\phi}{\phi - 1} \tag{48}$$

whereas in the version with nominal price rigidity

$$PQ_t = \frac{\phi}{\phi - 1} \frac{E_{t-1}\left(PX_t^* \Lambda_t^e\right)}{E_{t-1}\left(\dfrac{\Lambda_t^e}{S_t}\right)} \tag{49}$$

The expectations on the right-hand side depend only on exogenous and policy variables and can thus be calculated if those processes are known. The next block of equations includes

$$PI_t = \rho^{\rho} \left(1 - \rho\right)^{1-\rho} PM_t^{\rho} PQ_t^{1-\rho} \tag{50}$$

$$PA_t = \frac{CA_t^{-\vartheta^A} S_t}{\Lambda_t^e} \tag{51}$$

$$CP_t = \left(\frac{\Lambda_t^e}{S_t} PP_t \right)^{-\frac{1}{\vartheta^P}} \tag{52}$$

$$I_t = \frac{I^*_t}{S_t} \frac{E_t\left(\Lambda_{t+1}^e\right)}{E_t\left(\frac{\Lambda_{t+1}^e}{S_{t+1}}\right)} \tag{53}$$

Again, the right-hand side expectations are computable from the exogenous variables and policy processes.

Then the labour market equilibrium can be used to determine nominal wages as a function of previously solved-for variables.

$$1 = \left(\frac{\alpha \theta_t PX_t}{W_t} \right)^{\frac{1}{1-\alpha}} + \frac{\left[\eta^\eta (1-\eta)^{1-\eta} (1-\eta) PI_t\right]^{\frac{\eta\vartheta^N - \eta}{\vartheta^N}} S_t}{W_t^{\frac{\vartheta^N - \eta\vartheta^N + \eta}{\vartheta^N}} \Lambda_t^e} \tag{54}$$

This is an implicit equation that cannot be solved explicitly. If it is solved for W_t, the rest of the system can be computed recursively. Regarding this equation as a function of W_t (the other variables are predetermined and positive) then $\frac{\vartheta^N - \eta\vartheta^N + \eta}{\vartheta^N} > 0$ implies that the right-hand side is monotonically decreasing. It approaches ∞ as W_t approaches 0 and approaches 0 as W_t approaches ∞. Thus there is a unique solution.

$$LT_1 = \left(\frac{\alpha \theta_t PX_t}{W_t} \right)^{\frac{1}{1-\alpha}} \tag{55}$$

$$QT_t = \theta_t LT_t^\alpha \tag{56}$$

$$PN_t = \eta^\eta (1-\eta)^{1-\eta} W_t^\eta PI_t \tag{57}$$

$$CN_t = \left(\frac{\Lambda_t^e}{S_t} PN_t \right)^{-\frac{1}{\vartheta^N}} \tag{58}$$

$$LN_t = (1-\eta) \frac{PN_t}{W_t} CN_t \tag{59}$$

$$YN_t = \eta \frac{PN_t}{PI_t} CN_t \tag{60}$$

$$D_t = \left[\frac{PI_t \Lambda_t^e}{S_t} \left(1 - \beta \left(1 - \delta^D\right) E_t \left(\frac{PI_{t+1} \Lambda_{t+1}^e S_t}{PI_t \Lambda_t^e S_{t+1}} \right) \right) \right]^{-\frac{1}{\vartheta^D}} \tag{61}$$

The expectation $E_t\left(\frac{PI_t \Lambda_{t+1}^e S_t}{PI_t \Lambda_t^e S_{t+1}}\right)$ can be computed again from the basic processes.

When the rental price of durables is defined as

$$PR_t = PI_t \left(1 - \beta \left(1 - \delta^D\right) E_t \left(\frac{PI_{t+1} \Lambda_{t+1}^e S_t}{PI_t \Lambda_t^e S_{t+1}} \right) \right) \tag{62}$$

the rest of the system may be written as

$$ID_t = D_t - \left(1 - \delta^D\right)D_{t-1} \tag{63}$$

$$Y_t = YN_t + ID_t + CP_t \tag{64}$$

$$X_t = \left(1 - \rho\right)\frac{PI_t}{PQ_t}Y_t \tag{65}$$

$$M_t = \rho\frac{PI_t}{PM_t}Y_t \tag{66}$$

$$Z_t = QT_t - X_t \tag{67}$$

3.6.1 Justifying the solution

The above formulas refer to a subset of the variables of the model. We have not imposed any feasibility constraints on the equilibrium processes. To make economic sense, the resulting equilibrium must satisfy transversality or no-Ponzi game conditions for both the private sector and the government. We also left open the problem of portfolio decisions; in other words the behaviour of capital markets. To argue for the validity of the (sub)solution obtained, we proceed via several steps.

1. How does the long-run financial position of the private sector behave for a given demand policy? It is clear that different demand policies would lead to quite different present values of net future spending. In other words, any randomly selected aggregate demand policy that satisfies the implementability condition is very unlikely to result in a feasible equilibrium.

2. Can the government make any private income spending plan feasible? The answer is yes, provided that the government has the ability to adjust (lump-sum) taxes appropriately. With no restriction on taxes, the government can make a transfer each period that balances private sector accounts so that no borrowing or lending by households occurs.

3. Is this enough to make the proposed equilibrium feasible for the whole economy? Even though transfers between the government and the private sector do not affect the overall budget constraint, an arbitrary (implementable) aggregate demand policy may still be non-feasible for the economy as a whole. However, the government has another instrument, namely real government spending. Suppose the government spends on importables, which may even enter the household utility function in an additive way and not influence the marginal utilities of privately purchased goods. Then the government can adjust its own spending so that the current account is balanced at each period.[5]

Thus one can prove that there are, and in fact must be many, government policies that support an equilibrium for a given (implementable) aggregate demand policy. These policies also imply zero asset trade in equilibrium. In fact, in equilibrium models with government spending it is obviously the case that government spending has an impact on private consumption. Thus, simply stated, our assumption is as follows: the government can control aggregate demand constrained by the implementability condition. If we were interested in optimal policy analysis we should specify the welfare consequences of fluctuations in government spending, or try to model a realistic tax structure. These considerations are not necessary for our present purposes. Also, it may be feared that some demand policies would have widely unrealistic consequences for the path of government spending and taxes. Therefore, if one wished to carry out numerical calculations with the model, attention should be restricted to policies that are reasonable in a vague sense; ie they are easily interpretable demand policies involving parameters that do not result in extreme fluctuations in either expenditures or taxes.

[5] Non-negativity constraints should of course be observed.

3.7 Price formulas

Above we showed how the model can be solved recursively. Here, we derive the formulas for prices expressed as functions of policy and exogenous variables as far as it is possible or convenient.

Food prices are

$$PA_t = \frac{CA^{-\vartheta_\alpha} S_t}{\Lambda_t^e} \tag{68}$$

or

$$pa_t = -\vartheta_a ca_t - \lambda_t^e + s_t \tag{69}$$

where lower case letters denote logs of the corresponding variable.

Thus, food prices are determined by monetary policy, demand policy and the agricultural supply shock, with the latter's impact depending on the ϑ_a parameter. When this is large (ie foodstuffs are less luxurious), supply shocks have larger impacts on both absolute and relative food prices.

Flexible durable prices can be expressed as

$$PI_t = \rho^\rho (1-\rho)^{1-\rho} PM_t^{*\rho} \left(\frac{\phi}{\phi - 1} PX_t^* \right)^{1-\rho} S_t \tag{70}$$

or in logs

$$pi_t = \log\left(\rho^\rho (1-\rho)^{1-\rho}\right) + \rho pm_t^* + (1-\rho)\log\left(\frac{\phi}{\phi-1}\right) + (1-\rho) px_t^* + s_t \tag{71}$$

With full price flexibility, demand policy has no impact on durable prices, though exchange rates have a unitary elasticity. In this case, import and export prices contribute to durable price inflation with weights corresponding respectively to the import share and 1 minus the import share in the intermediate sector. The $(1-\rho)\log\left(\frac{\phi}{\phi-1}\right)$ term means that durable prices have a higher level when there is a larger domestic distortion as expressed by the mark-up term in the pricing formula for tradable production.

In the case of price rigidity, (70) becomes

$$PI_t = \rho^\rho (1-\rho)^{1-\rho} PM_t^{*\rho} \left(\frac{\phi}{\phi-1} \frac{E_{t-1}\left(PX_t^* \Lambda_t^e\right)}{E_{t-1}\left(\frac{\Lambda_t^e}{S_t}\right)} \right)^{1-\rho} S_t^\rho \tag{72}$$

A second-order approximation (exact in the case of log normality) gives

$$\begin{aligned} pi_t = {} & \log\left(\rho^\rho (1-\rho)^{1-\rho}\right) + \rho pm_t^* + (1-\rho)\log\left(\frac{\phi}{\phi-1}\right) + \rho s_t + \\ & (1-\rho)\left(E_{t-1}\left(px_t^* + s_t\right) + \frac{1}{2}\mathrm{var}_{t-1} px_t^* + \frac{1}{2}\mathrm{var}_{t-1} s_t + \mathrm{cov}_{t-1}\left(\lambda_t^e, px_t^*\right) - \mathrm{cov}_{t-1}\left(\lambda_t^e, s_t\right)\right) \end{aligned} \tag{73}$$

The direct impact of the exchange rate is blunted, as it manifests itself only via the import price. However, both monetary and demand policy expectations matter. The expected exchange rate has a direct positive impact and exchange rate variability adds to the price level as well. We can call monetary and fiscal policies *parallel* if devaluations tend to be accompanied by expansionist aggregate demand policies. Parallelism means that $\mathrm{cov}_{t-1}\left(\lambda_t^e, s_t\right)$ is negative in our model. Thus tradables prices become relatively higher when policy exhibits parallelism.

The price that enters the first-order condition (the demand function) for durables involves the rental price. In the flexible case this may be written as

$$PR_t = \left(\rho^{\rho} (1-\rho)^{1-\rho} PM_t^{*\rho} \left(\frac{\phi}{\phi-1} PX_t^* \right)^{1-\rho} S_t \right) x \tag{74}$$

$$\left(1-\left(1-\delta^D\right) E_t \left(\frac{\beta \Lambda_{t+1}^e}{\Lambda_t^e} \left[\frac{PM_{t+1}^{*\ \rho} PX_{t+1}^{*1-\rho}}{PM_t^{*\rho} PX_t^{*1-\rho}} \right] \right) \right)$$

If we use the notation $H_t = 1-\left(1-\delta^D\right) E_t \left(\frac{\beta \Lambda_{t+1}^e}{\Lambda_t^e} \left[\frac{PM_{t+1}^{*\ \rho} PX_{t+1}^{*1-\rho}}{PM_t^{*\rho} PX_t^{*1-\rho}} \right] \right)$, then taking logs yields

$$pr_t = \log\left(\rho^{\rho} (1-\rho)^{1-\rho}\right) + \rho pm_t^* + (1-\rho) \log\left(\frac{\phi}{\phi-1} \right) + (1-\rho) px_t^* + s_t + h_t \tag{75}$$

The h_t term can be approximated as

$$-h_t \approx \log\left(1-\delta^D\right) + \log\beta + E_t\left(\lambda_{t+1}^e - \lambda_t^e\right) + \rho E_t\left(pm_{t+1}^* - pm_t^*\right) + (1-\rho) E_t\left(px_{t+1}^* - px_t^*\right) +$$

$$\frac{1}{2}\left[\mathrm{var}_t\left(\lambda_{t+1}^e\right) + \rho\, \mathrm{var}_t\left(pm_{t+1}^*\right) + (1-\rho) \mathrm{var}_t\left(px_{t+1}^*\right) \right] + p\, \mathrm{cov}_t\left(\lambda_{t+1}^e, pm_{t+1}^*\right) + (1-\rho) \mathrm{cov}_t\left(\lambda_{t+1}^e, px_{t+1}^*\right)$$

$$+ \rho(1-\rho) \mathrm{cov}_t\left(px_{t+1}^*, pm_{t+1}^*\right), \tag{76}$$

where there is a factor (omitted) that is time-invariant if h_t is stationary as in Clarida (1996). Here, h_t is the deviation of the rental price from the purchase price. Interestingly, in the price flexibility case this deviation does not depend on exchange rate policy. However, it depends on demand policy. An expected contraction (ie a positive $E_t\left(\lambda_{t+1}^e - \lambda_t^e\right)$) depresses the rental price by making the possession of the durable good more valuable in the future. Expected increases in export and import prices have the same qualitative effect. In addition, the demand policies that depend on foreign prices make a contribution. Suppose, for example, that an increase in import prices is associated with a decline in aggregate demand (ie $\rho\, \mathrm{cov}_t\left(\lambda_{t+1}^e, pm_{t+1}^*\right)$ is positive). This decreases the rental price by making the purchase of the durable good a hedge against uncertainty. On the other hand, the plausible policy of an aggregate demand expansion following an export price increase would make the purchase a risky investment and would result in an increase in the rental price.

With price rigidity things get to be messy

$$PR_t = \left(\rho^{\rho} (1-\rho)^{1-\rho} PM_t^{*\rho} \left(\frac{\phi}{\phi-1} \frac{E_{t-1}\left(PX_t^* \Lambda_t^e\right)}{E_{t-1}\left(\frac{\Lambda_t^e}{S_t}\right)} \right)^{1-\rho} S_t^{\rho} \right) x \tag{77}$$

$$\left(1-\left(1-\delta^D\right) E_t \left(\frac{\beta \Lambda_{t+1}^e S_t^{1-\rho}}{\Lambda_t^e S_{t+1}^{1-\rho}} \left[\frac{PM_{t+1}^{*\ \rho} \left(\frac{E_t\left(PX_{t+1}^* \Lambda_{t+1}^e\right)}{E_t\left(\frac{\Lambda_{t+1}^e}{S_{t+1}}\right)} \right)^{1-\rho}}{PM_t^{*\rho} \left(\frac{E_{t-1}\left(PX_t^* \Lambda_t^e\right)}{E_{t-1}\left(\frac{\Lambda_t^e}{S_t}\right)} \right)^{1-\rho}} \right] \right) \right)$$

Now the log of the rental prices may be written as

$$pr_t = \log\left(\rho^{\rho}(1-\rho)^{1-\rho}\right) + \rho pm_t^* + (1-\rho)\log\left(\frac{\phi}{\phi-1}\right) + \rho s_t + (1-\rho)E_{t-1}\left(px_t^* + s_t\right) + \frac{1}{2}\mathrm{var}_{t-1}px_t^* +$$

$$\frac{1}{2}\mathrm{var}_{t-1}s_t + \mathrm{cov}_{t-1}\left(\lambda_t^e, px_t^*\right) - \mathrm{cov}_{t-1}\left(\lambda_t^e, s_t\right) + h_t \ . \tag{78}$$

The approximation of h_t yields

$$-h_t \approx \log\left(1-\delta^D\right) + \log\beta + E_t\left(\lambda_{t+1}^e + \rho pm_{t+1}^* - (1-\rho)s_{t+1}\right) -$$

$$\left(\lambda_t^e + \rho pm_t^* - (1-\rho)s_t\right) + \frac{1}{2}\mathrm{var}_t\left(\lambda_{t+1}^e + \rho pm_{t+1}^* - (1-\rho)s_{t+1}\right) +$$

$$(1-\rho)E_t\left(px_{t+1}^* + s_{t+1}\right) + \frac{1}{2}\mathrm{var}_t\, px_{t+1}^* + \frac{1}{2}\mathrm{var}_t\, s_{t+1}$$

$$+\mathrm{cov}_t\left(\lambda_{t+1}^e, px_{t+1}^*\right) - \mathrm{cov}_t\left(\lambda_{t+1}^e, s_{t+1}\right) - (1-\rho)E_{t-1}\left(px_t^* + s_t\right) + \frac{1}{2}\mathrm{var}_{t-1}px_t^*$$

$$+\frac{1}{2}\mathrm{var}_{t-1}s_t + \mathrm{cov}_{t-1}\left(\lambda_t^e, px_t^*\right) - \mathrm{cov}_{t-1}\left(\lambda_t^e, s_t\right) \tag{79}$$

Here the main novelty is that both exchange rate and exchange policy expectations play some role. Different exchange rate policies may imply either an increase or a decrease in the riskiness of investing in durables.

Public service prices are set by policymakers as

$$PP_t = F_{PP}\left(\Omega_{t-1}\right)E_{t-1}\left(PI_t\right) \tag{80}$$

The approximation results in

$$pp_t = f_{PP}\left(\Omega_{t-1}\right) + E_{t-1}\left(pi_t\right) + \frac{1}{2}\mathrm{var}_{t-1}\left(pi_t\right) \ . \tag{81}$$

One can see that expected intermediate good prices play a role here, and that unexpected changes in the exchange rate have no effect. This pricing assumption thus gives another source of price rigidity, irrespective of the pricing behaviour of tradable producers.

For wages one cannot give an explicit expression, but can determine the sign of the partial derivatives unambiguously. The implicit equation to be solved is

$$\left(\frac{\alpha\theta_t PX}{W_t}\right)^{\frac{1}{1-\alpha}} + \frac{\left[\eta^{\eta}(1-\eta)^{1-\eta}(1-\eta)PI_t\right]^{\frac{\eta\vartheta^N-\eta}{\vartheta^N}} S_t}{W_t^{\frac{\vartheta^N-\eta\vartheta^N+\eta}{\vartheta^N}}\Lambda_t^e} - 1 = 0$$

in both the flexible and rigid cases. However, the comparative statics are different in the two cases. When tradable prices are flexible

$$\partial W_t/\partial S_t = 1 \qquad \partial\frac{W_t}{S_t}\Big/\partial\theta_t > 0 \qquad \partial\frac{W_t}{S_t}\Big/\partial PX_t^* > 0 \qquad \partial\frac{W_t}{S_t}\Big/\partial PM_t^* > 0 \qquad \partial\frac{W_t}{S_t}\Big/\partial\Lambda_t^e < 0 \ .$$

In other words, the exchange rate has a unitary elasticity, and the foreign price, productivity and aggregate demand variables influence the euro wage directly; ie independently of exchange rate policy. All signs are just as expected. The main difference in the price rigidity case is that $\partial W_t/\partial S_t < 1$. The other derivatives should be written in terms of the nominal wage.

$$\partial W_t/\partial\theta_t > 0 \qquad \partial W_t/\partial PX_t^* > 0 \qquad \partial W_t/\partial PM_t^* > 0 \qquad \partial W_t/\partial\Lambda_t^e < 0 \ .$$

These signs can be interpreted as incipient changes in the euro wage as well but, in contrast to the price flexibility case, exchange rate policy can undo any of these. In general one can make a log linearisation that results in the following formula.

$$w_t = a_0 s_t + a_1 \log\theta_t + a_2 px_t^* + a_3 pm_t^* - a_3 \lambda_t^e \,, \tag{82}$$

where a_1, a_2, a_3 and a_4 are positive, time-varying and proportional to the corresponding derivatives, whereas a_0 equals 1 in the flexible case and is positive but less than 1 in the rigidity case.

Non-tradable prices in the case of full price flexibility may be expressed as

$$PN_t = \eta^\eta (1-\eta)^{1-\eta} W_t^\eta \left[\rho^\rho (1-\rho)^{1-\rho} \left(PM_t^*\right)^\rho \left(\frac{\phi}{\phi-1} PX_t^* \right)^{1-\rho} S_t \right]^{1-\eta} \tag{83}$$

The log linear approximation gives

$$pn_t = s_t + \log\left(\eta^\eta (1-\eta)^{1-\eta}\right) + \eta\left(a_1 \log\theta_t - a_3 \lambda_t^e\right) +$$

$$\log\left(\rho^\rho (1-\rho)^{1-\rho}\right) + (1-\rho)\log\frac{\phi}{\phi-1} + \left(\rho(1-\eta) + \eta a_3\right) pm_t^* + \left((1-\rho)(1-\eta) + \eta a_2\right) px_t^* \tag{84}$$

The formula is straightforward: non-tradable prices are influenced by those real variables that enter the formula for nominal wages, ie productivity and aggregate demand.

With rigid home tradable prices

$$PN_t = \eta^\eta (1-\eta)^{1-\eta} W_t^n \left[\rho^\rho (1-\rho)^{1-\rho} PM_t^{*\rho} \left(\frac{\phi}{\phi-1} \frac{E_{t-1}\left(PX_t^* \Lambda_t^e\right)}{E_{t-1}\left(\frac{\Lambda_t^e}{S_t}\right)} \right)^{1-\rho} S_t^\rho \right]^{1-\eta} \tag{85}$$

The approximation can be written as

$$pn_t = (\eta + \rho - \eta\rho) s_t + \log\left(\eta^\eta (1-\eta)^{1-\eta}\right) + \eta\left(a_1 \log\theta_t \, a_2 px_t^* - a_3 \lambda_t^e\right) + \log\left(\rho^\rho (1-\rho)^{1-\rho}\right) + (1-\rho)\log\frac{\phi}{\phi-1} +$$

$$\rho(1-\eta) pm_t^* + (1-\rho)\left(E_{t-1}\left(px_t^* + s_t\right) + \frac{1}{2}\mathrm{var}_{t-1} px_t^* + \frac{1}{2}\mathrm{var}_{t-1} s_t + \mathrm{cov}_{t-1}\left(\lambda_t^e, px_t^*\right) - \mathrm{cov}_{t-1}\left(\lambda_t^e, s_t\right)\right) \tag{86}$$

Again, the exchange rate's elasticity is less than 1. Also, uncertainty affects prices by generating risk premium terms. One can find again that parallel policies result in higher prices. Or, in other words, increased parallelism leads to higher inflation. This means, incidentally, that if policy turns from antiparallelism to something like no parallelism, inflation in the non-tradable sector would rise.

From the formulas it can be seen that the aggregate demand variable has an effect on relative prices, irrespective of price rigidity, and that exchange rate policy affects contemporaneous relative prices.

4. Analysis

4.1 Long-run volatility of the price level

Our model has no exact aggregate price index. Aggregate CPI indices used in practice are Laspeyres type indices where individual price indices are averaged by applying past relative expenditures as weights. This would give an exact price index only if preferences belonged to the generalised Leontief type (see Pollak (1983)). If relative expenditures change, then the impact of any of our four price indices on the aggregate CPI also changes. We can create plausible conditions under which, as the economy grows, the expenditure weights on food and public services converge to 0. For the sake of a simple demonstration let us suppose now that $\vartheta_N = 1$; ie non-tradable consumption enters the utility function as a logarithm. Then

$$PN_t\, CN_t = \frac{S_t}{\Lambda_t^e}$$

and

$$PA_t\, CA_t = \frac{CA_t^{1-\vartheta_a}\, S_t}{\Lambda_t^e}$$

imply that

$$\frac{PA_t\, CA}{PN_t CN_t} = CA_t^{1-\vartheta_a}$$

This ratio approaches 0 if the growth rate of the exogenous supply of food is lower than $\vartheta_a - 1$.

For the public service

$$PP_t\, CP_t = \Lambda_t^{e-\frac{1}{\vartheta^P}}\, S_t^{\frac{1}{\vartheta P}}\, PP_t^{1-\frac{1}{\vartheta P}}$$

Under the assumptions made on price formation in the public service sector, and on the assumption that foreign prices are stationary, one can rewrite this equation as

$$PP_t\, CP_t = \Lambda_t^{e-\frac{1}{\vartheta^P}}\, S_t\, G_t$$

where G_t is stationary.

Thus

$$\frac{PP_t\, CP_t}{PN_t\, CN_t} = G_t\, \Lambda_t^{e-\frac{1}{\vartheta^P}}$$

Then assuming that Λ_t^e grows at a negative rate is sufficient to have a vanishing share for expenditure on public services.

If the above conditions are fulfilled, the CPI volatility stemming from public service and food prices decreases as income and consumption grow. Thus even if these prices are more volatile than durable and non-tradable prices, the volatility of the overall index tends to approach the volatility of the latter two as the economy becomes richer.

4.2 Stabilisation of the price level

In our model, irrespective of the price flexibility assumption, exchange rate policy can achieve the stabilisation of any individual price index, except for that of the public service price. A brief inspection of the price formulas shows that all other prices depend on the nominal exchange rate. Thus, if we allow monetary policy to have the power to respond to shocks simultaneously, any CPI aggregate can be stabilised by appropriately adjusting the exchange rate. However, flexible exchange rates may have real effects in the model. In the following we will study this issue through thought experiments. Let us suppose that monetary policy responds in a price level stabilising manner to each of the five shocks. What are the real consequences of, and the possible trade-offs consequent upon, these policies? First we must ask what a price stabilising response means.

Initially, let us suppose that aggregate demand policies of the government are given. Inspecting the formulas demonstrates that for the export and import price and for the productivity variable an increase would lead to a price level increase if the exchange rate were fixed. Thus a strengthening of the exchange rate would be stabilising in these cases. On the other hand, a positive food production shock would result in a reduction in the price level, making depreciation the appropriate response. The foreign interest rate does not figure in any of the formulas; thus changes in it should not draw a response from monetary policy. However, it is clear from the implementability condition that aggregate demand policies are affected by foreign interest rate changes. Thus foreign interest rates may have an indirect effect on prices and we have to ask what this indirect effect is like.

The implementation condition

$$\Lambda_t^e = \beta I_t^* E_t\left(\Lambda_{t+1}^e\right)$$

has one certain implication. Expected aggregate demand must depend on the current foreign interest rate. The aggregate demand expectation has a role in the determination of several prices. It plays a role in the rental price equation, irrespective of price rigidity, and, in the case of rigid prices, influences tradable and non-tradable prices as well. The log linear approximations show, however, that only in the case of the rental price does $E_t\left(\Lambda_{t+1}^e\right)$ have a direct effect on the current price. Otherwise only covariance terms involving surprises are involved in the formulas. In the case of the rental price, a decrease in $E_t\left(\Lambda_{t+1}^e\right)$ (ie an expected increase in future aggregate demand) decreases current demand via a negative effect on the rental price. (This is just the effect of increasing the nominal interest rate on the demand for a capital (durable) good.) However, what is important for us here is the response of fiscal policy to a shock to foreign interest rates. We think of aggregate demand policies as substituting for capital markets to stabilise debt in the economy in question. Since for an indebted country the worsening of borrowing terms would naturally imply a restriction in demand, it is plausible to assume that a current shock to foreign simultaneously rates results in a higher Λ_t^e. It is easy to check that this response entails a reduction in all components of the CPI aggregate, except for public service prices, on which it has no effect. One can conclude that, because of these indirect effects, the appropriate price stabilising response of monetary policy would be to depreciate the nominal exchange rate following an increase in the foreign interest rate.

Now it is time to discard the assumption of constant fiscal policy in the face of shocks. One must ask how fiscal policy responds to the other four shocks, if it is supposed to behave in the manner hypothesised. The answers are plausible enough. Food supply shocks should leave aggregate demand undisturbed. Import price increases could invoke tightening, whereas export price and productivity increases would require more relaxed demand conditions. Changes in the policy mix that stabilise prices via monetary policy and debt via fiscal policy are shown in Table 1 for positive shocks to productivity, export prices, import prices, food supply and foreign interest rates.

Table 1

Type of stock	Policy response	
	Debt stabilising fiscal	**Price stabilising monetary**
Productivity shock	Ease	Tighten
Export price shock	Ease	Tighten
Import price shock	Tighten	Tighten
Food supply shock	Neither	Ease
Foreign rates shock	Tighten	Ease

In the case of import price shocks, it may be that fiscal tightening would achieve price stability without the intervention of monetary policy, or even overshoot so that price stability would require monetary easing.[6] The conjunction of debt and price stabilisation does not in general lead to parallelism. Only in the case of the import price shock do the two policies move in parallel.

6 It is worth noting that the interplay between responses to these shocks determines the risk premia that are present in three places in the model: the determination of the domestic interest rate, of rigid home tradable prices and of rental prices.

We have summarised how price stabilising monetary policies would work. Next we ask whether there are trade-offs; ie whether there exist negative side effects to such policies that may prevent their application. First, suppose that a negative food supply shock hits the economy and the exchange rate appreciates to restore price stability. Let us look for effects on tradable and import demand. Appreciation has a negative effect on the demand for public services, because of its predetermined nature. Via this it generates a decrease in the demand for tradables. With flexible prices no further effect exists, but with predetermined prices there is a reduction in the demand for non-tradables and durables alike. Thus price stabilisation would lead to an improvement in the current account, except for a change in the relative prices facing home producers. Because of price rigidity, relative prices are changed and imports become relatively cheaper than home tradables. The sign of the effect on the current account depends on the precise configuration of parameters. It is important to note that the effect on the demand for durable expenditures is variable, since it depends on the existing stock and thus on the history of shocks and responses. Whatever the net effect, it remains true that the current account would vary for no good reason at all.

Similar remarks are valid for the other cases as well. Take the example of a positive import price shock. From a debt stabilising point of view, an adjustment via import substitution would be needed. However, the strengthening of the exchange rate as a price stabilising response makes imports cheaper and further deteriorates the current account. Of course, the effects on total demand may make up for this, but this is a point where one could not make any judgements without knowing the parameters. The main point is that whereas changes in aggregate demand can unambiguously take the current account in the desired direction, monetary policy changes may not. In other words, monetary policy as a tool for managing the current account is just a second best solution. Hence, price stabilisation can have current account effects that may vary in time.

What happens if the exchange rate is fixed? In our model, all relative price changes would reflect real variation in exogenous conditions and in fiscal policy. Here, price rigidity coexists with nominal volatility. Hence, from a real allocation point of view, the fixed exchange rate solution seems to be superior. If price stability is also desirable, there is a genuine trade-off, the study of which would require the modelling of the costs of price level instability.

To summarise, we may state that to avoid fortuitous changes implied by using monetary policy to stabilise prices, one may resign oneself to accepting fluctuating prices. Striving for price stability would result in variation in domestic demand and in the current account in ways which depend on deep parameters, are time-varying, and may have no connection to the debt stabilisation objectives. Of course, one can make the course of fiscal policy subject to monetary policy's price stabilising intentions.

Still, fiscal policy may not be easily adjustable and, at any rate, it is limited by the implementability constraint. In sum, relative price variation may be a concern to policymakers and, in certain circumstances, could make them prefer some price level volatility.

4.3 Real interest rates

In the absence of a linear homogenous price aggregate, we have to define an own real interest rate for each good. The own interest rate of a good between *t* and *t+1* is the price of the good delivered in *t* in terms of the same good delivered in *t+1* minus 1. Then the own expected rate of interest of any good with price P_t can be written as

$$r_{t,t+1} = \frac{I_t P_t}{E_t\left(P_{t+1}\right)}$$

Assuming that a second-order approximation is valid, the effect of the real exchange rate on the slope of consumption can be written as

$$E_t\left(c_{t+1} - c_t\right) = \frac{1}{\vartheta}\left(i_t - \left(E_t\left(p_{t+1}\right) - p_t\right) + \log\left(\beta\right) + \frac{1}{2}\operatorname{var}_t\left(c_{t+1}\right) - \frac{1}{2}\operatorname{var}_t\left(p_{t+1}\right) + \operatorname{cov}_t\left(c_{t+1}, p_{t+1}\right)\right)$$

One can see that the slope depends on the parameter ϑ, with higher values implying a weaker effect on the expected change in consumption.

For any CPI aggregate we can have a corresponding real interest rate measure, the interpretation of which is, however, not clear at all. To become informed on this issue we first ask what the meaning of an interest rate change in our model is. Using the same approximation again we obtain

$$i_t = i_t^* - s_t + E_t s_{t+1} - \frac{1}{2}\operatorname{var}_t(s_{t+1}) + \operatorname{cov}_t\left(s_{t+1}, \lambda_{t+1}^e\right)$$

Assuming that the expectation terms are constant, there is a one-to-one relationship between the nominal exchange rate and the nominal interest rate. Ceteris paribus, an appreciation implies an immediate increase in the interest rate. Thus the real effects of changing the nominal interest rates are those of unexpected nominal exchange rate changes. We next ask whether these changes are associated with changes in the own real interest rates, and what the correlations between real interest rates, prices and demand are.

For foodstuffs, changes in the interest rate have no independent effect on consumption. Own rates are independent of the level of the nominal exchange rate and no effect via an interest rate demand price channel can exist. For instance, food prices can be stable while the exchange rate and the nominal interest rate fluctuate, resulting in real interest rate variations that have nothing to do with demand or price variation. For the other goods, price rigidity gives rise to real interest rate effects by altering demand. As public service prices are predetermined, a surprise appreciation gives rise to both an increase in the own real rate and a temporary decrease in demand. However, the demand change depends on the ϑ_P (intertemporal elasticity of substitution) parameter and may be rather small. Still, we have here a negative correlation between the real rate and demand. Suppose now that we have a monetary policy that strives to only partially undo any price shock. For concreteness' sake, let us assume that the price level is always set halfway between its no shock value and what the shock would imply if exchange rates were fixed. It is clear that this policy would imply a positive correlation between real interest rates and inflation.

For the non-tradable demand a negative relationship exists in the case of price rigidity, but its strength depends on the degree of implied price rigidity, and also on the preference parameter, ϑ^N. The situation is the most complicated for the rental price. Price rigidity causes a negative correlation between the real rate and demand, but the relationship depends on the amount of stocks; thus it is time-varying. What is also important here is that the relevant price for demand determination is the rental price and not the price of the tradable and durable good. Indeed, one can see from the relevant formulas that since price rigidity has two effects on the rental price, the change in the nominal exchange rate can be magnified. Again, an important conclusion is that the same real rate probably has time-varying effects, depending on the state of the economy.

Does a change in demand influence present or future prices? One way in which current monetary policy may affect future prices is via its effect on the future course of fiscal policy, the Λ_{t+1}^e variable. If we assume that fiscal policy is debt conscious, then a current exchange rate change that increases (decreases) foreign debt leads to an expected increase (decrease) in Λ_{t+1}^e and thereby to downward (upward) pressure on tomorrow's price level. However, this pressure depends on the net effect on imports of exchange rate policy, whose sign is, a priori, uncertain and time-varying.[7]

From the above analysis we have the following general observation about real rates in this economy. With exchange rate flexibility it is possible to obtain a fixed price level by changing nominal and, therefore, real interest rates. This is because prices and real interest rates are not correlated at all, even though real rates and demand are negatively correlated.

A few important conclusions emerge from the above discussion:

– If we want a useful indicator of the real interest rate, supply-determined prices should not be part of the corresponding price index.

[7] Maybe this uncertainty is the cause of a certain schizophrenia exhibited by central bankers. They sometimes wish for a weaker exchange rate and a higher interest rate simultaneously.

- As the demand effects of interest rates depend on intertemporal substitutability, any aggregate index ignoring this can be misleading.
- Because of the existence of durables, the effect of the interest rate on demand must be time-varying.
- A strong positive correlation between prices and real interest rates may signal only partial commitment to price stability.

4.4 Core inflation

Even the term core inflation reflects the desire of many to capture an inherent and presumably essential feature of reported inflation indices. By core inflation different authors refer to things that have similar, but not identical, properties (for a survey see Wynne (1999)). Supporters of one view claim that core inflation is a price index over which monetary policy exerts (direct) influence. On another view it is something which contains little temporary noise. The accurate definition of both criteria would require a model to determine if they are identical or what differences they contain. If an optimal core inflation measure is sought, one cannot ignore the question of what kind of information one intends to distil from it. Now we will consider the consequences of defining core inflation indices in our model. Roughly, our concern is to speculate on what sort of information a specific core index may provide or hide. Of the known domain of methodologies we will look at two general classes: core inflation obtained by excluding certain subgroups and outliers respectively.

4.4.1 Excluding certain subgroups

In order to define a core inflation index a frequently used expediency is to exclude food (or at least certain food) prices from the CPI. In our model, quite naturally, food prices contain information not only on supply shocks, but on demand and monetary policies as well. Thus excluding food prices may prevent one from judging whether monetary policy intends to stabilise the price level. However, our model also suggests that responding to supply shocks may have unwanted side effects, generating relative price and domestic demand variation. If this is thought to be a nuisance then monetary policy has a reason to neglect food supply shocks. This is a decision based on a judgement concerning the trade-off between the adverse effects of price level instability and real instability. The decision taken on this matter can tell us something about monetary policy.[8]

Another common expediency is to exclude public sector prices or changes in indirect taxes. There is a certain plausibility to the idea that these prices may provide useful information on future monetary policy and inflation as long as their pricing is motivated by price expectations (see Cecchetti and Groshen (2000)). However, in practice, the monthly variation of public service prices has been high in the transition economies, as noted above. Political considerations have caused another type of uncertainty. Though adjustments have usually been made once a year, this adjustment was not realised in every year. Thus in the catch-up period, the information provided by these prices may have been very noisy.

Excluding durable prices has probably never been considered. Still, this could be useful if, in certain circumstances, durable prices were a very noisy signal of true rental prices.[9] In principle, rental prices contain useful information on policy since they inform on expectations. Of course, an important empirical problem is whether the supply of durables is capable of adjusting promptly to shocks. In our model this is assumed, but on some real markets it may be far from the truth. In this case, rental prices may be similar to food prices.

Excluding non-tradable prices has not been considered either. In our model they are probably the most synthetic variable, containing information on many aspects of the economy. Therefore, this information may not be peculiar to monetary policy, and demand and tax policies may have substantial roles in determining non-tradable prices in the short run. On the other hand, technological shocks may

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8 Of course, the size of shocks can also be an influential motive to neglect these shocks.

9 If durable prices are included because they are related to transactions' demand for money, then it is not clear why house or share prices are not included in the price index to be watched or stabilised.

have permanent effects, making non-tradable prices less than perfect for signalling the stance of monetary policy.

In sum, one can have arguments for and against excluding the usual candidates from the CPI. One can even have reasons to exclude those subgroups that are normally considered to belong to the core. Concrete circumstances and purposes are decisive.

4.4.2 Excluding outliers

Trimming is hallowed as a method to filter out temporary noise, without the need to identify beforehand the possible origin of this noise. However, it has been argued that trimming may discard very good information if certain price-makers set prices according to expectations (see Bakshi and Yates (1998)). This may be a reason why infrequently changed public service prices should not be excluded at the time when they are changed, even if they seem to be outliers at that moment. An important aspect of our model is that a price stabilising monetary policy narrows the spread between predetermined public service prices and those that are at least partially responsive to exchange rate shocks immediately. In this sense, if policy is required to actively stabilise the price level, then large spreads of the distribution may signal that policy is not up to this task. On the other hand, the relative price of foodstuffs in terms of, say, non-tradables can be either larger or smaller, depending on the exact state of shocks. Thus we must know more about the nature of relative price changes before attributing them to noise. Indeed, it seems somewhat incredible to claim that central banks cannot do anything about food or energy prices (see, for instance, Blinder (1997)) that may become outliers more frequently than others. Still, as we have seen, central banks might have reasons to do little about these prices. This is, however, a signal of preferences and not of feasibility.

4.4.3 Discussion

We conclude that if one wants to justify adjustments to the price index or define an appropriate price index, one must be very specific about the structure of the economy and what is expected of monetary policy. We believe that core indices make sense as long as they have a useful function for policymakers or analysts, and not because they grasp some fundamental feature of never-changing reality. Thus the concept of core inflation cannot be seen as either a purely statistical phenomenon or a universal problem that is free of national characteristics. Let us look at a few practical examples.

1. Temporary changes in (non-processed) food supplies. A monetary policy response to such changes may cause large relative price fluctuations and unwarranted real volatility. Therefore, for a central bank caring about this, the right core inflation index should exclude these prices. At the same time, the operation of food markets may vary from country to country. While in some countries such supply shocks occur frequently and tend to cause significant changes in the CPI, in other countries they are supposed to be of no consequence. If the latter is the case, there is no reason to meddle with the price index.

2. Changes in taxation. The issue to resolve here is how persistent the effects on inflation of a specific tax change will be, given the reaction of markets. It also matters whether the real distortion caused by the tax change is of a type that will prompt monetary policymakers to take corrective action on the basis of their preferences and views on the operation of the economy. For example, an increase in labour costs is likely to speed up inflation, and not only in the very short run. An inflation-wary monetary policymaker will conclude from this that tightening may be in order. However, tightening may further aggravate the probable ensuing direct drop in profitability and take the economy towards recession. This is no trivial problem for a monetary policymaker seeking to achieve real economy objectives as well. Easing is another alternative, but the monetary policymaker may also decide not to make any moves. In the latter case, the effect of the tax change may be excluded from the core inflation index.

3. Fuel price changes. Experience has shown that changes in oil prices tend to be persistent, but lacking a permanent positive or negative trend. Central bankers may be faced with the genuine dilemma of whether to treat oil price changes the same way as changes in non-processed food prices or to take them seriously instead. The outcome of their pondering obviously depends on the exposure of a country's price level to fuel prices. Another consideration may be whether monetary policymakers wish to correct the changes in the terms of trade and in the real exchange rate that follow.

We would like to conclude by adding two further practical comments. First, in view of the fact that monetary policy objectives and the structure of the economy may change over time, a correct core inflation index, as well as the weights given to the individual components, may undergo a simultaneous change. Second, the relevance or irrelevance of a particular price shock to core inflation does not depend on whether the shock is temporary or not. The propagating mechanisms might make the most ephemeral shock persistent and relevant.

5. Conclusions

In this paper we investigated the consequences of introducing goods heterogeneity into a stochastic general equilibrium model for a small open economy. The behaviour of individual price series suggests that such an approach is required to answer certain questions. The main body of the study contains a model that serves to clarify ideas and enables us to derive the consequences of some hypotheses. For the sake of treatability we had to make several simplifying assumptions, the most heroic being that regarding fiscal policy. In essence, it is assumed that fiscal policy controls aggregate demand (under an implementability condition) via determining the euro marginal utility of households. What are the most likely shortcomings of the model and how do they influence the inferences we draw?

Excluding money and money demand from models used for the analysis of monetary policy has been in vogue for some time. This certainly makes an already complicated mathematical system more amenable to analysis. However, this may conceivably be misleading. One can give a justification for this neglect of money if one had money-in-the-utility function together with separability assumptions. We see little reason to include such a money demand formulation in the model. However, a plausible specification may bring important changes and new conclusions into the analysis and would be worth trying. This would also necessitate a numerical approach, towards which we want to move anyway.

Our modelling of fiscal policy may raise questions of consistency. A numerical solution of the model may also be useful in checking whether treating Λ_t^e as a (constrained) policy variable is consistent in the sense that one can establish a mapping between this variable and more traditional or fundamental fiscal policy variables.

The present model contains some features that depend on our modelling of price setting. Clearly, there are too few nominal and real rigidities in the model, and all inflation inertia must be attributed to (unexplained) policy inertia. Also, too many of the results may be driven by the assumption of foreign currency pricing of both exports and imports. However, this does not seem to be a totally implausible assumption for small developing economies.

The decomposition of the aggregate CPI as modelled here only partially answers the empirical description reported in Section 2. To make the model more useful for understanding the past and, possibly, for forecasting the future, it may be necessary to add new goods, like energy or commodities, as well as to relax the simplifying assumptions concerning the food sector. This step towards enhanced realism would also lead us to numerical simulations.

Having said that, we can summarise the most important lessons of this study, which may survive future refinements of the model.

First, it seems justified that the conjunction of non-homothetic preferences and permanent relative price changes raises the possibility that some of those notions we mentioned in the introduction should be modified. With respect to price stability, we found that at higher income levels, for any given monetary policy rule, CPI volatility probably declines, and that monetary policies striving for stabilising prices may involve fortuitous changes in real variables. (This result can be contrasted with the finding that stabilising prices is optimal in models with essentially homogenous goods, but differential costs to changing prices.) Concerning real interest rate indicators, we can conclude that they must be calculated with a careful view to differential intertemporal substitutability if they are to make any sense at all. Yet they may still not have the assumed correlation with inflation. Regarding the use of core inflation indices, goods heterogeneity may require other types of considerations than have been developed in the literature.

Second, we were able to derive some interesting hypotheses about price formation, such as those referring to risk premium terms, that may give rise to relative price changes distinct from those stipulated by Balassa-Samuelson type theories. And it turned out that the monetary and fiscal policy mix can modify durable prices; for instance parallelism (as defined above) may cause them to increase.

Third, we obtained assertions about policy. We found that parallelism is not consistent with price stability except when import price shocks are the overwhelming source of fluctuations. Also, trade-offs between price level stability and real stability may exist, and stabilising the price level may have uncertain effects on the current account. Furthermore, because of the existence of durables, or capital goods in general, the effect of monetary policy on demand must be time-varying. Finally, it appears that a strong positive correlation between prices and real interest rates may possibly signal only partial commitment to price stability.

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